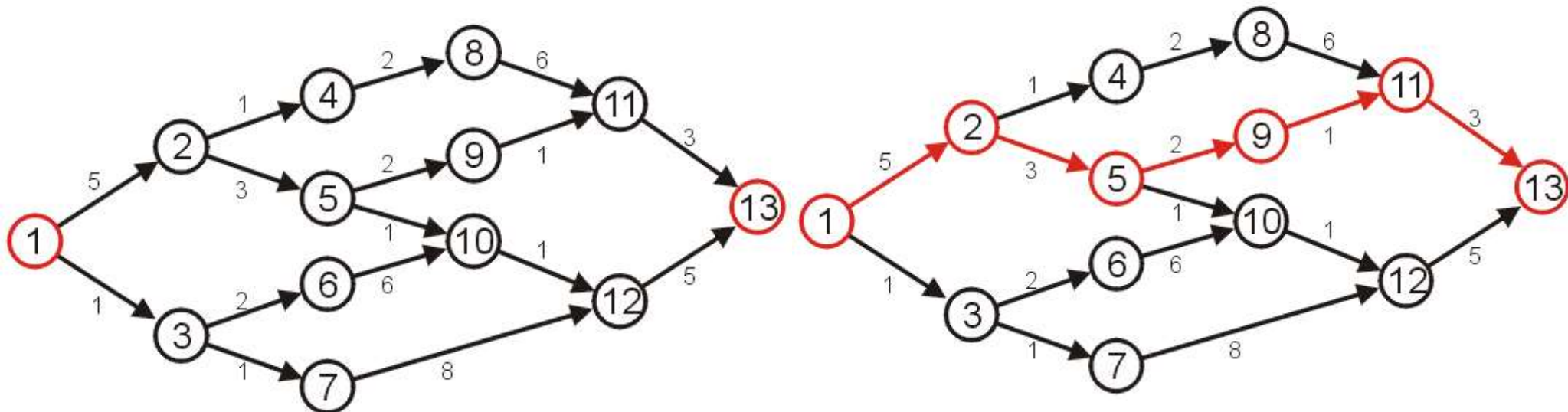


Data Structures

22. Shortest Path Trees

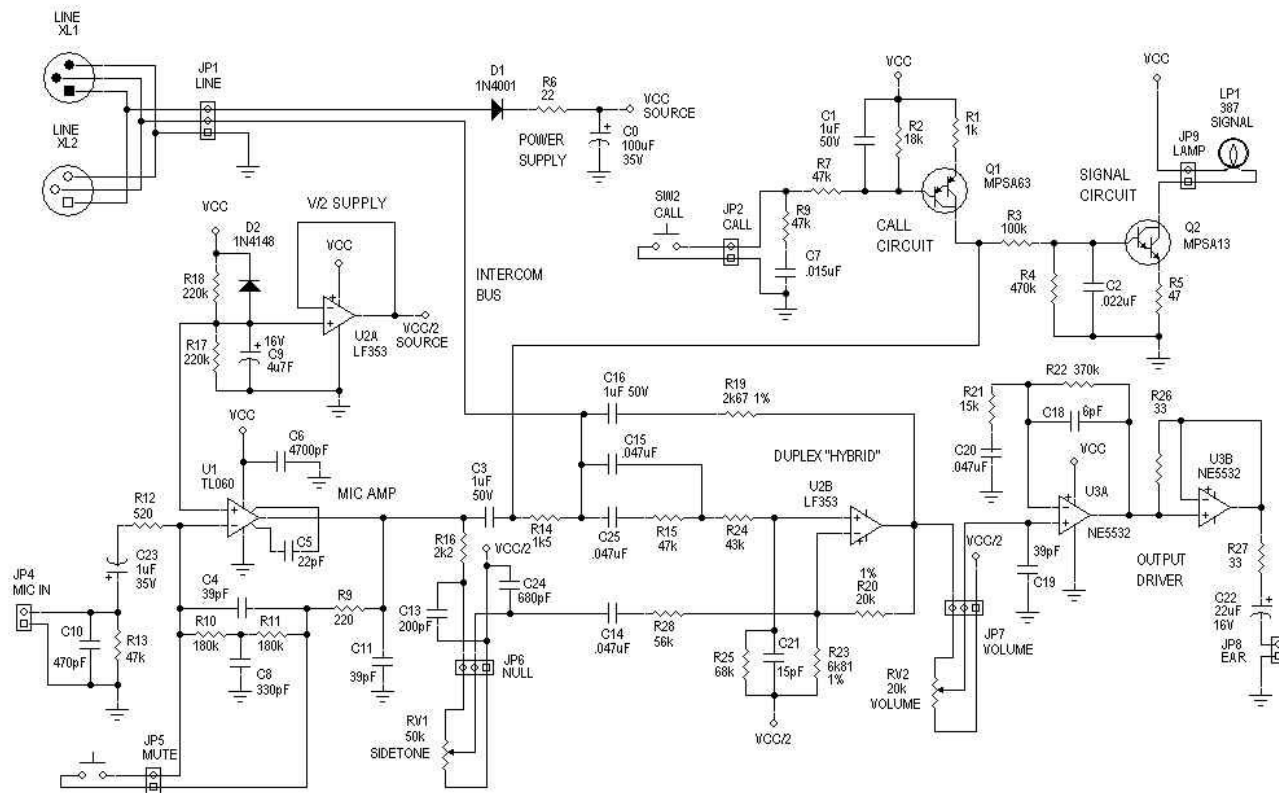
Shortest Path

- Given a weighted graph
 - Problem is to find the shortest path between two given vertices
- Length of a path in a weighted graph
 - Sum of the weights of each of the edges in that path
- **Example:** Shortest path from vertex 1 to vertex 13
 - Other paths exists but they are longer



Application – Circuit Design

- The time it takes for a change in input to affect an output depends on the shortest path

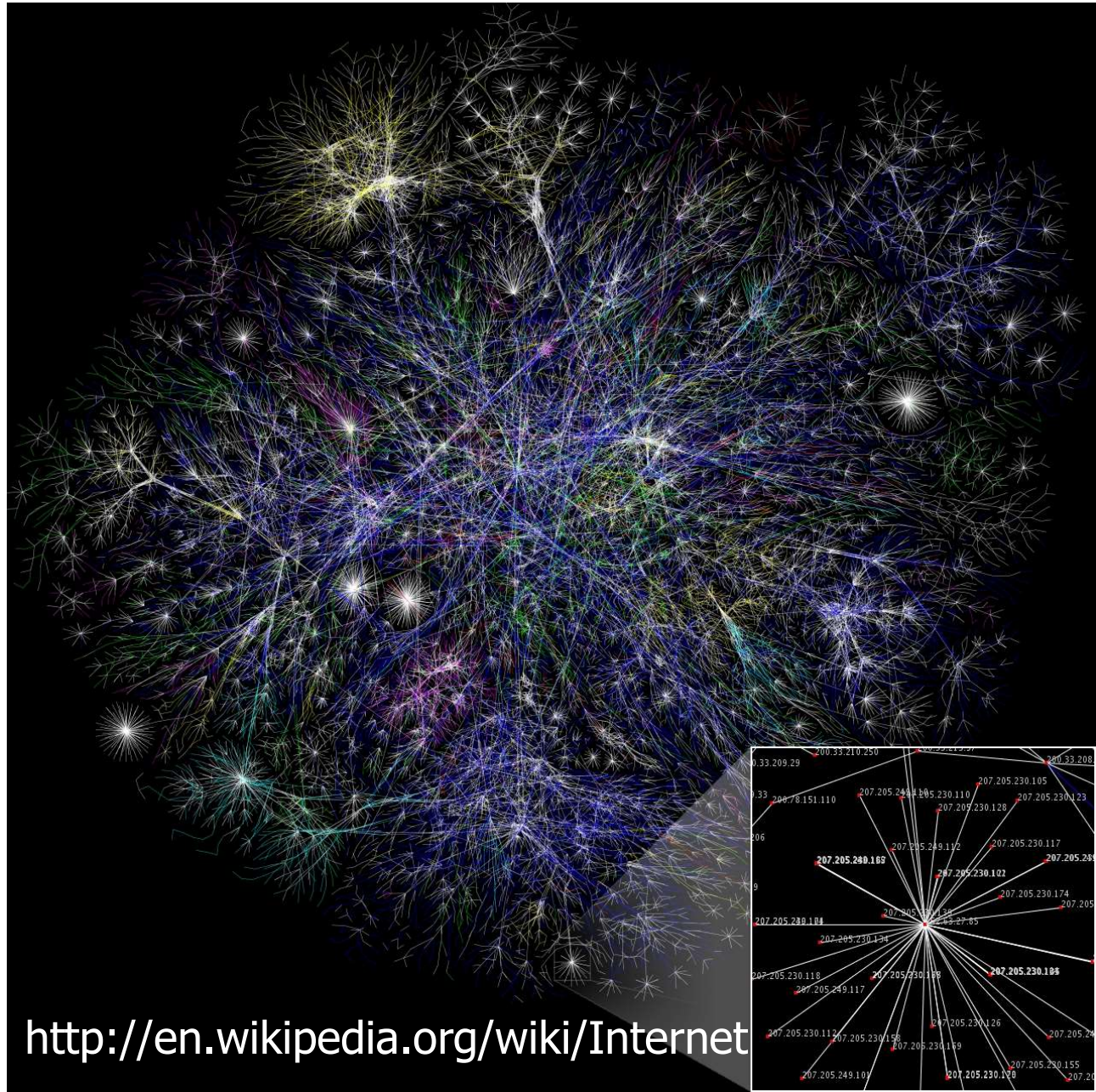


Application – Computer Networks

- The Internet is a collection of interconnected computer networks
 - Information is passed through packets
- Packets are passed from the source, through routers, to their destination
- Routers are connected to either:
 - Individual computers, or
 - Other routers
- These may be represented as graphs

Application – Computer Networks

- A visualization of the graph of the routers and their various connections through a portion of the Internet



Application – Computer Networks

- The path a packet takes depends on the IP address
- Metrics for measuring the shortest path may include
 - Low latency (minimize time)
 - Minimum hop count (all edges have weight 1)

Application – Traffic

- Find the shortest route between two points on a map
 - Shortest path, however, need not refer to distance...

from: University of Waterloo to: Rideau Hall - Google Maps - Mozilla Firefox

File Edit View History Bookmarks Tools Help

http://maps.google.ca/

G-Mail ECE UG CBC SE 240 WO UW Wireless Quest Schedule Maps Face

Saved Locations | Sign in | Help

Google Maps Canada

Search the map Find businesses Get directions

Search Results My Maps

New! Drag & drop the blue line to customize your route

☐ Avoid highways [Get reverse directions](#)

From: University of Waterloo
200 University Ave W,
Waterloo, ON

Drive: 553 km – about 5 hours 47 mins

1. Head southeast on Ring Rd toward University Ave W 17 m
2. Turn left at University Ave W 3.5 km
3. Take the ramp onto Conestoga Pkwy/HWY 85 S
Continue to follow Conestoga Pkwy 6.8 km
4. Take the exit onto HWY 8 E toward HWY 401 8.0 km
5. Take the exit onto HWY 401 E 65.8 km
6. Continue on HWY 401 Express E (signs for RTE 401 E/HWY 401 E/Airport) 55.1 km
7. Continue on HWY 401 E/MacDonald-Cartier Fwy 319 km
8. Take exit 721A to merge onto HWY 416 N toward Kemptville/Ottawa 74.7 km
9. Take the HWY 417 E/RTE 417 E exit toward Ottawa 1.6 km
10. Merge onto HWY 417 E/Trans Canada Hwy E 13.3 km
11. Take exit 118 to merge onto Nicholas St/Rue Nicholas 1.9 km
12. Turn right at Avenue Laurier Est/Laurier Ave E 0.4 km
13. Turn left at Avenue King Edward/King Edward Ave 1.3 km
14. Slight right to stay on Avenue King Edward/King Edward Ave 0.6 km
15. Turn right at Promenade Sussex/Sussex Dr 0.7 km

To: Rideau Hall
1 Sussex Dr, Ottawa, ON

[Add destination...](#)

These directions are for planning purposes only. You may find that construction projects, traffic, or other events may cause road conditions to differ from this map route.

Done

Variants of Shortest Path

Given a graph $G = (V, E)$

- **Single-source shortest paths**
 - Find shortest path from a given source vertex s to each vertex $v \in V$
- **Single-destination shortest paths**
 - Find shortest path to a given destination vertex t from each vertex v
- **Single-pair shortest path**
 - Find shortest path from u to v for given vertices u and v
- **All-pairs shortest-paths**
 - Find shortest path from u to v for every pair of vertices u and v

Single Source Shortest Path

Dijkstra's Algorithm

- **Problem:** From a given source vertex $s \in V$, find the shortest-paths and their weights $w(s, v)$ for all $v \in V$
- **Idea of the Algorithm**
 - Maintain a set S of vertices whose shortest-path distances from s are known
 - At each step add to S the vertex $v \in V - S$ whose distance estimate from s is minimal
 - Update the distance estimates of vertices adjacent to v

Dijkstra's Algorithm - Pseudocode

```
dist[s] = 0 // distance to source vertex is zero
p[s] = NULL
for all v ∈ V-{\s}
    dist[v] = ∞ // set all other distances to infinity
S = ∅ //S, the set of visited vertices is initially empty
Q = V //Q, the queue initially contains all vertices
while Q is not empty //while the queue is not empty
    u = mindistance(Q) //select the element of Q with the min distance
    S = S ∪ {u} // add u to list of visited vertices
    for all v ∈ neighbors[u]
        if dist[v] > dist[u] + w(u,v) //if new shortest path found
            dist[v] = dist[u] + w(u,v) //set new value of shortest path
            p[v] = u
```

Dijkstra's Vs. Prim's Algorithm

Dijkstra's Algorithm

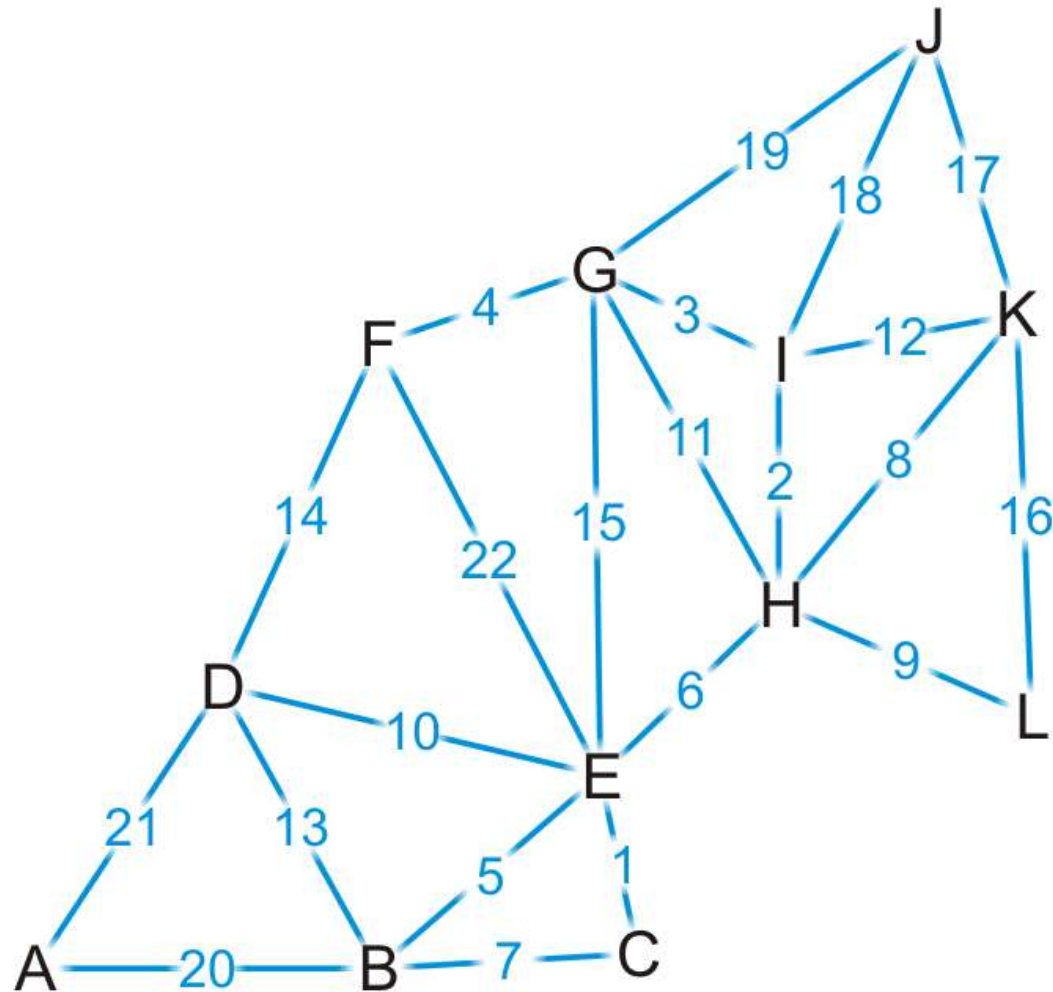
```
dist[s] = 0
p[s] = NULL
for all v ∈ V-{s}
    dist[v] = ∞
S = ∅
Q = V
while (Q not empty)
    u = mindistance(Q)
    S = S ∪ {u}
    for all v ∈ neighbors[u] && v ∈ V-S
        if dist[u] + w(u, v) < dist[v]
            dist[v] = dist[u] + w(u, v)
            p[v] = u
```

