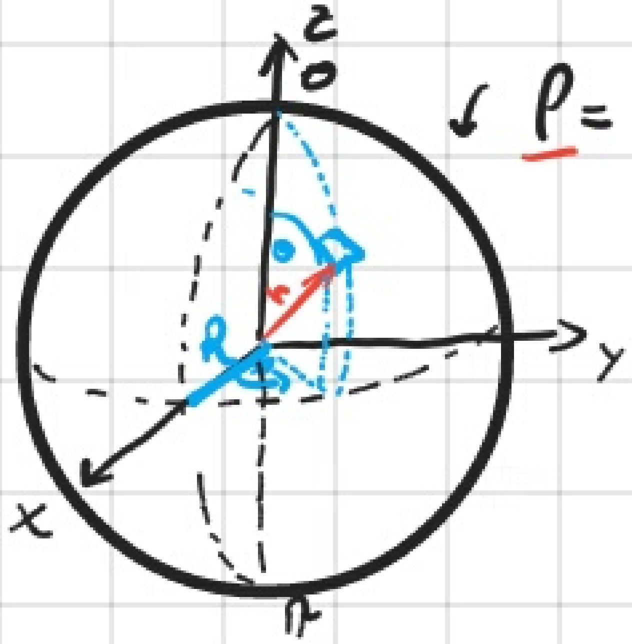
Ayudantía 3: (Guía Gauss)

(1) Ley de Gauss: $\oint \vec{E} \cdot d\vec{S} = \frac{q_{enc}}{\varepsilon_0}$

- $Q_T = ?$
- $\vec{E}(r) = ?$
- $V(r) = ?$



$\rho = \rho_0 \left(1 - r^2/R\right)$; $r \in [0, R]$

$$\boxed{\begin{aligned} dq &= \rho\, dV \\ dq &= \sigma\, dS \\ dq &= \lambda\, dr \end{aligned}}$$

(a)

$$dq = \rho\, dV$$

$$dq = \rho_0 \left(1 - r^2/R^2\right) \underbrace{(r^2 \sin\theta)}_{J_\Theta} dr\, d\theta\, d\phi \quad / \iiint$$

$$\int_0^{Q_T} dq = \int_0^R \int_0^\pi \int_0^{2\pi} \rho_0 \left(1 - \frac{r^2}{R^2}\right) r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$Q_T = \rho_0 \int_0^R \left(1 - \frac{r^2}{R^2}\right) r^2 dr \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\phi$$

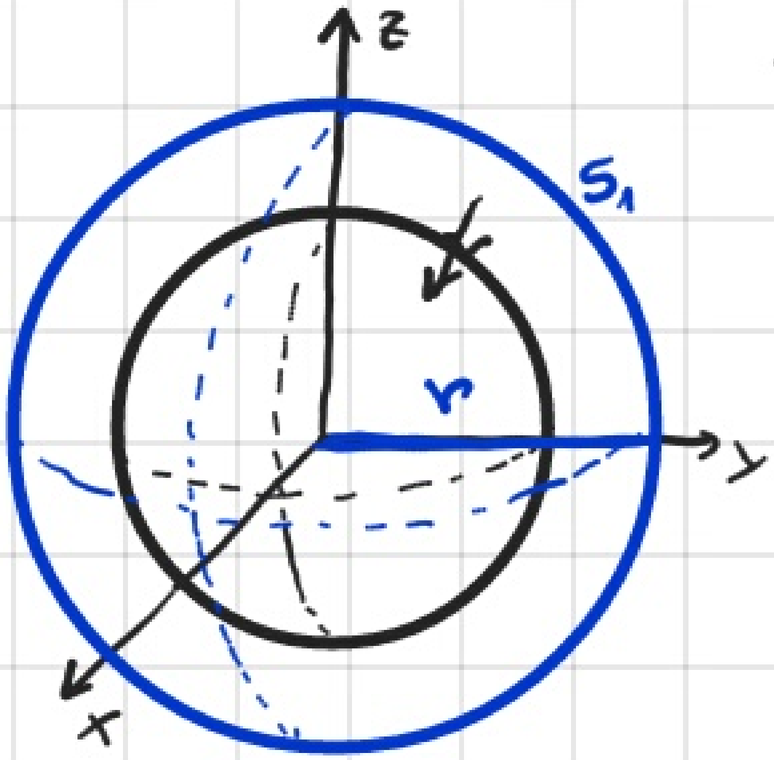
$$Q_T = \rho_0 \int_0^R \left(1 - \frac{r^2}{R^2}\right) r^2 dr \left(2\right)\left(2\pi\right)$$

