# A Decentralised Peer-Prediction Market

CS907 Dissertation Project - Final Report


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**Abstract**

A prediction market is a means of aggregating information about random variables and using the beliefs of its participants to calculate a prediction on the variable's outcome. Decentralised prediction markets differ from traditional ones in that they allow any user to specify the bets on which they would like to trade. In this project we implement a decentralised prediction market that uses participation from the community to both specify the available markets and then decide on their outcome by submission of reports from a group of arbiters. The mechanism we implement is incentive compatible, meaning it is always in a user's best interests to act truthfully: this is achieved by rewarding arbiters for truthful reporting and ensuring this payoff is never less than what they would gain by misreporting.

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# 1 Introduction

Prediction markets are exchange-traded markets[1] which allow users to trade on the outcomes of future events as opposed to traditional financial instruments. Users participate by placing bets and buying or selling shares in the markets these bets give rise to. Since users stake their own money, market prices should indicate the true beliefs of the userbase and the perceived likelihood the events have of occurring. Different users will have different beliefs and knowledge informing their decisions, hence prediction markets provide a means of aggregating information on events of interest using "the wisdom of the crowd". Shares in these markets are usually traded between \$0 and \$1, and for binary events a market will typically pay out \$1 for every share held for a positive outcome, and \$0 otherwise.

Traditional prediction markets are centralised in the sense that the system provides the possible bets on which to trade, and traders simply choose the price point and stake at which to participate. Since the system knows all possible bets beforehand it is simple for it to determine the outcomes of the markets and pay out the winnings accordingly. There are two issues with this centralised approach: firstly, it restricts the types of bets that can be made, since they must be explicitly offered by the market maker; secondly, it operates on trust – there is nothing to stop the central market maker from manipulating the system for their own gain. We aim for a system that avoids both of these issues.

In this project we implement a *decentralised* prediction market in which the userbase itself defines the markets and determines their outcomes – these are decided by consensus among groups of users, known as arbiters. This removes the need for a trusted centre, however with no central moderator bets may become ambiguous or their outcomes subjective, and arbiters may still attempt to manipulate the outcome of the market for their own gain by submitting false reports. It is also important that users continue to act according to their true beliefs, so that we may gather useful information on the events. We base our design on the incentive compatible peer prediction mechanism introduced by Freeman, Lahaie, and Pennock [19], which allows us to crowdsource market outcomes while incentivising users to act truthfully in all stages of the prediction market.

The rest of this dissertation is structured as follows: in Section 2 we introduce the various types of prediction markets and different approaches to their implementation. In Section 3 we give an overview of existing markets and the current literature within the algorithmic game theory community. In Section 4 we state our goals for the project and justify our reasons for undertaking it. Section 5 covers the high-level design of the market on which we base our implementation and introduces some theoretical considerations we take into account. In Section 6 we discuss the details of our implementation of the decentralised market, including the tools we use to develop it. Section 7 covers our approach towards project management. Finally, in Section 8 we reflect on the project's successes and suggest areas for improvement.

[1] Markets in which all transactions are routed through a central source.

# 2 Background

## 2.1 The Information Aggregation Problem

Consider the following problem within algorithmic mechanism design known as the *information aggregation problem* [26, Ch. 26]. An individual known as the "aggregator" wishes to obtain a prediction about an uncertain variable which will be realised at some point in the future. There are a number of individuals known as the "informants" who each hold sets of information about the variable's outcome. The goal is to design a mechanism that extracts the relevant information from the informants and uses it to provide a prediction of the variable's realisation. In an ideal setting the mechanism should produce the same prediction as an omniscient forecast that has access to all information available to all informants. This may not be viable in practice, since each agent's information is private, and so the mechanism must incentivise them to act in the desired truth-telling manner.

A prediction market is one mechanism that can provide such a forecast. In this setting the aggregator creates a financial security whose payoff is tied to the outcome of the variable. In the simpler case of binary events, such a security may pay out \$1 for each share held if the variable has a "true" or "yes" outcome, and \$0 otherwise, however other types of markets can be created, including discrete ("will the result of the match be a home win, away win, or a draw?"), continuous ("what will be the highest measured temperature in Coventry in September?"), or any combination of these types. Informants are then able to participate in the market induced by the security by trading shares according to their beliefs: those who believe, for example, that global warming is real might buy shares at a given price in the market, "the global average temperature in 2020 will be higher than that in 2019", while deniers may be inclined to sell shares. The share price will be adjusted accordingly, and the aggregator can view the current price as the informant's combined belief of the outcome of the event.

In the *partition model of knowledge* there is a set $\Omega$ of possible states of the world and at any moment the world is in exactly one state $\omega \in \Omega$, though the informants do not know which. Each informant $i$ may, however, possess partial information regarding this true state, represented by a partition $\pi_i$ of $\Omega$. The agent knows in which subset of this partition the true world state lies, but does not know the exact member of which is true. Given $n$ agents, their combined information $\hat{\pi}$ is the coarsest common refinement of the partitions $\pi_1, \dots, \pi_n$.[2]

We also assume a common prior probability distribution $P \in \Delta^{\Omega}$ which describes the probabilities that all agents assign to the different world states before receiving any information. Once each agent receives their partial information, they form their posterior beliefs by restricting the common prior to the subset of their partition in which they know the true state to lie. A *forecast* is an estimate of the expected value of the function $f : \Omega \to \{0, 1\}$, known as an *event*, which equals one for exactly one subset of $\Omega$ and 0 otherwise.

As mentioned, there is an ideal "omniscient" forecast that uses the distribution $P$ restricted to the subset of $\hat{\pi}$ in which the true world state lies – this is difficult to achieve given the private nature of each agent's information. The goal is therefore to design a mechanism to incentivise agents to reveal their private information such that in equilibrium we achieve a forecast as close as possible to the omniscient one. Prediction markets offer the agents the chance of financial gain

[2] A partition $\alpha$ of a set $X$ is a refinement of a partition $\rho$ of $X$ if every element of $\alpha$ is a subset of some element of $\rho$. In this case $\alpha$ is *finer* than $\rho$ and $\rho$ is *coarser* than $\alpha$.

for revealing information on the expected value of $f(\omega)$, and the share price of the security can be interpreted as the collective forecast of the agents. In the remainder of the section we shall outline some of the different approaches to designing the market mechanism: the best choice will vary for the setting and will depend on what is tractable given the types of bets that are on offer. First we shall introduce combinatorial markets.

## 2.2 Combinatorial Prediction Markets

A *combinatorial* prediction market is one in which the total state space $\Omega$ is the product space of a collection of base events. Suppose knowing the outcome of a given event cannot be predicted with certainty even if the outcomes to all other base events are known, and we wish to provide the opportunity to trade on any outcome $\omega \in \Omega$: with a set of base events of size $|\mathcal{E}|$ we would have a total outcome space of size $2^{|\mathcal{E}|}$. With such a large outcome space it may therefore be impossible to even all list the securities available in such a market. Combinatorial markets can make use of "expressive" bidding languages that represent collections of securities succinctly, including *combined orders* and *compound orders*.

Combined orders allow a user to trade a collection of securities by specifying the securities they wish to buy or sell together as a bundle along with limit prices for each constituent security. If there is even a single security in the bundle that is not at least as "good"[3] as its limit price, then no trade is made. If an agent were to trade on these securities in a non-combinatorial market, they would need a trade to be executed on each one individually. Throughout the execution of such a sequence of trades, however, market prices would be subject to change, and in the worst case these fluctuations could reduce or even reverse the utility of participating in such trades. Combined orders therefore protect its participants from such risk. Calculating the assignment of securities to buyers in such a setting is as hard as the winner-determination problem faced by a combinatorial *auction*, which known to be NP-hard.

Compound orders are generalisations of combined orders and allow users to trade on any Boolean expression on the set of base events. Now again the size of the outcome space is $2^{|\mathcal{E}|}$, but now there are $2^{2^{|\mathcal{E}|}}$ subsets of these outcomes expressible with Boolean formulae. Agents now place orders by requesting $q$ shares of the security $S_{\phi|\psi}$ at share price $p$, where $S_{\phi|\psi}$ pays out \$1 if both Boolean formulae $\phi$ and $\psi$ are true, \$0 if only $\psi$ is true, and refunds the user if $\psi$ is false. These orders will yield a payoff $\gamma^{\langle\omega\rangle}$ depending on the state $\omega \in \Omega$, and can be written as:

$$\gamma^{\langle\omega\rangle} = q \cdot \mathbb{1}_{\omega\in\psi}(\mathbb{1}_{\omega\in\phi} - p)$$

in which $\omega \in \phi$ means outcome $\omega$ satisfies the Boolean formula $\phi$. This says that the order will receive an overall payoff of zero if the true world state $\omega$ does satisfy $\psi$, meaning the order is invalid and refunded to the user. Otherwise, the user will receive a payout of $(1-p)$ for each share they bought if $\omega$ satisfies $\phi$. If the event does not occur they will lose what they paid and receive a payoff of $-p$ for each of $q$ shares. Each user $i$ has a payoff vector $\gamma_i$ induced by submitting orders. It is the job of the auctioneer to determine which orders to accept. Let $\alpha_i \in \{0,1\}$ indicate whether the auctioneer accepts an order from user $i$. Since they will collect the money paid by the trader and payout their winnings according to each user's payoff vector,

[3] If we are buying we want a price less than or equal to the limit price, while the opposite is true if we are selling.

the auctioneer receives payoff:

$$\gamma_a = \sum_i -\alpha_i \gamma_i$$

The *indivisible matching problem* asks that, given a set of orders, does there exist a set of $\alpha_i \in \{0,1\}$ and $\sum_i \alpha_i \geq 1$ such that for any outcome $\omega$ the auctioneer receives payoff $\gamma_a^{\langle\omega\rangle} \geq 0$? In other words, the auctioneer is looking to accept some bundle of orders *without risk*.

**Example** Suppose $\mathcal{E} = \{X_1, X_2\}$ and there are two orders: agent 1 wishes to buy two shares of security $S_{X_1}$ at \$0.6 per share while agent 2 wishes to sell one share of security $S_{X_1,X_2}$ for \$0.2 per share. Assuming the auctioneer accepts both orders, the payoffs are as follows (note player 2's payoff vector is negated since they are selling):

$$\begin{aligned}
\gamma_1 &= \left\langle \gamma_1^{X_1,X_2} \quad \gamma_1^{X_1,\bar{X}_2} \quad \gamma_1^{\bar{X}_1,X_2} \quad \gamma_1^{\bar{X}_1,\bar{X}_2} \right\rangle = \left\langle 2(1-0.6) \quad 2(1-0.6) \quad 2(-0.6) \quad 2(-0.6) \right\rangle \\
\gamma_2 &= -\left\langle 1(1-0.2) \quad 1(-0.2) \quad 1(-0.2) \quad 1(-0.2) \right\rangle \\
\gamma_a &= \left\langle 0 \quad -1 \quad 1 \quad 1 \right\rangle
\end{aligned}$$

Accepting both orders is therefore not a valid solution to the indivisible matching problem since in the event that $\omega = \{X_1, \bar{X}_2\}$ the auctioneer would have to run a loss. In fact, finding a solution to the indivisible matching problem is NP-complete [26, Ch. 26]. Although we do not focus on combinatorial prediction markets in this project, it is useful to illustrate the challenges of designing efficient market mechanisms. In particular, we are concerned with low liquidity, which arises when there is too little participation in the market for a trade to be executed.

## 2.3 Trading Mechanisms

### 2.3.1 Continuous Double Auctions

Trading mechanisms are responsible for deciding which trades are executed, in what quantities, and at which price points. In the previous section, although users had to submit orders of a particular form, the auctioneer was free to accept any subset of the orders received: this section expounds upon *how* trades may be chosen to be executed. These are called market trading mechanisms, and include double auctions, market call auctions, and automated market makers. A Continuous Double Auction (CDA) is a mechanism in which buyers are matched with sellers of a particular security. The market maker keeps an order book that tracks the bids, submitted by those looking to buy, and the asks, submitted by those looking to sell. Traders arrive asynchronously and place orders, and when two opposite orders match the trade is executed. CDAs are traditionally used in highly liquid markets, such as the New York Stock Exchange, where there are many bids for a given ask and vice versa. An issue with this approach is that it relies on there always being a willing buyer and seller at a particular price and quantity in order to execute a trade. Prediction markets have far fewer participants than stock exchanges, and the problem of low liquidity is made even worse in combinatorial markets, in which a trader's attention is split among an exponential number of securities. This makes the likelihood that a buyer and seller are looking to trade on the same event exceptionally small. Prices may also not be informative of the true beliefs held by traders in CDAs: since all traders can see bids and asks as they are submitted sellers may be encouraged to ask less than what they truly believe to be a security's "true" value in order to undercut another seller and make a profit. This is not useful in a prediction market setting, where we want to elicit the true beliefs from users.

