AVL Trees

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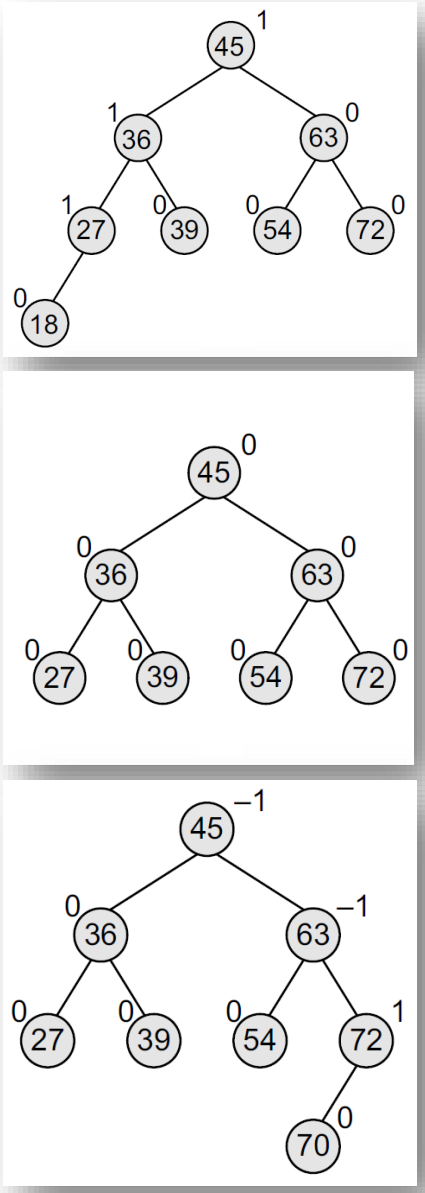
AVL Trees.

- AVL tree is a self-balancing binary search tree
  - AVL tree is designed by G.M. Adelson-Velsky and E.M. Landis in 1962
  - The heights of the two sub-trees of a node may differ by at most one

- The structure of an AVL stores an additional variable called the Balance Factor
  - Every node has a balance factor
  - The balance factor of a node is calculated by subtracting the height of its right sub-tree from the height of its left sub-tree
  - Every node has a balance factor of −1, 0, or 1

AVL Trees..

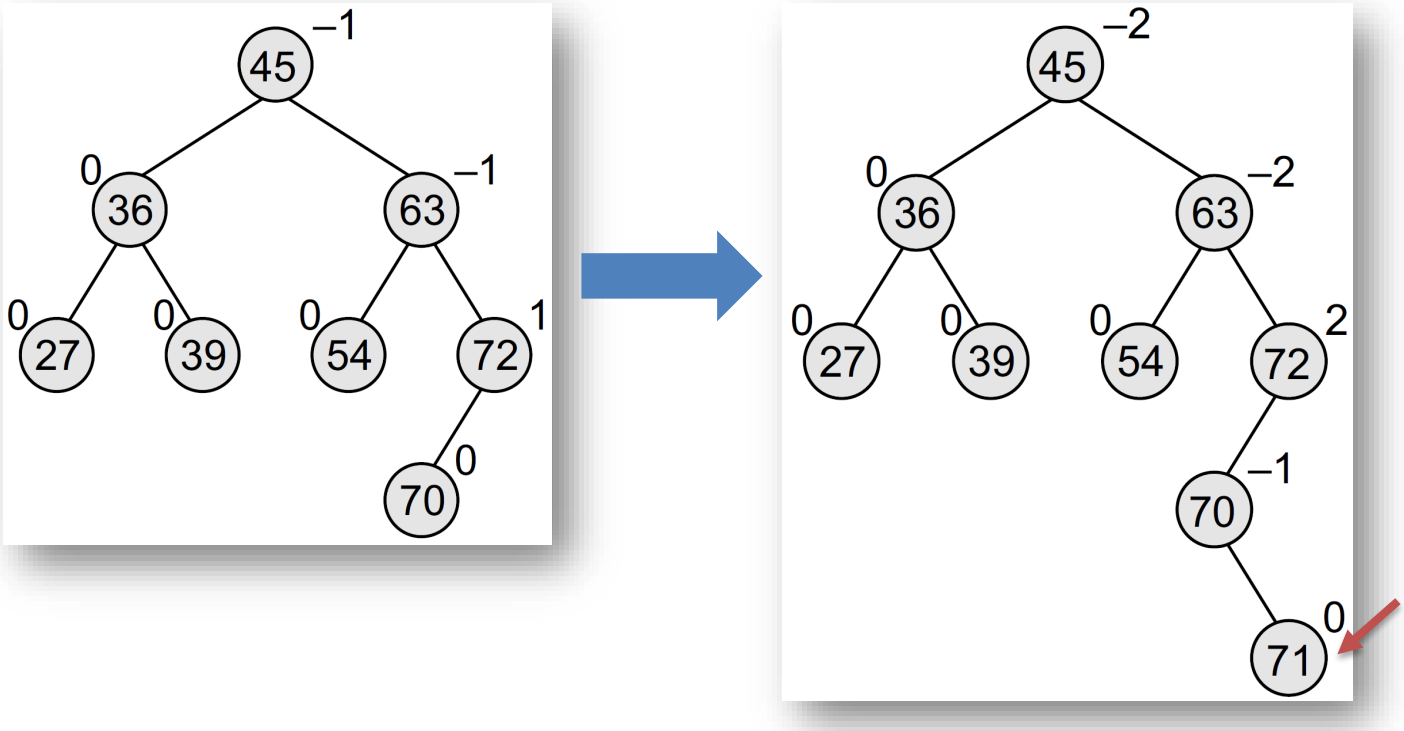
- If the balance factor of a node is 1, then it means that the left sub-tree of the tree is one level higher than that of the right sub-tree
  - Left-heavy tree
- If the balance factor of a node is 0, then it means that the height of the left sub-tree is equal to the height of the right sub-tree
  - Balance tree
- If the balance factor of a node is −1, then it means that the left sub-tree of the tree is one level lower than that of the right sub-tree
  - Right-heavy tree



Insertion

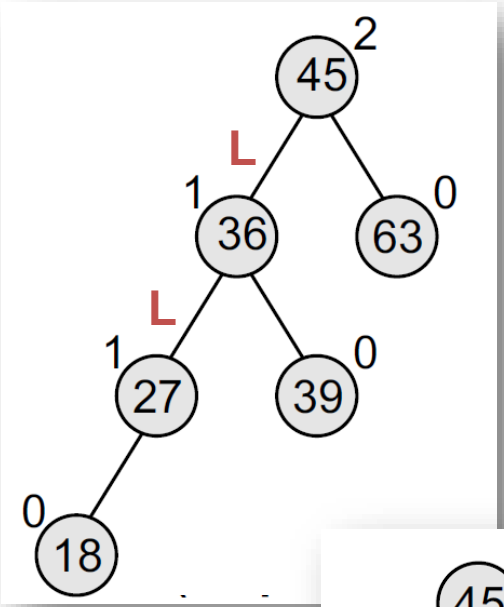
- In the AVL tree, the step of insertion is usually followed by an additional step of rotation
  - Rotation is done to restore the balance of the tree