Prim's Algorithm

```
Q = V[G];
for each u ∈ Q
    key[u] = ∞;
key[r] = 0;
p[r] = NULL;
while (Q not empty)
    u = ExtractMin(Q);
    for each v ∈ Adj[u]
        if (v ∈ Q and w(u, v) < key[v])
            p[v] = u;
            key[v] = w(u, v);
```

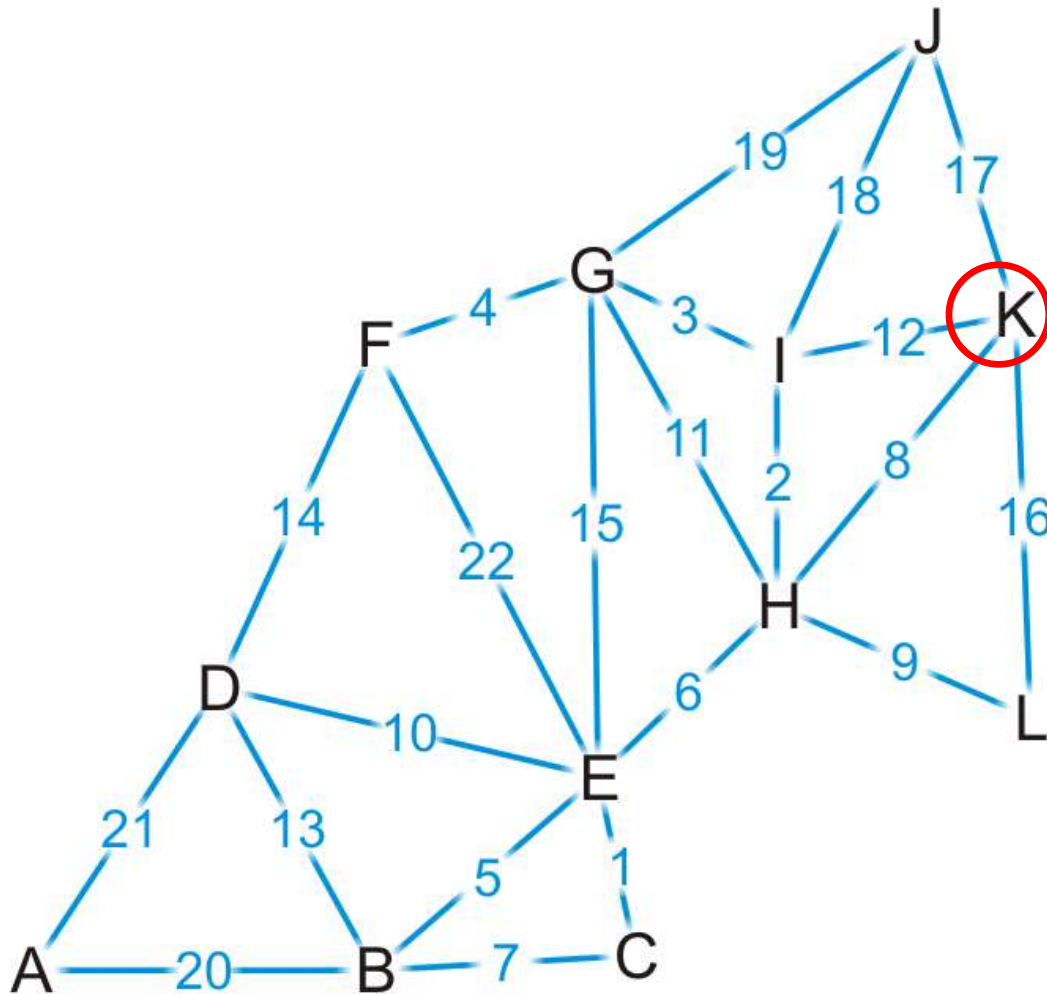
Dijkstra's Algorithm – Example

- Find the shortest path from K to every other vertex



Dijkstra's Algorithm – Example

- We visit vertex K

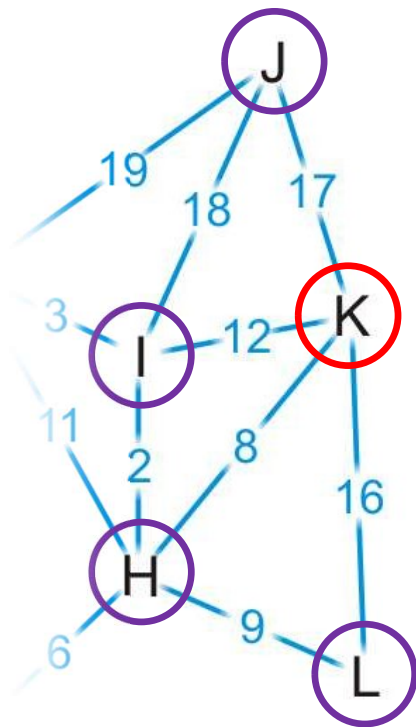


SPT

Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	∞	\emptyset
C	F	∞	\emptyset
D	F	∞	\emptyset
E	F	∞	\emptyset
F	F	∞	\emptyset
G	F	∞	\emptyset
H	F	∞	\emptyset
I	F	∞	\emptyset
J	F	∞	\emptyset
K	T	0	\emptyset
L	F	∞	\emptyset

Dijkstra's Algorithm – Example

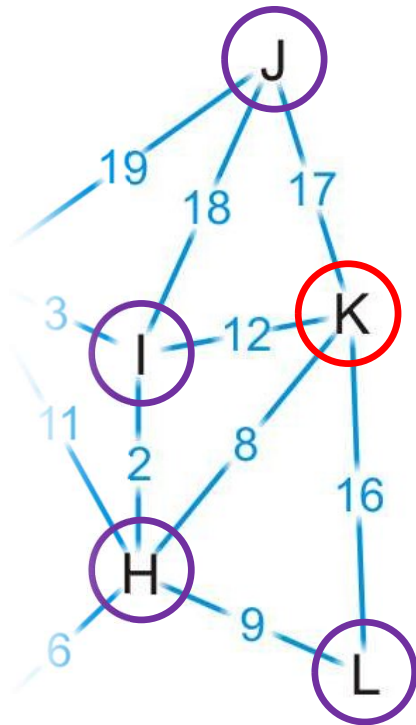
- Vertex K has four neighbors: H, I, J and L



Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	∞	\emptyset
C	F	∞	\emptyset
D	F	∞	\emptyset
E	F	∞	\emptyset
F	F	∞	\emptyset
G	F	∞	\emptyset
H	F	∞	\emptyset
I	F	∞	\emptyset
J	F	∞	\emptyset
K	T	0	\emptyset
L	F	∞	\emptyset

Dijkstra's Algorithm – Example

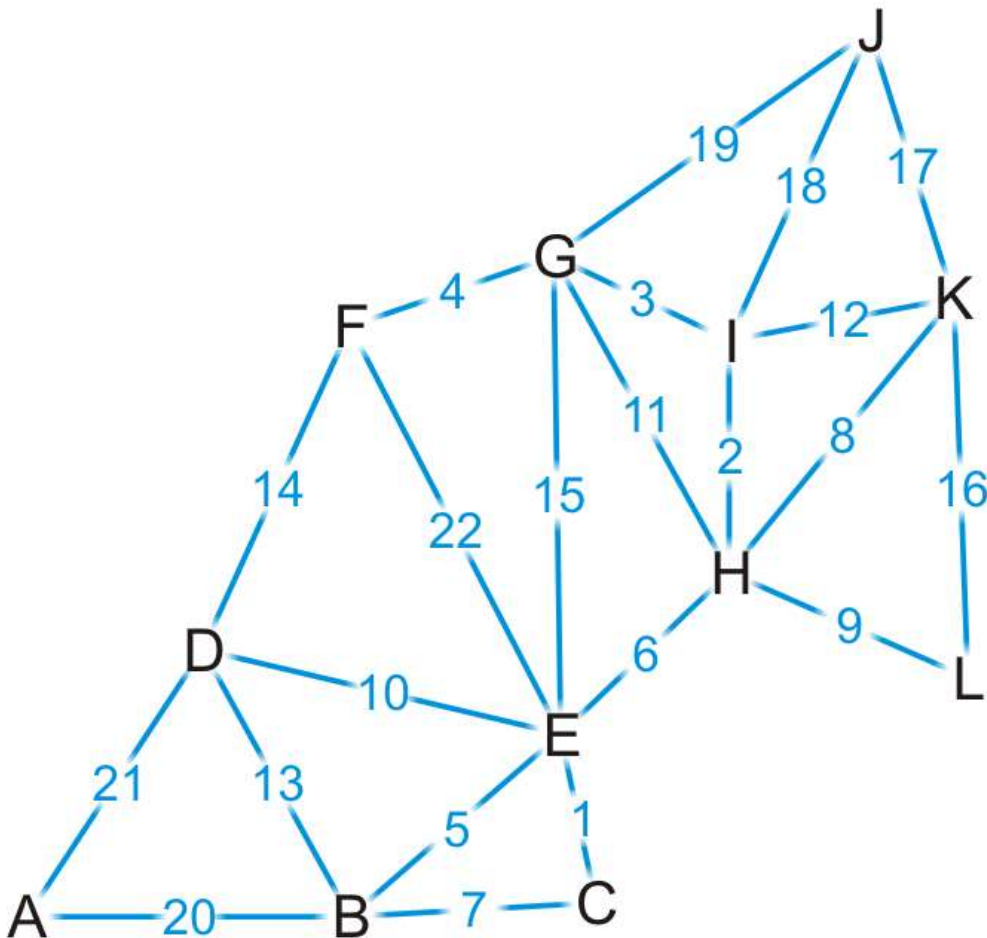
- We have now found at least one path to each of these vertices



Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	∞	\emptyset
C	F	∞	\emptyset
D	F	∞	\emptyset
E	F	∞	\emptyset
F	F	∞	\emptyset
G	F	∞	\emptyset
H	F	8	K
I	F	12	K
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

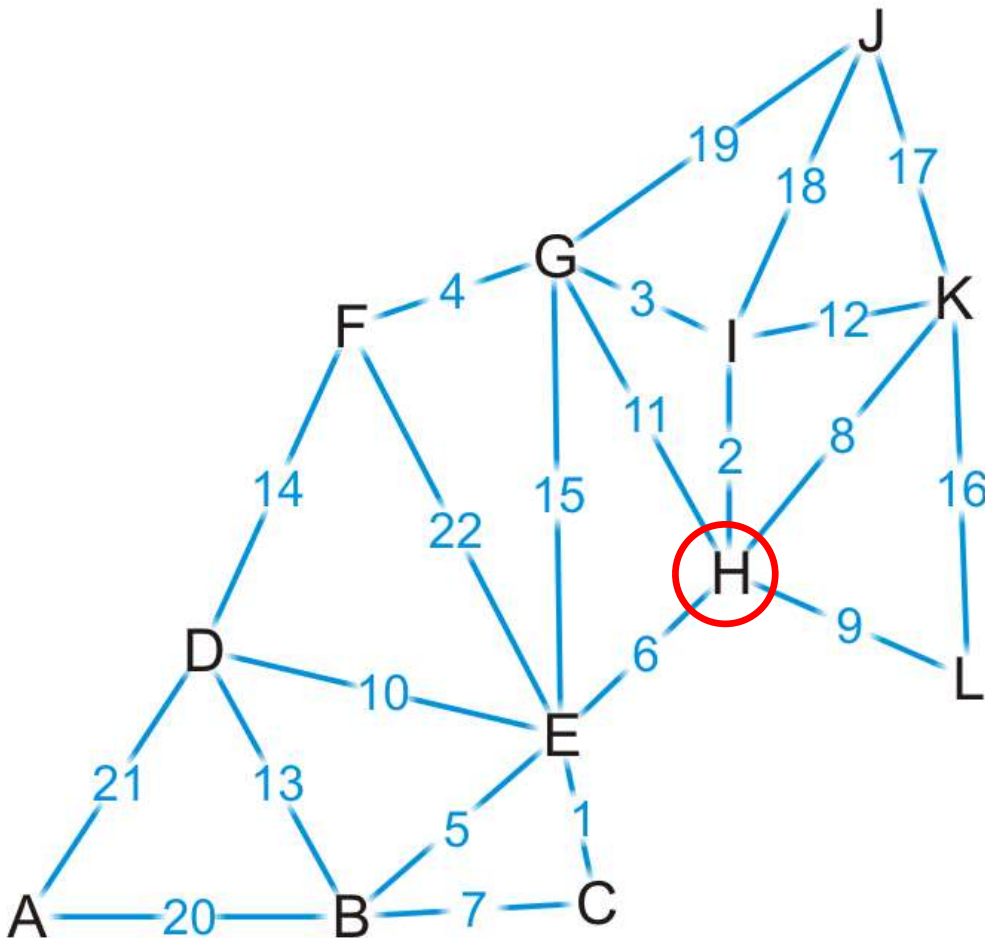
- We're finished with vertex K
 - To which vertex are we now guaranteed we have the shortest path?
 - mindistance(Q)



Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	∞	\emptyset
C	F	∞	\emptyset
D	F	∞	\emptyset
E	F	∞	\emptyset
F	F	∞	\emptyset
G	F	∞	\emptyset
H	F	8	K
I	F	12	K
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

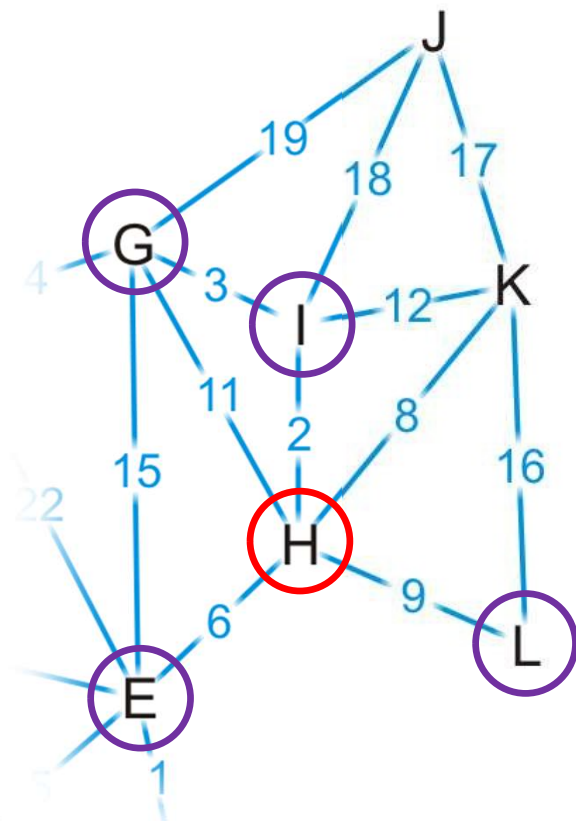
- We visit vertex H: the shortest path is (K, H) of length 8
 - Vertex H has four unvisited neighbors: E, G, I, L



Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	∞	\emptyset
C	F	∞	\emptyset
D	F	∞	\emptyset
E	F	∞	\emptyset
F	F	∞	\emptyset
G	F	∞	\emptyset
H	T	8	K
I	F	12	K
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

- Consider these paths:
 - (K, H, E) of length **8** + 6 = 14
 - (K, H, I) of length **8** + 2 = 10
- Which of these are shorter than any known path?

