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Optimum sizing of photovoltaic battery systems incorporating uncertainty through design space approach

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Abstract

Photovoltaic-battery system is an option for decentralized power generation for isolated locations receiving abundant sunshine. A methodology for the optimum sizing of photovoltaic-battery system for remote electrification incorporating the uncertainty associated with solar insolation is proposed in this paper. The proposed methodology is based on the design space approach involving a time series simulation of the entire system. The design space approach was originally proposed for sizing of the system with deterministic resource and demand. In the present paper, chance constrained programming approach has been utilized for incorporating the resource uncertainty in the system sizing and the concept of design space is extended to incorporate resource uncertainty. The set of all feasible design configurations is represented by a sizing curve. The sizing curve for a given confidence level, connects the combinations of the photovoltaic array ratings and the corresponding minimum battery capacities capable of meeting the specified load, plotted on an array rating vs. battery capacity diagram.The methodology is validated using a sequential Monte Carlo simulation approach with illustrative examples. It is shown that for the case of constant coefficient of variation of solar insolation, the set of sizing curves for different confidence levels may be represented by a generalized curve. Selection of optimum system configuration for different reliability levels based on the minimum cost of energy is also presented. The effect of ambient temperature on sizing a stand-alone photovoltaic-battery system is also illustrated through a representative example.

Keywords: Photovoltaic-battery system; Optimum system sizing; Chance constrained programming; Sizing curve; Design space; Uncertainty

1. Introduction

Photovoltaic-battery system is a sustainable option for isolated electrical power generation for locations receiving abundant sunshine. The design of such a remote electrification unit requires estimation of the capacities of the photovoltaic generator and the battery bank to satisfy a given demand. Incorporation of the variability associated with the solar insolation is important in the system sizing as it affects the performance and reliability of the overall system. In this paper, a methodology is proposed for the optimum sizing of a photovoltaic-battery system for a predefined system reliability level. It enables the construction of a sizing curve. The sizing curve connects the combinations of the photovoltaic array ratings and the corresponding minimum battery capacities capable of meeting the specified load, plotted on an array rating vs. battery capacity diagram. This helps in identifying the entire range of feasible system configurations or the design space satisfying a specified reliability level. Generation of

Abbreviations: ACC, annualized capital cost, Rs./y; *ALCC*, annualized life cycle cost, Rs./y; *AOM*, annual operation and maintenance cost, Rs./y; *COE*, cost of energy, Rs./kWh; *CRF*, capital recovery factor; *DOD*, depth of discharge; *LOLE*, loss of load expectation; *LOLP*, loss of load probability; Rs., Indian rupees (Rs. 45 ≈ US$1 in 2008).

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Nomenclature

A	photovoltaic array area, m^2
A^*	array area variable for the generalized sizing curve, m^2
A_{min}	minimum photovoltaic array area, m^2
B	battery bank capacity, Wh
B_{min}	minimum battery energy value, Wh
B_r	maximum battery energy value, Wh
C_0	capital cost, Rs.
d	discount rate
D	demand, W
D_{actual}	minimum deterministic demand considered in the chance constraint, W
E_{dem}	energy delivered, Wh
f	net charging/discharging efficiency
h	loss of load duration in the estimation of hourly confidence level
H	total duration considered in the estimation of hourly confidence level
I_g	global solar radiation intensity, W/m^2
I_d	diffuse radiation intensity, W/m^2
I_T	solar radiation intensity on tilted surface, W/m^2
n	life, years
P	power generated by the excess power generated by photovoltaic array, W
P_{du}	photovoltaic array, W
r_b	tilt factor for hourly beam radiation
t	loss of load duration in the estimation of *LOLE*
T	maximum time considered in the time series
T_a	ambient temperature, K
T_c	cell temperature, K
T_r	reference cell temperature, K
U_{PV}	heat transfer coefficient of the module, W/m^2 K
x_P	coefficient of variation of hourly solar insolation (and power)
z_α	Standard normal variate with a cumulative probability of α

Greek symbols

α	confidence level
α_h	hourly confidence level
$(\alpha\tau)$	transmissivity–absorptivity coefficient of the module
β	tilt of photovoltaic array
β_T	temperature coefficient of the photovoltaic array, K^{-1}
η_0	photovoltaic system efficiency
η_{0r}	photovoltaic system efficiency at reference cell temperature
η_c	charging efficiency
η_d	discharging efficiency
μ_P	mean photovoltaic power, W
μ_π	mean of loss of load expectation
ρ	reflectivity of ground surface
σ_P	standard deviation of photovoltaic power, W
σ_π	standard deviation of loss of load expectation
ϵ	coefficient of variation of loss of load expectation

the design space helps in selecting and optimizing an appropriate system configuration based on the desired objective. Kulkarni et al. (2007, 2008, 2009) introduced the concept of design space for optimum sizing of solar hot water systems. The representation of the design space has been illustrated for diesel generator-battery systems (Arun et al., 2008), photovoltaic-battery systems (Arun et al., 2007), as well as wind-PV hybrid systems (Roy et al., 2007). The objective of the present work is to include the uncertainty associated with solar insolation in designing a photovoltaic-battery system. It is presented as an extension to the design space methodology. System optimization is also illustrated based on the criterion of minimum cost of energy for a given reliability level.

Sizing of a photovoltaic-battery system involves the determination of the array rating (in terms of array area or the peak power, W_p) required and the capacity of the battery bank. The existing methods range from simple correlations to detailed mathematical models. Broadly these methods can be grouped into deterministic and stochastic approaches. In the deterministic approach, the uncertainty associated with the solar insolation is not considered. Simple procedures to obtain an independent design of the photovoltaic array and battery bank have been proposed (Bhuiyan and Asgar, 2003; Sandia National Lab, 2008). Correlations between the array capacity and the battery size have been reported (Barra et al., 1984; Bartoli et al., 1984). Egido and Lorenzo (1992) proposed a correlation between the normalized array capacity, storage size and loss of load probability (*LOLP*) for different locations in Spain. Such a correlation is site specific. Tsalides and Thanailakis (1986) used a simulation-based design of stand-alone photovoltaic systems for a remote village in northern Greece. Notton et al. (1996) studied the effect of time step, input and output power profile on the sizing of autonomous photovoltaic systems based on a simulation procedure. Sidrach-de-Cardona and Lopez (1998) proposed a simple simulation model for sizing photovoltaic systems. Techno-economic analysis of photovoltaic systems for various locations in Greece was reported by Kaldellis (2004). Hontoria et al. (2005) used an artificial neural network based methodology to obtain the sizing curve for stand-alone photovoltaic systems. Markvart et al. (2006) presented the sizing curve using superposition of contributions from individual climatic cycles of low daily solar radiation for a location in the south east of England.