- Insert a node with value 71 in a given AVL tree

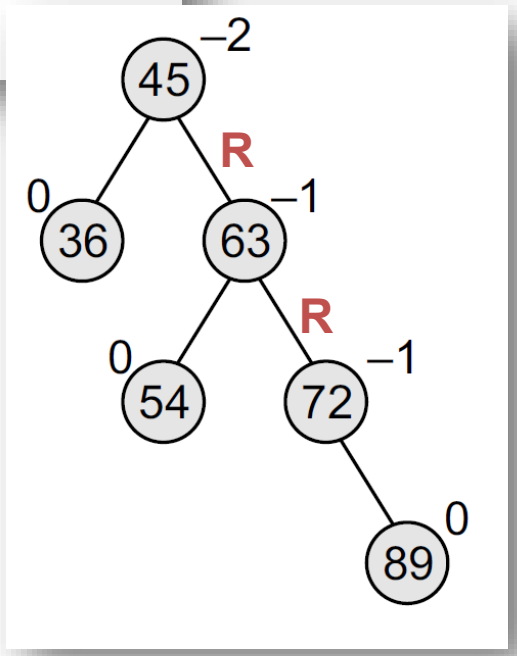


Rotations.

- LL rotation
  - The new node is inserted in the left sub-tree of the left sub-tree of the critical node

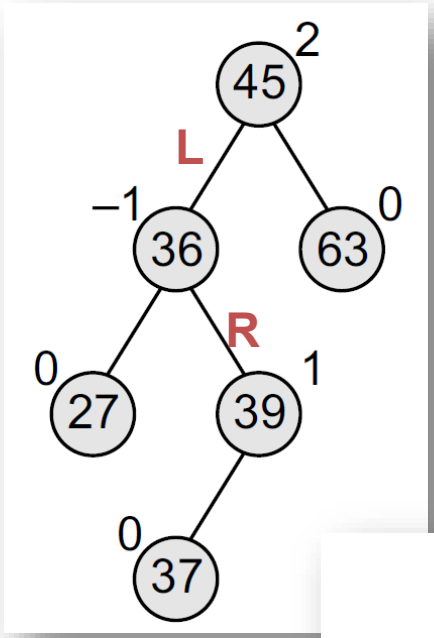


- RR rotation
  - The new node is inserted in the right sub-tree of the right sub-tree of the critical node

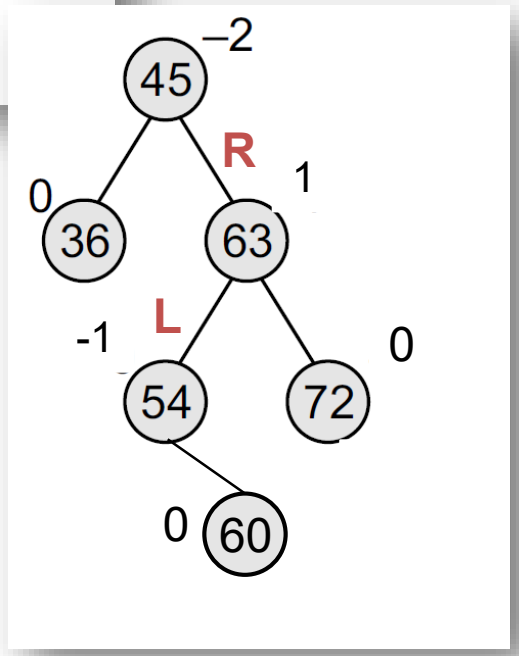


Rotations..

- LR rotation
  - The new node is inserted in the right sub-tree of the left sub-tree of the critical node