These problems arising from market low liquidity can be averted by using an automated market maker, in which a price maker is nearly always willing to accept both buy and sell orders at a certain price. Participation in the market will have an effect on these prices, the exact nature of which will be down to the market maker. This ensures that, if the price is desirable, participants are always able to make a trade. An automated market maker is not typically used in real-world markets since always assuming the opposite side to any trade would likely result in significant losses for "the house"; in play-money markets this is not such an issue as losses are less detrimental and have no real-world negative value. In general there are three properties that an automated market maker should satisfy in order to be of practical use: traders should have an incentive to participate in the market whenever their beliefs would change the price; the computation of market prices should be tractable; and the market's loss should be bounded. We present two options for implementing automated market makers: as a parimutuel market or using a Market Scoring Rule.

### 2.3.2 Parimutuel Markets

In parimutuel markets traders wager money their choice of outcomes from a mutually exclusive and exhaustive set. When the event's outcome is realised the total wagered money is split between those who wagered correctly, in proportion to the size of their bet. In order to accommodate traders selling shares prior to the outcome of the event, *dynamic* parimutuel markets incorporate a cost function that varies the price of a single share due to trading activity. An example of one such cost function is the share-ratio cost function. Suppose we have an outcome space $\Omega$ and let $q_j$ denote the total number of shares for event $j \in \Omega$. Let $\boldsymbol{q} = (q_1, \ldots, q_{|\Omega|})$ be the vector of outstanding shares of all contracts. The share-ratio cost function is:

$$C(\boldsymbol{q}) = \sqrt{\sum_j q_j^2}$$

A trader wishing to buy $q'_j - q_j$ shares of security $j$, thus changing the number of outstanding shares of $j$ from $q_j$ to $q'_j$, pays the market $C(\boldsymbol{q}_{-j}, q'_j) - C(\boldsymbol{q})$.[4] There is a corresponding price function $p_j$ that gives the price for purchasing an infinitesimal quantity of shares in security $j$ and is used to quote a share price to market participants:

$$p_j = \frac{q_j}{\sum_k q_k^2}$$

This share price $p_j$ should not be used to calculate the cost of a transaction since an agent's participation in the market will instantly change this value. The purpose of a prediction market is to elicit private information on some future event: we use the information we have on user participation in the market to compute probability $\pi_j$ of outcome $j$ as $\pi_j = p_j^2$.

[4]We use $\boldsymbol{q}_{-j}$ to denote the vector $(q_1, \ldots, q_{j-1}, q_{j+1}, \ldots, q_{|\Omega|})$.

### 2.3.3 Scoring Rule Markets

A scoring rule is used to assign probabilities to a set of mutually exclusive outcomes. When the scoring rule is proper[5], it can be converted to an automated market maker that uses a Market Scoring Rule (MSR) [20]. Again at the heart of a MSR is the cost function $C(\boldsymbol{q})$, which is a means of recording the total amount of money spent in the market by traders as a function of the total number of shares in circulation. Traders wishing to purchase $q'_j - q_j$ shares of security $j$ must again pay $C(\boldsymbol{q}_{-j}, q'_j) - C(\boldsymbol{q})$. Note that in both parimutuel markets and scoring rule markets, these rules also encode sell transactions, in which case $q'_j < q_j$. We cannot use $C$ directly to quote a share price to the user since we first need to know what quantity they to buy or sell: we use its derivative $p = \partial C / \partial q_j$ to again quote the cost for an infinitesimal quantity of shares. As an example, the quadratic scoring rule gives some reward $Q(\boldsymbol{r}, i)$ if the $i$th event occurs, given the probability vector $\boldsymbol{r}$. The cost function corresponding to the quadratic scoring rule is:

$$C(\boldsymbol{q}) = \frac{\sum_j q_j}{|\Omega|} + \frac{\sum_j q_j^2}{4b} + \frac{\left(\sum_j q_j\right)^2}{4b|\Omega|} - \frac{b}{|\Omega|}$$

Scoring rule markets typically pay out \$1 for each share held of a security whose outcome was positive, and \$0 otherwise. In contrast, parimutuel markets can have a different payoff per share since they pay out an equal portion of the total amount wagered per share held to winning shareholders. This project implements the former. The following section will outline some existing prediction markets as well as important results from the literature.

# 3 Literature Review

## 3.1 The Iowa Electronic Markets

The Iowa Electronic Markets (IEM) are real-money prediction markets developed by the University of Iowa [5] that have been running since 1988. They allow users to buy and sell contracts based on the outcome of U.S. political elections and economic indicators, and are currently offering markets for the winning party of the 2020 U.S. presidential election, the vote share between the Democratic and Republican parties in the 2020 U.S. presidential election, and the compositions of the houses of Congress, House of Representatives, and U.S. Senate after the outcome of the 2020 U.S. congressional elections. The number of markets offered is small and the topics are kept relevant to current events, meaning there is likely to be high liquidity for any security a user wishes to trade in. This has allowed the markets to predict the results of political elections with more accuracy and less error than traditional polls: for the presidential elections between 1988 and 2000, three-quarters of the time the IEM's market price on the day each poll was released was more accurate for predicting vote share than the poll itself [28, pg. 19]. These markets inspired similar markets in the forms of the Hollywood Stock Exchange, NewsFutures, and the Foresight Exchange report, which achieved similar successes despite not using real money.

[5] A scoring rule giving the highest expected reward for reporting the true distribution.

## 3.2 Combinatorial Markets

An issue with these markets is that they are restrictive in the bets they offer. Although this can be beneficial in that they provide a focused and liquid market in which to trade, they are less expressive. A combinatorial prediction market drastically increases the number of outcomes that can be predicted by offering securities on propositions that can be combined in various ways. One example of such a market is *Predictalot* [11], a combinatorial prediction market developed by Yahoo! that allowed users to trade securities in the 2010 NCAA Men's Division I Basketball Tournament. The tournament sees the top 64 teams play 63 games in a knockout competition, yielding a total outcome space of size $2^{63}$. *Predictalot* then kept track of the odds, computing them by scanning through all of the predictions made by users. This application served as the original inspiration for this project to explore prediction markets. Using a Market Scoring Rule for such a market would involve computing a summation over the entire outcome space $\Omega$, an intractable, #P-hard problem akin to counting the number of subsets in a list of integers that sum to zero. Instead, they use importance sampling, a technique for estimating properties of a particular probability distribution using only samples generated from a different distribution. This "naive" approach is then improved upon by the work of Dudík, Lahaie, and Pennock [17], who use convex optimisation and constraint generation to develop a tractable market maker. This approach is a compromise between treating all securities as independent and a fully combinatorial, "ideal" market maker, but still allows for information to be gathered among related securities. This allows for them to compute odds that make sense: for example, a large bet on a team to win the entire tournament would also increase their odds at every other stage in the tournament, since they must win these to even reach the final. Kroer, Dudík, Lahaie, and Balakrishnan [24] use integer programming to remove trades which are always profitable and incur no risk. In other words, they ensure all trades are arbitrage-free. On top of achieving bounded loss, a crucial element behind a market mechanism operating in the real world and avoiding bankruptcy, avoiding arbitrage is desirable as it leads to more accurate forecasts: since users cannot make risk-free profits, they are forced to bet according to their true beliefs.

## 3.3 Decentralised Markets

All examples so far have involved a centralised market mechanism. These types of systems involve a central authority providing the securities upon which users may trade and then verifying their outcome. *Decentralised* markets allow the users to specify the securities themselves and trade shares in them. Several examples of decentralised markets exist, and they are often implemented with cryptocurrencies. Peterson et al. [27] study the setting and implement the oracle at the heart of *Augur* [1], a decentralised prediction market built upon the Ethereum blockchain, which launched in 2018. It allows users to offer predictions on any topic, and markets may be either categorical, which are similar to binary markets in which the winner takes all, or scalar, which offer users a spectrum of outcomes in which to invest. As in many decentralised markets, outcomes of events are then resolved by the users, and in *Augur* users are incentivised to report truthfully by way of paying reporting fees: users back their report by depositing tokens, and token holders are then entitled to the trading fees generated. Although this persuades against manipulation, it has not been shown whether this system achieves any theoretical guarantees of truthful reporting.

As can often be the case with real-money markets, the platform had quickly devolved into an assassination market [15] – originally this referred to the case where users created markets on the deaths of certain people, which then incentivised their assassination. A user could stand to profit by placing a bet on the exact time of their death, and ensure this bet was profitable by assassinating the subject. More generally this refers to the users of a prediction market having the ability to influence a market's outcome and acting on this opportunity. Another issue with *Augur* is the option to report a market's outcome as "invalid": this is for the case where the user-made bet is too ambiguous to decided, such as, "Bayern Munich will play well against Paris Saint Germain".

Other decentralised markets based on cryptocurrencies exist, including *Omen* [9] and *Hivemind* [3]. The former is similar to *Augur* in that it allows users to create markets for any bet they like and whose outcomes are not decided by the system itself. Whereas *Augur* uses a reputation system whereby users back their report of a market's outcome with $REP tokens, *Omen* asks the market creator to supply an "oracle" through which the outcome can be determined. This oracle can even be *Augur*. Although this may solve the "invalid" outcome option for ambiguous bets, it may introduce bias into the process of outcome determination. For example, suppose a user creates a market for "The Democratic nominee will tell a lie during tonight's debate" and lists the oracle as the conservative news channel, Fox News. Users would then trade on how they think the oracle will report the outcome, and not what they believe the outcome will be themselves. An important aspect of decentralised markets must therefore be that the outcome is determined by the community, not a single source.

In contrast to *Augur*, which implements a traditional order book using Ethereum, *Omen* uses an automated market maker to provide liquidity to its securities. As we have discussed, automated market makers can be implemented via Market Scoring Rules. Hanson [20] shows that we can use any strictly proper scoring rule to implement an automated market maker: with such scoring rules, agents maximise their expected utility by truthfully revealing their predictions. In particular, in this project we implement the peer prediction market introduced by Freeman, Lahaie, and Pennock [19], which specifies a mechanism to trade bets and crowdsource outcome determination. Market outcomes are decided by asking users for reports, similarly to *Augur*. All they require is that the rule is strictly proper, giving plenty of choice to study the effects different rules have on user and price behaviour. While the choice of scoring rule is less important than the mechanism by which market outcomes are determined, a recent work by Liu, Wang, and Chen [25] introduces scoring rules for the setting where the aggregator has access only to user reports, which they call Surrogate Scoring Rules (SSRs). This appears to be an interesting avenue to further explore and adapt to a prediction market. One assumption they make, however, seems incompatible with the decentralised setting in that they require all events to be independent. Given that users can create a market for *any* bet, this condition is impossible to ensure. Since SSRs can be strictly proper under certain conditions, they may be suitable as the Market Scoring Rule in [19].

Other prediction markets existed in *PredictIt* [12] and *InTrade* [6], both offering markets for various political and economic events. However, both experienced disputes largely related to the wording of the available to trade and the ambiguity in their resolution. For example, a bet offered on *PredictIt* was, "Who will be Senate-confirmed Secretary of State on March 31, 2018?", and although Rex Tillerson was fired in the middle of March, he was officially the secretary of state until midnight of the 31st, leading to confusion among users in what they are trading on and therefore inaccuracies in the predictions it elicited. This will be a problem inherent to any prediction market that allows users to specify their own bets, and while the work of Freeman et al. [19] mitigates the problem of determining the outcome by setting it to the proportion of users reporting a “yes” outcome, this does nothing to penalise the initial creator for introducing an ambiguous market.

