

# Mecánica de Fluidos

## Introducción

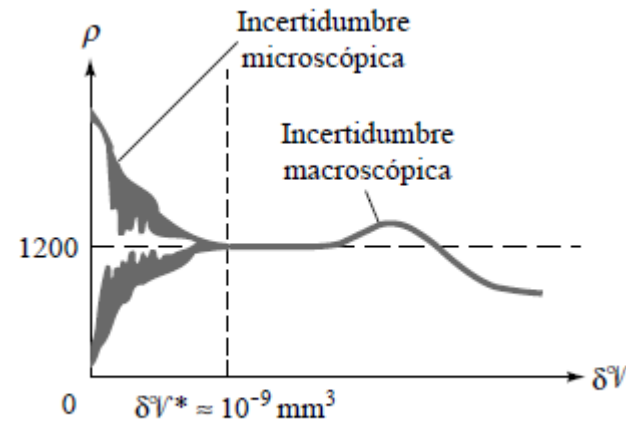
# ¿Qué es un fluido?

- Sustancia que se deforma continuamente ante un esfuerzo de cizalle.
- Líquidos: fuerzas intermoleculares grandes, generalmente incompresible
- Gases: fuerzas intermoleculares pequeñas, compresibles

# Hipótesis del continuo

- Densidad macroscópica ( $\Delta V > 10^{-9} \text{ mm}^3$ )

- Densidad microscópica



- “Las variaciones en las propiedades de un fluido son funciones suficientemente suaves para usar el cálculo diferencial”

# Propiedades de los Fluidos

- Presión  $p$  : esfuerzo de compresión de un fluido estático ( $\text{Pa}=\text{N}/\text{m}^2$ )
- Temperatura  $T$ : relacionada con la energía interna de un fluido ( $^{\circ}\text{K}$ )
- Energía interna  $U$  ( $\text{J}=\text{N}*\text{m}$ ;  $u:=U/\text{m}$ ;) )
- Entalpía  $H$  ( $H:=U+PV$ ;  $h=u+p/\rho$ )
- Entropía  $S$
- Calor específico  $C_{V,P}$  ( $\text{J}/(\text{kg}^{\circ}\text{K})$ ;  $c=C/\text{m}$ )
- Viscosidad  $\mu$ : resistencia a fluir de un fluido ( $\text{kg}/(\text{m}*\text{s})$ )
- Conductividad térmica  $\kappa$  ( $\text{W}/(^{\circ}\text{K}*\text{m})$  )

# Ecuaciones de estado

- Gases:

Ideales

$$p = \rho RT, \quad R = 287 \text{ J} / (\text{kg}^\circ K)$$

Van der Waals

$$\left( p + \frac{n^2 a}{V^2} \right) (V - nb) = nRT$$

- Líquidos

Incompresibles

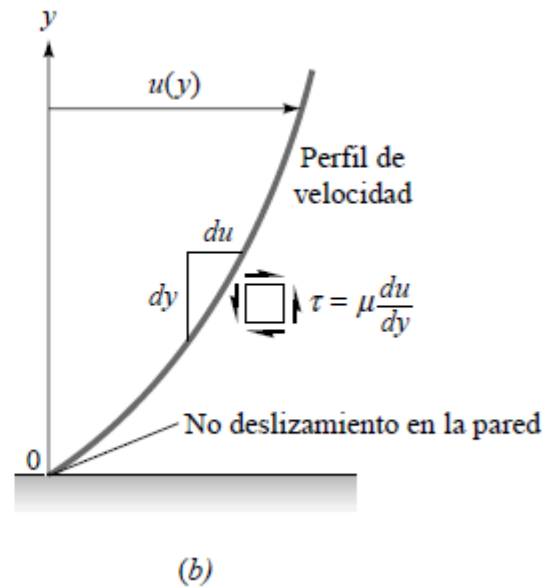
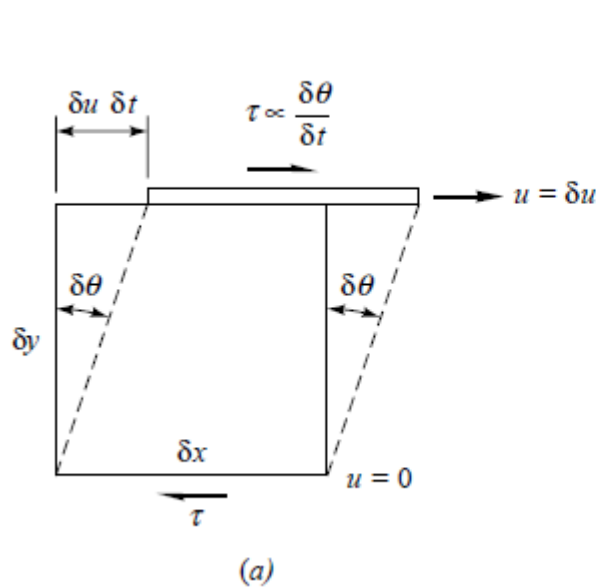
$$\rho = cte, c_p = c_v = cte, dh = c_p dT$$