$$Q_T = 4\pi \rho_0 \int_0^R \left(r^2 - \frac{r^4}{R^2}\right) dr$$

$$Q_T = 4\pi\rho_0 \left(\frac{r^3}{3}\Big|_0^R - \frac{r^5}{5} \cdot \frac{1}{R^2}\Big|_0^R \right)$$

$$Q_T = 4\pi\rho_0 \left(\frac{R^3}{3} - \frac{R^3}{5}\right)$$

$$\boxed{Q_T = \frac{8\pi\rho_0}{15} R^3}$$ ✓



- Sea la region $r \in [R, \infty[$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{enc}}{\varepsilon_0}$$

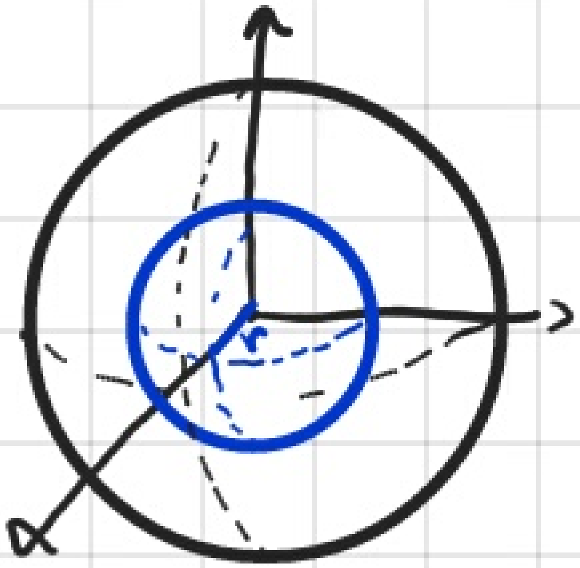
$$\int_{S_1} E\hat{r} \cdot r^2 \operatorname{sen}\theta \, d\phi \, d\theta \, \hat{r} = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Q_T}{\varepsilon_0}$$

$$E = \frac{1}{4\pi r^2 \varepsilon_0} \cdot \frac{8\pi \rho_0 R^3}{15}$$

$$E = \frac{2\rho_0}{15\varepsilon_0} \frac{R^3}{r^2} \rightsquigarrow \vec{E}(r) = \frac{2\rho_0}{15\varepsilon_0} \frac{R^3}{r^2} \hat{r}$$

$r \in [R, \infty[$



- Sea la region $r \in [0, R]$

$$\oint \vec{E}(r) \cdot d\vec{s} = \frac{q_{enc}}{\varepsilon_0} \quad \Bigg/ \quad \begin{array}{l} \vec{E}(\vec{r}) = E\hat{r} \\ d\vec{s} = r^2 \operatorname{sen}\theta \, d\phi \, d\theta \, \hat{r} \end{array}$$

$$E \, 4\pi r^2 = \frac{1}{\varepsilon_0} \int_V \rho \, dV$$

$$E = \frac{1}{4\pi r^2 \varepsilon_0} \int_0^r \int_0^\pi \int_0^{2\pi} \rho_0 \left(1 - \frac{r^2}{R^2}\right) r^2 \operatorname{sen}\theta \, dr \, d\phi \, d\theta$$

$$E = \frac{1}{\cancel{4\pi} r^2 \varepsilon_0} \cdot \cancel{4\pi} \int_0^r \rho_0 \left(1 - \frac{r^2}{R^2}\right) r^2 \, dr$$

$$= \frac{\rho_0}{r^2 \varepsilon_0} \left(\frac{r^3}{3} - \frac{r^5}{5R^2} \right)$$

$$= \frac{\rho_0}{\varepsilon_0} \left(\frac{r}{3} - \frac{r^3}{5R^2} \right)$$

$$\Rightarrow \quad \vec{E}(r) = \frac{\rho_0}{\varepsilon_0} \left(\frac{r}{3} - \frac{r^3}{5R^2} \right) \hat{r} \qquad r \in [0, R]$$

$V(r)$:

