Heaps

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Review

- Shortest path algorithms
  - Minimum spanning tree
    - Prim’s algorithm
    - Kruskal’s algorithm
  - Dijkstra’s algorithm

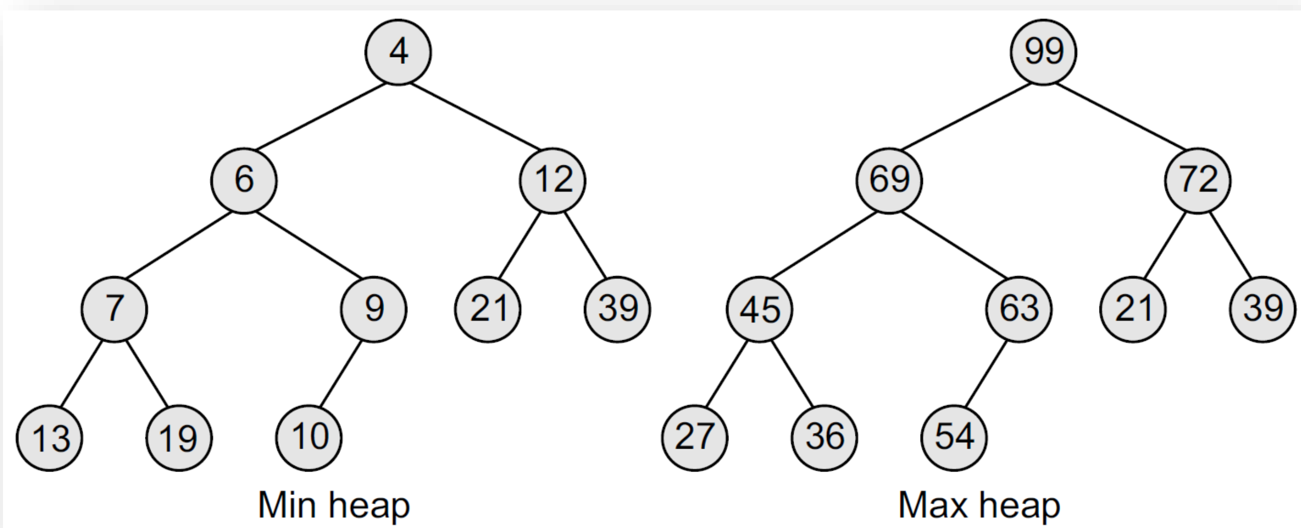
Binary Heap

- A **binary heap** is a complete binary tree in which every node satisfies the heap property
  - Min Heap