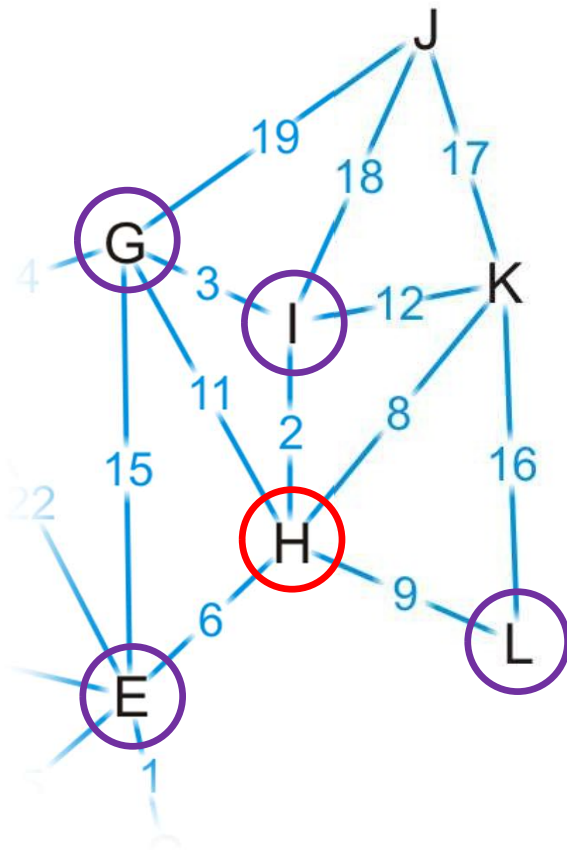


- (K, H, G) of length **8** + 11 = 19
- (K, H, L) of length **8** + 9 = 17

Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	∞	\emptyset
C	F	∞	\emptyset
D	F	∞	\emptyset
E	F	∞	\emptyset
F	F	∞	\emptyset
G	F	∞	\emptyset
H	T	8	K
I	F	12	K
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

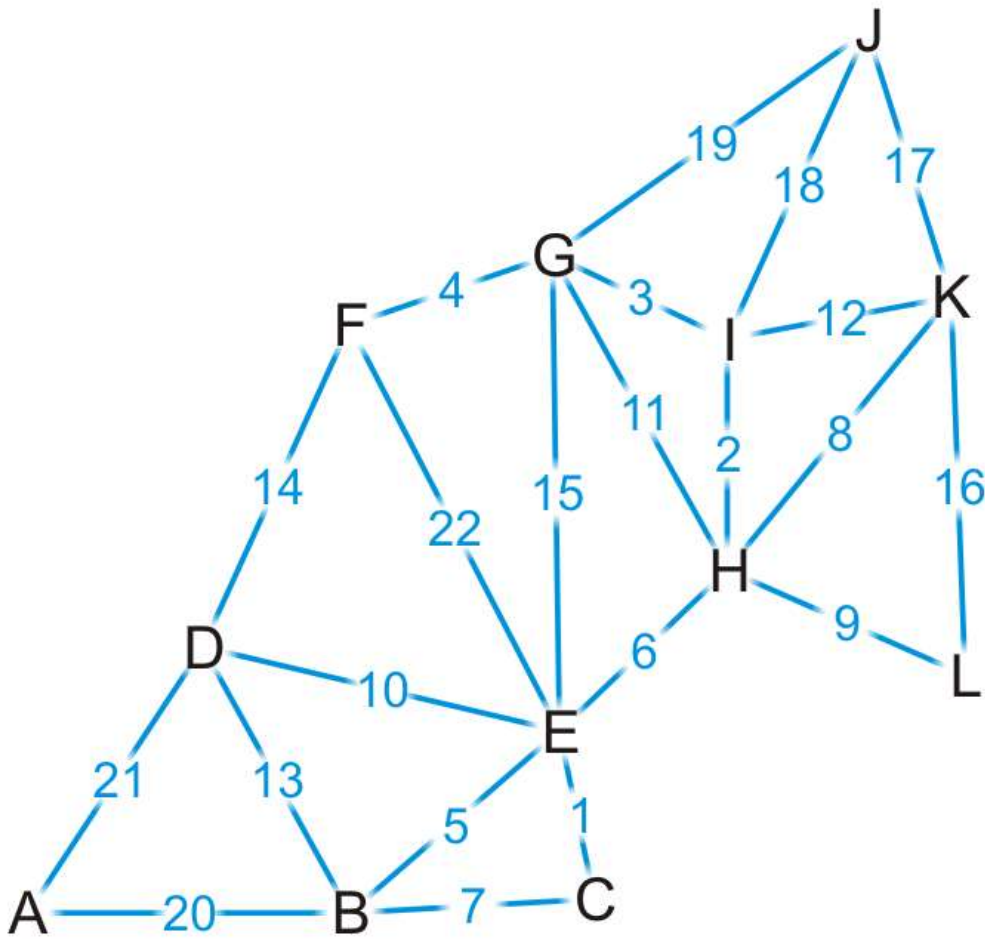
- We already have a shorter path (K, L), but we update the others



Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	∞	\emptyset
C	F	∞	\emptyset
D	F	∞	\emptyset
E	F	14	H
F	F	∞	\emptyset
G	F	19	H
H	T	8	K
I	F	10	H
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

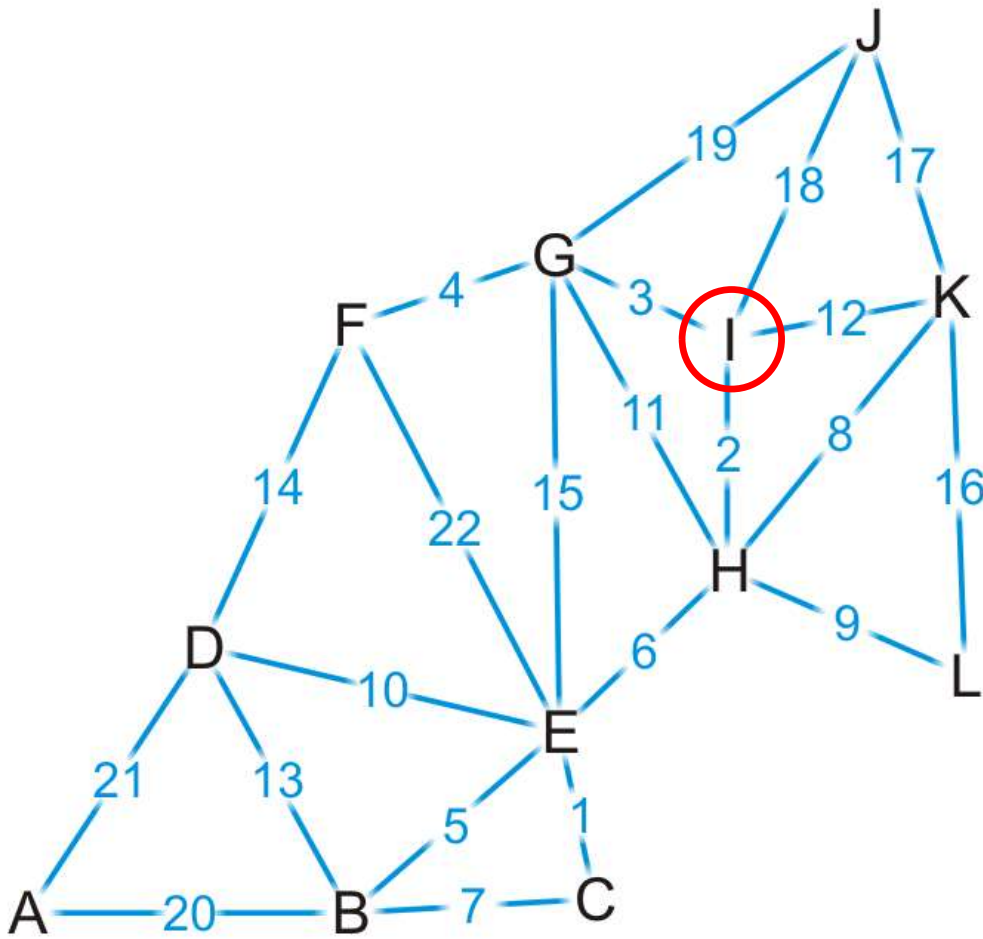
- We are finished with vertex H
 - Which vertex do we visit next?



Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	∞	\emptyset
C	F	∞	\emptyset
D	F	∞	\emptyset
E	F	14	H
F	F	∞	\emptyset
G	F	19	H
H	T	8	K
I	F	10	H
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

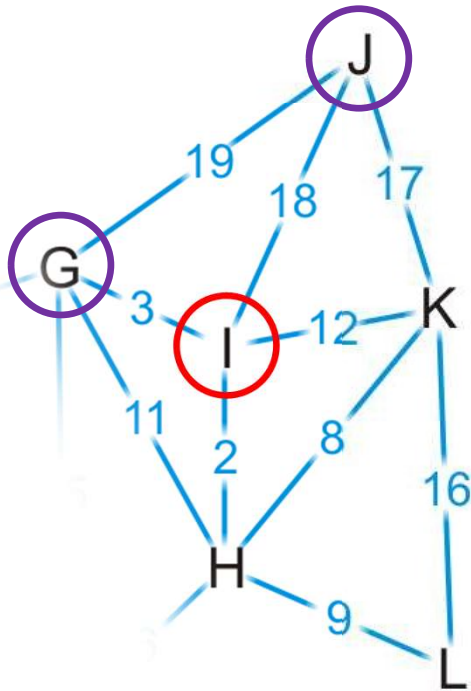
- The path (K, H, I) is the shortest path from K to I of length 10
 - Vertex I has two unvisited neighbors: G and J



Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	∞	\emptyset
C	F	∞	\emptyset
D	F	∞	\emptyset
E	F	14	H
F	F	∞	\emptyset
G	F	19	H
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

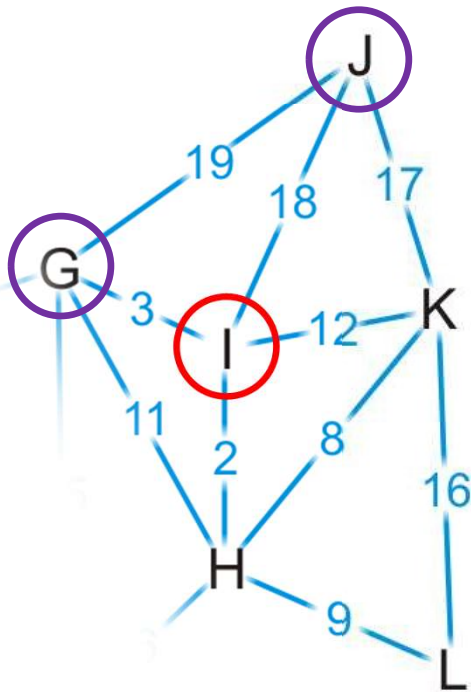
- Consider these paths:
 - (K, H, I, G) of length $10 + 3 = 13$
 - (K, H, I, J) of length $10 + 18 = 28$



Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	∞	\emptyset
C	F	∞	\emptyset
D	F	∞	\emptyset
E	F	14	H
F	F	∞	\emptyset
G	F	19	H
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

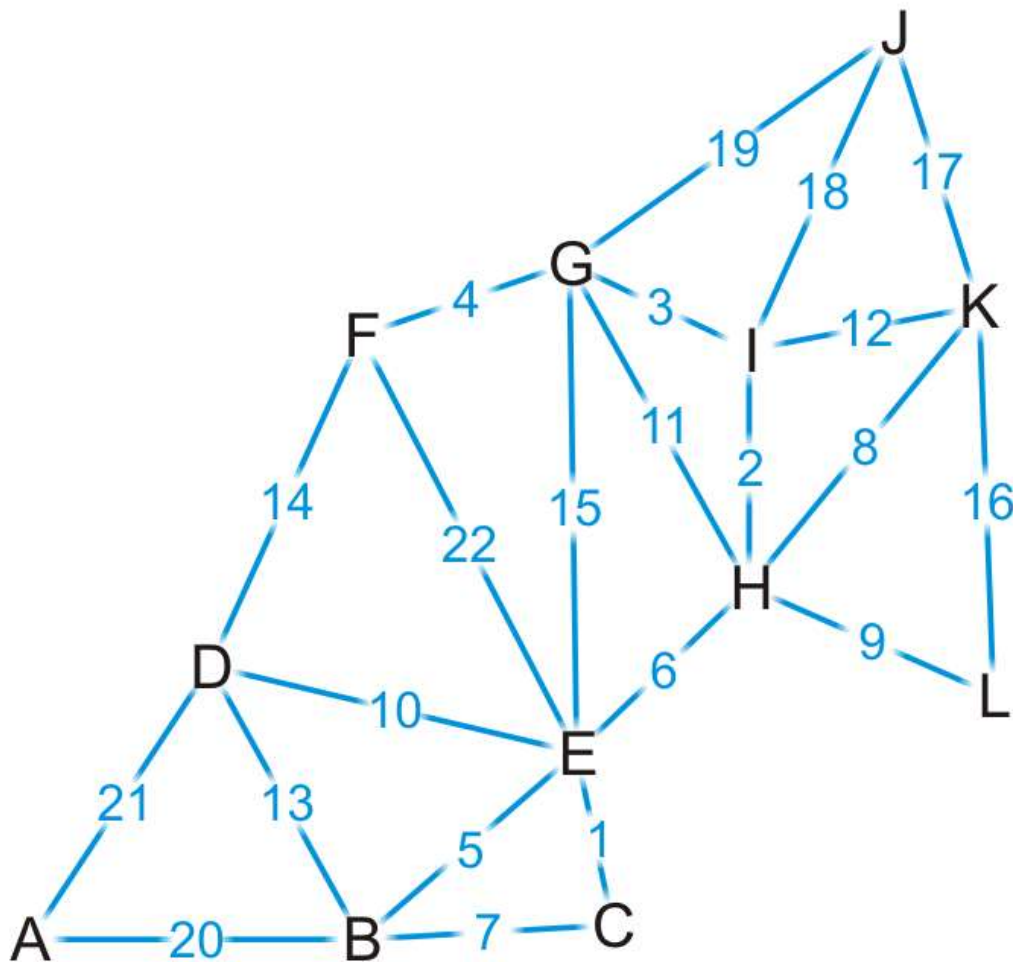
- We have discovered a shorter path to vertex G, but (K, J) is still the shortest known path to vertex J



Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	∞	\emptyset
C	F	∞	\emptyset
D	F	∞	\emptyset
E	F	14	H
F	F	∞	\emptyset
G	F	13	I
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

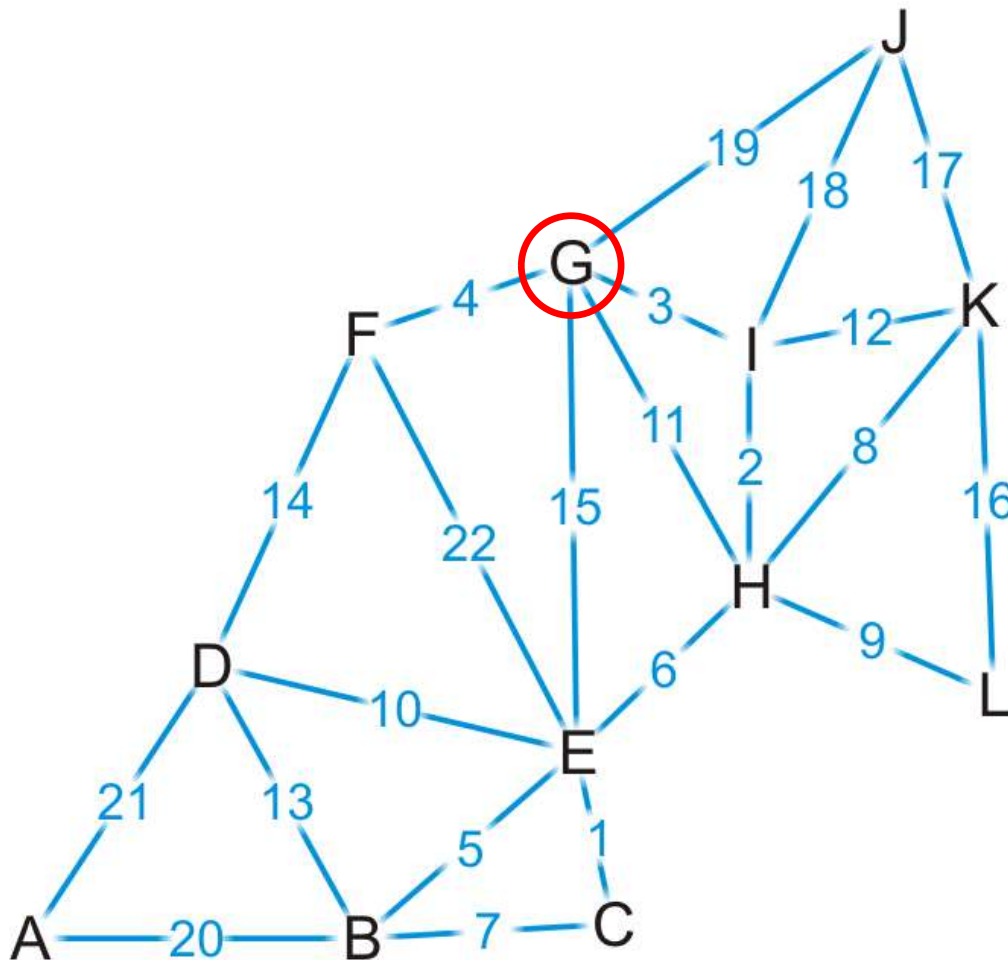
- Which vertex can we visit next?



Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	∞	\emptyset
C	F	∞	\emptyset
D	F	∞	\emptyset
E	F	14	H
F	F	∞	\emptyset
G	F	13	I
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

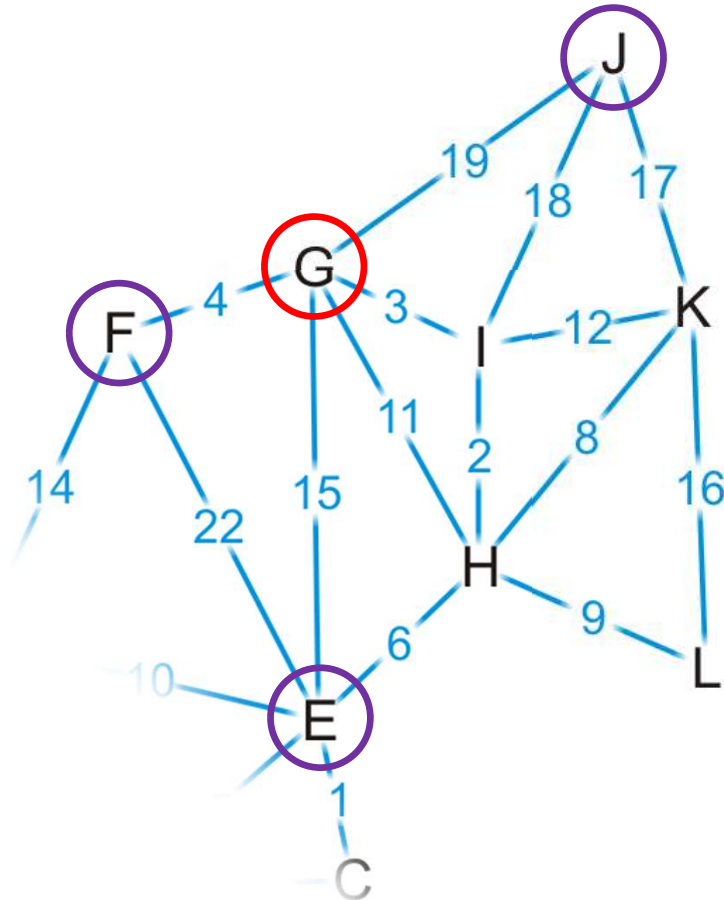
- The path (K, H, I, G) is the shortest path from K to G of length 13
 - Vertex G has three unvisited neighbors: E, F and J



Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	∞	\emptyset
C	F	∞	\emptyset
D	F	∞	\emptyset
E	F	14	H
F	F	∞	\emptyset
G	T	13	I
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

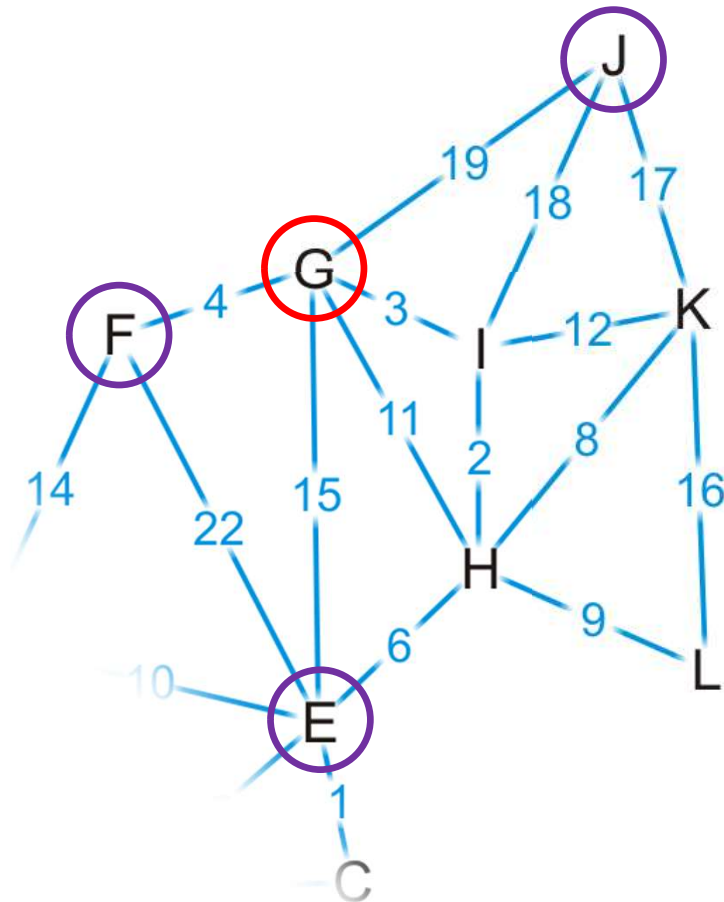
- Consider these paths:
 - (K, H, I, G, E) of length **13** + 15 = 28
 - (K, H, I, G, F) of length **13** + 4 = 17
 - (K, H, I, G, J) of length **13** + 19 = 32



Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	∞	\emptyset
C	F	∞	\emptyset
D	F	∞	\emptyset
E	F	14	H
F	F	∞	\emptyset
G	T	13	I
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

- We have now found a path to vertex F

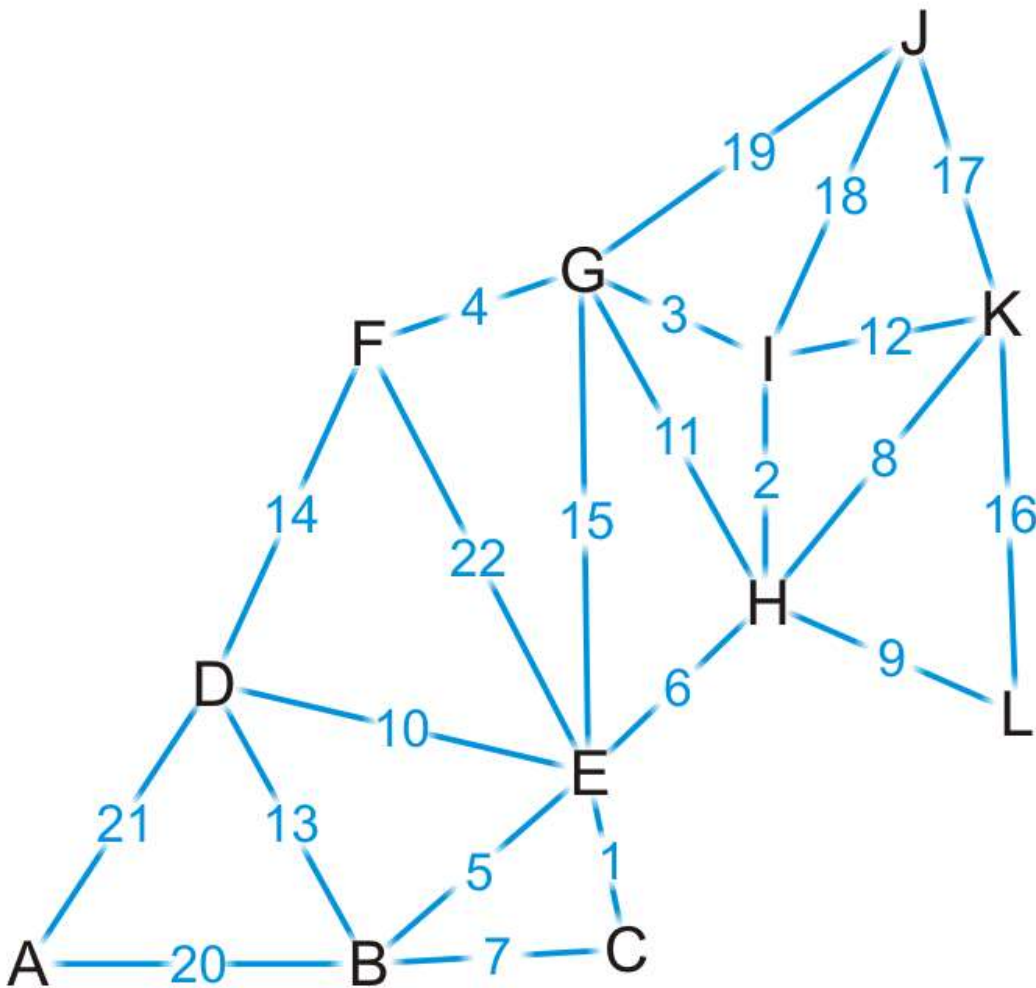


SPT

Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	∞	\emptyset
C	F	∞	\emptyset
D	F	∞	\emptyset
E	F	14	H
F	F	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

- Where do we visit next?

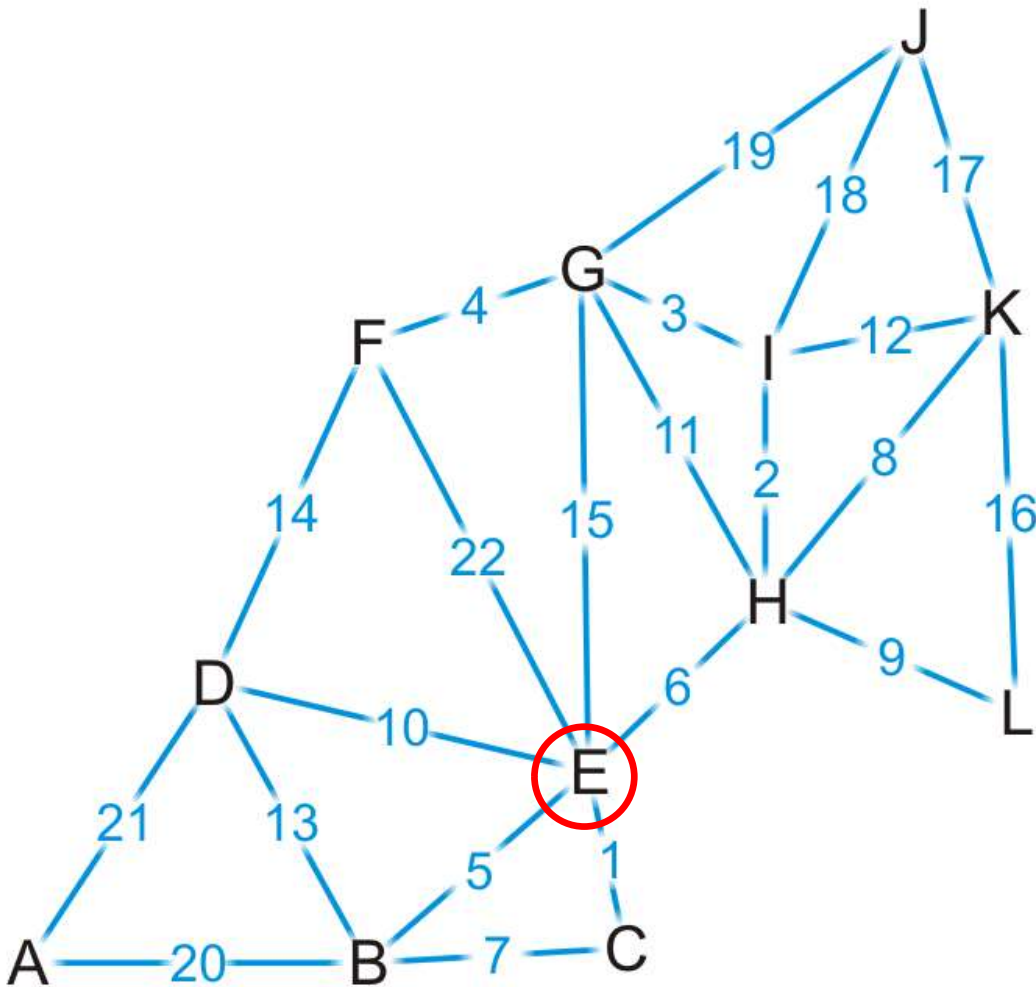


SPT

Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	∞	\emptyset
C	F	∞	\emptyset
D	F	∞	\emptyset
E	F	14	H
F	F	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

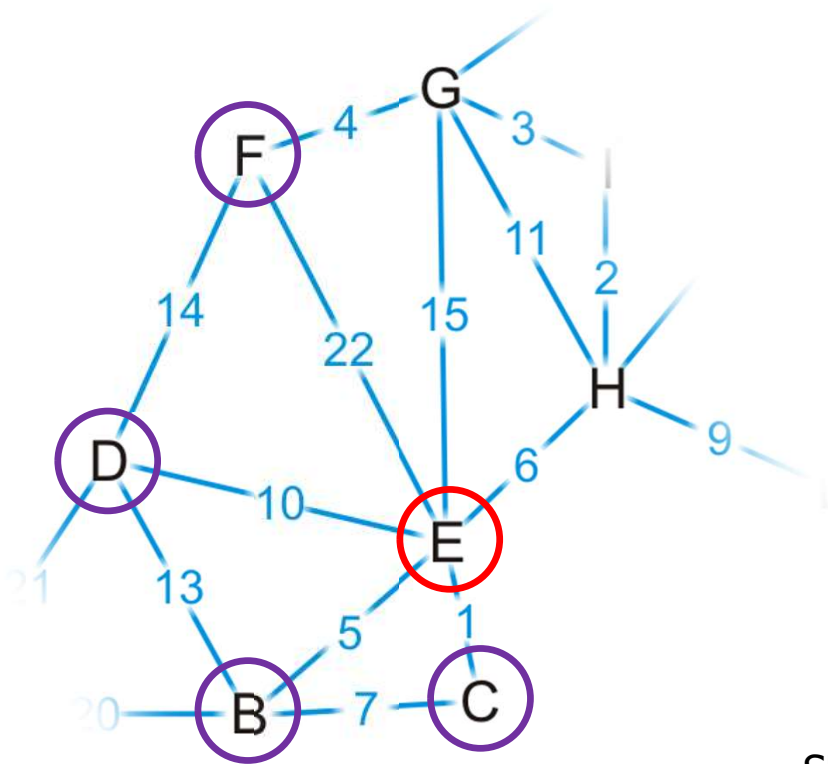
- The path (K, H, E) is the shortest path from K to E of length 14
 - Vertex G has four unvisited neighbors: B, C, D and F



Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	∞	\emptyset
C	F	∞	\emptyset
D	F	∞	\emptyset
E	T	14	H
F	F	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

- The path (K, H, E) is the shortest path from K to E of length 14
 - Vertex G has four unvisited neighbors: B, C, D and F

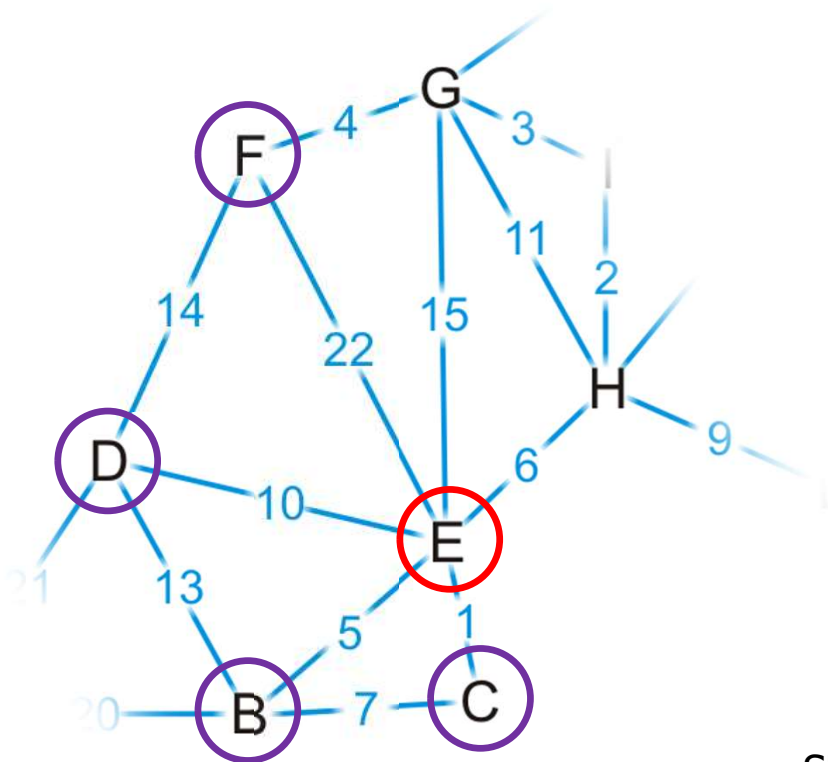


SPT

Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	∞	\emptyset
C	F	∞	\emptyset
D	F	∞	\emptyset
E	T	14	H
F	F	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

- Consider these paths:
 - (K, H, E, B) of length $14 + 5 = 19$
 - (K, H, E, C) of length $14 + 1 = 15$
 - (K, H, E, D) of length $14 + 10 = 24$
 - (K, H, E, F) of length $14 + 22 = 36$

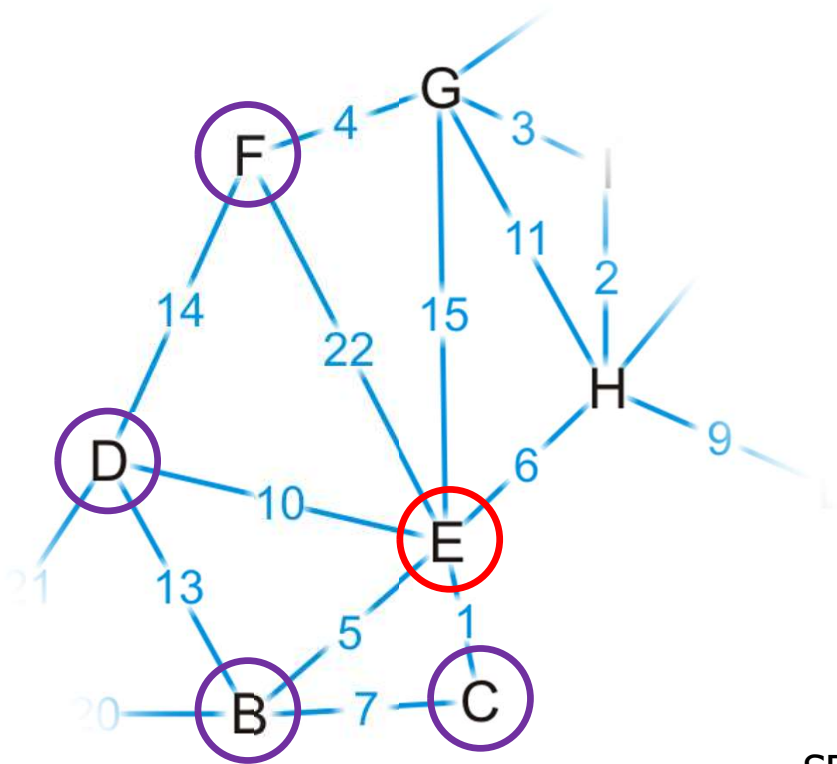


SPT

Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	∞	\emptyset
C	F	∞	\emptyset
D	F	∞	\emptyset
E	T	14	H
F	F	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

- We've discovered paths to vertices B, C, D

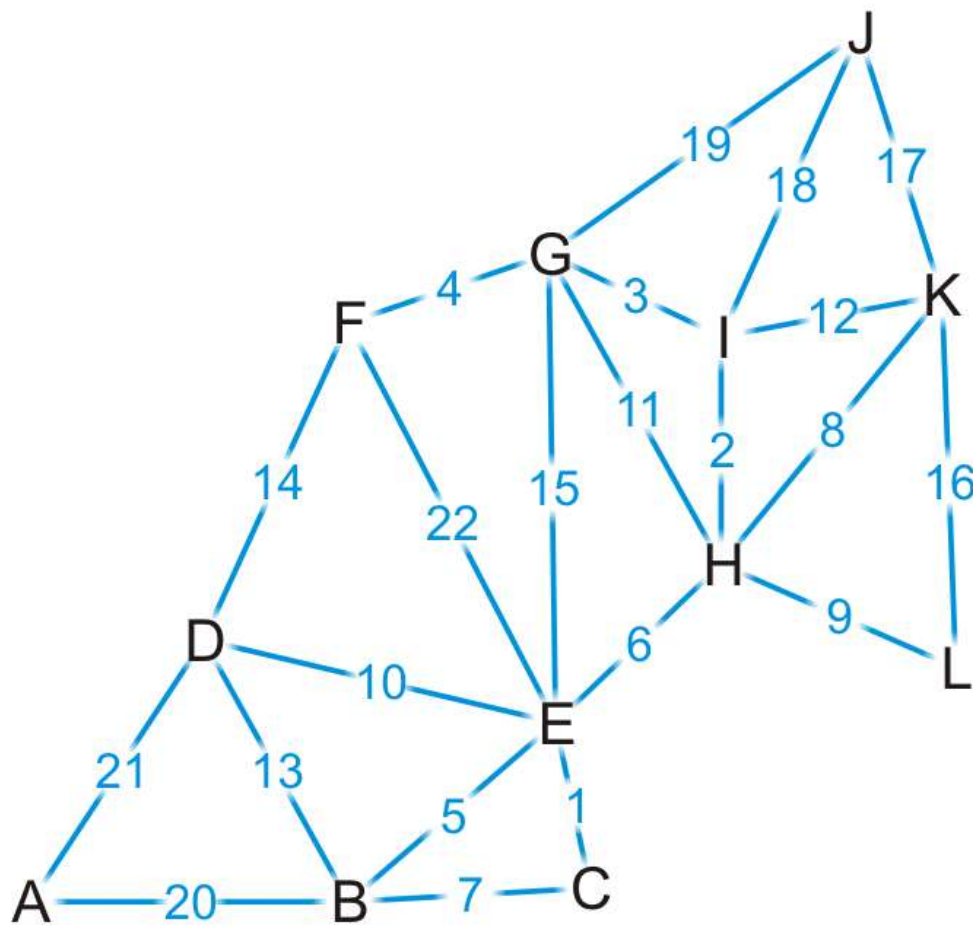


SPT

Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	19	E
C	F	15	E
D	F	24	E
E	T	14	H
F	F	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

- Which vertex is next?

