CHAPTER 15

Learning in Networks

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Abstract

We choose between alternatives without being fully informed about the rewards from different courses of action. In making our decisions, we use our own past experience and the experience of others. So the ways in which we interact – our social network – can influence our choices. These choices in turn influence the generation of new information and shape future choices. These considerations motivate a rich research programme on how social networks shape individual and collective learning. The present paper provides a summary of this research.

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1. INTRODUCTION

In a wide range of economic situations, individuals make decisions without being fully informed about the rewards from different options. In many of these instances, the decision problems are of a recurring nature and it is natural that individuals use their past experience and the experience of others in making current decisions. The experience of others is important for two reasons: One, it may yield information on different actions per se (as in the case of choice of new consumer products, agricultural practices, or medicines prescribed); and two, in many settings the rewards from an action depend on the choices made by others and so there is a direct value to knowing about other's actions (as in the case of which language to learn). This suggests that the precise way in which individuals interact can influence the generation and dissemination of useful information and that this could shape individual choices and social outcomes. In recent years, these considerations have motivated a substantial body of work on learning which takes explicit account of the structure of interaction among individual entities. The present paper provides a survey of this research.

I will consider the following framework: There is a set of individuals who are located on nodes of a network; the arcs of the network reflect relations between these individuals. Individuals choose an action from a set of alternatives. They are uncertain about the rewards from different actions. They use their own past experience, and also gather information from their neighbors (individuals who are linked to them) and then choose an action that maximizes individual payoffs. I start by studying the influence of network structure on individual and social learning in a pure information-sharing context. I then move on to a study of strategic interaction among players located in a network, i.e., interactions where an individual's actions alter payoffs of others. The focus will be on examining the relation between the network structure on the one hand, and the evolution of individual actions, beliefs and payoffs on the other hand. A related and recurring theme of the survey will be the relation between network structure and the prospects for the adoption of efficient actions.[1]

[1] This chapter builds on Goyal (2005, 2007a). The focus here is on analytical results and there will be no discussion of the large literature on agent based modeling and computational economics, which studies similar issues. For surveys of this work, see Judd and Tesfatsion (2005), Kirman and Zimmermann (2001)).

The following examples elaborate on the variety of applications, which follow within the scope of the general approach. We start with three examples involving pure information sharing.

Consumer choice: A consumer buying a computer chooses a brand without being fully informed about the different options. Since a computer is a major purchase, potential buyers also discusses the pros and cons of different alternatives with close friends, colleagues, and acquaintances. The importance of opinion leaders and mavens in the adoption of consumer goods has been documented in a number of studies (see e.g., Feick and Price, 1987; Kotler and Armstrong, 2004).

Medical innovation: Doctors have to decide on new treatments for ailments without complete knowledge of their efficacy and side-effects; they read professional magazines as well as exchange information with other doctors in order to determine whether to prescribe a new treatment. Empirical work suggests that location in inter-personal communication networks affects the timing of prescription while the structure of the connections between physicians influences the speed of diffusion of new medicines (for a pioneering study see on this see Coleman, 1966). There is also evidence that medical practices vary widely across countries and part of this difference is explained by the relatively weak communication across countries (see e.g., Taylor, 1979).

Agricultural practices: Farmers decide on whether to switch crops, adopt new seeds and alternative farming packages without full knowledge of their suitability for the specific soil and weather conditions they face. Empirical work shows that individuals use the experience of similar farmers in making critical decisions on adoption of new crops as well as input combinations (see e.g., Ryan and Gross (1943), Griliches, 1957; Conley and Udry, 2010).

We now turn to applications in which actions of others affect individual payoffs. We will focus on games of coordination and games of cooperation. The problem of coordination arises in its simplest form when, for an individual, the optimal course of action is to conform to what others are doing. The following three examples illustrate how coordination problems arise naturally in different walks of life.

Adoption of new information technology: Individuals decide on whether to adopt, say, a fax machine without full knowledge of its usefulness. This usefulness depends on the technological qualities of the product but clearly also depends on whether others with whom they communicate adopt a similar technology. Empirical work suggests that that there are powerful interaction effects in the adoption of information technology (Economides and Himmelberg, 1995).

Language choice: Individuals choose which language to learn at school as a second language. The rewards depend on the choices of others with whom they expect to interact. Empirical work suggests that changes in the patterns of interactions among individuals – for instance, a move from a situation in which groups are relatively isolated with little across-group interaction to one in which individuals are highly mobile and groups are more integrated – has played an important role in the extinction of several languages and the dominance of a few languages (see e.g., Watkins, 1991; Brenzinger 1998).

Social Norms: Individuals choose whether to be punctual or to be casual about appointment times. The incentives for being punctual are clearly sensitive to whether others are punctual or not. Casual observation suggests that in some countries punctuality is the norm while in others it is not.[2] Similarly, the decision on whether to stand in a queue or to jump it is very much shaped by the choices of others. Likewise, the arrangement of cutlery on a table is governed by norms which have evolved spatially and across time (for a study, see Elias, 1978). These examples illustrate the role of interaction externalities in shaping social outcomes.

The problem of cooperation arises when individual incentives lead to an outcome which is socially inefficient or undesirable. Such conflicts between individual incentive driven behavior and social goals are common in many situations. The following example illustrates this.

Provision of public goods: Individuals have the choice of exerting effort which is privately costly but yields benefits to themselves as well as to others. A simple example of this is proper maintenance of a personal garden that is also enjoyed by others. Another example is the participation of parents in school monitoring associations – such as governing bodies. In such contexts, it is often the case that the personal costs exceed the personal benefits but are smaller than the social benefits. Theoretical work argues that the structure of interaction between individuals – in particular whether one's acquaintances know each other – can be crucial in determining levels of public good provision (see e.g., Coleman, 1990).

I now place the material covered in this survey paper within a broader context. Repeated choice among alternatives whose relative advantages are imperfectly known is a common feature of many real life decision problems and so it is not surprising that the study of learning has been one of the most active fields of research in economics in the last two decades. Different aspects of this research have been surveyed in articles and books; see e.g., Blume and Easley (1995), Fudenberg and Levine (1998) Kandori (1997), Marimon (1997), Samuelson (1997), Vega-Redondo (1997), and Young (1998). The focus of the present survey will be on a very specific set of issues concerning the network structure of interaction and information flow on the one hand and the process of learning on the other hand.

Actions often yield outcomes which are informative about their true profitability and so the process of learning optimal actions has been extensively studied in the economics and decision theory literatures. The initial models focused on a the prospects of single decision maker learning the optimal action in the long run; see e.g., Berry and Fristedt

[2] For a formal model of punctuality as social norm, see Basu and Weibull (2002).

(1985), Rothschild (1974) and Easley and Kiefer (1988). In many of the applications (as in the examples mentioned earlier in the introduction) experimentation with different alternatives is expensive and it is natural to suppose that individuals will use the experience of others in making their own choices. However, if the outcomes of individual trials yield information which is shared with others, individual experimentation becomes a public good and so choice takes on a strategic aspect, even though actions of others do not matter for individual payoffs. This motivates a study of the dynamics of strategic experimentation; for an elegant analysis of this issue see e.g., Bolton and Harris (1999). In this line of research the actions and outcomes of any individual are commonly observed by everyone. By contrast, the focus of this chapter is on the differences in what individuals observe and how these differences affect the process of social learning.

Similarly, the study of coordination and cooperation problems has a long and distinguished tradition in economics. The problem of coordination has been approached in two different ways, broadly speaking. One approach views this to be a static game between players, and tries to solve the problem through introspective reasoning; Schelling (1960) introduced the notion of focal points in this context.[3] The second approach takes a dynamic perspective and seeks solutions to coordination problems via the gradual accumulation of precedent. This approach has been actively pursued in recent years; see Young (1998) for a survey of this work. The present chapter takes a dynamic approach as well and the focus here is how the network of interaction shapes the process of learning to coordinate.

The conflict between personal interest driven behavior and the social good has been a central theme in economics (and game theory). One approach to the resolution of this problem focuses on the role of repeated interaction between individuals. A self-interested individual may be induced to act in the collective interest in the current game via threats of punishments in the future from other players. This line of reasoning has been explored in models of increasing generality over the years and a number of important results have been obtained. For a survey of this work, see Mailath and Samuelson (2006). Most of this work takes the interaction between individuals to be centralized (an individual plays with everyone else) or assumes that interaction is based on random matching of players. A second approach to this problem focuses on the role of nonselfish individual preferences and nonoptimizing rules of behavior. This

[3] For recent work in this tradition see Bacharach (2006), and Sugden (2004). Also see Lewis (1969) for an influential study of the philosophical issues relating to conventions as solutions to coordination problems. On the applied side, the theory of network externalities is closely related to the problem of social coordination. This theory arose out of the observation that in many markets the benefits of using a product are increasing in the number of adopters of the same product (examples include fax machines and word processing packages). In this literature, the focus was on the total number of adopters and this work examined whether these consumption externalities will inhibit the adoption of new products. For a survey of this work, see Besen and Farrell (1994) and Katz and Shapiro (1994). The models of social coordination with local interaction presented in section 3.1 can be seen as an elaboration of this line of research.

approach uses empirical and experimental evidence as a motivation for the study of alternative models of individual behavior. The role of altruism, reciprocity, fairness and inequity aversion has been investigated in this line of work. Camerer (2003), and Fehr and Schmidt (2003) provide surveys of this research. The present chapter takes a dynamic approach to the study of cooperation and the interest is in understanding how decision rules and networks of interaction jointly shape individual behavior.

Section 2 presents the model and the main results for learning for the pure information sharing problem, while Section 3 takes up learning in strategic games played on networks. Section 4 concludes the paper.

2. NONSTRATEGIC INTERACTION

This section considers learning of optimal actions in a context where payoffs to an individual depend only on the actions chosen by him. We start with a model in which a set of individuals choose actions repeatedly: they observe the outcomes of their own actions as well as the actions and outcomes of their neighbors. We then study a simpler setting in which a sequence of individuals make one shot decisions.

2.1 Repeated choice and social learning

Consider a group of individuals who at regular intervals, individuals choose an action from a set of alternatives. They are uncertain about the rewards from different actions. So they use their own past experience and gather information from their neighbors, friends and colleagues, and then choose an action that maximizes individual payoffs. Three features are worth mentioning. One, individuals choose actions repeatedly and two, actions potentially) generate information on the value of the different alternatives. Thus the amount of information available to the group is endogenous and a function of the choices that individuals make. Three, individuals may have different neighbors and this will give them access to different parts of the information available in the group.[4]

It is convenient to present the model in three parts: the first part lays out the decision problem faced by individuals, the second part introduces notation concerning networks, while the third part discusses the dynamics. The presentation here is based on the work of Bala and Goyal (1998, 2001).[5]

Decision Problem: Suppose that time proceeds in discrete steps, and is indexed by $t = 1,2,\ldots$. There are $n \geq 3$ individuals in a society who each choose an action from

[4] There is an important strand of research in which a single individual makes one choice upon observing the history of past actions (and payoffs). I discuss models of sequential choice and social learning in section 2.3 below.

[5] In an early paper, Allen (1982) studied technology adoption by a set of individuals located on nodes of a graph, who are subject to local influences. This is close in spirit to the motivation behind the framework developed here. Her work focused on invariant distributions of actions, while the interest in this chapter is on the dynamic processes of learning that arise in different networks.

a finite set of alternatives, denoted by S_i. It assumed that all individuals have the same choice set, i.e., $S_i = S_j = A$, for every pair of individuals i and j. Denote by $a_{i,t}$ the action taken by individual i in time period t. The payoffs from an action depends on the state of the world θ, which belongs to a finite set Θ. This state of the world is chosen by nature at the start of the process and remains fixed across time. If θ is the true state and an individual chooses action $a \in A$ then he observes an outcome $y \in Y$ with conditional density $\phi(y, a; \theta)$ and obtains a reward $r(a, y)$. For simplicity, take Y to be a subset of $\mathcal{R}$, and assume that the reward function $r(a,.)$ is bounded. In some examples Y will be finite and we will interpret $\phi(y, a; \theta)$ as the probability of outcome y, under action a, in state θ.

Individuals do not know the true state of the world; their private information is summarized in a prior belief over the set of states. For individual i this prior is denoted by $\mu_{i,1}$. The set of prior beliefs is denoted by $\mathcal{P}(\Theta)$. To allow for the possibility of learning of any state of the world, it will be assumed that prior beliefs are interior, i.e., $\mu_{i,1}(\theta) > 0, \forall\theta$, and $\forall i \in N$. Given belief μ, an individual's one period expected utility from action a is given by

$$u(a, \mu) = \sum_{\theta\in\Theta}\mu(\theta)\int_Y r(a, y)\phi(y, a; \theta)dy. \tag{1}$$

The expected utility expression has a natural analogue in the finite Y case. In the basic model, it will be assumed that individuals have similar preferences which are reflected in a common reward function $r(\cdot, \cdot)$. Learning among neighbors with heterogeneous preferences is discussed subsequently.

Given a belief, μ, an individual chooses an action that maximize (one-period) expected payoffs. Formally, let $B : \mathcal{P}(\Theta) \rightarrow A$ be the one period optimality correspondence:

$$B(\mu) = \{a \in A | u(a, \mu) \geq u(a', \mu), \forall a' \in A\} \tag{2}$$

For each $i \in N$, let $b_i : \mathcal{P}(\Theta) \rightarrow A$, be a selection from the one period optimality correspondence B.

Let δ_θ represent point mass belief on the state θ; then $B(\delta_\theta)$ denotes the set of optimal actions if the true state is θ. A well-known example of this decision problem is the two-arm bandit.

Example 2.1 *The two-arm bandit.*

Suppose $A = \{a_0, a_1\}$, $\Theta = \{\theta_0, \theta_1\}$ and $Y = \{0, 1\}$. In state θ_1, action a_1 yields Bernoulli distributed payoffs with parameter $\pi \in (1/2, 1)$, i.e., it yields 1 with probability π, and 0 with probability $1 - \pi$. In state θ_0, action a_0 yields a payoff of 1 with probability $1 - \pi$, and 0 with probability π. Furthermore, in both states, action a_0 yields payoffs which are Bernoulli distributed with probability 1/2. Hence action a_1 is optimal in state θ_1, while action a_0 is optimal in state θ_0. The belief of an individual is a number $\mu \in (0, 1)$, which represents the probability that the true state is θ_1. The one period optimality correspondence is given by

$$B(\mu) = \begin{cases} a_1 & \text{if } \mu \geq 1/2 \\ a_0 & \text{if } \mu \leq 1/2 \end{cases}$$

An individual chooses an action $b_i(\mu_{i,1})$ and observes the outcome of his action; he also observes the actions and outcomes obtained by a subset of the others, viz., his *neighbors*. The notion of neighborhoods and related concepts are defined next.