- $r \in [R, \infty[$; $V(r \to \infty) \stackrel{!}{=} 0$

$$V_r = -\int_{\infty}^{r} \vec{E}(r) \cdot d\vec{r} + \cancel{V(r\to\infty)}^{\to 0}$$

$$= -\int_{\infty}^{r} \frac{2\rho_0}{15\varepsilon_0}\frac{R^3}{r^2}\hat{r} \cdot dr\,\hat{r}$$

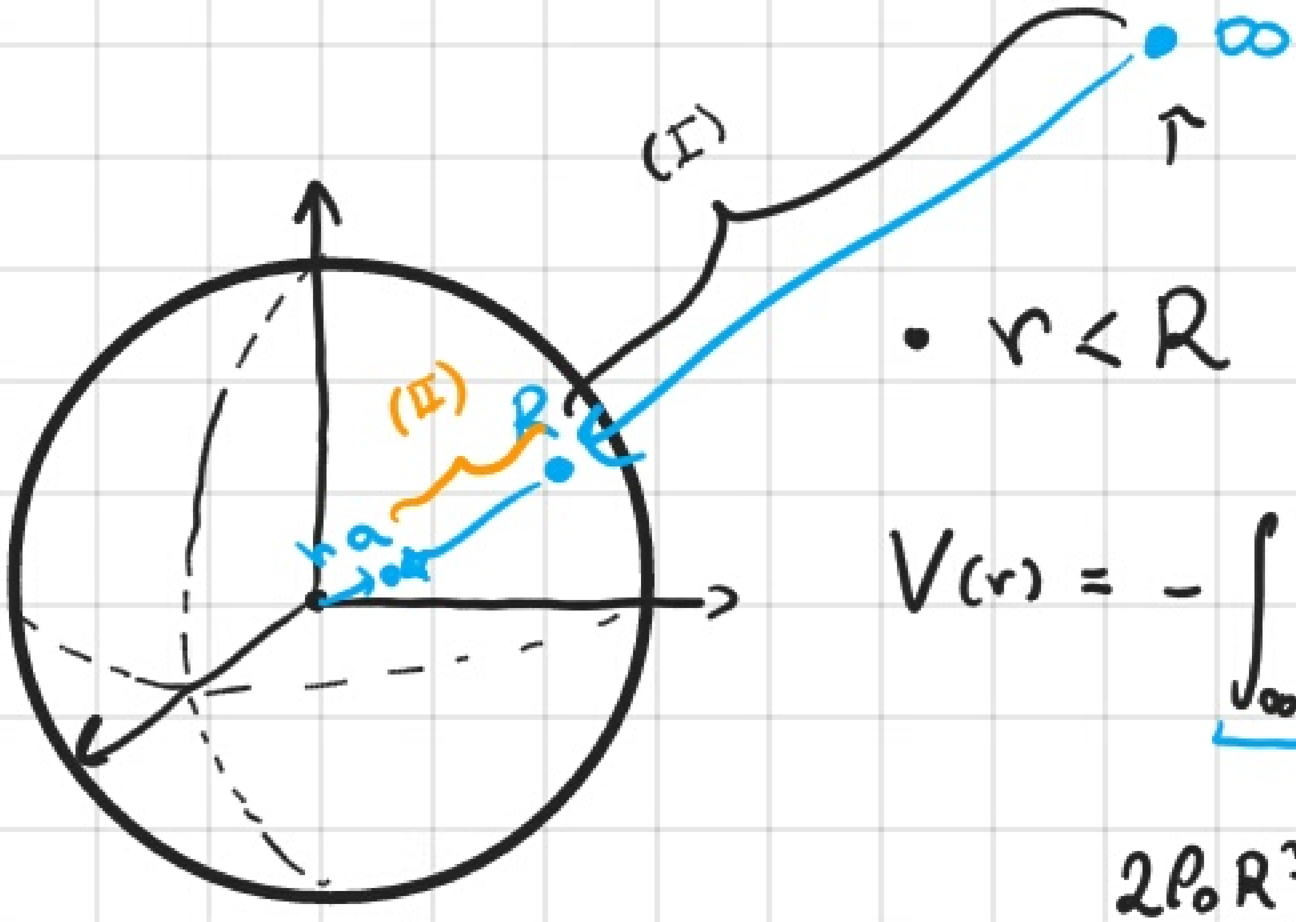
$$= -\frac{2\rho_0 R^3}{15\varepsilon_0}\int_{\infty}^{r} \frac{1}{r^2}\,dr$$

$$= -\frac{2\rho_0 R^3}{15\varepsilon_0}\left(-\frac{1}{r}\right)\Big|_{\infty}^{r}$$