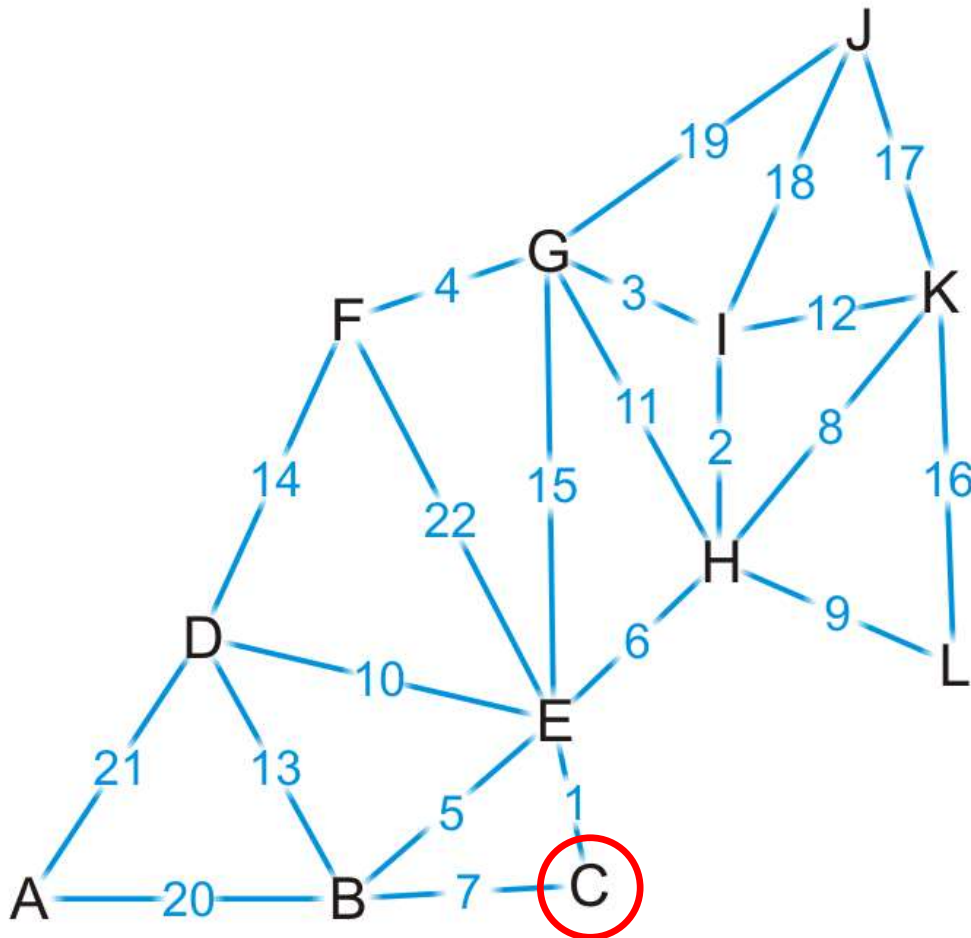


SPT

Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	19	E
C	F	15	E
D	F	24	E
E	T	14	H
F	F	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

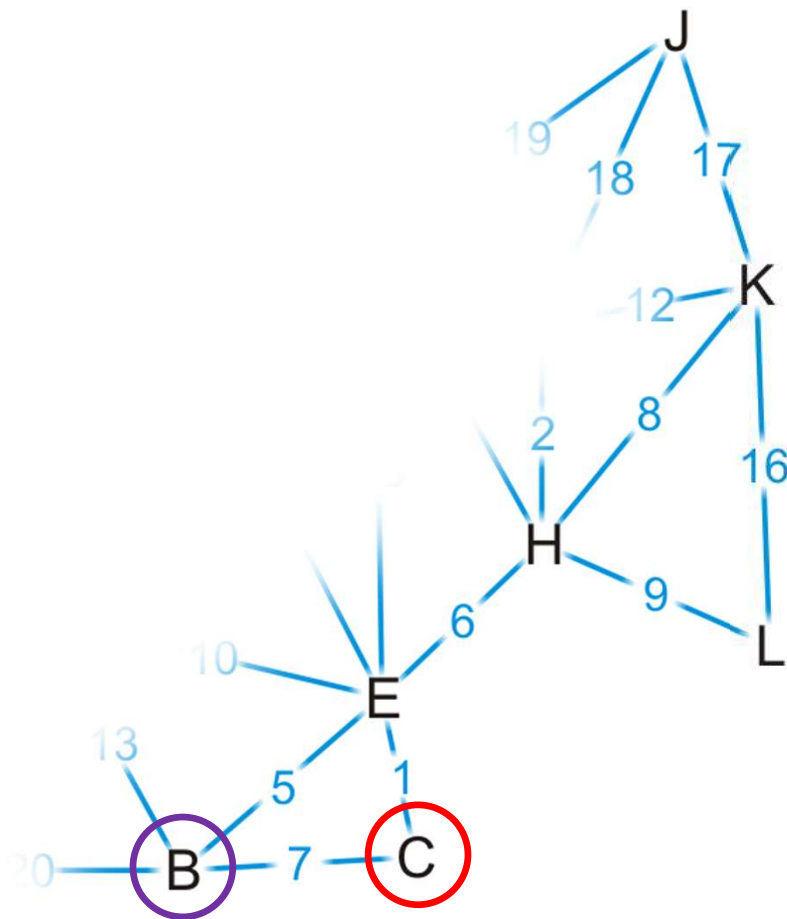
- We've found that the path (K, H, E, C) of length 15 is the shortest path from K to C
 - Vertex C has one unvisited neighbor, B



Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	19	E
C	T	15	E
D	F	24	E
E	T	14	H
F	F	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

- The path (K, H, E, C, B) is of length $15 + 7 = 22$
 - We have already discovered a shorter path through vertex E

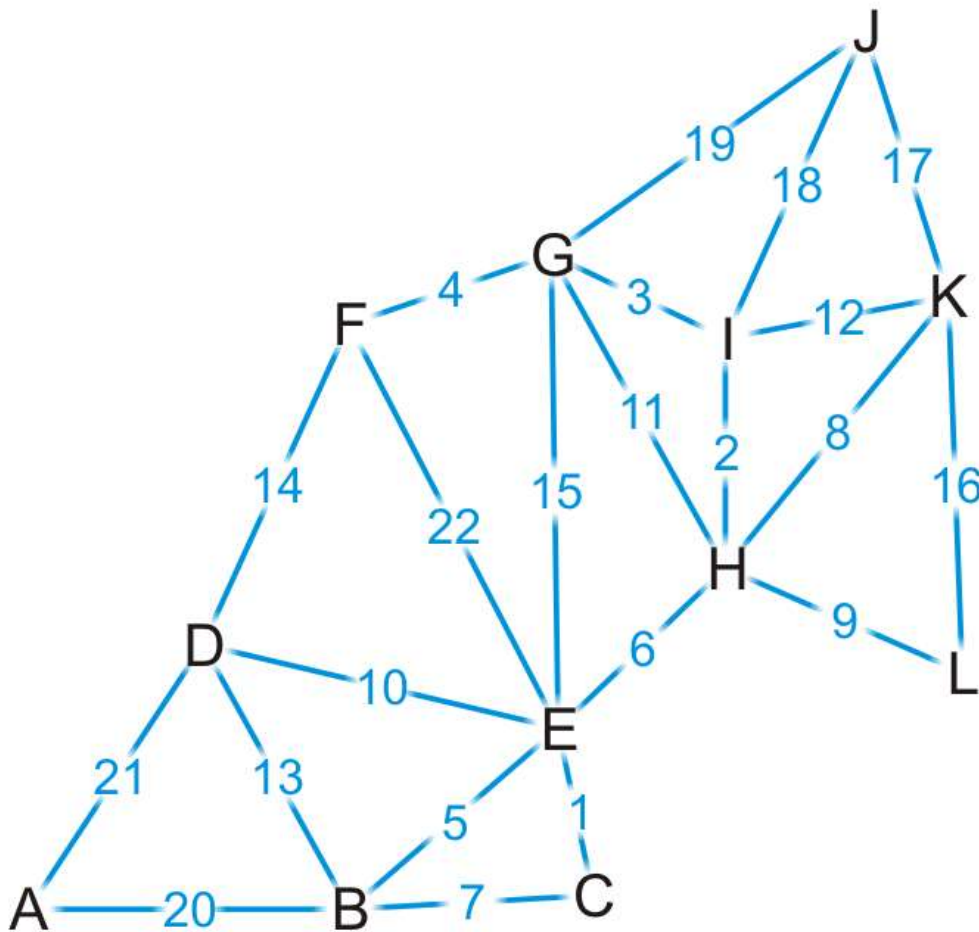


SPT

Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	19	E
C	T	15	E
D	F	24	E
E	T	14	H
F	F	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

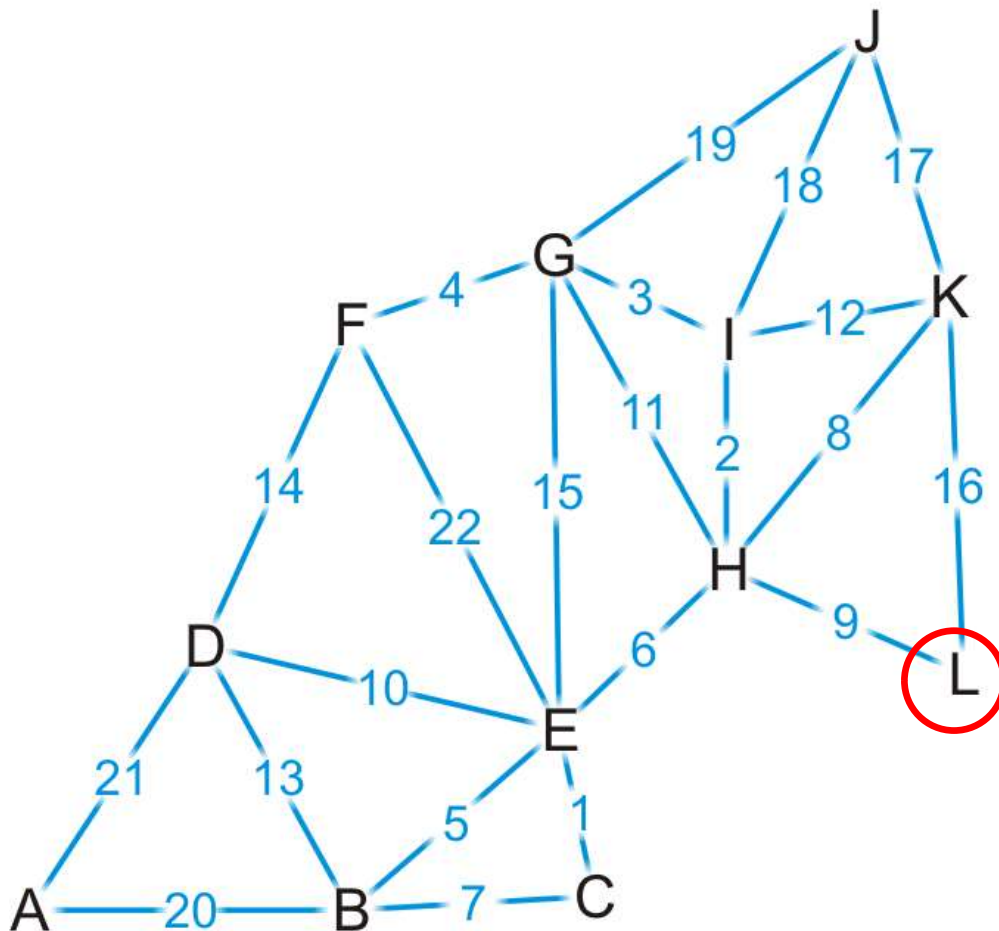
- Where to next?



Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	19	E
C	T	15	E
D	F	24	E
E	T	14	H
F	F	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	F	16	K

Dijkstra's Algorithm – Example

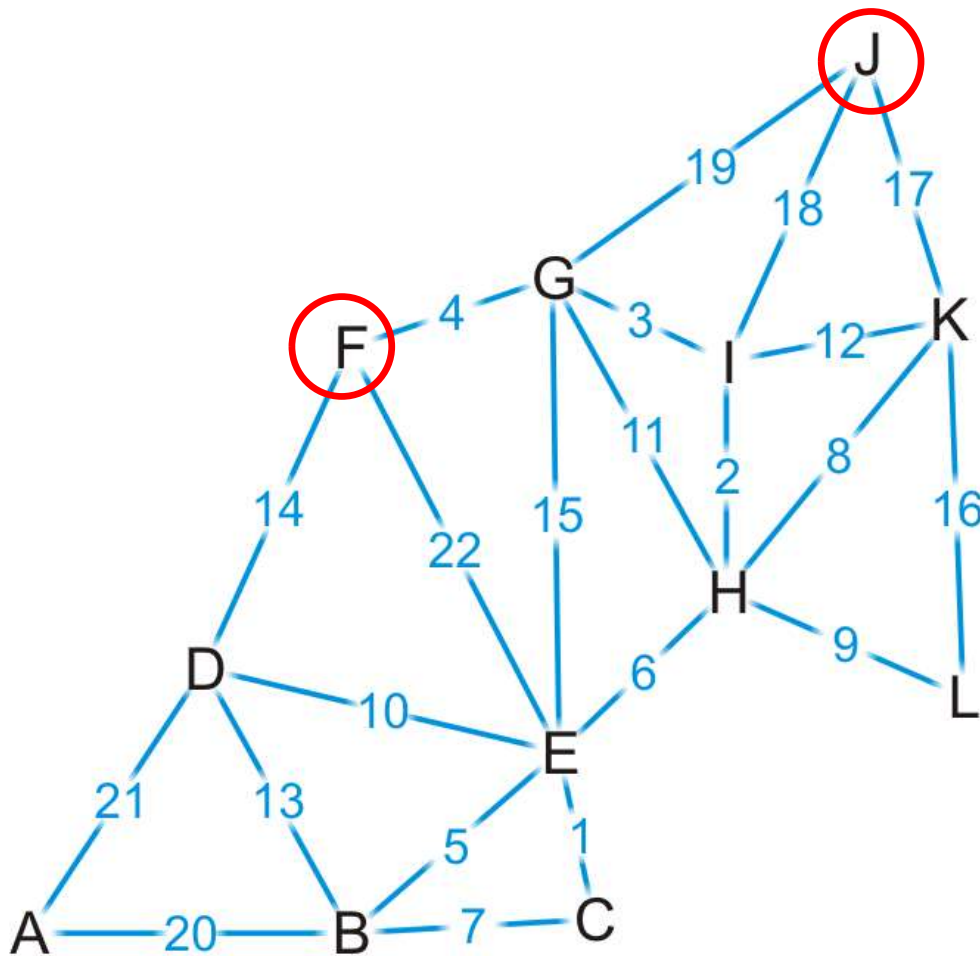
- We now know that (K, L) is the shortest path between these two points
 - Vertex L has no unvisited neighbors



Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	19	E
C	T	15	E
D	F	24	E
E	T	14	H
F	F	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	T	16	K

Dijkstra's Algorithm – Example

- Where to next?
 - Does it matter if we visit vertex F first or vertex J first?