Probabilistic approaches of sizing photovoltaic system account the effect of the solar insolation variability in the system design. Bucciarelli (1984) proposed a sizing method treating storage energy variation as a random walk. The probability density for daily increment or decrement of storage level was approximated by a two-event probability distribution. The method was further extended to account for the effect of correlation between day to day insolation values (Bucciarelli, 1986). Bucciarelli's method was modified by Gordon (1987) and Bagul et al. (1996) where the storage energy transitions were approximated by three-events as compared to the two-event probability. Bucciarelli's (1984) method and the three-event probability approaches (Gordon, 1987; Bagul et al., 1996) relate the array size and battery capacity for constant loss of load probability (*LOLP*) through closed form equations. However, it is observed that for normalized array size greater than about 1.2, both array and battery size tends to increase for the same *LOLP*, which restricts its usability to a specific range (Egido and Lorenzo, 1992). Groumpos and Papageorgiou (1987) proposed an optimal sizing scheme for stand-alone photovoltaic battery system for a given value of *LOLP* considering the daily insolation and its standard deviation and had illustrated the method for a village power system in Arizona, USA.

Several software tools are also available for the design of photovoltaic-battery systems. RETScreen (CETC, 2004) is useful for preliminary system sizing of isolated power systems. Hybrid2 (RERL, 2007) developed by National Renewable Energy Laboratory, USA is capable of performing the detailed time series simulation of photovoltaic power systems. The Hybrid Optimization Model for Electric Renewables or HOMER, in short (NREL, 2007), uses hourly simulations for arriving at optimal system sizing of isolated power systems. However, most of the available software tools identify and simulate a single design option. They generally do not generate and evaluate the range of possible design options. Also, the effect of non-linearity in the system modeling and the randomness associated with the design variables needs to be accounted in such tools.

Simulation models are data intensive but provide a framework for performing parametric studies and system optimization. It is beneficial to incorporate the uncertainty effects on design variables in such a framework. Chance constrained programming is a useful tool applicable for studying mathematical models with random variables. The approach was introduced by Charnes and Cooper (1959) based on the basic concept of constraints in the mathematical model complying with specified values of probability. The methodology has been applied to deal with uncertainty in several disciplines of engineering. It has been used to account the effect of uncertainty in problems of structural optimization (Rao, 1980), hydro system design (Changchit and Terrell, 1993; Azaiez et al., 2005), financial risk management (Aseeri and Bagajewicz, 2004), process engineering (Li et al., 2008) etc. In this paper, chance constrained programming approach is combined with the design space approach to design a stand-alone photovoltaic system.

In this paper, a generic methodology is presented to generate the design space for different reliability levels for photovoltaic-battery systems using chance constrained programming. It is developed as an extension to the deterministic sizing approach based on the time series simulation of the entire system. Comparisons with existing methods are also given for the deterministic and stochastic approaches. The system reliability is validated through sequential Monte Carlo simulation of the system. It is observed that the sizing offered by the proposed scheme offers a conservative system design. Under the simplified assumption of a constant coefficient of variation of the solar insolation, it is shown that a single generalized sizing curve may be obtained. It enables system sizing for various confidence levels using a single curve. The proposed methodology is demonstrated through illustrative examples.

2. Design space generation with deterministic approach

The system sizing methodology for photovoltaic-battery systems for a given load and solar insolation profile following a deterministic approach is discussed in this section. The schematic of a photovoltaic-battery system configuration is shown in Fig. 1. The components include photovoltaic modules, battery bank, charge controller and inverter. The proposed methodology employs a time series simulation based on the energy balance of the overall system. The power generated by the photovoltaic array at any given time t is given by:

$$P(t) = \eta_0 A I_T \tag{1}$$

where η_0 is the photovoltaic system efficiency, A is the total array area (m^2) and I_T the total instantaneous radiation incident on the array (W/m^2). Efficiency of a photovoltaic array depends on the material of construction, cell configuration, processing technology, operating temperature, so-

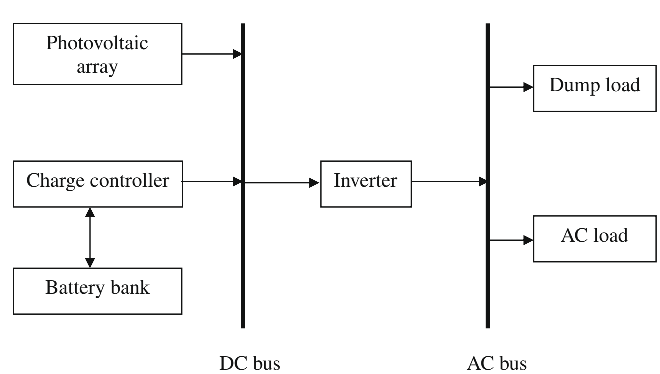

Fig. 1. Schematic of a photovoltaic-battery system for remote electrification.

lar insolation, operating current and voltage of the array, etc. Solar radiation incident on the array is calculated using the equation (Sukhatme, 1997):

$$I_T = (I_g - I_d)r_b + I_d\left(\frac{1+\cos\beta}{2}\right) + I_g\left(\frac{1-\cos\beta}{2}\right)\rho \quad (2)$$

The net power flow across the storage is accounted considering the efficiencies for the power conversion during charging and discharging processes. The rate of change of energy stored (dQ_B/dt) in the battery bank is proportional to the net power generated. The power generated is represented as the difference between the power generated by the photovoltaic array (P) and the power supplied to the load (D).

$$\frac{dQ_B}{dt} = (P(t) - D(t))f(t) \quad (3)$$

where $f(t)$ represents the efficiencies associated with the charging and discharging processes at the time step.

$$\begin{aligned} f(t) &= \eta_c \quad \text{whenever } P(t) \geqslant D(t) \\ f(t) &= \frac{1}{\eta_d} \quad \text{whenever } P(t) < D(t) \end{aligned} \quad (4)$$