Directed Networks: Each individual is located (and identified with) a distinct node of a network. A link between two individuals i and j is denoted by g_{ij}, where $g_{ij} \in \{0, 1\}$. In the context of information networks, it is natural to allow for the possibility that individual i observes individual j, but the reverse does not hold. This motivates a model of links which are *directed:* if $g_{ij} = 1$ then there is flow of information from j to i, but I will allow for $g_{ji} = 0$ even when $g_{ij} = 1$. In Figure 1, there are 3 players, 1, 2 and 3, and $g_{1,3} = g_{3,1} = g_{2,1} = 1$. A directed link from i to j, $g_{ij} = 1$ is represented as an arrow that ends at j.

There is a *directed* path from j to i in g either if $g_{ij} = 1$ or there exist distinct players $j_1, \ldots, j_m$ different from i and j such that $g_{i,j_1} = g_{j_1,j_2} = \ldots = g_{j_m,j} = 1$. For example, in Figure 1 there is a directed path from player 3 to player 2, but the converse is not true. The notation "$j \xrightarrow{g} i$" indicates that there exists a (directed) path from j to i in g. Define $N_i(g) = \left\{k | i \xrightarrow{g} k\right\} \cup \{i\}$ as the set of players that i accesses either directly or indirectly in g, while $\eta_i(g) \equiv |N_i(g)|$ is the number of people accessed. The length of a path between i and j is simply the number of intervening links in the path. The distance between two players i and j in a network g refers to the length of the shortest directed path between them in the network g, and is denoted by $d_{i,j}(g)$.

Let $N_i^d(g) = \{k \in N | g_{i,k} = 1\}$ be the set of individuals with whom i has a direct link in network g. This set $N_i^d(g)$ will be referred to as the *neighbors* of i in network g. Define $\eta_i^d(g) \equiv |N_i^d(g)|$ as the *out-degree* of individual i. Analogously, let $N_{-i}^d(g) = \{k \in N | g_{ki} = 1\}$ be the set of people who observe i and define $\eta_{-i}^d(g) \equiv |N_{-i}^d(g)|$ as the *in-degree* of individual i.

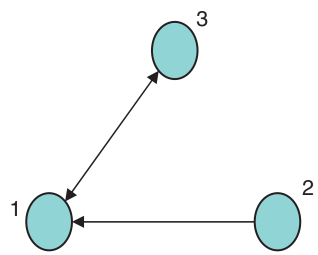

Figure 1 Directed information network.

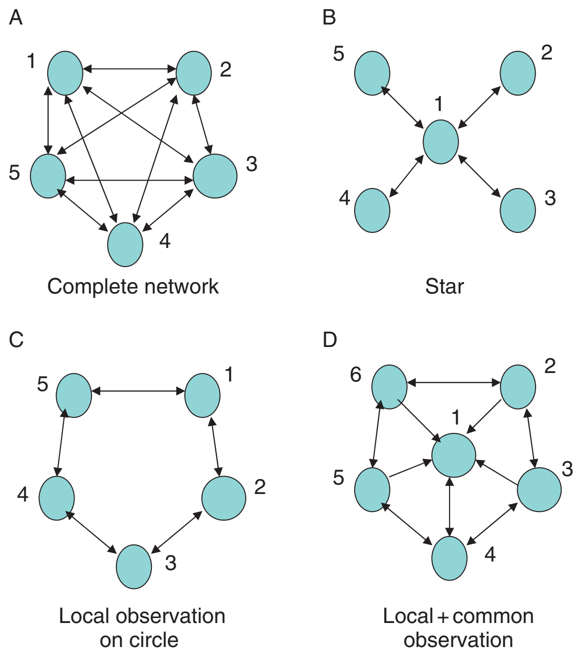

Figure 2 Simple information networks.

A network g is said to be connected if there exists a path between any pair of players i and j. The analysis will focus on connected networks. This is a natural class of networks to consider since networks that are not connected can sometimes be viewed as consisting of a set of connected subnetworks, and then the analysis I present can be applied to each of the connected subnetworks. For instance, consider the networks in Figure 2; each of them is connected. The last network, which combines local and common observation reflects a situation in which individuals gather information from their local neighborhoods and supplement it with information from a common source. The star represents a situation in which the in-degree and out-degree of an individual are equal, but the in-degree of the central node is higher than that of the peripheral node. By contrast, the network in Figure 2D represent a situation in which there is significant asymmetry between the in-degree and the out-degree of the central node, while the other nodes have in-degree equal to the out-degree. The central individual only observes one other node but is observed by five others.[6]

Empirical evidence on networks: The classic early work of Lazarsfeld, Berelson, and Gaudet 1948 and Katz and Lazersfeld 1955 investigated the impact of personal contacts

[6] There are, of course, networks for which the distinction between connectedness and disconnectedness is too coarse. As an example, suppose person 1 observes 2, who observes 3, and so on, until person $n-1$, who observes n. Clearly this network is not connected. However, the connected components are singletons. Learning (and payoffs) in this network is likely to exhibit very different features from learning in an empty network (which is also disconnected).

and mass media on voting and consumer choice with regard to product brands, films and fashion changes. They found that personal contacts play a dominant role in disseminating information which in turn shapes individuals' decisions. In particular, they identified 20% of their sample of 4000 individuals as the primary source of information for the rest. Similarly, Feick and Price 1987 found that 25% of their sample of 1400 individuals acquired a great deal of information about food, household goods, nonprescription drugs and beauty products and that they were widely accessed by the rest.

Research on virtual social communities reveals a similar pattern of communication. Zhang, Ackerman and Adamic 2007 study the Java Forum: an on-line community of users who ask and respond to queries concerning Java. They identify 14000 users and find that 55% of these users only ask questions, 12% both ask and answer queries, and about 13% only provide answers.[7]

The empirical research has highlighted a number of common features of social networks: *one*, the distribution of connections is very unequal. For instance, most web sites (over 90%) have fewer than 10 other web-sites linking to them (this is their in-degree), but at the same time there exist web-sites, such as google.com, bbc.com, and cnn.com, which have hundreds of thousands of in-coming links. Communication networks in rural communities have been found to exhibit a similar inequality: most people in a village talk to their relatives and neighbors and a small set of highly connected villagers leading to a skewed distribution of in-degrees (Rogers, 2003).

The *second* feature is the asymmetry between in-degree and out-degree of a node. In many contexts, there exist a small set of nodes which have a very large in-degree and relatively small out-degree, while most other nodes have an out-degree which is larger than their in-degree. This pattern of connections arises naturally if individuals have access to local as well as some common/public source of information. For instance, in agriculture, individual farmers observe their neighboring farmers and all farmers observe a few large farms and agricultural laboratories. Similarly, in academic research, individual researchers keep track of the work of other researchers in their own narrow field of specialization and also try and keep abreast of the work of pioneers/intellectual leaders in their subject more broadly defined.

Dynamics: In period 1 each individual starts by choosing an action $b_i(\mu_{i,1})$: in other words, we assume that individuals are myopic in their choice of actions. This myopia assumption is made for simplicity: it allows us to abstract from issues concerning strategic experimentation and to focus on the role of the network in shaping social learning.[8] At the end of the period, every individual i observes the outcome of his own actions. She also observes the actions and outcomes of each of his neighbors, $j \in N_i^d(g)$.

[7] Adar and Huberman 2000 report similar findings with regard to provision of files in the peer-to-peer network, Gnutella.

[8] We conjecture that the arguments developed in Theorem's 1–3 below carry over to a setting with far sighted players, so long as optimal decision rules exhibit a cut-off property in posterior beliefs (as identified for instance in example 2.1).

Individual i uses this information to update his prior $\mu_{i,1}$, and arrive at the prior for period 2, $\mu_{i,2}$. She then makes a decision in period 2, and so on.

In principle, the choices of an individual $j \in N_i^d$, reveal something about the priors (and hence private information) of that individual and over time will also reveal something about the actions and experience of his neighbors. However, in updating his priors, it will be assumed that an individual does not take into account the fact that the actions of his neighbors potentially reveal information about what these neighbors have observed about their neighbors. The main reason for this assumption is tractability. The study of social learning in the presence of inferences about the neighbors of neighbors is an important subject; see remarks below and at the end of this section on this issue.

I now describe the space of outcomes and the probability space within which the dynamics occur as the notation is needed for stating the results. The details of the construction are provided in the appendix. The probability space is denoted by $(\Omega, \mathcal{F}, P^\theta)$, where Ω is the space of all outcomes, $\mathcal{F}$ is the σ field and P^θ is a probability measure if the true state of the world is θ. Let P^θ, be the probability measure induced over sample paths in Ω by the state $\theta \in \Theta$.

Let Θ be endowed with the discrete topology, and suppose $\mathcal{B}$ is the Borel σ–field on this space. For rectangles of the form $\mathcal{T} \times H$, where $\mathcal{T} \subset \Theta$, and H is a measurable subset of Ω, let $P_i(\mathcal{T} \times H)$ be given by

$$P_i(\mathcal{T} \times H) = \sum_{\theta \in \mathcal{T}} \mu_{i,1}(\theta) P^\theta(H). \tag{3}$$

for each individual $i \in N$. Each P_i extends uniquely to all $\mathcal{B} \times \mathcal{F}$. Since every individual's prior belief lies in the interior of $\mathcal{P}(\Theta)$, the measures $\{P_i\}$ are pair wise mutually absolutely continuous. All stochastic processes are defined on the measurable space $(\Theta \times \Omega, \mathcal{B} \times \mathcal{F})$.

A typical sample path is of the form $\omega = (\theta, \omega')$, where θ is the state of nature and ω' is an infinite sequence of sample outcomes:

$$\omega' = ((y_{i,1}^a)_{a \in A, i \in N}, (y_{i,2}^a)_{a \in A, i \in N}, \ldots), \tag{4}$$

with $y_{i,t}^a \in Y_{i,t}^a \equiv Y$. Let $C_{i,t} = b_i(\mu_{i,t})$ denote the action of individual i in period t, $Z_{i,t}$ the outcome of this action, and let $U_{i,t} = u(C_{i,t}, \mu_{i,t})$ be the expected utility of i with respect to his own action at time t. Given this notation the posterior beliefs of individual i in period $t+1$ are:

$$\mu_{i,t+1}(\theta | g) = \frac{\prod_{j \in N_i^d(g) \cup \{i\}} \phi(Z_{j,t}; C_{j,t}; \theta) \mu_{i,t}(\theta)}{\sum_{\theta' \in \Theta} \prod_{j \in N_i^d(g) \cup \{i\}} \phi(Z_{j,t}; C_{j,t}; \theta) \mu_{i,t}(\theta)}. \tag{5}$$

The interest is in studying the influence of the network g on the evolution of individual actions, beliefs, and utilities, $(a_{i,t}, \mu_{i,t}, U_{i,t})_{i \in N}$, over time.

Remark 1: The above formula is one way in which individual i incorporates information from own and neighbors' experience. There are simpler alternatives. For instance, in period $t+1$, an individual may apply Bayes' Rule own experience to update belief $\mu_{i,t}$ to arrive at an interim belief $\hat{\mu}_{i,t+1}$ and then take a weighted average of this interim belief and period t belief of neighbors to arrive at an overall posterior belief $\mu_{i,t+1}$. For a study of learning with such updating rules see Jadbabaie, Sandroni and Tahbaz-Salehi (2010). Observe that in this model if there is no new information coming in every period, then learning entails averaging of initial opinions across neighbors, as in the DeGroot (1972) model. I refer to this as naive learning and discuss it in Section 2.2 below.

Remark 2: An alternative and general formulation of the problem of (fully) Bayesian learning is developed in a recent paper by Mueller-Frank (2010). He considers a finite set of agents who each have a commonly known partition of the states of the world. Individuals are located in networks and observe the actions of their neighbors. At the start, each agent chooses an action that is optimal with respect to his initial partition. As individuals observe their neighbors, they refine their partitions on the state of the world. The paper develops a number of results on how network structure and levels of common knowledge (with regard to rationality and strategies) matter for the properties of long run actions.

Main results: Individual actions are an optimal response to beliefs, which in turn evolve in response to the information generated by actions; thus, the dynamics of actions and beliefs feed back on each other. Over time, as an individual observes the outcomes of own actions and the actions and outcomes of neighbors, his beliefs will evolve depending on the particularities of his experience. However, it seems intuitive that as time goes by, and his experience grows, additional information should have a smaller and smaller effect on his views of the world. This intuition is captured by the following result, due to Bala and Goyal (1998), which shows that the beliefs and utilities of individuals converge.

Theorem 2.1 *The beliefs of individuals converge, in the long run. More precisely, there exists* $Q \in \mathcal{B} \times \mathcal{F}$ *satisfying* $P_i(Q) = 1$, *for all* $i \in N$, *and random vectors* $\{\mu_{i,\infty}\}$ *such that* $\omega \in Q \Rightarrow \lim_{t\to\infty}\mu_{i,t}(\omega) = \mu_{i,\infty}(\omega)$. *The utilities of individuals converge:* $\lim_{t\to\infty} U_{i,t}(\omega) = U_{i,\infty}(\omega)$, *for every* $i \in N$, *with probability 1.*

The convergence of beliefs follows as a corollary of a well known mathematical result, the Martingale Convergence Theorem (see e.g., Billingsley, 1985). Next consider the convergence in utilities. Let $A_i(\omega)$ be the set of actions that are chosen infinitely often by individual i along sample path ω. Note that since the set of actions is finite, this set is always nonempty. It is intuitive that an action $a \in A_i(\omega)$ must be optimal with respect to the long run beliefs. Moreover, since the different states in the limiting beliefs are not distinguished by the actions, these actions must yield the same utility in each of the states that are in the support of the limit belief $\mu_{i,\infty}(\omega)$. These observations yield convergence of individual utility.

