$$[H] = \int_V k(t)\left(\left\{\frac{\partial\{N\}}{\partial x}\right\}\left\{\frac{\partial\{N\}}{\partial x}\right\}^T + \left\{\frac{\partial\{N\}}{\partial y}\right\}\left\{\frac{\partial\{N\}}{\partial y}\right\}^T\right)dV$$

$$\begin{bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix} = \frac{1}{\det[J]} \begin{bmatrix} \frac{\partial y}{\partial \eta} & -\frac{\partial y}{\partial \xi} \\ -\frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \xi} \end{bmatrix} \begin{bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix} = 6400 \begin{bmatrix} 0{,}0125 & 0 \\ 0 & 0{,}0125 \end{bmatrix} \begin{bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \end{bmatrix}$$

$$\frac{dN1}{d\xi} = -0.25(1-\eta)$$

$$\frac{dN2}{d\xi} = 0.25(1-\eta)$$

$$\frac{dN3}{d\xi} = 0.25(1+\eta)$$

$$\frac{dN4}{d\xi} = -0.25(1+\eta)$$

$$\frac{dN1}{d\eta} = -0.25(1-\xi)$$

$$\frac{dN2}{d\eta} = -0.25(1+\xi)$$

$$\frac{dN3}{d\eta} = 0.25(1+\xi)$$

$$\frac{dN4}{d\eta} = 0.25(1-\xi)$$

Dla pc1

0,0125	0
0	0,0125

det[j] = 0,00015625

1/det[j] = 6400

$$\begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix} = \begin{bmatrix} 80 & 0 \\ 0 & 80 \end{bmatrix} \begin{bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \end{bmatrix}$$

Dla pc1

$$[H] = \int_V k(t)\left(\left\{\frac{\partial\{N\}}{\partial x}\right\}\left\{\frac{\partial\{N\}}{\partial x}\right\}^T + \left\{\frac{\partial\{N\}}{\partial y}\right\}\left\{\frac{\partial\{N\}}{\partial y}\right\}^T\right)dV$$

$$\begin{bmatrix}\frac{\partial N_i}{\partial x}\\ \frac{\partial N_i}{\partial y}\end{bmatrix} = \begin{bmatrix}80 & 0\\ 0 & 80\end{bmatrix}\begin{bmatrix}\frac{\partial N_i}{\partial \xi}\\ \frac{\partial N_i}{\partial \eta}\end{bmatrix}$$

	dN1/dξ	dN2/dξ	dN3/dξ	dN4/dξ
pc1	-0,39434	0,394338	0,105662	-0,10566
pc2	-0,39434	0,394338	0,105662	-0,10566
pc3	-0,10566	0,105662	0,394338	-0,39434
pc4	-0,10566	0,105662	0,394338	-0,39434

	dN1/dη	dN2/dη	dN3/dη	dN4/dη
pc1	-0,39434	-0,10566	0,105662	0,394338
pc2	-0,10566	-0,39434	0,394338	0,105662
pc3	-0,10566	-0,39434	0,394338	0,105662
pc4	-0,39434	-0,10566	0,105662	0,394338

$$\frac{dN_1}{d\mathrm{X}} = 80 * \frac{dN_1}{d\xi} + 0{,}0 * \frac{dN_1}{d\eta} = 80 * (-0{,}39434) + 0{,}0 * (0{,}39434) = -31{,}547$$

$$\frac{dN_2}{d\mathrm{X}} = 80 * \frac{dN_2}{d\xi} + 0{,}0 * \frac{dN_2}{d\eta} = 80 * (0{,}39434) + 0{,}0 * (-0{,}10566) = 31{,}547$$

pc	dN1/dx	dN2/dx	dN3/dx	dN4/dx
1	-31,547	31,54701	8,452995	-8,452995
2	-31,547	31,54701	8,452995	-8,452995
3	-8,453	8,452995	31,54701	-31,54701
4	-8,453	8,452995	31,54701	-31,54701

pc	dN1/dy	dN2/dy	dN3/dy	dN4/dy
1	-31,547	-8,453	8,45299	31,547
2	-8,45299	-31,547	31,547	8,45299
3	-8,45299	-31,547	31,547	8,45299
4	-31,547	-8,453	8,45299	31,547

