Demo: Convergence of Spectral Radii

The purpose of these experiments is to treat the the spectral radii as data. That is do statistically analysis on our data, and attempt to gain information so that we can prove some of these phenomena.

In[3]:=

```
(* Upload Code for Working With Random Matrices*)
PacletDirectoryLoad["/home/lodewyk/Documents/Wolfram/MyPaclets/LodewykJansenvanRensburg__Ap
Needs["LodewykJansenvanRensburg`ApproximateSpectra`"];
```

Generating Haar Unitaries

In[5]:=

```
(* Test with a 1000x1000 matrix *)
(* Run to generate new unitaries*)
n = 1500; (*750 nice size to work with*)

Un1 = RandomVariate[CircularUnitaryMatrixDistribution[n]];
Un2 = RandomVariate[CircularUnitaryMatrixDistribution[n]];
Un3 = RandomVariate[CircularUnitaryMatrixDistribution[n]];

G1 = (1/Sqrt[n])*RandomVariate[GaussianUnitaryMatrixDistribution[n]];
G2 = (1/Sqrt[n])*RandomVariate[GaussianUnitaryMatrixDistribution[n]];

(* Pass this into AnalyzeTupleSpectrum *)
U = {Un1, Un2};
G = {G1,G2};
```

Example 1: Seeing what happens to average spectral radii, and variance of spectral radii as the size d-> infty.

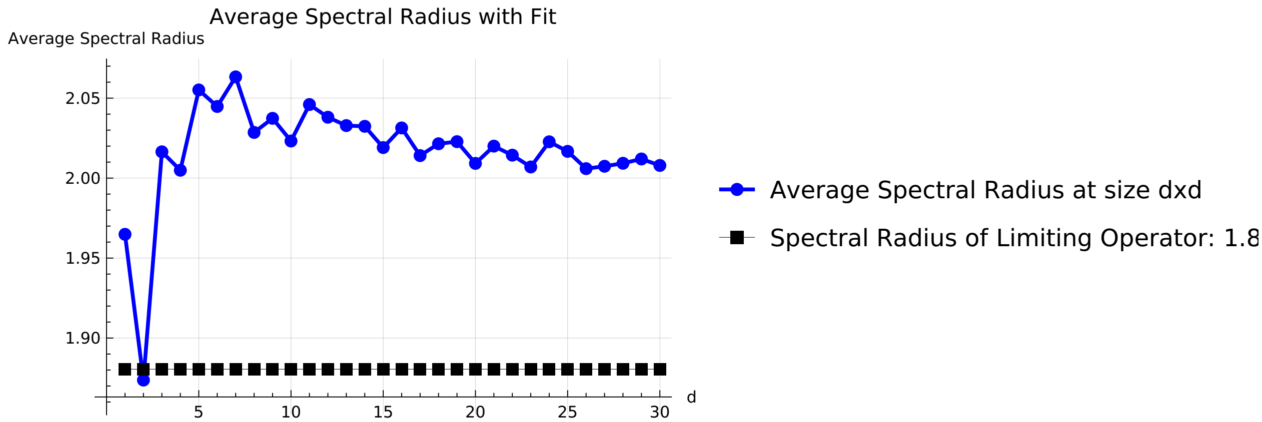
In[50]:=

```
(* Parameters *)
k = 2; (* Size of Coefficient Matrix*)
g = 2; (* Length of tuple, must be 2 for AnalyzeTupleSpectrum*)

SampleSize = 200; (* Average over this amount of matrices *)
MatrixStopSize = 30; (* Max size of matrices *)
MatrixStarSize = 1; (* Min size of matrices *)

(* Randomly Generate Coefficient Matrix*)
A = (1/2)*RandomSelfAdjointMatrixTuple[k, g];
A = RandomComplex[{},{g,k,k}];
PrintTuple[A]
(* Plots the Averaged Spectral Radius as size d grows *)
SpectralRadiusFitPlot[A, SampleSize, MatrixStarSize, MatrixStopSize]
(* Plots the Variance in Our Data for large d *)
SpectralRadiusVarianceFitPlot[A, SampleSize, MatrixStarSize, MatrixStopSize]

(* Plots an approximation of the distribution of the spectral radius at a fixed size*)
(* What does this do? Given a fixed matrix size d, is computes numRealization times the
and plots the historgram of bins. That is the area under the histrogram represents the
a given interval. *)
d = 50;
numRealization = 1000;
SpectralRadiusHistogramPlotWithFit[A, numRealization, 50]

(* Plots the spectrum of one realization *)
AnalyzeTupleSpectrum[A,U,PlotStyle → "2D"]
```