These equations relate the rate of change of energy of the storage with the input power, demand power and the power conversion efficiencies during charging/discharging. The inverter efficiency is accounted in the estimation of the demand. The change in stored energy over a time period of Δt, may be expressed as follows:

$$Q_B(t+\Delta t) = Q_B(t) + \int_t^{t+\Delta t}(P(t) - D(t))f(t)dt \quad (5)$$

For a relatively small time period, Eq. (5) may be approximated as:

$$Q_B(t+\Delta t) = Q_B(t) + (P(t) - D(t))f(t)\Delta t \quad (6)$$

During the system operation over the time period Δt, whenever the energy supplied by the array is greater than the demand ($P(t) > D(t)$), the surplus energy is used to charge the battery bank. Depending on the solar radiation input, if the energy delivered by the photovoltaic array is lower than the load, the battery meets the deficit in energy demand. The load can be met if the battery has not reached its depth of discharge and in that case, the stored chemical energy is converted into electrical energy. It is assumed that the charging and discharging takes place with a constant efficiency. The self-discharge loss for the battery is assumed to be negligible. The minimum array area required and the corresponding storage capacity for meeting the specified load may be obtained by solving Eq. (6) over the entire duration. To solve Eq. (6), required inputs are the expected load curve and the solar radiation profile over a specific time interval (e.g., daily, seasonal or yearly data), photovoltaic array conversion efficiency and the nominal charging and discharging efficiencies of the battery bank-converter system. In the generalized methodology, to obtain the minimum array area, a numerical search is performed that satisfies the energy balance (6) with the following additional constraints:

$$Q_B(t) \geqslant 0 \quad \forall t \quad (7)$$

$$Q_B(t=0) = Q_B(t=T) \quad (8)$$

Eq. (7) ensures that the battery energy level is always non-negative, while Eq. (8) represents the repeatability of the battery state of energy over the time period. The repeatability condition implies that there is no net energy supplied to or drawn from the battery bank over the time horizon. It is assumed that the load is recurring in the same pattern after time T. The required battery bank capacity (B_r) is obtained as:

$$B_r = \frac{max\{Q_B(t)\}}{DOD} \quad (9)$$

where DOD is the allowable depth of discharge of the battery.

Hence the value of the minimum array area ($A = A_{min}$) and the corresponding capacity of the battery bank (B_r) may be obtained. Any array area, higher than the minimum, is capable of supplying the load. However, the capacity of the battery bank would be lower due to the increase in power input. It is important from the designers' perspective to identify all the feasible combinations for the array area and the corresponding storage capacity. To identify the minimum storage capacity for a given array area, the energy balance Eq. (3) may be modified as follows:

$$\frac{dQ_B}{dt} = (P(t) - P_{du}(t) - D(t))f(t) \quad (10)$$

Here $P_{du}(t)$ represents the excess power from the array that to be dumped. In reality, dump loads are not usually installed for stand-alone PV systems. It mathematically represents the excess power for the photovoltaic array which is not really generated as the charge regulator disconnects the photovoltaic generator from the DC bus. It may be noted that $P_{du}(t)$ is a non-negative variable introduced in Eq. (3) when the array area is greater than the minimum required.

The simulations to obtain the minimum storage capacity are carried out for different values of the array area ($A > A_{min}$). For each value of array area (A) considered, the corresponding minimum battery bank capacity is obtained by minimizing the required storage capacity Eq. (9). The optimization variables are the initial battery energy, $Q_B(t=0)$ and the excess power $P_{du}(t)$. The combinations of the different array areas and the corresponding minimum storage capacities may be plotted on an array area (or array rating) vs. battery bank capacity diagram which is defined as the sizing curve of the system. The sizing curve represents the minimum storage capacity required for a given array rating. The sizing curve divides the entire space into feasible and infeasible regions. The region above the sizing curve represents the feasible region

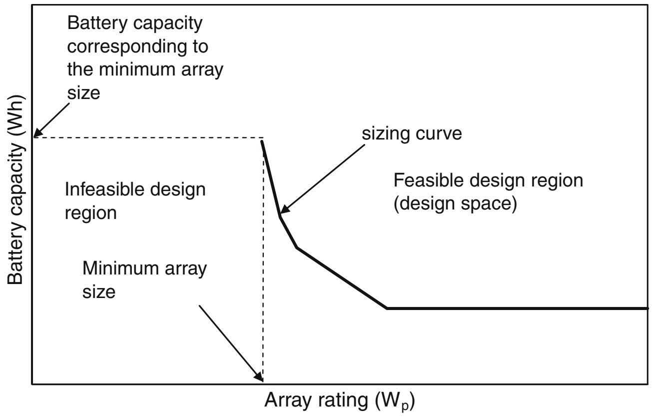

Fig. 2. Typical sizing curve and design space for a photovoltaic battery system.

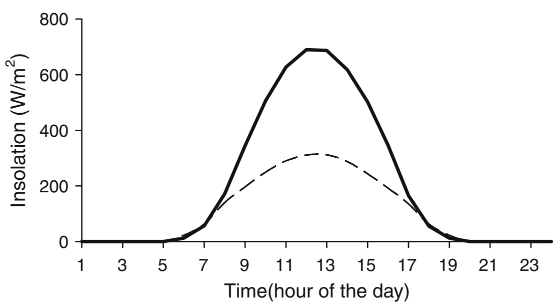

Fig. 3. Variation in the mean (solid line) and standard deviation (dashed line) of the hourly solar insolation on the array surface for an averaged day.

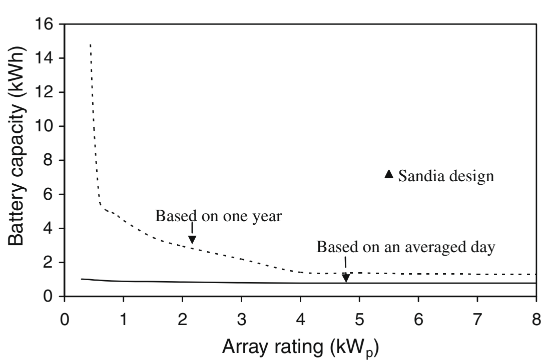

Fig. 4. Sizing curves and design space for example 1.

Table 1
Input parameters for the system sizing and optimization.

Photovoltaic array efficiency, %	10
Inverter efficiency, %	90
Net charging efficiency, %	85
Net discharging efficiency, %	85
Battery depth of discharge, %	70
Temperature coefficient of module, K^{-1}	0.0044
Reference temperature, °C	25
Heat transfer coefficient, W/m^2K	28.8
Transmissivity–absorptivity coefficient	0.9

as any combination of array area and battery capacity represents a feasible design option. The entire feasible region including the sizing curve is the design space for a given problem. A typical sizing curve and associated design space are shown in Fig. 2.