Obliczanie macierzy H dla pierwszego punktu całkowania

$$[H] = \int_V k(t)\left(\left\{\frac{\partial\{N\}}{\partial x}\right\}\left\{\frac{\partial\{N\}}{\partial x}\right\}^T + \left\{\frac{\partial\{N\}}{\partial y}\right\}\left\{\frac{\partial\{N\}}{\partial y}\right\}^T\right) dV$$

pc	dN1/dx	dN2/dx	dN3/dx	dN4/dx		pc	dN1/dy	dN2/dy	dN3/dy	dN4/dy
1	-31,547	31,54701	8,452995	-8,452995		1	-31,547	-8,453	8,45299	31,547
2	-31,547	31,54701	8,452995	-8,452995		2	-8,45299	-31,547	31,547	8,45299
3	-8,453	8,452995	31,54701	-31,54701		3	-8,45299	-31,547	31,547	8,45299
4	-8,453	8,452995	31,54701	-31,54701		4	-31,547	-8,453	8,45299	31,547

Obliczanie macierzy H dla pierwszego punktu całkowania

$$[H] = \int_V k(t)\left(\left\{\frac{\partial\{N\}}{\partial x}\right\}\left\{\frac{\partial\{N\}}{\partial x}\right\}^T + \left\{\frac{\partial\{N\}}{\partial y}\right\}\left\{\frac{\partial\{N\}}{\partial y}\right\}^T\right) dV$$

pc	dN1/dx	dN2/dx	dN3/dx	dN4/dx		pc	dN1/dy	dN2/dy	dN3/dy	dN4/dy
1	-31,547	31,54701	8,452995	-8,452995		1	-31,547	-8,453	8,45299	31,547

$$[H] = \int_V 30\left(\begin{Bmatrix}-31{,}54\\31{,}54\\8{,}45\\-8{,}45\end{Bmatrix}\{-31{,}54 \quad 31{,}54 \quad 8{,}45 \quad -8{,}45\} + \begin{Bmatrix}-31{,}54\\-8{,}45\\8{,}45\\31{,}54\end{Bmatrix}\{-31{,}54 \quad -8{,}45 \quad 8{,}45 \quad 31{,}54\}\right) dV$$

$$[H] = \int_V 30\Big($$

995,21	-995,21	-266,66	266,66
-995,21	995,21	266,66	-266,66
-266,66	266,66	71,45	-71,45
266,66	-266,66	-71,45	71,45

$+$

995,21	266,66	-266,66	-995,21
266,66	71,45	-71,45	-266,66
-266,66	-71,45	71,45	266,66
-995,21	-266,667	266,66	995,21

$\Big)\, dV$

Obliczanie macierzy H dla pierwszego punktu całkowania

$$[H] = \int_V k(t)\left(\left\{\frac{\partial\{N\}}{\partial x}\right\}\left\{\frac{\partial\{N\}}{\partial x}\right\}^T + \left\{\frac{\partial\{N\}}{\partial y}\right\}\left\{\frac{\partial\{N\}}{\partial y}\right\}^T\right)dV$$

$[H] = \int_V 30($

995,21	-995,21	-266,66	266,66
-995,21	995,21	266,66	-266,66
-266,66	266,66	71,45	-71,45
266,66	-266,66	-71,45	71,45

$+$

995,21	266,66	-266,66	-995,21
266,66	71,45	-71,45	-266,66
-266,66	-71,45	71,45	266,66
-995,21	-266,667	266,66	995,21

$) dV$

dV realizujemy poprzez przemnożenie wyniku przez Jakobian przekształcenia tego punktu całkowania

$[H] = 30 * ($

995,21	-995,21	-266,66	266,66
-995,21	995,21	266,66	-266,66
-266,66	266,66	71,45	-71,45
266,66	-266,66	-71,45	71,45