Compresibles

$$\frac{p}{p_a} = (B + 1) \left( \frac{\rho}{\rho_a} \right)^n - B$$

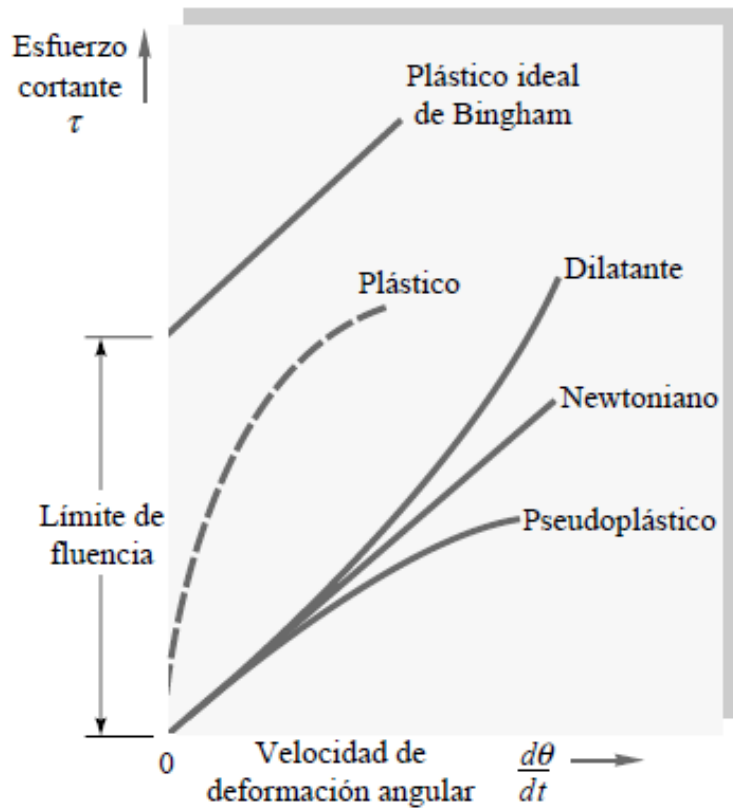
$n, B$ : constantes experimentales

# Tipos de fluidos

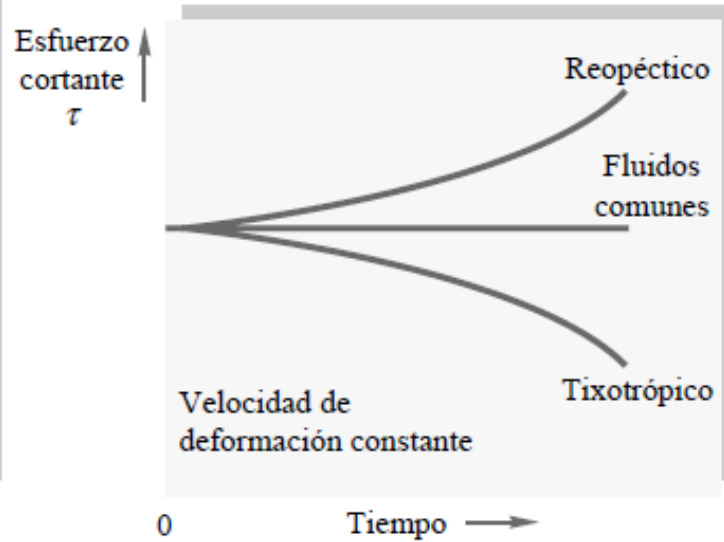


$\tan \delta\theta = \frac{\delta u \delta t}{\delta y}$  en el límite  $\delta t \rightarrow 0$  se obtiene  $\frac{d\theta}{dt} = \frac{du}{dy}$

Se define la velocidad de cizalle o *shear rate*  $\dot{\gamma} = \frac{du}{dy}$



(a)



(b)

$$\tau = \mu \frac{du}{dy}; \text{ Newtoniano}$$

$$\tau = \tau_B + \mu_B \frac{du}{dy} \quad \text{si} \quad \tau > \tau_B$$

Bingham

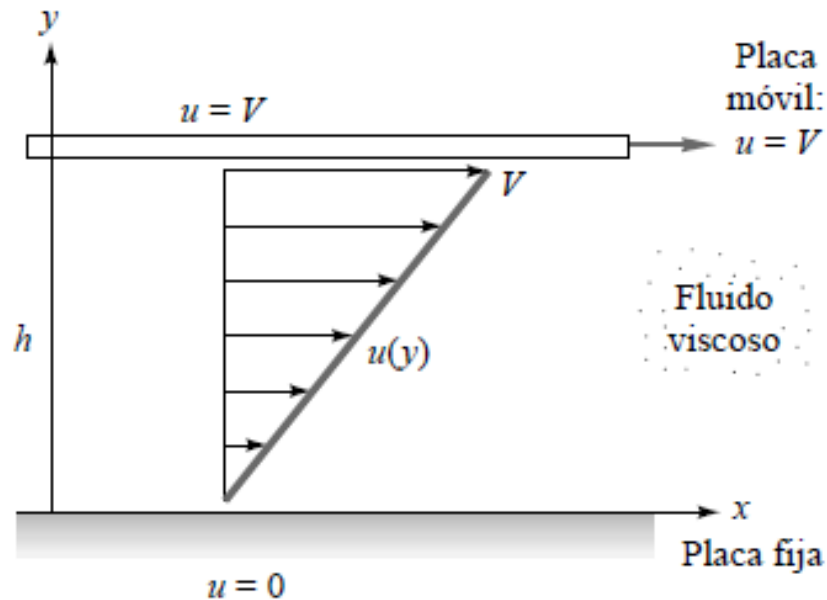
$$\frac{du}{dy} = 0 \quad \text{si} \quad \tau \leq \tau_B$$

