Christian-Albrechts-Universität zu Kiel **SS 2017**
Mathematisches Seminar
Prof. Dr. Mathias Vetter
Ole Martin, Adrian Theopold Sheet 08

Computational Finance

Exercises for participants of the programm **'Quantitative Finance'**

C-Exercise 24 (Using antithetic variables to reduce the variance of MC-estimators)
Write a scilab function

```
[V0, epsilon] = BS_EuOption_MC_AV (S0, r, sigma, T, K, M, g)
```

that computes the initial price of a European option with payoff $g(S_T)$ at time T in the Black-Scholes model via the Monte-Carlo approach with $M \in \mathbb{N}$ samples. Use the method of antithetic variables to reduce the variance of the estimator. In addition, the function shall return the radius ε of a confidence interval that contains the true price with a probability of approximately 95% (cf. Section 5.1).

Test your function for

$$S(0) = 100, \quad r = 0.05, \quad \sigma = 0.2, \quad T = 1, \quad K = 100, \quad M = 100000, \quad g(x) = (K-x)^+$$

and compare the result to the exact value (cf. the formula below (3.23) in the lecture notes) and the plain Monte Carlo simulation (cf. C-Exercise 21).

Hint: Modify the function of C-Exercise 21 appropriately.

C-Exercise 25QF (Using control variables to reduce the variance of MC-estimators)
Write a scilab function

```
V0 = BS_EuOption_MC_CV (S0, r, sigma, T, K, M, g)
```

that computes the initial price of a European option with payoff $g(S(T))$ in the Black-Scholes model via the Monte-Carlo approach with $M \in \mathbb{N}$ samples. Use the stock as control variate to reduce the variance of the estimator. To this end, estimate in a first Monte-Carlo simulation with M samples the optimal value

$$\frac{\mathrm{Cov}(g(S(T)), S(T))}{\mathrm{Var}(S(T))}.$$

Test your function for the European call with payoff $g(x) = (x-K)^+$ and parameters

$$S(0) = 120, \quad r = 0.05, \quad \sigma = 0.2, \quad T = 1, \quad K = 100, \quad M = 100000,$$

and compare the result to the exact value (cf. formula (3.23) from the lecture notes) and the plain Monte-Carlo simulation (cf. C-Exercise 21).

Please turn over.

C-Exercise 26 (Pricing a deep out-of-the-money European call option by Monte-Carlo with importance sampling)
Consider a Black-Scholes model with parameters S_0, r, $\sigma > 0$. The goal is to approximate by the Monte-Carlo method the fair price V_0 of an European call option on the stock with strike $K \gg S_0$ at maturity T.

Write a Scilab function

```
[V0, CIl, CIr] = BS_EuCall_MC_IS (S0, r, sigma, K, T, mu, N,
                                  alpha)
```

that approximates the price of the European call option via Monte-Carlo based on $N \in \mathbb{N}$ samples and additionally returns the left and right boundary of an asymptotic α-level confidence interval. Use a new random variable $Y \sim N(\mu, 1)$ for the importance sampling method.

Test your function for $S_0 = 100$, $r = 0.05$, $\sigma = 0.2$, $K = 200$, $T = 1$, $N = 10000$, $\alpha = 0.95$ and plot your estimator $V0$ in dependence on μ.

Hint: Experiment with the range of μ such that you can see visible changes in the variance of your estimator.

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

Submit until: Fri, 23.06.2017, 10:00
Discussion: Tue, 26/28.06.2017