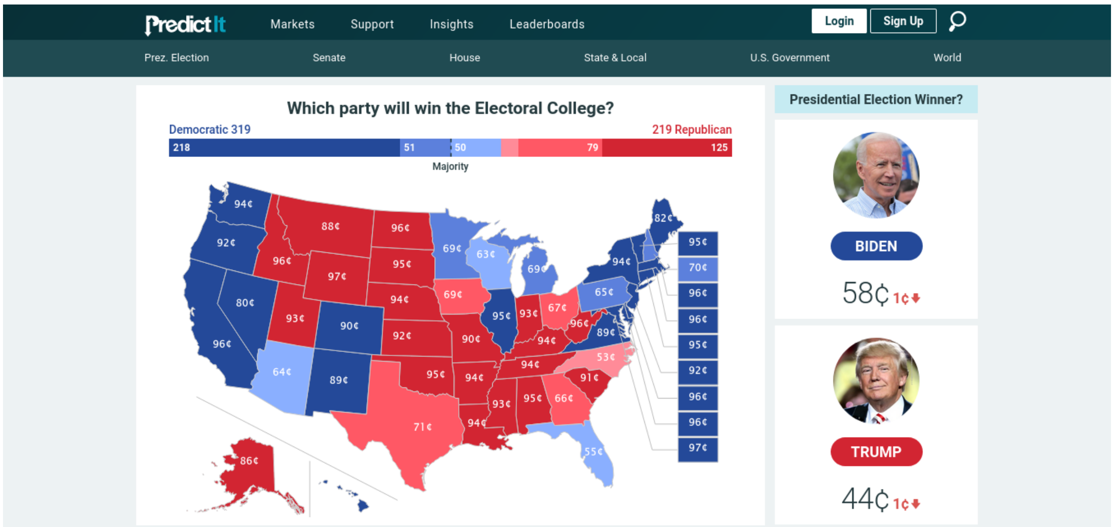



Figure 1: The *PredictIt* prediction market for the 2020 U.S. presidential election. As of May 21, 2021 Joe Biden is perceived to be more likely to become President.

### 3.4 A Note on *Predictalot*

As we will cover in the following section, the goals of this project are more concerned with the game-theoretic aspects of implementing a decentralised prediction market. This involves encouraging users to act in the desired manner, which in our case means “telling the truth” about both their beliefs on an event’s outcome and their observation of the result. Since combinatorial prediction markets are often centralised, meaning bets come straight from the system and not its users, all users have to do is choose to participate in the market at what they view as the correct price, hence there is much less opportunity for manipulation. For this reason, combinatorial prediction markets are more concerned with the computational aspects of offering combinatorial securities to its traders, and in particular how to compute accurate prices on combinations of interrelated securities. Both decentralised and combinatorial settings are interesting in their own right, and both can achieve an added level of expressiveness compared to a traditional prediction market such as the IEM. Here we will briefly describe approaches taken to improving the efficiency of pricing in the *Predictalot* prediction market.

We will first discuss the hardness of using a Market Scoring Rule to calculate prices in a combinatorial market. The class NP is the set of decision problems for which, given an input to the problem and a proposed solution, the “certificate”, verification that the certificate is indeed a correct solution can be done in polynomial time. By self-reducibility, the search version of the problem is no harder than the decision version. The class #P consists of *functions* that count the number of solutions to NP search problems. A function $g$ is #P-hard if, for every function

$f$ in #P, given an oracle for $g$ we can compute $f$ in polynomial time – that is, computing $g$ is at least as hard as any other function $f$ in #P. Chen et al. [16] show that computing the price and cost functions of a MSR in a combinatorial market is #P-hard, even when traders are restricted to betting on the disjunctions or conjunctions of two events. They do this via a reduction to the #P-hard function #2-SAT, which counts the number of satisfying assignments of a CNF formula, where each clause has two literals.

Dudík et al. [17] restrict the bidding language to achieve tractability. This is done by firstly calculating prices as if markets were independent then detecting and minimising arbitrage. Opportunities for arbitrage are detected using constraint generation, in which arbitrage occurs when the price falls below a certain level, and this is minimised using optimisation methods. This does not always completely remove all arbitrage opportunities. Kroer et al. [24] apply a similar approach by interleaving the execution of trades with the removal of arbitrage via constraint generation, and in fact use the method of Dudík et al., their *Linear Constraint Market Maker (LCMM)*, in order to first remove "easy" arbitrage prices. They then feed their generated integer program (IP) constraints to an algorithm which attempts to solve it. Since solving IPs is NP-hard, and with such a large outcome space there may not be time to solve the IP in time before a new trade is submitted, in which case they interrupt this step. Regardless of whether it was interrupted, they guarantee non-negative profit for every trade. When tested on data from *Predictalot*, they found an improvement in the accuracy of their odds over those computed by the LCMM alone, and allowing 30 minutes to solve the IPs yielded trades that could execute in 5 hours, that originally took 22 days. This shows the huge computational task involved in computing accurate prices in combinatorial markets, and they identify that further speedup can be obtained by not solving the IPs optimally, but instead using local search to find solutions that are "good enough".

# 4 Goals

## 4.1 Core Features

The goal of this project is to implement a truthful decentralised prediction market in which users may specify the bets on which to trade. The market outcomes will be decided by peer prediction, in which events are settled by a subset of the users known as arbiters. A user may even act as an arbiter in a market in which they themselves may hold a stake. Specifically, we seek to:

1. create a web application on which users can create custom bets and trade on these markets
2. implement a trading mechanism that allows users to buy and sell shares in the user-made securities using play money
3. crowdsource outcome determination using reports from arbiters who may hold positions in the market
4. incentivise truthful behaviour at all stages in the mechanism

These goals will largely be achieved by implementing the mechanism outlined by Freeman et al. [19], albeit with several practical modifications. The first three goals cover core functionality of any decentralised prediction market, while the fourth is concerned with the tuning of system parameters, in order to ensure that users do not manipulate the mechanism. Thus, users are discouraged from attempting to "game" the system for personal gain as it is in their best interests to be truthful.

Other non-essential features but highly desirable for strong user experience include asynchronous communication with the server in order to display up-to-date pricing information to the user without a page refresh, and the automated closing of markets. Both of these features would make the system straightforward and intuitive to use. Moreover, it would allow the system to run independently since we need not monitor and close markets by hand, meaning the market's functioning is only influenced by the community, one of the key points of implementing a decentralised market.

As we will discuss in more detail in Section 5, one aspect of the mechanism is the assumption that the system knows the signal error rates when user's receive news about a market's outcome. This is unrealistic in practice, since we have no way of knowing how users learn about the outcome of an event, nor the accuracy of this source's reporting (particularly important for highly subjective bets). Hence we also look to calculate an arbiter's average signal accuracy based on their past reporting history, instead of having them estimate its accuracy each time they submit a report. This leaves less opportunity to game the system, the entire point of implementing this prediction market mechanism.

## 4.2 Stretch Features

With more time, there are plenty of additional features that could be implemented to render the system more intuitive and usable. These include the option to create different types of markets, particularly categorical ones since they would function similarly to binary markets but allow multiple related markets to be expressed more succinctly. Furthermore, the option to create and sort markets by categories would help users offer their information more readily if they are especially interested in a certain topic, say politics or sport.

A useful feature to implement would be the tracking of price histories for each security. This would enable graphs to be generated so that users could be more informed on how the forecast of an event has changed over time and would bring participation in the market more in line with what traders would experience in a real exchange.

Finally, an issue with the mechanism of Freeman et al. as it stands is that it does not directly punish users for creating markets on ambiguous bets. Traders may become confused about the wording, leading to different interpretations of the security and could result in users trading on different assumptions – this is not useful for information aggregation. Although the negative effects are somewhat mitigated in that the very same community that trades in the security also decides on its outcome, there is no system in place to specifically encourage clear bets. This is something that could be improved and would be useful in avoiding the market becoming swamped with overly subjective wagers.

## 4.3 Motivation

As discussed, there are already numerous prediction markets that exist in the literature. The Iowa Electronic Markets are the longest running and arguably most successful, but the options offered to the users on which to trade are too restrictive. This is a similar issue among all centralised markets, including InTrade, PredictIt, and the Hollywood Stock Exchange. While their success can be attributed to their narrow focus on a particular topic, it seems more interesting to be able to aggregate information from a wider variety of themes, sacrificing perhaps some predictive accuracy for more widespread forecasts to be made.

Decentralised prediction markets are not a new concept, however current examples in the literature lack in the functionality they offer. In the case of *Omen*, while they allow any user to create a market, they rely on a single oracle to determine the outcome of the event. This leaves a single point of failure in the system and opens it to manipulation. For example, listing a biased news source as the oracle could have a significant effect on the event's outcome. Although this is mitigated somewhat by displaying to traders the oracle chosen, this still encourages them to trade on how they believe the oracle will report the market and not necessarily the market itself. The mechanism by which *Augur* determines market outcomes appears to be an improvement over this, in which multiple reporters from the community back their report of the market outcome with $REP tokens, thus implementing a form of reputation system. However, it does not deal with ambiguous bets elegantly, offering the option for a report a market's outcome as "invalid". Given the inevitability of such markets in a decentralised setting, this is a key weakness to *Augur*.

It is therefore well justified to implement the decentralised peer prediction mechanism of Freeman et al. This not only allows users to create markets for any event they see fit, but also crowdsources market outcomes by relying on reports from the community. Thus instead of relying on a single source of information, which leaves it vulnerable to reporting biases, it gets a more complete picture of how the users themselves, the ones interacting with the market, observed the outcome. This can average out the biases present in any one news source. This also seems to be a better method to deal with ambiguity than in *Augur*, since the outcome of the market can be influenced by a reporter's interpretation of a wager. Another issue is that *Augur* has not been shown to achieve any theoretical guarantees, which should be a key consideration in an environment in which rational selfish agents are interacting. This provides a good opportunity to apply a game-theoretic approach, and the market we implement is incentive compatible, meaning it it in a user's best interests to report market outcomes truthfully. Although the mechanism is not budget balanced, it can be fully subsidised by a trading fee on each transaction, further making it practical and self-sufficient.

# 5 Design

## 5.1 Mechanism Overview

As mentioned our design of the prediction market is based on the peer prediction mechanism proposed by Freeman et al. [19]. In this section we will outline the main ideas presented in their work and give an overview of the peer prediction mechanism.

We are interested in setting up a prediction market for outcome of a binary random variable $X \in \{0, 1\}$. We will use the terms "market", "stock", and "security" interchangeably throughout to refer to the entity comprising a wager, such as "Arsenal will beat Tottenham", and a deadline – these two pieces of information are all we need to represent the event $X$. The mechanism is divided into two main stages: the market stage, where users may buy and sell shares in the securities whose deadlines have not yet passed; and the arbitration stage, where a subset of the users report on the outcome of the security and the payout price per share is computed. In a traditional prediction market on binary events, if the market's outcome was positive then stakeholders with long positions will then be paid out \$1 for each share they own while those with short positions will buy back their shares at a price of \$1 per share. Similarly, if the outcome was negative, long users will have lost money since they receive no money back from their initial investment, while short users will profit as they must "buy back" their shares at a price of \$0 per share. This market is different in that market outcomes are set as the proportion of arbiters that reported a positive outcome. Therefore, even if a user has gone long on a security, this must have been at or below the right price to make a profit, since it is not necessarily the case that 100% of the users reporting on the market agree on its outcome.

Since we rely heavily on user participation for the mechanism to run correctly, it is important that users act in the desired manner. This mechanism incentivises users to act truthfully in two aspects: firstly, users are encouraged to trade on their belief of the market's realised outcome, rather than, for example, how a specific news source will report on it, since the outcome is determined entirely by the community and each arbiter will have access to their own news sources of varying biases; secondly, it is in a user's best interests to report on market outcomes truthfully, since they can receive no better payoff by attempting to manipulate the system, regardless of whether they hold a position in the market. Therefore, we are able to gather accurate public sentiment on the event itself as well as its outcome, and ambiguous securities are dealt with more gracefully.

These considerations also allow us to achieve certain useful guarantees. For example, in order to incentivise reporters to act truthfully, we must pay them more than what they would otherwise gain from attempting to manipulate the system. We can use this knowledge, coupled with what we know stakeholders are expecting to be paid out, to bound the amount we must pay to ensure incentive compatibility. Thus we can ensure that the system is sustainable and the system's loss is bounded.

In the following sections, we shall outline the mechanism we implement from a theoretical standpoint.

## 5.2 Market Stage

### 5.2.1 Trading Mechanism

The market stage allows users to create markets for any bet they desire and specifies how the share price reacts according to user participation. As we implement a decentralised market, we place no restriction on the bet that can be placed other than that its outcome must be binary. As we mention in Section 2.3, we implement a scoring rule market, in which users must pay $C(\boldsymbol{q}_{-j}, q'_j) - C(\boldsymbol{q})$ to change the total number of outstanding shares of security $j$ from $q_j$ to $q'_j$. Since market prices are dynamic we quote instantaneous share price to users with its derivative $p_j$ – this is also used to calculate the trading fee on a given transaction, which raises funds to pay arbiters in the following stage of the mechanism.

