PAUTA Listado 11

1) El Torque depende de:
$$\begin{cases} \text{Radio}: R \\ \text{Velocidad angular}: w \\ \text{Viscosidad}: \mu \\ \text{Ángulo del cono}: \theta \end{cases}$$

es decir, $M = M(R, w, \mu, \theta)$

* Unidades:
- Torque: $N \cdot m = \dfrac{Kg \cdot m^2}{s^2} \Rightarrow M \cdot L^2 \cdot T^{-2}$
- Radio: $m \Rightarrow L$
- Velocidad angular: $\dfrac{rad}{s} \Rightarrow T^{-1}$
- Viscosidad: $Pa \cdot s = \dfrac{Kg}{m \cdot s} \Rightarrow M \cdot L^{-1} \cdot T^{-1}$
- Ángulo: no tiene dimensiones

M	R	w	μ	θ
$M \cdot L^2 \cdot T^{-2}$	L	T^{-1}	$M \cdot L^{-1} \cdot T^{-1}$	$-$

$$\boxed{\begin{aligned} n &= 5 \\ j &= 3 \\ k &= 2 \end{aligned}}$$

$\therefore$ Se deben encontrar 2 adimensionales: Π_1 y Π_2

- $\boxed{\Pi_1 = \theta}$ (ángulo del cono)

- $\Pi_2 = M \cdot R^a \cdot w^b \cdot \mu^c \Rightarrow (M \cdot L^2 \cdot T^{-2})(L^a)(T^{-1})^b \cdot (M \cdot L^{-1} \cdot T^{-1})^c = M^0 L^0 T^0$

$$\left.\begin{aligned} L&: \ a - c + 2 = 0 \\ T&: \ -b - c - 2 = 0 \\ M&: \ c + 1 = 0 \end{aligned}\right\}$$

Resolviendo el sistema, se obtiene: $\boxed{a = -3}$ $\boxed{c = -1}$ $\boxed{b = -1}$

$$\boxed{\Pi_2 = \frac{M}{R^3 \cdot \mu \cdot W}}$$

2) La Resistencia F depende de:
- densidad del fluido : ρ
- Viscosidad del fluido : μ
- Velocidad de flujo : v
- Largo característico del cuerpo : L_c

Las dimensiones de las variables son:

F	ρ	μ	v	L_c
MLT^{-2}	ML^{-3}	$ML^{-1}T^{-1}$	LT^{-1}	L

$$\boxed{\begin{matrix} n = 5 \\ J = 3 \\ K = 2 \end{matrix}}$$

$$\Pi_1 = F \cdot \mu^a \cdot \rho^b \cdot L_c^c \Rightarrow M \cdot L \cdot T^{-2} \cdot M^a L^{-a} \cdot T^{-a} \cdot M^b L^{-3b} \cdot L^c = M^0 L^0 T^0$$

$$\left.\begin{aligned} M&: 1 + a + b = 0 \\ L&: 1 - a - 3b + c = 0 \\ T&: -2 - a = 0 \end{aligned}\right\}$$

Resolviendo el sistema, se obtiene:

$\boxed{a = -2}$ $\boxed{b = 1}$ $\boxed{c = 0}$

$$\therefore \boxed{\Pi_1 = \frac{F \cdot \rho}{\mu^2}}$$

$$\Pi_2 = V \cdot \mu^a \cdot \rho^b \cdot L_c^c \Rightarrow L^1 T^{-1} \cdot M^a L^{-a} T^{-a} \cdot M^b \cdot L^{-3b} \cdot L^c = M^0 L^0 T^0$$

$$\left.\begin{aligned} M&: a + b = 0 \\ L&: 1 - a - 3b + c = 0 \\ T&: -1 - a = 0 \end{aligned}\right\}$$

Resolviendo el sistema, se tiene:

$\boxed{a = -1}$ $\boxed{b = 1}$ $\boxed{c = 1}$

$$\therefore \Pi_2 = \frac{V \cdot L_c \cdot \rho}{\mu} \quad \text{(número de Reynolds)}$$

Del Teorema de Π, se tiene qle : $\Pi_1 = \Pi_1(\Pi_2)$

Con los valores de la tabla y de enunciado, se obtiene :

