1) $\boxed{f(x,y)} = \begin{cases} \dfrac{3x^2y}{x^2-xy+y^2} & (x,y) = (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ f:

2) $\forall (x,y) \neq (0,0)$:

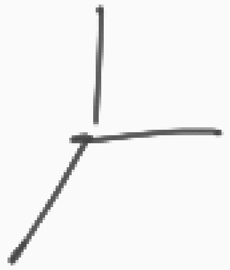
$$0 \leq \left| f(x,y) - 0 \right| = \left| \frac{3x^2y}{x^2-xy+y^2} \right| = \frac{3x^2|y|}{|(x^2-xy+\frac{1}{4}y^2)+\frac{3}{4}y^2|} = \frac{3x^2|y|}{(x-\frac{1}{2}y)^2+\frac{3}{4}y^2}$$

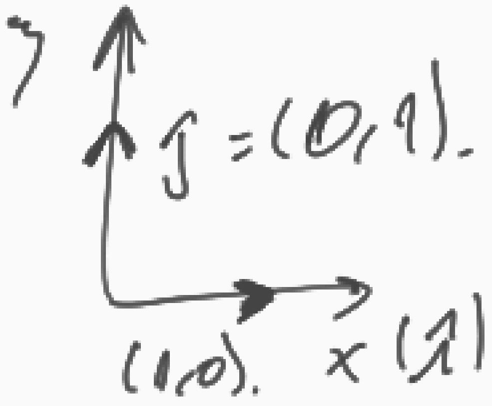
$$\leq 3x^2y. \xrightarrow{(x,y)\to(0,0)} 0$$

por teorema del sandwich, el limite es 0 y f es conti. en (0,0).

f es composición de funciones continuas en $\mathbb{R}^2$, luego f es continua en $\mathbb{R}^2 \setminus \{(0,0)\}$

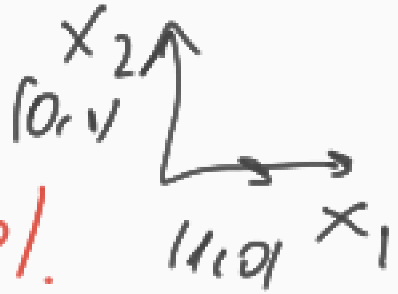
- $|x_i| \le \|\vec{x}\|$ $\forall \vec{x} \in \mathbb{R}^n$.
- $|\cos(\vec{x})| \le 1$
- $|\sin(\vec{x})| \le 1$
- $|\sin x| \le |x|$. $z \to k = (0,0,1)$





$$\frac{xy}{x^2+y^2} \le \frac{\|(x,y)\| \cdot \|(x,y)\|}{\|(x,y)\|^2} = \frac{\cancel{\|(x,y)\|^2}}{\cancel{\|(x,y)\|^2}} = 1$$

x_1, x_2



dirección canónica $\hat{\imath} = (1,0)$.

$x \to \hat{\imath} = (1,0,0)$ . . . $y \to \hat{j} = (0,1,0)$.

2) $$\frac{\partial f}{\partial x}(0,0) = \lim_{h \to 0} \frac{f((0,0)+h(1,0)) - \cancel{f(0,0)}^{\,0}}{h} = \lim_{h \to 0} \frac{f(h,0)}{h}$$

$x \to (1,0) = e_1$ $$= \lim_{h \to 0} \frac{0}{h} = 0.$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{h \to 0} \frac{f((0,0)+h(0,1)) - \cancel{f(0,0)}^{\,0}}{h} = \lim_{h \to 0} \frac{f(0,h)}{h}$$

$$= \lim_{h \to 0} \frac{0}{h} = 0. \; ; \; \nabla f(0,0) = \left(\frac{\partial f}{\partial x}(0,0), \frac{\partial f}{\partial y}(0,0)\right) = (0,0)$$

3) $\frac{\partial f}{\partial x}(x,y) = \frac{6xy(x^2-xy+y^2) - 3x^2y(2x-y)}{(x^2-xy+y^2)^2}$

$\frac{\partial f}{\partial y}(x,y) = \frac{3x^2(x^2-xy+y^2)^2 - 3x^2y(-x+2y)}{(x^2-xy+y^2)^2}$

4) $\frac{\partial f}{\partial \hat{u}}(0,0)$, $\hat{u} \in \mathbb{R}^2$, $\hat{u} = (a,b) \in \mathbb{R}^2$, $\|\hat{u}\| = 1$.