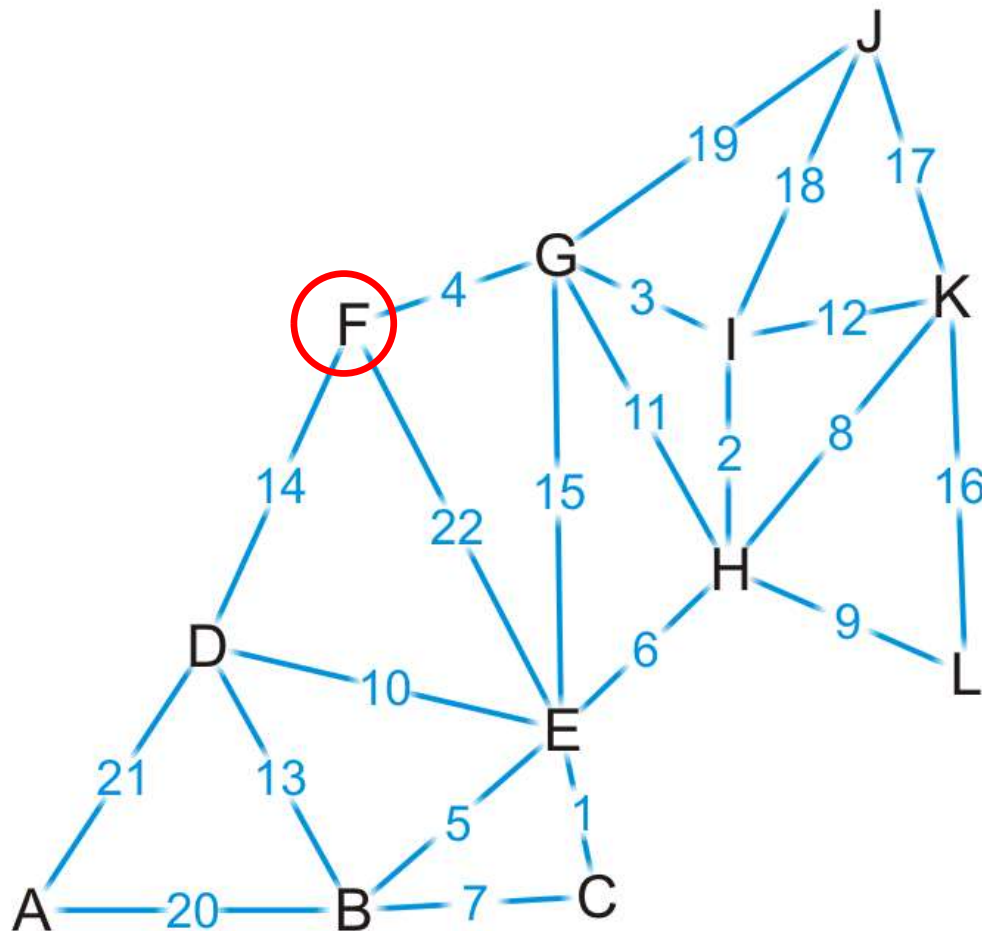


SPT

Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	19	E
C	T	15	E
D	F	24	E
E	T	14	H
F	F	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	T	16	K

Dijkstra's Algorithm – Example

- Let's visit vertex F first
 - It has one unvisited neighbor, vertex D

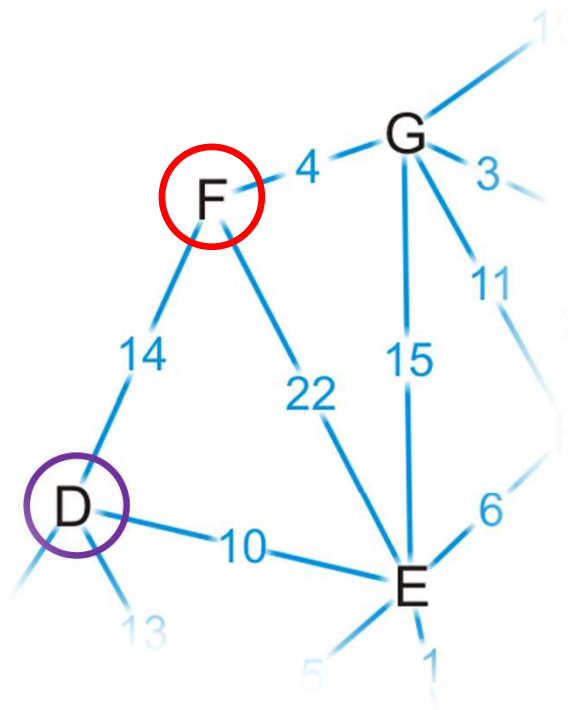


SPT

Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	19	E
C	T	15	E
D	F	24	E
E	T	14	H
F	T	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	T	16	K

Dijkstra's Algorithm – Example

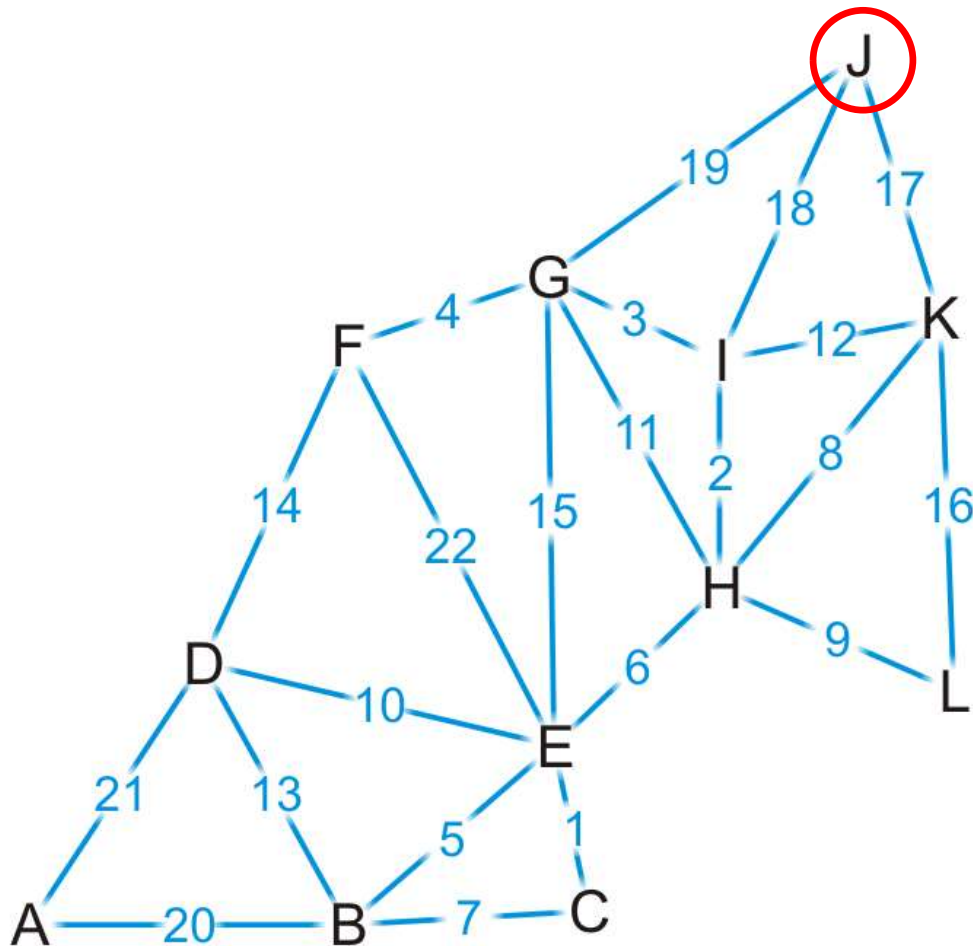
- The path (K, H, I, G, F, D) is of length **17** + 14 = 31
 - This is longer than the path we've already discovered



Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	19	E
C	T	15	E
D	F	24	E
E	T	14	H
F	T	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	F	17	K
K	T	0	\emptyset
L	T	16	K

Dijkstra's Algorithm – Example

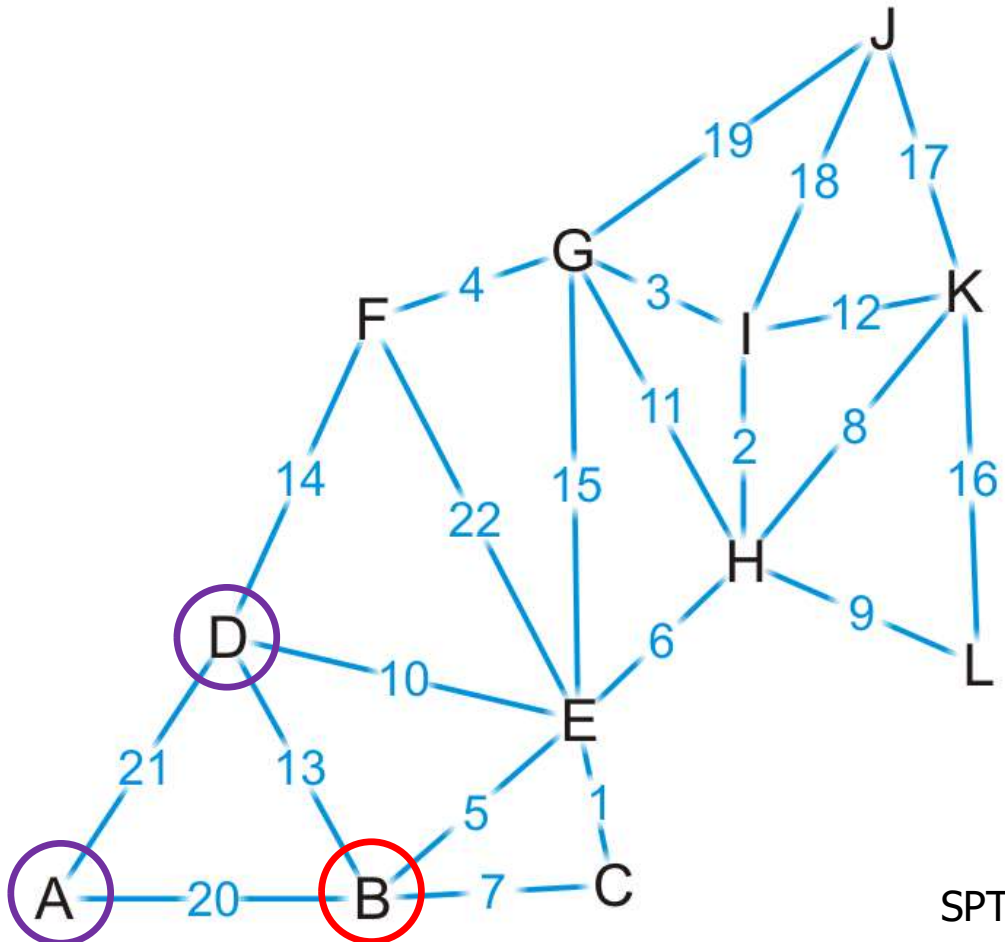
- Now we visit vertex J
 - It has no unvisited neighbors



Vertex	S	Distance	Parent
A	F	∞	\emptyset
B	F	19	E
C	T	15	E
D	F	24	E
E	T	14	H
F	T	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	T	17	K
K	T	0	\emptyset
L	T	16	K

Dijkstra's Algorithm – Example

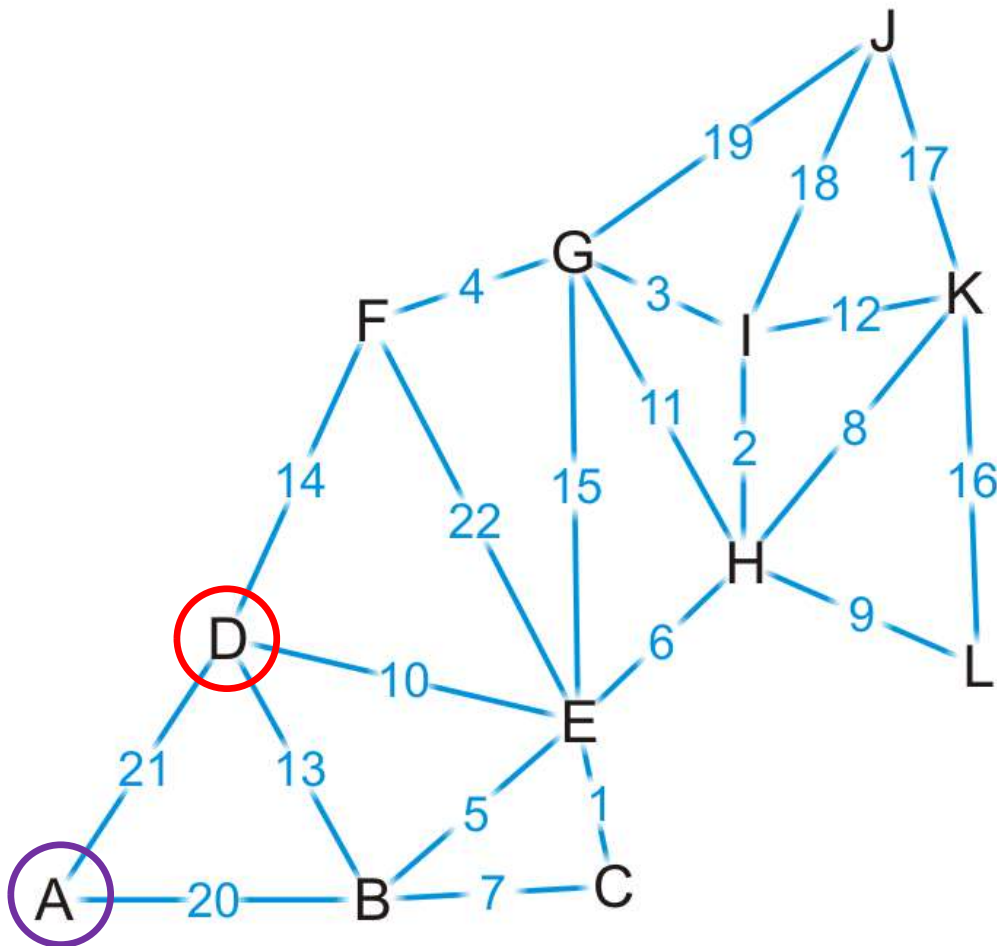
- Next we visit vertex B, which has two unvisited neighbors:
 - (K, H, E, B, A) of length **19** + 20 = 39
 - (K, H, E, B, D) of length **19** + 13 = 32
- We update the path length to A



Vertex	S	Distance	Parent
A	F	39	B
B	T	19	E
C	T	15	E
D	F	24	E
E	T	14	H
F	T	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	T	17	K
K	T	0	∅
L	T	16	K

Dijkstra's Algorithm – Example

- Next we visit vertex D
 - The path (K, H, E, D, A) is of length $24 + 21 = 45$
 - We don't update A

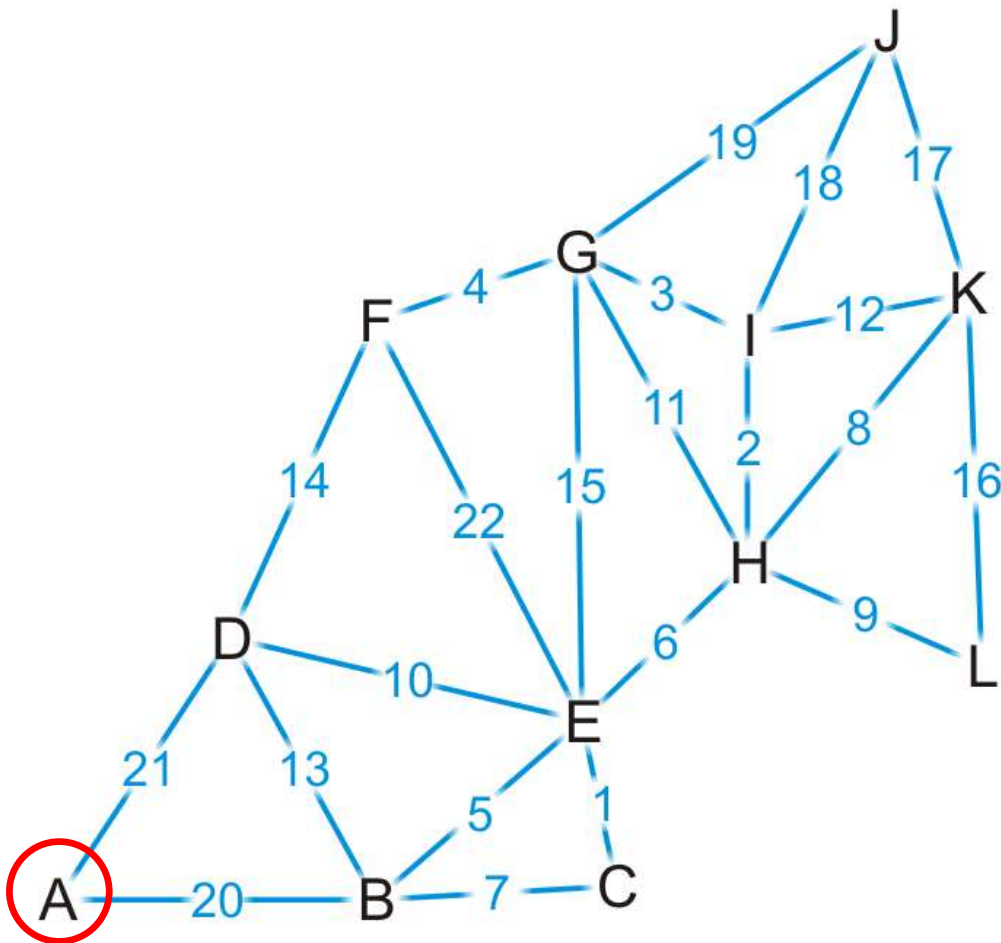


SPT

Vertex	S	Distance	Parent
A	F	39	B
B	T	19	E
C	T	15	E
D	T	24	E
E	T	14	H
F	T	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	T	17	K
K	T	0	Ø
L	T	16	K