$$\tau = m \left( \frac{du}{dy} \right)^n \quad \text{Ley de Potencia}$$

$n > 1$  Dilatante

$n < 1$  Pseudoplástico

# Flujo entre placas paralelas



Si el fluido es Newtoniano  $\left( \mu \frac{du}{dy} = \tau \right)$  y

considerando EE ( $\Rightarrow \tau$  es cte por  $\sum \vec{F}$ )

$$\Rightarrow \frac{du}{dy} = \text{constante}$$

Integrando se llega a:

$$u(y) = a + by$$

Con condiciones de contorno

$$u(y = 0) = 0$$

$$u(y = h) = V$$

$$\Rightarrow u(y) = V \frac{y}{h}$$

**Observación: Esta expresión es válida para Velocidades bajas**



# Unidades

**Tabla 1.1.** Dimensiones primarias en los sistemas SI y británico.

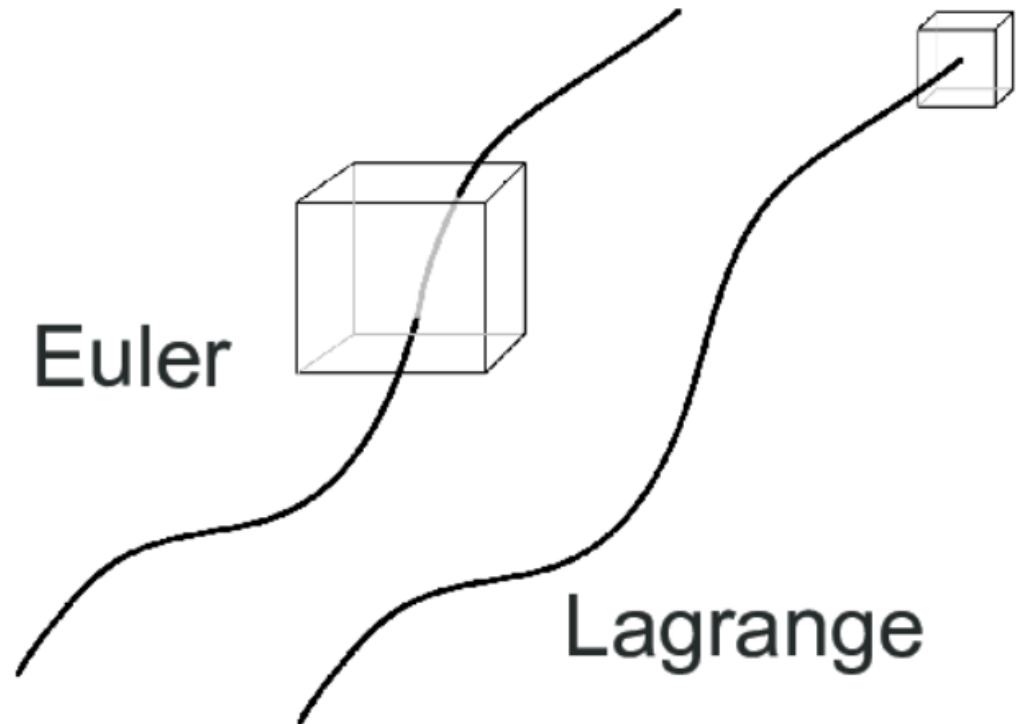
Dimensión primaria	Unidad SI	Unidad británica	Factor de conversión
Masa $\{M\}$	Kilogramo (kg)	Slug	1 slug = 14,5939 kg
Longitud $\{L\}$	Metro (m)	Pie (ft)	1 ft = 0,3048 m
Tiempo $\{T\}$	Segundo (s)	Segundo (s)	1 s = 1 s
Temperatura $\{\Theta\}$	Kelvin (K)	Rankine ( $^{\circ}\text{R}$ )	1 K = 1,8 $^{\circ}\text{R}$

**Tabla 1.2.** Dimensiones secundarias en Mecánica de Fluidos.

Dimensión secundaria	Unidad SI	Unidad británica	Factor de conversión
Área $\{L^2\}$	$\text{m}^2$	$\text{ft}^2$	$1 \text{ m}^2 = 10,764 \text{ ft}^2$
Volumen $\{L^3\}$	$\text{m}^3$	$\text{ft}^3$	$1 \text{ m}^3 = 35,315 \text{ ft}^3$
Velocidad $\{LT^{-1}\}$	$\text{m/s}$	$\text{ft/s}$	$1 \text{ ft/s} = 0,3048 \text{ m/s}$
Aceleración $\{LT^{-2}\}$	$\text{m/s}^2$	$\text{ft/s}^2$	$1 \text{ ft/s}^2 = 0,3048 \text{ m/s}^2$
Presión o esfuerzo $\{ML^{-1}T^{-2}\}$	$\text{Pa} = \text{N/m}^2$	$\text{lbf/ft}^2$	$1 \text{ lbf/ft}^2 = 47,88 \text{ Pa}$
Velocidad angular $\{T^{-1}\}$	$\text{s}^{-1}$	$\text{s}^{-1}$	$1 \text{ s}^{-1} = 1 \text{ s}^{-1}$
Energía, calor, trabajo $\{ML^2T^{-2}\}$	$\text{J} = \text{N} \cdot \text{m}$	$\text{ft} \cdot \text{lbf}$	$1 \text{ ft} \cdot \text{lbf} = 1,3558 \text{ J}$
Potencia $\{ML^2T^{-3}\}$	$\text{W} = \text{J/s}$	$\text{ft} \cdot \text{lbf/s}$	$1 \text{ ft} \cdot \text{lbf/s} = 1,3558 \text{ W}$
Densidad $\{ML^{-3}\}$	$\text{kg/m}^3$	$\text{slugs/ft}^3$	$1 \text{ slug/ft}^3 = 515,4 \text{ kg/m}^3$
Viscosidad $\{ML^{-1}T^{-1}\}$	$\text{kg}/(\text{m} \cdot \text{s})$	$\text{slugs}/(\text{ft} \cdot \text{s})$	$1 \text{ slug}/(\text{ft} \cdot \text{s}) = 47,88 \text{ kg}/(\text{m} \cdot \text{s})$
Calor Específico $\{L^2T^{-2}\Theta^{-1}\}$	$\text{m}^2/(\text{s}^2 \cdot \text{K})$	$\text{ft}^2/(\text{s} \cdot ^{\circ}\text{R})$	$1 \text{ m}^2/(\text{s}^2 \cdot \text{K}) = 5,980 \text{ ft}^2/(\text{s} \cdot ^{\circ}\text{R})$