$\frac{\partial f}{\partial \hat{u}}(0,0) = \lim_{h \to 0} \frac{f((0,0)+h\hat{u}) - f(0,0)}{h} = \lim_{h \to 0} \frac{f(ha,hb)}{h}$

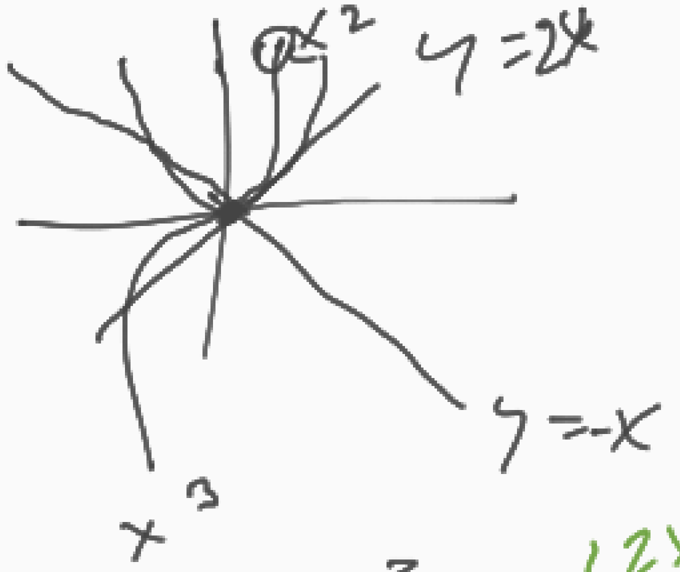
$= \lim_{h \to 0} \left[\frac{\frac{3a^2bh^3}{a^2h^2 - abh^2 + b^2h^2}}{h} \right] = \lim_{h \to 0} \frac{3a^2b\cancel{h^3}}{\cancel{h^3}(a^2-ab+b^2)}$

$= \frac{3a^2b}{a^2-ab+b^2}$.

$\frac{3a^2bh^3}{a^2h^2}$

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{y-x^2}.$$

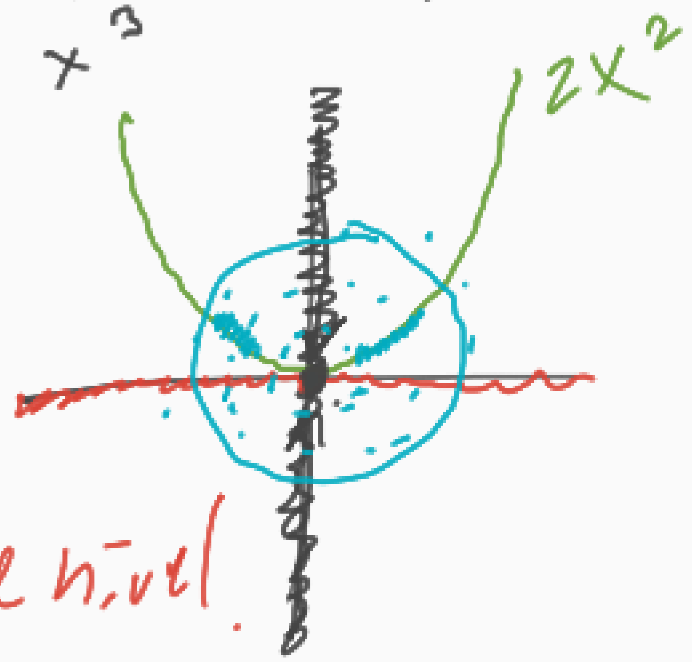
$y = mx^2.$



Sean $T_1 = \{(x,y) : x = 0, y \in \mathbb{R}\}.$
$(0,0) \in T_1'$. $T_2 = \{(x,y)\ y = 0,\ x \in \mathbb{R}\}.$
$y\ (0,0) \in T_2'$ y $(0,0) \in \underline{T_1' \cap T_2'}$.

$$\lim_{\substack{(x,y)\to(0,0)\\(x,y)\in T_1}} f(x,y) = \lim_{y\to 0} \frac{0}{y-0} = 0.$$

$$\lim_{\substack{(x,y)\to(0,0)\\(x,y)\in T_2}} f(x,y) = \lim_{x\to 0} \frac{x^2}{0-x^2} = -1.$$



Curvas de nivel.

