Christian-Albrechts-Universität zu Kiel **SS 2017**
Mathematisches Seminar
Prof. Dr. Mathias Vetter
Ole Martin, Adrian Theopold Sheet 02

Computational Finance

Exercises for participants of the programme **'Quantitative Finance'**

C-Exercise 05 (Numerical convergence of put option prices in the binomial model)
In the binomial model with $M \in \mathbb{N}$ steps that approximates the risk-neutral dynamics of a Black-Scholes model with parameters S_0, r, $\sigma > 0$ and time horizon $T > 0$, the fair initial price V_0 of a European put option on the stock with maturity T and strike $K > 0$ is given by

$$V_0 = Ke^{-rT}\mathrm{b}_{M,p}(a-1) - S_0\mathrm{b}_{M,\tilde{p}}(a-1)$$

with $a := \left\lceil \frac{\log(K/S_0) - M\log(d)}{\log(u/d)} \right\rceil$, $\tilde{p} := p\frac{u}{e^{r\Delta t}}$ and $\mathrm{b}_{N,q}(\cdot)$ denoting the cumulative distribution function of the binomial distribution with $N \in \mathbb{N}$ trials and success probability $q \in [0,1]$. $\lceil x \rceil$ is the smallest integer bigger or equal to x for $x \in \mathbb{R}$.

a) Write a Scilab function

```
V0 = BinMod_EuPut_CF (S0, r, sigma, T, M, K)
```

that computes the put option price in the approximating binomial model.

b) For $S_0 = 100$, $r = 0.05$, $\sigma = 0.2$, $T = 1$, $K = 100$, plot the put option price in the binomial model in dependence on the number of steps M in the range $[10, \ldots, 500]$.

Useful scilab commands: `log`, `sqrt`, `cdfbin`, `ceil`, `plot`

C-Exercise 06 (American put option in the CRR model)
Write a scilab function

```
V_0 = CRR_AmPut (S_0, r, sigma, T, M, K)
```

that computes and returns an approximation to the price of an American put option with strike $K > 0$ and maturity $T > 0$ in the Black-Scholes model with initial stock price $S(0) > 0$, interest rate $r > 0$ and volatility $\sigma > 0$. Use the binomial method as presented in the course with $M \in \mathbb{N}$ time steps.

Test your algorithm with

$$S(0) = 100,\ r = 0.03,\ \sigma = 0.24,\ T = 3/4,\ M = 500,\ K = 95.$$

Useful scilab commands: `exp`, `sqrt`, `max`, `for`

T-Exercise 07 Let X be a Brownian motion with drift $\mu \in \mathbb{R}$ and diffusion coefficient $\sigma > 0$, i.e. $X(t) = \mu t + \sigma W(t)$ where W denotes a SBM. Moreover, we consider the following functions:

$$\begin{aligned} f_1(x) &:= x^3 \\ f_2(x) &:= \exp(x) \\ f_3(x) &:= 6x + 2 \end{aligned}$$

(a) For $i = 1, 2, 3$ represent the process $f_i(X(t))$ as an Itô process, i.e. in the form

$$d(f_i(X(t))) = \ldots dt + \ldots dW(t).$$

(b) For $i = 1, 2, 3$ calculate the associated quadratic variation process $[f_i(X)]_t$.

(c) Calculate the covariation process $[f_1(X), f_3(X)]_t$.

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

Submit until: Fri, 05.05.2017, 10:00
Discussion: Mon, 08.05.2017 and Wed, 10.05.2017