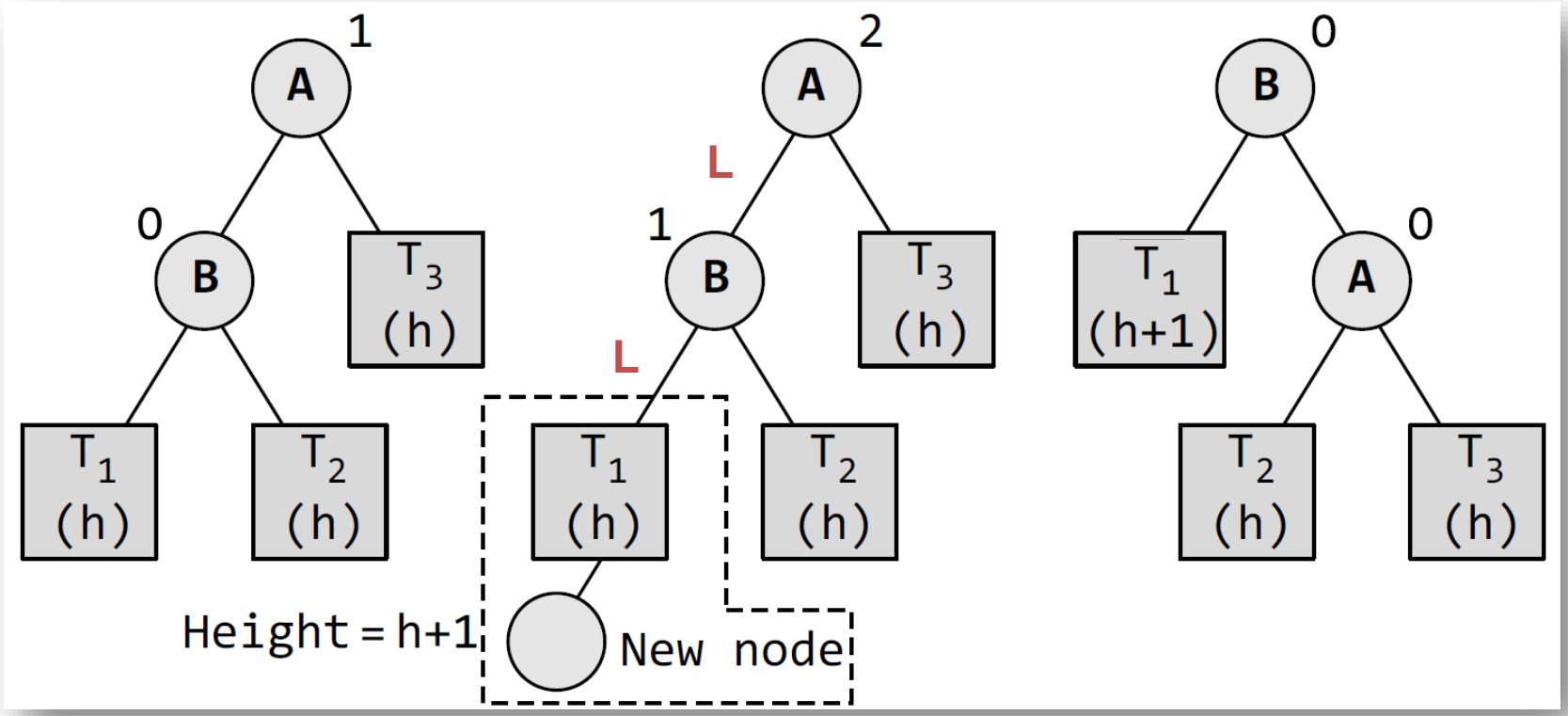


- RL rotation
  - The new node is inserted in the left sub-tree of the right sub-tree of the critical node



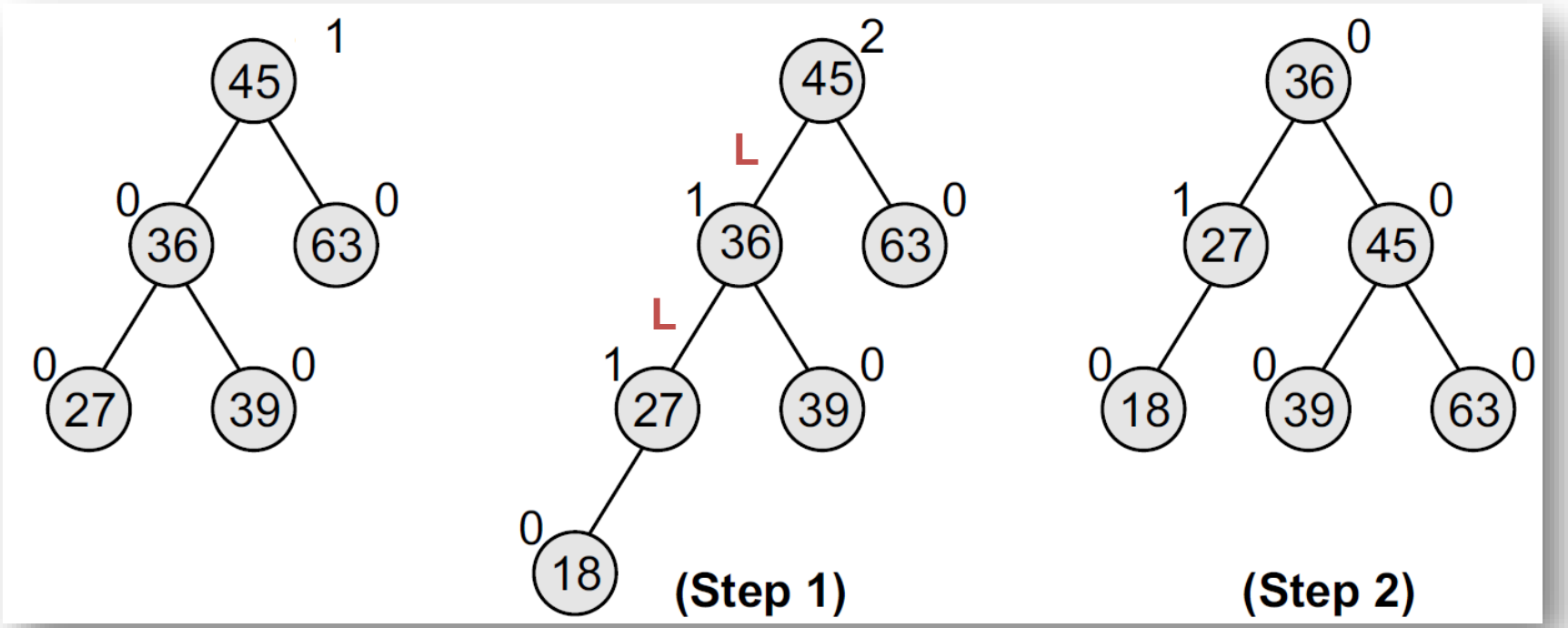
LL Rotation.

- By LL Rotation
  - Node B becomes the root, with T1 and A as its left and right child
  - T2 and T3 become the left and right sub-trees of A



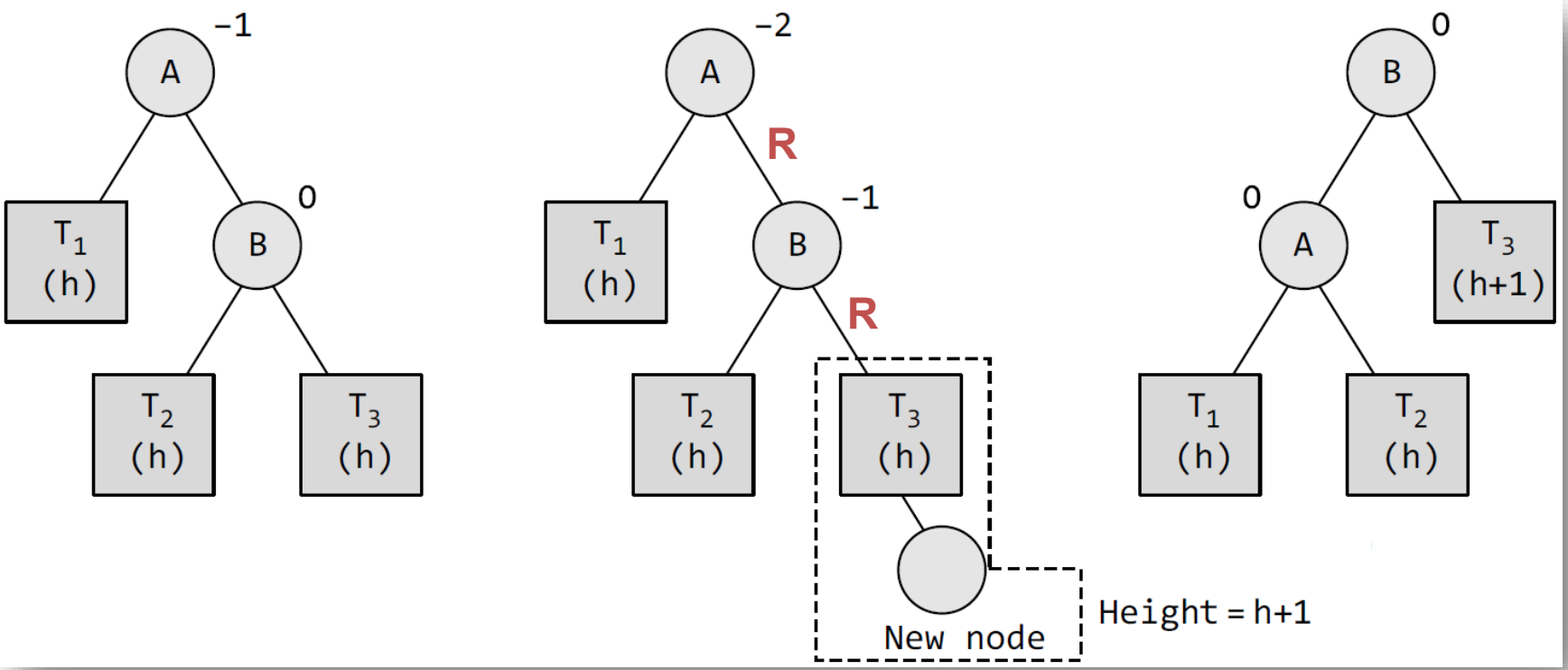
LL Rotation..