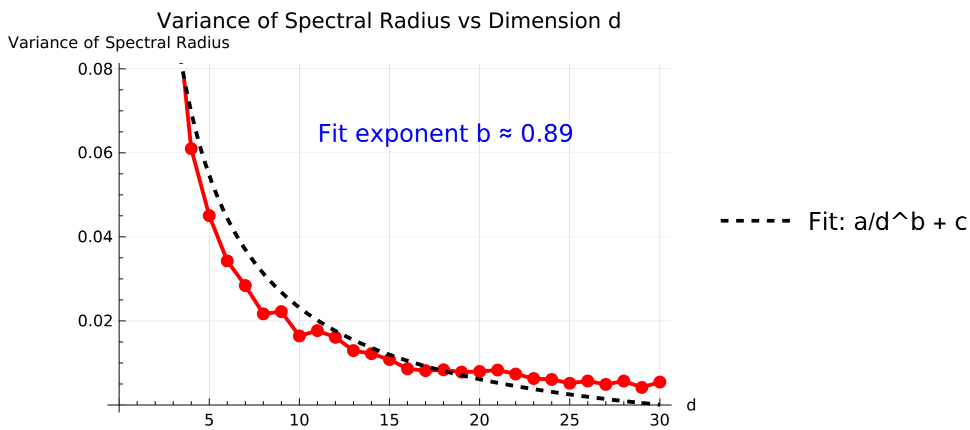
$$A1 = \begin{pmatrix} 0.243941 + 0.886397\,i & 0.671406 + 0.299742\,i \\ 0.761501 + 0.0682416\,i & 0.582714 + 0.566853\,i \end{pmatrix}$$

$$A2 = \begin{pmatrix} 0.766778 + 0.0623638\,i & 0.543836 + 0.721837\,i \\ 0.0236018 + 0.451677\,i & 0.554193 + 0.345989\,i \end{pmatrix}$$

