

# Database Systems

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# Relational Algebra

# DBMS Architecture

How does a SQL engine work ?

- SQL query → relational algebra plan
- Relational algebra plan → Optimized plan
- Execute each operator of the plan

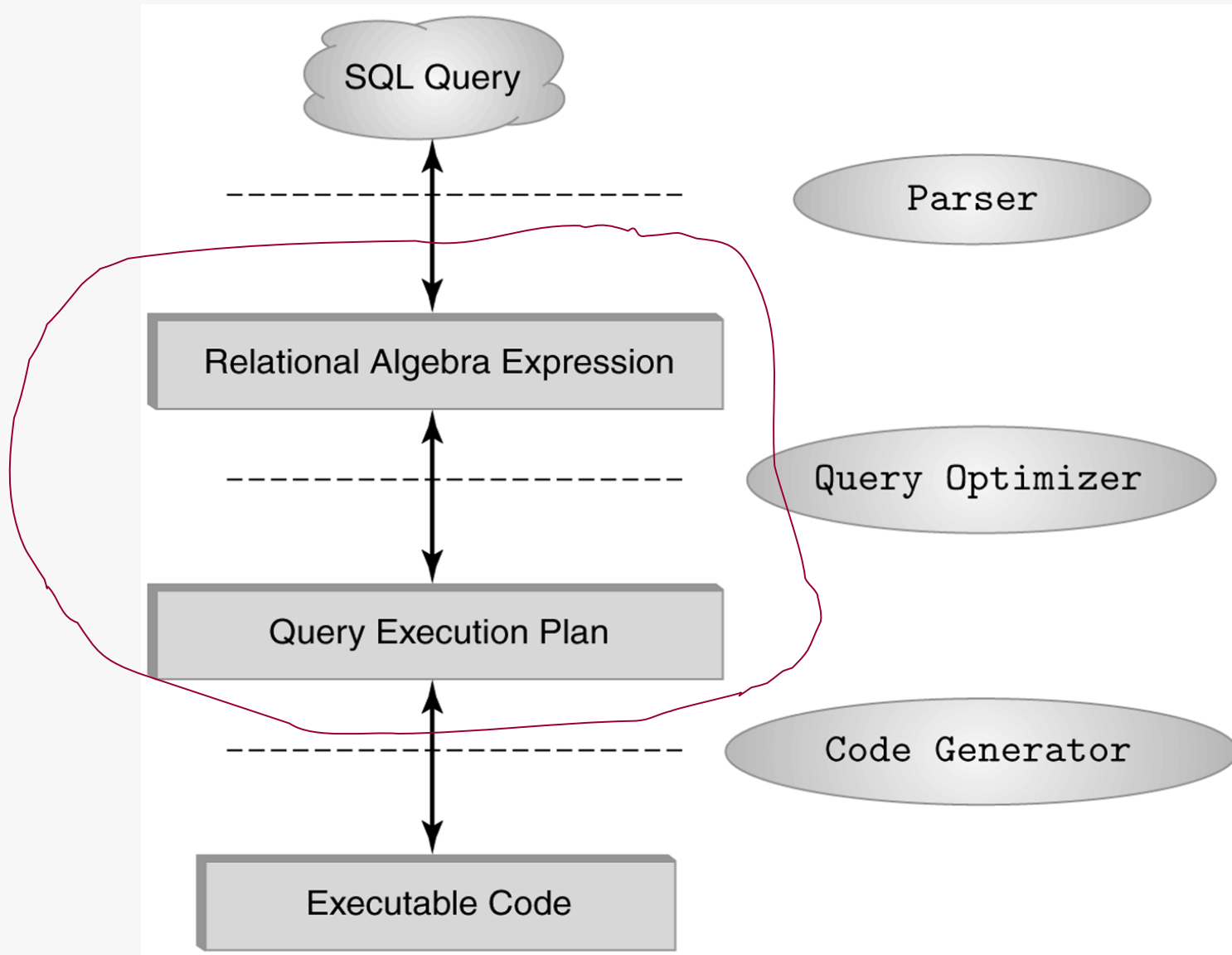
# What is an Algebra?