- Example for LL Rotation
  - Insert 18 in a given AVL tree



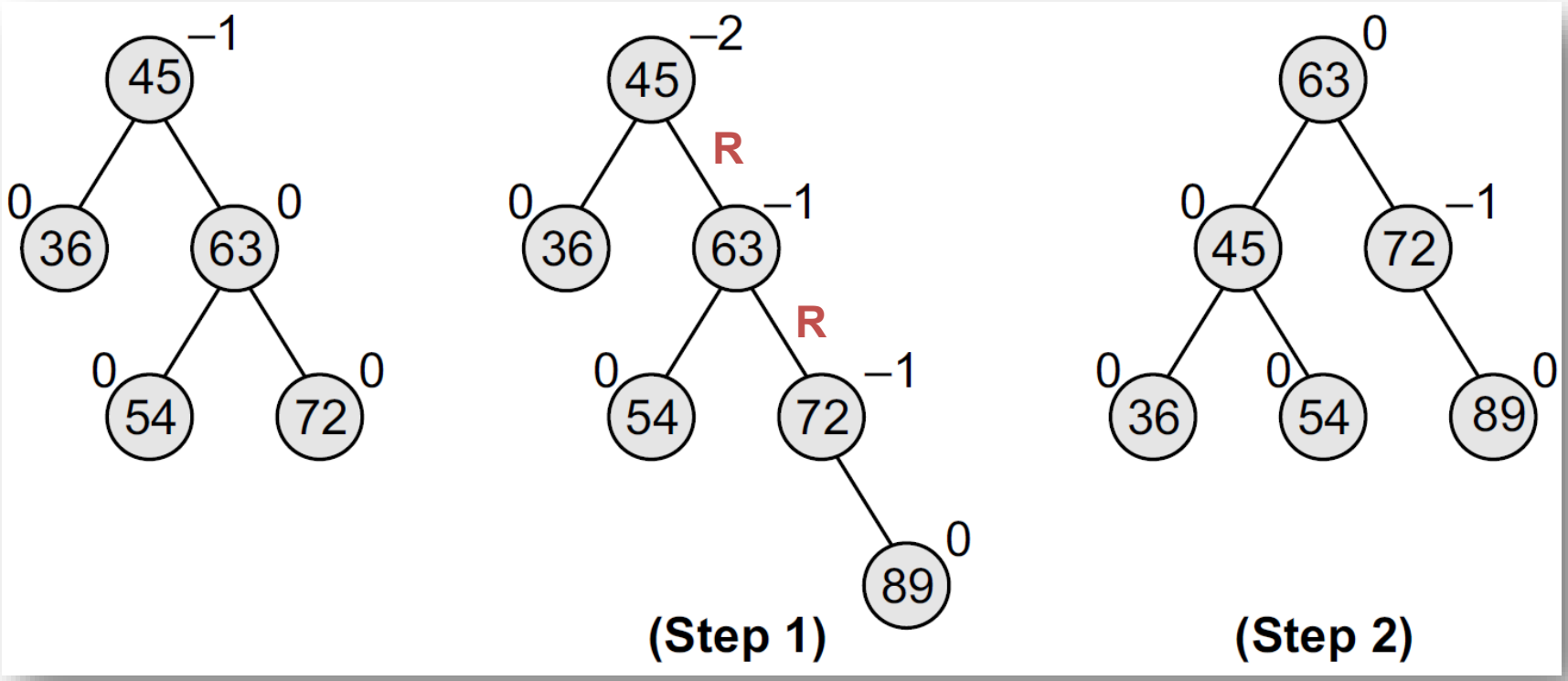
RR Rotation.

- In the context of RR rotation
  - Node B becomes the root, with A and T3 as its left and right child
  - T1 and T2 become the left and right sub-trees of A