In what follows, I will take θ_1 to be the true state of nature. Note that

$$Q^{\theta_1} = \{\omega = (\theta, \omega')|\theta = \theta_1\} \tag{6}$$

has P^{θ_1} probability 1.[9] It will be assumed that the strong law of large numbers holds on Q^{θ_1}.[10] All statements of the form 'with probability 1' are with respect to the measure P^{θ_1}.

We will now turn to a key question in this literature: Do individuals choose the right action and earn the maximal possible earn the state in the long run?

It is useful to begin an analysis of this question by examining the learning problem for a single agent in the two-arm bandit. Suppose the true state is θ_1 and action a_1 is the optimal action. A well-known result from the statistical literature says that starting from any prior belief $\mu_{i,\,0}$, there is a positive probability that an agent will switch to action a_0 and remain at that action for ever. Let us now suppose that individuals choose between the two arms of the bandit and observe their own outcomes as well as outcomes of a subset of the other members of the group. Suppose individual i starts with beliefs $\mu_{i,0}$ and let $\mu_{i,0} < 1/2$ except for agent 1. Fix $\mu_{1,0} > 1/2$. Define k to be the smallest number such that if agent 1 observes k outcomes of 0 with action a_1 then his posterior $\mu_{1,k} < 1/2$. Suppose the trials of agent 1 do yield a k long sequence of 0's. It is now easy to verify that in a connected society every agent i will have beliefs $\mu_{i,k} < 1/2$ at time k. But observe that different individuals will have started with different priors and observed different experiences. So the posterior beliefs will typically differ across agents. Observe that once everyone switches to action a_0, no further information is being generated and so individual beliefs will remain different in the long run. What about the long run utilities? The following result, due to Bala and Goyal (1998), responds to this question.

Theorem 2.2 *If the society is connected then every individual gets the same long run utility:* $U_{i,\,\infty}(\omega) = U_{j,\,\infty}(\omega)$ *for every pair of individuals* $i,j \in N$, *with probability one.*

The key observation here is that, if i observes the actions and outcomes of j then he must be able to do as well as j in the long run. While this observation is intuitively plausible, the formal arguments underlying the proof are quite complicated. The principal reason for the complication is that individual i observes the actions and corresponding outcomes of a neighbor j, but does *not* observe the actions and outcomes of the neighbors of j. The claim that i does as well as j if he observes j then rests on the idea that all payoff relevant information that j has gathered is (implicitly) reflected in the choices that he makes, over time. In particular, if j chooses a certain action in the long run then this action must be the best action for him, conditional on all his information. However, individual i observes these actions and the corresponding outcomes and can therefore do as well as j by simply imitating j.

[9] There is a slight abuse of notation here; the domain of the definition of P^{θ_1} is Ω and not $\Theta \times \Omega$.
[10] For a statement of the strong law of large numbers, see e.g., Billingsley (1985).

The final step in the proof shows that this payoff improvement property must also be true if person i observes j indirectly, via a sequence of other persons, i_1, i_2,..i_m. The above argument says that i does as well as i_1 who does as well as i_2, and so on until i_m does as well as j. The final step is to note that in a connected society there is an information path from any player i to any player j.

This result shows that in any connected society, local information transmission is sufficient to ensure that every person gets the same utility in the long run. Connected societies cover a wide range of possible societies and this result is therefore quite powerful. However, it leaves open the question of whether individuals are choosing the optimal action and earning the maximal possible utility in the long run.

To study this question it is useful to fix a true state and an optimal action. Let θ_1 is the true state of the world and let $B(\delta_{\theta_1})$ be the set of optimal actions, corresponding to this state. Social learning is said to be *complete* if for all $i \in N, A^i(\omega) \subset B(\delta_{\theta_1})$, on a set of sample paths which has probability 1 (with respect to the true state θ_1). The analysis of long run learning rests on the informativeness of actions. An action is said to be fully informative if it can help an individual distinguish between all the states: if for all θ, $\theta' \in \Theta$, with $\theta \neq \theta'$,

$$\int_Y |\phi(y; a, \theta) - \phi(y; a, \theta')| dy > 0. \tag{7}$$

By contrast, an action a is uninformative if $\phi(., a; \theta)$ is independent of θ. In Example 2.1 above, action a_0 is uninformative while action a_1 is fully informative.

In any investigation of whether individuals choose the optimal action in the long run, it is necessary to restrict beliefs. To see why this is so, consider Example 2.1 above. If everyone has priors such that the uninformative action is optimal then there is no additional information emerging in the society and so an individual using Bayes' updating will retain his priors and everyone will therefore choose the suboptimal action forever. Optimism in the prospects of a new technology is by itself, not sufficient. The structure of connections is also important for learning to take place. This is illustrated with the help of the following two examples.

Example 2.2 *Incomplete Learning.*

Suppose that the decision problem is as in Example 2.1 and suppose that everyone is optimistic, i.e., $\mu_{i,1}$ $(\theta_1) > 1/2$. Moreover, for concreteness, assume that beliefs satisfy the following condition.

$$\inf_{i \in N} \mu_{i,1} > \frac{1}{2}; \quad \sup_{i \in N} \mu_{i,1} < \frac{1}{1 + x^2} \tag{8}$$

where $x = (1 - \pi)/\pi \in (0, 1)$. These restrictions incorporate the idea that individuals are optimistic about the unknown action but there is an upper bound on their optimism. From the optimality correspondence formula given above, it follows that every person chooses a_1 in period 1. Suppose that individuals are arranged around a circle and observe their

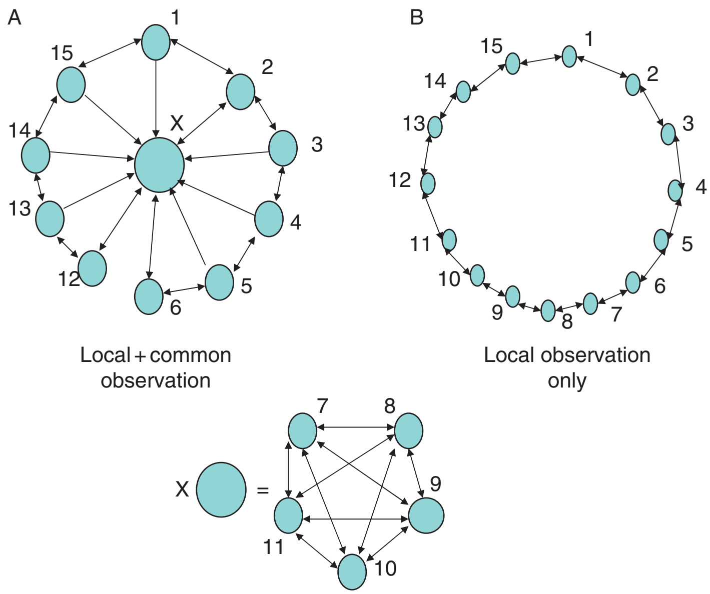

Figure 3 Local + Common Information.

neighbors and a set of common individuals: i.e., $N_i^d = \{i-1, i+1\} \cup \{7, 8, 9, 10, 11\}$. Figure 3 illustrates such a society.[11]

We now show that *there is a strictly positive probability of incomplete learning* in this society. The argument is quite simple: suppose that every person in the commonly observed set is unlucky in the first period and gets an outcome of 0. Consider any individual i and note that this person can get at most three positive signals from his immediate neighborhood. Thus any person in this society will have a minimum residual of two negative signals on the true state. Given the assumptions on the priors, this negative information is sufficient to push the posteriors below the critical cut-off level of 1/2 and this will induce a switch to action a_0 in period 2, for everyone. From then on, no further information is generated and so everyone chooses the suboptimal action a_0 forever. Notice that this argument does not use the size of the society and so there is an upper bound on the probability of learning, which is smaller than one, *irrespective of the size of the society*.

To appreciate the reasons underlying the failure of information aggregation in the above example, note that if θ_1 is the true state then, in a large society, roughly a fraction π (where $\pi > 1/2$) of individuals will receive a payoff of 1 from the action a_1 and a fraction $(1-\pi)$ (where $1-\pi < 1/2$) of people will receive a payoff of 0. Example 2.2 thus illustrates how a few common signals can block out and overwhelm a vast amount of locally available positive information on action a_1. One way of exploring the role of

[11] This Figure is only illustrative of the general structure, and must be interpreted with care: in particular, individual 6 observes 5 and 7–11, 7 observes 6 and 7–11, individuals 8–10 only observe 7–11, while individual 11 observes 7–11 and 12.

network structure is by altering the relative size of local and common information. This is the route taken in the next example.

Example 2.3 *Complete Learning.*

Consider again the decision problem as in Example 2.1 and suppose that information flows via observation of immediate neighbors who are located around a circle: for every i, $N_i^d = \{i-1, i+1\}$. Thus this society is obtained from the earlier one in Example 2.2 by deleting a large number of communication connections. Figure 2.1B illustrates this new society. What are the prospects of learning in such a society? First, fix an individual i and note that since θ_1 is the true state of the world, actions a_1 is actually the optimal action. This means that there is a positive probability that a sequence of actions a_1 will on average generate positive information forever. This means that starting with optimistic priors, individual i will persist with action a_1, forever, if he were isolated, with positive probability.

Analogous arguments show that similar sequences of actions can be constructed for each the neighbors of player i, $i-1$ and $i+1$. Exploiting independence of actions across players, it follows that the probability of the three players $i-1$, i, and $i+1$ receiving positive information on average is strictly positive. Denote this probability by $q > 0$. Hence the probability of individual i choosing the suboptimal action a_0 is bounded above by $1-q$. Finally, note that along this set of sample paths, the experience of other individuals outside the neighborhood cannot alter the choices of individual i. So we can construct a similar set of sample paths for individual $i+3$, whose information neighborhood is $\{i+2, i+3, i+4\}$. From the independent and identical nature of the trials by different individuals, it can be deduced that the probability of this sample of paths is $q > 0$ as well. Note next that since individuals i and $i+3$ do not share any neighbors, the two events, $\{i$ does not try optimal action$\}$ and $\{i+3$ does not try optimal action$\}$ are independent. This in turn means that the joint event that *neither of the two try the optimal action* is bounded above by $(1-q)^2$. In a society where $N_i^d = \{i-1, i+1\}$, and given that $q > 0$, it now follows that learning can be made arbitrarily close to 1, by suitably increasing the number of individuals.

Example 2.3 illustrates in a simple way in which the architecture of connections between individuals in a society can determine whether a society adopts the optimal action in the long run. It also helps in developing a general property of networks that facilitates learning of optimal actions. In the society of Example 2.2 (see Figure 2.1A) with a set of commonly observed individuals, the positive information generated on the optimal actions in different parts of the society is overturned by the negative information generated by this common observed group. By contrast, in a society with only local ties, negative information does arise over time but it cannot overrule the positive information generated in different parts of the society. This allows the positive local information to gain a foothold and eventually spread across the whole society.

The critical feature of the society in Example 2.3 (in Figure 2.1B) is the existence of individuals whose immediate neighborhood is distinct. This leads naturally to the idea of *local independence*. Two individuals i and j are locally independent if $N_i^d(g) \cup \{i\} \cap N_j^d(g) \cup \{j\} = \emptyset$. Moreover, a player i has optimistic prior beliefs if the set of optimal actions under the prior belief $B(\mu_{i,1}) \subset B(\delta_{\theta_1})$. The following general result on networks and social learning, due to Bala and Goyal (1998), can now be stated.

Theorem 2.3 *Consider a connected society. In such a society, the probability that everyone chooses an optimal action in the long run can be made arbitrarily close to 1, by increasing the number of locally independent optimistic players.*

The proof of this result is provided in the appendix. The arguments in the proof extend the intuition underlying Example 2.3, to allow for an arbitrary number of actions, as well as more general outcomes spaces.

Theorem 2.3 and Examples 2.2 and 2.3 have a number of interesting implications which are worth elaborating on. The *first* remark is about the relation with the strength of weak ties hypothesis due to Mark Granovetter (see Granovetter, 1974).[12] In Granovetter's theory, society is visualized as consisting of a number of groups that are internally tightly linked but have a few links across them. In one interpretation, the links across the groups are viewed as weak ties and Granovetter's idea is that weak ties are strong in the sense that they are critical for the flow of new ideas and innovations across the groups in a society. The above result can be interpreted as showing that in societies with this pattern of strong ties (within groups) and weak ties (across groups), the weak ties between the groups do carry valuable information across groups and therefore play a vital role in sustaining technological change and dynamism in a society.

The *second* remark is about what Examples 2.2 and 2.3 and Theorem 2.3 tell us about the generation and diffusion of information in real world networks. Our description of information networks in Section 2.1 suggests that they exhibit very unequal distribution of in-degrees. It is intuitively plausible that in networks with such unequal in-degree distribution a few highly connected nodes have the potential to startwaves of diffusion of ideas and technologies. The formal arguments presented above suggest, somewhat disturbingly, that these waves can lead to mass adoption of actions or ideas whose desirability is contradicted by large amounts of locally collected information. Moreover, due to the broad adoption of such actions, information generation about alternative actions is seriously inhibited and so suboptimal actions can persist.

The *third* remark is about the impact of additional links on the prospects of diffusion of a desirable action. On the one hand, Examples 2.2 and 2.3 together shows that adding links in a network can actually lower the probability of a society learning to choose the optimal action. However, if links are added to the network in Figure 2.1A

[12] For a general formulation of the role of social relation in economic activity, see Granovetter (1985).

eventually we will arrive at the complete network: clearly, the probability of adopting optimal action can be increased to 1, in a complete network by simply increasing the number of nodes. These observations suggest that the impact of additional links depends very much on the initial network and how the links are added. Can we say anything about the marginal value of different links in a network? The above discussion about strength of weak ties suggests that links which act as *bridges* between distinct groups in a society will be very valuable. However, an assessment of the marginal value of links in more general networks appears to be a open question.