- A language based on operators and a domain of values
- Operators map values taken from the domain into other domain values
- Hence, an expression involving operators and arguments produces a value in the domain
- When the domain is a set of all relations (and the operators are as described later), we get the *relational algebra*
- We refer to the expression as a *query* and the value produced as the *query result*

# Relational Algebra

- Five operators:
  - Union:  $\cup$
  - Difference:  $-$
  - Selection:  $\sigma$
  - Projection:  $\Pi$
  - Cartesian Product:  $\times$
- Derived or auxiliary operators:
  - Intersection, complement
  - Joins (natural, equi-join, theta join, semi-join)
  - Renaming:  $\rho$

# The Role of Relational Algebra in a DBMS



# 1. Union and 2. Difference

- $R1 \cup R2$
- Example:
  - $\text{ActiveEmployees} \cup \text{RetiredEmployees}$
- $R1 - R2$
- Example:
  - $\text{AllEmployees} - \text{RetiredEmployees}$

# What about Intersection ?

- It is a derived operator
- $R1 \cap R2 = R1 - (R1 - R2)$
- Also expressed as a join (will see later)
- Example
  - `UnionizedEmployees`  $\cap$  `RetiredEmployees`



# Union Compatible Relations

- Two relations are *union compatible* if
  - Both have same number of columns
  - Names of attributes are the same in both
  - Attributes with the same name in both relations have the same domain
- Union compatible relations can be combined using *union*, *intersection*, and *set difference*

# Example

Tables:

Person (*SSN, Name, Address, Hobby*)

Professor (*Id, Name, Office, Phone*)

are not union compatible.

But

$\pi_{Name}(\text{Person})$  and  $\pi_{Name}(\text{Professor})$   
are union compatible so

$\pi_{Name}(\text{Person}) - \pi_{Name}(\text{Professor})$   
makes sense.

# 3. Selection

- Returns all tuples which satisfy a condition
- Notation:  $\sigma_c(R)$
- Examples
  - $\sigma_{\text{Salary} > 40000}(\text{Employee})$
  - $\sigma_{\text{name} = \text{"Smith"}}(\text{Employee})$
- The condition  $c$  can be  $=, <, \leq, >, \geq, <>$

# Select Operator

- Produce table containing subset of rows of argument table satisfying condition

$$\sigma_{condition}(relation)$$

- Example:

Person

<i>Id</i>	<i>Name</i>	<i>Address</i>	<i>Hobby</i>
1123	John	123 Main	stamps
1123	John	123 Main	coins
5556	Mary	7 Lake Dr	hiking
9876	Bart	5 Pine St	stamps

$\sigma_{Hobby='stamps'}(Person)$

<i>Id</i>	<i>Name</i>	<i>Address</i>	<i>Hobby</i>
1123	John	123 Main	stamps
9876	Bart	5 Pine St	stamps

# Selection Condition

- Operators:  $<$ ,  $\leq$ ,  $\geq$ ,  $>$ ,  $=$ ,  $\neq$
- Simple selection condition:
  - *$\langle attribute \rangle operator \langle constant \rangle$*
  - *$\langle attribute \rangle operator \langle attribute \rangle$*
- *$\langle condition \rangle AND \langle condition \rangle$*
- *$\langle condition \rangle OR \langle condition \rangle$*
- NOT  *$\langle condition \rangle$*

# Selection Condition - Examples

- $\sigma_{Id > 3000 \text{ OR } Hobby = 'hiking'} (Person)$
- $\sigma_{Id > 3000 \text{ AND } Id < 3999} (Person)$
- $\sigma_{NOT(Hobby = 'hiking')} (Person)$
- $\sigma_{Hobby \neq 'hiking'} (Person)$

SSN	Name	Salary
1234545	John	200000
5423341	Smith	600000
4352342	Fred	500000

$\sigma_{\text{Salary} > 400000}$  (Employee)

SSN	Name	Salary
5423341	Smith	600000
4352342	Fred	500000

## 4. Projection

- Eliminates columns, then removes duplicates
- Notation:  $\Pi_{A1, \dots, An}(R)$
- Example: project social-security number and names:
  - $\Pi_{SSN, Name}(Employee)$
  - Output schema: Answer(SSN, Name)