Dijkstra's Algorithm – Example

- Finally, we visit vertex A
 - It has no unvisited neighbors and there are no unvisited vertices left
 - We are done

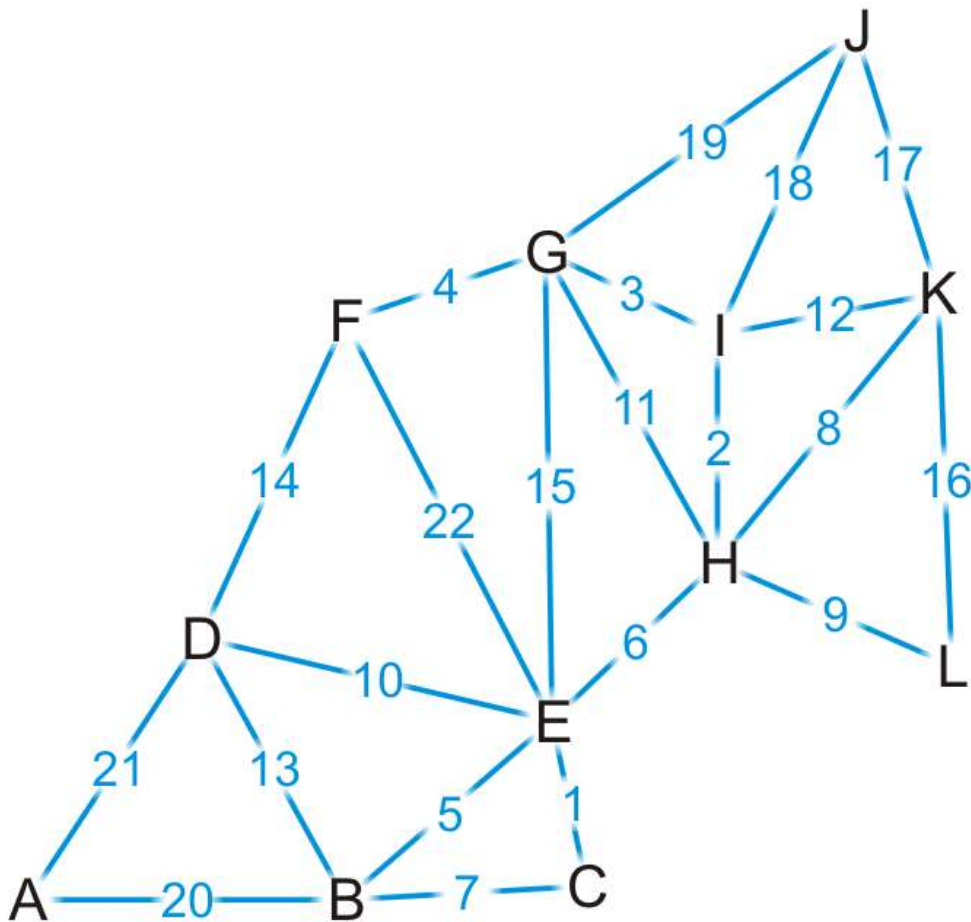


SPT

Vertex	S	Distance	Parent
A	T	39	B
B	T	19	E
C	T	15	E
D	T	24	E
E	T	14	H
F	T	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	T	17	K
K	T	0	Ø
L	T	16	K

Dijkstra's Algorithm – Example

- Thus, we have found the shortest path from vertex K to each of the other vertices

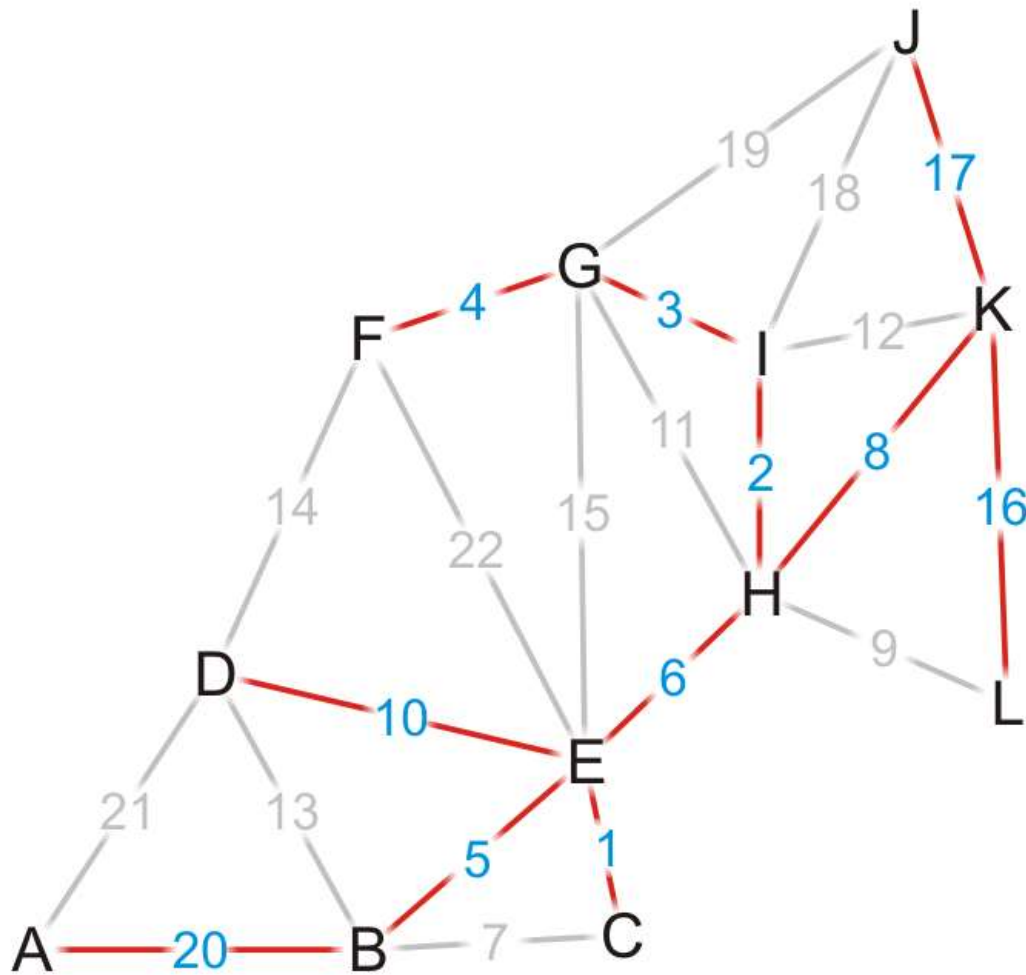


SPT

Vertex	S	Distance	Parent
A	T	39	B
B	T	19	E
C	T	15	E
D	T	24	E
E	T	14	H
F	T	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	T	17	K
K	T	0	Ø
L	T	16	K

Dijkstra's Algorithm – Example

- Using the previous pointers, we can reconstruct the paths

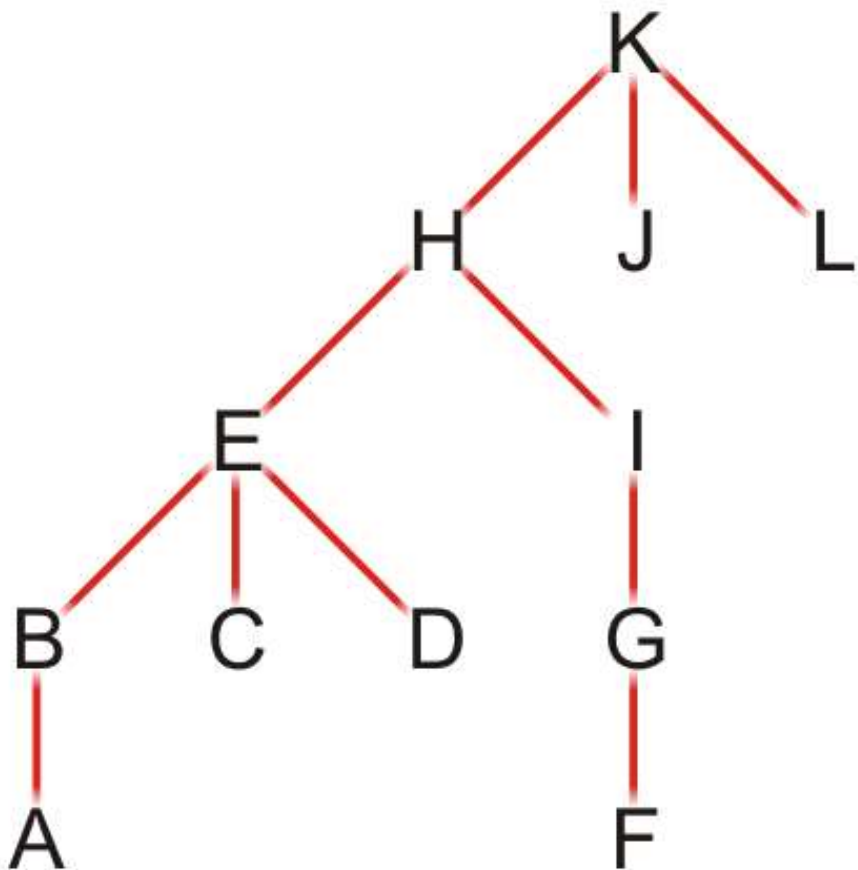


SPT

Vertex	S	Distance	Parent
A	T	39	B
B	T	19	E
C	T	15	E
D	T	24	E
E	T	14	H
F	T	17	G
G	T	13	I
H	T	8	K
I	T	10	H
J	T	17	K
K	T	0	Ø
L	T	16	K

Dijkstra's Algorithm – Example

- The table defines a rooted parental tree
 - The source vertex K is at the root
 - The previous pointer is the parent of the vertex in the tree

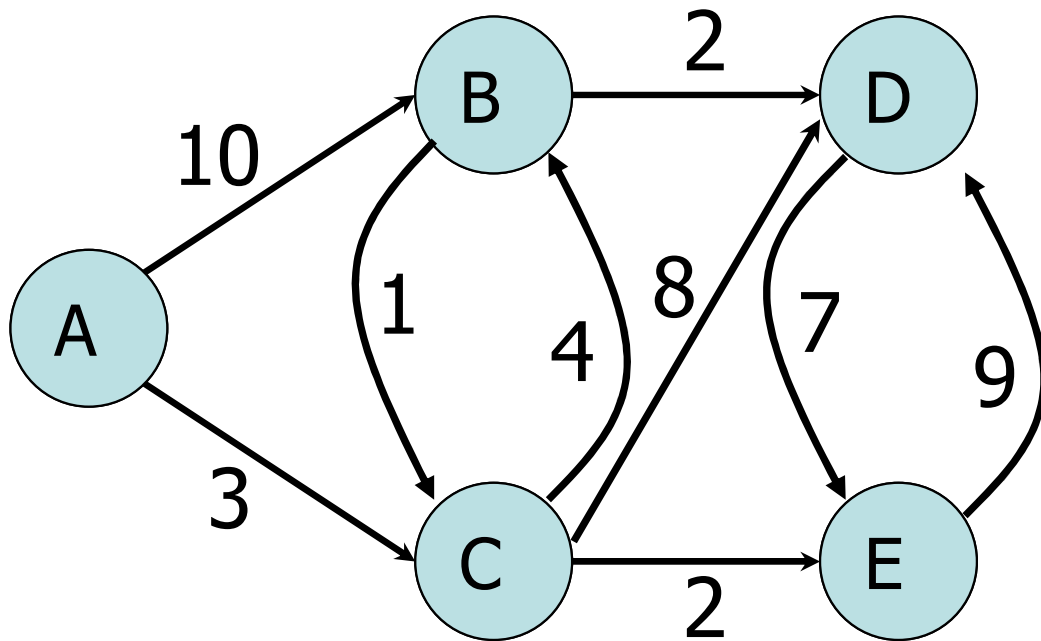


Vertex	Previous
A	B
B	E
C	E
D	E
E	H
F	G
G	I
H	K
I	H
J	K
K	Ø
L	K

Comments on Dijkstra's Algorithm

- If at some point, all unvisited vertices have a distance ∞ ?
 - This means that the graph is unconnected
 - We have found the shortest paths to all vertices in the connected subgraph containing the source vertex
- To find the shortest path between vertices v_j and v_k ?
 - Apply the same algorithm, but stop when visiting vertex v_k
- Does the algorithm change if graph is directed?
 - No

Dijkstra's Algorithm – Example



Initialization

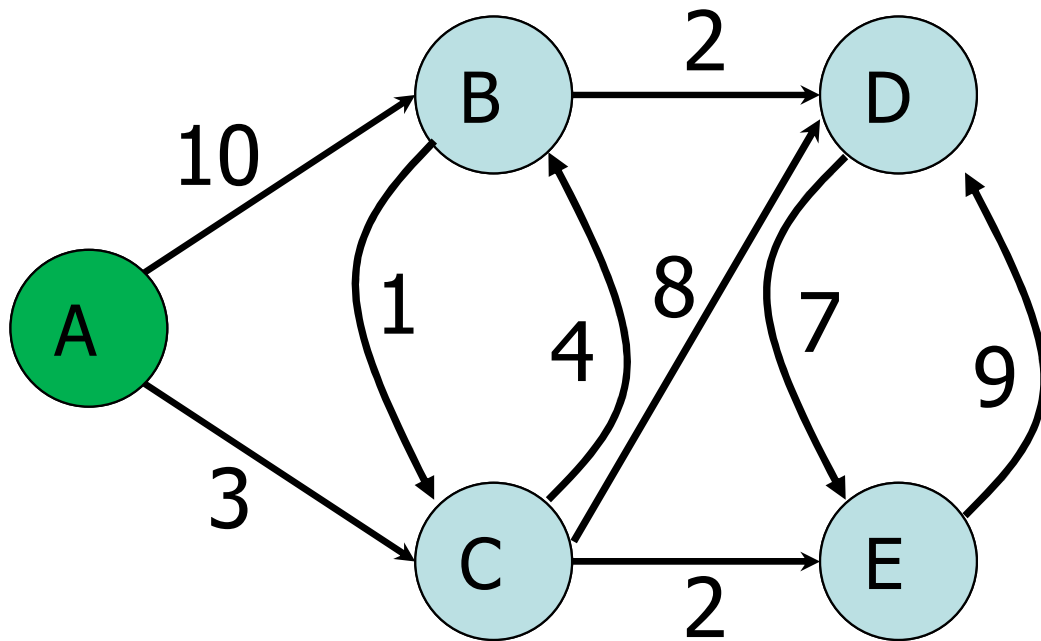
S: {}

Q:

A	B	C	D	E
0	∞	∞	∞	∞

Vertex	Distance	Parent
A	0	∅
B	∞	∅
C	∞	∅
D	∞	∅
E	∞	∅

Dijkstra's Algorithm – Example



$A \leftarrow \text{EXTRACT-MIN}(Q)$

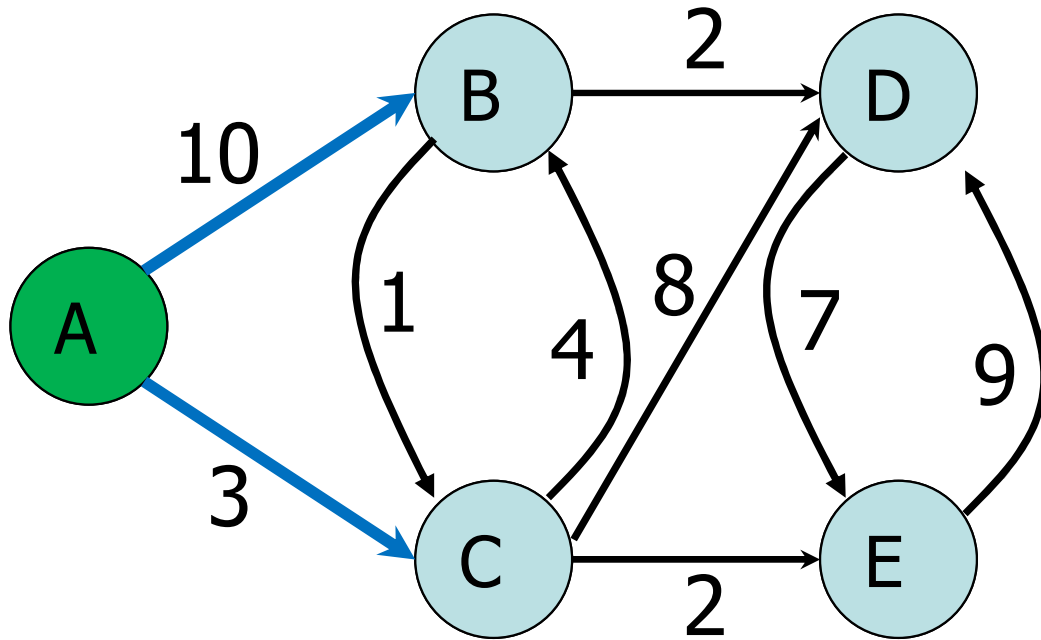
$S: \{A\}$

Q:

A	B	C	D	E
0	∞	∞	∞	∞

Vertex	Distance	Parent
A	0	\emptyset
B	∞	\emptyset
C	∞	\emptyset
D	∞	\emptyset
E	∞	\emptyset

