Stacks

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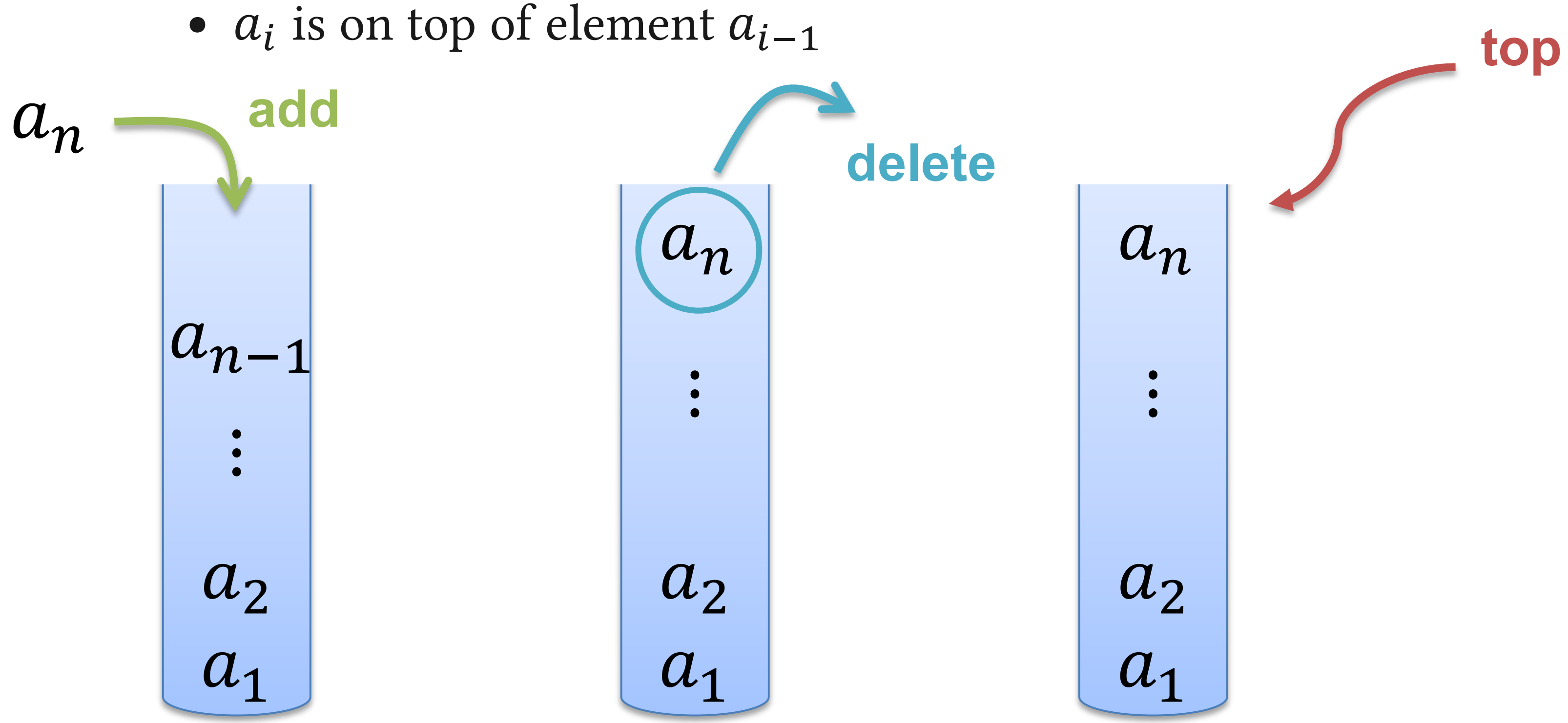
Review

- Array

- 2D Array = Matrix
  - Row-Major
  - Column-Major
  - Upper-Triangular
  - Lower-Triangular