$$If\ B\ is\ a\ child\ of\ A, then\ key(B)\ \geq\ key(A)$$

  - Max Heap

$$If\ B\ is\ a\ child\ of\ A, then\ key(A)\ \geq\ key(B)$$

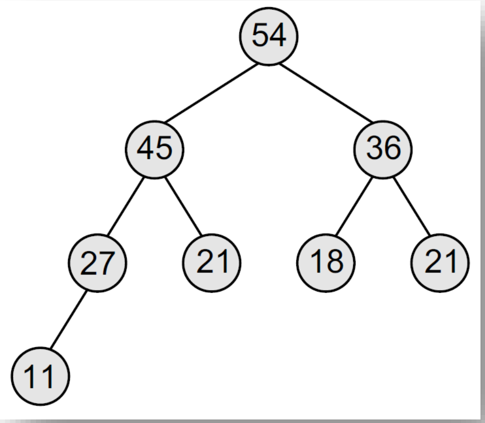


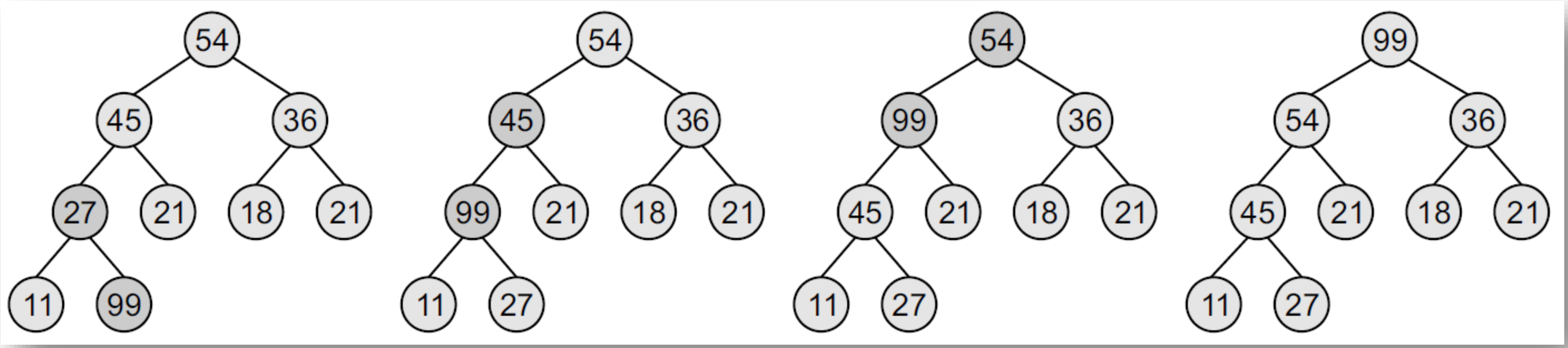
Heap – Insertion

- Inserting a new value into the heap is done in the following two steps:
  - Consider a max heap H with n elements
  1. Add the new value at the bottom of H
  2. Let the new value rise to its appropriate place in H

Example

- Consider a max heap and insert 99 in it





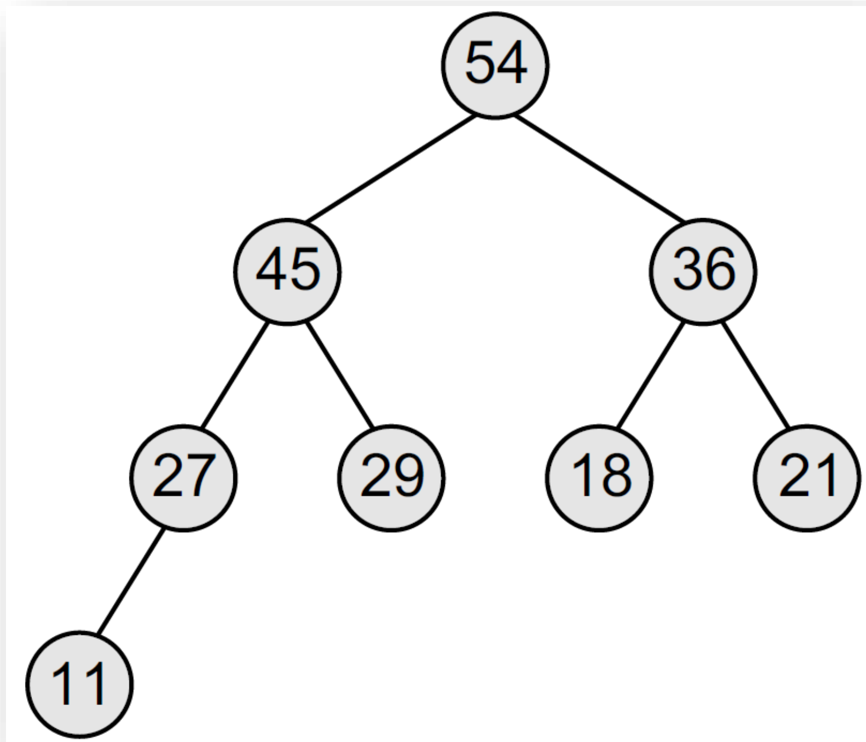
Heap – Deletion

- An element is always deleted from the root of the heap

- Consider a max heap H having n elements, deleting an element from the heap is done in the following three steps:
  1. Replace the root node's value with the last node's value
  2. Delete the last node
  3. Sink down the new root node's value so that H satisfies the heap property

