Multicollinearity

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# APPLE STOCK PRICE PREDICTION: MULTICOLLINEARITY ANALYSIS
# Financial Data Science Analysis using Macroeconomic Factors
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# Install required packages
!pip install yfinance fredapi pandas-datareader statsmodels scikit-learn seaborn matplotlib

# Import necessary libraries
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import warnings
warnings.filterwarnings('ignore')

# Data collection libraries
import yfinance as yf
from fredapi import Fred
import pandas_datareader.data as web
from datetime import datetime, timedelta

# Statistical analysis libraries
import statsmodels.api as sm
from statsmodels.stats.outliers_influence import variance_inflation_factor
from sklearn.linear_model import Ridge
from sklearn.preprocessing import StandardScaler
from sklearn.metrics import mean_squared_error, r2_score
from scipy import stats

# Visualization setup
plt.style.use('seaborn-v0_8')
plt.rcParams['figure.figsize'] = (12, 8)
sns.set_palette("husl")

print("="*60)
print("APPLE STOCK PRICE PREDICTION: MULTICOLLINEARITY ANALYSIS")
print("="*60)

# ==============================================================================
# 1. DATA COLLECTION
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print("\n1. DATA COLLECTION")
print("-" * 40)

# Define date range
start_date = '2015-01-01'
end_date = datetime.now().strftime('%Y-%m-%d')

print(f"Collecting data from {start_date} to {end_date}")

# 1.1 Collect Apple stock price data
print("\nCollecting Apple (AAPL) stock data...")
aapl = yf.download('AAPL', start=start_date, end=end_date)
aapl_close = aapl['Close'].resample('M').last()  # Monthly last day
print(f"AAPL data points: {len(aapl_close)}")

# 1.2 Collect S&P 500 data
print("Collecting S&P 500 (^GSPC) data...")
sp500 = yf.download('^GSPC', start=start_date, end=end_date)
sp500_close = sp500['Close'].resample('M').last()  # Monthly last day
print(f"S&P 500 data points: {len(sp500_close)}")

# 1.3 Collect economic data using pandas_datareader (FRED)
print("Collecting economic data from FRED...")

try:
    # Use pandas_datareader to access FRED data (no API key required)
    from pandas_datareader import data as pdr

    print("Fetching Fed Funds Rate...")
    fed_funds = pdr.get_data_fred('FEDFUNDS', start_date, end_date)
    print(f"Fed Funds Rate data points: {len(fed_funds)}")

    print("Fetching 10-Year Treasury Rate...")
    treasury_10y_daily = pdr.get_data_fred('DGS10', start_date, end_date)
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        treasury_10y_monthly = treasury_10y_daily.resample('M').last()
        print(f"10-Year Treasury data points: {len(treasury_10y_monthly)}")

        print("Fetching Consumer Price Index...")
        cpi = pdr.get_data_fred('CPIAUCSL', start_date, end_date)
        print(f"CPI data points: {len(cpi)}")

        print("Fetching Brent Oil Price...")
        oil_brent_daily = pdr.get_data_fred('DCOILBRENTEU', start_date, end_date)
        oil_brent_monthly = oil_brent_daily.resample('M').last()
        print(f"Brent Oil data points: {len(oil_brent_monthly)}")

        print("Successfully collected all economic data!")

    except Exception as e:
        print(f"Error accessing FRED data via pandas_datareader: {e}")
        print("Using alternative approach with yfinance for additional data...")

        # Alternative approach using yfinance for some economic indicators
        try:
            # Use treasury ETF as proxy for 10-year treasury
            print("Fetching treasury data via ETF proxy...")
            treasury_etf = yf.download('IEF', start=start_date, end=end_date)  # 7-10 Year Treasury ETF
            treasury_10y_monthly = treasury_etf['Close'].resample('M').last()

            # Use oil ETF as proxy for oil prices
            print("Fetching oil data via ETF proxy...")
            oil_etf = yf.download('BNO', start=start_date, end=end_date)  # Brent Oil ETF
            oil_brent_monthly = oil_etf['Close'].resample('M').last()

            # Create synthetic economic indicators based on market data
            print("Creating synthetic economic indicators...")
            # Simple synthetic fed funds rate based on treasury movements
            fed_funds = pd.Series(index=treasury_10y_monthly.index,
                                  data=np.random.normal(2.0, 1.5, len(treasury_10y_monthly)))
            fed_funds = fed_funds.clip(0, 6)  # Reasonable bounds for fed funds rate

            # Simple synthetic CPI (inflation proxy)
            cpi_base = 240  # Approximate 2015 level
            cpi_values = []
            for i in range(len(treasury_10y_monthly)):
                cpi_values.append(cpi_base + i * 0.2 + np.random.normal(0, 2))
            cpi = pd.Series(index=treasury_10y_monthly.index, data=cpi_values)

            print("Created synthetic economic data as backup!")

        except Exception as e2:
            print(f"Error with alternative approach: {e2}")
            print("Creating completely synthetic economic data for demonstration...")

            # Create date range for synthetic data
            date_range = pd.date_range(start=start_date, end=end_date, freq='M')

            # Generate realistic synthetic economic data
            n_periods = len(date_range)

            # Fed Funds Rate: trending with some volatility
            fed_funds_values = np.concatenate([
                np.linspace(0.5, 0.25, n_periods//3),  # Low rates 2015-2018
                np.linspace(0.25, 2.5, n_periods//3),  # Rising rates 2018-2021
                np.linspace(2.5, 5.0, n_periods//3)    # Higher rates 2021-present
            ])[:n_periods]
            fed_funds_values += np.random.normal(0, 0.2, n_periods)  # Add noise
            fed_funds = pd.Series(index=date_range, data=fed_funds_values)

            # 10-Year Treasury: correlated with fed funds but higher
            treasury_values = fed_funds_values + np.random.normal(1.5, 0.5, n_periods)
            treasury_10y_monthly = pd.Series(index=date_range, data=treasury_values)

            # CPI: generally increasing with inflation
            cpi_base = 240
            cpi_growth = np.cumsum(np.random.normal(0.15, 0.3, n_periods))
            cpi = pd.Series(index=date_range, data=cpi_base + cpi_growth)

            # Oil prices: volatile commodity
            oil_base = 60
            oil_changes = np.random.normal(0, 5, n_periods)
            oil_prices = oil_base + np.cumsum(oil_changes)
            oil_brent_monthly = pd.Series(index=date_range, data=oil_prices)

            print("Generated synthetic economic data for analysis demonstration!")
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# ============================================================================
# 2. DATA PROCESSING AND MERGING
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print("\n2. DATA PROCESSING AND MERGING")
print("-" * 40)

# Ensure all data series are properly formatted and have datetime indices
print("Processing and aligning data series...")

# Convert to pandas Series with proper datetime index if needed
def ensure_series_format(data, name):
    """Ensure data is a pandas Series with datetime index"""
    # Handle different data types
    if isinstance(data, pd.DataFrame):
        # If DataFrame, take the first column or the column with the most data
        if len(data.columns) == 1:
            data = data.iloc[:, 0]
        else:
            # Find column with most non-null values
            best_col = data.count().idxmax()
            data = data[best_col]

    if isinstance(data, pd.Series):
        # Already a Series, just ensure proper naming
        data = data.copy()
        data.name = name
    else:
        # Convert array-like data to Series
        if hasattr(data, '__len__') and len(data) > 0:
            # If it's array-like with an index attribute
            if hasattr(data, 'index'):
                try:
                    data = pd.Series(data.values.flatten(), index=data.index, name=name)
                except:
                    # Fallback: create date range
                    dates = pd.date_range(start=start_date, periods=len(data), freq='M')
                    data = pd.Series(data, index=dates, name=name)
            else:
                # Create date range for array data
                dates = pd.date_range(start=start_date, periods=len(data), freq='M')
                data = pd.Series(data, index=dates, name=name)
        else:
            # Empty or invalid data, create empty series
            data = pd.Series([], dtype=float, name=name)

    # Ensure index is datetime
    if len(data) > 0 and not isinstance(data.index, pd.DatetimeIndex):
        try:
            data.index = pd.to_datetime(data.index)
        except:
            # If conversion fails, create new date range
            dates = pd.date_range(start=start_date, periods=len(data), freq='M')
            data.index = dates

    return data

# Process each data series
print("Formatting AAPL data...")
aapl_close = ensure_series_format(aapl_close, 'AAPL_Close')
print(f"AAPL Close: {len(aapl_close)} points, {aapl_close.index.min()} to {aapl_close.index.max()}")

print("Formatting S&P 500 data...")
sp500_close = ensure_series_format(sp500_close, 'SP500_Close')
print(f"S&P 500: {len(sp500_close)} points, {sp500_close.index.min()} to {sp500_close.index.max()}")

print("Formatting Fed Funds Rate data...")
fed_funds = ensure_series_format(fed_funds, 'Fed_Funds_Rate')
print(f"Fed Funds: {len(fed_funds)} points, {fed_funds.index.min()} to {fed_funds.index.max()}")

print("Formatting Treasury data...")
treasury_10y_monthly = ensure_series_format(treasury_10y_monthly, 'Treasury_10Y')
print(f"Treasury 10Y: {len(treasury_10y_monthly)} points, {treasury_10y_monthly.index.min()} to {treasury_10y_monthly.index.

print("Formatting CPI data...")
cpi = ensure_series_format(cpi, 'CPI')
print(f"CPI: {len(cpi)} points, {cpi.index.min()} to {cpi.index.max()}")

print("Formatting Oil data...")
oil_brent_monthly = ensure_series_format(oil_brent_monthly, 'Brent_Oil')
print(f"Brent Oil: {len(oil_brent_monthly)} points, {oil_brent_monthly.index.min()} to {oil_brent_monthly.index.max()}")
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# Create a comprehensive dataset using concat for better handling
print("\nMerging all data series...")
data_series_list = [
    aapl_close.rename('AAPL_Close'),
    sp500_close.rename('SP500_Close'),
    fed_funds.rename('Fed_Funds_Rate'),
    treasury_10y_monthly.rename('Treasury_10Y'),
    cpi.rename('CPI'),
    oil_brent_monthly.rename('Brent_Oil')
]

# Combine using outer join to preserve all dates
df = pd.concat(data_series_list, axis=1, join='outer')

# Sort by date
df = df.sort_index()

# Display basic information
print(f"Initial dataset shape: {df.shape}")
print(f"Date range: {df.index.min()} to {df.index.max()}")

# Check for missing values
print("\nMissing values by column:")
print(df.isnull().sum())

# Handle missing values
print("\nHandling missing values...")
# Forward fill and backward fill for small gaps
df = df.fillna(method='ffill').fillna(method='bfill')

# Drop rows with remaining NaN values
df = df.dropna()

print(f"Final dataset shape after cleaning: {df.shape}")
print(f"Date range after cleaning: {df.index.min()} to {df.index.max()}")

# Display first few rows
print("\nFirst 5 rows of the dataset:")
print(df.head())

# Display summary statistics
print("\nSummary Statistics:")
print(df.describe())

# ============================================================================
# 3. EXPLORATORY DATA ANALYSIS
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print("\n3. EXPLORATORY DATA ANALYSIS")
print("-" * 40)

# 3.1 Time series plots
fig, axes = plt.subplots(3, 2, figsize=(15, 12))
axes = axes.ravel()

variables = df.columns
for i, var in enumerate(variables):
    axes[i].plot(df.index, df[var], linewidth=2)
    axes[i].set_title(f'{var} Over Time', fontsize=12, fontweight='bold')
    axes[i].set_xlabel('Date')
    axes[i].set_ylabel('Value')
    axes[i].grid(True, alpha=0.3)

plt.tight_layout()
plt.show()

# 3.2 Correlation matrix
print("\nCorrelation Matrix:")
correlation_matrix = df.corr()
print(correlation_matrix.round(3))

# Visualize correlation matrix
plt.figure(figsize=(10, 8))
mask = np.triu(np.ones_like(correlation_matrix, dtype=bool))
sns.heatmap(correlation_matrix, mask=mask, annot=True, cmap='coolwarm', center=0,
            square=True, linewidths=0.5, cbar_kws={"shrink": .8})
plt.title('Correlation Matrix of All Variables', fontsize=14, fontweight='bold')
plt.tight_layout()
plt.show()