Nota : $\lim_{r\to\infty} \frac{1}{r} = 0$

$$V(r) = \frac{2\rho_0 R^3}{15\varepsilon_0 r}$$

$$\boxed{\begin{aligned} V_a - V_b &= \int_a^b \vec{E} \cdot d\vec{r} \\ V_a &= -\int_{ref}^{a} \vec{E} \cdot d\vec{r} + V_{ref} \end{aligned}}$$



• $r < R$

$$V(r) = \underbrace{-\int_{\infty}^{R} \vec{E}(r \geq R)\cdot d\vec{r}}_{(I)} \underbrace{- \int_{R}^{r} \vec{E}(r<R)\cdot d\vec{r}}_{(II)}$$

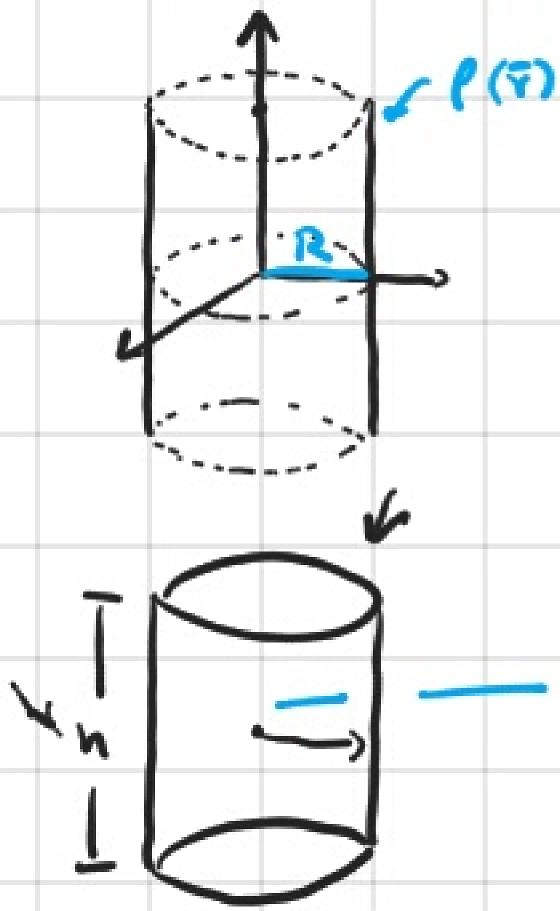
$$\frac{2\rho_0 R^3}{15\varepsilon_0 r} - \int_R^r \frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3} - \frac{r^3}{5R^2}\right) dr$$

$$\frac{2\rho_0 R^3}{15\varepsilon_0 r} - \frac{\rho_0}{\varepsilon_0}\left(\frac{1}{6}\cdot r^2\Big|_R^r - \frac{1}{20R^2}\cdot r^4\Big|_R^r\right)$$

$$V(r) = \frac{2\rho_0 R^3}{15\varepsilon_0 r} - \frac{\rho_0}{\varepsilon_0}\left(\frac{1}{6}(r^2 - R^2) - \frac{1}{20R^2}(r^4 - R^4)\right)$$

(3) $\bar{r} \in [0,R]$ (a)

