Basics

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$
$$\mu = E(W) = E(a+bX) = a + bE(X)$$
$$\sigma^2 = Var(X) = E[X^2] - \mu_x^2$$
$$\sigma^2 = \sum_{all\ i} \frac{(x_i - \bar{x})^2}{n-1}$$
$$\sigma = StdDev(X) = \sqrt{\sigma^2}$$
$$Var(X+c) = Var(X)$$
$$Var(cX) = c^2 Var(X)$$
$$Var(X+Y) \neq Var(X) + Var(Y)$$

Chebyshev's Rule:
$$1 - (\frac{1}{k^2})$$
yields the % of data that falls with k std deviations.

Covariance and Correlation

Covariance gives direction.
Correlation gives direction and strength.
Both are *linear.*
$$Cor = \frac{Cov}{\sigma_x \cdot \sigma_y}$$

Return on Portfolio

$$Avgreturn = w_1\bar{r}_1 + (1-w_1)\bar{r}_2$$
$$r_{12} = \frac{\sigma_{12}}{\sigma_1\sigma_2}$$
$$\sigma^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2 \cdot Correlation$$
$$Stockreturn = \alpha + \beta \cdot Indexreturn$$

Probabilities

$0 \leq$ All probabilities ≤ 1
Mutually exclusive and Exhaustive

$$P(\overline{A}) = 1 - P(A)$$
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

Joint vs. Marginal probabilities
Independent if $P(A|B) = P(A)$
For Independent only: $P(E \text{ and } F) = P(E) \cdot P(F)$

	B	$\overline{B}$
A	$P(AandB) = P(B)P(A\|B)$	$P(Aand\overline{B}) = P(\overline{B})P(A\|\overline{B})$
$\overline{A}$	$P(\overline{A}andB) = P(\overline{A})P(B\|\overline{A})$	$P(\overline{A}and\overline{B}) = P(\overline{A})P(\overline{B}\|\overline{A})$

Random Variables

$$P(X \leq x) \rightarrow CDF$$
$$E(cX) = c \cdot E(X)$$
$$E(X+c) = E(X) + c$$
$$E(X+Y) = E(X) + E(Y)$$

Discrete Probability Distributions

$$\mu_x = E(X) = \sum_{all\ x_i} x_i P(X = x_i)$$
$$\sigma_x^2 = Var(X) = \sum_{all\ x_i} (x_i - \mu_x)^2 P(X = x_i)$$
$$= E[X^2] - (\mu_x)^2$$

	Chebyshev Any Prob	Empircal Normal
$P(\mu - \sigma < x < \mu + \sigma)$	≥ 0	68%
$P(\mu - 2\sigma < x < \mu + 2\sigma)$	$\geq 75\%$	95%
$P(\mu - 3\sigma < x < \mu + 3\sigma)$	$\geq 89\%$	100%

$$W = a + bX$$
$$\mu_W = E(W) = E(a+bX)$$
$$= a + bE(X)$$
$$= a + b\mu_x$$
$$Var(W) = Var(a+bX)$$
$$= b^2\sigma_x^2$$

Binomial Distribution

- n independent trials
- binary result
- same probability of success
- total number of successes

$$X \sim B(n,p)$$
$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$
$$P(X = x) = \frac{n!}{x!(n-x)!} p^x q^{(n-x)}$$
$$\mu_x = E(X) = n \cdot p$$
$$\sigma^2 = Var(X) = n \cdot p \cdot q = n \cdot p \cdot (1-p)$$

Shape of distribution depends on p, n.
Small p, left skewed. Large p, right skewed

all success	p^n
all failure	$(1-p)^n$
at least one failure	$1 - p^n$
at least one success	$1 - (1-p)^n$

Joint Distribution

$$P_{x,y} = P(X = x \text{ and } Y = y)$$

Independent if **for all values**:
$P_{x,y}(X = x \text{ and } Y = y) = P_X(x)P_Y(y)$
Conditional Distribution:
$P_{X|Y}(X = x|Y = y) = \frac{P_{X,Y}(x,y)}{P_Y(y)}$
Conditional Expectation:
$E(X|Y = y) = \sum_{allx} x \cdot P(X = x|Y = y)$
Covariance:

$$\sigma_{XY} = \sum_{i=1}^{N} [x_i - E(X)][(y_i - E(Y)]P(x_i, y_i)$$
$$Cov(X,Y) = E(XY) - E(X)E(Y)$$

Correlation:

$$\rho = \frac{\sigma_{X,Y}}{\sigma_X \sigma_Y}$$

If X and Y are **independent**:

$$E(X+Y) = E(X) + E(Y)$$
$$Var(X+Y) = Var(X) + Var(Y)$$
$$Cov(X,Y) = 0 \text{because} E(XY) = E(X)E(Y)$$

If X and Y are **not independent**:

$$E(X+Y) = E(X) + E(Y)$$
$$Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$$

Most general combination of random variables:

$$E((a+bX) + (c_dY)) = a + bE(X) + c + dE(Y)$$
$$Var((a+bX) + (c+dY)) = b^2Var(X) + d^2Var(Y) + 2bdCov(X,Y)$$

Continuous Probability Distribution

$P(X = x) = 0$ because it is always possible to be more precise.
Probability of an interval: $P(a \leq X \leq b) = F_x(b) - F_x(a)$

Uniform Distribution

$$
\begin{aligned}
E(X) =& \frac{(a+b)}{2} \\
Var(X) =& \frac{(b-a)^2}{12}
\end{aligned}
$$

y axis should be a fraction to make area $= 1$

Normal Distribution

$$
\begin{aligned}
X \sim& \mathcal{N}(\mu, \sigma^2) \\
Z =& \frac{X-\mu}{\sigma} \sim \mathcal{N}(0,1) \\
P(a \leq X \leq b) =& P[(a-\mu) \leq (X-\mu) \leq (b-\mu)] \\
=& P[\frac{(a-\mu)}{\sigma} \leq \frac{(X-\mu)}{\sigma} \leq \frac{(b-\mu)}{\sigma}] \\
=& P[\frac{(a-\mu)}{\sigma} \leq Z \leq \frac{(b-\mu)}{\sigma}]
\end{aligned}
$$

Checking Normality:

- mean and median close
- $IQR \sim 1.33\sigma$
- $range \sim 6\sigma$
- skewness $= 0$
- expected distributions (see table)