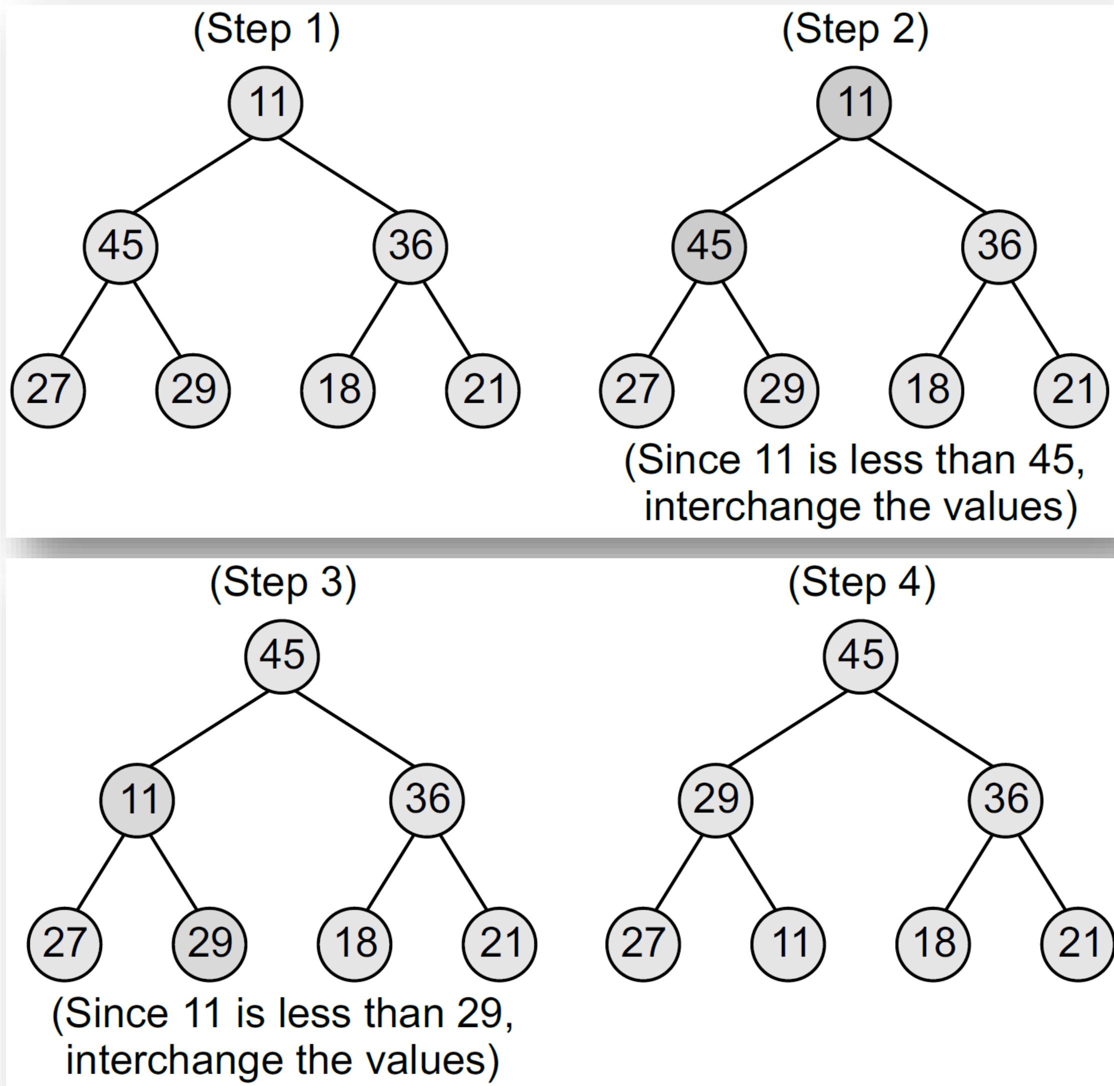
Example.