$(r \leq R)$



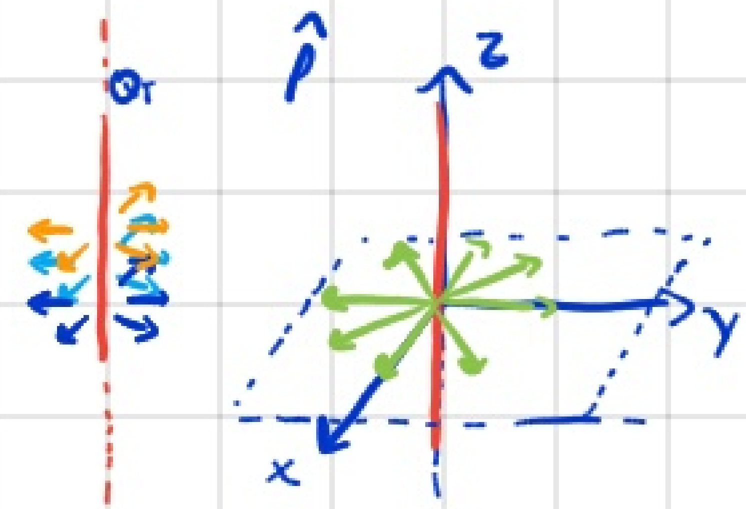
$$Q_T = \int_V \rho \, dV$$

$$Q_T = \int_0^R \int_0^{2\pi} \int_0^h (ra) r \, dr \, d\phi \, dz$$

$$= a \, 2\pi h \int_0^R r^2 \, dr$$

$$= a \frac{2\pi}{3} R^3 h$$

$\vec{E}(r) = E(r)\hat{\rho}$



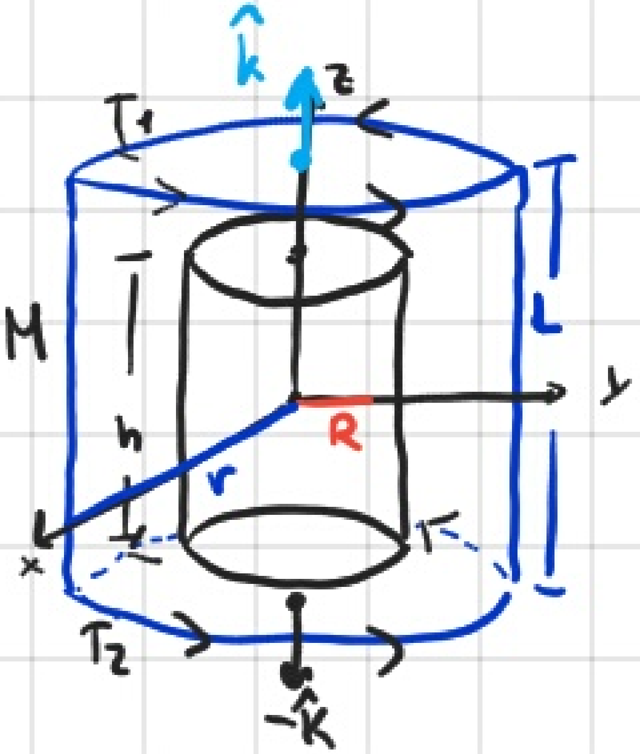
$Q_T : r \geq R$

$$Q_T = \int_V \rho \, dV$$

$$= \int_V (0) \, dV$$

$$= 0$$

(b)



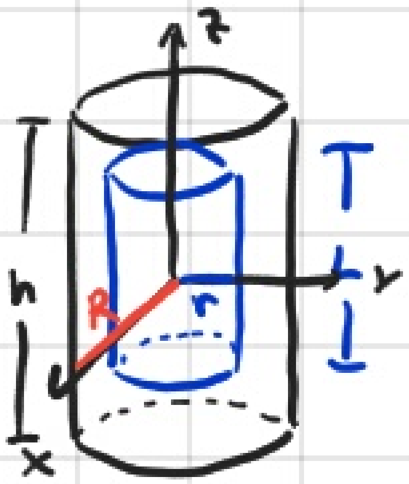
$$\oint \vec{E}(r)\cdot d\vec{s} = \frac{q_{en}}{\varepsilon_0}$$

$$\int_{T1} \vec{E}(r)\cdot d\vec{s}_{T_1} + \int_{M} \vec{E}(r)\cdot d\vec{s}_{TM} + \int_{T2} \vec{E}(r)\cdot d\vec{s}_{T2}$$

$\vec{E}(r) = E(r)\hat{\rho}$

$$\int_{T_1} \cancelto{0}{E(r)\hat{\rho}\cdot dS_{T_1}\hat{k}} + \int_{M} E(r)\hat{\rho}\cdot dS_M\hat{\rho} - \int_{T_2} \cancelto{0}{E(r)\hat{\rho}\cdot ds_{T_2}\hat{k}}$$

$$\int_{M} E(r)\, ds_M = \frac{q_{enc}}{\varepsilon_0}$$



$\vec{E}(r) \sim \vec{E}(r<R)$, $\vec{E}(r>R)$ Tarea

$V \sim$!!