SSN	Name	Salary
1234545	John	200000
5423341	John	600000
4352342	John	200000

$\Pi_{\text{Name,Salary}}(\text{Employee})$

Name	Salary
John	20000
John	60000

# Project Operator

- Produces table containing subset of columns of argument table

$$\pi_{attribute\ list}(relation)$$

- Example:

Person

<i>Id</i>	<i>Name</i>	<i>Address</i>	<i>Hobby</i>
1123	John	123 Main	stamps
1123	John	123 Main	coins
5556	Mary	7 Lake Dr	hiking
9876	Bart	5 Pine St	stamps

$\pi_{Name, Hobby}(\text{Person})$

<i>Name</i>	<i>Hobby</i>
John	stamps
John	coins
Mary	hiking
Bart	stamps

# Project Operator

- Example:

Person

<i>Id</i>	<i>Name</i>	<i>Address</i>	<i>Hobby</i>
1123	John	123 Main	stamps
1123	John	123 Main	coins
5556	Mary	7 Lake Dr	hiking
9876	Bart	5 Pine St	stamps

$\pi_{Name,Address}(\text{Person})$

<i>Name</i>	<i>Address</i>
John	123 Main
Mary	7 Lake Dr
Bart	5 Pine St

Result is a table (no duplicates); can have fewer tuples than the original

# Expressions

$\pi_{Id, Name} (\sigma_{Hobby='stamps' \text{ OR } Hobby='coins'} (Person) )$

<i>Id</i>	<i>Name</i>	<i>Address</i>	<i>Hobby</i>
1123	John	123 Main	stamps
1123	John	123 Main	coins
5556	Mary	7 Lake Dr	hiking
9876	Bart	5 Pine St	stamps

Person

<i>Id</i>	<i>Name</i>
1123	John
9876	Bart

Result

## 5. Cartesian Product

- Each tuple in R1 with each tuple in R2
- Notation:  $R1 \times R2$
- Example:
  - Employee  $\times$  Dependents
- Very rare in practice; mainly used to express joins

## Cartesian Product Example

### Employee

Name	SSN
John	999999999
Tony	777777777

### Dependents

EmployeeSSN	Dname
999999999	Emily
777777777	Joe

### Employee x Dependents

Name	SSN	EmployeeSSN	Dname
John	999999999	999999999	Emily
John	999999999	777777777	Joe
Tony	777777777	999999999	Emily
Tony	777777777	777777777	Joe

# Cartesian Product

- If  $R$  and  $S$  are two relations,  $R \times S$  is the set of all concatenated tuples  $\langle x, y \rangle$ , where  $x$  is a tuple in  $R$  and  $y$  is a tuple in  $S$ 
  - $R$  and  $S$  need not be union compatible.
  - *But  $R$  and  $S$  must have distinct attribute names. Why?*
- $R \times S$  is expensive to compute. But why?

$A$	$B$	$C$	$D$
x1	x2	y1	y2
x3	x4	y3	y4

$R$                        $S$

$A$	$B$	$C$	$D$
x1	x2	y1	y2
x1	x2	y3	y4
x3	x4	y1	y2
x3	x4	y3	y4

$R \times S$

# Relational Algebra

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# Renaming

- Changes the schema, not the instance
- Notation:  $\rho_{B_1, \dots, B_n}(R)$
- Example:
  - $\rho_{\text{LastName}, \text{SocSocNo}}(\text{Employee})$
  - Output schema:  
Answer(LastName, SocSocNo)

# Renaming Example

## Employee

Name	SSN
John	999999999
Tony	777777777

## $\rho_{LastName, SocSocNo}$ (**Employee**)

LastName	SocSocNo
John	999999999
Tony	777777777

# Example – Second Method

Transcript (*StudId*, *CrsCode*, *Semester*, *Grade*)

Teaching (*ProfId*, *CrsCode*, *Semester*)

$$\pi_{StudId, CrsCode}(\text{Transcript})[StudId, CrsCode1]$$
$$\times \pi_{ProfId, CrsCode}(\text{Teaching})[ProfId, CrsCode2]$$

This is a relation with 4 attributes:

*StudId*, *CrsCode1*, *ProfId*, *CrsCode2*



Thank  
You

A blue paper cutout of the words "Thank You" in a stylized, rounded font. The text is white with a blue outline. The cutout is hanging from a thin brown string that is tied in a loop at the top. The background is white.