$f(x,y) = k = 1$

$\frac{x^2}{y-x^2} = 1 \Leftrightarrow x^2 = y - x^2.$

$\Leftrightarrow \boxed{y = 2x^2}.$

$\therefore \underset{\sim}{\lim} f(x,y)$ no existe.

2) $f(x,y) = \begin{cases} \dfrac{x^3+y^3}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

$|x_i| \leq \|\bar{x}\|.$

$(1,2)$

$1 \leq \sqrt{5}$ $2 \leq \sqrt{5}.$

en $(1,1)$ f es continua por ser comp. de fun. continuas.

$\forall (x,y) \neq (0,0)$:

$$0 \leq |f(x,y) - 0| = \left| \frac{x^3+y^3}{x^2+y^2} \right| = \frac{|x^3+y^3|}{x^2+y^2} \leq \frac{\boxed{|x|^3+|y|^3}}{x^2+y^2}$$

$$\leq \frac{2\|(x,y)\|^{\cancel{3}}}{\cancel{\|(x,y)\|^2}} \xrightarrow{(x,y) \to (0,0)} 0.$$

$$\leq \frac{|x| \cdot |x| \cdot |x| + |y|^3}{\|(x,y)\|^2}$$

$$\frac{\|(x,y)\|^{\cancel{3}} + \|(x,y)\|^{\cancel{3}}}{\cancel{\|(x,y)\|^2}} = 0$$

$\sqrt{x^2+y^2} = \|(x,y)\|^2$

Diferenciabilidad

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h \to 0} \frac{f((0,0)+h(1,0)) - \overset{0}{\cancel{f(0,0)}}}{h} = \lim_{h \to 0} \frac{f(h,0)}{h} = \lim_{h \to 0} \frac{\dfrac{\cancel{h^3} - 0^3}{\cancel{h^2} - 0^2}}{\cancel{h}}$$

$$= 1.$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{h \to 0} \frac{f((0,0)+h(0,1)) - \overset{0}{\cancel{f(0,0)}}}{h} = \lim_{h \to 0} \frac{f(0,h)}{h} = \lim_{h \to 0} \frac{\dfrac{h^3}{h^2}}{h} = 1$$

$$\therefore \nabla f(0,0) = \left(\frac{\partial f}{\partial x}(0,0), \frac{\partial f}{\partial y}(0,0) \right) = (1,1).$$

$$\boxed{\lim_{(h,k)\to(0,0)} \frac{f((h,k)-(0,0)) - \nabla f(0,0)\cdot(h,k) - \cancel{f(0,0)}^{0}}{\|(h,k)\|} = 0.}$$

condición de dif.

$$\lim_{(h,k)\to(0,0)} \frac{f(h,k) - (1,1)\cdot(h,k)}{\sqrt{h^2+k^2}} = \lim_{(h,k)\to(0,0)} \frac{\frac{h^3+k^3}{h^2+k^2} - h - k}{\sqrt{h^2+k^2}}$$

$$= \lim_{(h,k)\to(0,0)} \frac{\cancel{h^3}+\cancel{k^3}\cancel{-h^3} - hk^2 - kh^2 \cancel{-k^3}}{(h^2+k^2)\sqrt{h^2+k^2}} = \lim_{(h,k)\to(0,0)} \frac{-hk^2-kh^2}{(h^2+k^2)\sqrt{h^2+k^2}}$$

$h=k$

$$= \lim_{k\to 0} \frac{-k^3-k^3}{2k^2\sqrt{2k^2}} = \lim_{k\to 0} \frac{-\cancel{2k^3}}{\cancel{2k^2}\cdot\sqrt{2}\,|k|} = \lim_{k\to 0} \frac{-k}{\sqrt{2}\,|k|} \begin{matrix} \nearrow 1 \\ \searrow -1 \end{matrix}$$

$$= \begin{cases} 1, & k\to 0^- \\ -1, & k\to 0^+ \end{cases}$$

$\therefore$ f no es diferenciable en $(0,0)$.

$f \in C^1(A)$.

$f \in C^1 \Rightarrow f$ es dif.

Si f es dif. $\Rightarrow \dfrac{\partial f}{\partial \hat{u}}(x_0, y_0) = \nabla f(x_0, y_0) \cdot \hat{u}$

la dirección de máximo crecimiento es en la dirección del $\dfrac{\nabla f(x_0, y_0)}{\|\nabla f(x_0, y_0)\|}$.