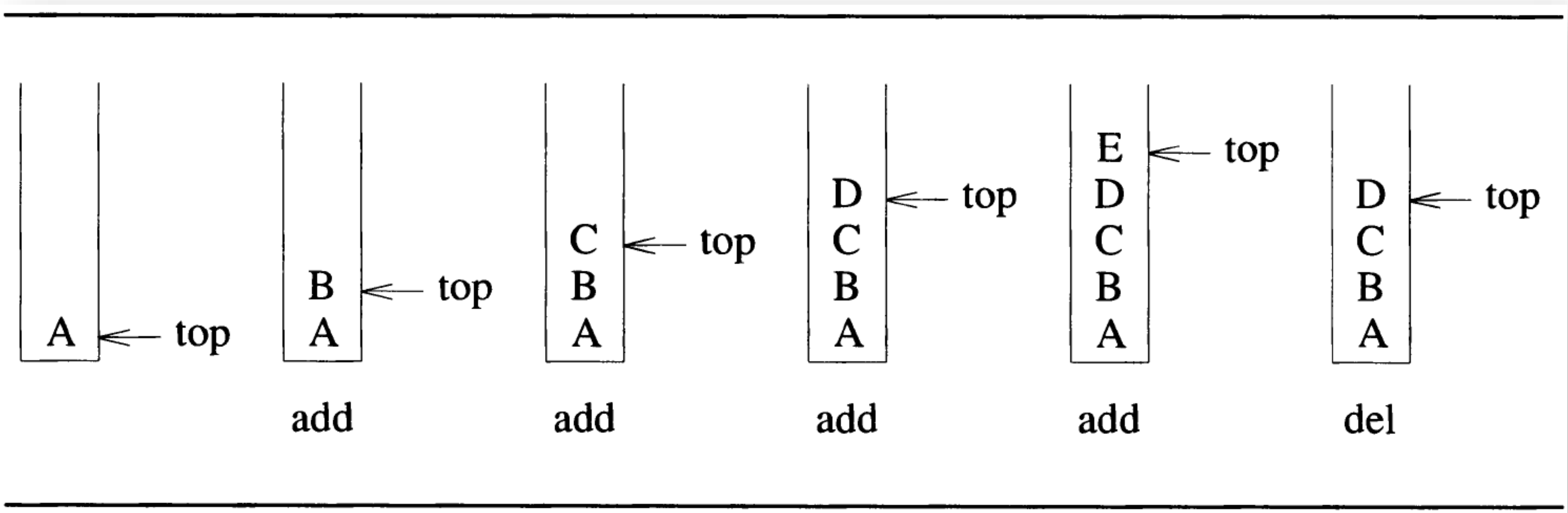
Stacks.

- A **stack** is an **ordered** list in which insertions and deletions are made at one end called the **top**
  - Given a stack $S = (a_1, a_2, \ldots, a_n)$
    - a_1 is the bottom element
    - a_n is the top element
    - a_i is on top of element a_{i-1}

