**Christian-Albrechts-Universität zu Kiel** **SS 2017**
Mathematisches Seminar
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# Computational Finance

Exercises for participants of the programme **'Quantitative Finance'**

**C-Exercise 01**

Write a scilab function

```
Vn = capital(V0, r, n, c)
```

that computes and returns the capital $V_n$ if an interest of $r > 0$ has been paid on the initial endowment $V_0 > 0$ for $n \in \mathbb{N}$ years. If $c = 1$, the variable $r$ refers to a continuous rate (i.e., $V_n = V_0 e^{rn}$), and if $c = 0$, it refers to a simple rate (i.e., $V_n = V_0(1+r)^n$). Test your function for

$$V_0 = 1000, \quad r = 0.05, \quad n = 10, \quad c = 0.$$

*Useful scilab commands:* `if`, `elseif`, `else`, `exp`

**C-Exercise 02**

We are given two vectors $x = (x_1, \dots, x_n)^\top$ and $y = (y_1, \dots, y_n)^\top$ of the same length, where $^\top$ denotes the transpose of a vector or matrix. We consider the problem of finding numbers $\vartheta_1$, $\vartheta_2$, $\vartheta_3$ and $\vartheta_4$ such that the approximation

$$y_k \approx \vartheta_1 x_k^3 + \vartheta_2 x_k^2 + \vartheta_3 x_k + \vartheta_4, \quad k = 1, \dots, n,$$

holds as closely as possible. The solution $\widehat{\vartheta} = \left(\widehat{\vartheta}_1, \widehat{\vartheta}_2, \widehat{\vartheta}_3, \widehat{\vartheta}_4\right)^\top$ to this problem is given by $\widehat{\vartheta} = (X^\top X)^{-1} X^\top y$ with

$$X := \begin{pmatrix} x_1^3 & x_1^2 & x_1 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ x_n^3 & x_n^2 & x_n & 1 \end{pmatrix}.$$

The approximated $y$-values are then given by $\widehat{y} := X\widehat{\vartheta}$. Write a scilab function

```
[y_hat, theta_hat] = cubic_regression (x, y)
```

that computes $\widehat{\vartheta}$ and $\widehat{y}$ for vectors $x = (x_1, \dots, x_n)^\top$ and $y = (y_1, \dots, y_n)^\top$. Test your function for

$$x = (0, 1, 2, 3, 4)^\top, \quad y = (1, 0, 3, 5, 8)^\top.$$

Plot $x$ against $y$ (as points) and the function $u \mapsto \vartheta_1 u^3 + \vartheta_2 u^2 + \vartheta_3 u + \vartheta_4$ (as a line plot) in a common graph for this example.

*Useful scilab commands:* `ones`, `inv`, `plot`, `title`, `xlabel`, `ylabel`, `legend`

**C-Exercise 03**

Let $s_1, \ldots, s_N$ denote a time series of, e.g., stock prices on days $1, \ldots, N$. The *logarithmic return* (log-return) of the stock on day $n \in \{2, \ldots, N\}$ is given by

$$l_n := \log\left(\frac{s_n}{s_{n-1}}\right) = \log(s_n) - \log(s_{n-1}).$$

Assuming 250 trading days per year, the *annualized empirical mean* of log-returns is given by

$$\hat{\mu} = \frac{250}{N-1} \sum_{k=2}^{N} l_k$$

and the *annualized empirical standard deviation* of log-returns is given by

$$\hat{\sigma} = \sqrt{\frac{250}{N-2} \sum_{k=2}^{N} \left(l_k - \frac{\hat{\mu}}{250}\right)^2}.$$

Write a scilab function

```
function lr = log_returns (data)
```

that computes and returns the time series of log-returns for the time series given in `data`.

On the web page of the course you find the file `time_series_dax.csv` containing a time series of daily DAX data. Import this time series and test your function with it. Visualize the time series of log-returns in a plot. Compute and display the annualized empirical mean and standard deviation of the log-returns.

*Useful scilab commands:* `diff`, `log`, `csvRead`, `mean`, `variance`, `disp`, `string`

**T-Exercise 04**

We want to price an American put option with strike price $K = 1.1$ and time to maturity being three years. For this purpose we want to utilize a CRR model with $M = 3$ equally spaced time periods, $S(0) = 1$, $\sigma^2 = 0.2$ and an annual interest rate of 5%.

a) Draw and calculate the corresponding CRR model by hand (of course you can still use a calculator) and write beneath each point the corresponding price of the option (please round on four position after the decimal point after each calculation).

b) Calculate the replicating portfolio for the first time period $\varphi_0(1), \varphi_1(1)$.

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

**Submit until:** Fri, 28.04.2017, 10:00
**Discussion:** 02/03.05.2017,