Cellular Automata and Computational Universality

July 30, 2018

Thomas Archbold
University of Warwick
T.Archbold@warwick.ac.uk

Abstract

Cellular automata are discrete models with the ability to not only give rise to beautiful, intricate patterns, but also to be used as powerful tools of computation, with applications in cryptography, error-correction coding, and simulation of computer processors, to name a few. They also raise profound questions about the nature of our reality, asking whether our universe could be one such automaton. This paper provides a discussion into these automata, exploring the various power and limitations of a number of specific rule sets, covering John Conway's well-known "Game of Life" to the more obscure "Langton's ant" and "Wireworld". In particular, it explores the notion of computational universality, or Turing completeness, an automaton's ability to simulate any conceivable computation, and considers their potential in the context of solving two specific problems, the Firing Squad Synchronization Problem and the Majority Problem. Existing solutions to these problems are explored, and their existing avenues for optimization are discussed. In order to fully appreciate the complex structures that can arise from such simple beginnings, this project also presents software to visualize and probe further into the nature of the automata highlighted.

1 Introduction

A cellular automaton is a discrete model of computation which consists of a finite collection of "cells", each in one of a finite number of states. The state of each of these cells may evolve over the discrete progression of time in discrete steps, and may do so according to a deterministic set of rules specifying the state to which each cell is to progress, taking into account the states of cells in its neighbourhood. Their inherently discrete nature allows for strong analogies to be made with digital computers, and gifts them the ability to simulate digital processes and the potential to solve problems in this area.

Consider the cellular automaton defined by the simple rule:

$$a_i^{t+1} = a_{i-1}^t + a_{i+1}^t \mod 2$$

For any automaton, in each time step the rule is applied to all cells in the automaton instantaneously and simultaneously. In this case, our set of states is $0, 1$, and for each cell we look at the cells immediately preceding and succeeding it: if exactly one of them is in state 1, then this cell will be in state 1 in the next time step. Otherwise it will be in state 0.

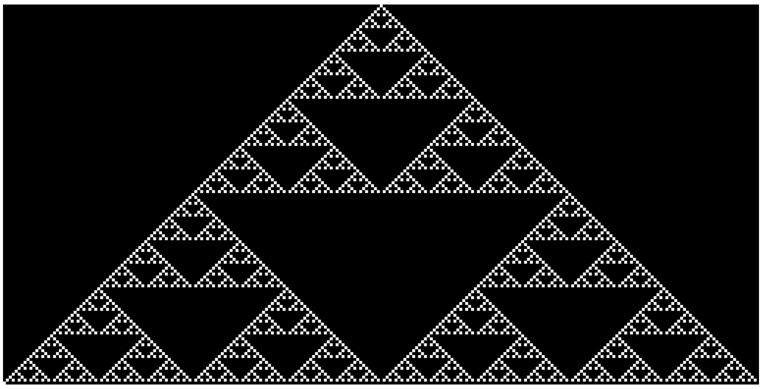

Figure 1:
??

2

3 Examples

4 Applications

5 Computational Universality

6 Firing Squad Synchronization Problem

7 Majority Problem

8