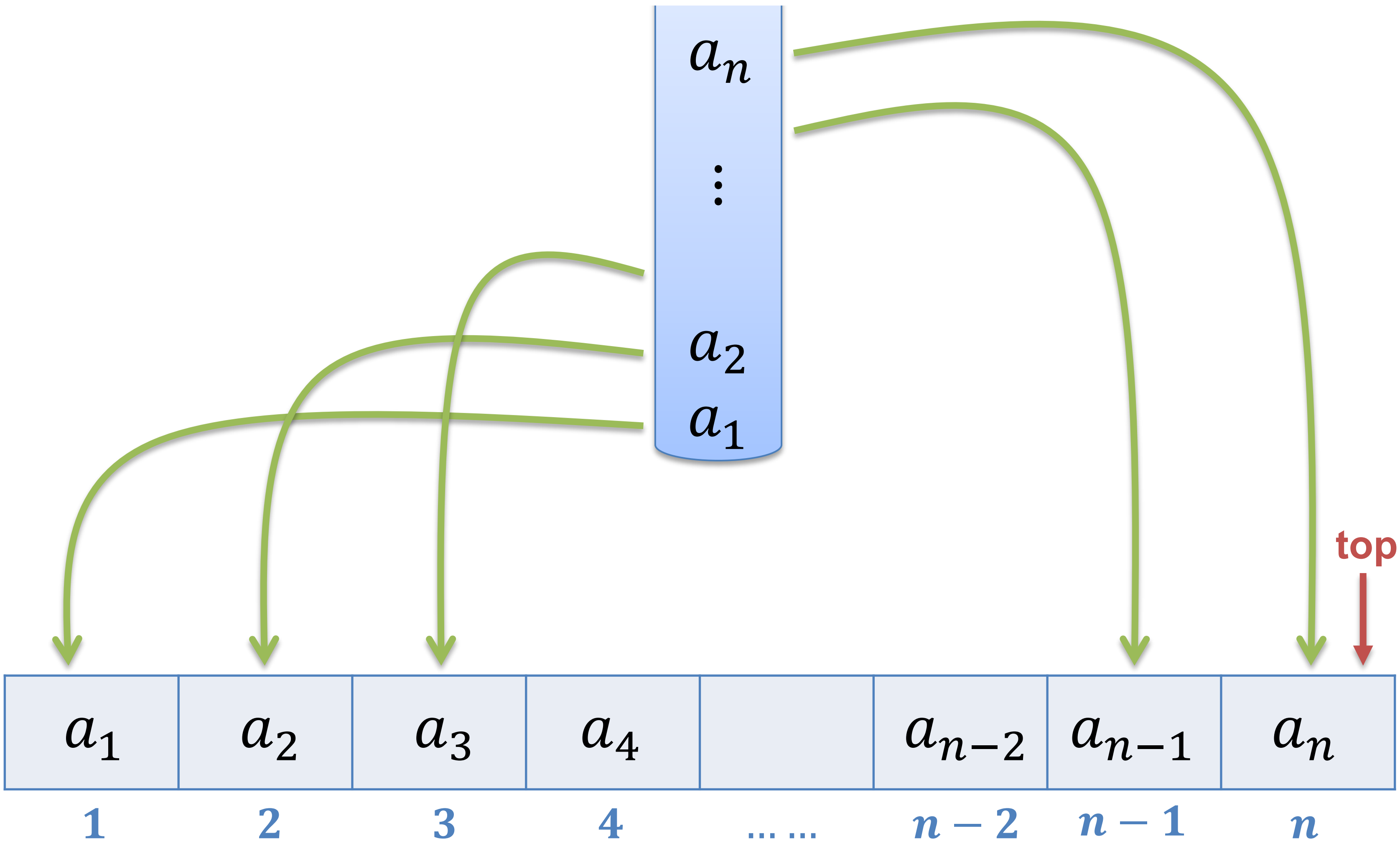


Stacks..

- By the definition of stack, if we add the elements A, B, C, D, E to the stack, in that order, then E is the first element we delete from the stack
  - **Last-In-First-Out**



Leverage Array to Implement Stack



Stack Permutation

- Given a sequence of elements and a empty stack, if a permutation can be generated by these elements and the stack, the permutation is called "stack permutation"

- For a given sequence of elements $\{A, B, C\}$, please write down its stack permutation
  - ABC
    - push A, pop A, push B, pop B, push C, pop C
  - ACB
  - BAC
    - Push A, push B, pop B, pop A, push C, pop C
  - BCA
  - CBA
    - Push A, push B, push C, pop C, pop B, pop A

Expressions

- When pioneering computer scientists conceived the idea of higher-level programming languages, they were faced with many hurdles
  - How to generate machine-language instructions to evaluate an arithmetic expression

$$A \div B - C + D \times E - A \times C$$

Operator (運算子) Operand (運算元)

  - The first problem with understanding the meaning of an expression is to decide in what order the operations are carried out
    - Specify the order by using parentheses

$$A \div (B - C) + D \times (E - A) \times C$$

$$(A \div B) - C + (D \times E) - (A \times C)$$

priority	operator
1	unary minus
2	**not**
3	$*$, /, **div**, **mod**, **and**
4	+, −, **or**, **xor**
5	<, <=, =, <>, >=, >, **in**

Infix & Postfix Notations

- If e is an expression with operators and operands, the conventional way of writing e is called **infix**
  - The operators come **in-between** the operands

$$A \div B - C + D \times E - A \times C$$

- The **postfix** form of an expression calls for each **operator to appear after its operands**

$$AB \div C - DE \times + AC \times -$$

operation	postfix
$T_1 := A/B$	$T_1C{-}DE{*}{+}AC{*}{-}$
$T_2 := T_1{-}C$	$T_2DE{*}{+}AC{*}{-}$
$T_3 := D{*}E$	$T_2T_3{+}AC{*}{-}$
$T_4 := T_2{+}T_3$	$T_4AC{*}{-}$
$T_5 := A{*}C$	$T_4T_5{-}$
$T_6 := T_4{-}T_5$	T_6

Infix to Postfix

- It is simple to describe an algorithm for producing postfix from infix

(1) Fully parenthesize the expression.