2.1. Example 1: deterministic system sizing

The system sizing methodology is illustrated with an example in this section. For a systematic comparison, the sizing results reported by Notton et al. (1996) have been utilized. The proposed methodology helps in identifying the minimum system configurations as well as the set of all feasible design configurations, called the design space.

A constant load of 42 W prevailing over the entire day is considered for the analysis (Notton et al., 1996). The sizing curves are generated for Ajaccio (41°55′N, 8°39′E) using the same input parameters as reported by Notton et al. (1996). For illustration, the sizing curve is generated initially based on a representative day. The data for the representative day is obtained as the averaged value of the 18 years hourly data recorded for the site. The variation of solar insolation in terms of the hourly mean and standard deviation for the averaged day is shown in Fig. 3. Furthermore, the sizing curve is also generated for a period of one year to account and illustrate the hour by hour variation and seasonal effects in the solar insolation. Following the proposed procedure, the system sizing curves are generated and presented in Fig. 4. The various input parameters used in the sizing are given in Table 1.

Many of the common sizing methods utilize daily or monthly mean values for solar radiation and load. The Sandia National Lab (2008) guidelines follow a deterministic approach for the system design taking monthly mean values of the daily insolation and daily load data to arrive at the system sizing. It follows a worst-scenario approach with respect to the resource and provides a unique design point. The solar radiation data corresponding to the minimum insolation month is used for sizing the array. The system sizing obtained using the Sandia guidelines (2008) is represented as a single design point in the feasible design space for comparison. It is observed that Sandia guidelines (2008) provide a conservative estimate for the battery capacity as it is based on the number of days of autonomy. For the present example, for the same array rating, the battery capacity given by Sandia guidelines (2008) comes out to be about 9.2 times the battery capacity given by the proposed approach based on an averaged day. Though Sandia guidelines (2008) allow a quick estimation, it provides a single solution which may lead to sub-optimal capacity utilization as the array and battery capacity are sized independently. As the Sandia guidelines (2008) utilize the worst-case design methodology, it results in an extremely conservative design with very high reliability of the overall system. In the proposed methodology, a time step simulation of the entire system has been performed to obtain system sizing close to actual requirement. However, the major improvement offered by the proposed methodology is a set of feasible solutions, compared to a single point solution obtained using Sandia guidelines (2008). It may also be noted that the over sizing the components does not permit understanding of the trade-offs. As the proposed methodology identifies the entire range of feasible solution, it is possible to optimize the system configuration based on the relative cost of each component.

It is seen that the sizing curve obtained using the one year hourly solar insolation data provides a higher system capacity as compared to that obtained with the averaged day. The system sizing obtained in this case accounts for all the minimum insolation periods which prevail in the year. It is desirable to use the yearly data for the resource and the demand as the seasonality associated with these variables is properly captured in the sizing process. However, it may be noted that in many situations such extensive database of solar insolation and load may not be available and the system planning has to be based on the available data for the site. It is illustrated with this example that the proposed sizing methodology is not limited by the availability of the resource and demand data.

To compare the proposed methodology with the results presented by Notton et al. (1996), the same input data set as in the original work is used for system simulation using the proposed model equations. It may be noted that the system sizing differs from that obtained considering an averaged day as a detailed data set for the solar insolation is now used. In Fig. 5, the sizing curves obtained using the proposed system equations taking one year solar radiation data are plotted along with the results reported by Notton et al. (1996).

The designer has the option of selecting an appropriate configuration from the feasible design region (or the design space) according to the requirements of additional capacity reserve and days of autonomy. Any configuration located on the sizing curve as well as inside the design space would be capable of meeting the demand. The design space which contains all the feasible configurations thus provides a framework for system optimization based on an appropriate objective function.

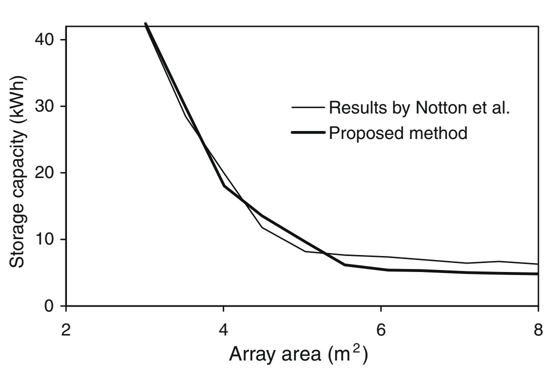

Fig. 5. Comparison of the proposed method with results reported by Notton et al. (1996).

3. Generation of the design space with probabilistic approach

The design space methodology is extended to incorporate the effects of uncertainty in solar insolation. The hourly solar input is treated as a stochastic variable. The system is sized for a specified reliability level. Chance constrained programming approach has been used to solve this optimization problem under uncertainty.

The hourly solar input is assumed to follow a normal distribution with known mean and standard deviation. This is a simplifying assumption for illustrating the proposed methodology. It is possible to evaluate the error involved in the analysis under such an assumption. The load is assumed to be deterministic over the time step. The source uncertainty is incorporated through a probabilistic constraint. The system has to cater to at least the specified deterministic demand depending on the probability of power generated by the photovoltaic array. The chance constraint relating the probability of the demand, $D(t)$, being met by the photovoltaic system is given as:

$$Prob[D(t) \geqslant D_{actual}(t)] \geqslant \alpha \tag{11}$$

where D_{actual} is the deterministic demand to be met over the time step and α the specified reliability of compliance of the constraint or the confidence level. The demand met by the system, $D(t)$, in Eq. (11) can be replaced by the system energy balance (6). After algebraic manipulation, the chance constraint for the demand can be written as:

$$Prob\left[\frac{Q_B(t+\Delta t)}{f(t)\Delta t} - \frac{Q_B(t)}{f(t)\Delta t} + D_{actual}(t) \leqslant P(t)\right] \geqslant \alpha \tag{12}$$

Eq. (12) is a chance constraint incorporating the random variable. The deterministic equivalent for the energy balance in terms of the demand (D_{actual}), mean array power $\mu_{P(t)}$ and standard deviation of array power $\sigma_{P(t)}$ is obtained as:

$$\frac{Q_B(t+\Delta t)}{f(t)\Delta t} - \frac{Q_B(t)}{f(t)\Delta t} \leqslant (\mu_{P(t)} - D_{actual}(t) - \sigma_{P(t)} z_\alpha) \tag{13}$$

where z_α is the inverse of the cumulative normal probability distribution corresponding to the required confidence level α with zero mean and unity standard deviation. Expressing the deterministic equivalent in terms of the battery energy values for any time step considering the excess power $P_{du}(t)$, we get:

$$Q_B(t+\Delta t) = Q_B(t) + [\mu_{P(t)} - D_{actual}(t) - \sigma_{P(t)} z_\alpha - P_{du}(t)] f(t)\Delta t \tag{14}$$

The storage capacity for a given array area capable of meeting the specified load and meeting the reliability requirements may be obtained by solving Eq. (14) over the entire duration. The required battery capacity for a given array size is obtained using Eq. (9) following the procedure described in the previous section.