The *fourth* remark is about a potential trade-off between the possibility of learning and the speed of learning. A society with common pool of observations has quick but inefficient convergence, whereas the society with pure local learning exhibits slower speed of learning but the probability of learning is higher. A similar trade-off is also present in Ellison and Fudenberg (1993), who study a spatial model of learning, in which the payoffs are sensitive to location. They suppose that there is a continuum of individuals arranged along a line. Each individual has a window of observation around himself (this is similar to the pure local learning network considered above). They consider a choice between two technologies, and suppose that technology A (B) is optimal for all locations to the right (left) of 0. For individual i, the window is an interval given by $[i - w, i + w]$, for some $w \in \mathcal{R}_{+}$. Each individual chooses the action which yields a higher average payoff in this window. Suppose that, at the start, there is a boundary point $x_0 > 0$, with technology A being adopted to the right of x_0, and technology B being adopted to the left of x_0. Ellison and Fudenberg show that the steady state welfare is decreasing in the size of the interval. Thus, smaller intervals are better from a long term welfare point of view. However, if w is small then the boundary moves slowly over time and if the initial state is far from the optimum then this creates a trade-off: increasing w leads to a short-term welfare gain but a long-term welfare loss.

Diversity: The discussion now turns to the third question posed in the introduction: what is the relation between the structure of connections and the prospects for diversity of actions in a society? Theorem 2.2 says that in a connected society all individuals will obtain the same utility. If there is a unique optimal action for every state this implies that all individuals will choose the same action as well. In case there are multiple optimal actions, however, the result does not say anything about conformism and diversity. To get an impression of the issues involved, start with a society that is split into distinct complete components. Now the level of integration of the society can be measured in terms of the number of cross-group links. Figure 4 presents three such societies, with varying levels of integration. Bala and Goyal (2001) study the probability of diversity as a function of the level of integration. They find that diversity can occur with positive probability in a partially integrated society but that the probability of diversity is zero in a fully integrated society (i.e., the complete network). A characterization of networks which allow for diversity appears to be an open problem.

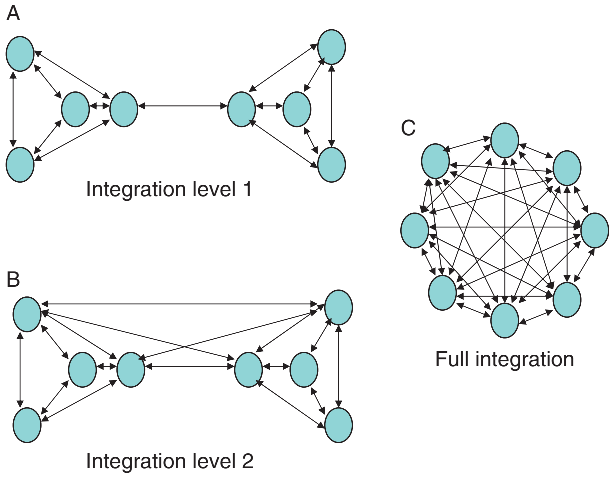

Figure 4 Network integration and social conformism.

Preference heterogeneity: This result on conformism, and indeed all the results reported so far, are obtained in a setting where individuals have identical preferences. However, individuals often have different preferences which are reflected in a different ranking of products or technologies. This raises the question: What are the implications of individual preference heterogeneity for the three results, Theorems 2.1–2.3, obtained above?

It is easy to see that the considerations involved in Theorem 2.1 do not depend in any way on the structure of interaction, so the convergence of beliefs and utilities will obtain in the more general setting with differences in preferences as well. This leads to a study of the long run distribution of utilities. In a society with individuals who have different preferences, the analogue of Theorem 2.2 would be as follows: *in a connected society, all individuals with the same preferences obtain the same utility*. Bala and Goyal (2001) show that this conjecture is false. They construct an example in which preference differences can create information blockages that impede the transmission of useful information and thereby sustain different utility levels (and also different actions) for individuals with similar preferences.

This example motivates a stronger notion of connectedness: *group-wise* connectedness. A society is said to be group-wise connected if for every pair of individuals i and j of the same preference type, either j is a neighbor of i, or there exists a path from j to i with all members of the path being individuals having the same preference as i and j. Bala and Goyal (2001) then show that the conjecture on equal utilities for members of the same preference type obtains in societies which satisfy this stronger connectedness requirement. In a setting with a unique optimal action for every preference type, this result also implies conformity in actions within preference types.

In the above discussion on heterogeneity, we have kept the proximity in preferences and proximity in social connections separate. In some contexts, it is possible that network proximity and distance on other dimensions are related. A large body of research argues that social relations such as friendships exhibit 'homophily': people tend to make friends with others from the same race and gender. For a recent attempt at understanding the ways in which homophily in networks affects social learning, see Golub and Jackson (2010b).

Sources of information and decision rules: The framework developed in Section 2.1 was motivated by the idea that social proximity is an important factor in determining the sources of information that individuals rely upon in making decisions. While this motivation appears to be sound and is supported by a range of empirical work, the framework developed to study social learning above is a little rigid in the following sense. In many situations, individuals get aggregate data on relative popularity of different actions in the population at large. Measures of popularity are potentially useful because they may yield information on average quality of the product. This type of information is not considered the analysis above. One way to model these type of ideas is to suppose that individuals get to observe a sample of other people chosen at random. They use the information obtained from the sample – which could relate to the relative popularity of different actions, or actions and outcomes of different actions – in making his choices. This approach has been explored by Ellison and Fudenberg (1993, 1995), among others. These papers examine the ways in which the size of the sample, the type of information extracted from the sample, and the decision rule that maps this information into the choice of individuals affects social learning.

In recent work, Gale and Kariv (2003) study a model with fully rational individuals who get a private signal at the start and in subsequent periods get to observe the actions of their neighbors, who in turn observe the actions of their neighbors and so on. In this setting, the choice of actions can potentially communicate the private information generated at the start of the process. Individual actions, however, do not generate fresh information unlike the model discussed in Section 2.1. However, full rationality of individuals makes the inference problem quite complicated and they are obliged to focus on small societies to get a handle on the dynamics of learning. In particular, they show that beliefs and utilities of individual individual's converge. This finding is similar to Theorem 2.1 reported above. They also show that in a connected society every individual chooses the same action and obtains the same utility. Thus social conformism obtains in the long run. The ideas behind this result mirror those discussed in Theorem 2.2 above.

Changes in decision rules will have an impact on the learning process and the outcomes may well be different with non-Bayesian rules of thumb such as imitation of the best or popularity weighted schemes. In a recent paper, Chatterjee and Xu (2004) explore learning from neighbors under alternative bounded rational decision rules.

2.2 Naive learning

The previous section focused on a model of Bayesian learning and established a number of results on the relation between the evolution of beliefs, actions, and payoffs and the structure of the network. The complexity of the learning process meant that I was able to obtain only relatively simple results concerning the effects of networks. This section simplifies the learning process and this will allow us to derive sharper results with regard to the implications of networks. The exposition in this section draws on the work of DeMarzo, Vayanos and Zweibel (2003) and Golub and Jackson (2010a).

There are n individuals, each of whom starts with a belief at date 0, a number p_i^0. In period 1 each individual i updates his belief, by taking an average of the beliefs of others and his own belief. She assigns weight $\omega_{ij} \in [0, 1]$ to each $j \in N$, with $\sum_{j \in N} w_{ij} = 1$. This yields p_i^1, for $i \in N$. Define for any $t \geq 0$, the vector of beliefs at start of that period, $\mathbf{p}^t = \{p_1^t, p_2^t, p_3^t, .. p_n^t\}$. Let $\mathbf{W}$ be the $n \times n$ stochastic matrix defined by the weights which individuals assign to each others opinions. The belief revision process repeats itself over time $t = 2, 3, 4, \ldots$ I study the evolution of $\mathbf{p}^t$ in relation to the matrix $\mathbf{W}$.

As in the previous section, our interest will be in connected societies; in the present context a society is said to be connected if for every pair of players i and j, either $\omega_{ij} > 0$ or there is a intermediate sequence of individuals $i_1, i_2, .. i_k$ such that $w_{ii_1}, \ldots . w_{i_k j} > 0$. DeMarzo et al. (2003) prove the following result.

Theorem 2.4 *Suppose that the matrix W comes from a connected society and that $w_{ii} > 0$ for all i. Then*

1. *The influence of j on i converges:* $\lim_{t \to \infty} w_{ij}^t = w_j$.
2. *The beliefs converge $p_i^t \to p^*$, for all $i \in N$, where $p^* = wp^0$.*
3. *The social influence vector $\boldsymbol{w} = (w_1, w_2, w_3, \ldots, w_n)$, is defined as the unique solution to* $\mathbf{w} \cdot \mathbf{W} = \mathbf{w}$.

The society is connected and so the Markov process is irreducible, and since $\omega_{ii} > 0$, the Markov chain is also a periodic. It is well known that such a Markov chain has a unique stationary distribution, i.e., there exists a unique probability distribution w such that $wW = w$. Moreover, starting from any state i, the probability of being in any state j at date t converges to ω_j, as $t \to \infty$.

I now turn to the prospects of complete learning in different types of networks. To study this issue, it is useful to define θ as the true state, and let individual prior belief be $p_i^0 = \theta + \rho_i$, which is distributed i.i.d with mean equal to true state θ and variance given by $0 \leq \sigma^2 < \infty$. Under what circumstances will updated individual beliefs approach the truth as n gets large?

To get some intuition for this result, let us start with a consideration of society in which every individual assigns an equal weight to every other individual. If $\omega_{ij} = 1/n$ for every i and j, then at end of period 1, with n individuals, average belief is $p_j^{1,n} = \theta + 1/n \sum_{i \in N} \rho_i$, for all $j \in N$. It follows from the law of large numbers that this belief converges to the truth as n gets large. I now present an example taken from Golub and Jackson (2010), which illustrates how network structure matters for learning.

Example: Consider a star network in which all spokes only observe the center while the center observes all the spokes. For a society with $n = 4$ the matrix is given in Table 1:

Table 1

	1	2	3	4
1	1-δ	$\delta/3$	$\delta/3$	$\delta/3$
2	1-ε	ε	0	0
3	1-ε	0	ε	0
4	1-ε	0	0	ε

It is easy to show that for general n, with 1 at the center the social influence vector is:

$$w_1 = \frac{1-\varepsilon}{1-\varepsilon+\delta}; \quad w_j = \frac{\delta}{(n-1)(1-\varepsilon+\delta)}, \quad j \neq 1 \tag{9}$$

Finally, I observe that the limit belief is given by:

$$\theta + w_1\rho_1 + \frac{\delta}{(n-1)(1-\varepsilon+\delta)}\sum_{j=2}^{n}\rho_j \tag{10}$$

does not converge to θ generally, since $w_1\ \rho_1 \neq 0$, irrespective of n. ■

Let w_i^n be the limit social influence and p_i^n be the limit belief of person i in a network with n individuals. Golub and Jackson (2010) build on the above example to prove the following result.

Theorem 2.5 *Let* $\boldsymbol{W}_n$ *be a sequence of connected societies, as n varies, in which* $w_{ii} > 0$, *for all* $i \in N$. *Then*

$$\text{plim}_{n\to\infty}\sum_{i\in A_n} w_i^n p_i^n = \theta, \tag{11}$$

if and only if $\lim_{n\to\infty} \max_i w_i^n \to 0$.

This result shows that vanishing social influence is both necessary as well as sufficient in a context of naive learning to guarantee that complete learning obtains in a large society. This result also nicely complements our analysis of incomplete learning in the previous section. It shows how bounds on social influence are necessary for complete social learning.

2.3 Sequence of single choices

In the framework studied in Section 2.1, it was assumed that an individual observes all the actions as well as the outcomes of the actions of his direct neighbors. There is an extensive literature that has studied a simpler formulation in which individuals move only once but they learn by observing the history of past behavior. Broadly speaking, there are two approaches here. One, where individuals observe actions and outcomes from the past; here the actions generate information and so the friction arises when individuals stop choosing actions which are informative. Two, where individuals receive individual private signals but observe only the actions from the past; here the source of friction is that individual actions may not reflect private information.

Observation of past actions and outcomes: Consider a sequence of agents entering at discrete points in time $t = 1,2,3\ldots.$ Each agent enters with a prior belief, observes the entire history of actions and outcomes of actions of all past agents, updates his priors, and then chooses an action which is optimal with respond to the posterior beliefs. Do individual agents eventually learn the truth and choose the optimal action? I follow Bala and Goyal (1994, 1995) in the discussion below.

Given that there is only one agent at any point in time, I can drop the subscript i in the notation I developed in repeated action-learning model. As before let $\theta_1 \in \Theta$ be the true state. Suppose that prior beliefs at point of entry of agent are given by μ_t^0. Moreover, let these prior beliefs be drawn from some distribution $\mathcal{D}$ on the set of possible priors. I will suppose that D admits priors, which place point mass on any of the states $\theta \in \Theta$. I shall refer to this as the heterogeneous prior assumption. Agent t enters with prior belief μ_t^0, and updates this prior upon observing the history of past actions and outcomes using Bayes' Rule. Let $a_t = b(\mu_t)$ denote the action in period t, Z_t the outcome of this action, and let $U_t = u(a_t, \mu_t)$ be the expected utility with respect to his own action at time t. Given this notation the posterior beliefs of individual in period t are:

$$\mu_t(\theta|Z_t) = \frac{\prod_{t'=1}^{t-1}\phi(Z_{t'}; a_{t'}; \theta)\mu_t^0(\theta)}{\sum_{\theta'\in\Theta}\prod_{t'=1}^{t-1}\phi(Z_{t'}; a_{t'}; \theta)\mu_t^0(\theta')}. \tag{12}$$

The interest is in studying the evolution of individual actions, beliefs, and utilities, (a_t, μ_t, U_t), over time. Let δ_{θ_1} refer to a belief that the true state of the world is θ_1. Let $B(\delta_{\theta_1})$ be the set of optimal actions with this point mass belief. Let S_t be the number of times that an optimal action $x \in B(\delta_{\theta_1})$ has been chosen until time t. I shall say that actions converge in probability if for every $\varepsilon > 0$,

$$\lim_{t\to\infty} P(|1 - \frac{S_t}{t}| < \varepsilon) = 1. \tag{13}$$

The following result from Bala and Goyal (1995) characterizes long run learning.

