PROBLEMA 1

$$v \in X \Rightarrow x_k = \frac{v}{k} \to 0, \quad k\left(\frac{v}{k} - 0\right) = v \Rightarrow v \in T(X;0)$$

$$\Rightarrow \exists \lambda_k > 0 \ \exists x_k \to 0, \ x_k \in X \text{ tq}$$

$\lambda_k x_k = \lambda_k (x_k - 0) \to v$. Como $\lambda_k x_k \in X$,

se tiene $v \in \overline{X}$

$$\therefore \quad X \subseteq T(X;0) \subseteq \overline{X}$$

PROBLEMA 2

$$h(x) = \begin{cases} 4x^3 - 3x^4 & x \geq 0 \\ 4x^3 + 3x^4 & x < 0 \end{cases}$$

$$h'(x) = \begin{cases} 12x^2 - 12x^3 & \text{si } x > 0 \\ 12x^2 + 12x^3 & \text{si } x \leq 0 \end{cases}$$

$$h''(x) = \begin{cases} 24x - 36x^2 & x > 0 \\ 24x + 36x^2 & x < 0 \end{cases}$$

$x_0 > 0$:
$$x_0 - \frac{12x_0^2(1 - x_0)}{12x_0(2 - 3x_0)} = -x_0$$

$$\Rightarrow x_0 = \frac{3}{5} = 0.6$$

$$x_k = (-1)^k x_0 \quad k \in \mathbb{N}$$

completar con lo visto en clase.

PROBLEMA 3

$x^0 = (10,-5)$

Se resuelve

1° $\min \{ \nabla f(x^0)^T t : \max\{|t_1|,|t_2|\} \le 1 , x^0 + t \in C \}$

$\nabla f(x^0)^T t = 14t_1 - 88t_2$, sujeto a :

$5 \le 10 + t_1 \le 10$ y $-5 \le t_2 - 5 \le 5$ y $\|(t_1,t_2)\|_\infty \le 1$

$\Leftrightarrow -1 \le t_1 \le 0$ y $0 \le t_2 \le 1$.

La solución es $t_1 = -1$, $t_2 = 1$, i.e., $t^0 = (-1,1)$

Paso

$\min \{ f(x^0 + \lambda t^0) : \lambda \geq 0 , x^0 + \lambda t^0 \in C \}$

$x^0 + \lambda t^0 = (10-\lambda, \lambda - 5) \in C$

$\Leftrightarrow 0 \le \lambda \le 5$

$f(x^0 + \lambda t^0) = (10-\lambda)^2 + 4(\lambda-5)^2 - 6(10-\lambda) - 48(\lambda-5) + 100$

$\Rightarrow \lambda^0 = 5$

$\Rightarrow x^1 = x^0 + \lambda^0 t^0 = (5,0)$

$f(5,0) = 95 < 480 = f(10,-5)$

2° $\min \{ \nabla f(x^1)^T t : \|(t_1,t_2)\|_\infty \le 1 , x^1 + t \in C \}$

$\nabla f(x^1)^T t = 4t_1 - 48t_2$, sujeto a :

$5 \le 5 + t_1 \le 10$ y $-5 \le t_2 \le 5$ y $\|(t_1,t_2)\|_\infty \le 1$

$\Leftrightarrow 0 \le t_1 \le 1$ y $-1 \le t_2 \le 1$

La solución es $t_1 = 0$, $t_2 = 1$, i.e., $t^1 = (0,1)$

Paso

$\min \{ f(x^1 + \lambda t^1) : \lambda \geq 0 , x^1 + \lambda t^1 \in C \}$

$x^1 + \lambda t^1 = (5,\lambda) \in C \Leftrightarrow 0 \le \lambda \le 5$

$f(x^1+\lambda t^1) = 5^2 + 4\lambda^2 - 6(5) - 48\lambda + 100$

$\Rightarrow \lambda^1 = 5$

$\Rightarrow x^2 = x^1 + \lambda^1 t^1 = (5,5)$

$f(5,5) = -45$

3° $\nabla f(x^2)^T t = 4t_1 - 8t_2$ sujeto a

$0 \leq t_1 \leq 1$ y $-1 \leq t_2 \leq 0$

Luego $\nabla f(x^2)^T t \geq 0$

En consecuencia x^2 es óptimo.