$+$

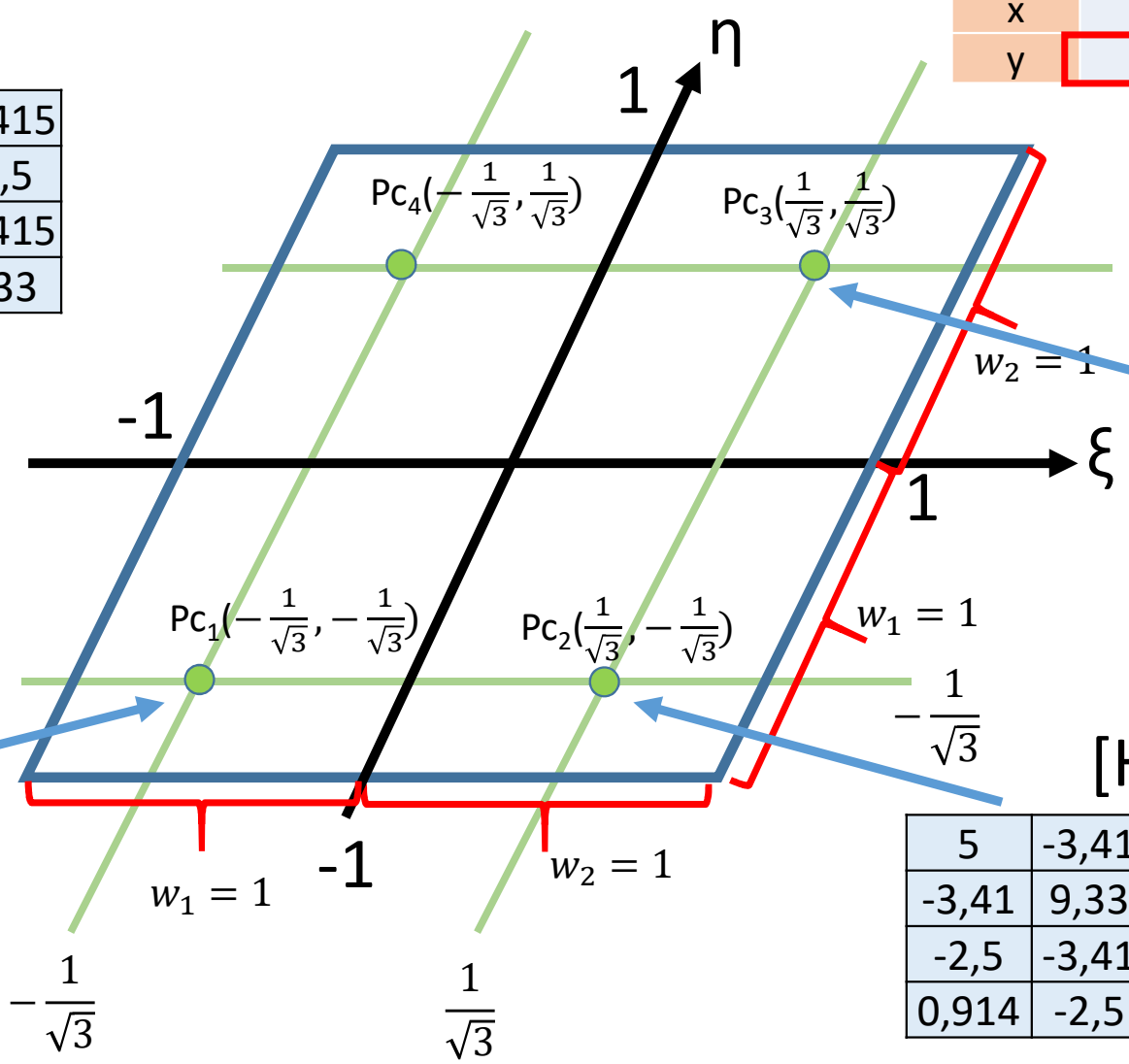
995,21	266,66	-266,66	-995,21
266,66	71,45	-71,45	-266,66
-266,66	-71,45	71,45	266,66
-995,21	-266,667	266,66	995,21

$) * \ 0{,}000156$

ID	1	2	3	4
x	0	0,025	0,025	0
y	0	0	0,025	0,025

[H]pc4

5	0,915	-2,5	-3,415
0,915	0,67	0,915	-2,5
-2,5	0,915	5	-3,415
-3,415	-2,5	-3,415	9,33