It may be noted that z_α represents the inverse of the cumulative normal probability distribution corresponding to the required confidence level α with zero mean and unity standard deviation. The normal distribution is symmetric with respect to its mean. The mean of the standard normal distribution is obtained with a cumulative probability of 0.5. Therefore, it may be observed from Eq. (14) that the system sizing obtained for $\alpha = 0.5$ corresponds to the deterministic case, as $z_\alpha = 0$ in this case. The combinations of the different array ratings and the corresponding minimum storage requirements may be plotted on an array rating vs. battery capacity diagram which is the sizing curve for the confidence level (α). The confidence level and the loss of load expectation (*LOLE*) are approximately related by the following expression:

$$LOLE \approx \frac{(1-\alpha)\Delta t}{T} \tag{15}$$

where Δt is the time step for the simulation and T is the total time frame considered for the analysis. It may be noted that the above relation is accurate for resource profiles with a sharp peak.

3.1. Validation through Monte Carlo simulation

Sequential Monte Carlo method helps in simulating the occurrence of random events over time, recognizing the underlying statistical properties of the system. Using this approach it is possible to model the time dependence of the random variables involved in the analysis of a system. Monte Carlo simulation method is used for validating the results obtained using the chance constrained approach for the photovoltaic-battery system sizing. The methodology of Monte Carlo simulation is represented in Fig. 6. The estimated values of confidence level and *LOLE* obtained by Monte Carlo simulation for different system configurations from the design space are compared with that given by the chance constrained approach.

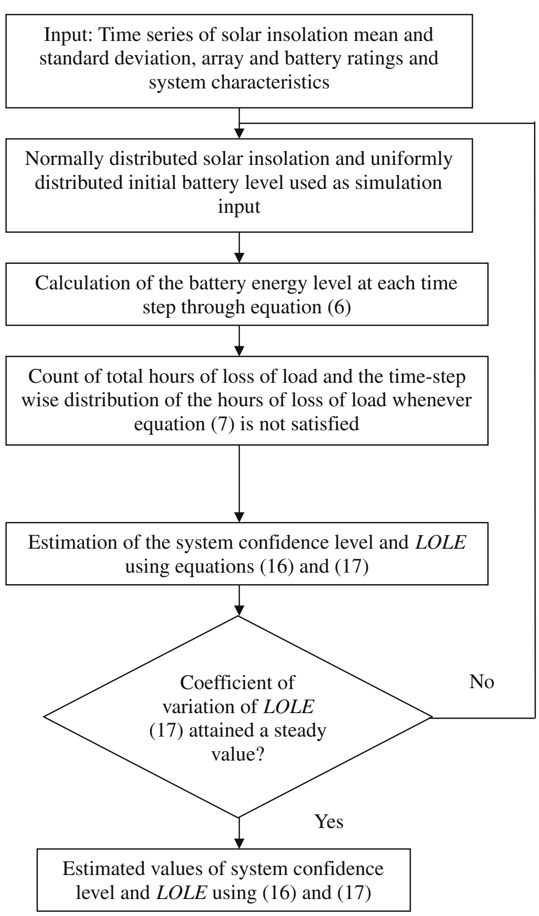

Fig. 6. Monte Carlo simulation for system reliability estimation.

The sizing curve and design space are generated following the proposed methodology, for specified confidence levels. For checking the system reliability predicted by the design space, specific array–battery bank configurations are selected from the design space. The selected system is simulated using the system energy balance (6). However, the hourly insolation is made to randomly vary in the simulation. The random insolation values used in the simulation are sampled from a normal distribution with specified mean and standard deviation for the time step. For comparison, the data set used is such that the mean and standard deviation data of the hourly insolation correspond to the same values as used in the chance constrained model for deriving the sizing curve and the design space. The initial battery level is assumed to follow a uniform distribution based on the minimum and the maximum energy capacity of the battery. The hourly simulations are carried out for the span of a year (using 8760 hourly data points), the battery energy level (Q_B) is checked with the minimum permissible battery energy level (B_{min}) at each hour. The system confidence level (α) is estimated using the following equation:

$$\alpha_h \approx 1 - \frac{\sum h}{H} \tag{16}$$

where $\sum h$ corresponds to the total duration when there is a loss of load (i.e., $Q_B(t) < B_{min}$) for the specified hour during the day. H is the total number of specified hours considered in that interval (i.e., 365 h for a yearly simulation) for hourly simulation in the specified time horizon. The minimum value of (α_h) obtained for each of the 24 h time period would correspond to the system confidence level (α). Furthermore, the system *LOLE* is estimated as:

$$LOLE \approx \frac{\sum t}{T} \tag{17}$$

where $\sum t$ corresponds to the total duration when there is loss of load (i.e., $Q_B(t) < B_{min}$) over the entire time frame and T the total time horizon of the simulation (i.e., 8760 h). Fraction of time load is not met represents the *LOLP* of the system. Eq. (17) represents the *LOLP* of the power system by calculating the fraction of time load is not met. It may be noted that this equation is different from Eq. (15). In Eq. (15), the approximate relation between the confidence level and *LOLE* is established. The confidence level and *LOLE* is computed at each iteration by randomly varying the hourly insolation and the initial battery level. The simulation is terminated when the estimated *LOLE* value for a system achieves a specified degree of confidence. The system reliability is estimated as the mean of the results obtained over the repeated simulations. The coefficient of variation (ε) is used as the parameter for deciding the stopping criterion,

$$\varepsilon = \frac{\sigma_\pi}{\mu_\pi} \tag{18}$$

where μ_π is the estimated mean of the *LOLE* and σ_π is the estimated standard deviation. The simulation is stopped when the value of coefficient of variation attains a reasonably steady value over different iterations.