(2) Move all operators so that they replace their corresponding right parentheses.

(3) Delete all parentheses.

  - Take $A \div B - C + D \times E - A \times C$ for example
    - $(\Big(\big(((A \div B) - C\big) + (D \times E)\Big) - (A \times C))$
    - $(\Big(\big(((A \div B) - C\big) + (D \times E)\Big) - (A \times C))$
    - $AB \div C - DE \times +AC \times -$

Infix & Prefix Notations

- If e is an expression with operators and operands, the conventional way of writing e is called **infix**
  - The operators come **in-between** the operands

$$A \div B - C + D \times E - A \times C$$

- In the **prefix** form of an expression, the **operators precede their operands**
  - Take $A \div B - C + D \times E - A \times C$ for example
    - $((((A \div B) - C) + (D \times E)) - (A \times C))$
    - $((((A \div B) - C) + (D \times E)) - (A \times C))$
    - $-+-ABC \times DE \times AC$

infix	prefix
$A * B / C$	$/*ABC$
$A / B - C + D * E - A * C$	$- + - /ABC * DE * AC$
$A * (B + C) / D - G$	$-/* A + BCDG$

Examples.

- Given a infix expression $A + B \times C - D \div E$, please write down the prefix and postfix expressions

$$A + B \times C - D \div E$$

$$\big((A + (B \times C)\big) - (D \div E))$$

  - Prefix

$$\big((A + (B \times C)\big) - (D \div E))$$

$$- +A \times BC \div DE$$

  - Postfix

$$\big((A + (B \times C)\big) - (D \div E))$$

$$ABC \times +DE \div -$$

Examples..

- Given a infix expression $(A + B) \times C \div (D - E \div F)$, please write down the prefix and postfix expressions

$$\Big(\Big((A + B) \times C\Big) \div (D - (E \div F))\Big)$$

  - Prefix

$$\div \times +ABC - D \div EF$$

  - Postfix

$$AB + C \times DEF \div - \div$$

Examples...

- Given a infix expression $A \wedge \neg(B > C) \vee (D \vee \neg E)$, please write down the prefix and postfix expressions