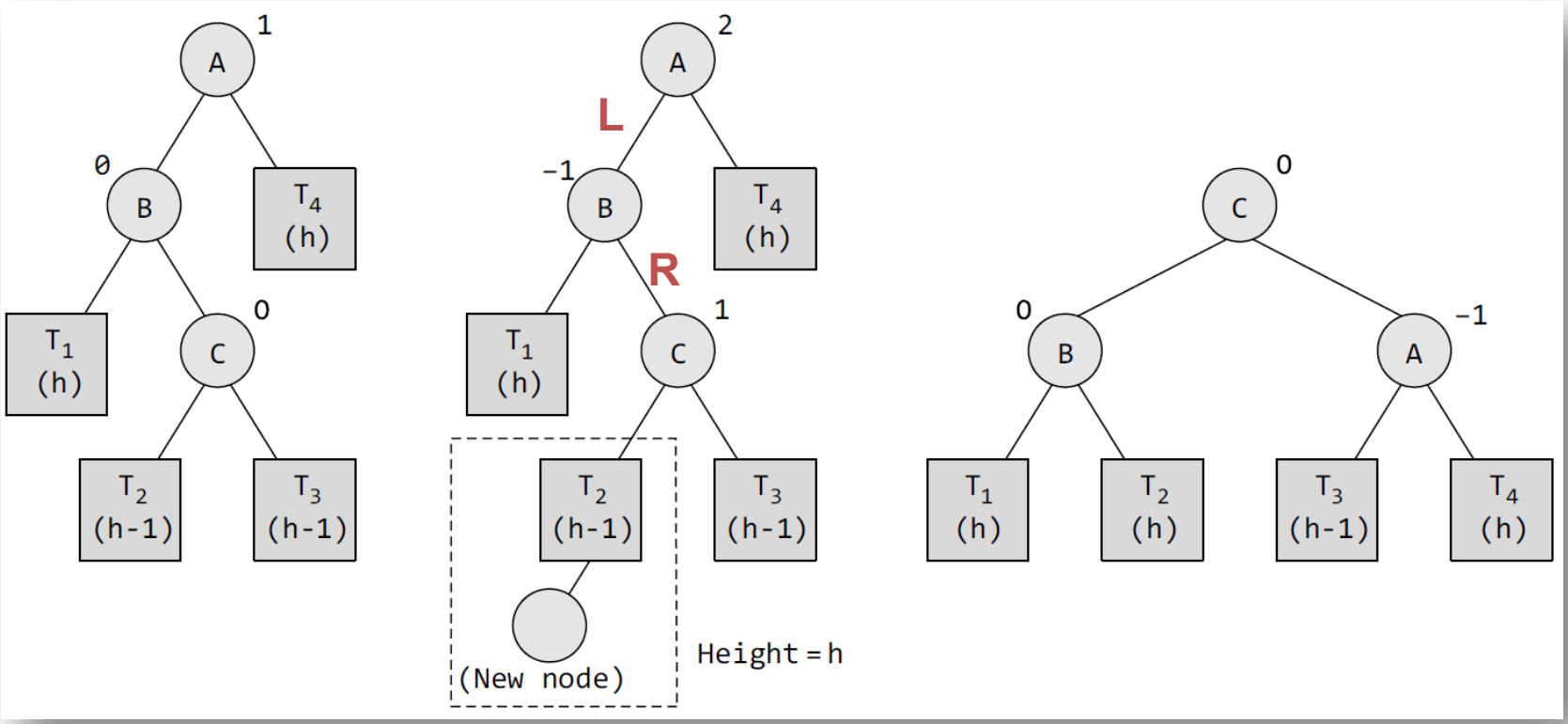
RR Rotation..

- Example for RR Rotation
  - Insert 89 in a given AVL tree



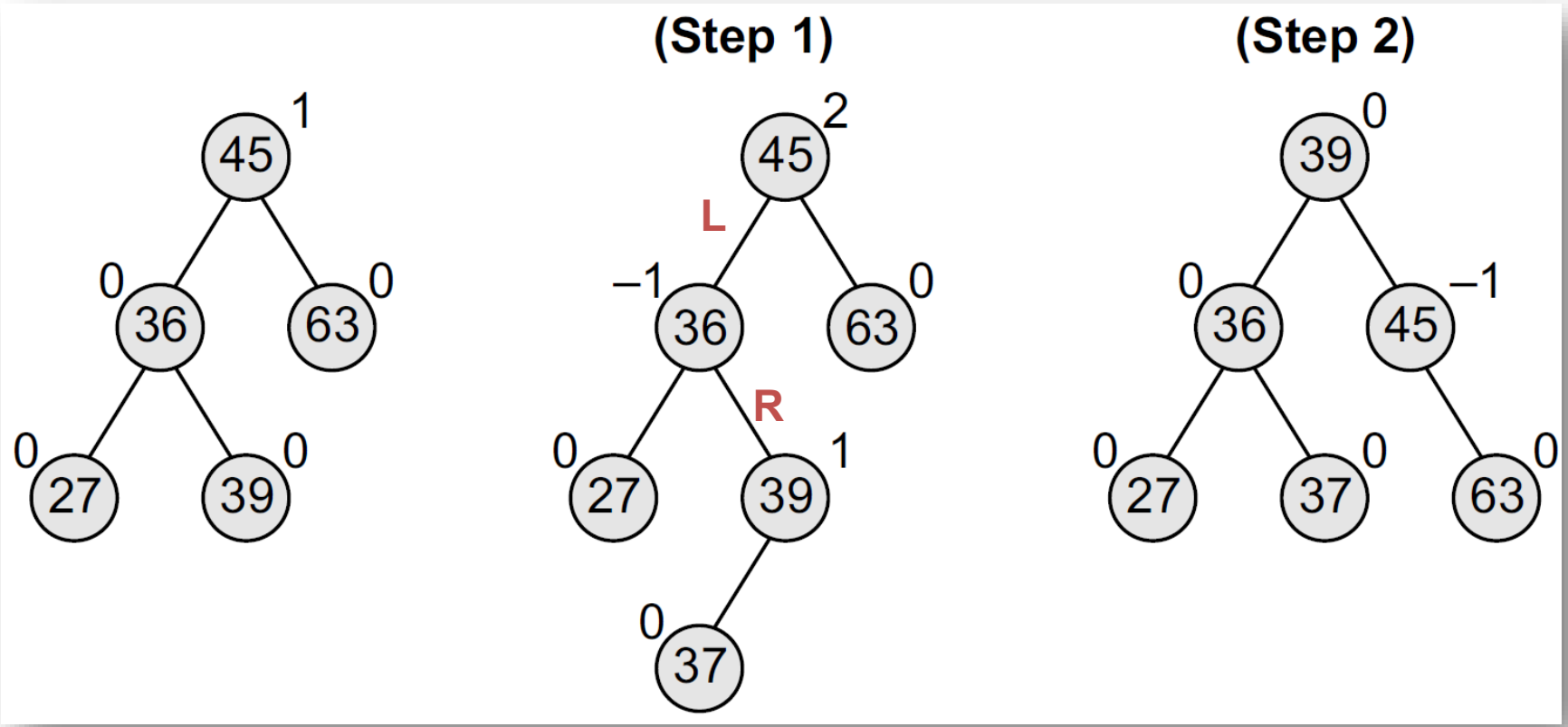
LR Rotation.

- By LR rotation
  - Node C becomes the root, with B and A as its left and right children
  - Node B has T1 and T2 as its left and right sub-trees and T3 and T4 become the left and right sub-trees of node A