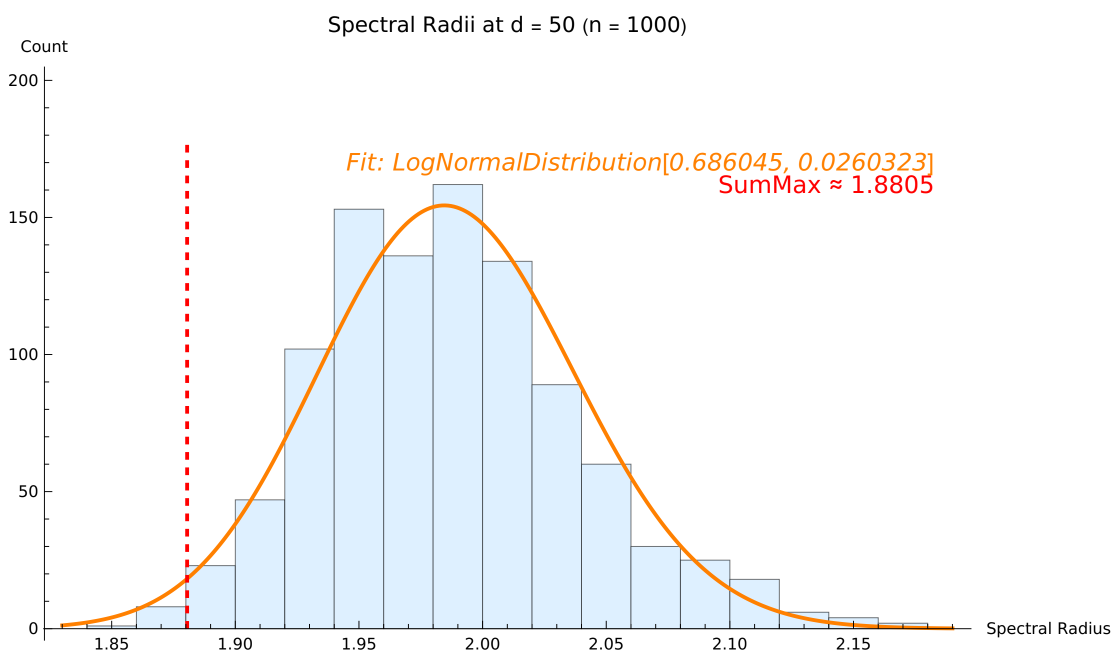
Out[58]=



Out[59]=



Out[62]=



```
==========================================================================================
 AnalyzeTupleSpectrum:
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Sqrt of eig values of  A1bar ⊗ A1 + A2bar ⊗ A2: {1.88052, 0.91409, 0.499371, 0.499371}
Sqrt of eig values of A1bar ⊗ A1 - A2bar ⊗ A2 : {1.3108, 1.3108, 0.596922, 0.340268}

Sqrt of eig values of  A1bar ⊗ A1 + A2bar ⊗ A2, and A1bar ⊗ A1 - A2bar ⊗ A2:
{0.340268, 0.499371, 0.596922, 0.91409, 1.3108, 1.88052}
==========================================================================================
```
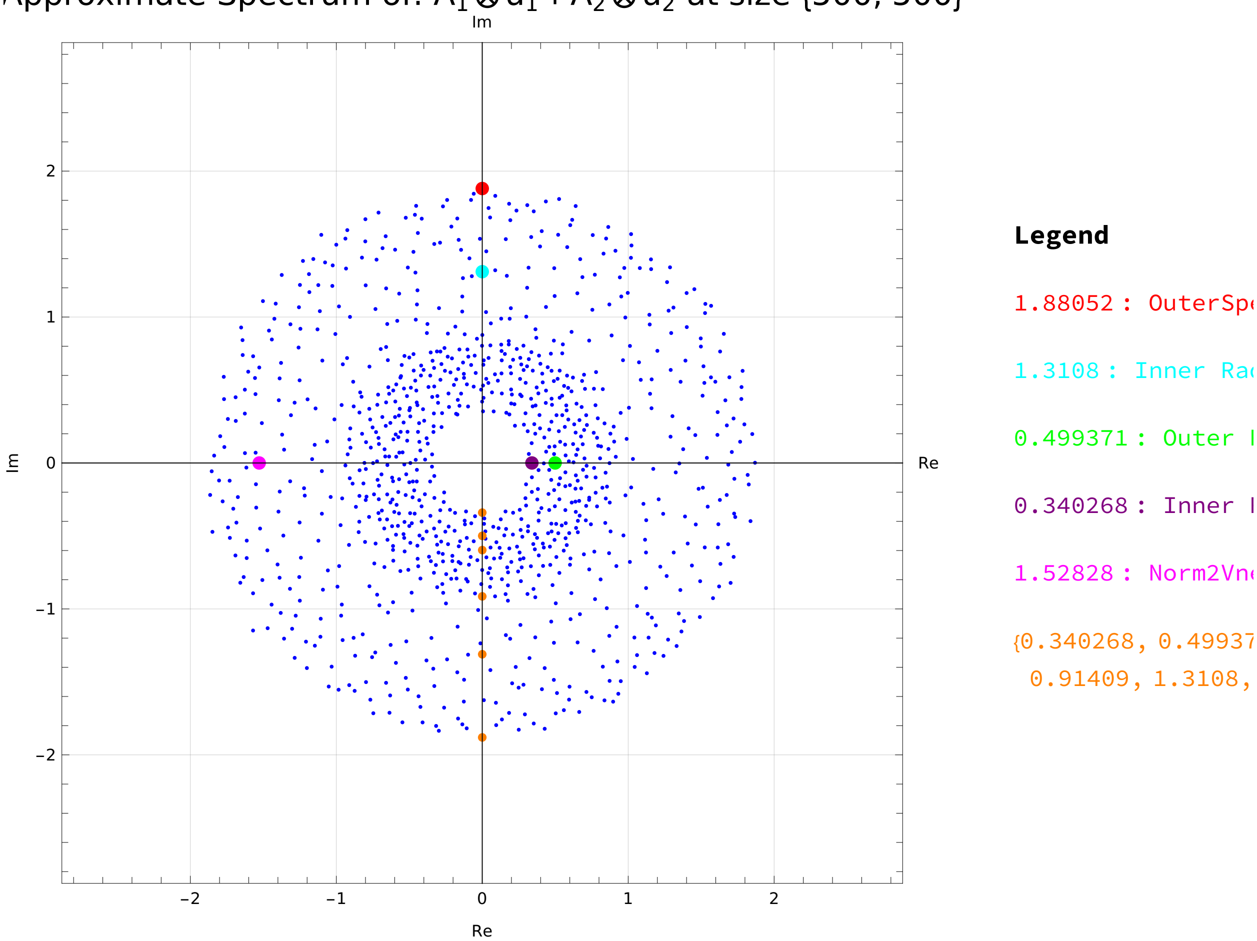