Theorem 2.6 *Suppose that the distribution $\mathcal{D}$ respects the heterogeneity property. The sequence of actions a_t converges in probability to the set $G(\delta_{\theta_1})$. The utilities U_t converge in probability to $U(x, \delta_{\theta_1})$, where $x \in G(\delta_{\theta_1})$. Moreover, actions fail to converge to an optimal action, almost surely.*

First observe that heterogeneity is necessary for learning to obtain. To see why this is true consider the two arm bandit considered in Example 2.1 above. If every agent entered with the same prior belief (say) $\mu > 1/2$ there exists a finite sequence of 0's which will lead to posterior falling below the threshold 1/2. Once this happens, every agent will choose action 0, forever after. More generally, if priors of all agents are bounded above by a number $x < 1$ then a variant of the above argument can be used to demonstrate that there is a positive probability of incomplete learning. Second, note that if there is no upper bound on prior beliefs: but then for a period t, there exist a range of prior beliefs such that an agent with these beliefs will choose an action that is entirely independent of the history of past actions and outcomes. In other words, along almost all sample paths, the suboptimal action is chosen infinitely often. So actions fail to converge to optimal actions, almost surely.

However, as time goes by, and the informative action is chosen infinitely often, sufficient information on the true state is revealed: this implies that the set of beliefs which can be insensitive to past history shrinks. The theorem stated above tells us that the set of beliefs shrinks sufficiently quickly to ensure that there is convergence to optimal action in probability.

Observation of past actions: In many interesting economic contexts, there may be limited opportunity for direct communication and so individuals learn from others by observing their actions. Let us briefly sketch such a model. There is a single sequence of privately informed individuals who take one action each. Before making his choice an individual gets to observe the actions of all the people who have made a choice earlier. The actions of his predecessors *potentially* reveal their private information. An individual can therefore use the information revealed via the actions of others along with his own private information to make decisions. I will refer to this process as observational social learning. This model was introduced in Banerjee (1993) and Bikhchandani, Hirshleifer and Welch (1992); for a general treatment, see Smith and Sorensen (2000). An extensive literature has grown around this basic model. I will discuss some of the basic insights but a comprehensive survey is outside the scope of the present chapter.[13] The principal question is: do individuals eventually learn and choose the optimal action?

I first discuss the basic insight of the early papers by Banerjee (1993) and Bikhchandani, Hirshleifer and Welch (1992). Consider a setting in which private signals are equally accurate and individuals assign equal weight to their own and the signal of others. To fix ideas suppose that there are two actions and two states. For simplicity, suppose that in state 1 action 1 is optimal, while in state 0 action 0 is optimal. Suppose that initially agents believe that the states are equally likely. At point of entry, agent in period t, observes a private signal: probability of signal x when true state is x is q, where $q > 1/2$. The probability that signal is x when true state is $y \neq x$ is $1 - q < 1/2$. Assume that signals are drawn independently, conditional on the true state in every period. Now suppose that the first two individuals observe a signal in favor of state (and hence action) 1. They will both choose action 1. Consider agent 3 who observes this sequence of 1's. Given that the information from others is as accurate as his own,

[13] For an elegant survey of the research on observational learning see, Chamley (2004).

two signals in favor of state 1 will overrule his own signal in favor of action 0. So agent 3 will also choose action A, and this is his choice irrespective of his own signal. In that case, his action does not convey any information about his signal. In particular, agents 4 onward are in the same situation as agent 3. So they too will ignore their own private information and choose action 1. Thus the sequence of individuals may *herd* on action 1. Observe that this argument is independent of whether 1 is in fact an optimal action. So I have shown that there is a strictly positive probability that society may herd on the wrong action. Finally, observe that private signals arrive independently (and exogenously) over time: so eventually there will *always* be enough information to infer the optimal action. This illustrates how observational learning may fail to aggregate private information.

The discussion in the previous section suggests a possible way out of this inefficient herding: suppose agents draw signals that are heterogeneous and have different levels of accuracy. This will induce private beliefs which vary across agents. In particular, if some agents receive very 'strong' signals – signals which make one state much more likely as compared to the other state – then they may choose to ignore past observations and choose an action which reflects their private signal. Suppose private belief about state 0, given by μ_t^0 ranges between $\underline{\beta}$ and $\overline{\beta}$. I shall say that beliefs are bounded if there are numbers $\underline{\beta} > 0$ and $\overline{\beta} < 1$. Beliefs are unbounded if $\underline{\beta} = 1 - \overline{\beta} = 0$. Following our discussions after Theorem 2.6, it is easy to possible to verify that if agents have bounded beliefs then inefficient herding may occur, while if beliefs are unbounded then observational learning will lead to efficient choice of actions eventually; for a general analysis of this problem see Smith and Sorensen (2000).

In a recent paper, Acemoglu et al. (2010), introduce social networks in a single sequence setting with observational learning. They propose the following natural model. Suppose that agent at time t can draw a sample from the past, $N_t \subseteq \{1, 2, ..t-1\}$. Let this sample be drawn with some probability distribution $\mathcal{L}_t$. Some examples of such distributions are:

1. $L_t(\{1, 2, \ldots t-1\}) = 1$: this corresponds to the standard model in which every agent observes the entire past history of actions.
2. $L_t(\{t-1\}) = 1$: every agent observes only the immediately preceding agent.
3. L_t assigns equal probability to picking every subset of the past sequence of agents.

Acemoglu et al. (2010) obtain several interesting results with regard to asymptotic learning. I will focus on the setting with unbounded beliefs. Recall, that if observation window is the entire past history then the arguments above (from Bala and Goyal (1995) and Smith and Sorensen (2000) ensure that actions converge in probability to optimal actions. So the interest is in examining what is the minimum information needed to ensure learning.

Their result develops a sufficient condition on social networks which ensures that actions converge to optimal action in probability. A simple example illustrates the key idea: suppose that there is a positive probability that for all $t \geq 2$, $L_t(1) = p > 0$. Suppose that this agent Mr. 1 chooses action 1. Under the assumption of unbounded beliefs I know that any point there is a possibility of an agent with extremal signals and hence private beliefs which sharply favor one state over the other. But under our hypothesis there is a strictly positive probability that such an agent observes a single agent, Mr. 1, who has chosen action 1. It is then easy to see that this agent will choose an action that depends solely on his private signal. As beliefs arise independently over time and as observation neighborhoods are independent across agents, it follows that there is a strictly positive probability that agents will choose action in line with their private belief. This prevents asymptotic learning.

To rule this problem out, Acemoglu et al. (2010) develop the property of *expanding observations* in social networks. A social network is said to satisfy expanding observations if for all $k \in \mathcal{N}$,

$$\lim_{t \to \infty} L_t(\max_{b \in N_t} b < k) = 1. \tag{14}$$

If the network does not satisfy this property then it is has nonexpanding observations. Expanding observations rules out the example discussed above in which every agent samples agent 1 with strictly positive probability. The following theorem shows expanding an observation suffices to ensure asymptotic learning.

Theorem 2.7 *Assume that beliefs satisfy unbounded private beliefs property and network L_t satisfies expanding observations. Then actions converge in probability to the optimal action.*

The proof of this result rests on a set of arguments. First, the authors establish a generalized improvement principal. Suppose every agent t gets to observe one person from the past: then there is a strict increase in the probability of Mr. t making the correct choice, as compared to the person he observes. This argument builds on the 'welfare improvement' principle in Banerjee and Fudenberg (2004) and the 'imitation' principle across neighbors in Bala and Goyal (1998). The second step shows that this improvement principle can be extended to allow for multiple observations. The third step exploits expanding observations to infer that later agents will have access to new information and so the expected utilities must converge to the maximum possible value, i.e., actions must converge to the optimal one.

It is worth discussing the relationship between Example 2.2, Theorem 2.3 and Theorem 2.7. Note that the key obstacle to complete learning in the repeated action setting is the asymmetry in observation: there is a small group of agents who observe few others but are observed by everyone. In Theorem 2.7 by contrast, the expanding observations property of social networks ensures that agents eventually assign zero probability on any fixed set of early agents. This ensures that new information arrives into the system and ensures long run learning.

3. STRATEGIC INTERACTION

In this section, I will study situations in which the payoffs to an individual depend on his own action as well as the action of others players. The choice of actions of others is imperfectly known and this creates uncertainty about the payoffs that an individual can hope to earn. As in the previous sections, time proceeds at discrete points; at every point, an individual gets an opportunity to choose an action, with some probability. In choosing her action, she takes into account the past history of actions of others, forms some view of how others will move in the current period, and then chooses an action which is 'optimal' for herself. The set of actions taken together defines the profile of actions for the current period. This in turn shapes the behavior of actions over time. The focus will be on the relation between the network of interaction between individuals and the dynamics of actions and utilities. I will consider two classical types of games: games of coordination and games of cooperation. I start with the former.

3.1 Coordination games

It is useful to start with a description of a two action coordination game among two players. Denote the players by 1 and 2 and the possible actions by α and β. The rewards to a player depend on her own action and the action of the other player. These rewards are summarized in the following pay-off matrix.

Table 2

1 \ 2	α	β
α	a, a	d, e
β	e, d	b, b

At the heart of coordination games are two basic ideas: one, there are gains from individuals choosing the same action; and two, rewards may differ depending on which actions the two players coordinate on. These considerations motivate the following restrictions on the payoff parameters.

$$a > d; b > d; d > e; a + d > b + e. \tag{15}$$

These restrictions imply that there are two (pure strategy) Nash equilibria of the game: $\{\alpha, \alpha\}$ and $\{\beta, \beta\}$ and that coordinating on either of them is better than not coordinating at all.[14] The assumption that $a + d > b + e$ implies that if a player places equal probability on her opponent playing the two actions then it is strictly better for her to choose α. In other words, α is the *risk-dominant* action in the sense of Harsanyi and Selten (1988). It is important to note that α can be risk-dominant even if it is not efficient (that is even if $b > a$). Indeed, one of the important considerations in the research to date has been relative salience of riskiness versus efficiency. Given the restrictions on the payoffs, the two equilibria are *strict* in the sense that the best response

[14] In principle, players can want to coordinate on action combinations $\{\alpha, \beta\}$ or $\{\beta, \alpha\}$; games with such equilibria may be referred to as anti-coordination games. See Bramoulle (2007) for a study of network effects in this class of games.

in the equilibrium yields a strictly higher payoff than the other option. It is well known that strict equilibria are robust to standard refinements of Nash equilibrium (see e.g., Van Damme, 1991).

The focus of the analysis is on the effects of local interactions. I study local interaction in terms of neighborhoods within a model of networks. Suppose, as before, that the $N = \{1, 2, \ldots, n\}$ players are located on the nodes of an undirected network $g \in \mathcal{G}$, where $\mathcal{G}$ is the set of all possible undirected networks on n nodes. I will assume that a player i plays the coordination game with each of her neighbors. Recall that $N_i(g) = \{j \in N | g_{i,j} = 1\}$ refers to the set of players with whom i is linked in network g. Three networks of interaction – the complete network, the star and local interaction among players located around a circle – will be extensively used in this chapter. For easy reference they are presented in Figure 5.

As before, s_i denotes the strategy of player i and $S_i = \{\alpha, \beta\}$ the strategy set.[15] Let $S = \prod_{i \in N} S_i$ denote the set of all strategy profiles in the game and let s refer to a typical member of this set. In the two player game, let $\pi(x, y)$ denote the payoffs to player i when this player chooses action x, while her opponent chooses action y. The payoffs to a player i in network g, from a strategy s_i, given that the other players are choosing s_{-i} are:

$$\prod_i (s_i, s_{-i} \,|\, g) = \sum_{j \in N_i(g)} \pi(s_i, s_j) \tag{16}$$

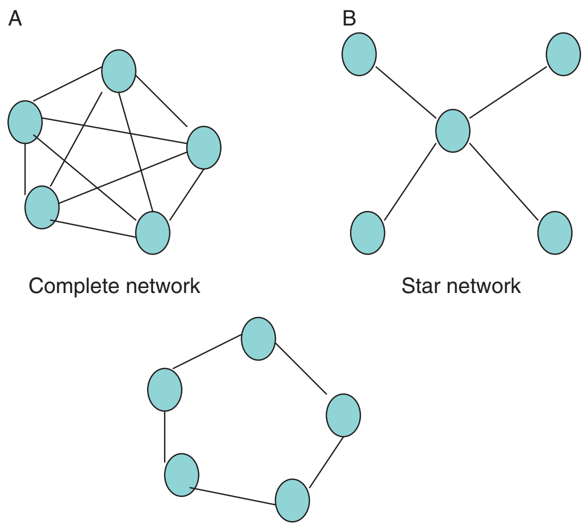

Figure 5 Simple networks.

[15] In the present model, individual choices refer to languages, word processing packages, norms for driving etc. For an influential formulation in which individuals choose where to locate on a network and their payoffs depend on who locates next to them, see Schelling (1975).

This formulation reflects the idea that a player i interacts with each of the players in the set $N_i(g)$. The players N, their interactions summarized by a network g, the set of actions for each player $S_i = \{\alpha, \beta\}$, and the payoffs 16 (where $\pi(x, y)$ satisfies 15) together define a social coordination game.

3.1.1 Multiple equilibria

This section starts by describing some Nash equilibria that can arise under different network structures. First, note that the strategy profile $s_i = x$, for all $i \in N$, where $x \in \{\alpha, \beta\}$ is a Nash equilibrium for every possible network structure. This is easily checked given the restrictions on the payoffs. Are there other types of equilibria which display diversity of actions and how is their existence affected by the structure of interaction? To get a sense of some of the forces driving conformism and diversity, it is useful to consider a class of societies in which there are several groups and intra-group interaction is more intense as compared to inter-group interaction. Figure 6 presents network structures with two groups that capture this idea. The number of cross-group links reflect the level of integration of the society.

Simple calculations reveal that equilibria with a diversity of actions are easy to sustain, in societies with low levels of integration, but that such equilibria cannot be sustained in the fully integrated society, i.e., in the complete network.

The above observations are summarized in the following result.