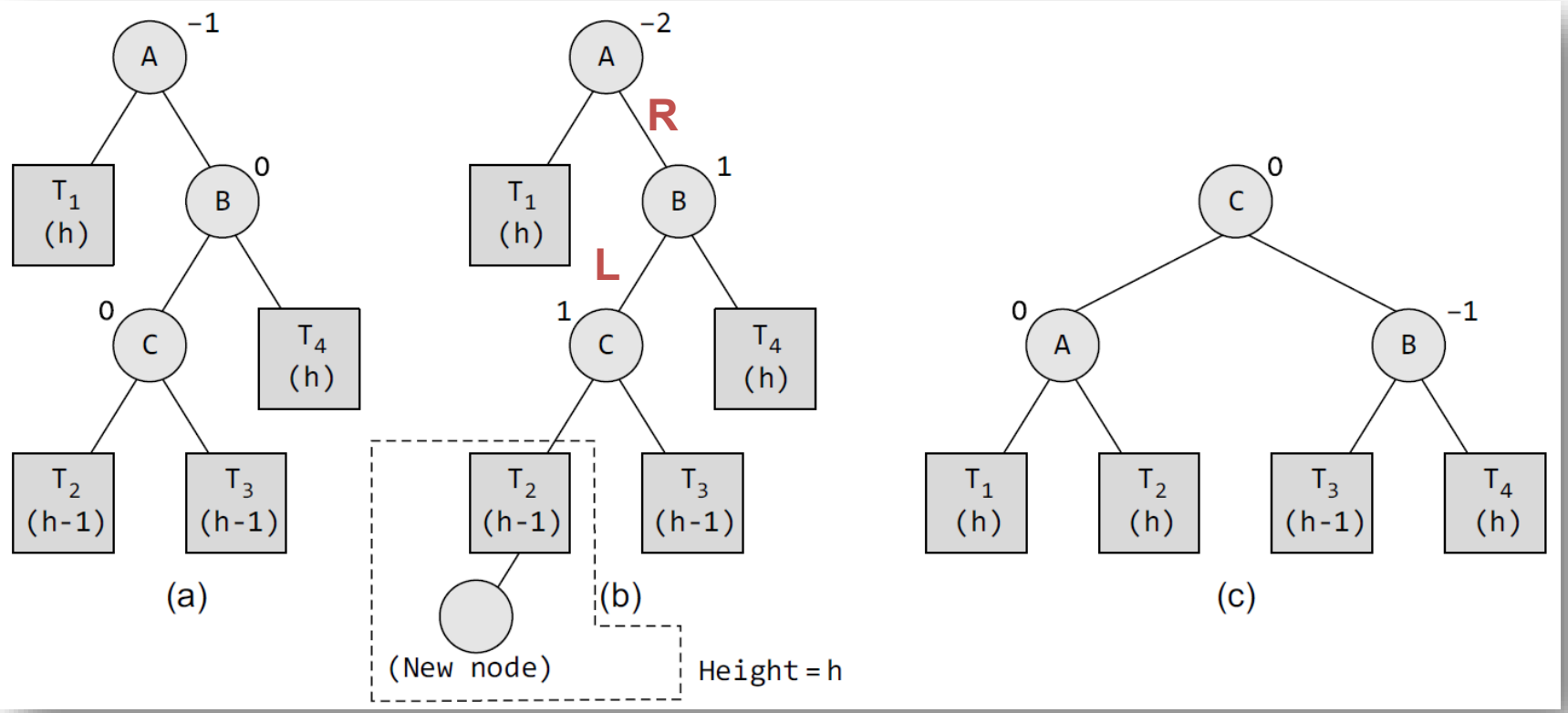
LR Rotation..

- Example for RR Rotation
  - Insert 37 in a given AVL tree



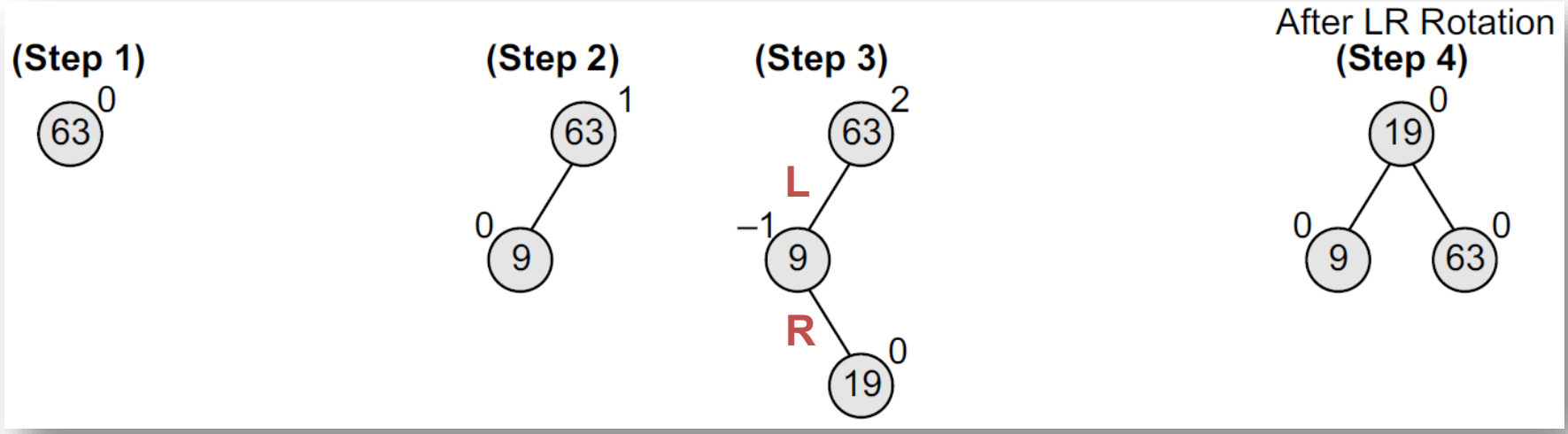
RL Rotation

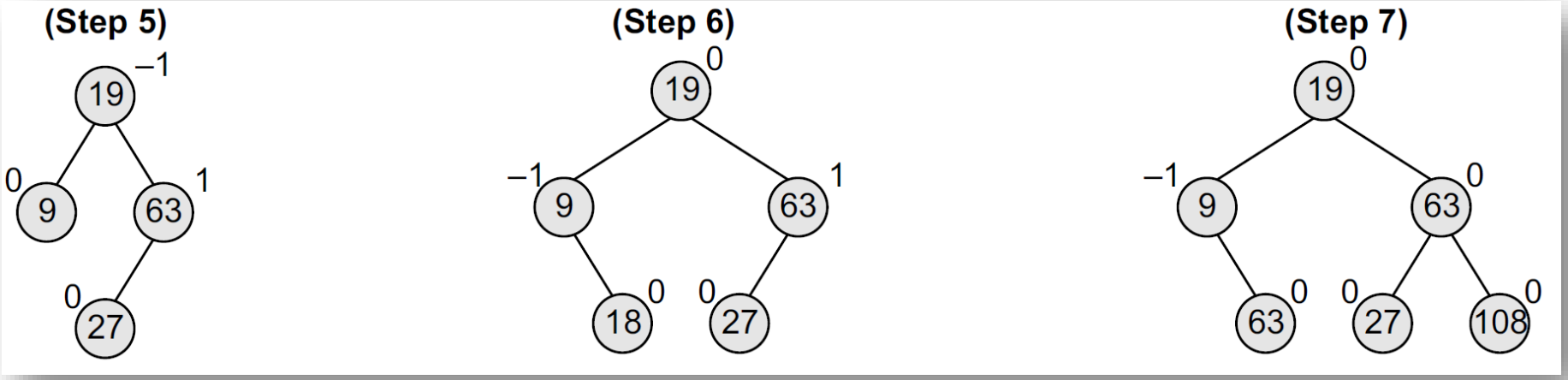
- By RL rotation
  - Node C becomes the root, with A and B as its left and right children
  - Node A has T1 and T2 as its left and right sub-trees and T3 and T4 become the left and right sub-trees of node B



Example.