3.2. Example 2: system sizing for specified reliability

In this section the application of the proposed methodology for system sizing and generation of the sizing curves for a desired level of confidence is illustrated. The same input data set and system parameters as in example 1 are used in the analysis. The additional data set used is the hourly standard deviation of the solar insolation (Fig. 3). The sizing curves are obtained for a representative average day for typical values of confidence levels (Fig. 7). The results obtained through the chance constrained model are validated through the sequential Monte Carlo simulation. It may be noted that the hourly insolation inputs to the Monte Carlo simulation are generated using the random number generation that follows the normal distribution with the mean and standard deviation provided in Fig. 3. Specific configurations selected from the sizing curve are simulated to estimate the system reliability in terms of confidence levels and approximate values of *LOLE*. The confidence level and *LOLE* values obtained for representative configurations selected from the sizing curve based on the Monte Carlo simulations are given in Tables 2–5.

For all cases, it is observed that the Monte Carlo simulation of the system predicts a higher reliability compared to the chance constrained model. This indicates that the proposed model offers a conservative design for the system.

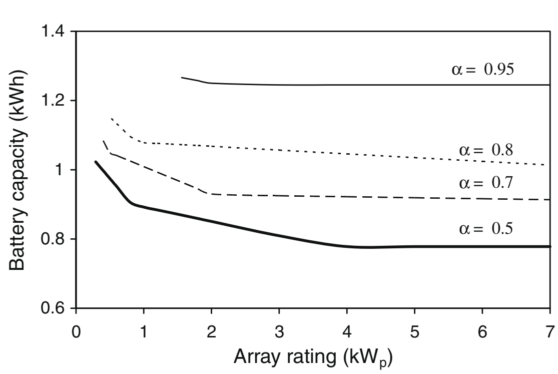

Fig. 7. Sizing curves and the design space for various values of confidence levels (example 2).

The system sizing obtained with the proposed method is compared with the two-event probability (Bucciarelli, 1984, 1986) method and three-event probability (Gordon, 1987; Bagul et al., 1996) method for a given reliability level. The sizing curve based on an averaged day for a confidence level of 0.5 (*LOLE* of 0.021) and the system sizing curves with the two-event probability (Bucciarelli, 1984, 1986) and three-event probability (Gordon, 1987; Bagul et al., 1996) approaches are represented in Fig. 8. Compared to the proposed sizing curve, two-event and three-event probability based sizing curves have regions where the analytic equations are not valid giving a physically inconsistent sizing. It reconfirms the observations of Egido and Lorenzo (1992). It is observed that if the uncertainty of solar insolation at individual hour is incorporated in the design, the over sizing of the system can be avoided, maintaining a desired level of reliability. It may be noted that in two-event and three-event probability methods, the uncertainty in solar insolation is taken over a daily time scale.

3.3. Generalized sizing curve

For locations where the standard deviation of the solar insolation does not significantly vary with time a generic representation of the sizing curve is possible. Under the condition of constant coefficient of variation, the set of all sizing curves for different confidence levels may be combined into a single generalized sizing curve. The coefficient of variation for the array power (x_P) is given by:

$$x_P = \frac{\sigma_{P(t)}}{\mu_{P(t)}} \tag{19}$$

Whenever the coefficient of variation for the array power is constant, Eq. (14) may be rewritten as:

$$Q_B(t+\Delta t) = Q_B(t) + [\eta_0 \mu_{I_T}(t) A^* - D_{actual}(t) - P_{du}(t)] f(t) \Delta t \tag{20}$$

where

$$A^* = A(1 - x_P z_\alpha) \tag{21}$$

Under this condition, a single curve may be plotted between the modified area (A^*) and the battery capacity which is the sizing curve corresponding to $\alpha = 0.5$. The system sizing for different reliability levels may be quickly determined from such a plot. For a desired confidence level, the corresponding array area (A) may be found from Eq. (21). Using the input data for example 2, the generalized sizing curve is plotted in Fig. 9 for a constant coefficient of variation value (x_P) of 0.45 for the array power. As an example, for determining the minimum battery capacity corresponding to an array area of 20 m^2, meeting a confidence level of 0.8, the value of A^* is 12.43 m^2. The required minimum battery capacity may be found directly from the battery capacity vs. the modified area plot (Fig. 9). Corresponding to the value of 12.43 m^2, the required minimum battery capacity is 882.2 Wh. The mini-

Table 2
Comparison of system confidence level (α) with Monte Carlo simulation for representative configurations from the design space with $\alpha = 0.5$.

Array area (m^2)	Battery capacity (Wh)	Confidence level (Monte Carlo)
6	952	0.6274
8	906	0.5616
10	892	0.5836
12	884	0.6493
14	875	0.6164
16	867	0.6712
18	859	0.6877
20	851	0.6849

Table 3
Comparison of system confidence level (α) with Monte Carlo simulation for representative configurations from the design space with $\alpha = 0.7$.

Array area (m^2)	Battery capacity (Wh)	Confidence level (Monte Carlo)
6	1041	0.9616
8	1025	0.9699
10	1009	0.9836
12	993	0.9753
14	977	0.9781
16	961	0.9808
18	945	0.9671
20	930	0.9753

Table 4
Comparison of system confidence level (α) with Monte Carlo simulation for representative configurations from the design space with $\alpha = 0.8$.

Array area (m^2)	Battery capacity (Wh)	Confidence level (Monte Carlo)
6	1133	0.9973
8	1096	0.9973
10	1079	0.9945
12	1076	0.9973
14	1074	0.9973
16	1072	0.9973
18	1070	0.9973
20	1068	0.9973

Table 5
Comparison of system confidence level (α) with Monte Carlo simulation for representative configurations from the design space with $\alpha = 0.95$.

Array area (m^2)	Battery capacity (Wh)	Confidence level (Monte Carlo)
16	1265	0.9973
18	1257	0.9973
20	1250	0.9973
22	1245	0.9973
24	1245	0.9973
26	1245	1
28	1245	1
30	1245	1
32	1245	1

mum battery capacity corresponding to array area of 20 m^2 and confidence level of 0.95 is also indicated on Fig. 9.

4. System optimization

The sizing curve identifies the design space which helps the designer in selecting an optimum system configuration based on the desired objective. In this section, the procedure for selecting an optimum system configuration is discussed. The cost of energy (*COE*), which accounts for the capital as well as the operating costs associated with the system, is chosen as an appropriate economic parameter to evaluate and to optimize the system configuration. It is calculated as:

$$COE = \frac{ALCC}{E_{dem}} \tag{22}$$

The annualized life cycle cost (*ALCC*) includes the annualized capital cost and the annual operating and maintenance costs. The annual operating and maintenance cost is taken as 1% of the total capital cost. E_{dem} is the total annual energy to be delivered by the system. The annualized capital cost (*ACC*) is calculated as

$$ACC = \sum_i C_0 i \times CRF_i \tag{23}$$

where, the capital recovery factor (*CRF*) is a function of the life (n) and the discount rate (d).

$$CRF_i = \frac{d(1+d)^{n_i}}{(1+d)^{n_i} - 1} \tag{24}$$

C_{0i} is the capital cost of the ith system component corresponding to the photovoltaic module, battery bank, inverter, and balance of system. CRF_i is the capital recovery factor for the ith component and it is a function of the discount rate (d) and life of the component (n_i).