Theorem 3.1 *Consider a social coordination game. A strategy profile in which everyone plays the same action is a Nash equilibrium for every network $g \in \mathcal{G}$. If the network is complete, then these are the only possible Nash equilibria. If the network is incomplete then there may exist equilibria with a diversity of actions as well.*

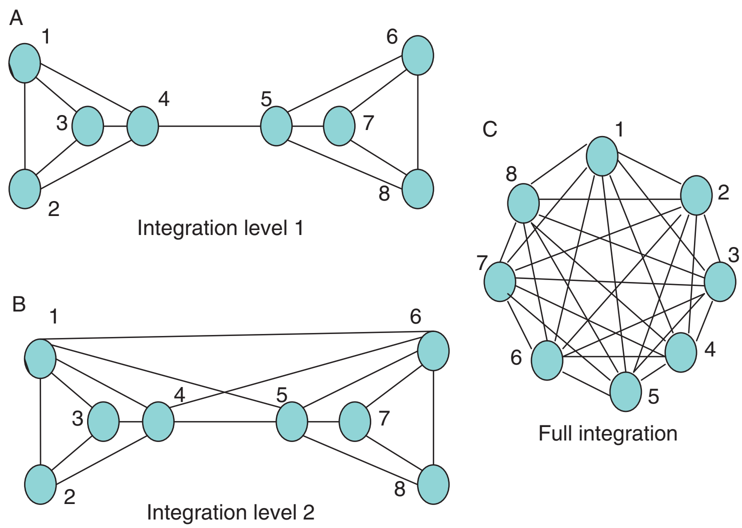

Figure 6 Levels of network integration.

This result yields two insights: one, multiple equilibria with social conformism exist for every possible network of interaction, and two, social diversity can arise in equilibrium, but the possibility of such outcomes depends on the network architecture. The result does raise a natural question: does diversity become less likely as I add links to a network? The following example illustrates why this may not be true in general.

Example 3.1 *Density of networks and diversity of actions*

Consider a star network with 8 nodes. Let node 1 be the central node. Observe that in the star network, any equilibrium must involve conformism: this is because in equilibrium every periphery node must choose the same action as the central node. Now supplement this network by adding three links between three peripheral nodes 1, 2 and 3, such that they now constitute a complete (sub) network. Now it is easy to see that if the payoffs from the two actions are similar then there is an equilibrium in which these three nodes choose action β, while the rest of the players all choose action α. Thus adding links to a network can sometimes facilitate the emergence of action diversity. Also note that the outcome with a single action remains an equilibrium in the new network. △

These observations lead to an examination of the robustness of different equilibria and how this is in turn related to the structure of interaction.

3.1.2 Dynamic stability and equilibrium selection

Assume that time is a discrete variable and indexed by $t = 1,2,3, \ldots$ In each period, with probability $p \in (0,1)$, a player gets an opportunity to revise her strategy. Faced with this opportunity, player i chooses an action which maximizes her payoff, under the assumption that the strategy profile of her neighbors remains the same as in the previous period. If more than one action is optimal then the player persists with the current action. Denote the strategy of a player i in period t by s_i^t. If player i is not active in period t then set $s_i^t = s_i^{t-1}$. This simple best-response strategy revision rule generates, for every network g, a transition probability function, $P_g(ss') : S \times S \to [0, 1]$, which governs the evolution of the state of the system s^t. A strategy profile (or state), s, is absorbing if the dynamic process cannot escape from the state once it reaches it, i.e., $P_g(ss) = 1$. The interest is in the relation between absorbing states and the structures of local interaction.

The first step in the analysis is the following convergence result and the characterization of the limiting states.

Theorem 3.2 *Consider a social coordination game. Starting from any initial strategy profile, s^0, the dynamic process s^t converges to an absorbing strategy profile in finite time, with probability 1. There is an equivalence between the set of absorbing strategy profiles and the set of Nash equilibria of the static social game.*[16]

[16] The relation between the Nash equilibria of the social coordination game and the equilibria of the original 2×2 game has been explored in Mailath, Samuelson and Shaked (1997). They show that the Nash equilibria of the static social game is equivalent to the set of correlated equilibria of the 2×2 game. Ianni (2001) studies convergence to correlated equilibria under myopic best response dynamics.

The equivalence between absorbing states and Nash equilibria of the social game of coordination is easy to see. The arguments underlying the convergence result are as follows: start at some state s_0. Consider the set of players who are not playing a best response. If this set is empty then the process is at a Nash equilibrium profile and this is an absorbing state of the process, as no player has an incentive to revise her strategy. Therefore, suppose there are some players who are currently choosing action α but would prefer to choose β. Allow them to choose β, and let s_1 be the new state of the system (this transition occurs with positive probability, given the decision rules used by individuals). Now examine the players doing α in state s_1 who would like to switch actions. If there are some such players then have them switch to β and define the new state as s_2. Clearly, this process of having the α players switch will end in finite time (since there are a finite number of players in the society). Let the state with this property be $\hat{s}$. Either there will be no players left choosing α or there will be some players choosing α in $\hat{s}$. In the former case the process is at a Nash equilibrium. Consider the second possibility next. Check if there are any players choosing β in state $\hat{s}$, who would like to switch actions. If there are none then the process is at an absorbing state. If there are some β players who would like to switch then follow the process as outlined above to reach a state in which there is no player who wishes to switch from β to α. Let this state be denoted by $\bar{s}$. Next observe that no player who was choosing α (and did not want to switch actions) in $\hat{s}$ would be interested in switching to β. This is true because the game is a coordination game and the set of players choosing α has (weakly) increased in the transition from $\hat{s}$ to $\bar{s}$. Hence the process has arrived (with positive probability) at a state in which no player has any incentive to switch actions. This is an absorbing state of the dynamics; since the initial state was arbitrary, and the above transition occurs with positive probability, the theory of Markov chains says that the transition to an absorbing state will occur in finite time, with probability 1.

An early result on convergence of dynamics to Nash equilibrium in regular networks (where every player has the same number of neighbors) is presented in Anderlini and Ianni (1996). In their model, a player is randomly matched to play with one other player in her neighborhood. Moreover, every player gets a chance to revise her move in every period. Finally, a player who plans to switch actions can make an error with some probability. They refer to this as noise on the margin. With this decision rule, the dynamic process of choices converges to a Nash equilibrium for a class of regular networks. The result presented here holds for all networks and does not rely on mistakes for convergence. Instead, the above result exploits inertia of individual decisions and the coordination nature of the game to obtain convergence.

Theorem 3.2 shows that the learning process converges. This result also says that every Nash equilibrium (for a given network of interaction) is an absorbing state of the process. This means that there is no hope of selecting across the variety of equilibria identified earlier in Proposition 3.1 with this dynamic process. This finding motivates a study of relative stability of different equilibria. There are a number of different approaches which have been adopted in the literature. I will examine the robustness with respect to small but repeated perturbations, i.e., stochastic stability of outcomes. For a general study of dynamic stability of equilibria, see Samuelson (1997) and Young (1998).

The ideas underlying stochastic stability can be informally described as follows. Suppose that s and s' are the two absorbing states of the best-response dynamics described above. Given that s is an absorbing state, a movement from s to s' requires an error or an experiment on the part of one or more of the players. Similarly, a movement from s' to s requires errors on the part of some subset of players. I will follow standard practice in this field and refer to such errors/experiments as *mutations*. The state s is said to be stochastically stable if it requires relatively more mutations to move from s to s' as compared to the other way around. If it takes the same number of mutations to move between the two states, then they are both stochastically stable.

Formally, suppose that, occasionally, players make mistakes, experiment, or simply disregard payoff considerations in choosing their strategies. Assume that, conditional on receiving a revision opportunity at any point in time t, a player chooses her strategy at random with some small *mutation* probability $\varepsilon > 0$. Given a network g, and for any $\varepsilon > 0$, the mutation process defines a Markov chain that is a periodic and irreducible and, therefore, has a unique invariant probability distribution; denote this distribution by μ_g^ε.[17] The analysis will study the support of μ_g^ε as the probability of mistakes becomes very small, i.e., as ε converges to 0. Define $\lim_{\varepsilon \to 0} \mu_g^\varepsilon = \hat{\mu}_g$. A state s is said to be *stochastically stable* if $\hat{\mu}_g(s) > 0$. This notion of stability identifies states that are relatively more stable with respect to such mutations.[18] I will now examine the effects of the network of interaction, g, on the set of stochastically stable states. I will consider the complete network, local interaction around the circle, and the star network.

Example 3.2 *The complete network.*

This example considers the complete network in which every player is a neighbor of every other player. Suppose that player 1 is deciding on whether to choose α or β. It is easy to verify that at least $k = (n-1)(b-d)/[(a-e)+(b-d)]$ players need to choose α, for α to be optimal for player 1 as well. Similarly, the minimum number of players needed to induce player 1 to choose β is given by $l = (n-1)(a-e)/[(a-e)+(b-d)]$. Given the assumption that $a + d > b + e$ it follows that $k < n/2 < l$. If everyone is choosing α then it takes l mutations to transit to a state where everyone is choosing β; likewise, if everyone is choosing β then it takes k mutations to transit to a state where everyone is choosing α. From the general observations on stochastic

[17] This follows from standard results in the theory of Markov chains, see e.g., Billingsley (1985).

[18] These ideas have been applied extensively to develop a theory of equilibrium selection in game theory. The notion of stochastic stability was introduced into economics by Kandori, Mailath and Rob (1993) and Young (1993).

stability above, it then follows that in the complete network, everyone choosing the risk-dominant action α is the unique stochastically stable outcome. ■

Example 3.3 *Local interaction around a circle.*

This example considers local interaction with immediate neighbors around a circle and is taken from Ellison (1993). Suppose that at time $t-1$ every player is choosing β. Now suppose that two adjacent players i and $i+1$ choose action α at time t, due to a mutation in the process. It is now easy to verify that in the next period, $t+1$ the immediate neighbors of i and $i+1$, players $i-1$ and $i+2$ will find it optimal to switch to action α (this is due to the assumption that α is risk-dominant and $a+d > b+e$). Moreover, in period $t+2$ the immediate neighbors of $i-1$ and $i+2$ will have a similar incentive, and so there is a process under way which leads to everyone choosing action α, within finite time. On the other hand, if everyone is choosing α then $n-1$ players must switch to β to induce a player to switch to action β. To see why this is the case, note that a player bases her decision on the actions of immediate neighbors, and so long as at least one of the neighbors is choosing α the optimal action is to choose α. It then follows that everyone choosing the risk-dominant action α is the unique stochastically stable state. ■

The simplicity of the above arguments suggests the following conjecture: the risk-dominant outcome obtains in all networks. This conjecture is false as the following example illustrates.

Example 3.4 *The star network.*

This example considers interaction on a star; recall that a star is a network in which one player has links – and hence interacts – with all the other $n-1$ players, while the other players have no links between them. This example is taken from Jackson and Watts (2002). Suppose that player 1 is the central player of the star network. The first point to note about a star network is that there are only two possible equilibrium configurations, both involving social conformism. A study of stochastically stable actions therefore involves a study of the relative stability of these two configurations. However, it is easily verified that in a star network a perturbation that switches the action of player 1 is sufficient to get a switch of all the other players. Since this is also the minimum number of mutations possible, it follows that both states are stochastically stable! ■

Examples 3.2–3.4 show that network structure has an important bearing on the nature of stochastically stable states. They also raise two types of questions. The first question pertains to network structure. Is it possible to identify general features of networks that sustain conformism and diversity, respectively, and also if some networks favor one type of conformism while other networks facilitate a different type of conformism? The second question relates to the decision rules. Is the role of interaction structures sensitive to the formulation of decision rules and the probability of mutations? The first question appears to be an open one;[19] Section 3.1.3 takes up the second question.

[19] There is a small experimental literature on coordination games which studies specific structures of local interaction; see e.g., Cassar (2007), Berninghaus et al. (2002).

I conclude this section by discussing the effects of the network of interaction on the rates of convergence of the dynamic process. From a practical point of view, the invariant distribution $\hat{\mu}_g$ is only meaningful if the rate of convergence of the dynamics is relatively quick. In the above model, the dynamics are Markovian and if there is a unique invariant distribution then standard mathematical results suggest that the rate of convergence is exponential. In other words, there is some number $\rho < 1$ such that the probability distribution of actions at time t, σ^t, approaches the invariant distribution σ^* at a rate approximately given by ρ^t. While this result is helpful, it is easy to see that this property allows a fairly wide range of rates of convergence, depending on the value of ρ. If ρ is close to 1 then the process is essentially determined by the initial configuration σ_0 for a long period, while if ρ is close to 0 then initial conditions play a less important role and dynamics shape individual choices quickly. The work of Ellison (1993) directed attention to the role of interaction structure in shaping the rate of convergence. He argued that in a complete network transition between strict Nash equilibria based on mutations would take a very long time in large populations since the number of mutations needed is of the order of the population. By contrast, as Example 3.3 showed under local interaction around a circle, a couple of mutations (followed by best responses) are sufficient to initiate a transition to the risk-dominant action. Thus local interaction leads to dramatically faster rates of convergence to the risk-dominant action.[20]

3.1.3 Related themes

The study of social coordination with local interaction has been a very active field of research and a number of themes have been explored. This section discusses two strands of this work. One, I consider other decision rules and two, I discuss the implications of different initial configurations.

Alternative Decision Rules: In the discussion above, I started with a myopic best response decision rule. I then complemented it with small but persistent mutations and looked at what happens as the probability of mutations becomes small. I now discuss alternatives to the best response rule with equiprobable mutations. A first step in this exercise is to consider an alternative formulation of decision rules in which individual experimentation is more sensitive to payoff losses. In any period t, an individual i located in network g is drawn at random and chooses (say) α according to a probability distribution, $p_i^{\gamma}(\alpha|s^t, g)$, where $\gamma > 0$ and s^t is the strategy profile at time t.

$$p_i^{\gamma}(\alpha|s^t, g) = \frac{e^{\gamma \Pi_i(\alpha, s_{-i}^t|g)}}{e^{\gamma \Pi_i(\alpha, s_{-i}^t|g)} + e^{\gamma \Pi_i(\beta, s_{-i}^t|g)}} \tag{17}$$

[20] While local interaction has dramatic effects on rates of convergence it is worth observing that it entails repeated interaction among neighbors. Repeated interaction however makes the bounded and myopic rational rules of behavior in the dynamic models less plausible.

This is referred to as the log-linear response rule; in the context of coordination games, this rule was first studied by Blume (1993). Note that for large values of γ the probability distribution will place most of the probability mass on the best response action. Define $\Delta_i(s\,|\,g) = \Pi_i(\beta, s_{-i}\,|\,g) - \Pi_i(\alpha, s_{-i}\,|\,g)$. Then for large γ the probability of action α is:

$$p_i^{\gamma}(\alpha|s^t, g) = \frac{e^{-\gamma\Delta_i(s^t|g)}}{1 + e^{-\gamma\Delta_i(s^t|g)}} \cong e^{-\gamma\Delta_i(s^t|g)}. \tag{18}$$

This expression says that the probability of not choosing the best response is exponentially declining in the payoff loss from the deviation. The analysis of local learning in coordination games when individuals use the log-linear decision rule is summarized in the following result, due to Young (1998).