- Construct an AVL tree by inserting the following elements in the given order: 63, 9, 19, 27, 18, 108, 99, 81





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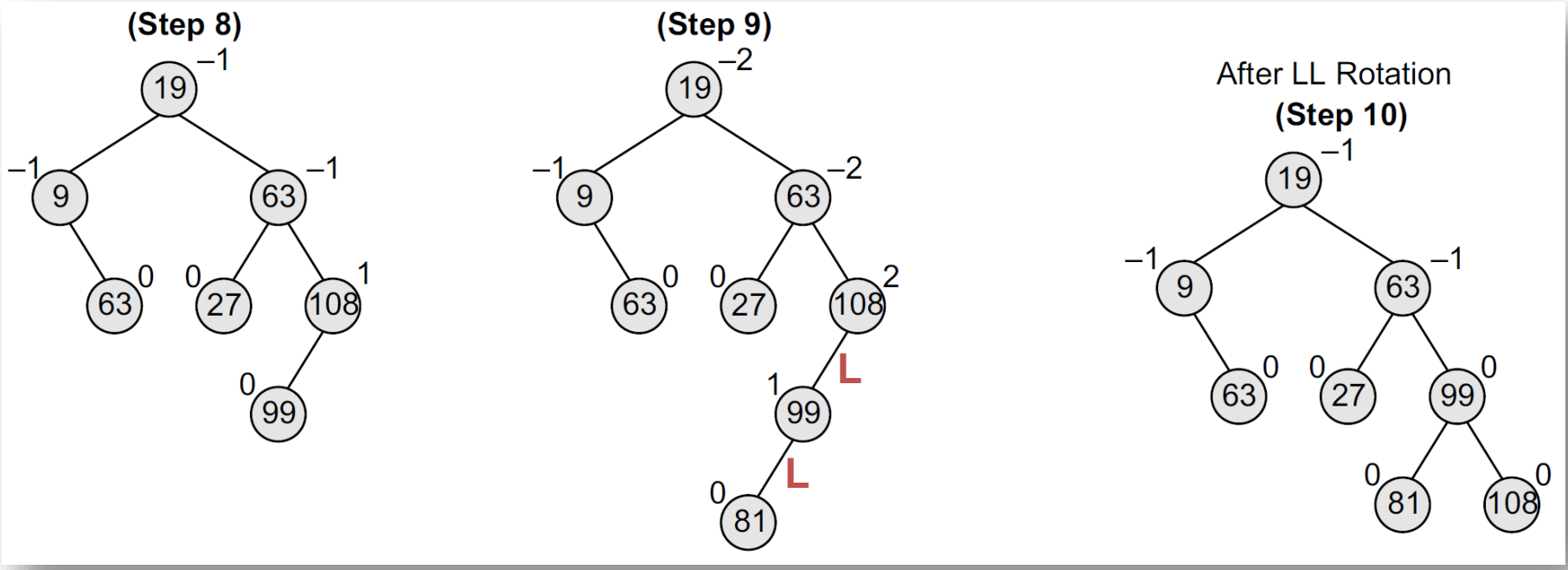
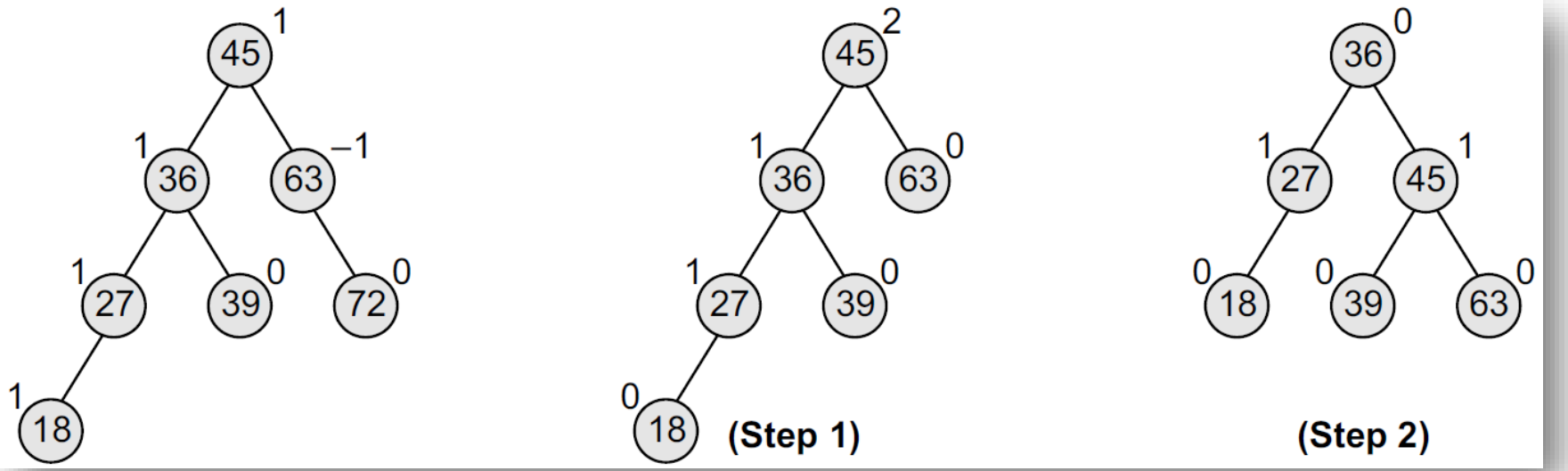


Deletion

- Deletion of a node in an AVL tree may disturb the balance of the tree
  - To rebalance the AVL tree, we need to perform rotations!
  - R rotation
  - L rotation

