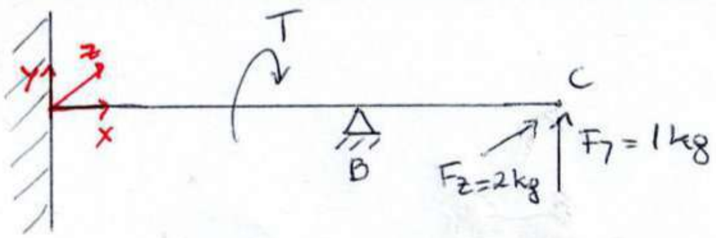


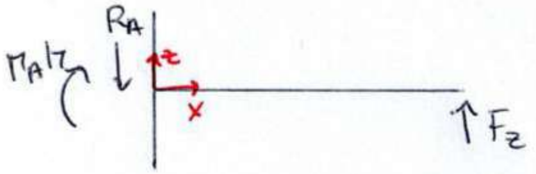
$T = 1000\ kg \cdot mm$
$L_{AB} = 0.6\ m$
$L_{AC} = 1\ m$
$\varnothing = 10\ mm$

Solución

a)

Plano xz

$\Sigma F_z = 0 \Rightarrow R_A|_z = F_z = 2\ kg$



$\circlearrowleft + \Sigma M_A|_y = 0 \Rightarrow F_z \cdot L - M_A|_y = 0$

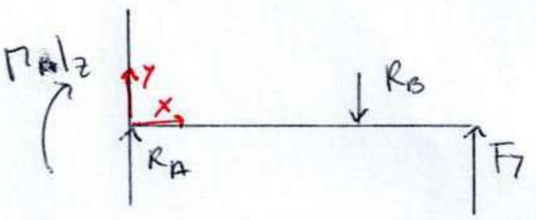
$\Rightarrow M_A|_y = F_z \cdot L = 2kg \cdot 1m = 2k$

Plano xy

$\Sigma F_y = 0 \Rightarrow -R_A + R_B = F_y = 1\ kg$ (i)

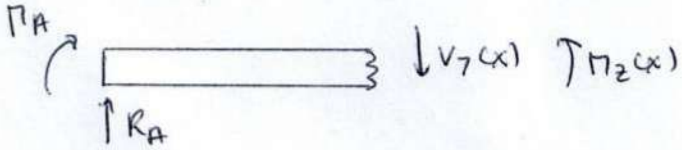
$\circlearrowleft + \Sigma M_A|_z = 0 \Rightarrow -R_B \cdot 600mm + F_y \cdot 1000mm - M$

$M_A|_z = 1000 - 600 R_B$ (ii)



En este caso, nos encontramos con 2 ec y 3 incógnitas. Necesitamos una 3° ec, la podemos obtener mediante Ec. de la Elástica o Momento de área.

Mediante Ec. de la Elástica



$\circlearrowleft + \Sigma M_x = 0$

$M_A + R_A \cdot x - M_z(x) = 0$
$M_z(x) = M_A + R_A \cdot x$

$$EI \frac{d^2y}{dx} = M_A + R_A x$$

$$EI \frac{dy}{dx} = M_A x + \frac{R_A x}{2} + C_1$$

$\frac{dy}{dx} = 0,\ x = 0$

$$EI \cdot 0 = 0 + 0 + C_1 \rightarrow C_1 = 0$$

$$EI\ y = \frac{M_A x^2}{2} + \frac{R_A x^3}{6} + C_2$$

$$EI \cdot 0 = \frac{M_A \cdot 0}{2} + \frac{R_A \cdot 0^3}{6} + C_2 \rightarrow C_2 = 0.$$

$$\Rightarrow EI\ y(x) = \frac{M_A x^2}{2} + \frac{R_A x^3}{6}$$

$$y_B(x=600)=0 \rightarrow 0 = \frac{R_A(600)^3}{6} + \frac{M_A(600)^2}{2} = 0$$

$$\Rightarrow R_A = -\frac{M_A|_z}{200} \quad (iii)$$

De (i), (ii), (iii) :