Out[63]=

Approximate Spectrum of: $A_1 \otimes u_1 + A_2 \otimes u_2$ at size {500, 500}



To study the two characteristic polynomials.

In[64]:=
```
AnalyzeCharPoly[A, True];
PrintTuple[A]
```

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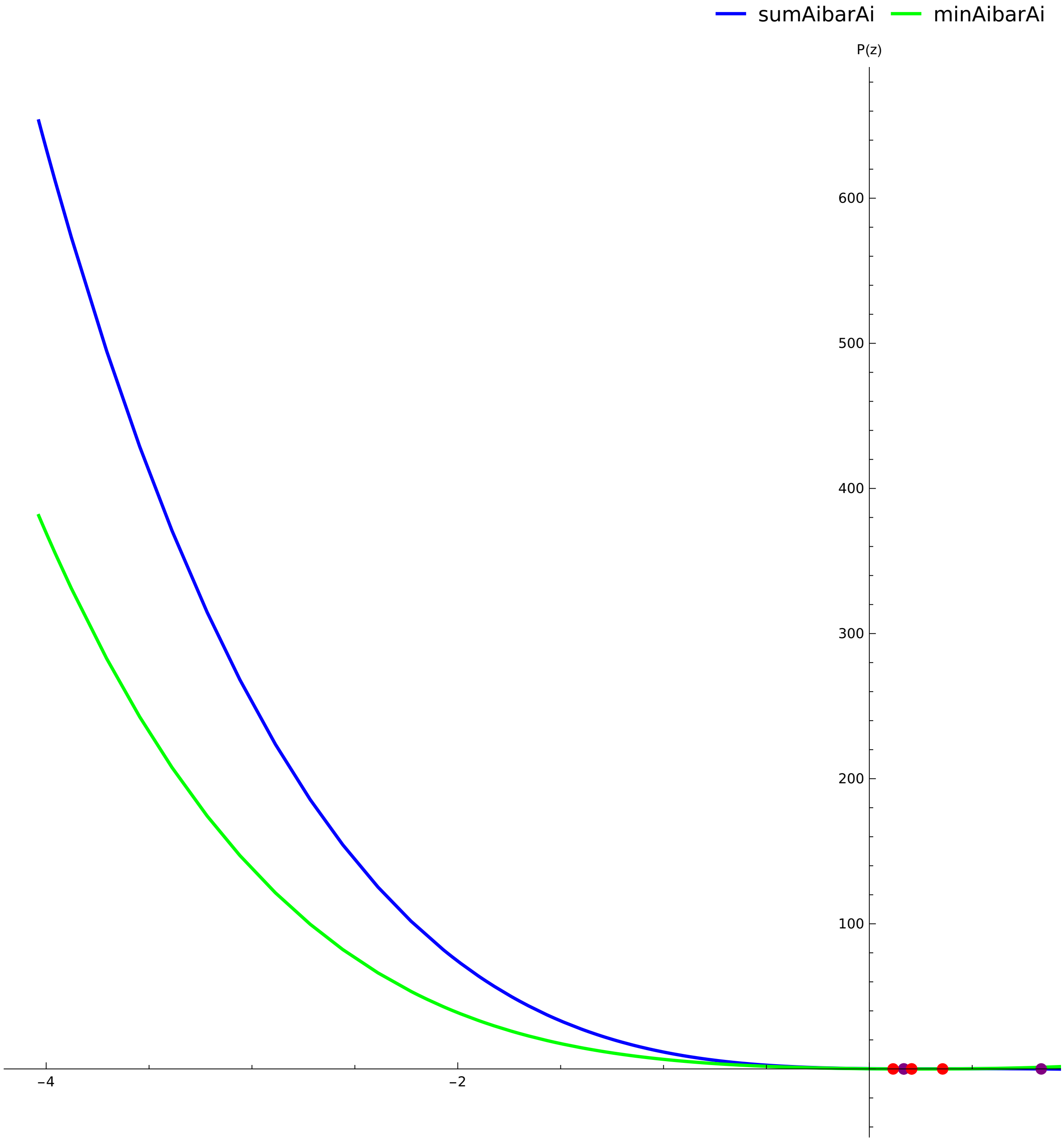
Analyzing Characteristic Polynomial of
A1bar ⊗ A1 + A2bar ⊗ A2 and A1bar ⊗ A1 - A2bar ⊗ A2 :