- Delete the root node's value from a given max heap H



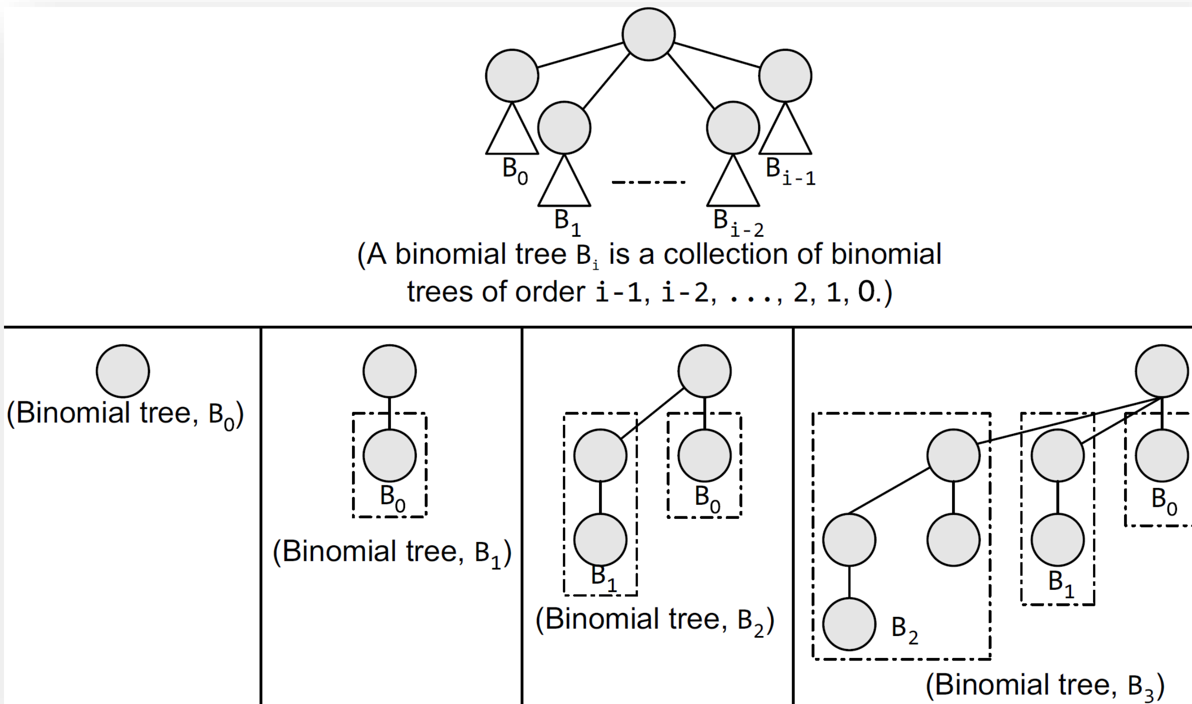
Example..