# Tipo de descripción del movimiento

- **Euleriana:** estudia el movimiento como el flujo de un campo
- **Lagrangeana:** estudia el movimiento como un conjunto de partículas materiales



# Cantidades cinemáticas

$$\vec{r} = \int \vec{V} dt \quad \text{velocidad;} \quad \vec{a} = \frac{d\vec{V}}{dt} \quad \text{aceleración}$$

$$Q = \int (\vec{V} \cdot \hat{n}) dA \quad \text{caudal;} \quad \vec{\Omega} = \frac{1}{2} \nabla \times \vec{V} \quad \text{velocidad angular}$$

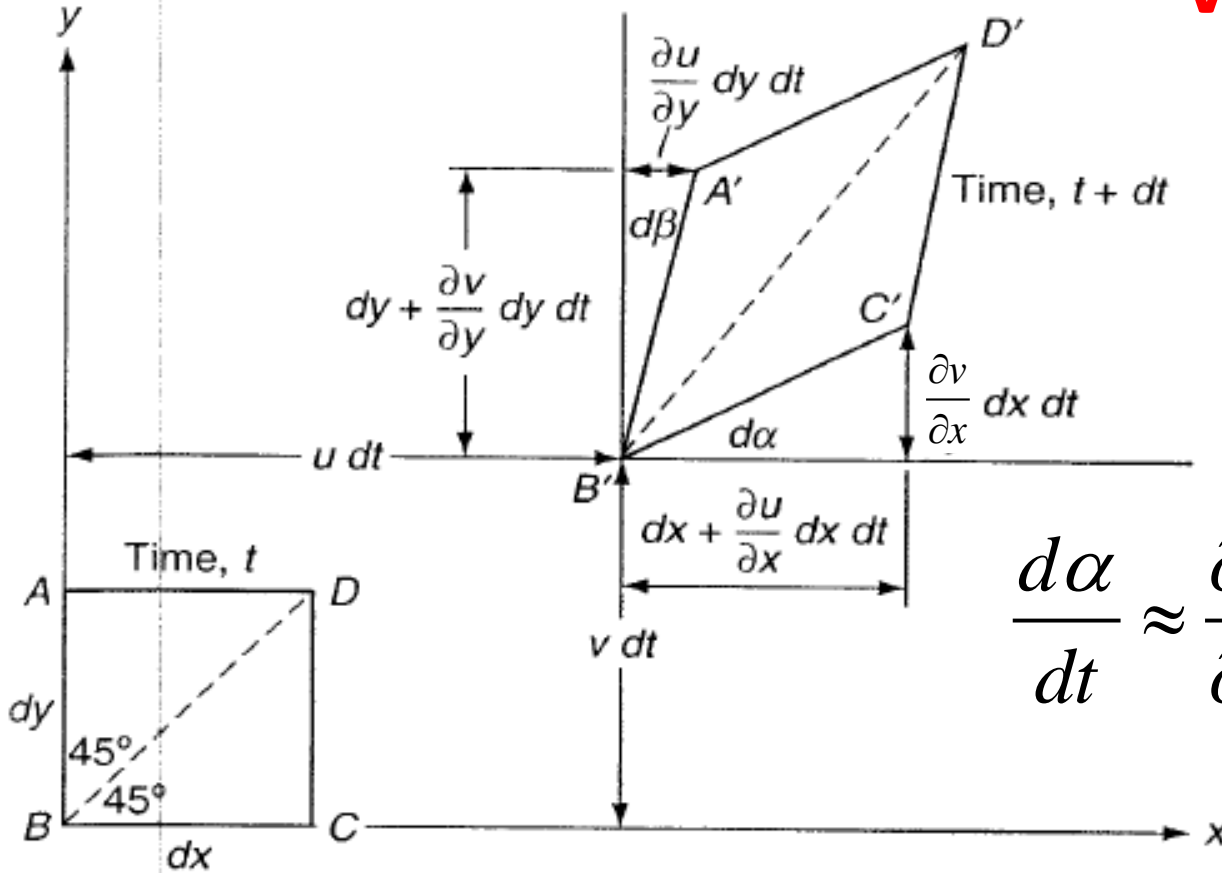
En coordenadas cartesianas:

$$\vec{V}(\vec{r}, t) = u(x, y, z, t) \hat{i} + v(x, y, z, t) \hat{j} + w(x, y, z, t) \hat{k}$$

Observación:

$$\begin{aligned} \frac{d\vec{V}}{dt} = & \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial t} \\ & + \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial v}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial v}{\partial z} \cdot \frac{\partial z}{\partial t} \\ & + \frac{\partial w}{\partial t} + \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial t} \end{aligned}$$