### 5.2.2 Trading fees

In addition to calculating transaction costs and share prices via the MSR, the market stage is responsible for implementing trading fees. As mentioned, the mechanism we implement is not budget balanced, meaning the market must be subsidised in order to pay arbiters for submitting outcome reports. Trading fees raise these subsidies. For any given market, buying shares will push the share price $p$ upwards towards \$1, while selling shares will push it towards \$0. There are two types of transactions that a user may be involved in: one in which a trader is increasing their risk, and one in which a user is liquidating shares it has previously bought or sold. For example, suppose Alice holds ten shares in a particular security: if she were to sell up to and including ten shares she would simply be liquidating shares that have already been sold to her, while if she were to buy additional shares or sell more than ten, then she would be increasing her risk. Risk transactions are defined analogously for a user buying shares. Trading fees are only imposed on transactions in which a user increases their risk, and can be viewed as a fee on their worst-case loss. Specifically, for fixed system parameter $f$ and for transactions in which a user increases their risk, a buy transaction that pushes share price to $p$ incurs an additional charge of $fp$, while a sell transaction that pushes share price to $p$ incurs additional charge of $f(1-p)$. These fees also allow us to bound the maximum payout we must pay to a stakeholder, since they bound share prices away from \$0 and \$1. Users may trade shares in a given security as long as they have enough funds to make the transaction (including the fee), and as long as the deadline has not yet passed. After the market expires, stakeholders' positions are final and we then determine the outcome of the market via peer prediction in the arbitration stage.

## 5.3 Arbitration Stage

### 5.3.1 Outcome Reporting

The arbitration stage is concerned with determining the perceived outcome of the event $X$ from a subset of the community, known as arbiters, who offer reports on the outcome they observed. Specifically, each arbiter $i$ receives a private signal $x_i \in \{0,1\}$ that tells them the result of the event – this is analogous to reading the news, watching the match, even hearing about it from a friend, and will vary from market to market. The arbiter then submits a report $\hat{x}_i \in \{0,1\}$ to the system that tells it what they believe to be the outcome. Since the signal they receive is private information, we have no way of determining whether this report is what they truly observed or whether they are lying. Instead, we incentivise arbiters to act truthfully by paying them a reward if their report agrees with another randomly chosen arbiter: for this we implement the "1/prior with midpoint" mechanism, which we will detail below.

Once all reports have been collected and the arbiters paid, the outcome of the market $\hat{X} \in [0,1]$ is set to the proportion of arbiters that reported a positive outcome. This differs from traditional prediction markets, in which shares of a security will pay out \$1 if the event occurred, and \$0 otherwise. Stakeholders are then paid out in the usual manner, where those with long positions are paid out $\hat{X}$ for each share owned, while those with short positions must buy them back at $\hat{X}$ per share. This should not change how traders view the security: if they have information telling them the event will occur they will still buy into the market if the share price is appropriate, while if they believe the event is unlikely they will continue to sell. The mechanism simply accommodates for the possibly ambiguous bets made by the community.

### 5.3.2 1/prior mechanism

We use a modified version of the 1/prior payment mechanism to reward users for submitting reports on an event's outcome and to incentivise truth telling behaviour. The original version was conceived by Jurca and Faltings [22, 23] as a means of rewarding arbiters for participation in opinion polls, another means of crowdsourcing a forecast in which users submit probabilistic estimates for the likelihood of events to occur. Witkowski [29] then generalised this to pay out different amounts depending on the signals reported by paired arbiters. For arbiters $i$ and $j$ with reports $\hat{x}_i$ and $\hat{x}_j$, the 1/prior mechanism pays a reward $u(\hat{x}_i, \hat{x}_j)$ as follows:

$$u(\hat{x}_i, \hat{x}_j) = \begin{cases} k\mu & \text{if } \hat{x}_i = \hat{x}_j = 0 \\ k(1-\mu) & \text{if } \hat{x}_i = \hat{x}_j = 1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

In this, $k$ is a parameter and $\mu$ is the common prior belief that $X = 1$. A suitable value to use for $\mu$ in our case is the closing price of the market: if users feel the event is likely to occur they will buy shares of it, pushing the share price towards \$1, and if they feel it is unlikely it will be pushed towards \$0. Everyone can see this price, and if it differs from a user's beliefs they will participate in the market and alter the price accordingly, making it a sensible choice for the common prior.

The modification introduced by Freeman et al. [19] to the 1/prior mechanism is simple and requires two additional values. Let $\mu_1^i$ be the probability that, given that agent $i$ receives a positive signal of the event's outcome, another randomly chosen user also receives a positive signal. Similarly, let $\mu_0^i$ be the probability that, given that agent $i$ receives a negative signal, another randomly chosen user receives a positive signal. We require a common value for these "update" probabilities across all agents, so we define the $\mu_1$ and $\mu_0$ as follows:

$$\begin{aligned} \mu_1 &:= \min_i \mu_1^i \\ \mu_0 &:= \max_i \mu_0^i \end{aligned} \tag{2}$$

The modified payment mechanism is now simply equation (1) with $\mu$ replaced by $(\mu_1 + \mu_0)/2$. This is the "1/prior-with-midpoint" mechanism and guarantees that the incentives for arbiters are always the same, no matter the signal they receive. The arbiter with the greatest incentive to misreport – that is, an arbiter with a large stake in the market in which they are reporting – has this incentive weakly decreased by using replacing $\mu$ with the midpoint $(\mu_1 + \mu_0)/2$ in the payment rule.

In particular, suppose arbiter $i$ holds a position of $n_i$ securities in the market. We can ensure truthful reporting is a best response for $i$ by setting the 1/prior-with-midpoint parameter $k$ to the appropriate value such that they will receive weakly greater reward from the payment mechanism than they would by misreporting. With $m$ arbiters and $\delta = \mu_1 - \mu_0$, truthful reporting for arbiter $i$ is a best response if:

$$k \geq \frac{2|n_i|}{m\delta} \tag{3}$$

## 5.4 Tools

In order to write our web application, we will need a platform on which to host the server, the ability to define webpages, and a means of interacting with a database for persistent storage. There are a number of useful packages provided by Quicklisp that allows all of this functionality to be implemented in Lisp. Using a single language to write the entire application – as opposed to, say, a combination of HTML, PHP, MySQL, and JavaScript – encourages a cleaner and more flexible implementation and allows us to make full use of the tools available to the language. A key advantage to using Lisp in particular is its focus on extensibility, and specifically its powerful macro system allows us to abstract away unnecessary details and write more generalised code. We discuss this in greater detail in Section 6.

Our prediction market is written in the widely-used Common Lisp dialect of the Lisp family of programming languages, and we use the Steel Bank Common Lisp (SBCL) compiler and runtime environment to develop the code. Most importantly, Common Lisp is well-supported by Quicklisp, which provides the following packages that enable us to write the prediction market:

- Hunchentoot [4]
- CL-WHO [2]
- Mito [7]
- SXQL [14]
- Parenscript [10]
- Smackjack [13]

Hunchentoot provides the environment on which we host the server and provides automatic session handling, allowing us to implement a login system, easy access of HTTP GET and POST parameters submitted via HTML forms, and a simple interface through which to define webpage handlers. To generate the webpages themselves we use CL-WHO[6], which translates Lisp expressions into HTML strings that we then pass to the appropriate Hunchentoot functions. The structure of a Lisp program maps well to that expected by an HTML file, while allowing us to dynamically generate pages through the use of macros, making Lisp a suitable choice for this purpose.

Mito is an Object Relational Mapper (ORM) that provides an interface with which we can connect and interact with a Relational Database Management System from within the Lisp environment. We opt for a MySQL backend, simply due to its familiarity, though this choice is largely immaterial given our simple requirements. We can compose more complex MySQL statement using SXQL, and this integrates well with Mito so that the two are effectively used as one library.

Parenscript allows us to incorporate JavaScript into the site with the intention of improving user experience. Currently this allows us to perform client-side validation of form data, to ensure information arrives at the server in the correct format and that all the necessary data is there before sending it. It also enables us to use the Smackjack library, allowing us to have

[6]**C**ommon **L**isp **W**ith **H**TML **O**utput.

asynchronous communication between the client and server. Particularly important in any real-time market, this ensures all prices displayed to the user are current and the user is interacting with an up-to-date state of the system.

We use the Ngrok [8] utility throughout the project's development to tunnel ports on our local machine to public URLs, to ensure our market not only functions locally but continues to do so on different machines and with multiple users at once. Finally, Git and Github have been used extensively for version control and remote storage and backup.

# 6 Implementation

In this section we shall detail our implementation of the trading and peer prediction mechanism. The system consists of five independent parts:

1. Database
2. Trading
3. Arbitration
4. Server
5. User experience

## 6.1 Database

### 6.1.1 Table Definitions

We define three tables that handle all interactions with the database: `USER`, which stores all users in the system and their remaining budget; `SECURITY`, which is stores the wager, deadline, number of shares, and final outcome for every user-created security; and `USER-SECURITY`, which is responsible for the many-to-many relationship that users may have with securities, and allows us to store user positions as well as reports once the deadline has passed. The table definitions are as follows:

| **User** | *name* | *budget* |
|---|---|---|

| **Security** | *bet* | *shares* | *deadline* | *outcome* |
|---|---|---|---|---|

| **User-security** | *user* | *security* | *shares* | *report* | *positive belief* | *negative belief* |
|---|---|---|---|---|---|---|

Table 1: Table definitions in our database

An entry in the `USER` table consists of a username and a budget. All users currently start with \$100, which is of no real consequence since the market uses play-money. Currently there is no requirement to supply a password when logging in; this has been done to speed up the testing process, and if required in the future this will be a simple addition to make.

Securities are also simple entities to store, whose fields fully describe a market: *bet* stores a string that specifies the wager being made; *shares* stores the total number of shares (referred to as $q$ in Section 2.3); *deadline* holds the date and time by which trading is to stop; and *outcome* is

```
(deftable user-security ()
          ((user :col-type user)
           (security :col-type security)
           (shares :col-type :integer
                   :initform 0)
           (report :col-type (or :integer :null))
           (positive-belief :col-type (or :double :null))
           (negative-belief :col-type (or :double :null))))
```

Listing 1: Defining the `USER-SECURITY` table in Mito

initially null and eventually set to the payout price per share, or the fraction of arbiters reporting a positive outcome, once the deadline has passed.

The `USER-SECURITY` table represents the many-to-many mapping between users and securities, and is used to store a user's position in a given market as well as the outcome they have reported for it, if they have acted as an arbiter. The columns are as follows: *user* holds a reference to an entry in the `USER` table; *security* holds a reference to an entry in the `SECURITY` table; *shares* stores the user's position in this market if they have one, otherwise 0; *report* stores the user's report on the outcome, if they are an arbiter for this security; and *positive belief* and *negative belief* represent how reliable a user's signals are, based on their previous reporting history. The manner in which we use the last two fields will be discussed in greater depth in Section 6.3.

#### 6.1.2 Database Interface

We provide the interface with which we interact with the database in `database.lisp`. We first initialise the database and connect to it, and then define the tables as in the previous section. We opt to use MySQL as the backend simply as it was already installed on the development machine, as well as some prior familiarity with the language. Tables are then created using the `deftable` macro supplied by Mito: syntactically, this is similar to vanilla Common Lisp's `defstruct` macro. Listing 1 shows how we define the columns and their associated datatypes. The macro defines the default accessors[7], the slots `created_at` and `updated_at`, and a primary key `id` if none is specified. As the listing shows, we can specify a previously defined table as the column datatype, in this case `user` and `security`, in order to model the foreign key relation in a straightforward manner.

Insertion is similarly straightforward: to insert a new entry into a table we simply create an instance of the structure that is implicitly defined when calling `deftable` then call `create-dao`. To retrieve records from the database we can use either `select-dao` or `find-dao`: the former returns all records satisfying the criteria provided, while the latter returns the first match.

We design the interface to the database so that no custom queries need to be created outside of `database.lisp`. This ensures the code interacting with it can be kept as clean and simple as possible. We use another library from Quicklisp, SXQL, in order to build the more complex queries to the database. For example, Listing 2 shows how we retrieve all the securities whose deadline is yet to pass, in order of first to last to expire. We use the SXQL functions `where`, `:>`, `order-by`, and `:asc` that expand into the corresponding MySQL code for Mito to execute.