- Delete the root node's value from a given max heap *H*



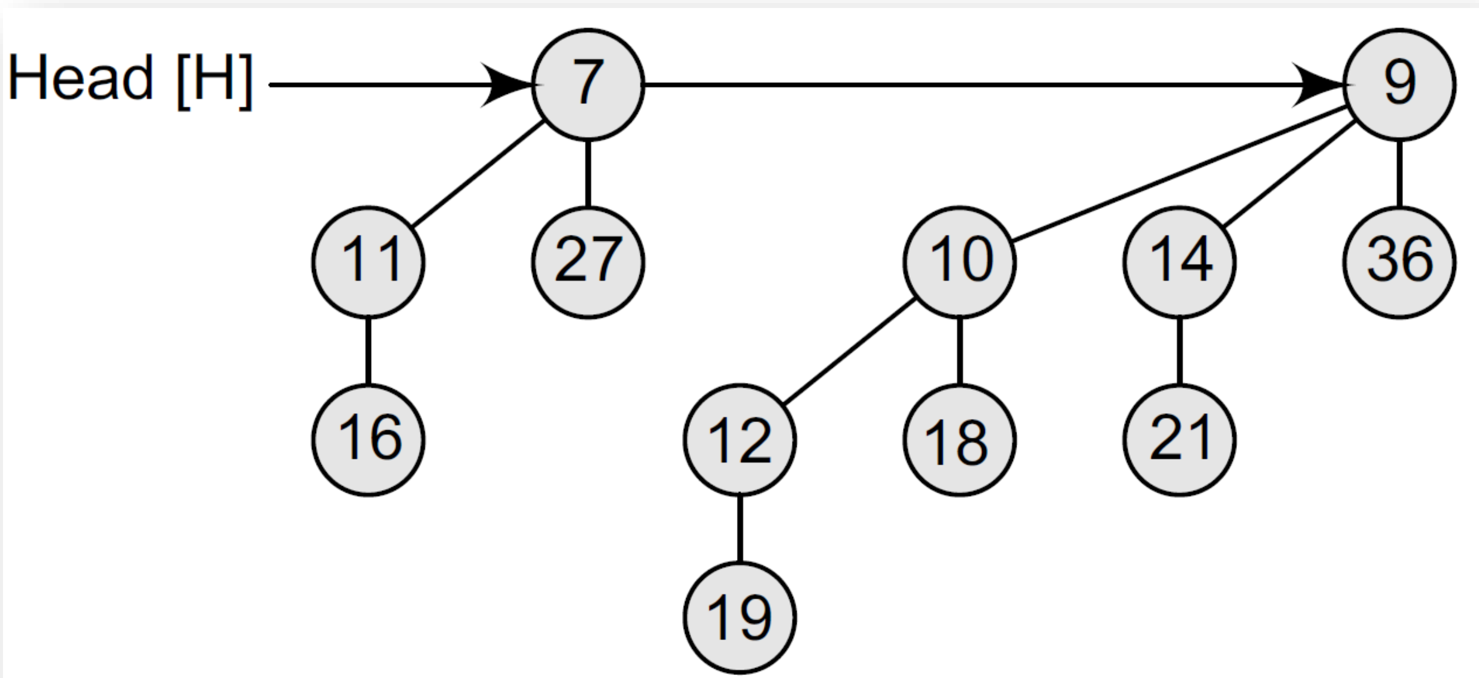
Binomial Tree

- A binomial tree is an ordered tree
  - A binomial tree B_i with order i has 2^i nodes
    - A binomial tree of order 0 has a single node
  - The height of a binomial tree B_i is i
  - A binomial tree of order i has a root node whose children are the root nodes of binomial trees of order $i-1,\ i-2, \cdots,\ 2,\ 1,$ and 0