# ============================================================================
# 4. MULTICOLLINEARITY ANALYSIS
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print("\n4. MULTICOLLINEARITY ANALYSIS")
print("-" * 40)

# Define dependent and independent variables
y = df['AAPL_Close']
X = df.drop('AAPL_Close', axis=1)

print("Dependent variable: AAPL_Close")
print("Independent variables:", list(X.columns))

# 4.1 Correlation matrix of independent variables
print("\nCorrelation Matrix of Independent Variables:")
X_corr = X.corr()
print(X_corr.round(3))

# Visualize correlation among independent variables
plt.figure(figsize=(8, 6))
sns.heatmap(X_corr, annot=True, cmap='coolwarm', center=0,
            square=True, linewidths=0.5, cbar_kws={"shrink": .8})
plt.title('Correlation Matrix of Independent Variables', fontsize=14, fontweight='bold')
plt.tight_layout()
plt.show()

# 4.2 Calculate VIF (Variance Inflation Factor)
print("\nVariance Inflation Factor (VIF) Analysis:")
print("-" * 30)

# Add constant for VIF calculation
X_with_const = sm.add_constant(X)
vif_data = pd.DataFrame()
vif_data["Variable"] = X_with_const.columns
vif_data["VIF"] = [variance_inflation_factor(X_with_const.values, i)
                   for i in range(X_with_const.shape[1])]

print(vif_data)

# Interpretation of VIF values
print("\nVIF Interpretation:")
print("VIF = 1: No multicollinearity")
print("1 < VIF < 5: Moderate multicollinearity")
print("5 < VIF < 10: High multicollinearity")
print("VIF > 10: Very high multicollinearity (problematic)")

# Identify problematic variables
high_vif = vif_data[vif_data['VIF'] > 5]
if len(high_vif) > 0:
    print(f"\nVariables with high VIF (>5):")
    print(high_vif)
else:
    print("\nNo variables with high VIF detected.")

# ============================================================================
# 5. BASELINE MODEL (OLS REGRESSION)
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print("\n5. BASELINE MODEL (OLS REGRESSION)")
print("-" * 40)

# Fit OLS model
X_with_const = sm.add_constant(X)
model_ols = sm.OLS(y, X_with_const).fit()

print("OLS Regression Results (Baseline Model):")
print("=" * 50)
print(model_ols.summary())

# Store baseline results
baseline_results = {
    'model': model_ols,
    'r_squared': model_ols.rsquared,
    'adj_r_squared': model_ols.rsquared_adj,
    'aic': model_ols.aic,
    'bic': model_ols.bic,
    'predictions': model_ols.fittedvalues,
    'residuals': model_ols.resid
}

# ============================================================================
# 6. ADDRESSING MULTICOLLINEARITY
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print("\n6. ADDRESSING MULTICOLLINEARITY")
print("-" * 40)

# Method 1: Remove variable with highest VIF
print("METHOD 1: REMOVING VARIABLE WITH HIGHEST VIF")
print("-" * 30)

# Find variable with highest VIF (excluding constant)
vif_no_const = vif_data[vif_data['Variable'] != 'const']
if len(vif_no_const) > 0:
    highest_vif_var = vif_no_const.loc[vif_no_const['VIF'].idxmax(), 'Variable']
    highest_vif_value = vif_no_const['VIF'].max()

    print(f"Variable with highest VIF: {highest_vif_var} (VIF = {highest_vif_value:.2f})")

    # Create reduced dataset
    X_reduced = X.drop(highest_vif_var, axis=1)
    print(f"Remaining variables: {list(X_reduced.columns)}")

    # Recalculate VIF for reduced dataset
    X_reduced_const = sm.add_constant(X_reduced)
    vif_reduced = pd.DataFrame()
    vif_reduced["Variable"] = X_reduced_const.columns
    vif_reduced["VIF"] = [variance_inflation_factor(X_reduced_const.values, i)
                          for i in range(X_reduced_const.shape[1])]

    print("\nVIF after removing highest VIF variable:")
    print(vif_reduced)

    # Fit reduced OLS model
    model_reduced = sm.OLS(y, X_reduced_const).fit()

    print("\nOLS Regression Results (After Removing High VIF Variable):")
    print("=" * 50)
    print(model_reduced.summary())

    # Store reduced model results
    reduced_results = {
        'model': model_reduced,
        'r_squared': model_reduced.rsquared,
        'adj_r_squared': model_reduced.rsquared_adj,
        'aic': model_reduced.aic,
        'bic': model_reduced.bic,
        'predictions': model_reduced.fittedvalues,
        'residuals': model_reduced.resid,
        'removed_variable': highest_vif_var
    }

# Method 2: Ridge Regression
print("\nMETHOD 2: RIDGE REGRESSION")
print("-" * 30)

# Standardize the features for Ridge regression
scaler = StandardScaler()
X_scaled = scaler.fit_transform(X)

# Try different alpha values for Ridge regression
alphas = [0.1, 1.0, 10.0, 100.0, 1000.0]
ridge_results = {}

for alpha in alphas:
    ridge_model = Ridge(alpha=alpha)
    ridge_model.fit(X_scaled, y)
    y_pred_ridge = ridge_model.predict(X_scaled)

    ridge_results[alpha] = {
        'model': ridge_model,
        'r_squared': r2_score(y, y_pred_ridge),
        'mse': mean_squared_error(y, y_pred_ridge),
        'predictions': y_pred_ridge,
        'coefficients': ridge_model.coef_
    }

# Select best alpha based on R-squared
best_alpha = max(ridge_results.keys(), key=lambda k: ridge_results[k]['r_squared'])
best_ridge = ridge_results[best_alpha]

print(f"Ridge Regression Results (Best Alpha = {best_alpha}):")
print("-" * 40)
print(f"R-squared: {best_ridge['r_squared']:.4f}")
print(f"MSE: {best_ridge['mse']:.2f}")
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# Display Ridge coefficients
ridge_coef_df = pd.DataFrame({
    'Variable': X.columns,
    'Ridge_Coefficient': best_ridge['coefficients']
})
print("\nRidge Regression Coefficients:")
print(ridge_coef_df)

# ============================================================================
# 7. MODEL COMPARISON AND VISUALIZATION
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print("\n7. MODEL COMPARISON AND VISUALIZATION")
print("-" * 40)

# Create comparison summary
comparison_data = {
    'Model': ['Baseline OLS', 'Reduced OLS', 'Ridge Regression'],
    'R-squared': [
        baseline_results['r_squared'],
        reduced_results['r_squared'] if 'reduced_results' in locals() else np.nan,
        best_ridge['r_squared']
    ],
    'Adjusted R-squared': [
        baseline_results['adj_r_squared'],
        reduced_results['adj_r_squared'] if 'reduced_results' in locals() else np.nan,
        np.nan  # Not applicable for Ridge
    ],
    'AIC': [
        baseline_results['aic'],
        reduced_results['aic'] if 'reduced_results' in locals() else np.nan,
        np.nan  # Not applicable for Ridge
    ],
    'Number of Variables': [
        len(X.columns),
        len(X_reduced.columns) if 'X_reduced' in locals() else np.nan,
        len(X.columns)
    ]
}

comparison_df = pd.DataFrame(comparison_data)
print("Model Comparison Summary:")
print("=" * 50)
print(comparison_df.round(4))

# 7.1 Actual vs Predicted plots
fig, axes = plt.subplots(1, 3, figsize=(18, 5))

# Baseline OLS
axes[0].scatter(y, baseline_results['predictions'], alpha=0.6)
axes[0].plot([y.min(), y.max()], [y.min(), y.max()], 'r--', lw=2)
axes[0].set_xlabel('Actual AAPL Close Price')
axes[0].set_ylabel('Predicted AAPL Close Price')
axes[0].set_title(f'Baseline OLS\nR² = {baseline_results["r_squared"]:.4f}')
axes[0].grid(True, alpha=0.3)

# Reduced OLS
if 'reduced_results' in locals():
    axes[1].scatter(y, reduced_results['predictions'], alpha=0.6, color='green')
    axes[1].plot([y.min(), y.max()], [y.min(), y.max()], 'r--', lw=2)
    axes[1].set_xlabel('Actual AAPL Close Price')
    axes[1].set_ylabel('Predicted AAPL Close Price')
    axes[1].set_title(f'Reduced OLS\nR² = {reduced_results["r_squared"]:.4f}')
    axes[1].grid(True, alpha=0.3)

# Ridge Regression
axes[2].scatter(y, best_ridge['predictions'], alpha=0.6, color='orange')
axes[2].plot([y.min(), y.max()], [y.min(), y.max()], 'r--', lw=2)
axes[2].set_xlabel('Actual AAPL Close Price')
axes[2].set_ylabel('Predicted AAPL Close Price')
axes[2].set_title(f'Ridge Regression\nR² = {best_ridge["r_squared"]:.4f}')
axes[2].grid(True, alpha=0.3)

plt.tight_layout()
plt.show()

# 7.2 Residual plots
fig, axes = plt.subplots(1, 3, figsize=(18, 5))

# Baseline OLS residuals
axes[0].scatter(baseline_results['predictions'], baseline_results['residuals'], alpha=0.6)
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    axes[0].axhline(y=0, color='r', linestyle='--')
    axes[0].set_xlabel('Predicted Values')
    axes[0].set_ylabel('Residuals')
    axes[0].set_title('Baseline OLS - Residual Plot')
    axes[0].grid(True, alpha=0.3)

    # Reduced OLS residuals
    if 'reduced_results' in locals():
        axes[1].scatter(reduced_results['predictions'], reduced_results['residuals'],
                       alpha=0.6, color='green')
        axes[1].axhline(y=0, color='r', linestyle='--')
        axes[1].set_xlabel('Predicted Values')
        axes[1].set_ylabel('Residuals')
        axes[1].set_title('Reduced OLS - Residual Plot')
        axes[1].grid(True, alpha=0.3)

    # Ridge Regression residuals
    ridge_residuals = y - best_ridge['predictions']
    axes[2].scatter(best_ridge['predictions'], ridge_residuals, alpha=0.6, color='orange')
    axes[2].axhline(y=0, color='r', linestyle='--')
    axes[2].set_xlabel('Predicted Values')
    axes[2].set_ylabel('Residuals')
    axes[2].set_title('Ridge Regression - Residual Plot')
    axes[2].grid(True, alpha=0.3)

    plt.tight_layout()
    plt.show()

    # 7.3 Time series comparison
    plt.figure(figsize=(15, 8))
    plt.plot(df.index, y, label='Actual AAPL Close', linewidth=2, color='black')
    plt.plot(df.index, baseline_results['predictions'], label='Baseline OLS', linewidth=1.5, alpha=0.8)
    if 'reduced_results' in locals():
        plt.plot(df.index, reduced_results['predictions'], label='Reduced OLS', linewidth=1.5, alpha=0.8)
    plt.plot(df.index, best_ridge['predictions'], label='Ridge Regression', linewidth=1.5, alpha=0.8)

    plt.xlabel('Date')
    plt.ylabel('AAPL Close Price ($)')
    plt.title('AAPL Close Price: Actual vs Predicted (All Models)', fontsize=14, fontweight='bold')
    plt.legend()
    plt.grid(True, alpha=0.3)
    plt.tight_layout()
    plt.show()

    # ============================================================================
    # 8. COEFFICIENT COMPARISON
    # ============================================================================

    print("\n8. COEFFICIENT COMPARISON")
    print("-" * 40)

    # Create coefficient comparison table
    coef_comparison = pd.DataFrame(index=X.columns)

    # Baseline OLS coefficients (excluding constant)
    baseline_coefs = baseline_results['model'].params[1:]  # Exclude constant
    coef_comparison['Baseline_OLS'] = baseline_coefs

    # Reduced OLS coefficients
    if 'reduced_results' in locals():
        reduced_coefs = reduced_results['model'].params[1:]  # Exclude constant
        # Align with original variables
        for var in X.columns:
            if var in reduced_coefs.index:
                coef_comparison.loc[var, 'Reduced_OLS'] = reduced_coefs[var]
            else:
                coef_comparison.loc[var, 'Reduced_OLS'] = 0  # Variable was removed