# Velocidad angular y Vorticidad



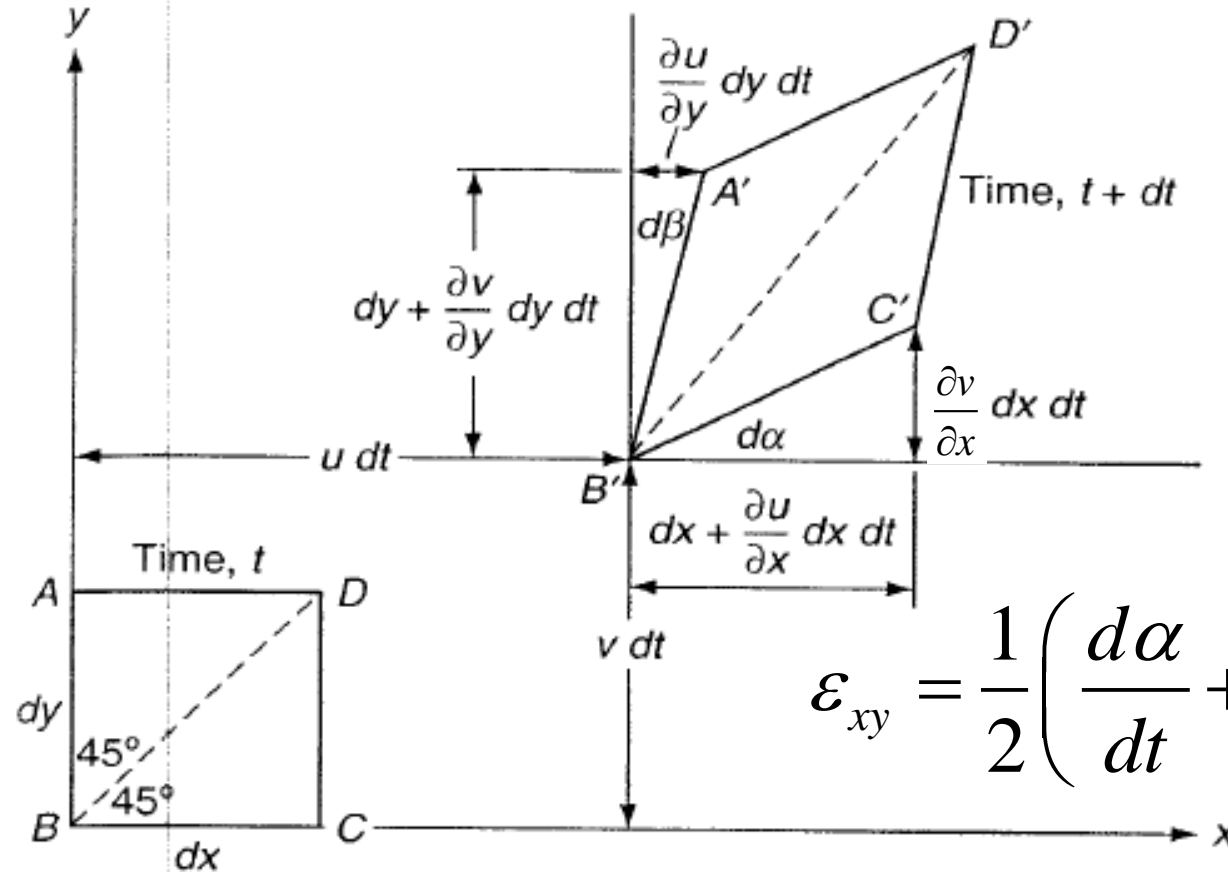
$$\dot{\vec{\Omega}} = \begin{pmatrix} \dot{\Omega}_x \\ \dot{\Omega}_y \\ \dot{\Omega}_z \end{pmatrix} := \frac{\vec{\omega}}{2}$$

$$\frac{d\alpha}{dt} \approx \frac{\partial v}{\partial x} \quad \frac{d\beta}{dt} \approx \frac{\partial u}{\partial y}$$

Tasa de rotación:  $\frac{d\Omega_z}{dt} := \frac{1}{2} \left( \frac{d\alpha}{dt} - \frac{d\beta}{dt} \right) = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$

$$\frac{d\Omega_x}{dt} := \frac{1}{2} \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right); \quad \frac{d\Omega_y}{dt} := \frac{1}{2} \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right)$$

# Tensor Vel. Deformación



Shear strain rate  $\epsilon_{xy}$ :  
tasa reducción de  
ángulo entre  $\perp$

$$\epsilon_{xy} = \frac{1}{2} \left( \frac{d\alpha}{dt} + \frac{d\beta}{dt} \right) \approx \frac{1}{2} \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)$$

