

Computational Finance

Exercises for participants of the programme 'Quantitative Finance'

C-Exercise 27 (Simulating paths of geometric Brownian motion)

Consider a geometric Brownian motion defined via

$$dX(t) = \mu X(t)dt + \sigma X(t)dW(t), \quad X(0) = X_0,$$

with $\mu, X_0 \in \mathbb{R}$, $\sigma > 0$ and a Brownian motion $W(t)$. Write a function

```
[X_exact, X_Euler, X_Milshtein]=Sim_Paths_GeoBM(X0, mu, sigma, T, N)
```

that simulates the discrete path approximation $X(0), X(T/N), X(2T/N), \dots, X(T)$ using the exact solution for the geometric Brownian motion SDE (`X_exact`), using the Euler method (`X_Euler`) and using the Milshtein method (`X_Milshtein`). Use the same simulations of $W(kT/N) - W((k-1)T/N)$, $k = 1, \dots, N$, for all three methods.

Test the function for

```
X0=100, mu=0.05, sigma=0.2, T=2
```

and $N = 10, 100, 1000, 10000$. For each value of N plot all three simulated paths into a single plot.

Useful scilab commands: subplot

Please turn over.

C-Exercise 28 (Valuation of European options in the Heston model using the Euler method)

Write a scilab function

```
[V0, c1, c2] = Heston_EuCall_MC_Euler (S0, r, gamma0, kappa,  
                                     lambda, sigma_tilde, T, g, M, m)
```

that computes the price $V(0)$ of a European option with payoff $g(S_T)$ and maturity T in the Heston model via the Monte-Carlo method using M samples together with the 95%-confidence interval.

To this end, approximate the paths by the Euler method with a grid of m equidistant points in time by implementing your pseudo code from T-Exercise 29.

Test your function for the European call option using the same parameters as in C-Exercise 17, $M = 10000$ and $m = 250$. Compare your result with the result from C-Exercise 17.

T-Exercise 29 (Valuation of options in the Heston model using the Euler method)

Extend the Euler method for the Black-Scholes model presented from chapter 5.5 suitably such that it can generate paths of the price process in the Heston model.

- (a) Write your solution for the path simulation in pseudo code e.g. as presented in the lecture notes.
- (b) Write an algorithm in pseudo code that approximates using the Monte-Carlo method the price of an European up-and-out call option on the stock S with strike price K and barrier $B > K$. I.e., the option pays off at maturity T the amount

$$V(T) = 1_{\{S(t) < B \text{ for all } t \in [0, T]\}} (K - S(T))^+.$$

You may use your result from part a).

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where `##` denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

Submit until: Fri, 30.06.2017, 10:00
Discussion: Mon/Wed, 03/05.07.2017