Π_1	$1{,}37 \times 10^{10}$	$1{,}34 \cdot 10^{10}$	$1{,}31 \times 10^{10}$	$1{,}28 \times 10^{10}$	$1{,}27 \times 10^{10}$
Π_2 (R_e)	143298,9	181601,2	229387,8	267599,7	296297,7

Si se grafican estos puntos utilizando escala logarítmica y aplicando ajuste potencial (por enunciado), se tiene:

$$\boxed{\Pi_1 = 5 \cdot 10^{10} \cdot R_e^{-0{,}111}}$$

Luego, con la función obtenida se puede predecir la resistencia para el caso dado :

considerando :

$L_c = 15 \, in = 1{,}25 [ft]$

$V = 0{,}6 \, m/s = 1{,}968 [ft/s]$

$\rho = 1{,}94 [slug/ft^3]$

$\mu = 2{,}09 \times 10^{-5} [slugs/ft \cdot s]$

$$\Pi_1 = \frac{F \cdot \rho}{\mu^2}$$

$$R_e = \frac{\rho \cdot V \cdot L_c}{\mu} = 228344{,}49$$

$$\Pi_1 = 5 \times 10^{10} \left(\frac{\rho \cdot V \cdot L_c}{\mu}\right)^{-0{,}111} \Rightarrow \boxed{\Pi_1 = 1{,}271 \cdot 10^{10}}$$

despejando F ; $F = \dfrac{\Pi_1 \cdot \mu^2}{\rho} \Rightarrow \boxed{F = 2{,}86 [lbf]}$

3) $Q = Q(\alpha, g, y)$

Dimensiones:

Q	α	g	y
L^3T^{-1}	$-$	$L \cdot T^{-2}$	L

$$\boxed{\begin{matrix} n = 4 \\ j = 2 \\ K = 2 \end{matrix}}$$

$$\boxed{\Pi_1 = \alpha}$$

$$\Pi_2 = Q \cdot g^a \cdot y^b \Rightarrow L^3 T^{-1} \cdot L^a T^{-2a} \cdot L^b = L^0 \cdot T^0$$

$L: \; 3 + a + b = 0$

$T: \; -1 - 2a = 0$

Resolviendo, se obtiene: $\boxed{a = -1/2}$ $\boxed{b = -5/2}$

$$\therefore \; \Pi_2 = \frac{Q}{g^{1/2} \cdot y^{5/2}}$$

, De enunciado se tiene que $\Pi_2 = \Pi_2(\Pi_1)$

y (m)	0,1	0,2	0,3	0,4
Q (m^3/s)	0,0022	0,013	0,035	0,073
Π_2	0,22	0,23	0,22	0,23

$\Rightarrow$ el valor es casi constante

$\therefore \; \Pi_2 = 0{,}22$

Ahora, con $\alpha = 55°$ y $y = 3{,}2$ (m)

$$\Pi_2 = \frac{Q}{g^{1/2} \cdot y^{5/2}} \Rightarrow Q = \Pi_2 \cdot g^{1/2} \cdot y^{5/2}$$

$$\boxed{Q = 12{,}62 \; [m^3/s]}$$

4.-) Se tiene que: $M = M(W, R, N, \alpha)$

donde:

M	W	R	N	α
ML^2T^{-2}	T^{-1}	L	$ML^{-1}T^{-1}$	$-$

$\boxed{\begin{array}{l} n = 5 \\ j = 3 \\ k = 2 \end{array}}$

Por lo que se deben buscar 2 adimensionales.

- $\boxed{\Pi_1 = \alpha}$

- $\Pi_2 = M \cdot W^a \cdot R^b \cdot N^c = [ML^2T^{-2}] \cdot [T^{-a}] \cdot [L^b] \cdot [M^c \cdot L^{-c} \cdot T^{-c}] = M^0 L^0 T^0$

$$\left.\begin{array}{l} M: \ 1 + c = 0 \\ L: \ 2 + b - c = 0 \\ T: \ -2 - a - c = 0 \end{array}\right\} \text{Resolviendo: } \boxed{\begin{array}{l} c = -1 \\ b = -3 \\ a = 1 \end{array}}$$

$$\Rightarrow \boxed{\Pi_2 = \frac{M}{W \cdot R^3 \cdot N}}$$

II) Se puede expresar: $2N_{inicial} = N_f$; $2W_{inicial} = W_{final}$:

$$\frac{M_{inicial}}{W_i \cdot R^3 \cdot N_i} = \frac{M_{final}}{W_f \cdot R^3 \cdot N_f} = \frac{M_{final}}{2W_i \cdot R^3 \cdot 2N_i} = \frac{M_{final}}{4 W_i \cdot R^3 \cdot N_i}$$

$$\Rightarrow \boxed{M_{inicial} = \frac{M_{final}}{4}}$$

5.) $\frac{\Delta P}{L} = f(V, D, \rho, \mu)$

$\frac{\Delta P}{L}$	V	D	ρ	μ
$ML^{-2}T^{-2}$	LT^{-1}	L	ML^{-3}	$ML^{-1}T^{-1}$

$$\boxed{\begin{array}{l} n=5 \\ j=3 \\ k=2 \end{array}}$$

Se buscan 2 adimensionales.

- $\Pi_1 = \frac{\Delta P}{L} \cdot D^a \cdot \rho^b \cdot \mu^c = (ML^{-2}T^{-2})(L^a)(M^b L^{-3b})(M^c L^{-c} T^{-c}) = M^0 L^0 T^0$

$$\left.\begin{array}{ll} L: & a - 3b - c - 2 = 0 \\ M: & b + c + 1 = 0 \\ T: & -c - 2 = 0 \end{array}\right\} \text{Resolviendo: } \boxed{\begin{array}{l} a = 3 \\ b = 1 \\ c = -2 \end{array}} \Rightarrow \Pi_1 = \frac{\Delta P}{L} \cdot \frac{D^3 \cdot \rho}{\mu^2}$$

- $\Pi_2 = V \cdot D^a \cdot \rho^b \cdot \mu^c = (LT^{-1})(L^a)(M^b L^{-3b})(M^c L^{-c} T^{-c}) = M^0 L^0 T^0$

$$\left.\begin{array}{ll} L: & a - 3b - c + 1 = 0 \\ M: & b + c = 0 \\ T: & -c - 1 = 0 \end{array}\right\} \boxed{\begin{array}{l} a = 1 \\ b = 1 \\ c = -1 \end{array}} \Rightarrow \Pi_2 = \frac{D \rho V}{\mu} = Re$$

b) $A = \pi \cdot r^2 = 5{,}027 \cdot 10^{-3} \, [m^2]$; $V = \frac{Q}{A}$

Q [m³/s]	0,005	0,01	0,015	0,020
ΔP [Pa]	5800	20300	42100	70800
V [m/s]	0,995	1.99	2,98	3,98

Π_1	5,93 E 7	2,07 E 8	4,30 E 8	7,24 E 8
Π_2	7,94 E 4	1,59 E 5	2,38 E 5	3,18 E 5

- Gráfico Log - Log:

c) $\Pi_1 = b \cdot (\Pi_2)^K \quad / \log$

$\log(\Pi_1) = \log(b) + K \log(\Pi_2)$, es de la forma: $y = m + Kx$

$$\Rightarrow K = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\log(4{,}3E8) - \log(5{,}93E7)}{\log(2{,}38E5) - \log(7{,}94E4)} \approx 1{,}8 = K$$

$$\Rightarrow \log(4{,}3E8) = \log(b) + 1{,}8 \log(2{,}38E5)$$

$b = 0{,}09$

$$\therefore \boxed{\Pi_1 = 0{,}09(Re)^{1{,}8}}$$

$Q = 50 [m^3/h] = 0{,}0139 [m^3/s]$

d) $A = \pi \cdot r^2 = \pi (0{,}025)^2 = 1{,}96 \cdot 10^{-3} [m^2]$

$$Q = V \cdot A \Rightarrow V = \frac{0{,}0139}{1{,}9 \times 10^{-3}} = 7{,}08 [m/s]$$

$$\Pi_2 = Re = \frac{D \rho V}{\mu} = \frac{0{,}05 \cdot 804 \cdot 7{,}08}{1{,}92E-3} = 148237{,}5$$

$$\Pi_1 = 0{,}09\,(148237{,}5)^{1.8} = 1{,}83E8$$

$$\Rightarrow \Delta p = \frac{\Pi_1 \, L \, \nu^2}{D^3 \cdot \rho} = \frac{1{,}83E8 \times 200 \times (1{,}92\,E{-3})^2}{(0{,}05)^3 \cdot 804} = \boxed{1{,}34\,E6\ [Pa]}$$

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