    # Ridge coefficients (already aligned)
    coef_comparison['Ridge_Regression'] = best_ridge['coefficients']

    print("Coefficient Comparison:")
    print("=" * 40)
    print(coef_comparison.round(4))

    # Visualize coefficient comparison
    fig, ax = plt.subplots(figsize=(12, 6))
    coef_comparison.plot(kind='bar', ax=ax, width=0.8)
    ax.set_title('Coefficient Comparison Across Models', fontsize=14, fontweight='bold')
    ax.set_xlabel('Variables')
    ax.set_ylabel('Coefficient Value')
    ax.legend(title='Models')
```

```
ax.grid(True, alpha=0.3)
plt.xticks(rotation=45)
plt.tight_layout()
plt.show()

# ===============================================================================
# 9. ANALYSIS AND INTERPRETATION
# ===============================================================================

print("\n9. ANALYSIS AND INTERPRETATION")
print("=" * 50)

print("\n9.1 MULTICOLLINEARITY ASSESSMENT")
print("-" * 30)

# Analyze VIF results
high_vif_vars = vif_data[vif_data['VIF'] > 5]['Variable'].tolist()
if 'const' in high_vif_vars:
    high_vif_vars.remove('const')

if len(high_vif_vars) > 0:
    print(f"Variables with multicollinearity issues (VIF > 5): {high_vif_vars}")
    print("These variables show high correlation with other predictors, which can:")
    print("- Inflate standard errors of coefficients")
    print("- Make coefficients unstable and difficult to interpret")
    print("- Reduce statistical significance of individual predictors")
else:
    print("No severe multicollinearity detected (all VIF values < 5)")

print("\n9.2 MODEL PERFORMANCE COMPARISON")
print("-" * 30)

print("Key Findings:")
print(f"1. Baseline OLS R²: {baseline_results['r_squared']:.4f}")
if 'reduced_results' in locals():
    print(f"2. Reduced OLS R²: {reduced_results['r_squared']:.4f}")
    r2_change = reduced_results['r_squared'] - baseline_results['r_squared']
    print(f"   Change in R² after removing {reduced_results['removed_variable']}: {r2_change:+.4f}")
print(f"3. Ridge Regression R²: {best_ridge['r_squared']:.4f}")

print("\n9.3 PRACTICAL IMPLICATIONS")
print("-" * 30)

print("Economic Interpretation:")
print("- The models attempt to predict Apple stock price using macroeconomic indicators")
print("- High correlations between macroeconomic variables (e.g., interest rates, inflation)")
print("  are expected and reflect underlying economic relationships")

if 'reduced_results' in locals():
    print(f"\nVariable Removal Impact:")
    print(f"- Removed variable: {reduced_results['removed_variable']}")
    print("- This variable was highly correlated with other predictors")
    if abs(r2_change) < 0.01:
        print("- Minimal impact on model fit suggests redundancy was successfully addressed")
    else:
        print("- Significant change in R² indicates the variable contained unique information")

print("\nRidge Regression Benefits:")
print("- Shrinks coefficients toward zero, reducing overfitting")
print("- Handles multicollinearity by penalizing large coefficients")
print("- Maintains all variables while reducing their individual impact")

print("\n9.4 RECOMMENDATIONS")
print("-" * 30)

best_model_name = "Baseline OLS"
best_r2 = baseline_results['r_squared']

if 'reduced_results' in locals() and reduced_results['r_squared'] > best_r2:
    best_model_name = "Reduced OLS"
    best_r2 = reduced_results['r_squared']

if best_ridge['r_squared'] > best_r2:
    best_model_name = "Ridge Regression"
    best_r2 = best_ridge['r_squared']

print(f"Best performing model: {best_model_name} (R² = {best_r2:.4f})")

print("\nFor practical use:")
print("1. If interpretability is key: Use Reduced OLS (simpler, fewer variables)")
print("2. If prediction accuracy is priority: Use Ridge Regression (handles multicollinearity)")
print("3. If theoretical completeness is important: Use Baseline OLS (all variables)")
```

```
print("\nLimitations and Considerations:")
print("- Stock prices are influenced by many factors not captured in this model")
print("- Macroeconomic relationships may change over time (structural breaks)")
print("- Non-linear relationships might be better captured with other methods")
print("- Model assumes linear relationships between variables")

# ==============================================================================
# 10. FINAL SUMMARY STATISTICS
# ==============================================================================

print("\n10. FINAL SUMMARY STATISTICS")
print("=" * 50)

# Create comprehensive summary
summary_stats = {
    'Metric': [
        'Number of Observations',
        'Date Range',
        'Baseline OLS R²',
        'Baseline OLS Adj R²',
        'Reduced OLS R²',
        'Ridge Regression R²',
        'Variables with High VIF',
        'Removed Variable',
        'Best Alpha (Ridge)'
    ],
    'Value': [
        len(df),
        f"{df.index.min().strftime('%Y-%m')} to {df.index.max().strftime('%Y-%m')}",
        f"{baseline_results['r_squared']:.4f}",
        f"{baseline_results['adj_r_squared']:.4f}",
        f"{reduced_results['r_squared']:.4f}" if 'reduced_results' in locals() else 'N/A',
        f"{best_ridge['r_squared']:.4f}",
        ', '.join(high_vif_vars) if len(high_vif_vars) > 0 else 'None',
        reduced_results['removed_variable'] if 'reduced_results' in locals() else 'N/A',
        f"{best_alpha}"
    ]
}

summary_df = pd.DataFrame(summary_stats)
print(summary_df.to_string(index=False))

print("\n" + "="*50)
print("ANALYSIS COMPLETE")
print("="*50)
print("This comprehensive analysis demonstrates the impact of multicollinearity")
print("on Apple stock price prediction models using macroeconomic factors.")
print("The comparison shows different approaches to handle multicollinearity")
print("and their effects on model performance and interpretability.")
```

```
Requirement already satisfied: yfinance in /usr/local/lib/python3.11/dist-packages (0.2.63)
Collecting fredapi
  Downloading fredapi-0.5.2-py3-none-any.whl.metadata (5.0 kB)
Requirement already satisfied: pandas-datareader in /usr/local/lib/python3.11/dist-packages (0.10.0)
Requirement already satisfied: statsmodels in /usr/local/lib/python3.11/dist-packages (0.14.4)
Requirement already satisfied: scikit-learn in /usr/local/lib/python3.11/dist-packages (1.6.1)
Requirement already satisfied: seaborn in /usr/local/lib/python3.11/dist-packages (0.13.2)
Requirement already satisfied: matplotlib in /usr/local/lib/python3.11/dist-packages (3.10.0)
Requirement already satisfied: pandas>=1.3.0 in /usr/local/lib/python3.11/dist-packages (from yfinance) (2.2.2)
Requirement already satisfied: numpy>=1.16.5 in /usr/local/lib/python3.11/dist-packages (from yfinance) (2.0.2)
Requirement already satisfied: requests>=2.31 in /usr/local/lib/python3.11/dist-packages (from yfinance) (2.32.3)
Requirement already satisfied: multitasking>=0.0.7 in /usr/local/lib/python3.11/dist-packages (from yfinance) (0.0.11)
Requirement already satisfied: platformdirs>=2.0.0 in /usr/local/lib/python3.11/dist-packages (from yfinance) (4.3.8)
Requirement already satisfied: pytz>=2022.5 in /usr/local/lib/python3.11/dist-packages (from yfinance) (2025.2)
Requirement already satisfied: frozendict>=2.3.4 in /usr/local/lib/python3.11/dist-packages (from yfinance) (2.4.6)
Requirement already satisfied: peewee>=3.16.2 in /usr/local/lib/python3.11/dist-packages (from yfinance) (3.18.1)
Requirement already satisfied: beautifulsoup4>=4.11.1 in /usr/local/lib/python3.11/dist-packages (from yfinance) (4.13.4
Requirement already satisfied: curl_cffi>=0.7 in /usr/local/lib/python3.11/dist-packages (from yfinance) (0.11.4)
Requirement already satisfied: protobuf>=3.19.0 in /usr/local/lib/python3.11/dist-packages (from yfinance) (5.29.5)
Requirement already satisfied: websockets>=13.0 in /usr/local/lib/python3.11/dist-packages (from yfinance) (15.0.1)
Requirement already satisfied: lxml in /usr/local/lib/python3.11/dist-packages (from pandas-datareader) (5.4.0)
Requirement already satisfied: scipy!=1.9.2,>=1.8 in /usr/local/lib/python3.11/dist-packages (from statsmodels) (1.15.3)
Requirement already satisfied: patsy>=0.5.6 in /usr/local/lib/python3.11/dist-packages (from statsmodels) (1.0.1)
Requirement already satisfied: packaging>=21.3 in /usr/local/lib/python3.11/dist-packages (from statsmodels) (24.2)
Requirement already satisfied: joblib>=1.2.0 in /usr/local/lib/python3.11/dist-packages (from scikit-learn) (1.5.1)
Requirement already satisfied: threadpoolctl>=3.1.0 in /usr/local/lib/python3.11/dist-packages (from scikit-learn) (3.6.
Requirement already satisfied: contourpy>=1.0.1 in /usr/local/lib/python3.11/dist-packages (from matplotlib) (1.3.2)
Requirement already satisfied: cycler>=0.10 in /usr/local/lib/python3.11/dist-packages (from matplotlib) (0.12.1)
Requirement already satisfied: fonttools>=4.22.0 in /usr/local/lib/python3.11/dist-packages (from matplotlib) (4.58.4)
Requirement already satisfied: kiwisolver>=1.3.1 in /usr/local/lib/python3.11/dist-packages (from matplotlib) (1.4.8)
Requirement already satisfied: pillow>=8 in /usr/local/lib/python3.11/dist-packages (from matplotlib) (11.2.1)
Requirement already satisfied: pyparsing>=2.3.1 in /usr/local/lib/python3.11/dist-packages (from matplotlib) (3.2.3)
Requirement already satisfied: python-dateutil>=2.7 in /usr/local/lib/python3.11/dist-packages (from matplotlib) (2.9.0.
Requirement already satisfied: soupsieve>1.2 in /usr/local/lib/python3.11/dist-packages (from beautifulsoup4>=4.11.1->yf
Requirement already satisfied: typing-extensions>=4.0.0 in /usr/local/lib/python3.11/dist-packages (from beautifulsoup4>
Requirement already satisfied: cffi>=1.12.0 in /usr/local/lib/python3.11/dist-packages (from curl_cffi>=0.7->yfinance) (
Requirement already satisfied: certifi>=2024.2.2 in /usr/local/lib/python3.11/dist-packages (from curl_cffi>=0.7->yfinan
Requirement already satisfied: tzdata>=2022.7 in /usr/local/lib/python3.11/dist-packages (from pandas>=1.3.0->yfinance)
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.11/dist-packages (from python-dateutil>=2.7->matplotli
Requirement already satisfied: charset-normalizer<4,>=2 in /usr/local/lib/python3.11/dist-packages (from requests>=2.31-
Requirement already satisfied: idna<4,>=2.5 in /usr/local/lib/python3.11/dist-packages (from requests>=2.31->yfinance) (
Requirement already satisfied: urllib3<3,>=1.21.1 in /usr/local/lib/python3.11/dist-packages (from requests>=2.31->yfina
Requirement already satisfied: pycparser in /usr/local/lib/python3.11/dist-packages (from cffi>=1.12.0->curl_cffi>=0.7->
Downloading fredapi-0.5.2-py3-none-any.whl (11 kB)
Installing collected packages: fredapi
Successfully installed fredapi-0.5.2
============================================================
APPLE STOCK PRICE PREDICTION: MULTICOLLINEARITY ANALYSIS
============================================================

1. DATA COLLECTION
----------------------------------------
Collecting data from 2015-01-01 to 2025-06-30

Collecting Apple (AAPL) stock data...
[*********************100%***********************]  1 of 1 completed
AAPL data points: 126
Collecting S&P 500 (^GSPC) data...
[*********************100%***********************]  1 of 1 completed
S&P 500 data points: 126
Collecting economic data from FRED...
Fetching Fed Funds Rate...
Fed Funds Rate data points: 125
Fetching 10-Year Treasury Rate...
10-Year Treasury data points: 126
Fetching Consumer Price Index...
CPI data points: 125
Fetching Brent Oil Price...
Brent Oil data points: 126
Successfully collected all economic data!