$$R_A = 1 \text{ kg}$$
$$M_A|_z = -200 \text{ kg}\cdot\text{mm}$$
$$R_B = 2 \text{ kg}$$

$$\rightarrow y(x) = \frac{-200x^2}{2} + \frac{x^3}{6}$$

El máximo esfuerzo normal principal, $\sigma(1)|_{máx}$, se produce en el empotramiento y vale:

$$\sigma(1)|_{máx} = \frac{\sigma_{xx}}{2}\Big|_{flexión\ máx} + \sqrt{\left(\frac{\sigma_{xx}}{2}\Big|_{flexión\ máx}\right)^2 + \left(\frac{16T}{\pi D^3}\right)^2}$$

Buscaremos el valor de $\sigma_{xx}|_{flexión\ máx}$

$$\sigma_{xx}|_{flexión\ máx} = \frac{|M_z|\,y^* + |M_y|\sqrt{R^2-y^{*2}}}{I}$$

Sabemos que $M_z|_{máx}$ y $M_y|_{máx}$ se producen en la misma sec y tienen un valor de:

$$M_z(x) = M_A|_z + R_A|_y \cdot x \qquad M_y(x) = M_A|_y - R_A|_z \cdot x$$

$$M_z(x=0) = M_A|_z = -200 \text{ kg mm} \qquad M_y(x=0) - 2000 \text{ kg}\cdot\text{mm}$$

Por tanto

$$\sigma_{xx}|_{flexión\ máx} = \frac{200\,y^* + 2000\sqrt{5^2-y^{*2}}}{I}$$

Derivamos para obtener y^*

$$\frac{d\sigma_{xx}}{dy^*} = 0 \Longrightarrow \frac{1}{I}\left[200 + \frac{2000\,y^*}{\sqrt{25-y^{*2}}}\right] = 0$$

$$\Rightarrow \frac{2000\,y^*}{\sqrt{25-y^{*2}}} = 200$$

$$\Rightarrow \frac{y^*}{\sqrt{25-y^{*2}}} = 0.1 \quad /()^2$$

$$\rightarrow \frac{y^*}{\sqrt{25-y^{*2}}} = 0.01$$

$$\Rightarrow y^* = 0.49$$
$$z^* = 4.98$$

$$I = \frac{\pi \cdot R^4}{4} = \frac{\pi\,(5)^4}{4} = 490.9\ mm^4$$

Por tanto:

$$\sigma_{xx} = \frac{200\,(0.49) + 2000\,(4.98)}{490.9}$$

$$\sigma_{xx}|_{flex\,máx} = 20.47\ kg/mm^2$$

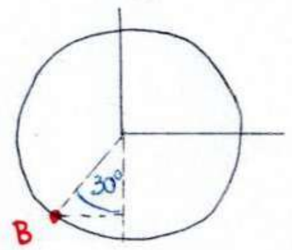
Finalmente:

$$\sigma(1)|_{máx} = \frac{20.47}{2} + \sqrt{\left(\frac{20.47}{2}\right)^2 + \left(\frac{16 \cdot 1000}{\pi \cdot 10^3}\right)^2}$$

$$\sigma(1)|_{máx} = 21.7\ kg/mm^2$$

b) $\sigma_{xx}|_{flex\,máx} = 20.47\ kg/mm^2$

c) $\sigma_{ij}|_B = ?$



$$\tau_{x\theta} = \frac{16\,T}{\pi\,D^3} = \frac{16 \cdot 1000}{\pi\,(10)^3} = 5{,}1\ kg/mm^2$$

$$y|_B = 5 \cos 30° = 4.3\ mm$$

$$z|_B = 5 \operatorname{sen} 30° = 2.5\ mm$$

Calcularemos cada una de la componentes del tensor esfuerzo en el punto B. Partiremos por calcular σ_{xx}:

$$\sigma_{xx}|_B = \frac{M_z|_B \cdot y|_B + M_y|_B \cdot z|_B}{I}$$

Sabemos que

$$|M_z(x)| = M_A|_z + R_A|_y \cdot x$$
$$= -200 + x$$
$$= -200 + 600$$
$$= 400$$

$$|M_y(x)| = 2000 - 2x$$
$$= 2000 - 2 \cdot 600$$
$$= 800$$

$$\Rightarrow \sigma_{xx} = \frac{400 \cdot 4{,}3 + 800 \cdot 2{,}5}{490.9} = 7{,}6\ kg/mm^2$$

Ahora calcularemos las componentes de corte:

$\tau_{xy}|_{V_y}$ $\quad$ $\tau_{xy}|_{torsión}$

$\tau_{xz}|_{V_z}$ $\quad$ $\tau_{xz}|_{torsión}$

- Calculamos las componentes de corte generadas por V_y y V_z

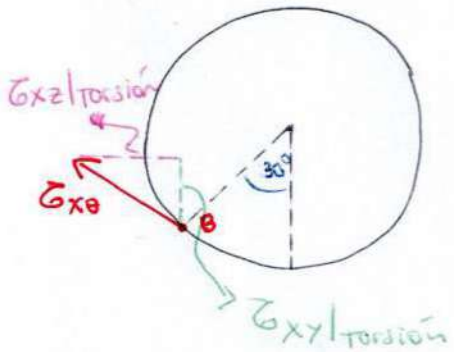
$$\tau_{xy}\Big|_{V_y} = \frac{V_y\, S(y)}{b\, I} \qquad \frac{S(y)}{b} = \frac{1}{3}\left(R^2 - y^2\right)$$

$$\Rightarrow \tau_{xy}\Big|_{V_y} = \frac{1}{I}\cdot\frac{1}{3}\left(R^2 - y^2\right) = \frac{1}{3}\,\frac{\left(5^2 - 4.3^2\right)}{490.9} = 4,4 \times 10^{-3}\ \mathrm{kg/mm^2}$$

$$\tau_{xz}\Big|_{V_z} = \frac{V_z\, S(z)}{b\, I} \qquad \frac{S(z)}{b} = \frac{1}{3}\left(R^2 - z^2\right)$$

$$\Rightarrow \tau_{xz}\Big|_{V_z} = \frac{2}{I}\,\frac{1}{3}\left(R^2 - z^2\right) = \frac{1}{3}\,\frac{\left(5^2 - 2,5^2\right)}{490.9} = 2,5 \times 10^{-2}\ \mathrm{kg/mm}$$

- Ahora, para finalizar, calculamos las componentes de corte generados por la torsión.



$$\tau_{x\theta} = 5,1\ \mathrm{kg/mm^2}$$

$$\Rightarrow \tau_{xy}\Big|_{torsion} = 5.1 \cos 60° = 2,6\ \mathrm{kg/mm^2}$$

$$\tau_{xz}\Big|_{torsion} = 5.1 \operatorname{sen} 60° = 4,4\ \mathrm{kg/mm^2}$$

$$\Rightarrow \tau_{xy}\Big|_{TOTAL} = \tau_{xy}\Big|_{torsion} - \tau_{xy}\Big|_{V_y} = 2,6 - 4,4\times 10^{-3} \approx 2,6\ \mathrm{kg/mm}$$

$$\tau_{xz}\Big|_{TOTAL} = \tau_{xz}\Big|_{torsión} + \tau_{xz}\Big|_{V_z} = 4,4 + 0.025 \approx 4,4\ \mathrm{kg/mm^2}$$

Finalmente

$$\sigma_{ij}\Big|_B = \begin{bmatrix} 7,6 & 2,6 & 4,43 \\ 2,6 & 0 & 0 \\ 4,43 & 0 & 0 \end{bmatrix} \mathrm{kg/mm^2} \;//$$