For any interaction with the database, we need to establish a connection prior to the trans-

[7] Functions for accessing members of a struct.

```
(defun get-expiring-securities (datetime)
  " return all securities ordered by deadline, most imminent first "
  (with-open-database
    (select-dao 'security
                (where (:> :deadline datetime))
                (order-by (:asc :deadline)))))
```

Listing 2: Retrieving active markets using Mito and SXQL

```
(defmacro with-open-database (&body code)
  " execute CODE without worrying about the connection "
  `(progn
     (connect-database)
     (let ((result (progn ,@code)))
       (disconnect-database)
       result)))
```

Listing 3: Defining our `with-open-database` macro

action and disconnect after it is complete. This gives us the opportunity to make use of Lisp's macro system: while it is a small example, since its use is so widespread it greatly reduces the number of lines and ensures we never forget to close a database connection. We define our macro `with-open-database` as in Listing 3. Since the final statement in a function or macro definition in Lisp is the value returned by that block, we are able to open the connection, execute arbitrary code and store the final result in the variable `result`, then disconnect from the database and return the result of the transaction.

In order to enable the transactions in the following sections, when we first initialise the database we create a user with the name "bank". All money that is then to be collected from or paid out to users is done so through this user, so that money is largely conserved. Although not so important for a play-money market, it does help for accounting purposes.

## 6.2 Trading

Trading allows us to gather public sentiment on the user-defined securities, and this part of the system is responsible for setting share prices, calculating the cost of transactions, and charging fees to raise funds for the arbitration stage, to ensure arbiters are incentivised to report truthfully. These features are implemented in the files `msr.lisp` and `market.lisp`.

We implement our automated market maker using a scoring rule market. In order to achieve the guarantees of Freeman, Lahaie, and Pennock's mechanism we must use a scoring rule that is strictly proper, which can then be implemented as a market maker based on a convex cost function. We use the commonly-used Logarithmic Market Scoring Rule (LMSR) created by Robin Hanson [21], whose cost function is defined as follows:

$$C(\boldsymbol{q}) = b \log \left( \sum_j e^{q_j/b} \right) \tag{4}$$

This assumes that each security $j$ is one of a collection of mutually exclusive and exhaustive outcomes. Since we are only dealing with binary events, we can compute a share price based only on the number of shares bought for the positive outcome, and assume that buying shares in the negative outcome is equivalent to selling shares in the positive outcome. In this case, we have $\boldsymbol{q} = (0,\, q_1 - q_0)$, where $q_0$ and $q_1$ are the quantity of shares bought by agents in the negative and positive outcomes, respectively. This gives us the following cost function for LMSR in the binary setting, where $q = q_1 - q_0$:

$$C_b(q) = b \log(1 + e^{q/b}) \tag{5}$$

In these cost functions, $b > 0$ appears as a parameter that allows us to control the responsiveness of $C$. A lower value of $b$ corresponds to a more sensitive share price, meaning the price will change more quickly for smaller transactions. It also controls the market's risk of loss: for markets with $|\Omega|$ outcomes it can be shown that the maximum loss incurred by the market maker is $b \log |\Omega|$. In our case, each market will lose at most $b \log 2$. Recall that to compute the actual cost to charge an agent for a transaction, we compute $C_b(q') - C_b(q)$ for an agent wishing to take the total quantity of shares from $q$ to $q'$, and this also encodes sell transactions. The share price function is the derivative of the cost function, which is in our case:

$$p_b(q) = \frac{e^{q/b}}{1 + e^{q/b}} \tag{6}$$

Upon market creation a user is only given the option to buy a positive number of shares – otherwise, it would make more sense for the user to create a market for the opposite outcome. After this, the custom security is created and is open for all other users to trade in. The share price reflects the strength of the community's opinion on the event's outcome: for example, at $q = 0$ then the community is exactly split on whether the event will have a positive outcome, and appropriately $p_b(0) = \$0.50$ for any $b$. Meanwhile, for $q = 20$ and $b = 10$ this means twenty more shares have been bought than sold in the market and would yield a quoted share price of $p_{10}(20) \approx \$0.88$. Users feel more confident that the event will be positive, thus pushing the price upwards. Suppose that upon seeing this price increase, an agent wishes to buy ten more shares in the market: they would then be required to pay $C_{10}(30) - C_{10}(20) = 10 \log(\frac{1+e^3}{1+e^2}) \approx \$9.22$. Such an action in the market would push the share price to $p(30) \approx \$0.95$, plus any fees. Figure 2 shows the interface for creating a new market, while Figure 3 shows an example of the transaction summary that a user is presented with after having done so.

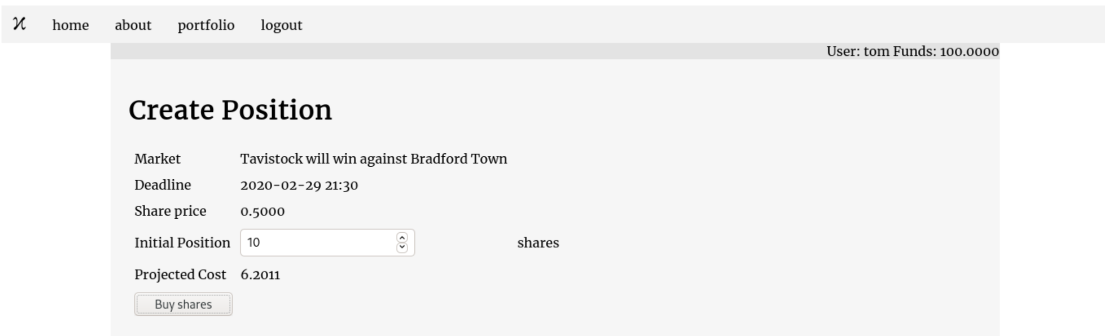



Figure 2: The interface for creating a new market

Fees are only charged on transactions where an agent increases their risk: this means they

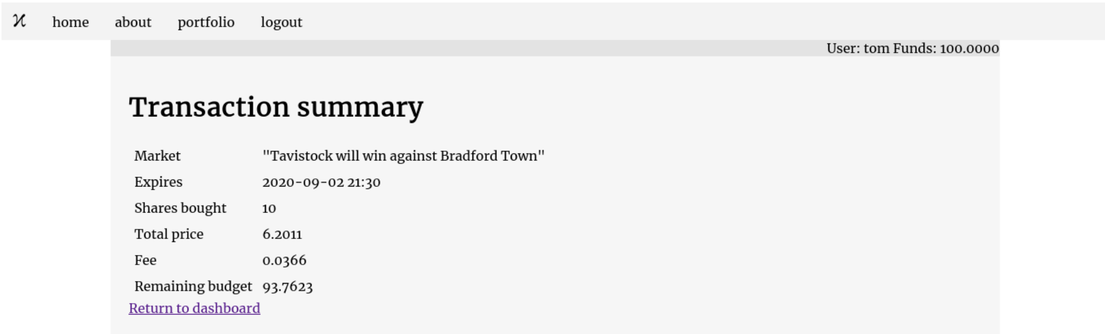



Figure 3: Users are presented with a transaction summary upon creating a new market

are buying or selling a greater number of shares than their current position. For example, a user owning ten shares and selling five would incur no extra cost since they are liquidating a position they have already invested in, while selling more than ten or buying additional shares would incur an additional cost. The fee serves a secondary purpose in bounding the price of the security away from \$0 and \$1, as a potential trader would be spending more than \$1 or less than \$0 to buy or sell the shares. For example, even if an agent were to buy an infinitesimal number of shares, meaning their transaction cost would be given by the share price function $p$, if the share price was \$0.99 and the transaction fee was set to 5%, they would be required to pay $\$0.99 \cdot 1.05 = 1.0395$, greater than the maximum possible payout. This allows us to bound the total number of securities that will exist for a given market, and hence its maximum payout. In our market we implement a fixed trading fee of 5% on all risk transactions.

Obviously, we need to take into account a trader's budget when they are looking to make a transaction and if they cannot afford one, deny them the trade. This is simple for buy transactions, where the cost must not exceed their budget. For a short sell, we need to ensure that the user will be able to cover the expense of buying back the shares in the worst case – that is, the event occurs and they are required to buy back their position at \$1 per share. Hence the function, `sufficient-funds?`, ensures a user can only go short on a security if their budget plus the amount they are paid for shorting is larger than the size of their short position.

## 6.3 Arbitration

### 6.3.1 Computing signal reliability

The arbitration stage is where we resolve market outcomes using arbiter reports and pay out winnings to, or demand payment from, stakeholders in the security. Since the mechanism incentivises arbiters to act truthfully even if an arbiter themselves holds a stake in the market, we decide to let anyone opt in to reporting on the outcome. Once a market's deadline has passed the security will be listed as an expired market and the user is presented with the option to act as an arbiter, as in Figure 4 under "Unresolved Markets". After doing so, they may then input their observed signal, or indeed a lie, as Figure 5 shows. Once the required number of reports have been collected, we move onto rewarding arbiters for submitting reports and computing the final outcome of the market.

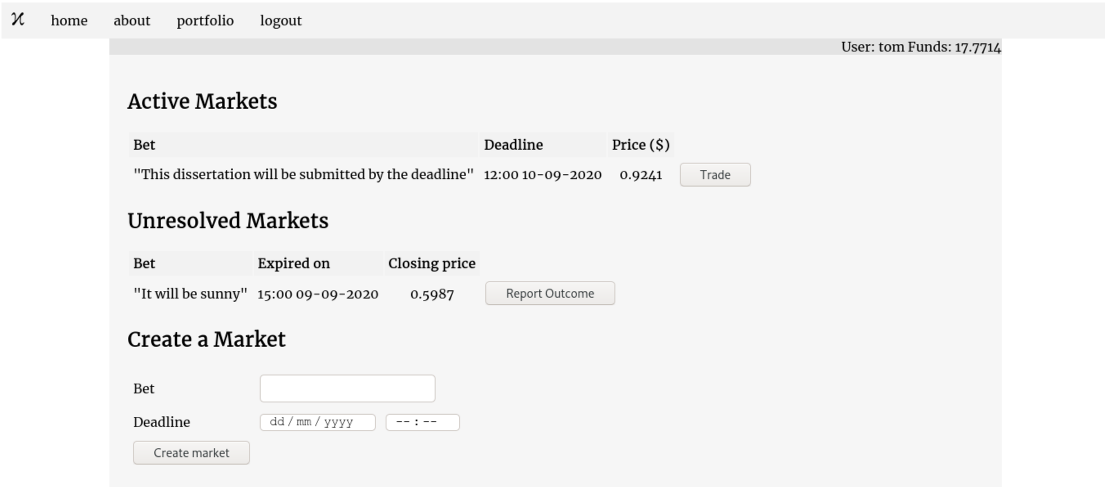



Figure 4: Users can trade in active markets or report on those that have expired

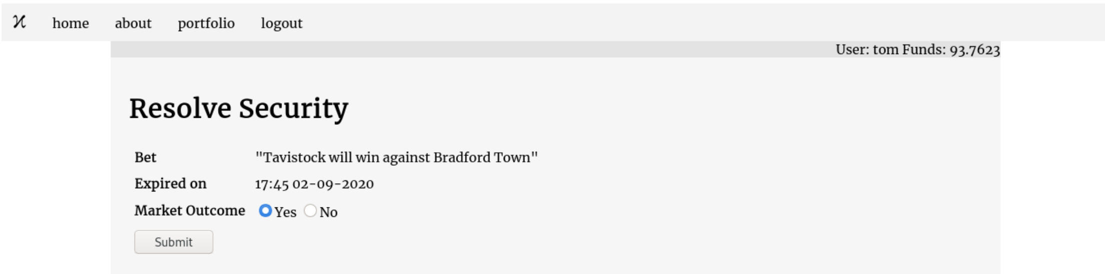



Figure 5: The interface by which arbiters report market outcomes

Arbiters are rewarded by being paid a certain amount of money only if their report agrees with that of another randomly chosen arbiter. The reward is determined by the 1/prior-with-midpoint mechanism, where instead of using the common prior probability $\mu$ we use the update probabilities $\mu_1$ and $\mu_0$. Recall that these are the probability that, given that arbiter $i$ receives a positive signal, so too does another randomly chosen arbiter $j$, and the probability that, given that $i$ receives a negative signal, another randomly chosen arbiter $j$ receives a positive signal. Since we cannot assume arbiters to act truthfully, this is more complicated than simply counting the types of reports received. Instead, we use information about signal error rates $\Pr[x_i = 1|X = 1]$ and $\Pr[x_i = 1|X = 0]$ based on past market outcomes and past reporting behaviour for each arbiter $i$. This is a fairly involved process: first we gather all securities that have been reported on in the past by $i$ and whose outcome has been determined; we then iterate through each one of these returned securities and determine the report that $i$ submitted as well as its outcome, and push this into a list of results. At the end of this stage we have a list `((security report outcome) ...)` that describes the arbiter's report and the market's true (peer-determined) outcome for each security for which they have acted as an arbiter. The code implementing this is shown in Listing 4.