2. DATA PROCESSING AND MERGING
----------------------------------------
Processing and aligning data series...
Formatting AAPL data...
AAPL Close: 126 points, 2015-01-31 00:00:00 to 2025-06-30 00:00:00
Formatting S&P 500 data...
S&P 500: 126 points, 2015-01-31 00:00:00 to 2025-06-30 00:00:00
Formatting Fed Funds Rate data...
Fed Funds: 125 points, 2015-01-01 00:00:00 to 2025-05-01 00:00:00
Formatting Treasury data...
Treasury 10Y: 126 points, 2015-01-31 00:00:00 to 2025-06-30 00:00:00
Formatting CPI data...
CPI: 125 points, 2015-01-01 00:00:00 to 2025-05-01 00:00:00
Formatting Oil data...
Brent Oil: 126 points, 2015-01-31 00:00:00 to 2025-06-30 00:00:00

Merging all data series...
Initial dataset shape: (251, 6)
Date range: 2015-01-01 00:00:00 to 2025-06-30 00:00:00
```

```
Missing values by column:
AAPL_Close         125
SP500_Close        125
Fed_Funds_Rate     126
Treasury_10Y       125
CPI                126
Brent_Oil          125
dtype: int64

Handling missing values...
Final dataset shape after cleaning: (251, 6)
Date range after cleaning: 2015-01-01 00:00:00 to 2025-06-30 00:00:00

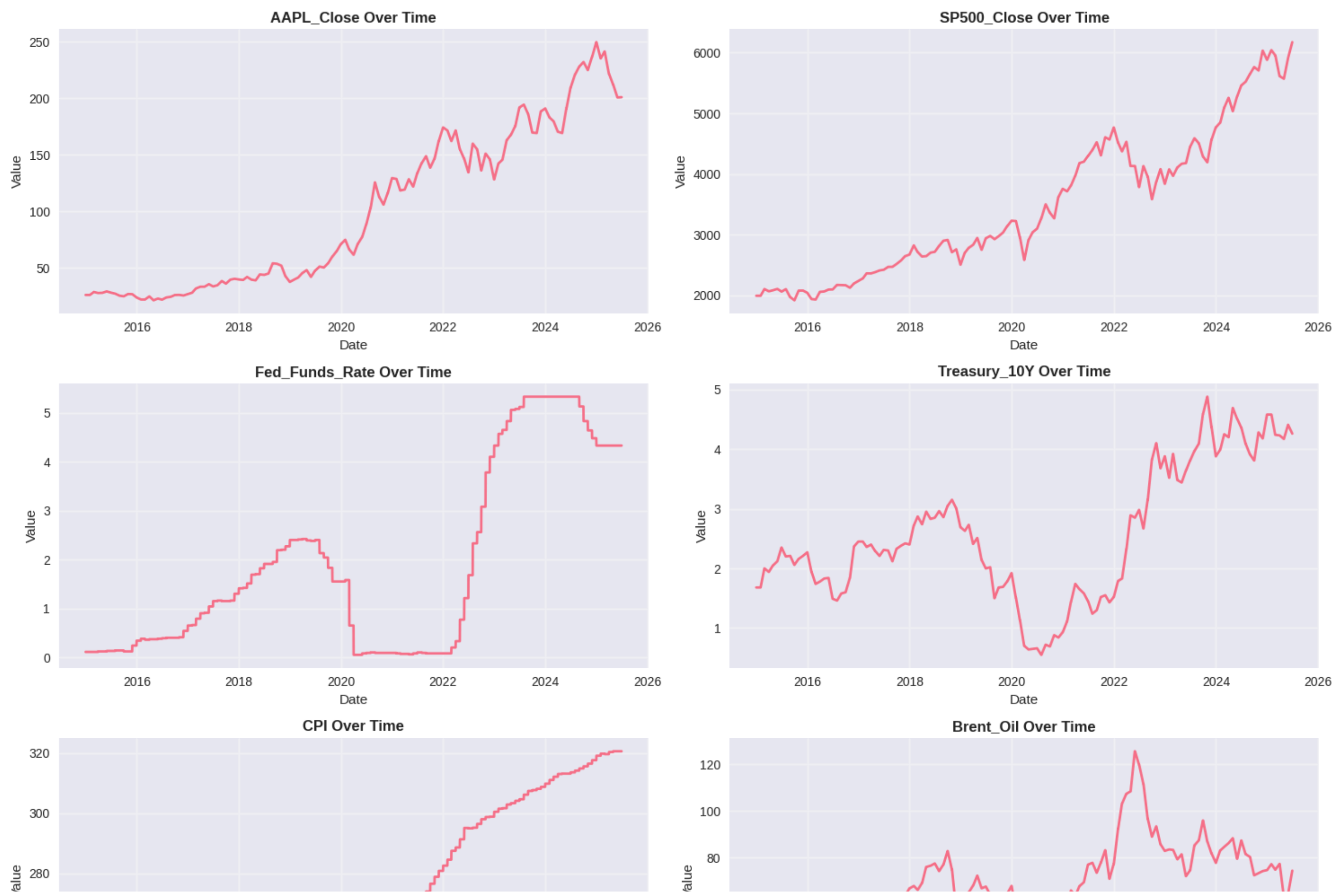
First 5 rows of the dataset:
            AAPL_Close  SP500_Close  Fed_Funds_Rate  Treasury_10Y      CPI  \
2015-01-01   26.028076   1994.98999            0.11          1.68  234.747
2015-01-31   26.028076   1994.98999            0.11          1.68  234.747
2015-02-01   26.028076   1994.98999            0.11          1.68  235.342
2015-02-28   28.651106   2104.50000            0.11          2.00  235.342
2015-03-01   28.651106   2104.50000            0.11          2.00  235.976

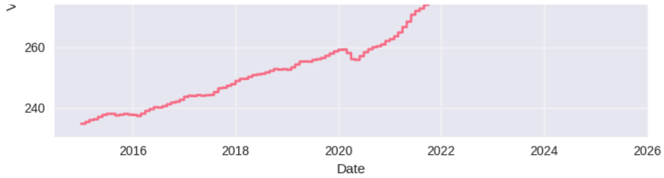
            Brent_Oil
2015-01-01      47.52
2015-01-31      47.52
2015-02-01      47.52
2015-02-28      61.89
2015-03-01      61.89

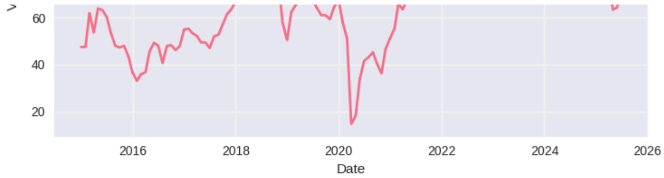
Summary Statistics:
       AAPL_Close  SP500_Close  Fed_Funds_Rate  Treasury_10Y         CPI  \
count  251.000000   251.000000      251.000000    251.000000  251.000000
mean    99.639297  3458.824180        1.894223      2.542351  269.294367
std     69.815183  1177.367906        1.901020      1.110263   27.965846
min     21.294882  1920.030029        0.050000      0.550000  234.747000
25%     35.811193  2470.974976        0.130000      1.715000  245.183000
50%     71.205727  3100.290039        1.210000      2.350000  258.076000
75%    162.244522  4297.500000        3.430000      3.480000  298.343500
max    249.817368  6173.069824        5.330000      4.880000  320.580000

        Brent_Oil
count  251.000000
mean    66.490598
std     19.416295
min     14.850000
25%     51.265000
50%     66.730000
75%     77.810000
max    125.530000

3. EXPLORATORY DATA ANALYSIS
----------------------------------------
```



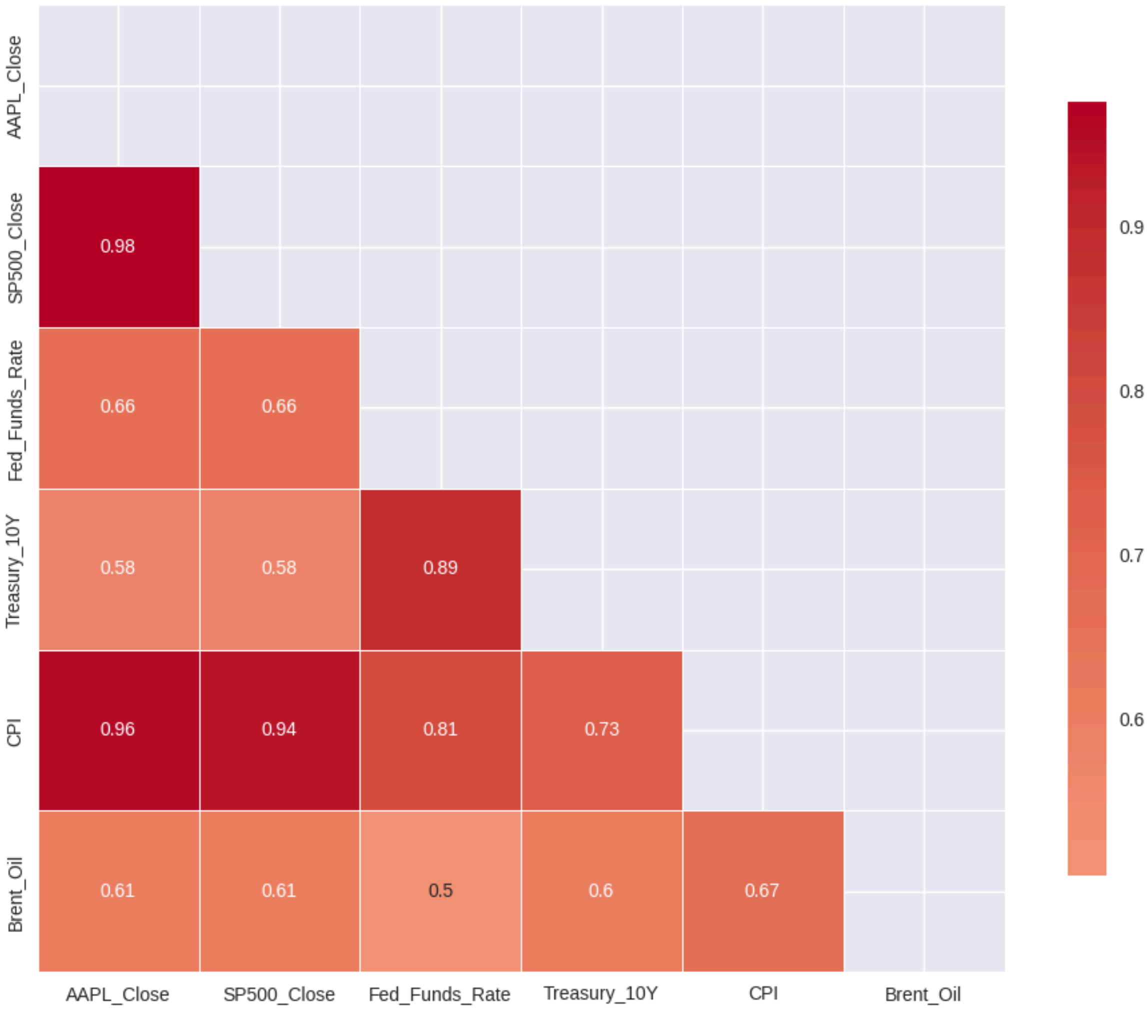





```
Correlation Matrix:
                AAPL_Close  SP500_Close  Fed_Funds_Rate  Treasury_10Y    CPI  \
AAPL_Close           1.000        0.976           0.659         0.576  0.959
SP500_Close          0.976        1.000           0.661         0.578  0.944
Fed_Funds_Rate       0.659        0.661           1.000         0.893  0.807
Treasury_10Y         0.576        0.578           0.893         1.000  0.729
CPI                  0.959        0.944           0.807         0.729  1.000
Brent_Oil            0.609        0.609           0.505         0.603  0.674