Theorem 3.3 *Consider a social coordination game on a connected network g. Suppose that in each period one individual is picked at random to revise choices. In revising choices this individual uses the log-linear response rule. Then the stochastically stable outcome is a state in which every player chooses the risk-dominant action.*

This result tells us that if the mutation probabilities are payoff sensitive in a strong form – the probability of choosing an action is exponentially declining in payoff losses associated with it – then the network structure has no effects on the long run distribution of actions. To get some intuition for the result it is useful to discuss the dynamic process in the star network. In that example, the simplest way to get a transition is via a switch in the action of the central player. In the standard model, with payoff insensitive mutations, the probability of the central player making a switch from α to β is the same as the other way around. By contrast, under the log-linear response rule, matters are very different. If there are many peripheral players, then there is a significant difference in the payoff losses involved and the probability of switching from α to β is significantly smaller than the probability of switching from β to α. This difference is crucial for obtaining the above result.

The mutation structure has been the subject of considerable research over the years. In an influential paper, Bergin and Lipman (1996) showed that any outcome could be supported as stochastically stable if the order of magnitudes for mutations was different across different actions. This 'anything is possible' result has provoked several responses and two of them are worth discussing here. The first response interprets mutations as errors, and says that these errors can be controlled at some cost. This argument has been developed in van Damme and Weibull (2002). This paper shows that incorporating this cost structure leads back to the risk-dominant equilibrium. This line of research has been further extended to cover local interaction on general weighted graphs by Baron, Durieu, Haller, and Solal (2002). A second response is to argue that risk-dominance obtains for any possible mutation rule, if some additional conditions are satisfied. In this vein, a recent paper by Lee, Szeidl and Valentinyi (2003) argues that given any state dependent mutation process, under local interaction on a 2 dimensional torus the dynamics select for the risk-dominant action, provided the number of players is sufficiently large.

I turn next to a consideration of decision rules that are different in a more fundamental way from the best response principle. A simple and widely studied rule of thumb is *imitation: choose an action that yields the highest payoffs, among all the actions that are currently chosen by all others.*[21] Robson and Vega-Redondo (1996) study this rule in the context of social coordination games, and show that, taken together with random matching, it leads to the efficient action being the unique stochastically stable action.

Role of initial configuration: In the discussion of the dynamics above, no restrictions were placed on the initial configuration of actions. A number of papers have examined the nature of dynamics under restrictions on initial configuration. Two approaches are discussed here as they illustrate quite different types of restrictions on initial conditions. The first one uses a random process to determine the initial configuration: every individual independently chooses an action with some probability. This random choice determines the starting point of the dynamic process. Lee and Valentinyi (2000) study the spread of actions on a 2-dimensional lattice starting with this random assignment of initial actions. They show that if individuals use the best-response rule then all players choose the risk-dominant action eventually.

The second approach considers the following problem: If I start with a small group of players choosing an action, what are the features of the network that allow for this behavior to be taken up by the entire population? Goyal (1996) consider diffusion in the context of specific networks; Morris (2000) shows that maximal contagion occurs when local interaction is sufficiently uniform and there is slow neighbor growth, i.e., the number of players who can be reached in d steps does not grow exponentially in d.

Endogenous networks: So far I have assumed that the network of interaction is fixed and given. In this section, I briefly discuss some work which endogenizes the network. I present a simple model in which players choose their partners and choose an action in the coordination games they play with their different partners. This framework allows us to endogenize the nature of interaction and study the effect of partner choice on the way players coordinate their actions in the coordination game. The issue of endogenous structures of interaction on coordination was explored in early papers by Ely (2002), Mailath, Samuelson and Shaked (1997), and Oechssler (1997). They use a framework in which players are located on islands. Moving from an island to another implies severing all ties with the former island and instead playing the game with all the players in the new island. Thus, neighborhoods are made endogenous via the choice of islands. In my exposition here, I will follow the approach of Droste, Gilles and Johnson (1999), Goyal and Vega-Redondo (2005), and Jackson and Watts (2002), in which individuals create links and thereby shape the details of the network of interaction.

[21] This rule is also used in the study of altruism in Section 3.2.

As before I shall suppose that $N = \{1, 2, \ldots, n\}$ is the set of players, where $n \geq 3$. Each player has a strategy $s_i = \{g_i, a_i\} \in \mathcal{G}_i$, where g_i refers to the links that she forms while $a_i \in A_i$ refers to the choice of action in the accompanying coordination game. Any profile of link decisions $g = (g_1, g_2 \ldots g_n)$ defines a directed network. Given a network g, I say that a pair of players i and j are directly linked if at least one of them has established a linked with the other one, i.e. if $\max\{g_{ij}, g_{ji}\} = 1$.[22] To describe the pattern of players' links, I shall take recourse to our earlier notation and define $\hat{g}_{ij} = max\{g_{ij}, g_{ji}\}$ for every pair i and j in N. I refer to g_{ij} as an active link for player i and a passive link for player j. I will say that a network g is essential if $g_{ij}g_{ji} = 0$, for every pair of players i and j. Also, let $G^c(M) \equiv \left\{g : \forall i, j \in M, \hat{g}_{ij} = 1, g_{ij}g_{ji} = 0\right\}$ stand for the set of complete and essential networks on the set of players M. Given any profile $s \in S$, I shall say that $s = (g, a) \in S^h$ for some $h \in \{\alpha, \beta\}$ if $g \in G^c$ and $a_i = h$ for all $i \in N$.

Every player who establishes a link with some other player incurs a cost $c > 0$. Given the strategies of other players, $s_{-1} = (s_1, \ldots s_{i-1}, s_{i+1}, \ldots s_n)$, the payoffs to a player i from playing some strategy $s_i = (g_i, a_i)$ are given by:

$$\Pi_i(s_i, s_{-i}) = \sum_{j \in N^d(i;\hat{g})} \pi(a_i, a_j) - \mu^d(i; g) \cdot c \tag{19}$$

The following result, due to Goyal and Vega-Redondo (2005), provides a complete characterization of stochastically stable social networks and actions in the coordination game.

Theorem 3.4 *Suppose (15) holds and $b > a$. There exists some $\bar{c} \in (e, a)$ such that if $c < \bar{c}$ then the long run network is complete while all players choose action α, while if $\bar{c} < c < b$ then the long run network is complete and all players choose action β. Finally, if $c > b$ then the long run network is empty and actions are undetermined.*

This result illustrates that the *dynamics* of link formation play a crucial role in the model. I observe that the only architecture that is stochastically stable (within the interesting parameter range) is the complete one, although players' behavior in the coordination game is *different* depending on the costs of forming links. However, if the network were to remain fixed throughout, standard arguments indicate that the risk-dominant action must prevail in the long run (cf. Kandori, Mailath and Rob, 1993). Thus, it is the link formation *process* that, by allowing for the *co*-evolution of the links and actions, shapes individual behavior in the coordination game.

I now briefly provide some intuition on the sharp relationship found between the *costs* of forming links and the corresponding behavior displayed by players in the coordination game. On the one hand, when the cost of forming links is small, players wish to be linked with everyone irrespective of the actions they choose. Hence, from an individual perspective, the relative attractiveness of different actions is quite *insensitive* to what is the

[22] This approach to link formation builds on the work of Goyal (1993) and Bala ad Goyal (2000).

network structure faced by any given player at the time of revising her choices. In essence, a player must make her fresh choices as if she were in a complete network. In this case, therefore, the risk-dominant (and possibly) inefficient convention prevails since, under complete connectivity, this convention is harder to destabilize (through mutations) than the efficient but risk-dominated one. By contrast, if costs of forming links are high, individual players choose to form links only with those who are known (or perceived) to be playing the same action. This lowers the strategic uncertainty in the interaction and thus facilitates the emergence of the efficient action.

I conclude by discussing the relation between the above results and the earlier work of Ely (2002), Mailath, Samuelson and Shaked (1997), Oechssler (1997), and Bhaskar and Vega-Redondo (2004). The basic insight flowing from the earlier work is that, if individuals can easily separate/insulate themselves from those who are playing an inefficient action (e.g., the risk-dominant action), then efficient "enclaves" will be readily formed and eventually attract the "migration" of others who will adopt the efficient action eventually. One may be tempted to identify *easy* mobility with *low* costs of forming links. However, the considerations involved in the two approaches turn out to be very different. This is evident from the differences in the results: recall that in the network formation approach, the risk-dominant outcome prevails if the costs of forming links are small. There are two main reasons for this contrast. First, in the network formation approach, players do not *indirectly* choose their pattern of interaction with others by moving across a *pre-specified* network of locations (as in the case of player mobility). Rather, they construct *directly* their interaction network (with no exogenous restrictions) by choosing those agents with whom they want to play the game. Second, the cost of link formation is paid per link formed and thus becomes truly effective only if it is high enough. Thus it is precisely the restricted "mobility" that high costs induce which helps insulate (and thus protect) the individuals who are choosing the efficient action. If the costs of link formation are low, then the extensive interaction this facilitates may have the unfortunate consequence of rendering risk-dominance considerations decisive.

3.2 Games of cooperation

This section studies effects of interaction structure in situations where incentives of individuals are in conflict with socially desirable outcomes. Potential conflict between incentives of individuals and social desirable outcomes is clearly an important dimension of social and economic life and the importance of this problem has motivated an extensive literature on the evolution of social norms. This literature spans the fields of biology, computer science, philosophy, and political science, in addition to economics.[23] However, it seems that few analytical results with regard to the effects of network

[23] For a survey of work see e.g., Ullman-Margalit (1977), Axelrod (1997), Nowak and May (1992).

structure on social cooperation have been obtained. This presentation here will be based on Eshel, Samuelson, and Shaked (1998).

The provision of local public goods illustrates the economic issues very well: every individual contributes to an activity and all the individuals in her neighborhood benefit from it.[24] Suppose there are $N = \{1, 2 \ldots, n\}$ players (where n is assumed to be large) and each player has a choice between two actions C (contribute) and D (defect or not contribute). Let $s_i = \{C, D\}$ denote the strategy of player i and as usual let $s = \{s_1, s_2, s_3, \ldots, s_n\}$ refer to the strategy profile of the players. Define $n_i\,(C, s_{-i}\,|g)$ as the number of neighbors of player i in network g who choose C given the strategy profile s_{-i}. The payoffs to player i, in network g, from choosing C, given the strategy profile s_{-i} are:

$$\Pi_i(C, s_{-i}|g) = n_i(C, s_{-i}|g) - e. \tag{20}$$

where $e > 0$ is the cost associated with the (contribution) action C. On the other hand, the payoffs to player i from action D are given by:

$$\Pi_i(D, s_{-i}|g) = n_i(C, s_{-i}|g). \tag{21}$$

Since $e > 0$, it follows that action D strongly dominates action C. So, if players are payoff optimizers then they will never choose C. Thus it is necessary to have at least some players using alternative decision rules if there is to be any chance of action C being adopted.

3.2.1 Imitation and altruism

Suppose that all players use an *imitate the best action* rule: compare the average payoffs from the two actions across the different members of the population and choose the action that attains the higher average payoff. Moreover, if everyone chooses the same action then payoffs across different actions cannot be compared and an individual persists with the current action.

As in the previous sections consider a dynamic model in which time is discrete and indexed by $t = 1, 2, \ldots$ Suppose that in each period a player gets a chance to revise her strategy with some probability $p \in (0,1)$. This probability is independent across individuals and across time. Let the strategy profile at time t be denoted by s^t. The above decision rule along with an initial action profile, s^1, define a Markov process where the states of the process are the strategy profiles s. The probability of transition from s to s' is either 0 or 1. Recall that a state (or a set of states) is said to be absorbing if the process cannot escape from the state once it is reached. The interest is in the relation between the interaction structure and the nature of the absorbing state (or set of states) of the dynamic process.

[24] Clearing the snow in front of one's house, having a night light on, and controlling the level of noise pollution are some everyday activities that fit the description of local public goods.

Consider first the complete network. If both actions are being chosen in a society then it follows from simple computations that the average payoffs from choosing D are larger than the payoffs from choosing C.[25] This means that the outcome in which everyone chooses action D will obtain. Thus *starting from any initial configuration (except the extreme case where everyone is doing C), the dynamic process will converge to a state in which everyone chooses D*. This negative result on the prospects of contribution under the complete network leads to an exploration of local interaction.

So consider next the contribution game with local interaction around a circle.[26] Let $N_i(g) = \{i-1, i+1\}$ be the neighborhood of player i and let this interaction network be denoted by g^{circle}. The payoffs to player i in g^{circle} are given by $n_i(C, s_{-i}|g^{\text{circle}}) - e$ if player i chooses C and by $n_i(C, s_{-i}|g^{\text{circle}})$ if she chooses action D.

In the following discussion, to focus attention on interesting range of costs, it will be assumed that $e < 1/2$. Suppose that there is a string of 3 players choosing action C, and they are surrounded on both sides by a population of players choosing D. Given the decision rule, any change in actions can only occur at the boundaries. What are the payoffs observed by the player choosing action C on the boundary? Well, she observes one player choosing D with a payoff of 1, while she observes one player choosing action C with payoff $2 - e$. Moreover, she observes her own payoff of $1-e$, as well. Given that $e < 1/2$, it follows that she prefers action C. On the other hand, the player on the boundary choosing action D, observes one player choosing action D, with payoff 0, one player choosing action C with payoff $1-e$ and herself with a payoff 1. Given that $e < 1/2$, she prefers to switch to action C. This suggests that the region of altruists will expand. Note however that if everyone except one player is choosing action C, then the player choosing D will get a payoff of 2, and since this is the maximum possible payoff, this will induce her neighbors to switch to action D. However, as they expand, this group of egoists will find their payoffs fall (as the interior of the interval can no longer free ride on the altruists). These considerations suggest that a long string of players choosing action C can be sustained, while a long string of players choosing D will be difficult to sustain. These arguments are summarized in the following result, due to Eshel, Samuelson, and Shaked (1998).

