Christian-Albrechts-Universität zu Kiel **SS 2017**
Mathematisches Seminar
Prof. Dr. Mathias Vetter
Ole Martin, Adrian Theopold Sheet 05

Computational Finance

Exercises for participants of the programme **'Quantitative Finance'**

T-Exercise 14

Let W_1, W_2 be independent standard Brownian motions. Consider a market with three assets S_0, S_1, S_2, which follow the equations

$$\begin{aligned} S_0(t) &= 1, \\ \mathrm{d}S_1(t) &= S_1(t)\,(3\mathrm{d}t + \mathrm{d}W_1(t) - \mathrm{d}W_2(t))\,, \\ \mathrm{d}S_2(t) &= S_2(t)\,(1\mathrm{d}t - \mathrm{d}W_1(t) + \mathrm{d}W_2(t))\,. \end{aligned}$$

Construct an arbitrage in this market.
Hint: For an arbitrary self-financing strategy $\varphi = (\varphi_0, \varphi_1, \varphi_2)$, represent $\hat{V}_\varphi$ as Itō process. Then try to choose φ_2 such that the risk is eliminated.

T-Exercise 15 (Black-Scholes price of a forward start call)

A *forward start option* is an option that transforms at time T_0 to a European call option with strike $S(T_0)$, i.e., it pays off at maturity $T > T_0$ the amount

$$V(T) = (S(T) - S(T_0))^+ \,.$$

Determine the fair price process $v(t, S(t))$ and the perfect hedging strategy $\varphi(t) = (\varphi_0(t), \varphi_1(t))$ of the forward start option in the Black-Scholes model for all $t \in [0, T]$.
Hint: Recap the basic properties of conditional expectations.

C-Exercise 16 (Black-Scholes price of a down-and-out call)

a) Write a function

```
V_t = BS_Price_DownOut_Call (r, sigma, S_t, T, K, H, t)
```

that computes the price of a down-and-out call with maturity T, strike K and barrier H at time t given the stock price $S(t) = S_t$ in the Black-Scholes model with parameters $r, \sigma > 0$ according to the formula from the lecture notes.

b) For $r = 0.03$, $\sigma = 0.3$, $T = 1$, $H = 80$ plot V_t as a function of both t and S_t in the range $(t, S_t) \in [0,1] \times [70,130]$ for $K = 80, 90, 100, 120$.

Useful commands: `cdfnor`, `surf`, `subplot`, `xtitle`

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

Submit until: Fri, 26.05.2017, 10:00
Discussion: 29./31.05.2017