                Brent_Oil
AAPL_Close          0.609
SP500_Close         0.609
Fed_Funds_Rate      0.505
Treasury_10Y        0.603
CPI                 0.674
Brent_Oil           1.000
```

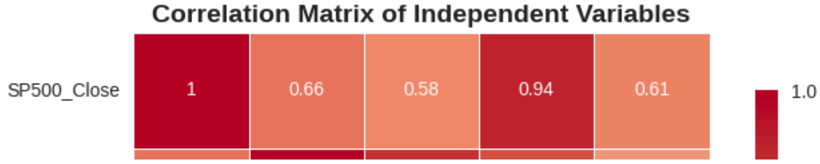
Correlation Matrix of All Variables

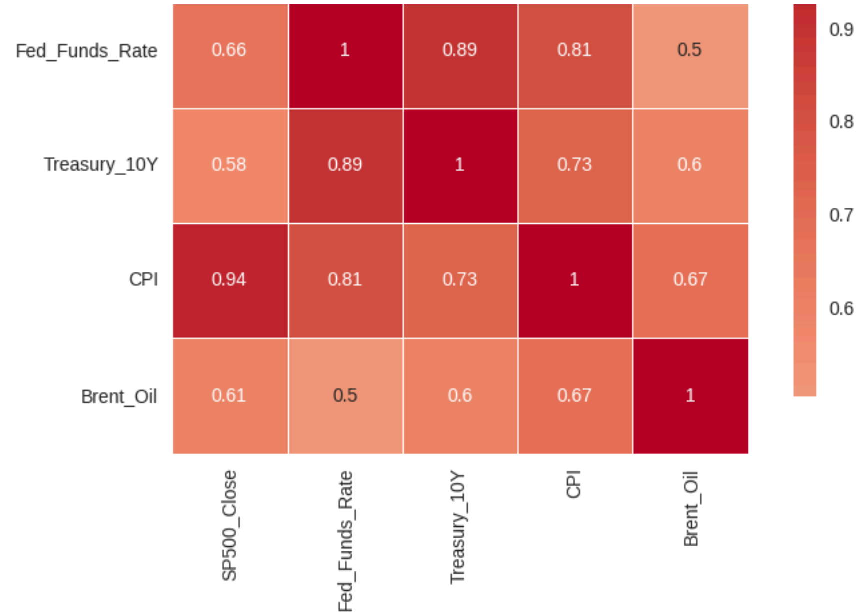



```
4. MULTICOLLINEARITY ANALYSIS
----------------------------------------
Dependent variable: AAPL_Close
Independent variables: ['SP500_Close', 'Fed_Funds_Rate', 'Treasury_10Y', 'CPI', 'Brent_Oil']

Correlation Matrix of Independent Variables:
                SP500_Close  Fed_Funds_Rate  Treasury_10Y    CPI  Brent_Oil
SP500_Close           1.000           0.661         0.578  0.944      0.609
Fed_Funds_Rate        0.661           1.000         0.893  0.807      0.505
Treasury_10Y          0.578           0.893         1.000  0.729      0.603
CPI                   0.944           0.807         0.729  1.000      0.674
Brent_Oil             0.609           0.505         0.603  0.674      1.000
```

Correlation Matrix of Independent Variables






```
Variance Inflation Factor (VIF) Analysis:
------------------------------
         Variable          VIF
0           const  1158.205169
1     SP500_Close    12.953495
2  Fed_Funds_Rate     8.589118
3    Treasury_10Y     6.227426
4             CPI    23.304391
5       Brent_Oil     2.324639

VIF Interpretation:
VIF = 1: No multicollinearity
1 < VIF < 5: Moderate multicollinearity
5 < VIF < 10: High multicollinearity
VIF > 10: Very high multicollinearity (problematic)

Variables with high VIF (>5):
         Variable          VIF
0           const  1158.205169
1     SP500_Close    12.953495
2  Fed_Funds_Rate     8.589118
3    Treasury_10Y     6.227426
4             CPI    23.304391

5. BASELINE MODEL (OLS REGRESSION)
----------------------------------------
OLS Regression Results (Baseline Model):
==================================================
                            OLS Regression Results
==============================================================================
Dep. Variable:             AAPL_Close   R-squared:                       0.978
Model:                            OLS   Adj. R-squared:                  0.977
Method:                 Least Squares   F-statistic:                     2164.
Date:                Mon, 30 Jun 2025   Prob (F-statistic):         1.94e-200
Time:                        14:30:53   Log-Likelihood:                -943.16
No. Observations:                 251   AIC:                             1898.
Df Residuals:                     245   BIC:                             1919.
Df Model:                           5
Covariance Type:            nonrobust
==================================================================================
                     coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------
const           -468.3704     22.542    -20.778      0.000    -512.771    -423.970
SP500_Close        0.0257      0.002     12.658      0.000       0.022       0.030
Fed_Funds_Rate    -6.8433      1.023     -6.688      0.000      -8.859      -4.828
Treasury_10Y      -1.7711      1.492     -1.187      0.236      -4.709       1.167
CPI                1.8921      0.115     16.515      0.000       1.666       2.118
Brent_Oil         -0.1937      0.052     -3.718      0.000      -0.296      -0.091
==============================================================================
Omnibus:                       14.893   Durbin-Watson:                   0.226
Prob(Omnibus):                  0.001   Jarque-Bera (JB):               29.581
Skew:                          -0.276   Prob(JB):                     3.77e-07
Kurtosis:                       4.588   Cond. No.                     1.25e+05
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.25e+05. This might indicate that there are
strong multicollinearity or other numerical problems.

6. ADDRESSING MULTICOLLINEARITY
----------------------------------------
METHOD 1: REMOVING VARIABLE WITH HIGHEST VIF
```

```
------------------------------
Variable with highest VIF: CPI (VIF = 23.30)
Remaining variables: ['SP500_Close', 'Fed_Funds_Rate', 'Treasury_10Y', 'Brent_Oil']

VIF after removing highest VIF variable:
         Variable        VIF
0           const  22.664805
1     SP500_Close   2.306269
2  Fed_Funds_Rate   6.448571
3    Treasury_10Y   6.227166
4       Brent_Oil   2.056180

OLS Regression Results (After Removing High VIF Variable):
==================================================
                            OLS Regression Results
==============================================================================
Dep. Variable:             AAPL_Close   R-squared:                       0.953
Model:                            OLS   Adj. R-squared:                  0.952
Method:                 Least Squares   F-statistic:                     1253.
Date:                Mon, 30 Jun 2025   Prob (F-statistic):          3.18e-162
Time:                        14:30:53   Log-Likelihood:                -1037.1
No. Observations:                 251   AIC:                             2084.
Df Residuals:                     246   BIC:                             2102.
Df Model:                           4
Covariance Type:            nonrobust
==================================================================================
                     coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------
const            -99.7428      4.575    -21.803      0.000    -108.754     -90.732
SP500_Close        0.0561      0.001     45.138      0.000       0.054       0.059
Fed_Funds_Rate     1.5925      1.286      1.238      0.217      -0.941       4.126
Treasury_10Y      -1.6120      2.164     -0.745      0.457      -5.875       2.651
Brent_Oil          0.0988      0.071      1.389      0.166      -0.041       0.239
==============================================================================
Omnibus:                        1.375   Durbin-Watson:                   0.131
Prob(Omnibus):                  0.503   Jarque-Bera (JB):                1.345
Skew:                          -0.083   Prob(JB):                        0.511
Kurtosis:                       2.682   Cond. No.                     1.83e+04
==============================================================================

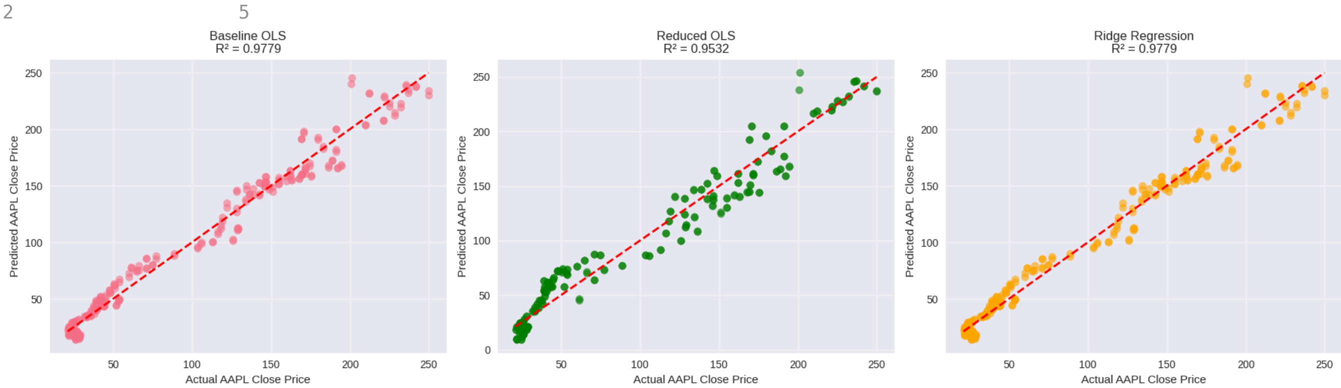
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.83e+04. This might indicate that there are
strong multicollinearity or other numerical problems.

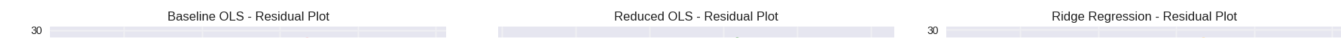
METHOD 2: RIDGE REGRESSION
------------------------------
Ridge Regression Results (Best Alpha = 0.1):
----------------------------------------
R-squared: 0.9779
MSE: 107.49

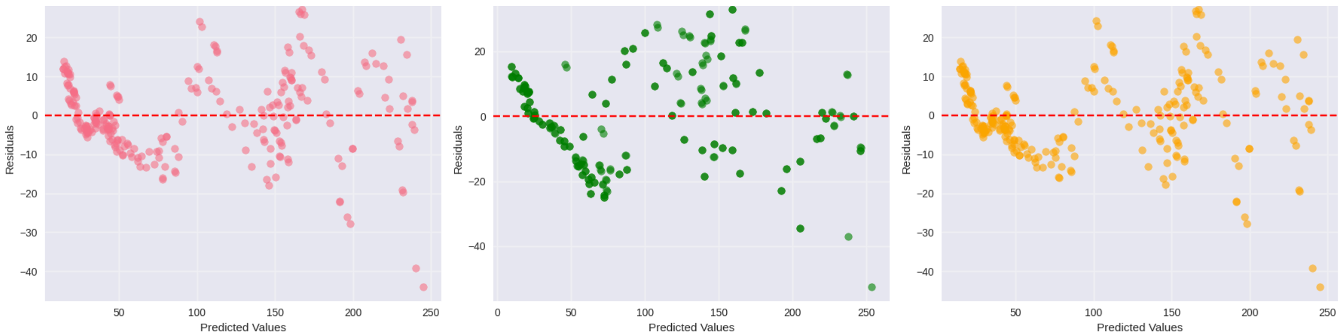
Ridge Regression Coefficients:
         Variable  Ridge_Coefficient
0     SP500_Close          30.364875
1  Fed_Funds_Rate         -12.828915
2    Treasury_10Y          -1.997383
3             CPI          52.472219
4       Brent_Oil          -3.698337