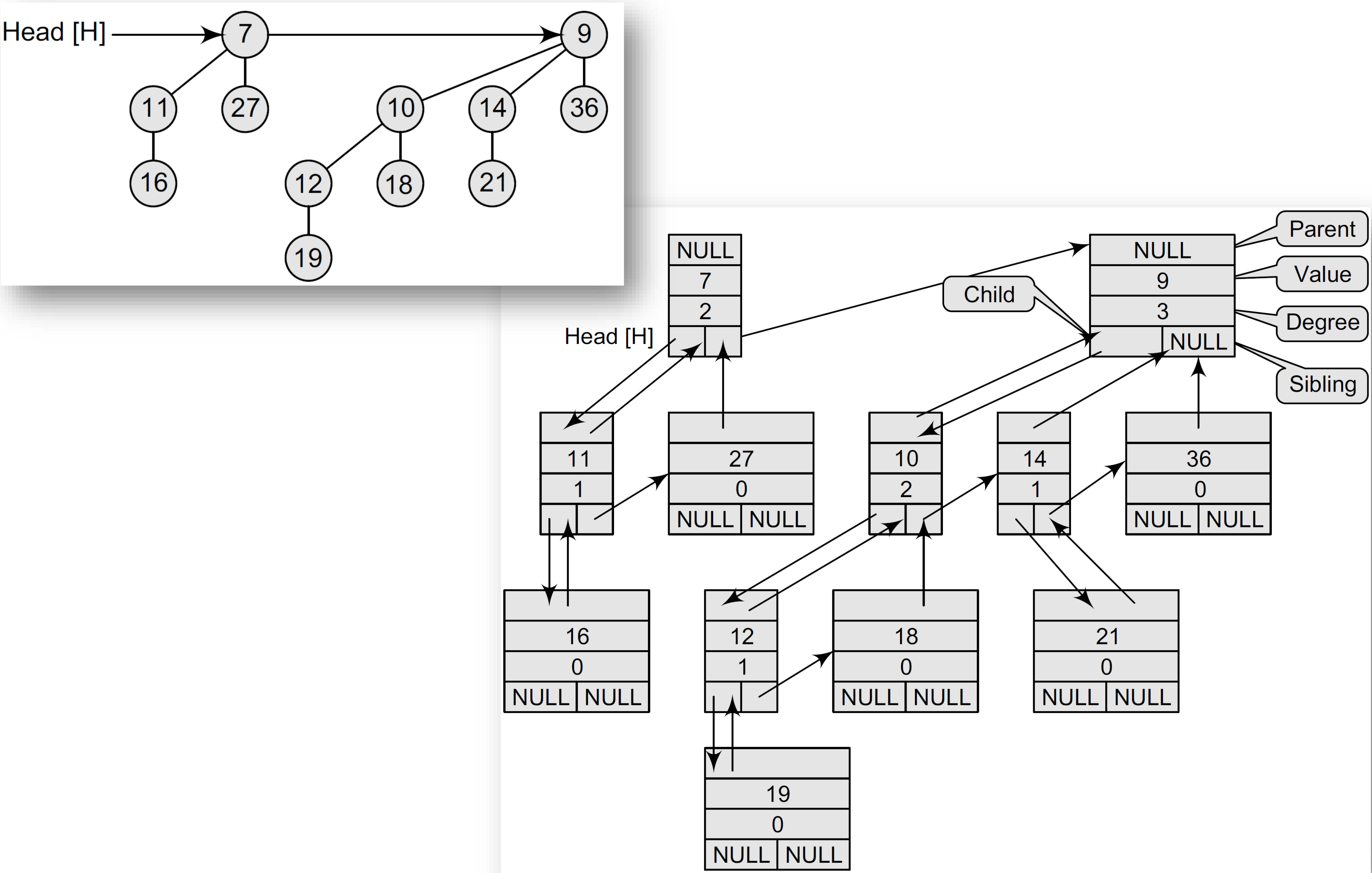


Binomial Heap

- A **binomial heap** H is a set of **binomial trees**
  - Every binomial tree in H satisfies the minimum heap property