==



========= Additional Info =============

Characteristic Polynomial of A1bar ⊗ A1 + A2bar ⊗ A2: $\left(0.183749 - 5.55112\times10^{-17}\,i\right) - \left(1.26207 - 1.11022\times10^{-15}\,i\right) z + \left(4.48209 - 1.33227\times10^{-15}\,i\right) z^2 - \left(4.70701 - 1.66533\times10^{-16}\,i\right) z^3 + z^4$

Matrix: A1bar ⊗ A1 + A2bar ⊗ A2 =

$$\begin{pmatrix} 1.43704+0.\,i & 0.891492-0.00243922\,i & 0.891492+0.00243922\,i & 1.35744+0.\,i \\ 0.292516-0.313481\,i & 1.09113-0.147502\,i & 0.870604+0.0461658\,i & 1.11229-0.00595137\,i \\ 0.292516+0.313481\,i & 0.870604-0.0461658\,i & 1.09113+0.147502\,i & 1.11229+0.00595137\,i \\ 0.78911+0.\,i & 0.651776+0.149743\,i & 0.651776-0.149743\,i & 1.08772+0.\,i \end{pmatrix}$$

Characteristic Polynomial of A1bar ⊗ A1 - A2bar ⊗ A2: $(0.121792+2.79608\times10^{-18}\,i)-(1.41069+4.13933\times10^{-17}\,i)\,z+(3.18769+2.58411\times10^{-16}\,i)\,z^2-(0.883576-7.39394\times10^{-16}\,i)\,z^3+z^4$

Matrix: A1bar ⊗ A1 - A2bar ⊗ A2 =

$$\begin{pmatrix} 0.253369+0.\,i & -0.0325443-1.04159\,i & -0.0325443+1.04159\,i & -0.276176+0.\,i \\ 0.199985-1.00321\,i & 0.198084-0.608972\,i & 0.192858-0.411038\,i & 0.0100086+0.4178\,i \\ 0.199985+1.00321\,i & 0.192858+0.411038\,i & 0.198084+0.608972\,i & 0.0100086-0.4178\,i \\ 0.379971+0.\,i & 0.313065+0.634044\,i & 0.313065-0.634044\,i & 0.234039+0.\,i \end{pmatrix}$$

========= End =============

$$A1 = \begin{pmatrix} 0.243941+0.886397\,i & 0.671406+0.299742\,i \\ 0.761501+0.0682416\,i & 0.582714+0.566853\,i \end{pmatrix}$$

$$A2 = \begin{pmatrix} 0.766778+0.0623638\,i & 0.543836+0.721837\,i \\ 0.0236018+0.451677\,i & 0.554193+0.345989\,i \end{pmatrix}$$

This shows that there is high probability that the spec-rad at step d is larger than the spectral radius of the limiting operator.

Example 2: Linear Combination in Gaussian Random Matrices.

What is interesting is that this shows for fix d, the probability of the spectral radius at level d, is less than true spectral radius is high. Where in the Haar unitary case, we get more cases where the true spectral radius is a lower bound.

In[27]:=
```
A = RandomComplex[{},{g,k,k}];
SpectralRadiusHistogramPlotWithFitGUE[A, 1000, 90]
```

Out[28]=