We then use this history to compute the probability that, assuming the arbiter has always

```lisp
(defun get-securities-reported-by-user (user)
  (with-open-database
    (select-dao 'security (inner-join 'user-security
                                      :on (:= :security.id
                                              :user-security.security-id))
                (where (:and (:= :user-security.user-id (user-id user))
                             (:not-null :security.outcome)
                             (:not-null :user-security.report))))))

(defun get-reporting-history (user)
  (let ((securities (get-securities-reported-by-user user))
        security-reports)
    (dolist (security securities)
      (with-open-database
        (let ((report (user-security-report
                       (find-dao 'user-security
                                 :user user
                                 :security security)))
              (outcome (security-outcome security)))
          (if outcome
            (push (list security report outcome) security-reports)))))
    security-reports))
```

Listing 4: Gathering a user's reporting history

```lisp
(defun calculate-positive-belief (reporting-history)
  ;; first only get the reports where the outcome was positive
  (let ((positive-outcomes (remove-if-not #'(lambda (x) (>= 0.5 (third x)))
                                          reporting-history))
        positive-reports)

    ;; now count the number of times the user reported a positive outcome
    (setf positive-reports (count T (mapcar #'(lambda (x) (equal 1 (second x)))
                                            positive-outcomes)))
    (if positive-outcomes
      (/ positive-reports (length positive-outcomes))
      1)))
```

Listing 5: Computing an arbiter's positive signal belief given their reporting history

acted truthfully, their signal has been correct in telling them the true outcome of the event. We refer to these probabilities, $\Pr[x_i = 1|X = 1]$ and $\Pr[x_i = 1|X = 0]$, as a arbiter $i$'s positive and negative signal beliefs as a remnant of a previous implementation. Listing 5 shows how we calculate an arbiter's positive belief, with a similar method applying for computing the negative belief: for the arbiter's history restricted to the securities whose outcome was reported as positive by the majority of arbiters, we count the number of times this arbiter also reported a positive outcome, thus giving us the arbiter's signal error probability given we know the outcome is (more likely to have been) positive. If there have been no positive outcomes, then we assume the arbiter has a perfect signal. Calculating the negative signal belief is done in a similar manner: first we collect items of the arbiter's reporting history where the security was mostly reported as a negative outcome, then we count the number of times this arbiter submitted a positive report. Again we assume in the lack of markets with negative outcomes that the arbiter has a perfect signal. We repeat this to compute the positive and negative signal beliefs for each arbiter.

We use this information about the reliability of each arbiter's signal, along with the prior probability $\mu$ that the event had a positive outcome, to compute the update probabilities $\mu_1$ and $\mu_0$, used in the 1/prior-with-midpoint mechanism. We first compute the update probabilities $\mu_1^i$

```
(let ((reports (mapcar #'second arbiter-reports)))
  ;; the payoff of each share held is the fraction of arbiters reporting 1
  (setf outcome (float (/ (count 1 reports) (length reports)))))
```

Listing 6: Computing the market outcome

```
(defun shuffle (lst)
  " shuffle LST randomly (without modifying it) "
  (let ((lst (copy-list lst)))
    (loop for i from (length lst) downto 2 do
          (rotatef (elt lst (random i))
                   (elt lst (1- i))))
    lst))

(defun random-pairing (lst)
  " randomly pair items from LST together if length(lst) is even "
  (if (evenp (length lst))
    (loop for (a b) on (shuffle lst) by #'cddr while b
          collect (list a b))))
```

Listing 7: Assigning arbiters to peers randomly

for each arbiter $i$, using a randomly chosen peer arbiter $j$, as follows:

$$\begin{aligned} \mu_1^i &= \Pr[x_j = 1 | x_i = 1] \\ &= \Pr[x_j = 1 | X = 0] \cdot \Pr[X = 0 | x_i = 1] + \Pr[x_j = 1 | X = 1] \cdot \Pr[X = 1 | x_i = 1] \end{aligned} \tag{7}$$

We use the same approach to compute $\mu_0^i$ for each $i$. The final values of $\mu_1$ and $\mu_0$ are then calculated by taking the minimum and maximum across all $\mu_1^i$ and $\mu_0^i$, respectively. Thus we have common update probabilities across all agents.

### 6.3.2 Rewarding the arbiters

We may now pair arbiters randomly to pay them via the 1/prior-with-midpoint mechanism. We first retrieve two lists from the database: the first is the list of all arbiter reports and is of the form `arbiter-reports = ((arbiter report) ...)`; the second is a list of all arbiter signal beliefs and is of the form `arbiter-beliefs = ((arbiter positive-belief negative-belief) ...)`. Since we will be pairing arbiters randomly and still need efficient access to these values, we then create two hash tables associating an arbiter to their report and their beliefs, giving us `report[i] = report` and `beliefs[i] = (positive-belief negative-belief)` for each $i$. The market's outcome is simply set to the proportion of arbiters that reported a positive outcome. At this point we pay out the appropriate winnings to stakeholders, paying out money to those who hold shares and demanding money from those who have gone short. Now that the market's outcome has been determined it will no longer be listed as an unresolved market on the user's dashboard.

We next compute the random pairing of arbiters. Lisp has no function to shuffle a list, so we implement the Fisher-Yates algorithm [18, pg. 26-27] ourselves as the `shuffle` function in Listing 7. The random pairing is then formed by walking through the shuffled list and collecting adjacent elements as pairs.

For each set of paired arbiters we then retrieve their reports and signal beliefs from the hash tables and compute, for each arbiter $i$ in the pair, the values of $\mu_1^i$ and $\mu_0^i$ according to

```
(defmacro define-url-fn ((name) &body body)
  " creates handler NAME and pushes it to *DISPATCH-TABLE* "
  `(progn
     ;; define the handler
     (defun ,name ()
       ,@body)

     ;; add the handler to the dispatch table
     (push (create-prefix-dispatcher
             ,(format NIL "/~(~A~)" name) ',name)
           *dispatch-table*)))
```

Listing 8: Macroising URL functions

equation (7), where the randomly chosen $j$ is simply the partner with whom they have been paired. We push these values to a list so we may then compute the minimum value of $\mu_1^i$ and maximum value of $\mu_0^i$ across all arbiters. Finally, to compute the smallest $k$ to satisfy inequality (3) we set $k = \max_i 2|n_i|/m\delta$, and use this to pay arbiters according to equation (1).

## 6.4 Server

The code from each of the separate areas of the market is then drawn together in the file `server.lisp`, in which we set up the web server, define the webpages, and call the functions from the different interfaces we provide.

We use Hunchentoot to host the web server. At startup this involves creating an instance of a Hunchentoot `easy-acceptor`, which opens up a port of our choosing to accept requests. Doing so also initialises the dispatch table, which is a list containing the functions to execute when the corresponding webpage is loaded. We can specify webpages to the dispatch table by using Hunchentoot's `create-prefix-dispatcher` function: this simply takes a URL and the function to execute when that URL is loaded. Again the opportunity to use Lisp's macro system arises: the code in Listing 8 shows a macro that defines a function and a URL of the same name, creates a dispatcher for them and pushes it to the dispatch table. This again not only saves rewriting repetitive code but also keeps the codebase clear – for example, the URL `/index` is served by a function called `index`. Now each time we wish to define a new webpage we need only to call `define-url-fn` followed by the name of the page and the code to execute.

We use CL-WHO to define the content of the webpages, which translates Lisp statements into strings of HTML. In order to achieve a consistent style while avoiding code duplication we define a macro describing a "standard page", an abridged version of which is given in Listing 9. This allows us to concisely define webpages with a similar look and feel, and only requires us to specify the content that makes the page unique via the `body` parameter.

Hunchentoot also automatically provides session handling – this is obviously important for allowing multiple users to be logged on at the same time and presenting the appropriate information to them. All we need to do is define the symbol `session-user` in the data structure for session handling, then when a user logs in we set this value to the matching user retrieved from the database with `(session-value 'session-user)`. This allows us to display consistent user information across different webpages within the same session, such as the user's budget or their portfolio of securities. Finally, users mostly interact with the market through forms: Hunchentoot provides the `get-parameter`, `post-parameter`, and `parameter` functions to retrieve the

```
(defmacro standard-page ((&key title) &body body)
  " template for a standard webpage "
  `(with-html-output-to-string
     (*standard-output* NIL :prologue T :indent T)
     (:html :xml\:lang "en"
            :lang "en
            (:head (:title title)
                   (:link :href "/style.css" :type "text/css" :rel "stylesheet")
                   ;; rest of preamble ... )
            (:body
              (:ul :id "navbar"
                   (:li ... ))
              (:div :class "container"
                    ...
                    (:div :class "content"
                          ,@body))))))
```

Listing 9: Macroising webpage definitions

values from these forms and send them to and from the different webpages.

## 6.5 User Experience

In Sections 6.1, 6.2, 6.3, and 6.4 we have detailed the manner in which we achieve the goals laid out in Section 4.1 which implement the core features of any prediction market, as well as the behaviour that makes the peer prediction mechanism of Freeman et al. unique. In this section we shall detail how we make our prediction market more user-friendly and intuitive to use.

The most important aspect behind making the web application responsive is the integration of asynchronous server communications so that we can display up-to-date information to the user without a page refresh, as well as trigger markets to close trading at the correct time. For this we use the Quicklisp libraries Parenscript coupled with Smackjack to translate Lisp code to JavaScript and allow us to communicate asynchronously with the server. We first use Parenscript for form validation on the client-side, and in particular to ensure that all required form fields are completed without needing to send the entire form to the server, only for it to potentially be incomplete. Parenscript provides the function `ps-inline`, which allows us to insert Lisp code, that will be converted to a valid JavaScript program, within the CL-WHO defined pages. We can therefore validate forms before being sent to the server with `(form :action "create-market" :method :POST :onsubmit (ps-inline ...))`. We implement form validation using a macro: Listing 10 shows the three functions we define to dynamically check for completed fields. The function `make-nonempty-check` simply writes Lisp code that, by the time it is called within `ps-inline` will expand to the JavaScript statement `field.value == 0`. The function `make-nonempty-list` allows us to simply specify a list of fields that we require to be non-empty and have it generate a collection of non-empty checks. Finally, the macro `nonempty-fields` calls the previous function and splices it from a list to successive arguments to the `or` function. This is all wrapped within a call to `ps-inline`, yielding something similar to:

```
if (a.value == "" || b.value == "" || ... ) {
    alert("Please fill in all required fields");
}
```

We use the Smackjack library in conjunction with Parenscript to implement AJAX. Similarly

```
(defun make-nonempty-check (field)
  `(equal (getprop ,field 'value) ""))

(defun make-nonempty-list (fields)
  (loop while fields
        collecting (make-nonempty-check (pop fields))))

(defmacro nonempty-fields (msg &rest fields)
  `(ps-inline
     (when (or ,@(make-nonempty-list fields))
       (if (equal ,msg "")
         (alert "Please fill in all required fields")
         (alert ,msg))
       (return false))))
```

Listing 10: Macro for ensuring all required fields are complete

to how we defined the webpage functions that execute when loading a specific URL in Section 6.4, we must also push our various AJAX functions to the dispatch table: Hunchentoot provides a special data structure, similar to the `easy-acceptor`, that handles all asynchronous calls in the form of the `ajax-processor`. We then create an AJAX dispatcher from this and push it to the dispatch table as before:

```
(push (create-ajax-dispatcher *ajax-processor*) *dispatch-table*)
```

We define all of our AJAX functions using the Smackjack macro `defun-ajax`. This allows us to write functions in Lisp which perform some computation on the server side and send a response back to the client asynchronously. These function definitions are the same as any other in Lisp, with additional information to specify the AJAX processor associated with it, the method by which the data will be sent to the client, and the format that the client should expect it in. We use only one processor, `*ajax-processor*`, which is initialised when the server starts, and we send all data via an HTTP POST request as a JSON string.