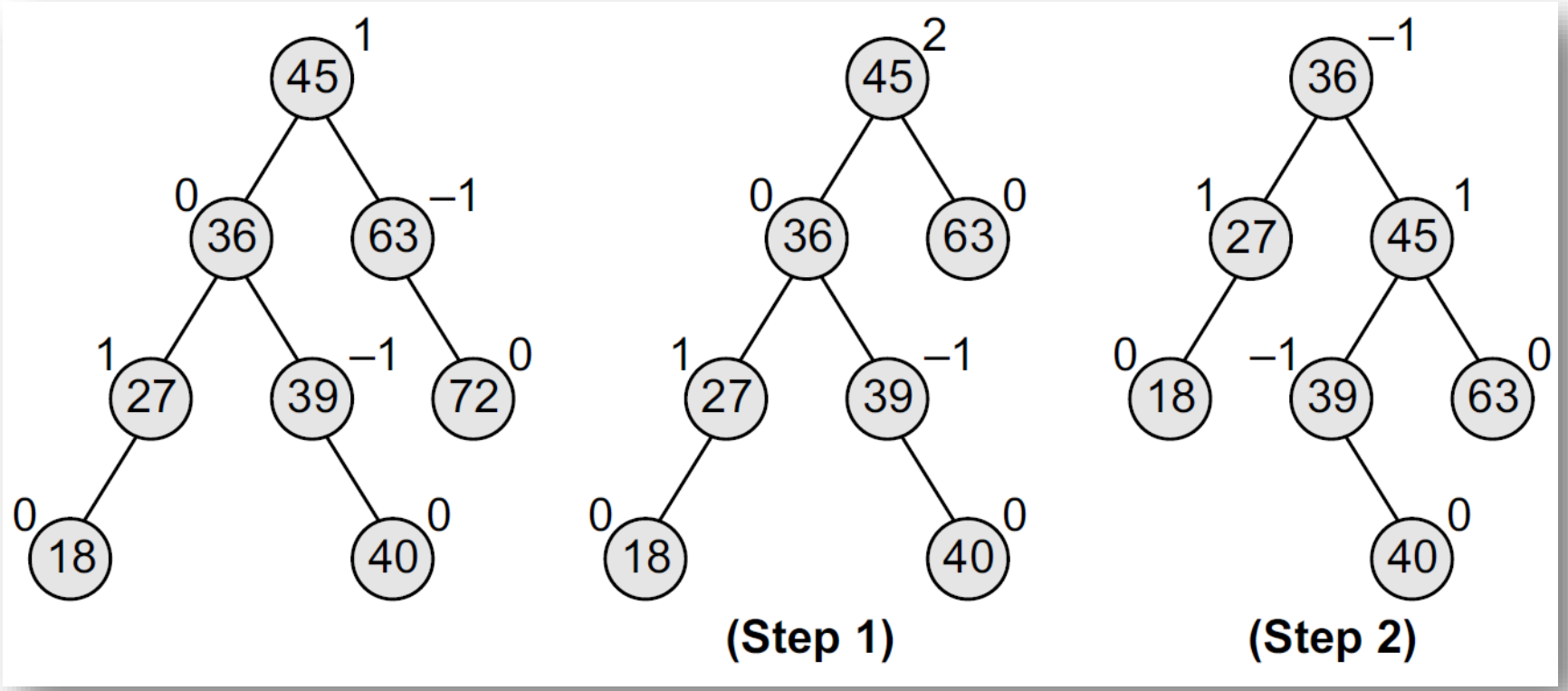
Examples – 1

- Delete 72 from a given AVL tree



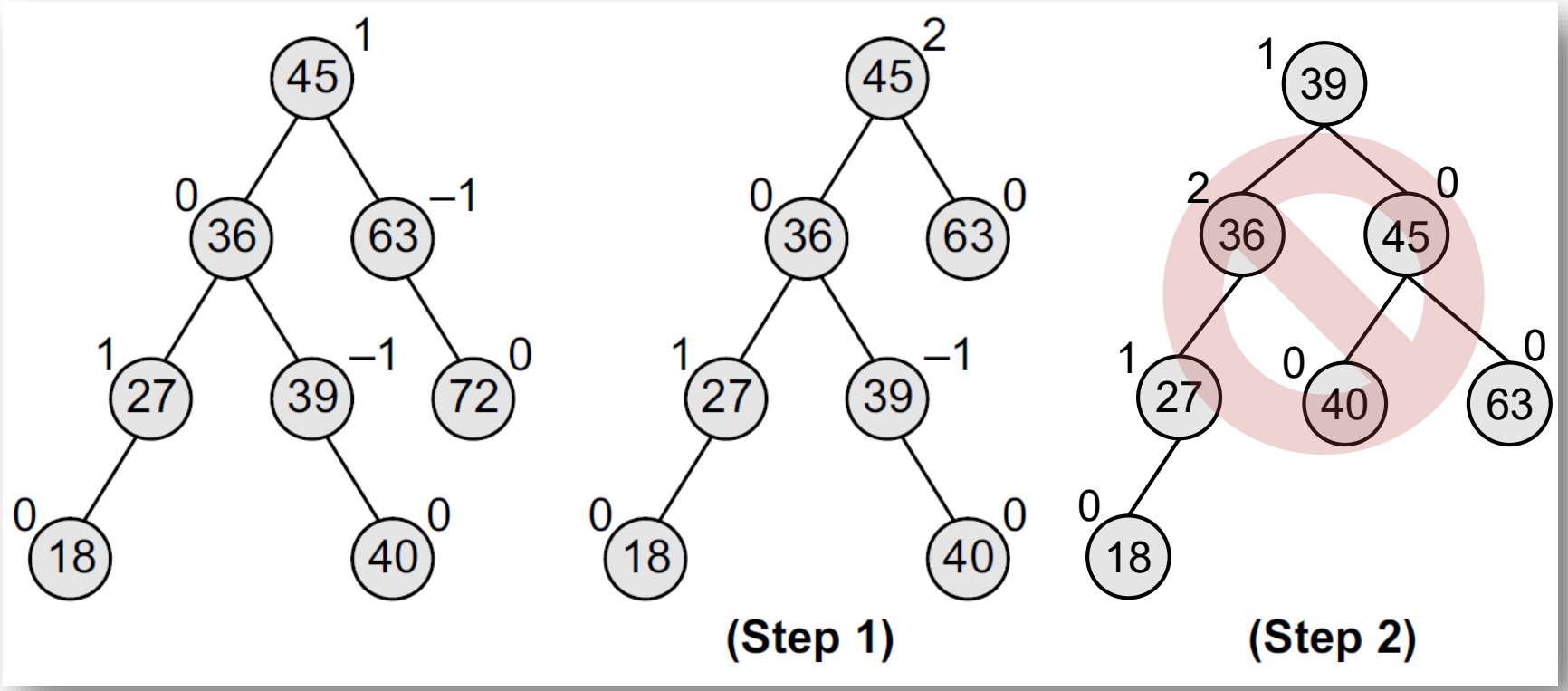
Examples – 2.

- Delete 72 from a given AVL tree



Examples – 2..

- Delete 72 from a given AVL tree



Questions?

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