Theorem 3.5 *Consider the contribution game with local interaction and suppose that $e < 1/2$. Absorbing sets are of two types: one, they contain a singleton state in which all players choose either action C or action D, and two, they contain states in which there are strings of C players*

[25] Note that in a complete network, with a strategy profile where K players choose C, the average payoffs to a C player are $K - 1 - e$, while the average payoffs to a D player are K.

[26] Tieman, Houba and Van der Laan (2000) consider a related model of cooperative behavior with local interaction in games with conflict. In their model, players are located on a network and play a generalized (many action) version of the prisoner's dilemma. They find that with local interaction and a tit-for-tat type decision rule, superior payoff actions that are dominated can survive in the population, in the long run.

of length 3 or more which are separated by strings of D players of length 2. In the latter case, at least 60% of the players choose action C (on average).

It is worth commenting on the relative proportions of C players in mixed configurations. First note that a string of D players cannot be of size 3 or longer (in an absorbing state). If it is then the boundary D players will each have two players choosing C on one side and a player choosing D who is surrounded by D players. It is easy to show that these boundary players will switch to C. Likewise, there have to be at least 3 players in each string of C players; otherwise, the boundary players will switch to D. These considerations put together yield the proportions mentioned in the result above.

Given the above arguments, it is easily seen that in any string of five players at least three players will remain with action C, forever. If players strategies are randomly chosen initially then it follows that the probability of such a string of C players can be made arbitrarily close to 1, by suitably increasing the number of players. This idea is summarized in the following result, due to Eshel, Samuelson, and Shaked (1998).

Theorem 3.6 *Consider the contribution game with local interaction and suppose that $e < 1/2$. Suppose that players' initial strategy choices are determined by independent, identically distributed variables where the probability of C and D is positive. Then the probability of convergence to an absorbing set containing states with at least 60% of C players goes to 1, as n gets large.*

This result shows that with local interaction around a circle, in large societies a majority of individuals will contribute, in the long run.[27]

So far the discussion on network effects has been restricted to the case of pure local interaction among agents located on a circle. This raises the question: how likely is contribution in more general structures of interaction? In the case of pure local interaction around a circle, the persistence and spread of cooperative behavior appears to be related to the presence of C players who are protected from D players and therefore earn sufficiently high payoffs so that other C players on the boundary persist with action C as well. In higher dimensional interaction (e.g., k-dimensional lattices) or asymmetric interaction (as in a star), this protective wall can be harder to create and this may make altruism more vulnerable. The following example illustrates this.

Example 3.5 *Altruism in a star network.*

First note that mixed configurations in which some individuals choose C while others choose D are not possible in a star. Therefore, only the two pure strategy configurations – everyone choosing C or everyone choosing D – are possible in an absorbing state. What is the relative robustness of these two absorbing states? As in the previous section, let us examine the stochastic stability of the two states. It is possible to move from a purely altruistic society to a purely egoist society, via a switch by the central player, followed by imitation by the rest. The reverse transition requires

[27] Eshel, Samuelson and Shaked (1998) also study stochastic stability of different absorbing sets. They show that the states identified in the above proposition are also the stochastically stable ones.

switching of action by at least three players, the central player and two peripheral players. Thus if interaction is on a star network then all individuals will choose D and contribution will be zero, in the long run.

These arguments taken along with the earlier discussion on the pure local interaction model suggest that the structure of interaction has profound effects on the levels of contribution that can be sustained in a society. These observations also lead back to a general question posed in the introduction: Are some interaction structures better at sustaining contributions (and hence efficient outcomes) as compared to other interaction structure and what is the relation between interaction structures and diversity of actions?

Existing work on this subject seems to be mostly based on simulations with special classes of networks such as lattices and regular graphs (Nowak and May, 1992; Nowak and Sigmund, 2005).[28] This work suggests that in the absence of mutations, altruism can survive in a variety of interaction settings. There is also an extensive literature in evolutionary biology on the emergence and persistence of altruistic traits in different species. In this work the spread of altruistic traits is attributed to greater reproductive success. This success leads to the larger set of altruists spilling into neighboring areas and this in turn leads to a growth of the trait over time (see e.g., Wynne-Edwards, 1986; Eshel and Cavalli-Sforza, 1982).

So far we have taken as given the network of interaction. Ostracism is a natural way punishment strategy; thus, forming and dissolving links may be one way to sustain cooperation. For an early attempt at studying cooperation in networks, see Raub and Weesie (1990); for recent attempts at modeling endogenous networks and cooperation, see Vega-Redondo (2006), Fosco and Mengel (2008) and Ule (2008).

3.2.2 Interaction and information neighborhoods

In the discussion in the previous section, it was assumed that players observe those with whom they play, in other words the neighborhood of interaction coincides with the neighborhood of information. In this section, I will briefly explore the role of this assumption.[29] I will proceed by considering an example: As before, individuals are located around a circle and interact with their immediate neighbors. However, each individual observes her own action and payoffs and the actions and payoffs of a subset of the population drawn randomly from the population. Specifically, when faced with a chance to revise actions, an individual gets to observe actions of the set $I_{i,t}$; this set always includes $\{i\}$, and in addition includes a subset of $N \backslash \{i\}$. Suppose that the probability of drawing $I_{i,t}$ is $P(I_{i,t}) > 0$, for any $I_{i,t} \subset N \backslash \{i\}$. Assume that the draw of samples is independent across players and across time periods. I will refer to this is *local*

[28] There is also a small experimental literature that examines cooperative behavior in games with local interaction; see Cassar (2007) for an overview of this work.

[29] For recent work on information and interaction neighborhoods in the context of coordination games, see Alos-Ferrer and Weidenholzer (2008).

plus random information. If $j \in I_{i,t}$, then i gets to see $\{s_{j,t-1}, \Pi_{j,t-1}\}$. In period t, if she has a choice then player i will choose $s_{i,t} = C$ if C yields a higher average payoff in her sample of informants; else she will choose D. The above rules define a stationary Markov process, with state space $S = \{C, D\}^n$. Let $P_s\, s'(g)$ be the transition matrix, given a network of interaction, g. The following result, due to Goyal (2007b), summarizes the analysis of stochastically stable states.

Theorem 3.7 *Consider the contribution game. Suppose interaction is with immediate neighbors around a circle, there is local plus random information and players follow the best rule. Then universal defection is the unique stochastically stable outcome.*

Let us start with the ALL C state and suppose 1 player switches to D. She earns 2, and this is the maximum possible payoff attainable in this setting. Now get individuals outside $N_1(g) = \{-1, 2\}$, to choose an action, one by one. Suppose that for any player $j \notin N_1(g)$, $I_{j,t} = \{j, 1\}$. This player compares maximum possible payoff of $2 - e$ from action C with a payoff of 2 from D which player 1 earns. So she will switch to D. Iterate on players one at a time. Finally, suppose players -1 and 2 in $N_1(g)$ move. They compare payoff of $-e$ from action C with payoff 2 from action D earned by player 1. Clearly, they will switch to D as well. I have thus shown that a single mutation followed by standard imitation dynamics suffices for the transition to all D state.

Next start with an ALL D state. Clearly a single mutation to action C will have no effect. The individual compares $-e$ from action C with a payoff 0 or a positive payoff from D and so she will switch back to action D. Moreover, no player choosing D has any incentive to move to C. Hence, it takes two or more mutations to transit from an all D state. Similar arguments can be developed for any mixed state of actions. So I have shown that it takes relatively fewer mutations to arrive at the all D state as compared to other states with action C. Thus the state of universal defection is uniquely stochastically stable.

A comparison of Theorem 3.7 with Theorem 3.6 highlights the important role of information radius in sustaining cooperation. Recall, that if information and interaction radius are the same, then Eschel, Samuelson and Shaked (1998) show that are absorbing states with mixed action configurations and in such a mixed configuration at least 60% of players choose C. By contrast, if interaction is local around with a circle but players have access to information drawn at random from the population, then universal defection is the only possible outcome. This discussion points to the need for a more general systematic study of models in which the neighborhood of information is allowed to vary generally and its effects on long run outcomes studied.

4. CONCLUDING REMARKS

I have examined the following general framework: there is a set of individuals who are located on nodes of a network; the arcs of the network reflect relations between these individuals. At regular intervals, individuals choose an action from a set of alternatives.

They are uncertain about the rewards from different actions. They use their own past experience, as well as gather information from their neighbors (individuals who are linked to them) and then choose an action that maximizes individual payoffs.

I first studied the influence of network structure on individual and social learning in a pure information-sharing context. I then moved on to a study of strategic interaction among players located in a network, i.e., interactions where actions alter payoffs of others. The focus was on the relation between the network structure on the one hand, and the evolution of individual actions, beliefs, and payoffs on the other hand. A related and recurring theme of the survey was the relation between network structure and the prospects for the adoption of efficient actions.

The work to date provides a number of insights about how networks – connectedness, centrality, dispersion in connections – and decision rules together shape individual behavior and welfare. While much progress has been made, there remain a number of important open problems.

A first remark concerns the formation of networks. Most of the work on communication and learning uses a framework where the network is exogenously given. The systematic study of information sharing and learning in endogenously evolving networks is an important subject for further research.[30]

A second remark concerns the role of interaction neighborhood and information neighborhood. Social networks reflect patterns of interaction among individuals and they also serve as a conduit for the transmission of useful information in a society. Traditionally, interaction and information have been conflated and this has allowed for a parsimony in the details of modeling. However, interaction and information are distinct and have rather different implications. Moreover, in applications this distinction is natural and deserves more attention.

The third remark is about the formulation of individual decision-making rules in models of networks. Strategic considerations and indirect inferences quickly become very complicated in a network setting: so both descriptive plausibility and tractability motivate simple individual decision rules. However, existing results already suggest that the role of networks may be sensitive to the precise assumptions on individual decision rules. The network irrelevance property under log-linear response rule noted in Theorem 3.3. and the network effects under best-response rule identified in Example 3.4 illustrate this point. More work is clearly needed before we have a systematic understanding of the ways in which networks and different decision rules combine to shape behavior and welfare.

A fourth and final remark is about the context of social leaning. In many interesting applications, learning takes place within a context where firms and governments have

[30] For recent attempts at modeling endogenous formation of networks and groups in the context of communication and information sharing, see Acemoglu et al. (2010) and Baccara and Yariv (2010).

an active interest in facilitating/impeding the flow of information and certain behaviors. As we develop a more complete understanding of the 'pure' problem of learning and evolution in networks it is important to integrate these insights with economic models of pricing, advertising and market competition.[31]

5. APPENDIX

It is important to construct the probability space within which the processes relating to Bayesian social learning take place. Recall that the probability space is denoted by $(\Omega, \mathcal{F}, P^\theta)$, where Ω is the space of all outcomes, $\mathcal{F}$ is the σ-field and P^θ is a probability measure if the true state of the world is θ. Let Θ be the set of possible states of the world and fix $\theta \in \Theta$ in what follows. For each individual $i \in N$, action $a \in A$, and time periods $t = 1,2,\ldots$ let $Y^a_{i,t}$ be the set of possible outcomes. For each $t = 1,2..$ let $\Omega_t = \prod_{i\in N}\prod_{a\in A} Y^a_{i,t}$ be the space of t^{th} outcomes across all individuals and all actions. For simplicity, we will assume that $Y^a_{i,t} = Y$. Ω_t is endowed with the product topology. Let $H_t \subset \Omega_t$, be of the form

$$H_t = \prod_{i\in N}\prod_{a\in A} H^a_{i,t} \tag{22}$$

where $H^a_{i,t}$ is a Borel subset of Y, for each $i \in N$ and $a \in A$. Define the probability P^θ_t of the set H_t as:

$$P^\theta_t(H_t) = \prod_{i\in N}\prod_{a\in A}\int_{H^a_{i,t}} \phi(y; a, \theta)\,dy \tag{23}$$

where ϕ is the density of y, given action a and state θ. P^θ extends uniquely to the σ-field on Ω_t generated by the sets of the form H_t. Let $\Omega = \prod_{t=1}^{\infty}\Omega_t$. For cylinder sets $H \subset \Omega$, of the form

$$H = \prod_{t=1}^{T} H_T \times \prod_{t=T+1}^{\infty} \Omega_t \tag{24}$$

let P^θ (H) be defined as $P^\theta(H) = \prod_{t=1}^{T} P^\theta_t(H_t)$. Let $\mathcal{F}$ be the σ-field generated by sets of the type given by (24). P^θ extends uniquely to the sets in $\mathcal{F}$. This completes the construction of the probability space $(\Omega, \mathcal{F}, P^\theta)$.

Let Θ be endowed with the discrete topology, and suppose $\mathcal{B}$ is the Borel σ–field on this space. For rectangles of the form $\mathcal{T} \times H$, where $\mathcal{T} \subset \Theta$, and H is a measurable subset of Ω, let $P_i(\mathcal{T} \times H)$ be given by

[31] For recent efforts in this direction, see Chatterjee and Dutta (2010), Colla and Mele (2010), Galeotti and Goyal (2009).

$$P_i(\mathcal{T} \times H) = \sum_{\theta \in \Theta} \mu_{i,1}(\theta) P^{\theta}(H). \tag{25}$$

for each individual $i \in N$. Each P_i extends uniquely to all $\mathcal{B} \times \mathcal{F}$. Since every individual's prior belief lies in the interior of $\mathcal{P}(\Theta)$, the measures $\{P_i\}$ are pairwise mutually absolutely continuous.

The σ-field of individual i's information at the beginning of time 1 is $\mathcal{F}_{i,1} = \{\emptyset, \Theta \times \Omega\}$. For every time period $t \geq 2$, define $\mathcal{F}_{i,t}$ as the σ-field generated by the past history of individual i's observations of his neighbors actions and outcomes, $(C_{j,1}, Z_{j,1})_{j \in N_i^d(g)}, \ldots\ldots, (C_{j,t-1}, Z_{j,t-1})_{j \in N_i^d(g)}$. Individuals only use the information on actions and outcomes of their neighbors, so the set classes $\mathcal{F}_{i,t}$ are the relevant σ-fields for our study. We shall denote by $\mathcal{F}_{i,\infty}$ the smallest σ-field containing all $\mathcal{F}_{i,t}$, for $t \geq 2$.

Recall that the objects of study are the optimal actions, $C_{i,t}$, the individual beliefs, $\mu_{i,t}$ and individual expected utilities $U_{i,t}$.

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