One manner in which we use this asynchronous communication is to quote a projected cost of a transaction to a user looking to trade. Since we require the number of shares a trader is looking to buy or sell in order to quote $C_b(q') - C_b(q)$, and we wish to do this without a page refresh, we implement the cost function as an AJAX function that takes the user's desired quantity of shares and the current total number of shares in the security to quote the cost. The interface we present to the user restricts them to entering positive quantities only and checking a button to specify whether to buy or sell (since entering a negative number to sell could be confusing), hence we also need the value of this radio button to compute the transaction cost. This function is given in Listing 11. The function is now defined within Smackjack's namespace: next we need to define the function that calls it asynchronously in JavaScript.

The JavaScript functions that call the AJAX functions need to be defined within the usual `script` tags, for which we simply wrap the appropriate Lisp code in the `(:script ...)` macro provided by Parenscript. For retrieving the transaction cost of a given trade, we define the function `ajax-transaction-cost-trade` as in Listing 12. This function is responsible for getting the quantity of shares the user has input, the current outstanding quantity of shares, and which radio button has been checked to specify whether they are buying or selling. This is achieved by the Parenscript macro `chain`, which chains the list of arguments following it into a list of function calls and attribute retrievals as required. Note that the two radio buttons specifying whether to

```
(defun-ajax ajax-transaction-cost-quantity (quantity q buying-p)
            (*ajax-processor* :method :POST :callback-data :json)

            (if (stringp quantity) (setf quantity (parse-integer quantity)))
            (if (stringp q) (setf q (parse-integer q)))
            (let ((q* (if buying-p
                          (+ q quantity)
                          (- q quantity))))
            (format NIL
                    "{\"newShares\" : ~D, \"oldShares\" : ~D, \"cost\" : ~4\$}"
                    q*
                    q
                    (msr:transaction-cost q* q))))
```

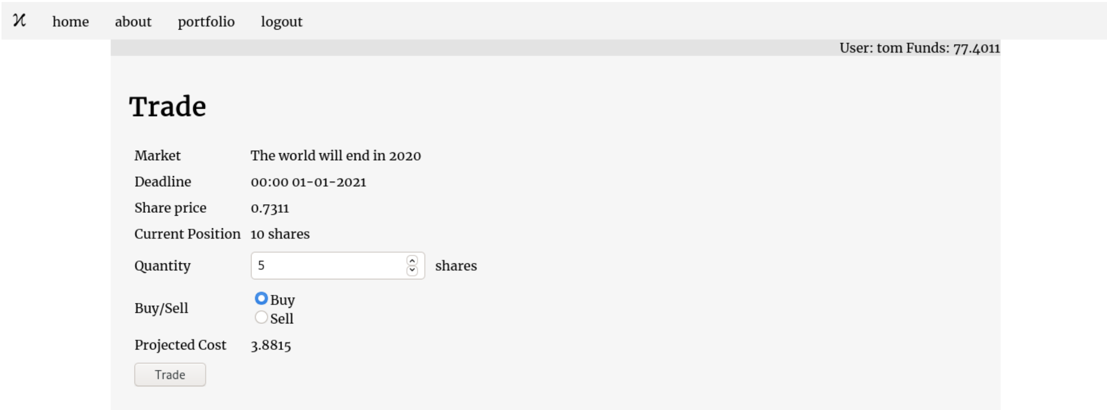
Listing 11: Defining an AJAX function for computing transaction cost using Smackjack

```
(ps
  (defun ajax-transaction-cost-trade ()
    " calculate transaction cost when trading in market "
    (let ((quantity (chain document (get-element-by-id :quantity) value))
          (q (chain document (get-element-by-id :old-shares) value))
          (radios (chain document (get-elements-by-name "buying")))
          checked
          buying-p)

      ;; find which radio button is checked
      (loop for option in radios do
            (if (@ option checked)
              (setf checked (@ option value))))

      ;; checked is equal to 1 if "buy" is selected
      (setf buying-p (equal checked 1))

      (chain smackjack (ajax-transaction-cost-quantity
                        quantity
                        q
                        buying-p
                        display-projected-cost)))))
```

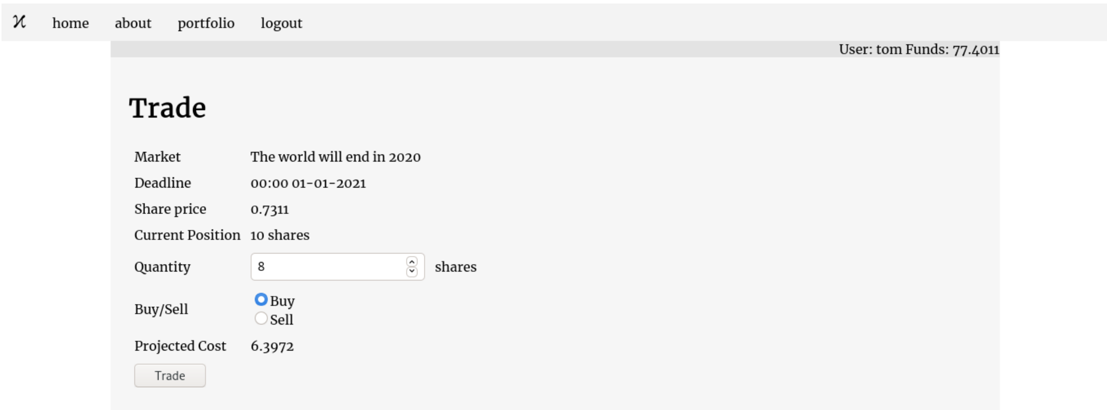
Listing 12: Calling the AJAX function asynchronously

buy or sell must necessarily have the same `id` and `name` field within the form, hence we cannot simply call `get-element-by-id` to retrieve the value. Instead, we call `get-element-by-names` to return both, then iterate through them to verify which one was checked. We then call the AJAX function we defined earlier from Smackjack, `ajax-transaction-cost-quantity`, to compute the transaction cost. This then triggers the callback function `display-projected-cost`, which actually updates the value in the appropriate table cell asynchronously. Listing 12 reflects the rest of this process.

Finally, we use AJAX to automatically close trading when a market's deadline has passed. We first define a Smackjack function that retrieves the most imminently expiring security, then compute the difference in seconds between the current time and its deadline. A JSON object is then returned containing this value, which we use to set a timer for the page to reload. We next define the Javsacript function `set-timer` which takes this JSON object and sets the page to reload after this interval. This process is given in Listing 13. This prevents user from trading shares after potentially knowing the outcome, and instead reloads the page and lists the market under "Unresolved Markets" and no longer active (see Figure 4).




(a)




(b)

Figure 6: Updating transaction costs asynchronously

```
(defun-ajax ajax-set-timer ()
            (*ajax-processor* :method :POST :callback-data :json)

            (let ((next-expiring (db:get-next-expiring-security
                                  (local-time:now)))
                  timer)
              (unless (equal next-expiring NIL)
                (setf timer (local-time:timestamp-difference
                              (db:security-deadline next-expiring)
                              (local-time:now))))
              (format NIL "{ \"security\" : ~S, \"seconds\" : ~D }"
                      (db:security-bet next-expiring)
                      (ceiling timer)))))

;; within a (:script) tag elsewhere ...
(ps
  (defun set-timer ()
    (chain smackjack
           (ajax-set-timer #'(lambda (response)
                               (set-timeout (lambda ()
                                              (chain location (reload)))
                                            (* (@ response seconds) 1000)))))))
```
Listing 13: Triggering the close of trading automatically

# 7 Project Management

## 7.1 Methodology

The project has been developed incrementally, with a focus on integrating new functionality completely before progressing to new features. This approach is well-suited to this project's design: since it consists of five separate areas which are drawn together at the end, it is possible to focus on implementing a feature within one area without it affecting the rest. As a result, testing has been performed throughout and ensures that a newer version of the project is never worse than its predecessor. Using Git and Github has been helpful in this regard, providing cloud storage and the ability to roll back to previous versions of the project if the current one is broken by a new feature.

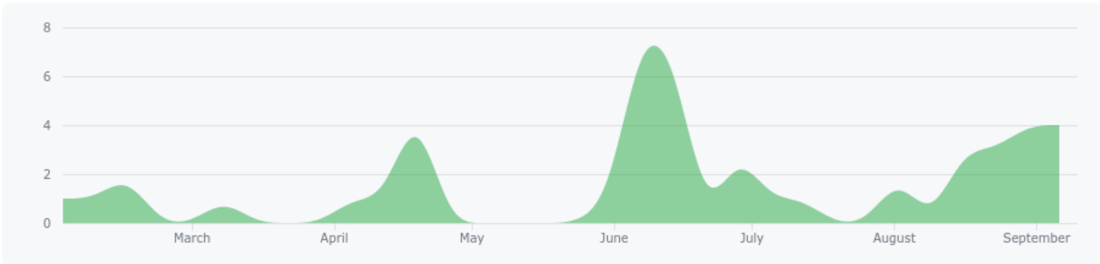



Figure 7: Commits to Github

Meetings have been taken with this project's supervisor, Professor Matthias Englert, to track progress. These were more regular towards the beginning of the project, though as it began to take shape the consistency of these meetings declined. As we will discuss in the following section, this is due in part to the exam period, during which most focus was diverted towards revision, however the original frequency was never picked up after this time as was initially planned. Moreover, it bears mentioning that the current situation regarding the coronavirus pandemic may also have had a part to play in the frequency of these meetings. Regardless, the author feels more initiative should have been taken on his end to ensure their consistency, even with the move online.

## 7.2 Scheduling

There have been few major issues with regard to the scheduling of this project, although it undergone a slight change from the timetable in its original conception, which was meant to be an implementation of a combinatorial prediction market similar to *Predictalot*. Figure 8 shows the schedule as it was planned at the time of our presentation on the project, which was when it was still in its early stages of development. In Figure 9 we detail the order in which tasks were actually carried out.

An initial prototype of the prediction market had been written in time for the presentation and at its core our current iteration is very similar to this early version with respect to the datatypes we define and the processes of trading and arbitration. However, it was entirely text-based and had no capacity for persistent storage, meaning it was unusable in a practical setting. At any rate its purpose was to showcase the mechanism in action and this is, at its core, very simple.

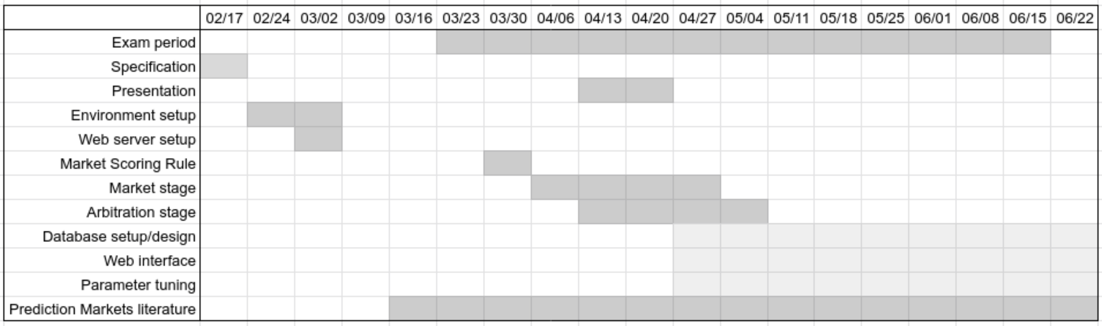



Figure 8: The project's timetable as it was initially planned

Work after this point was focused on translating this terminal-based implementation to operate on a barebones web server, with the first steps to achieve persistent storage via interaction with a database. Since Mito's `deftable` macro is functionally very similar to that of Common Lisp's `defstruct`, this conversion was relatively straightforward. Similarly, the code to set up and begin defining webpages in our implementation is simple and hence took little time. In the case of both the web server and the database, a significantly larger portion of time was spent deciding how to integrate features as they were developed into the system as a whole. As we try to keep our implementation modular there is the implicit requirement to not only get a feature working in isolation, but to then integrate it into the rest of the system successfully. This involved constantly updating the web and database interfaces, and is reflected in Figure 9, which shows much more overlap between tasks. Some issues were encountered trying to implement asynchronous communication with the server via Parenscript and Smackjack, and much time was spent getting this working. This highlights one of the drawbacks behind using both libraries: while they are somewhat well-documented with online reference manuals, it seems they are not widely used and hence have few practical examples from which to learn.

Worthy of note is the two-week delay from the original deadline for the interim report, arising from the university-wide implementation of a two-week extension to all assessed work. We decided to take advantage of this by spending an extra two weeks to implement the arbitration stage to a greater degree of completion, in order to have more material for discussion in the interim report. This in part stems from the fact that we had made less progress during and after the exam period as expected. This did not affect the overall progress of the project, however, since the two weeks worth of progress made during this time was simply borrowed from what would have been achieved in the two weeks after the original deadline.