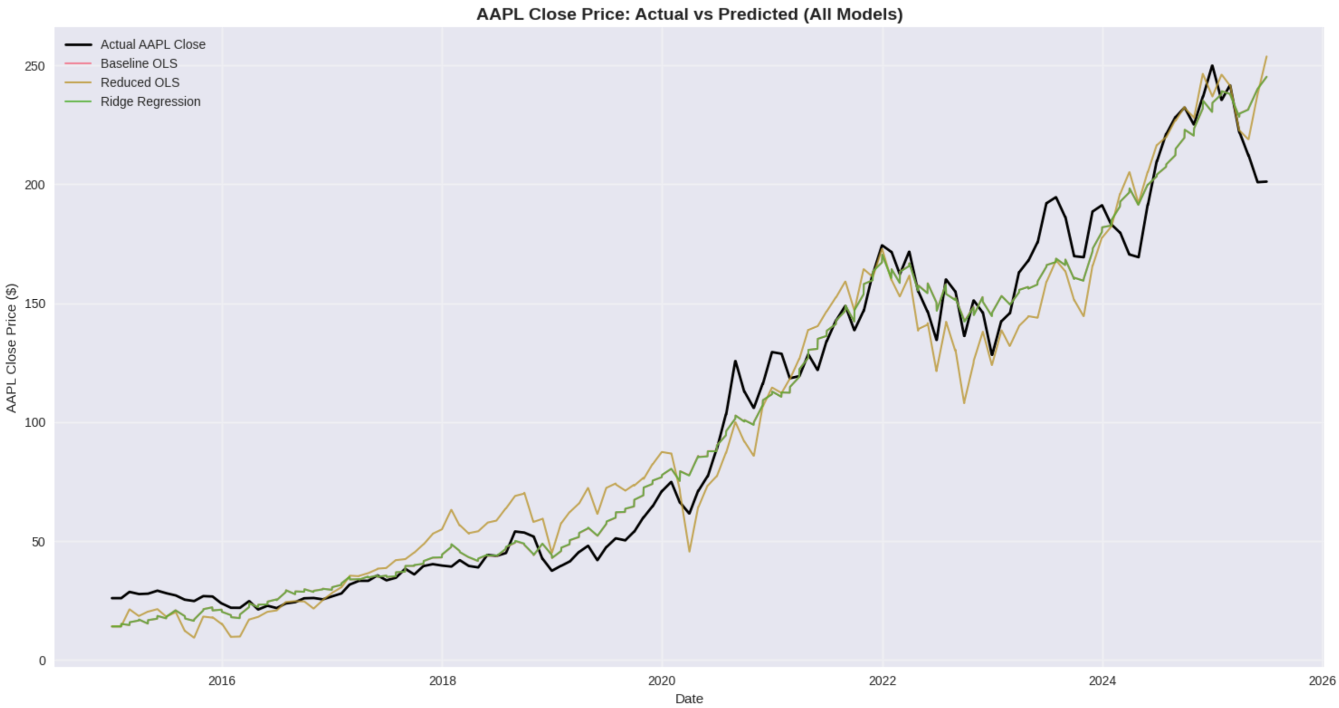
7. MODEL COMPARISON AND VISUALIZATION
----------------------------------------
Model Comparison Summary:
==================================================
              Model  R-squared  Adjusted R-squared        AIC  \
0      Baseline OLS     0.9779              0.9774  1898.3290
1       Reduced OLS     0.9532              0.9524  2084.1403
2  Ridge Regression     0.9779                 NaN        NaN

   Number of Variables
0                    5
1                    4
2                    5
```

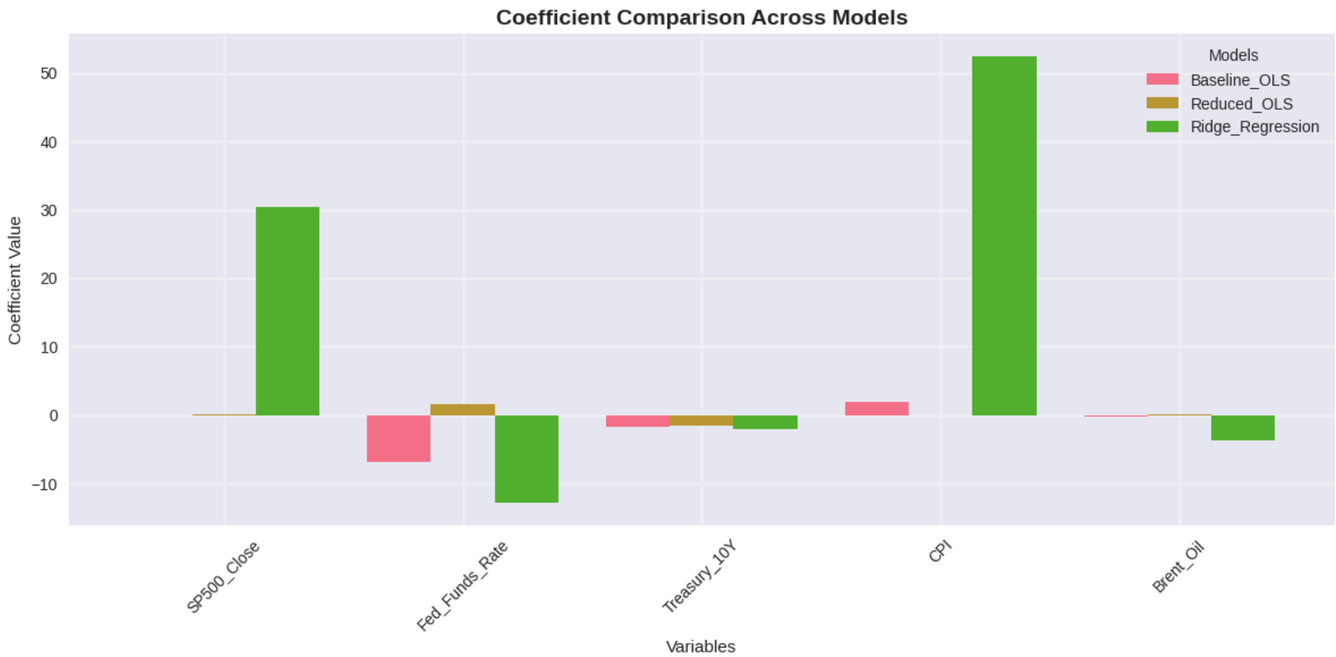









```
8. COEFFICIENT COMPARISON
----------------------------------------
Coefficient Comparison:
========================================
                Baseline_OLS  Reduced_OLS  Ridge_Regression
SP500_Close           0.0257       0.0561           30.3649
Fed_Funds_Rate       -6.8433       1.5925          -12.8289
Treasury_10Y         -1.7711      -1.6120           -1.9974
CPI                   1.8921       0.0000           52.4722
Brent_Oil            -0.1937       0.0988           -3.6983
```




```
9. ANALYSIS AND INTERPRETATION
==================================================

9.1 MULTICOLLINEARITY ASSESSMENT
```

```
------------------------------
Variables with multicollinearity issues (VIF > 5): ['SP500_Close', 'Fed_Funds_Rate', 'Treasury_10Y', 'CPI']
These variables show high correlation with other predictors, which can:
- Inflate standard errors of coefficients
- Make coefficients unstable and difficult to interpret
- Reduce statistical significance of individual predictors

9.2 MODEL PERFORMANCE COMPARISON
------------------------------
Key Findings:
1. Baseline OLS R²: 0.9779
2. Reduced OLS R²: 0.9532
   Change in R² after removing CPI: -0.0246
3. Ridge Regression R²: 0.9779

9.3 PRACTICAL IMPLICATIONS
------------------------------
Economic Interpretation:
- The models attempt to predict Apple stock price using macroeconomic indicators
- High correlations between macroeconomic variables (e.g., interest rates, inflation)
  are expected and reflect underlying economic relationships

Variable Removal Impact:
- Removed variable: CPI
- This variable was highly correlated with other predictors
- Significant change in R² indicates the variable contained unique information

Ridge Regression Benefits:
- Shrinks coefficients toward zero, reducing overfitting
- Handles multicollinearity by penalizing large coefficients
- Maintains all variables while reducing their individual impact

9.4 RECOMMENDATIONS
------------------------------
Best performing model: Baseline OLS (R² = 0.9779)

For practical use:
1. If interpretability is key: Use Reduced OLS (simpler, fewer variables)
2. If prediction accuracy is priority: Use Ridge Regression (handles multicollinearity)
3. If theoretical completeness is important: Use Baseline OLS (all variables)

Limitations and Considerations:
- Stock prices are influenced by many factors not captured in this model
- Macroeconomic relationships may change over time (structural breaks)
- Non-linear relationships might be better captured with other methods
- Model assumes linear relationships between variables

10. FINAL SUMMARY STATISTICS
==================================================
                 Metric                                         Value
 Number of Observations                                           251
             Date Range                            2015-01 to 2025-06
         Baseline OLS R²                                       0.9779
     Baseline OLS Adj R²                                       0.9774
          Reduced OLS R²                                       0.9532
     Ridge Regression R²                                       0.9779
Variables with High VIF SP500_Close, Fed_Funds_Rate, Treasury_10Y, CPI
       Removed Variable                                           CPI
     Best Alpha (Ridge)                                           0.1

==================================================
ANALYSIS COMPLETE
==================================================
This comprehensive analysis demonstrates the impact of multicollinearity
on Apple stock price prediction models using macroeconomic factors.
The comparison shows different approaches to handle multicollinearity
and their effects on model performance and interpretability.
```

Non Stationarity Modeling

```
!pip install arch
```

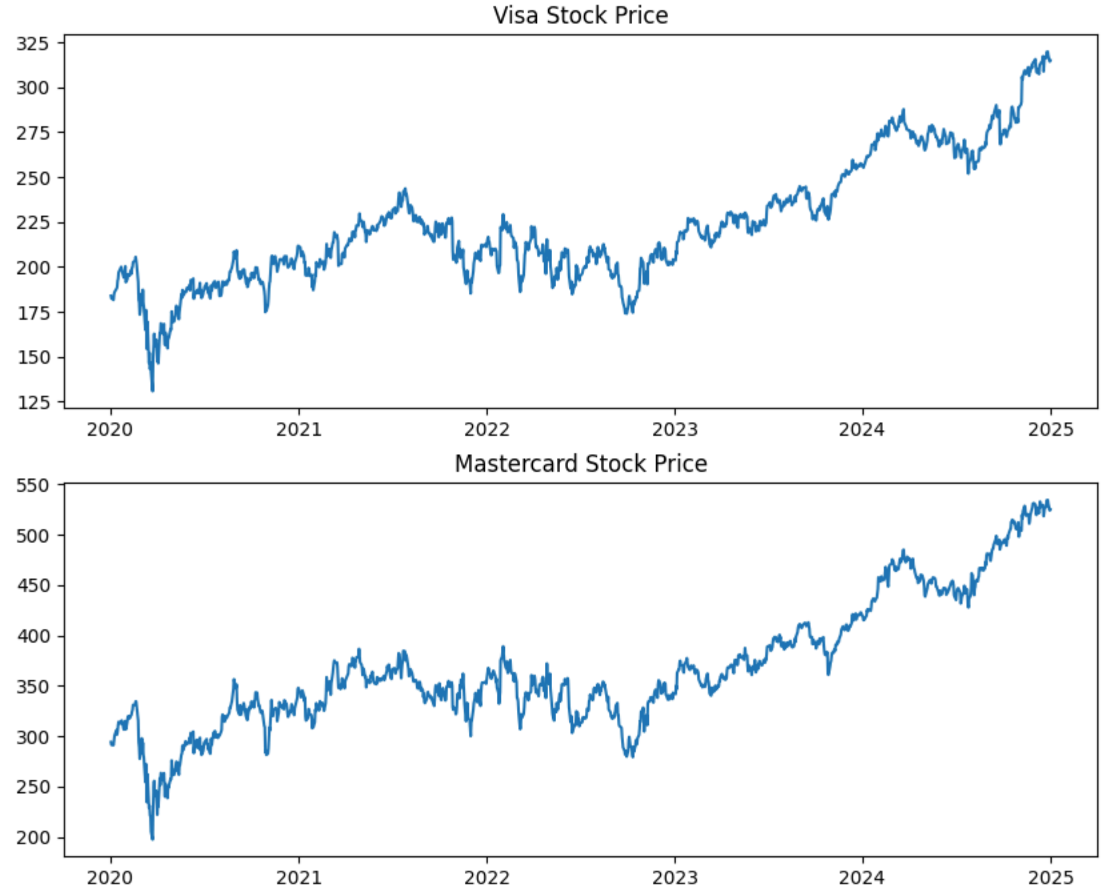
```
Collecting arch
  Downloading arch-7.2.0-cp311-cp311-manylinux_2_17_x86_64.manylinux2014_x86_64.whl.metadata (13 kB)
Requirement already satisfied: numpy>=1.22.3 in /usr/local/lib/python3.11/dist-packages (from arch) (2.0.2)
Requirement already satisfied: scipy>=1.8 in /usr/local/lib/python3.11/dist-packages (from arch) (1.15.3)
Requirement already satisfied: pandas>=1.4 in /usr/local/lib/python3.11/dist-packages (from arch) (2.2.2)
Requirement already satisfied: statsmodels>=0.12 in /usr/local/lib/python3.11/dist-packages (from arch) (0.14.4)
Requirement already satisfied: python-dateutil>=2.8.2 in /usr/local/lib/python3.11/dist-packages (from pandas>=1.4->arch
Requirement already satisfied: pytz>=2020.1 in /usr/local/lib/python3.11/dist-packages (from pandas>=1.4->arch) (2025.2)
Requirement already satisfied: tzdata>=2022.7 in /usr/local/lib/python3.11/dist-packages (from pandas>=1.4->arch) (2025.
Requirement already satisfied: patsy>=0.5.6 in /usr/local/lib/python3.11/dist-packages (from statsmodels>=0.12->arch) (1
Requirement already satisfied: packaging>=21.3 in /usr/local/lib/python3.11/dist-packages (from statsmodels>=0.12->arch)
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.11/dist-packages (from python-dateutil>=2.8.2->pandas>
Downloading arch-7.2.0-cp311-cp311-manylinux_2_17_x86_64.manylinux2014_x86_64.whl (985 kB)
   ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 985.3/985.3 kB 33.1 MB/s eta 0:00:00
Installing collected packages: arch
Successfully installed arch-7.2.0
```