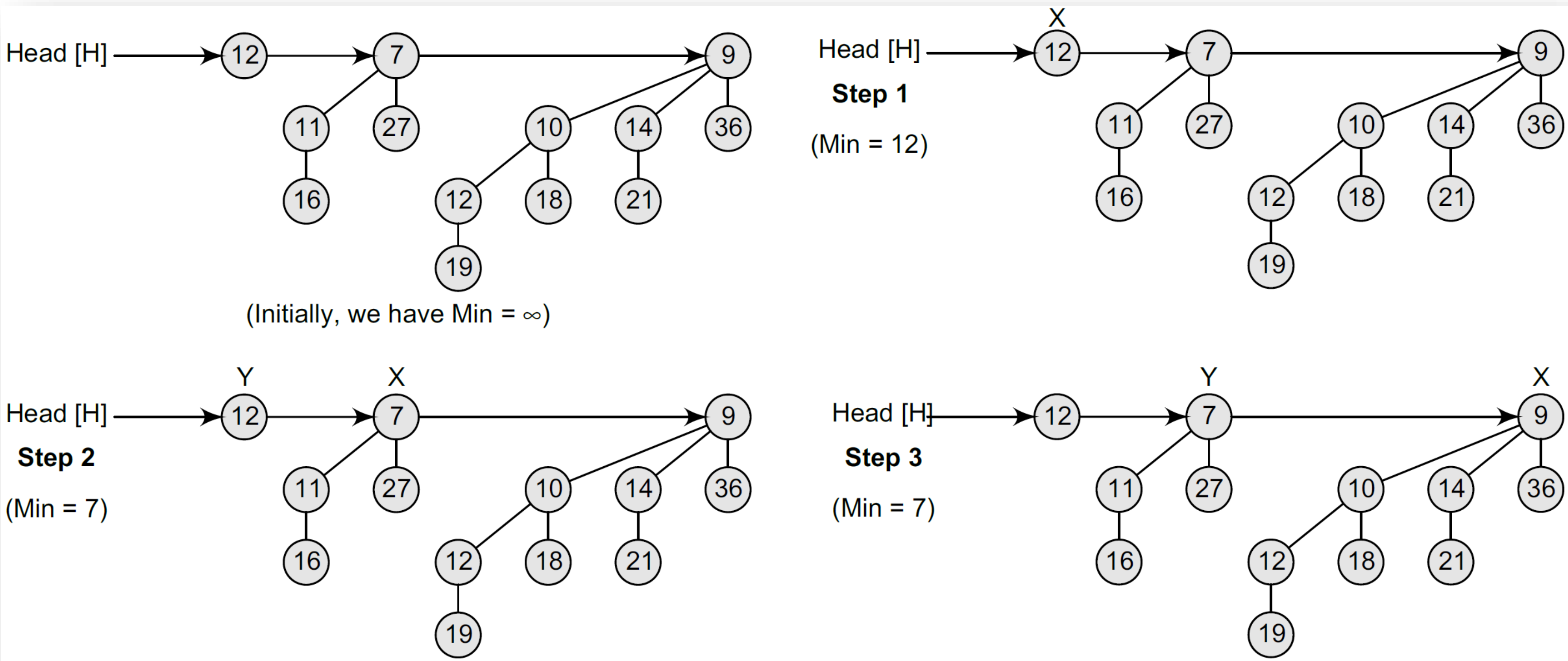


Binomial Heap with Linked List



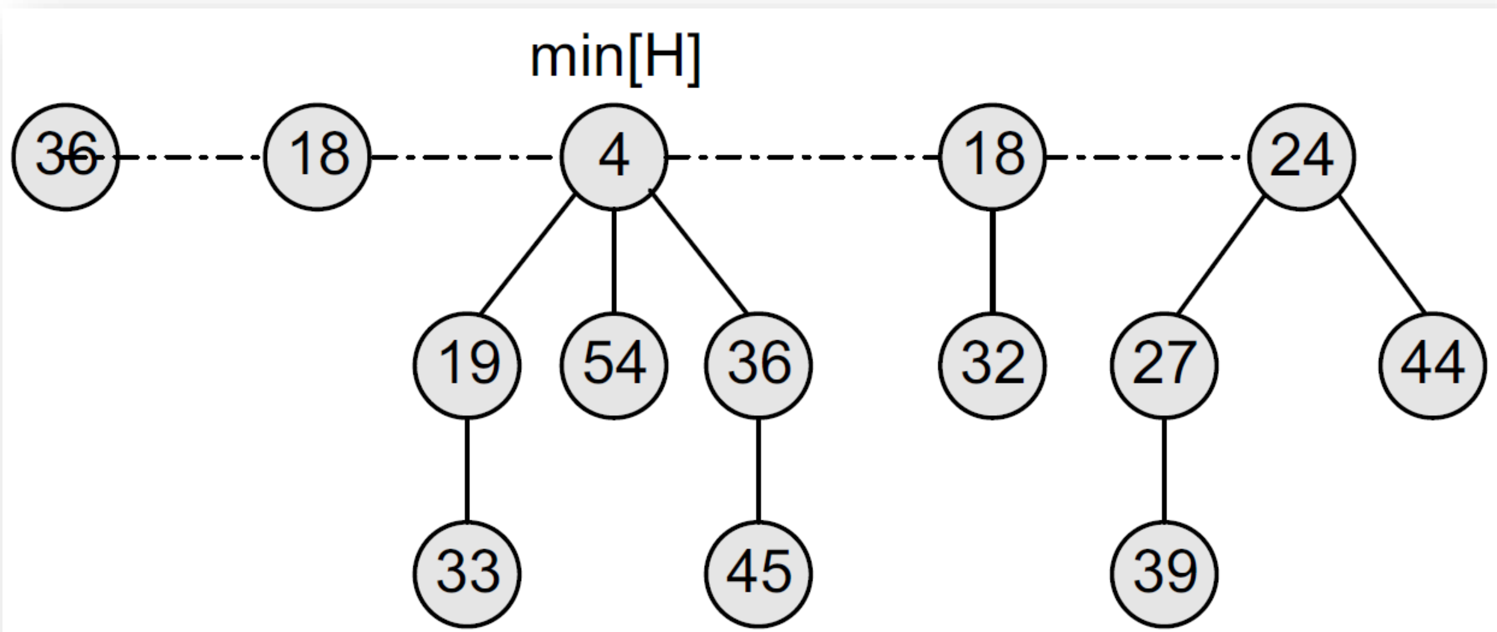
Minimum Value in Binomial Heap

- Since a binomial heap is heap-ordered, the node with the minimum value in a particular binomial tree will appear as a root node in the binomial heap



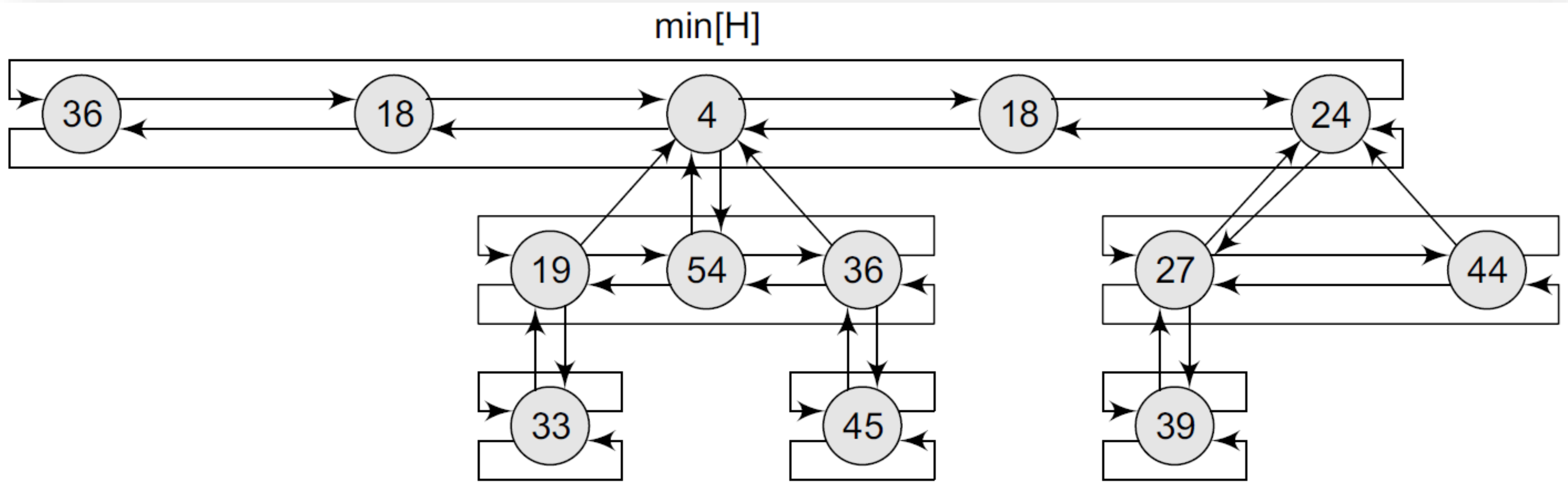
Fibonacci Heaps.

- A Fibonacci heap is a collection of trees
  - It is loosely based on binomial heaps
  - Fibonacci heaps differ from binomial heaps as they have a more relaxed structure
  - The trees in a Fibonacci heap are **not** constrained to be binomial trees



Fibonacci Heaps..

- Fibonacci heap H is generally accessed by a pointer called min[H] which points to the root that has a minimum value
  - If the Fibonacci heap H is empty, then min[H] $= NULL$



Schedule

12/3	Advanced Graphs	
12/5	Shortest Path Algorithms	
12/10		
12/12		
12/17		
12/19		
12/22	Make-up Class for 12/31?	
12/24		
12/26		
12/31	Bridge Holiday	
1/2	Study Day	
1/7	Final Exam	

Questions?

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