$$((A \wedge (\neg(B > C))) \vee (D \vee (\neg E)))$$

  - Prefix

$$((A \wedge (\neg(B > C))) \vee (D \vee (\neg E)))$$

$$\vee \wedge A \neg > BC \vee D \neg E$$

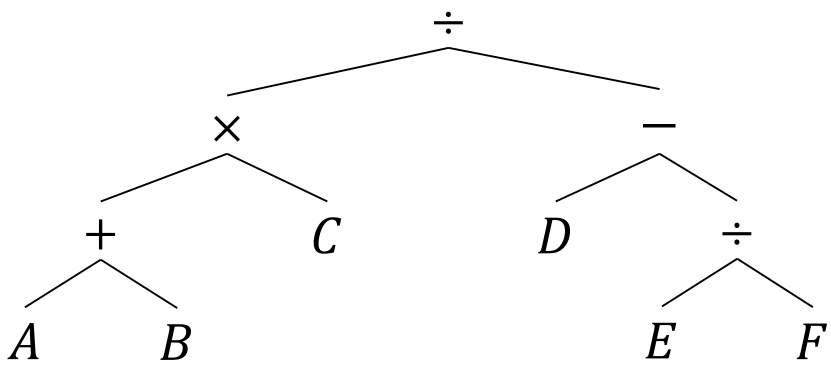
  - Postfix

$$((A \wedge (\neg(B > C))) \vee (D \vee (\neg E)))$$

$$ABC > \neg \wedge DE \neg \vee \vee$$

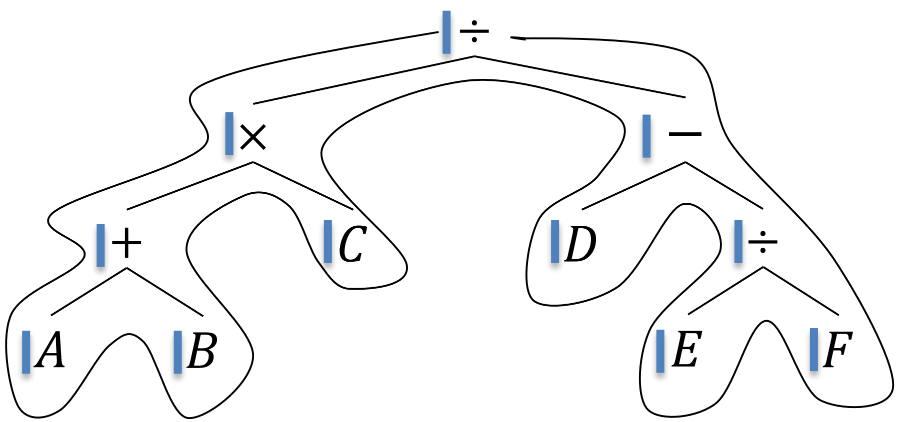
By Looking!.

- Given a infix expression $(A + B) \times C \div (D - E \div F)$, please write down the prefix and postfix expressions



By Looking!..

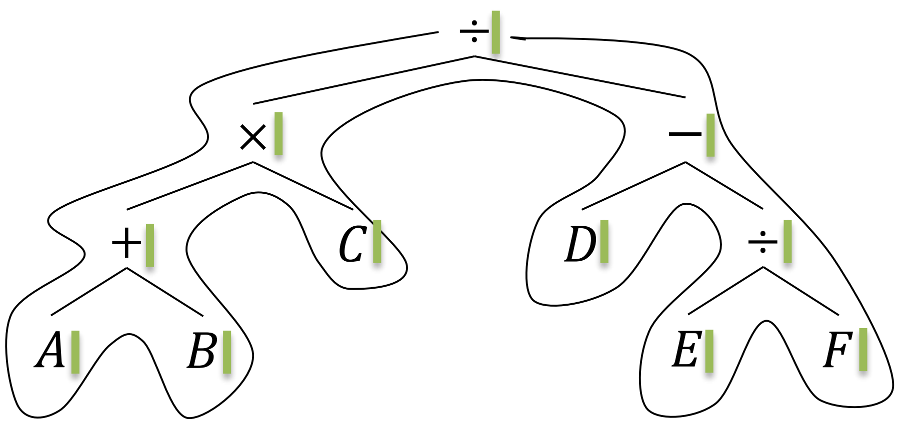
- Given a infix expression $(A + B) \times C \div (D - E \div F)$, please write down the prefix and postfix expressions
  - Prefix



$\div \times +ABC - D \div EF$

By Looking!...

- Given a infix expression $(A + B) \times C \div (D - E \div F)$, please write down the prefix and postfix expressions
  - Postfix



$AB + C \times DEF \div - \div$

Examples.

- Given a postfix expression $ABCD +\times E \div -$, please write down the infix expression
  - The pattern for postfix is $< operand_1, operand_2, operator > = operand_1 operator\ operand_2$

$$ABCD +\times E \div -$$

$AB\boxed{CD +}\times E \div -$	$C + D$
$A\boxed{BCD +\times} E \div -$	$B \times (C + D)$
$A\boxed{BCD +\times E \div} -$	$B \times (C + D) \div E$
$\boxed{ABCD +\times E \div -}$	$A - B \times (C + D) \div E$

Examples..

- Given a prefix expression $-A \div \times B + CDE$, please write down the infix expression
  - The pattern for prefix is $< operator, operand_1, operand_2 >= operand_1 operator\ operand_2$

$$-A \div \times B + CDE$$

$$-A \div \times B \boxed{+CD}E \qquad C + D$$

$$-A \div \boxed{\times B + CD}E \qquad B \times (C + D)$$

$$-A \boxed{\div \times B + CDE} \qquad B \times (C + D) \div E$$

$$\boxed{-A \div \times B + CDE} \qquad A - B \times (C + D) \div E$$

Questions?

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