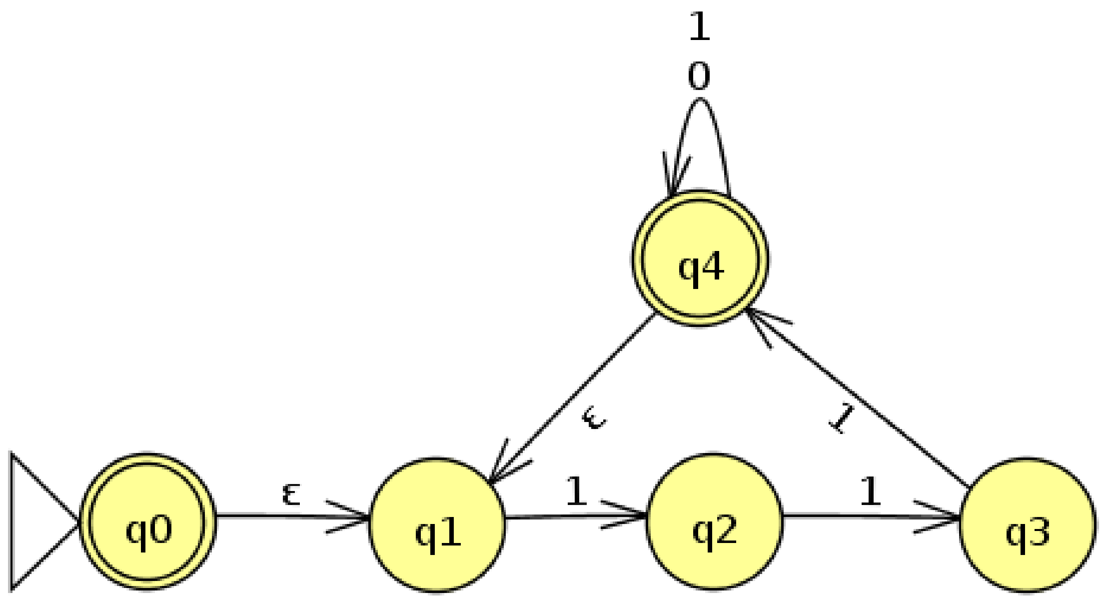
Homework 5

William Jagels

February 21, 2017

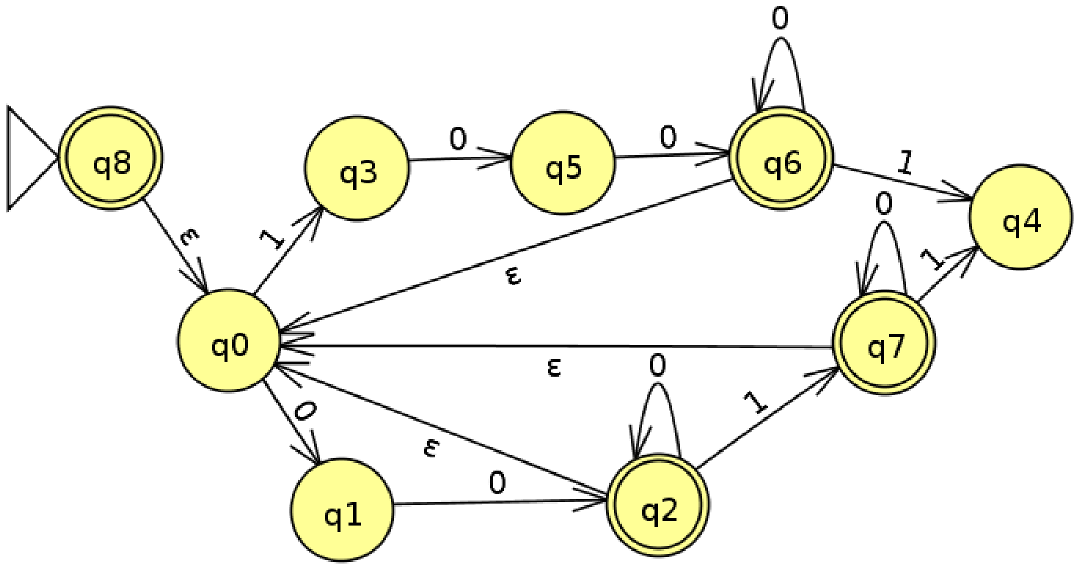
1. Exercise 1.10a

$\{w|w \text{ contains at least three 1s}\}*$



2. Exercise 1.10b

$\{w|w \text{ contains at least two 0s and at most one 1}\}*$



3. Exercise 1.29b

$A_2 = \{www | w \in \{a, b\}^*\}$

Assume A_2 is regular. Let $s = a^p b a^p b a^p b$ and divide into 3 pieces, $s = xyz, |xy| \leq p$. Therefore, xy only contains symbol a. As $|y| > 0$, set $y = a^k$, $k > 0$. $xy^2z = a^{p+k}ba^pba^pb$ results in a string not in A_2. This is a contradiction, and cannot be pumped.

