Christian-Albrechts-Universität zu Kiel **SS 2017**
Mathematisches Seminar
Prof. Dr. Mathias Vetter
Ole Martin, Adrian Theopold Sheet 06

Computational Finance

Exercises for participants of the programme **'Quantitative Finance'**

C-Exercise 17
Write a scilab function

```
Vt = Heston_EuCall_Laplace (St, r, gamma0, kappa, lambda,
                sigma_tilde, T, t, K, R)
```

that computes the price of a European call option in the Heston model at time t via the Laplace transform approach.

Test your function for

$$S(t) = 100, \quad r = 0.05, \quad \gamma(0) = 0.2^2, \quad \kappa = 0.5, \quad \lambda = 2.5,$$
$$\widetilde{\sigma} = 1, \quad T = 1, \quad t = 0, \quad K = 100, \quad R = 3.$$

Usefull scilab commands: `%i`, `sinh`, `cosh`

C-Exercise 18
Write a scilab function

```
V0 = BS_EuCall_FFT (S0, r, sigma, T, K, R, N, M)
```

that computes the initial price of a bunch of European call option with identical maturity T and strikes $K = (K_1, \ldots, K_n)$ in the Black-Scholes model via the fast Fourier transform approach. Use the formulas from Section 4.2 in the lecture notes. Test your function for

```
S0=100, r=0.05, sigma=0.2, T=1, K=(80,81,...,130), R=1.1, N=2^11,
                                M=50.
```

Usefull scilab commands: `fft`, `real`, `interpln`

Please turn page.

T-Exercise 19 (Merton's jump diffusion model)
In the Merton model the logarithmic stock price is of the form

$$X(t) = X_0 + \mu t + \sigma W(t) + \sum_{j=1}^{N(t)} Y_j$$

where W is a standard Brownian motion, $X_0, \mu \in \mathbb{R}$, $\sigma > 0$, $N(t)$ is a Poisson random variable with Parameter λt for $\lambda > 0$ and $Y_1, Y_2, \ldots$ are normally distributed with mean $m \in \mathbb{R}$ and variance $s^2 > 0$. Moreover $W, N(t), Y_1, Y_2, \ldots$ are all independent. Calculate the characteristic function

$$\chi_t(u) = \mathbb{E}[e^{iuX(t)}].$$

Hints:

a) *You may use without proof that the characteristic function of a random variable* $Z \sim \mathcal{N}(\mu, \sigma^2)$ *is of the form* $\mathbb{E}[\exp(iuZ)] = \exp(iu\mu - u^2\sigma^2/2)$.

b) *It holds* $\exp(\sum_{j=1}^{N(t)} Y_j) = \sum_{n=1}^{\infty} \mathbb{1}_{\{N(t)=n\}} \exp(\sum_{j=1}^{n} Y_j)$.

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

Submit until: Fri, 02.06.2017, 10:00
Discussion: 12./14.06.2017