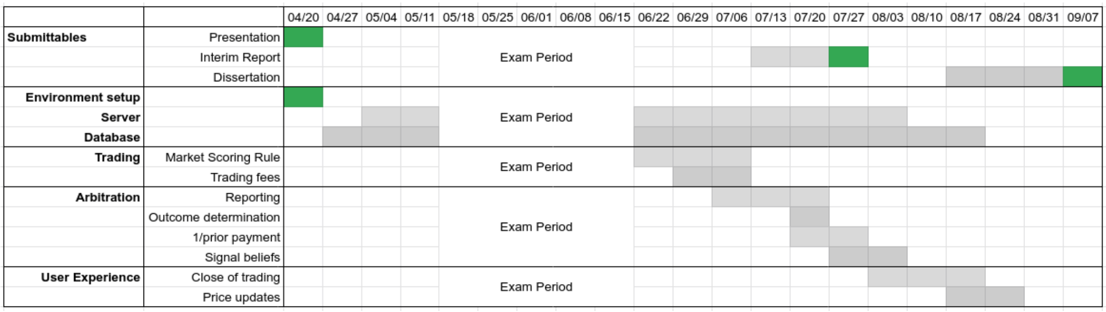



Figure 9: The realised schedule

## 7.3 Ethics

There is little ethical consideration required for the development of this project. All development and testing has been done independently, and all resources used to implement the system are available freely. Testing has been performed externally only to small extent, and even then only informally through gathering opinions amongst colleagues. As we have discussed has been a problem for existing prediction markets, there is one ethical issue that arises from using real money in prediction markets and this is their potential to inspire "assassination markets", where there is a real-life incentive to change the outcome of the market through committing actions of dubious character, such as assassinating the subject of a security speculating on the time of their death. We avoid testing the strength of our users' moral fibre by using fake money only, with no real-world value, meaning there is no incentive to act in such a way as to compromise one's integrity for monetary gain.

# 8 Evaluation

In this section we shall begin by reflecting on some of the successes of the project, followed by suggestions of improvements that could be made to our implementation were we to have more time. We conclude with a reflection on the project as a whole.

## 8.1 Successes

One obvious success of the project is that we have implemented the decentralised prediction market that we originally intended based on the work of Freeman et al. A key component of the market is that participants are incentivised to act in the desired way through paying each user a specific amount of money. This has required building a strong understanding of their paper to ensure everything has been implemented correctly so that we may achieve the theoretical guarantees that they outline. We have also managed to become more well-acquainted with the wider literature on prediction markets; a key goal motivation before choosing a project had been to gain more exposure to the field of algorithmic game theory, and in this regard the project has been a personal success.

Translating the mechanism Freeman et al. from a theoretical setting into a practical environment has required slightly modifying certain aspects of it to make it functional in the real world, especially with respect to the assumptions we can make. For example, the paper uses the following bound on the number of securities to calculate the minimum payment required in the arbitration stage to incentivise truth-telling:

$$k \geq \frac{2B(1+f)}{fm\delta} \tag{8}$$

in which $B$ is the maximum amount of money that any single agent can spend in the market. They use the above inequality instead of inequality (3) as they view knowledge about an upper bound on the number of securities that any single agent owns, $|n_i|$, as "an unsatisfying restriction". However, since we are the prediction market, we require knowledge about each agent's position in order to pay out winnings appropriately by the time the event has taken place. Hence we can use inequality (3) to calculate the minimum value of $k$, giving a more direct means of computing the parameter and a more exact bound.

Another modification we make is in our calculation of what we have referred to as a user's "positive signal belief" and "negative signal belief". These are so named because in an earlier version of the market, each time an arbiter was asked to report on an event's outcome they would be asked for an estimation of $\Pr[x_i = 1|X = 1]$, the probability that they receive a positive signal given the true result is positive; and $\Pr[x_i = 1|X = 0]$, the probability that they receive a positive signal given the event was truly negative. Freeman et al. assume knowledge of the news sources from which users gain their signals – for example, for some political event it is known that 10% of the population checks only liberal or left-leaning news sources, while 10% check only right-leaning sources. These signal probabilities therefore directly follow. In practice this is completely unrealistic, since users may check a variety of news sources at varying points on the political spectrum, so we cannot know from where exactly which proportions of the population are getting their information. Instead, we compute the signal beliefs using each arbiter's reporting history and the true outcomes of the events they have reported on. This gives us an idea of the signal each arbiter receives on average. However, this value will not change from market to market, meaning we are less well-equipped to deal with ambiguous wagers since an arbiter has no way to signify to the system that the bet was indeed unclear or open to interpretation. Having the value of $\delta$, the "update strength", vary between markets allows the $k$ from equation (1) adjust payments according to its perceived ambiguity. This issue is, however, somewhat mitigated by the likelihood that most bets submitted on our market will be unambiguous in nature.

One final variation that we make to the original mechanism is that we keep the trading fee $f$ fixed. The reason for this is twofold: firstly, it relies on $\delta$, which as we have just discussed does not vary for different markets as it is always calculated from an arbiter's history for *every* market. If our value of $\delta$ is just an estimate, it can be argued there is little sense in trying to set $f$ as precisely as possible. Secondly, we have not yet been able to calculate $f$ dynamically in the allotted time. Its calculation involves other variables and would likely require changes to various other parts of the system, in particular the database, which could jeopardise the system's current working state before submission. At any rate, this is something we wish to implement in the future as, regardless of how much $\delta$ changes, it guarantees that the mechanism generates enough revenue to pay arbiters without requiring outside subsidisation. We currently have $f$ set to 5% as Freeman et al. show empirically that this relatively high fee can subsidise ambiguous markets with low values of $\delta$ and high participation. However, since it is not set dynamically, there is no way of knowing whether it will be enough to cover arbitration costs – only once we find ourselves paying out more than we have will we know we have made a mistake. We can avert this crisis by overestimating the value of $f$ required, which is the solution we currently opt for, but this is certainly a less satisfying solution. Overall, we consider our implementation a success as the tweaks we make to the mechanism allow it to be realised in a practical setting that does not require unrealistic assumptions to be made.

Finally, while not necessarily important from an academic perspective, we consider the project a success for having been implemented in the Lisp programming language, with which we had had no experience prior to starting the project. It has at times rendered development more challenging and introduced obstacles that would not have ordinarily been there – for example, worrying about whether a Parenscript and Smackjack statement would translate to the correct JavaScript, as opposed to simply writing the correct JavaScript straight away – although the features of the language have also made it a more rewarding experience. In addition to harnessing the power of macros, its functional style along with its simple, consistent syntax has oftentimes allowed for the code to be written in a clearer, more concise manner.

## 8.2 Next steps

### 8.2.1 Additions

There are numerous improvements we can make to our market, and while we have implemented all necessary features of the paper on which we base the mechanism itself, such additions would be significant in increasing the usability of our system. These fall into two categories: those that add new functionality, and those that improve on the existing codebase.

A key feature missing from the system currently is the ability to plot graphs of a market's share price. This would greatly improve the usability of the system and would be an effective analytical tool to visualise a security's price movements over time. Since share price is a proxy for the community's beliefs on the outcome of the corresponding event, this would be useful in observing how public sentiment changes throughout a market's lifetime as a result of the agents receiving new information. It appears there is plenty of choice regarding libraries that would allow one to plot such graphs, such as ADW-Charting and CGN, which are both available through Quicklisp.

One way to generalise the mechanism we have implemented would be to allow for different types of events to be wagered on. Currently, users may only submit events whose outcome is either a "yes" or a "no" – this could be improved upon by allowing for categorical markets, in which traders are offered to buy or sell shares in more than two mutually exclusive outcomes. For example, a market could be created on the bet, "Which country will host the 2032 Olympic Games", with options to invest in Germany, Italy, the United States, and Canada. There are two issues with this that prevented it from being explored within the time-frame of this project. Firstly, a great deal of the analysis of [19] relies on the assumption that traders are participating in a binary market, hence much of the parameters would have to be derived again in order to successfully translate the incentive-compatible mechanism to the categorical setting. While this is feasible over a longer period, the idea came too late to be possible in the remaining time. Secondly, the codebase would need to be altered significantly to allow for this generalisation. It could be beneficial, since binary markets are just subsets of categorical ones, however the system would require a large rewrite to generate the HTML dynamically and deal with the added complexity.

Finally, a longer term goal would be to apply the approaches of Kroer et al. [24] to our market to translate it to a combinatorial setting. However, the extent to which this can be done appears to be very limited: a key component of the mechanisms they analyse is that the securities are related in some way, meaning a user's participation in one security can affect the odds of another. The degree to which a new security is related to other existing securities is controlled by the market creator, who for example sets the initial price and specifies the constraints used in solving the integer programs. This would be counter-productive in our setting since we rely on users to create the markets themselves: either we allow users to control these parameters, in which case we must incentivise them to act truthfully, or we specify them ourselves, which may be inaccurate and negates the point of implementing a decentralised market in the first place.

With more time, we would also look to implement more responsiveness from the system. This would include building on the JavaScript code already in place to have prices constantly updating, so that users are informed of price changes as they happen and they may execute trades at these up-to-date prices. Similarly, countdowns for markets whose deadlines are imminent would be useful in not only making the interface feel more alive but also in generating urgency and encouraging user participation.

### 8.2.2 Improvements

Currently we allow any user to create a market for an event of their choosing. We require that upon creating the market, they are only able to buy shares. This is a natural restriction that means users only make markets for events they believe will, more than likely, occur. However, there is nothing to stop them from simply negating the statement and creating a market for this. Although not a serious issue, this may lead to unnatural or unclear phrasing, in which case it would simply be better to allow them to go short initially. An issue related to this is that we do not verify the bets that users place, and therefore there could be duplicate markets on the same event whose prices do not match. Clearly buyers are encouraged to trade in the lower priced one, and sellers the higher priced one, leading to a discrepancy in the forecasts we make. We do not currently see a good solution to this problem.

An issue in our implementation is that of deciding when there are enough reports. Instead of randomly selecting users to act as arbiters, which may cause delays in determining the outcome of the market as we must wait for each arbiter to report, we allow any user to act as an arbiter, hopefully increasing the speed with which we can close markets. Thanks to how we set the parameter $k$ in equation (1), arbiters are incentivised to act truthfully regardless of whether they hold a stake in the market. However, it is undecided how the number of arbiters required should scale with the size of the userbase: it is currently set to two for ease of testing, though it will somehow need to increase with a growing community (up to a point). This in turn allows for finer control over the payout per share.

Finally, there are several sources of avoidable inefficiency in our code. Firstly, consider the fragments given in Listing 4. In order to get a user's reporting history we first retrieve all of the securities reported on by the user from the `SECURITY` table, then we iterate over these returned securities and retrieve the securities whose outcome has been determined: this involves executing one query for each parent record and another for each child record. This is known as the $N + 1$ query problem and is common among ORMs such as Mito. We can achieve equivalent behaviour for a fraction of the queries using eager loading, which executes a single query to retrieve each child record. Mito has a specific macro for this, and given this pattern's prevalence in our codebase, would make for a useful improvement. We use our `with-open-database` macro, given in Listing 3, to define many of the functions exposed by the database interface. Problems of inefficiency could arise when calling numerous of these functions sequentially, since a connection would be opened and closed for each one. It would be more efficient to connect only once, execute all transactions, and then disconnect. At present, we do not see a solution to this that keeps the benefits of macroising the interactions with the database in the first place.

## 8.3 Closing Remarks

Overall, this project has been successful and we have enjoyed building an understanding of the literature surrounding prediction markets, as well as gaining proficiency in Lisp. One criticism of the project is that it does not necessarily offer something new, since we base our design on an existing mechanism in order to achieve their guarantees. While we implement the peer prediction market and make necessary adaptations to make it suitable for real-world use, there are no novel results that arise from our work. The alternative, of proving some new theoretical result, would of course have been an unlikely task in the time frame, although the project could have perhaps benefited from being more ambitious. At any rate, what we implement here can be used as an effective tool to calculate forecasts on a wide array of bets and is open to further extension to increase the potential for real-world use.

We will leave our prediction market running for as long as possible after submission here: `https://1a091f61d58d.ngrok.io/index`. Many thanks go to Matthias for supervising this project, in suggesting to look at prediction markets, and for helpful advice throughout.

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