4.1. Example 3: optimal system sizing incorporating uncertainty

To illustrate the system optimization, example 2 has been considered. The cost data considered for the economic analysis are given in Table 6 (Kolhe et al., 2002; NREL, 2006). Based on the minimum cost of energy for the different configurations, the optimum configuration is identified

Table 6
Economic parameters considered for system optimization.

Discount rate, d%	10
Photovoltaic array cost, Rs./kW_p	150,000
Photovoltaic array life, years	20
Inverter cost, Rs./kW	18,000
Inverter life, years	10
Battery bank cost, Rs./kWh	4000
Battery bank life, years	5
Balance of system cost, % of the hardware cost	10
Balance of system life, years	10
Annual operation and maintenance cost, % of project cost	1

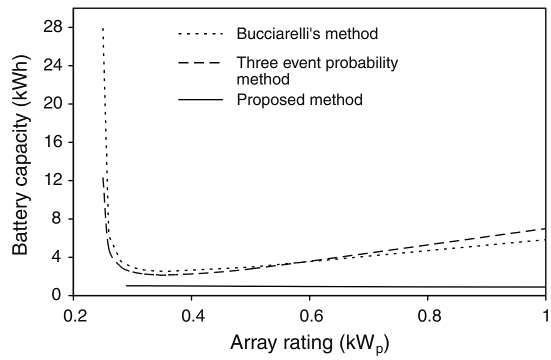

Fig. 8. Comparison of the proposed method with two-event and three-event probability methods for *LOLE* value of 0.021 (example 2).

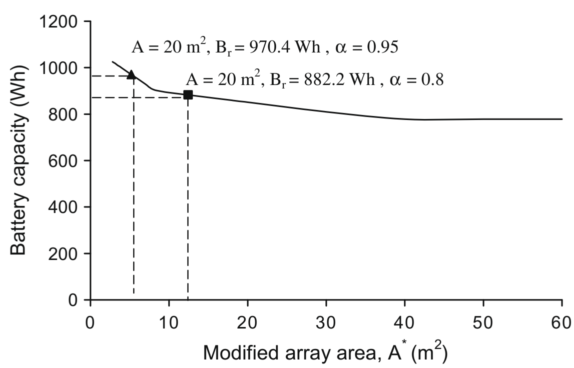

Fig. 9. Generalized sizing curve.

Table 7
Optimum cost of energy for different confidence levels (example 3).

Confidence level	Array rating (kW_p)	Battery capacity (kWh)	Cost of energy (Rs/kWh)
0.50	0.29	1.02	19.85
0.70	0.40	1.08	26.48
0.80	0.52	1.15	33.73
0.95	1.56	1.27	95.26

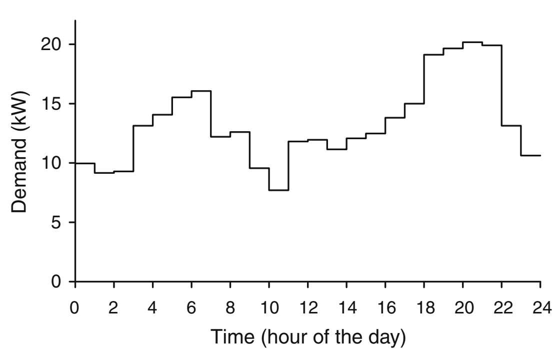

Fig. 10. Load curve for the rural location in Kerala, India (example 4).

and represented on the sizing curves for various values of confidence levels. The results are similar to those reported in Fig. 7 and not shown for brevity. The optimum cost of energy for different values of confidence levels are listed in Table 7. It is observed that for ensuring higher levels of reliability, with increase in the system capacities the associated cost of energy also increases. Depending on the reliability requirements and the availability of capital any appropriate configuration may be selected from the design space. It may be noted that the minimum cost of energy may be calculated by searching for the minimum value of cost of energy along the sizing curve for a specified confidence level. In absence of any discrete variables, optimum configuration must lie on the sizing curve. This is principle optimization approach adopted to calculate the optimum system configuration. The entire set of system equations is also optimized numerically using non-linear optimization technique to validate these results.

4.2. Example 4: optimal sizing for a remote indian location

The optimal system sizing, based on the estimated load profile and solar radiation profile for an isolated rural location in Western Ghat region of Kerala, India (Ashok, 2007), is presented here. The load curve shows a varying demand pattern over the day (Fig. 10). In previous examples, the effect of ambient temperature has not been considered explicitly. However, the proposed methodology is not limited by such an assumption. The effect of the ambient temperature of the sizing curve is demonstrated with this example. The solar insolation profiles as well as the ambient temperature used for the system sizing are represented in Fig. 11. The module efficiency has been modeled as a function of the cell temperature (Mattei et al., 2006):

$$\eta_0 = \eta_{0r}[1 - \beta_T(T_c - T_r)] \tag{25}$$

where η_{0r} is the module efficiency at the reference temperature (T_r), T_c is the cell temperature, and β_T is the temperature coefficient of the module. The cell temperature can be calculated based on the thermal energy balance of the module (Mattei et al., 2006).

$$T_c = \frac{U_{PV}T_a + I_T[(\alpha\tau) - \eta_{0r} - \beta_T\eta_{0r}T_r]}{U_{PV} - \beta_T\eta_{0r}I_T} \tag{26}$$

where U_{PV} is the heat transfer coefficient of the module, T_a is the ambient temperature, and $(\alpha\tau)$ is the effective transmissivity–absorptivity coefficient of the module. The input parameters considered for the system sizing and optimization, are as given in Tables 1 and 6.

Sizing curves have been plotted considering (i) only the nominal efficiency and (ii) by considering the temperature effect for confidence levels of 0.5 and 0.95, respectively (Fig. 12). It is observed that the effect of temperature on the sizing curve is pronounced for the lower array ratings. Further the sizing requirements are found to be higher for higher levels of system confidence when the temperature effect is accounted. Higher the ambient temperature, more area is required at higher confidence level. The comparison of system confidence levels and *LOLE* with those obtained with Monte Carlo simulation approach is presented in

Table 8. It is observed that the proposed methodology offers a conservative design.