4. Exercise 1.46a

$\{0^n 1^m 0^n | m, n \geq 0\}$

We can apply the pumping lemma with $s = 0^p 1 0^p$ and $s = xyz$. xy contains only zeroes. Assign $y = 0^k$ and $k \geq 0$. Now, $xy^0z = 0^{p-k}10^p$ cannot be in the language. Because of this, the language cannot be regular.

5. Exercise 1.46c

$\{w | w \in \{0, 1\}*$ is not a palindrome$\}$

In order to determine whether or not a string is a palindrome, we need to have memory. Reversing the string and comparing it to the original is impossible given that a finite automata has no memory. This would require infinitely many states, therefore the language is not regular.

6. Exercise 1.46d

$\{wtw | w, t \in \{0, 1\}^+\}$

We can apply the pumping lemma with $s = 0^p 1 0^p$ and $s = xyz$. xy contains only zeroes. Assign $y = 0^k$ and $k > 0$ due to the + operator. Now, $xy^0z = 0^{p-k}10^p$ cannot be in the language. Because of this, the language cannot be regular.

7. Exercise 1.47

Let $\Sigma = \{1, \#\}$ and let

$Y = \{w | w = x1\#x2\# \cdots \#x_k \text{ for } k \geq 0, \text{ each } x_i \in 1^*, \text{ and } x_i \neq x_j \text{ for } i \neq j\}$

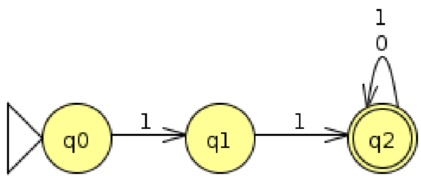
Prove that Y is not regular.

The language Y requires that there be a memory of encountered strings between delimiters. There would need to be infinitely many states to accomplish this, therefore this is not a regular language as a DFA is unable to recognize it.

8. Exercise 1.49

8.1. Part A

Let $B = \{1^k y | y \in \{0,1\}^* \text{ and } y \text{ contains at least } k \text{ 1s, for } k \geq 1\}$. Show that B is a regular language.



With the above NFA, we can recognize any number of 1s more than zero. Therefore, the language is regular. To require a different number of 1s, just add more states between $q0$ and $q2$.

8.2. Part B

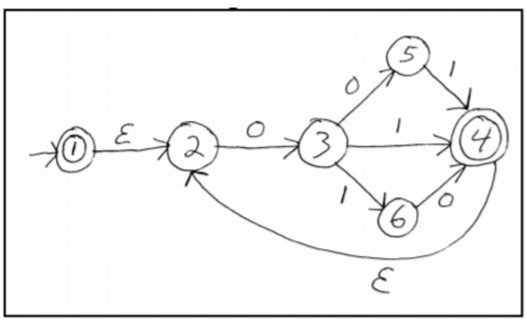
Let $C = \{1^k y | y \in \{0,1\}^* \text{ and } y \text{ contains at most } k \text{ 1s, for } k \geq 1\}$. Show that C isn't a regular language.

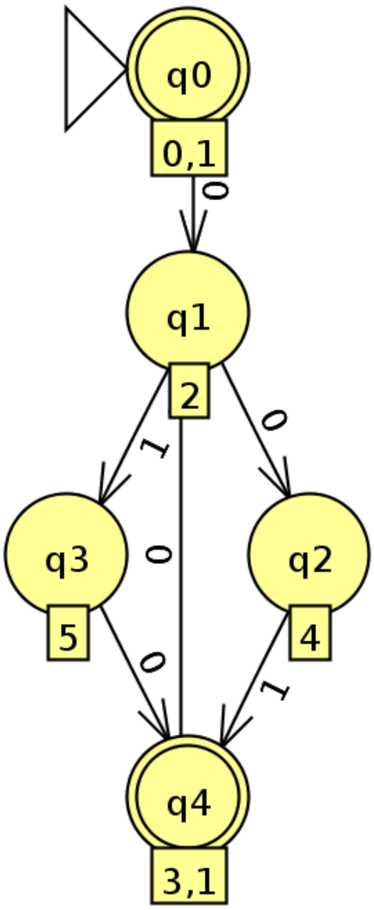
C is not a regular language because it would violate the pumping lemma. $s = 1^p 0^p 1^p \in C$, $s = xyz$, $|xy| \leq p$, $y = 1^i$, $i \geq 1$. Therefore, $xz = 1^{p-i} 0^p 1^p \in C$ which contradicts.

9. Show that $\{0^n 1^m 2^k | k \text{ divides } n+m\}$ is not regular

As there are infinitely many natural numbers, there are infinitely many ks, ns, and ms there could be. This language is infinite and therefore not regular.

10. Convert the following NFA to a DFA




q0
0,1
0
q1
2
1
0
0
q3
q2
5
4
0
1
q4
3,1