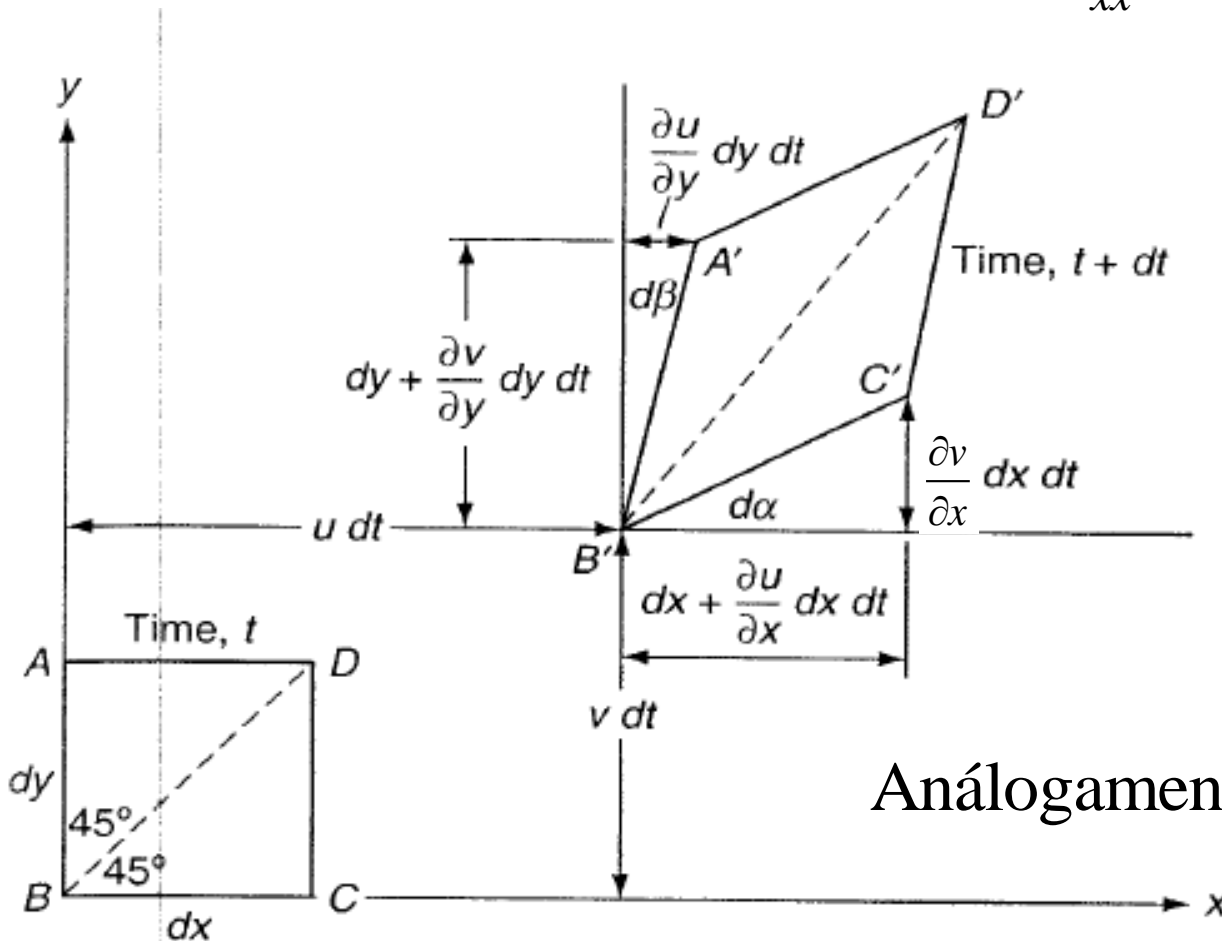
Análogamente:  $\epsilon_{yz} = \frac{1}{2} \left( \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right); \epsilon_{zx} = \frac{1}{2} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)$

# Tensor Vel. Deformación

Extensional strain rate  $\epsilon_{xx}$ :

tasa dilatación en dirección  $x$

$$\epsilon_{xx} dt = \frac{\partial u}{\partial x} dt \Rightarrow \epsilon_{xx} = \frac{\partial u}{\partial x}$$



Análogamente:  $\epsilon_{yy} = \frac{\partial v}{\partial y}$ ;  $\epsilon_{zz} = \frac{\partial w}{\partial z}$

# Tensor Velocidad de Deformación

$$\mathcal{E}_{ij} = \begin{pmatrix} \mathcal{E}_{xx} & \mathcal{E}_{xy} & \mathcal{E}_{xz} \\ \mathcal{E}_{yx} & \mathcal{E}_{yy} & \mathcal{E}_{yz} \\ \mathcal{E}_{zx} & \mathcal{E}_{zy} & \mathcal{E}_{zz} \end{pmatrix}$$

Para un fluido lineal (Newtoniano)  $\tau_{xy} = \mu \frac{V}{h} = 2\mu\mathcal{E}_{xy} = \mu \frac{du}{dy}$

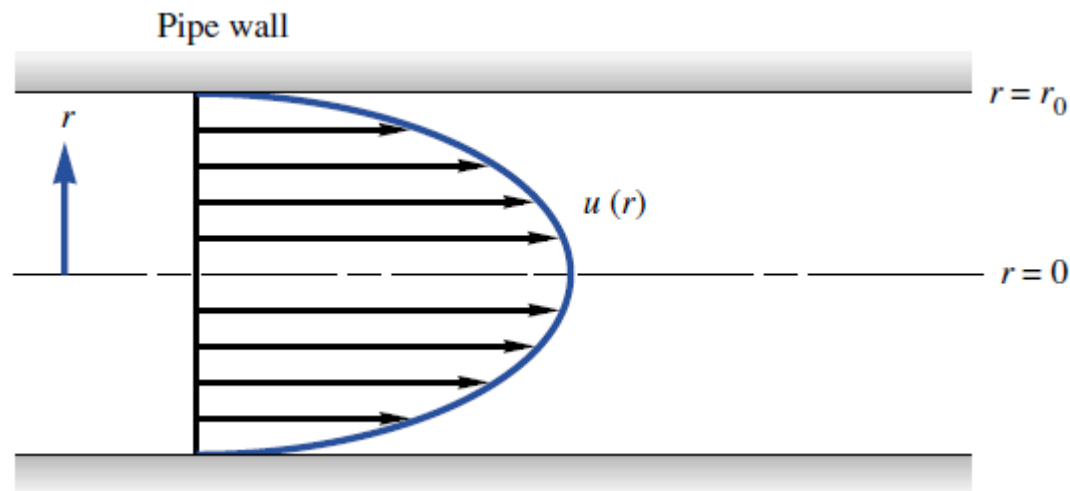
Para un fluido de Potencia  $\tau_{xy} = 2K\mathcal{E}_{xy}^n$