Optimum system configurations for these confidence levels are also determined based on the minimum cost of energy. The optimum cost of energy for a confidence level of 0.5 (*LOLE* of 0.02) is Rs. 21.26/kWh while for meeting a confidence level of 0.95 (*LOLE* of 0.002) it comes to be Rs. 47.37/kWh. When the temperature effect is accounted, the cost of energy for meeting a confidence level of 0.5 is Rs. 25.12/kWh while for meeting a confidence level of 0.95 it comes to be Rs. 65.66/kWh. For higher confidence level, the effect of temperature variation cannot be neglected.

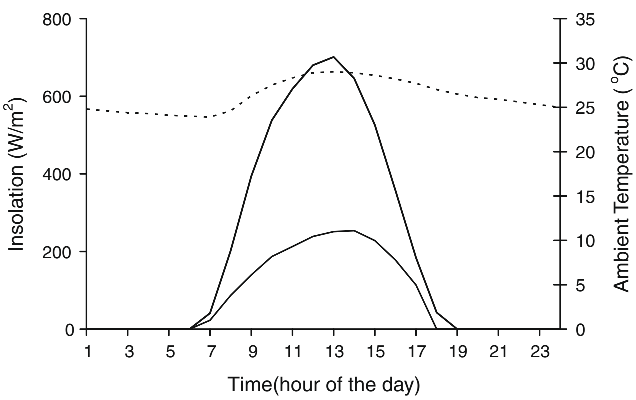

Fig. 11. Variation in the mean (thin curve) and standard deviation (bold curve) of the hourly solar insolation as well as the ambient temperature (dashed curve) for a representative day for the rural location in Kerala, India (example 4).

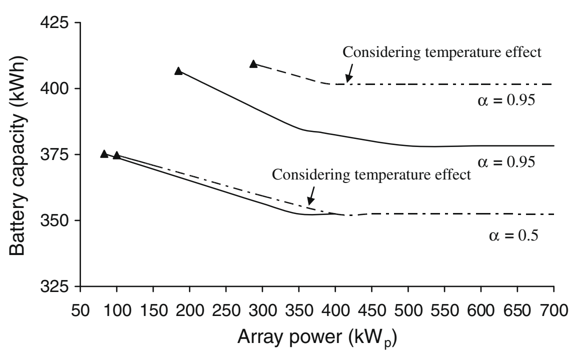

Fig. 12. Optimum system configurations for the rural location in Kerala, India (example 4).

The system performance can be compared considering the ratio of the catered load power to the maximum photovoltaic power that the generator could generate for the actual irradiance and temperature conditions. This is also known as the performance ratio. The average performance ratio of the optimum system for the confidence level of 0.5 is found to be 0.67. For the confidence level of 0.95, the value of performance ratio drops to 0.4 because of the higher system rating. Typically, as the system reliability increases the performance ratio of the PV system gets reduced.

5. Conclusions

The concept of design space for the optimum system sizing of photovoltaic-battery bank systems incorporating solar resource uncertainty is presented in this paper. Based on the proposed approach, for given resource, demand and system characteristics, a sizing curve may be plotted on the photovoltaic array rating vs. storage capacity diagram for a desired confidence level. The proposed methodology is thus capable of handling the uncertainty associated with the solar insolation at the design stage following a chance constrained programming approach. Generation of the design space offers flexibility in the overall design process as it identifies all the feasible configurations for meeting a given load and reliability. The methodology for obtaining the design space is based on the energy balance of the overall system. In deriving the discretised solution for the energy balance, the interval of integration has been considered to be finitely small so that demand and resource values are constant during that time and therefore the conversion efficiency values also remain constant over the time period. In the energy balance, discrete hourly time steps have been considered in all the illustrations as the data for resource and demand are generally available for hourly intervals. However, the proposed methodology is flexible to take up any time step. It may also be noted that though, for illustrating the method, averaged day profile for the solar

Table 8
The system confidence levels (α) and corresponding *LOLE* compared with the Monte Carlo simulation for representative configurations from the design space (example 4).

α = 0.5					α = 0.95				
Array area (m^2)	Without the effect of ambient temperature		Considering the effect of ambient temperature		Array area (m^2)	Without the effect of ambient temperature		Considering the effect of ambient temperature	
	Battery capacity (kWh)	Confidence level (Monte Carlo)	Battery capacity (kWh)	Confidence level (Monte Carlo)		Battery capacity (kWh)	Confidence level (Monte Carlo)	Battery capacity (kWh)	Confidence level (Monte Carlo)
1000	374	0.5534	–	–	2100	387	1	–	–
1200	372	0.6632	374	0.6438	2500	383	1	–	–
1500	369	0.7205	371	0.7589	3000	378	1	408	0.9973
2000	365	0.8164	367	0.8301	3600	374	1	404	1
3000	356	0.8438	359	0.8712	3800	373	1	403	1
4000	352	0.9041	352	0.8904	4000	373	1	402	1

resource and demand have been considered, the method is not limited by the extent of data used. Depending on the availability of data for the resource and the demand accurate design space may be generated. The approach can also be adopted incorporating alternate models for the system components as the basic methodology is simulation based.

In the chance constrained model, the hourly solar insolation is treated as a normally distributed random variable. This is a simplifying assumption but alternate models for the probability distribution may be considered for the analysis. It is shown that under the condition of constant coefficient of variation of hourly solar insolation (and hence power), the set of sizing curves for different confidence levels may be represented by a single curve. This helps in providing a quick estimation of the system sizing for any desired value of reliability.

Through examples, identification of the design space for typical demand-resource profiles has been presented. Systematic comparison is made with the existing results and current methodologies adopted for system design. The reliability levels given by the design space approach is validated through the sequential Monte Carlo simulation approach. It is seen that the generation of the design space for a specified reliability level would prevent the over sizing of systems. Larger capacity systems may be planned only in situations where very low values of *LOLE* are desired. Selection of optimum system based on minimum cost of energy is also discussed for different values of reliability. The methodology can be also used in the design of integrated isolated systems involving other sources of energy and storage. Thus, the design space approach is a useful tool for the system planning of isolated power systems as it provides the trade-offs between the cost of energy and the system reliability.

The effect of the ambient temperature of the sizing curve is demonstrated through an illustrative example. It is observed that the effect of temperature on the sizing curve is pronounced for the lower array ratings. As the array rating increases, the effect of ambient temperature on system sizing reduces for deterministic cases. For a high system reliability, the effect of ambient temperature on system sizing and the optimum cost of energy cannot be neglected.

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