Dijkstra's Algorithm – Example



Update all neighbors of A

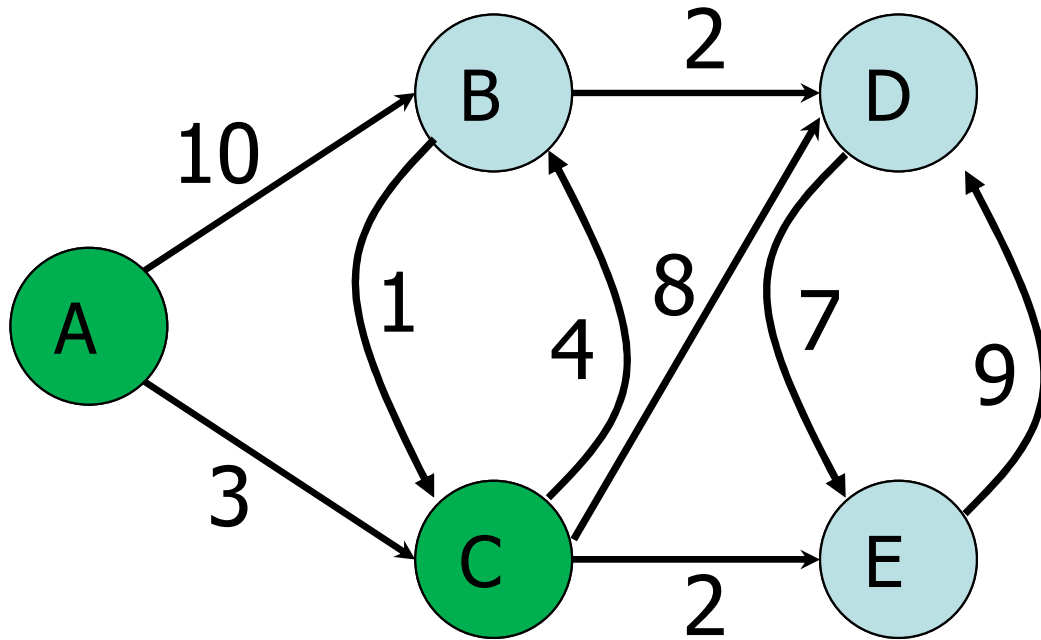
S: {A}

Q:

A	B	C	D	E
0	∞	∞	∞	∞
	10	3	∞	∞

Vertex	Distance	Parent
A	0	\emptyset
B	10	A
C	3	A
D	∞	\emptyset
E	∞	\emptyset

Dijkstra's Algorithm – Example



$C \leftarrow \text{EXTRACT-MIN}(Q)$

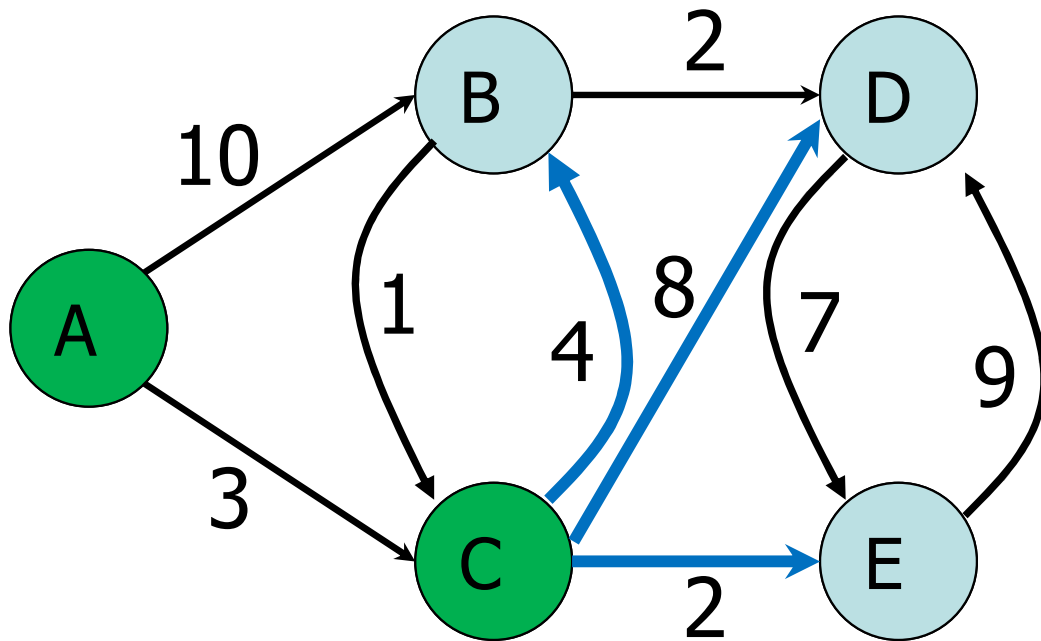
$S: \{A, C\}$

Q:

A	B	C	D	E
0	∞	∞	∞	∞
	10	3	∞	∞

Vertex	Distance	Parent
A	0	\emptyset
B	10	A
C	3	A
D	∞	\emptyset
E	∞	\emptyset

Dijkstra's Algorithm – Example



Update all neighbors of C

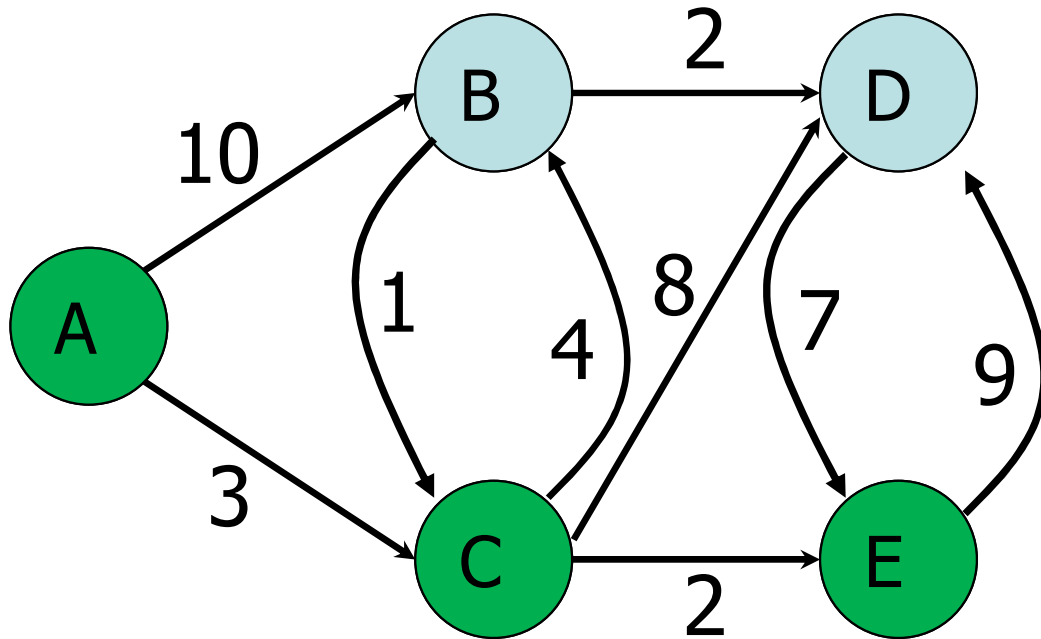
S: {A, C}

Q:

A	B	C	D	E
0	∞	∞	∞	∞
	10	3	∞	∞
	7		11	5

Vertex	Distance	Parent
A	0	\emptyset
B	7	C
C	3	A
D	11	C
E	5	C

Dijkstra's Algorithm – Example



$E \leftarrow \text{EXTRACT-MIN}(Q)$

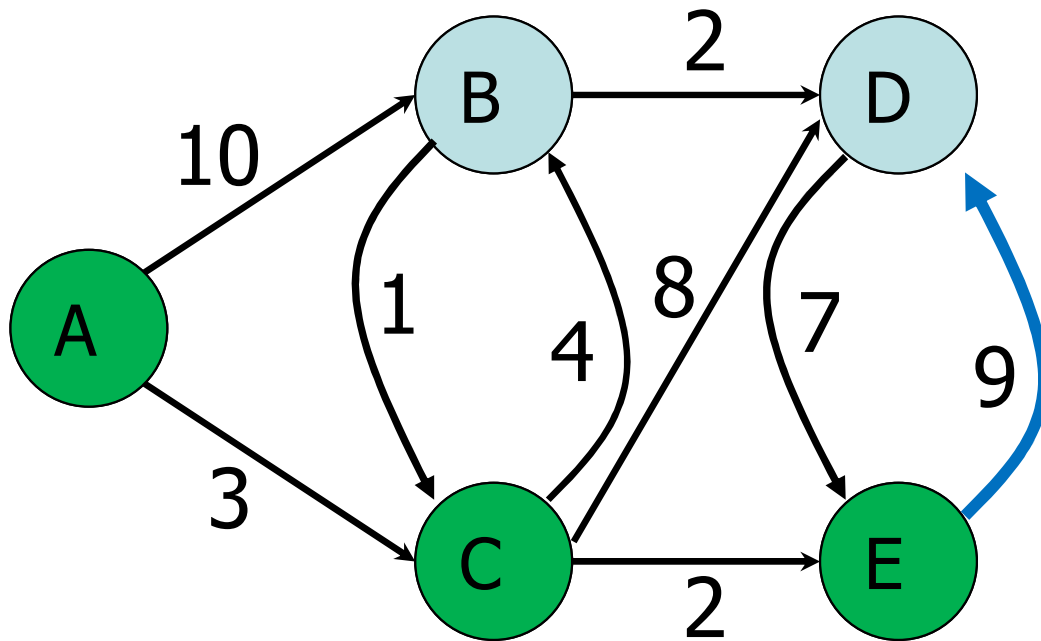
$S: \{A, C, E\}$

Q:

A	B	C	D	E
0	∞	∞	∞	∞
	10	3	∞	∞
	7		11	5

Vertex	Distance	Parent
A	0	\emptyset
B	7	C
C	3	A
D	11	C
E	5	C

Dijkstra's Algorithm – Example



Update all neighbors of E

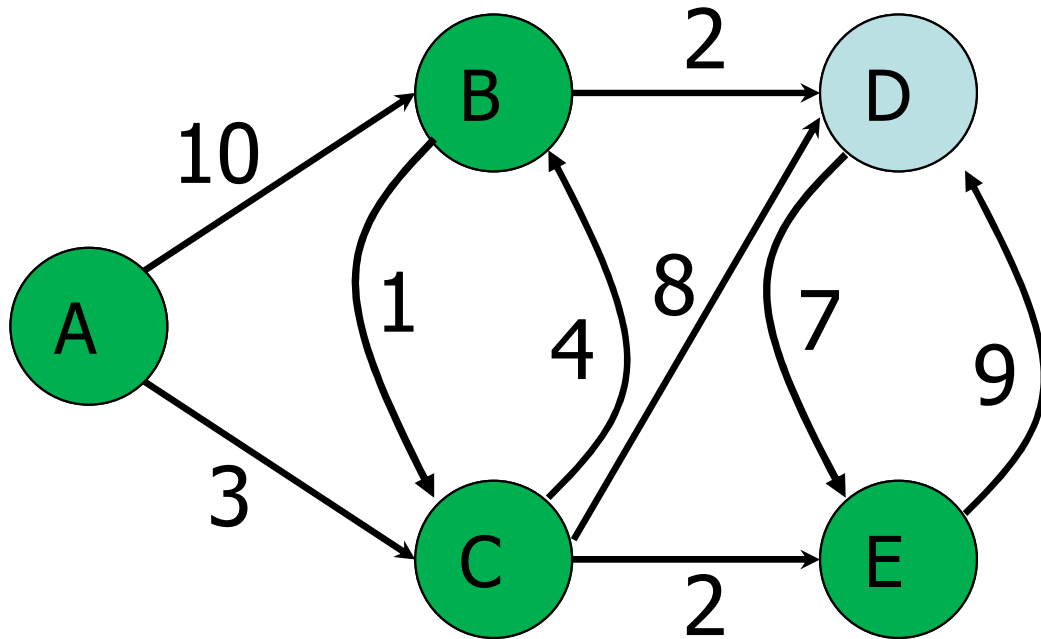
S: {A, C, E}

Q:

A	B	C	D	E
0	∞	∞	∞	∞
	10	3	∞	∞
	7		11	5
	7		11	

Vertex	Distance	Parent
A	0	\emptyset
B	7	C
C	3	A
D	11	C
E	5	C

Dijkstra's Algorithm – Example



$B \leftarrow \text{EXTRACT-MIN}(Q)$

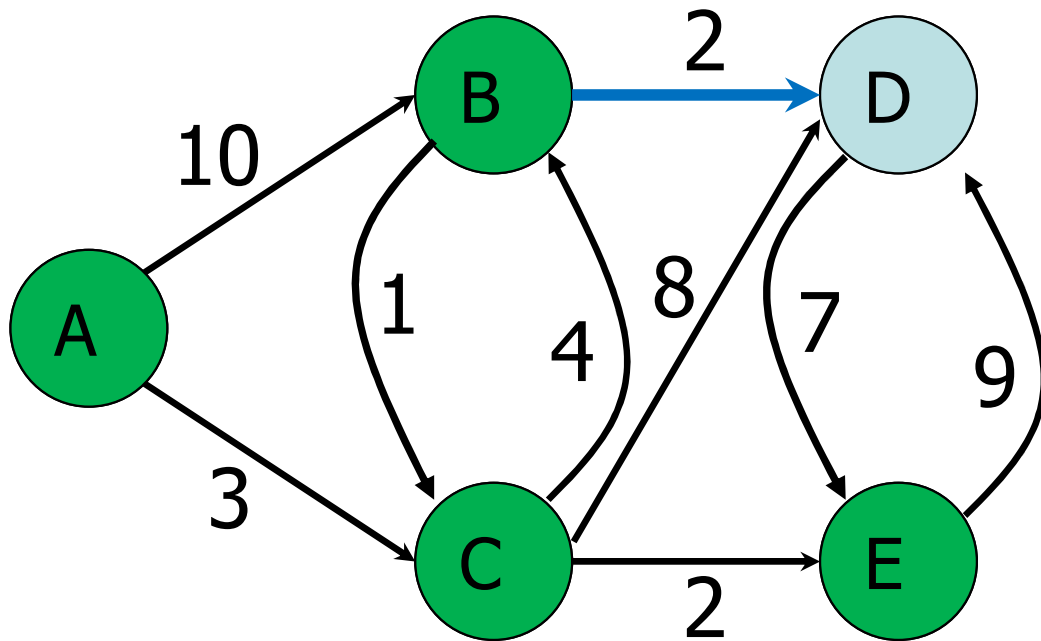
$S: \{A, C, E, B\}$

Q:

A	B	C	D	E
0	∞	∞	∞	∞
	10	3	∞	∞
	7		11	5
	7		11	

Vertex	Distance	Parent
A	0	\emptyset
B	7	C
C	3	A
D	11	C
E	5	C

Dijkstra's Algorithm – Example



Update all neighbors of B

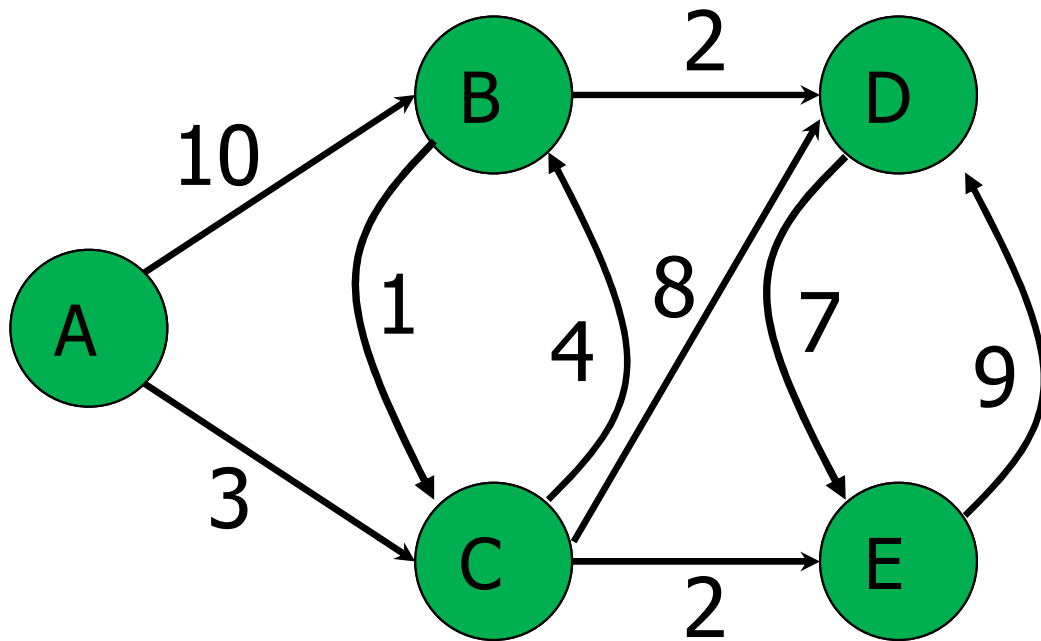
S: {A, C, E, B}

Q:

A	B	C	D	E
0	∞	∞	∞	∞
	10	3	∞	∞
	7		11	5
	7		11	
			9	

Vertex	Distance	Parent
A	0	\emptyset
B	7	C
C	3	A
D	9	B
E	5	C

Dijkstra's Algorithm – Example



$D \leftarrow \text{EXTRACT-MIN}(Q)$

$S: \{A, C, E, B, D\}$

Q:

A	B	C	D	E
0	∞	∞	∞	∞
	10	3	∞	∞
	7		11	5
	7		11	
			9	

Vertex	Distance	Parent
A	0	\emptyset
B	7	C
C	3	A
D	9	B
E	5	C

Negative Edges

- Dijkstra's algorithm is based on the greedy method
 - It adds vertices by increasing distance

Any Question So Far?