# Problemas

For low-speed (laminar) steady flow through a circular pipe, as shown in Fig. P1.12, the velocity  $u$  varies with radius and takes the form

$$u = B \frac{\Delta p}{\mu} (r_0^2 - r^2)$$

where  $\mu$  is the fluid viscosity and  $\Delta p$  is the pressure drop from entrance to exit. What are the dimensions of the constant  $B$ ?



**Solution:** Using Table 1-2, write this equation in dimensional form:

$$\{u\} = \{B\} \frac{\{\Delta p\}}{\{\mu\}} \{r^2\}, \quad \text{or:} \quad \left\{ \frac{L}{T} \right\} = \{B?\} \frac{\{M/LT^2\}}{\{M/LT\}} \{L^2\} = \{B?\} \left\{ \frac{L^2}{T} \right\},$$

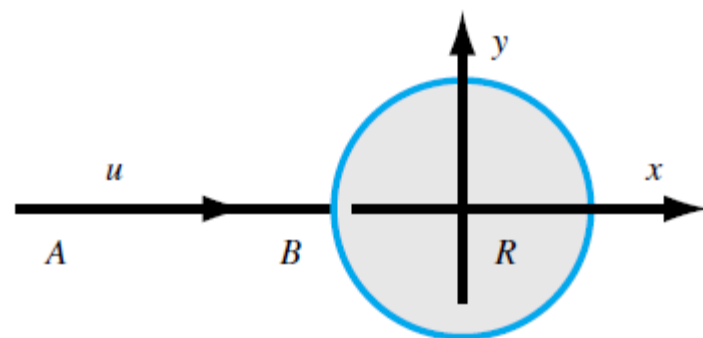
$$\text{or:} \quad \{B\} = \{L^{-1}\} \quad \text{Ans.}$$



According to the theory of Chap. 8, as a uniform stream approaches a cylinder of radius  $R$  along the symmetry line  $AB$  in Fig. P1.22, the velocity has only one component:

$$u = U_{\infty} \left( 1 - \frac{R^2}{x^2} \right) \text{ for } -\infty < x \leq -R$$

where  $U_{\infty}$  is the stream velocity far from the cylinder. Using the concepts from Ex. 1.5, find (a) the maximum flow deceleration along  $AB$  and (b) its location.



**Solution:** We see that  $u$  slows down monotonically from  $U_{\infty}$  at A to zero at point B,  $x = -R$ , which is a flow “stagnation point.” From Example 1.5, the acceleration ( $du/dt$ ) is

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0 + U_{\infty} \left( 1 - \frac{R^2}{x^2} \right) \left[ U_{\infty} \left( + \frac{2R^2}{x^3} \right) \right] = \frac{U_{\infty}^2}{R} \left( \frac{2}{\zeta^3} - \frac{2}{\zeta^5} \right), \quad \zeta = \frac{x}{R}$$

This acceleration is negative, as expected, and reaches a minimum near point B, which is found by differentiating the acceleration with respect to  $x$ :

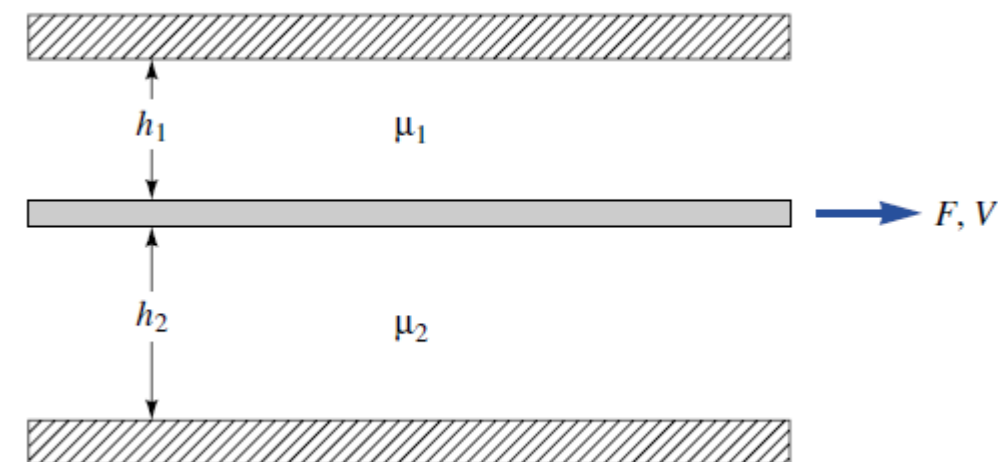
$$\frac{d}{dx} \left( \frac{du}{dt} \right) = 0 \quad \text{if } \zeta^2 = \frac{5}{3}, \quad \text{or } \frac{x}{R} \Big|_{\text{max decel.}} \approx -1.291 \quad \text{Ans. (b)}$$

$$\text{Substituting } \zeta = -1.291 \text{ into } (du/dt) \text{ gives } \frac{du}{dt} \Big|_{\text{min}} = -0.372 \frac{U_{\infty}^2}{R} \quad \text{Ans. (a)}$$

The density of (fresh) water at 1 atm, over the temperature range 0 to 100°C, is given in Table A.1. Fit these values to a least-squares<sup>†</sup> equation of the form  $\rho = a + bT + cT^2$ , with  $T$  in °C, and estimate its accuracy. Use your formula to compute the density of water at 45°C, and compare your result with the accepted experimental value of 990.1 kg/m<sup>3</sup>.