[H]pc3

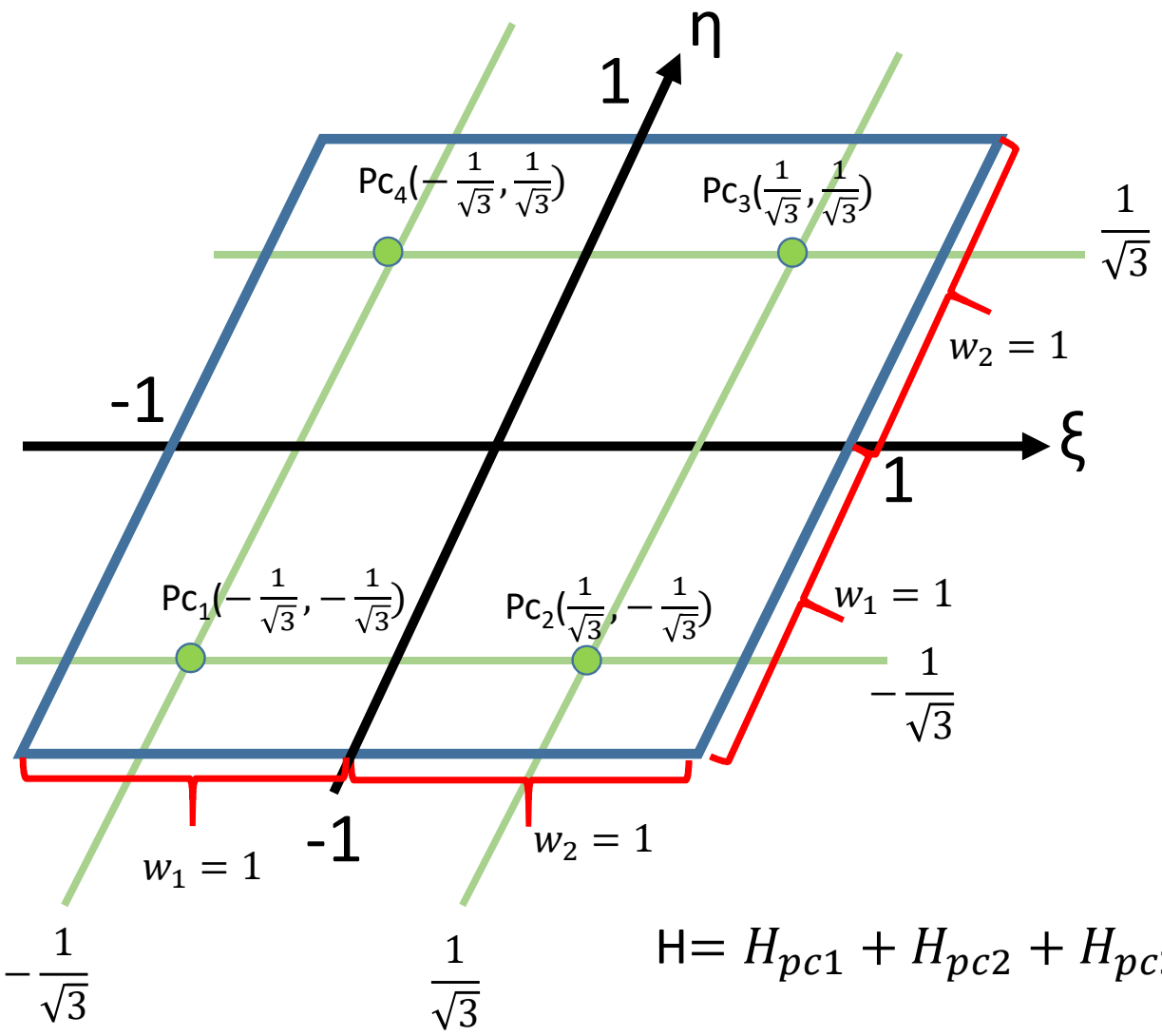
0,66	0,91	-2,5	0,916
0,91	5	-3,41	-2,5
-2,5	-3,41	9,33	-3,4
0,916	-2,5	-3,41	5

[H]pc1

9,33	-3,41	-2,5	-3,41
-3,41	5	0,91	-2,5
-2,5	0,91	0,66	0,91
-3,41	-2,5	0,91	5

[H]pc2

5	-3,41	-2,5	0,91
-3,41	9,33	-3,41	-2,5
-2,5	-3,41	5	0,91
0,914	-2,5	0,914	0,66



ID	1	2	3	4
x	0	0,025	0,025	0
y	0	0	0,025	0,025

$$H = H_{pc1} + H_{pc2} + H_{pc3} + H_{pc4} =$$

20	-5	-10	-5
-5	20	-5	-10
-10	-5	20	-5
-5	-10	-5	20

Zadanie domowe – zrealizuj całkowanie macierzy H

$$[H] = \int_V k(t)\left(\left\{\frac{\partial\{N\}}{\partial x}\right\}\left\{\frac{\partial\{N\}}{\partial x}\right\}^T + \left\{\frac{\partial\{N\}}{\partial y}\right\}\left\{\frac{\partial\{N\}}{\partial y}\right\}^T\right)dV$$

Implementacja pętli po elementach

- Obliczamy macierz Jakobiego, Jakobian i Jakobian odwrotny dla każdego punktu całkowania
- Obliczamy Macierz H w poszczególnych punktach całkowania
- Sumujemy macierze H z punktów całkowania

$$\mathrm{H} = H_{pc1} + H_{pc} + H_{pc3} + H_{pc4}$$