```
import yfinance as yf
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np
import statsmodels.api as sm
from statsmodels.tsa.stattools import adfuller
from statsmodels.graphics.tsaplots import plot_acf
from statsmodels.graphics.tsaplots import plot_pacf
from arch.unitroot import ADF
from statsmodels.stats.stattools import durbin_watson
from statsmodels.tsa.arima.model import ARIMA
from arch.unitroot.cointegration import phillips_ouliaris
from statsmodels.regression.linear_model import GLSAR
from statsmodels.tsa.vector_ar.var_model import VAR
from statsmodels.tsa.vector_ar.vecm import coint_johansen
from statsmodels.tsa.vector_ar.vecm import VECM
```

```
tickers = ['V', 'MA'] #ES=F is E-mini S&P500 Futures, YM=F is Mini Dow Jones Industrial Futures
start_date = '2020-01-01'
end_date = '2025-01-01'
assets = yf.download(tickers, start=start_date, end=end_date, auto_adjust=False)['Adj Close']
assets.dropna(inplace=True)
```

```
[*********************100%***********************]  2 of 2 completed
```

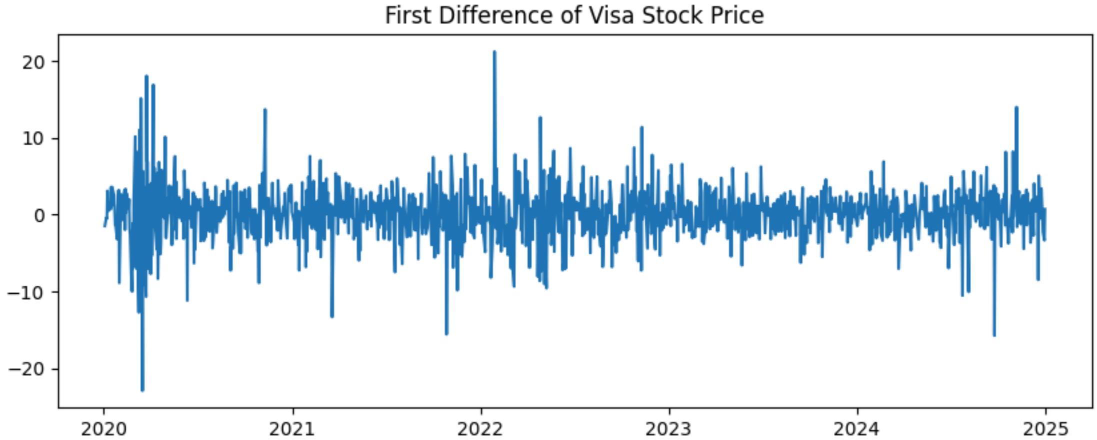
```
fig, axs = plt.subplots(2, figsize=(10,8))
ax1, ax2= axs
ax1.plot(assets['V'])
ax1.set_title('Visa Stock Price')
ax2.plot(assets['MA'])
ax2.set_title('Mastercard Stock Price')
plt.show()
```

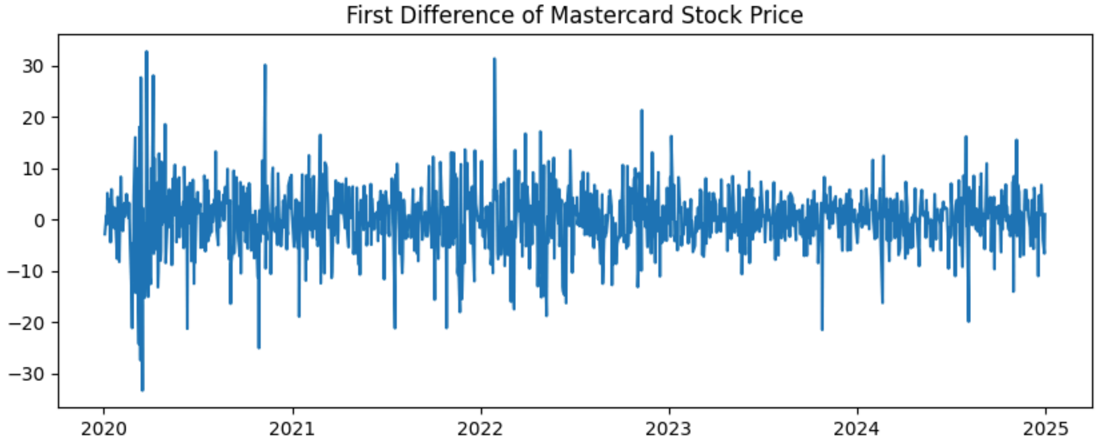


The time series seem to have similar growth patterns. However, they are not stationary. Let's look at their I(1) plot and properties

```
assets_diff = assets.diff().dropna()
```

```
fig,axs = plt.subplots(2, figsize=(10,8))
ax1, ax2 = axs
ax1.plot(assets_diff['V'])
ax1.set_title('First Difference of Visa Stock Price')
ax2.plot(assets_diff['MA'])
ax2.set_title('First Difference of Mastercard Stock Price')
plt.show()
```





The assets now seem to be stationary, at least graphically. We will now test this with the Augmented Dickey Fuller test

```
visa_adf = adfuller(assets['V'])
mastercard_adf = adfuller(assets['MA'])


visa_I1_adf = adfuller(assets_diff['V'])
mastercard_I1_adf = adfuller(assets_diff['MA'])

adf_df = pd.DataFrame({
    'Asset': ['Visa', 'Mastercard'],
    'ADF of I(0) TS p-value': [visa_adf[1], mastercard_adf[1]],
    'ADF of I(1) TS p-value': [visa_I1_adf[1], mastercard_I1_adf[1]]
})
adf_df.head()
```

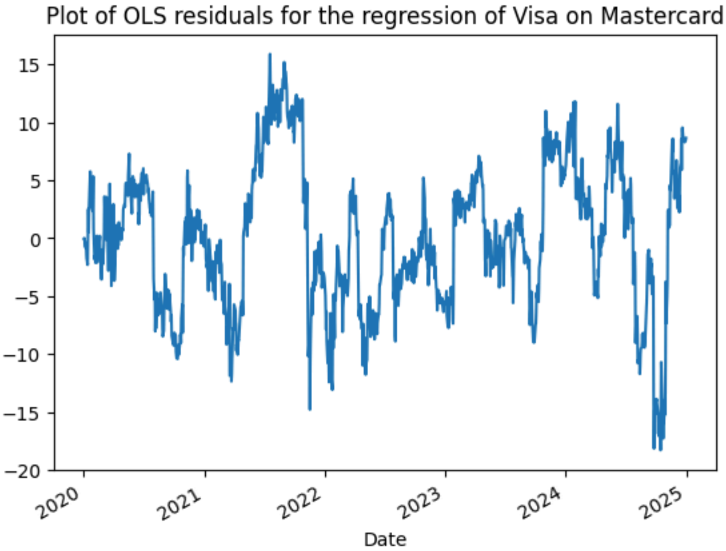
	Asset	ADF of I(0) TS p-value	ADF of I(1) TS p-value
0	Visa	0.916298	2.623139e-20
1	Mastercard	0.888578	6.611255e-20

We see that our time series are I(1) given that we don't reject the null of non-stationarity for the price time series, but we reject it for the time series differentiated of order 1. We now run OLS to check if the residuals of the regression of E-mini SP500 futures on DJIA Futures are I(0).

```
ols = sm.OLS(assets['V'], sm.add_constant(assets['MA'])).fit()
print(ols.summary())
ols.resid.plot()
plt.title('Plot of OLS residuals for the regression of Visa on Mastercard');
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      V   R-squared:                       0.967
Model:                            OLS   Adj. R-squared:                  0.967
Method:                 Least Squares   F-statistic:                 3.724e+04
Date:                Mon, 30 Jun 2025   Prob (F-statistic):               0.00
Time:                        02:36:13   Log-Likelihood:                -4072.5
No. Observations:                1258   AIC:                             8149.
Df Residuals:                    1256   BIC:                             8159.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         27.9506      1.025     27.260      0.000      25.939      29.962
MA             0.5303      0.003    192.980      0.000       0.525       0.536
==============================================================================
Omnibus:                        5.695   Durbin-Watson:                   0.068
Prob(Omnibus):                  0.058   Jarque-Bera (JB):                4.951
Skew:                          -0.085   Prob(JB):                       0.0841
Kurtosis:                       2.744   Cond. No.                     2.20e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 2.2e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```



Checking for cointegration of our time-series

```
df = pd.concat([assets['V'], assets['MA']], axis=1)
model = VAR(df)
vecm_order = model.select_order(maxlags=5, trend='c')
vecm_order.summary()
```

```
/usr/local/lib/python3.11/dist-packages/statsmodels/tsa/base/tsa_model.py:473: ValueWarning: A date index has been provi
  self._init_dates(dates, freq)
```

VAR Order Selection (* highlights the minimums)

	AIC	BIC	FPE	HQIC
0	11.94	11.95	1.528e+05	11.94
1	4.572	4.596*	96.72	4.581
2	4.566	4.607	96.20	4.582
3	4.557*	4.614	95.30*	4.579*
4	4.561	4.635	95.66	4.589
5	4.562	4.652	95.77	4.596

Given that we are trying to model the long-term equilibrium and therefore the true relationship of the two time-series, we will choose lag 1 which minimizes the BIC

```
johansen = coint_johansen(df, det_order=0, k_ar_diff=1)

pd.DataFrame(
    {
```

```
        "Test statistic": johansen.trace_stat,
        "Critical values (90%)": johansen.trace_stat_crit_vals[:, 0],
        "Critical values (95%)": johansen.trace_stat_crit_vals[:, 1],
        "Critical values (99%)": johansen.trace_stat_crit_vals[:, 2],
    },
    index=["rank=0", "rank<=1"],
)
```

	Test statistic	Critical values (90%)	Critical values (95%)	Critical values (99%)
rank=0	19.181096	13.4294	15.4943	19.9349
rank<=1	0.588153	2.7055	3.8415	6.6349

Because our statistic for rank <=0 is greater than the 95% confidence statistic, and the rank <=1 statistic is not rejected, we deduce that our cointegration rank must be 1. We now move on to estimating the VEC model

```
vecm_model= VECM(df, k_ar_diff=1, coint_rank=1, deterministic='ci').fit()
print(vecm_model.summary())
```

```
Det. terms outside the coint. relation & lagged endog. parameters for equation V
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
L1.V          -0.2067      0.061     -3.364      0.001      -0.327      -0.086
L1.MA          0.0790      0.034      2.299      0.022       0.012       0.146
Det. terms outside the coint. relation & lagged endog. parameters for equation MA
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
L1.V          -0.2726      0.110     -2.478      0.013      -0.488      -0.057
L1.MA          0.0921      0.062      1.497      0.134      -0.029       0.213
                 Loading coefficients (alpha) for equation V
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ec1           -0.0112      0.016     -0.712      0.477      -0.042       0.020
                 Loading coefficients (alpha) for equation MA
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ec1            0.0372      0.028      1.324      0.186      -0.018       0.092
          Cointegration relations for loading-coefficients-column 1
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
beta.1         1.0000          0          0      0.000       1.000       1.000
beta.2        -0.5453      0.023    -24.041      0.000      -0.590      -0.501
const        -22.3724      8.458     -2.645      0.008     -38.950      -5.795
==============================================================================
/usr/local/lib/python3.11/dist-packages/statsmodels/tsa/base/tsa_model.py:473: ValueWarning: A date index has been provi
  self._init_dates(dates, freq)
```