PROBLEMA 4

$$\min \{ \underbrace{2x_1 + 3x_2^2 + e^{2x_1^2 + x_2^2}}_{= f(x_1, x_2)} : (x_1, x_2) \in \mathbb{R}^2 \}$$

$f_1(x_1,x_2) = 2x_1 + 3x_2^2$, $f_2(x_1,x_2) = e^{2x_1^2+x_2^2}$

f_1 es convexa , f_2 es estrictamente convexa

$\Rightarrow$ $f = f_1 + f_2$ es estrictamente convexa

(x_1,x_2) es óptimo $\Leftrightarrow$ $\nabla f(x_1,x_2) = 0$

$\Leftrightarrow$ $4x_1 \cdot e^{2x_1^2} + 2 = 0$ y $x_2 = 0$.

- Se observa (en clase) $\exists! x_1 \in \mathbb{R}$ tq $e^{2x_1^2} = -\frac{1}{2x_1}$

$\Rightarrow$ $\exists!$ mínimo.

- $\lim_{x_1^2+x_2^2 \to +\infty} f(x_1,x_2) = +\infty$

La exponencial crece más rápido

$\{ \lim_{|x_1| \to +\infty} (2x_1 + e^{2x_1^2+x_2^2}) = +\infty \}$

2.2. Problema

Minimizar $f(x_1, x_2) = 2\,x_1 + e^{2\,{x_1}^2 + {x_2}^2} + 3\,{x_2}^2$ sujeto a $(x_1, x_2) \in \mathbb{R}^2$.

En la solución del problema utilizaremos el método de Fletcher-Reeves calculando $\lambda_k = \inf\{\nabla f(x^k + \lambda d^k)^t d^k = 0)\}$

Inicio:

Dado $x^0 = (1, 0)$, se define $d^0 = -\nabla f(1,0) = \begin{pmatrix} -31{,}5562 \\ 0 \end{pmatrix}$

Iteración General:

<u>Paso 1:</u>

$$\nabla f(x^0 + \lambda d^0) = \begin{pmatrix} 2 - e^{2(31{,}5562\lambda - 1)^2}(126{,}225\lambda - 4) \\ 0 \end{pmatrix}$$

$$\begin{aligned}
\lambda_0 &= \inf\{\nabla f(x^0 + \lambda d^0)^t d^k = 0)\} \\
&= \inf\left\{\begin{pmatrix} 2 - e^{2(31{,}5562\lambda - 1)^2}(126{,}225\lambda - 4) & 0 \end{pmatrix}\begin{pmatrix} -31{,}5562 \\ 0 \end{pmatrix} = 0)\right\} \\
&= \inf\left\{2 - e^{2(31{,}5562\lambda - 1)^2}(126{,}225\lambda - 4) = 0\right\} \text{[redacted]} \\
&= 0{,}04362196784
\end{aligned}$$

[redacted]

Definimos:

$$x^1 = x^0 + \lambda_0 d^0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 0{,}04362\begin{pmatrix} -31{,}5562 \\ 0 \end{pmatrix} = \begin{pmatrix} -0{,}3765 \\ 0 \end{pmatrix}$$

Se calcula

$$\nabla f(x^1) = \begin{pmatrix} -1{,}9480 \cdot 10^{-7} \\ 0 \end{pmatrix}$$

$$\nabla f(x^0) = \begin{pmatrix} 31{,}5562 \\ 0 \end{pmatrix}$$

$$\beta_0 = \frac{||\nabla f(x^1)||^2}{||\nabla f(x^0)||^2} = \frac{(-1{,}9480 \cdot 10^{-7})^2}{31{,}5562^2} = 3{,}8110 \cdot 10^{-17}$$

$$d^1 = -\nabla f(x^1) + \beta_0 d^0 = -\begin{pmatrix} -1{,}9480 \cdot 10^{-7} \\ 0 \end{pmatrix} + 3{,}8110 \cdot 10^{-17}\begin{pmatrix} -31{,}5562 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1{,}9480 \cdot 10^{-7} \\ 0 \end{pmatrix}$$

$\Rightarrow \lambda_1 = 0.1204984354$

$\Rightarrow x^2 = \begin{pmatrix} -0.3765445824 \\ 0 \end{pmatrix}$