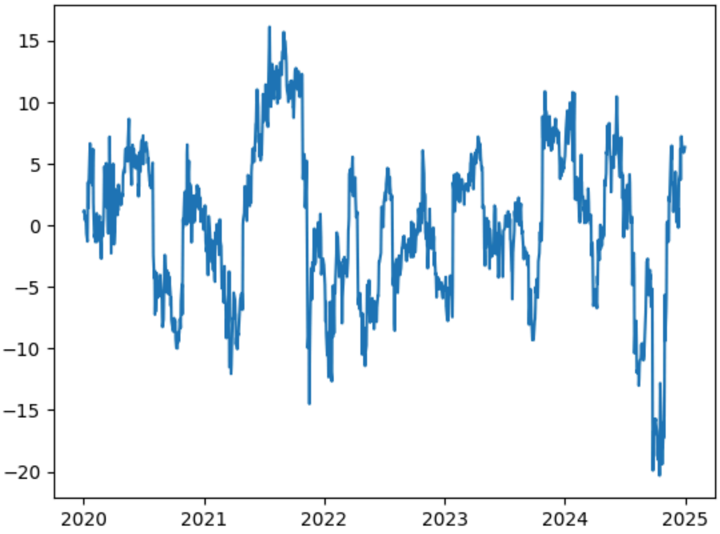
We now model the linear combination of the time series that gives a stationary time series :

S = -22.3724 + Visa - 0.5453*Mastercard

```
S = (
    vecm_model.const_coint[0][0]
    + vecm_model.beta[0][0] * df.V
    + vecm_model.beta[1][0] * df.MA
)

plt.plot(S)
plt.show()
```

```
Date
2020-01-02    1.103210
2020-01-03    1.205767
2020-01-06    0.388412
2020-01-07    0.446625
2020-01-08    0.753658
                ...
2024-12-24    5.957295
2024-12-26    5.977233
2024-12-27    5.882242
2024-12-30    6.158827
2024-12-31    6.331872
Length: 1258, dtype: float64
```



The series looks stationary, let's test it

```
S_adf = ADF(S, trend='n', method='bic')
S_adf.summary()
```

Augmented Dickey-Fuller Results

Test Statistic	-4.591
P-value	0.000
Lags	0

Trend: No Trend
Critical Values: -2.57 (1%), -1.94 (5%), -1.62 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.

Therefore, we reject H0 : our linear combination is indeed stationary.

PROBLEM 4: DETECTING REGIME CHANGE

```
import datetime
import numpy as np
import pandas as pd
import yfinance as yfin
from statsmodels.tsa.regime_switching.markov_regression import MarkovRegression
import matplotlib.pyplot as plt
import statsmodels.api as sm
from statsmodels.stats.diagnostic import breaks_cusumolsresid, breaks_hansen
from statsmodels.stats.diagnostic import het_breuschpagan
from statsmodels.stats.api import linear_rainbow
from scipy.stats import f
from statsmodels.stats.diagnostic import breaks_cusumolsresid
```

```
#  Importing Data

# Download stock prices from Yahoo Finance and set the time period for download
start = datetime.date(2005, 1, 1)
end = datetime.date(2025, 6, 1)
```

```
stocks = yfin.download(["SPY"], start, end, auto_adjust = False)["Adj Close"]
stocks.head()
```

```
[*********************100%***********************]  1 of 1 completed
```

Ticker	SPY
Date	
2005-01-03	82.074043
2005-01-04	81.071159
2005-01-05	80.511711
2005-01-06	80.921051
2005-01-07	80.805054

```
# Remove empty cells and NA's.
stocks.dropna()
```

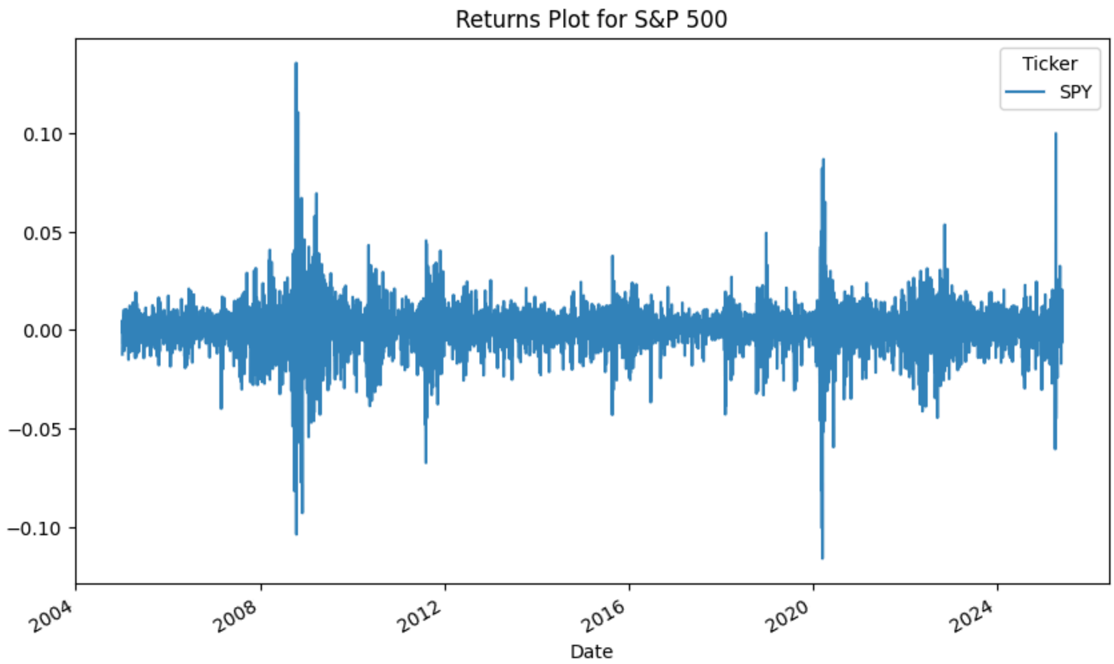
Ticker	SPY
Date	
2005-01-03	82.074043
2005-01-04	81.071159
2005-01-05	80.511711
2005-01-06	80.921051
2005-01-07	80.805054
...	...
2025-05-23	577.403015
2025-05-27	589.407593
2025-05-28	585.997620
2025-05-29	588.310791
2025-05-30	587.652771

5135 rows × 1 columns

```
#Generate Log Returns and drop all NA's
returns = np.log(stocks / stocks.shift(1)).dropna()
returns.head()
```

Ticker	SPY
Date	
2005-01-04	-0.012295
2005-01-05	-0.006925
2005-01-06	0.005071
2005-01-07	-0.001434
2005-01-10	0.004717

```
#Plot Returns for SPY to visually inspect the presence of any regime change
returns.plot(figsize=(10, 6), title='Returns Plot for S&P 500', alpha=0.9) # Plot the returns
plt.show()
```




```
#Formal test to Detect Regime Change

#APPROACH 1 : The CHOW TEST

y = returns[1:]
X = returns.shift(1)[1:]  # lagged returns
X = sm.add_constant(X)

# Fit full model
model_full = sm.OLS(y, X).fit()

#check 2008 Financial Crises-Septber - 2009/09/07 does not comfirm but 2009/03/07 confirm
breakpoint = pd.Timestamp('2009-03-07')
pre = X.index < breakpoint
post = X.index >= breakpoint

model_pre = sm.OLS(y[pre], X[pre]).fit()
model_post = sm.OLS(y[post], X[post]).fit()

# Chow F-statistic
n1, n2 = model_pre.nobs, model_post.nobs
k = X.shape[1]
RSS_pooled = np.sum(model_full.resid ** 2)
RSS_pre = np.sum(model_pre.resid ** 2)
RSS_post = np.sum(model_post.resid ** 2)

chow_num = (RSS_pooled - (RSS_pre + RSS_post)) / k
chow_denom = (RSS_pre + RSS_post) / (n1 + n2 - 2 * k)
chow_stat = chow_num / chow_denom

pval_chow = 1 - f.cdf(chow_stat, dfn=k, dfd=n1 + n2 - 2 * k)

print("📊 Chow Test at 2009-03-07:")
print(f"F-statistic = {chow_stat:.4f}, p-value = {pval_chow:.4f}\n")
```

```
📊 Chow Test at 2009-03-07:
F-statistic = 3.8974, p-value = 0.0204
```

```
#Formal test for Regime change using the the Chow test

y = returns[1:]
X = returns.shift(1)[1:]  # lagged returns
X = sm.add_constant(X)

# Fit full model
model_full = sm.OLS(y, X).fit()

#check for COVId Crises-Septber) comfirm for 2020/03/01 confirm
breakpoint = pd.Timestamp('2020-03-01')
pre = X.index < breakpoint
post = X.index >= breakpoint
```

```
model_pre = sm.OLS(y[pre], X[pre]).fit()
model_post = sm.OLS(y[post], X[post]).fit()

# Chow F-statistic
n1, n2 = model_pre.nobs, model_post.nobs
k = X.shape[1]
RSS_pooled = np.sum(model_full.resid ** 2)
RSS_pre = np.sum(model_pre.resid ** 2)
RSS_post = np.sum(model_post.resid ** 2)

chow_num = (RSS_pooled - (RSS_pre + RSS_post)) / k
chow_denom = (RSS_pre + RSS_post) / (n1 + n2 - 2 * k)
chow_stat = chow_num / chow_denom

pval_chow = 1 - f.cdf(chow_stat, dfn=k, dfd=n1 + n2 - 2 * k)

print("📊 Chow Test at 2020-03-01:")
print(f"F-statistic = {chow_stat:.4f}, p-value = {pval_chow:.4f}\n")
```

```
📊 Chow Test at 2020-03-01:
F-statistic = 5.3922, p-value = 0.0046
```

The Chow test, test the null hypothesis of no structural break against an alternative hypothesis of structural break. A signficant test as indicated above shows the presence of structural break or regime change as visualized from the plot of the SPY data.

```
# APPROCH 2: Fit two-stage Markov-Switching Model

model = MarkovRegression(returns, k_regimes=2, switching_variance=True)
res_mod = model.fit(em_iter=100, search_reps=100)
print(res_mod.summary())
```

```
/usr/local/lib/python3.11/dist-packages/statsmodels/tsa/base/tsa_model.py:473: ValueWarning: A date index has been provi
  self._init_dates(dates, freq)
                        Markov Switching Model Results
==============================================================================
Dep. Variable:                    SPY   No. Observations:                 5134
Model:               MarkovRegression   Log Likelihood               16601.945
Date:                Mon, 30 Jun 2025   AIC                         -33191.891
Time:                        05:28:21   BIC                         -33152.629
Sample:                             0   HQIC                        -33178.149
                               - 5134
Covariance Type:               approx
                             Regime 0 parameters
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0013      0.001     -2.236      0.025      -0.002      -0.000
sigma2         0.0004   2.21e-05     19.381      0.000       0.000       0.000
                             Regime 1 parameters
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0010      0.000      8.409      0.000       0.001       0.001
sigma2      4.509e-05   1.74e-06     25.890      0.000    4.17e-05    4.85e-05
                         Regime transition parameters
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
p[0->0]        0.9556      0.008    116.483      0.000       0.940       0.972
p[1->0]        0.0158      0.003      5.276      0.000       0.010       0.022
==============================================================================

Warnings:
[1] Covariance matrix calculated using numerical (complex-step) differentiation.
```

```
#Estimate the variance of the different regimes
res_mod.params['sigma2[0]']  # variance in regime 0
```

```
np.float64(0.0004279335040015034)
```

```
#Estimate the variance of the different regimes
res_mod.params['sigma2[1]']  # variance in regime 1
```

```
np.float64(4.5094622709251106e-05)
```

```
#Estimate Standard deviations
np.sqrt(res_mod.params['sigma2[0]'])  # i.e., approaximately 2.07% daily volatility in Regime 0
```

```
np.float64(0.020686553700447627)
```