$T, ^\circ\text{C}$	$\rho, \text{kg/m}^3$	$\mu, \text{N} \cdot \text{s/m}^2$	$\nu, \text{m}^2/\text{s}$
0	1000	1.788 E-3	1.788 E-6
10	1000	1.307 E-3	1.307 E-6
20	998	1.003 E-3	1.005 E-6
30	996	0.799 E-3	0.802 E-6
40	992	0.657 E-3	0.662 E-6
50	988	0.548 E-3	0.555 E-6
60	983	0.467 E-3	0.475 E-6
70	978	0.405 E-3	0.414 E-6
80	972	0.355 E-3	0.365 E-6
90	965	0.316 E-3	0.327 E-6
100	958	0.283 E-3	0.295 E-6

A thin plate is separated from two fixed plates by very viscous liquids  $\mu_1$  and  $\mu_2$ , respectively, as in Fig. P1.48. The plate spacings  $h_1$  and  $h_2$  are unequal, as shown. The contact area is  $A$  between the center plate and each fluid. (a) Assuming a linear velocity distribution in each fluid, derive the force  $F$  required to pull the plate at velocity  $V$ . (b) Is there a necessary *relation* between the two viscosities,  $\mu_1$  and  $\mu_2$ ?



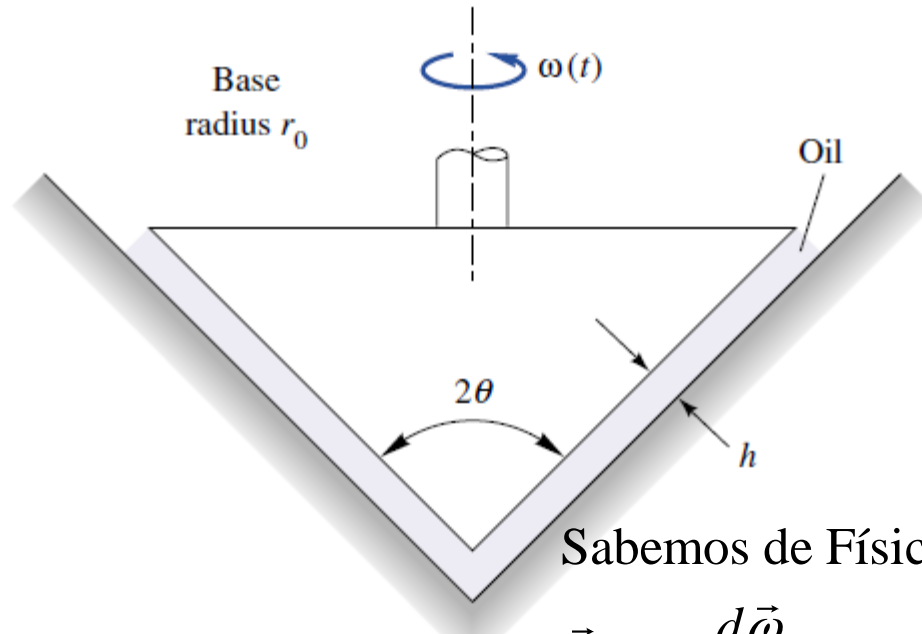
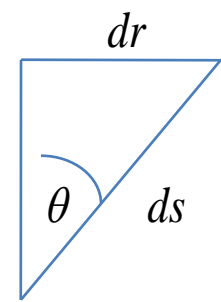
**Solution:** (a) Assuming a linear velocity distribution on each side of the plate, we obtain

$$F = \tau_1 A + \tau_2 A = \left( \frac{\mu_1 V}{h_1} + \frac{\mu_2 V}{h_2} \right) A \quad \text{Ans. (a)}$$

The formula is of course valid only for laminar (nonturbulent) steady viscous flow.

(b) Since the center plate separates the two fluids, they may have separate, unrelated shear stresses, and there is no necessary relation between the two viscosities.

A solid cone of angle  $2\theta$ , base  $r_0$ , and density  $\rho_c$  is rotating with initial angular velocity  $\omega_0$  inside a conical seat, as shown in Fig. P1.53. The clearance  $h$  is filled with oil of viscosity  $\mu$ . Neglecting air drag, derive an analytical expression for the cone's angular velocity  $\omega(t)$  if there is no applied torque.



$$\begin{aligned}\vec{T} &= \int \vec{r} \times \vec{F} = \int r \underbrace{\left( \mu \frac{\omega r}{h} \right)}_{\tau_w} dA_w \\ &= \iint r \left( \mu \frac{\omega r}{h} \right) (r d\phi ds) \\ &= \int_{r=0}^{r_0} \int_{\phi=0}^{2\pi} r \left( \mu \frac{\omega r}{h} \right) \left( r d\phi \frac{dr}{\sin \theta} \right) = \frac{\pi \mu \omega r_0^4}{2h \sin \theta}\end{aligned}$$

Sabemos de Física II (o Mecánica) que:

$$\vec{T} = -I \frac{d\vec{\omega}}{dt} \quad (\text{signo - xq el torque va en contra del movimiento})$$

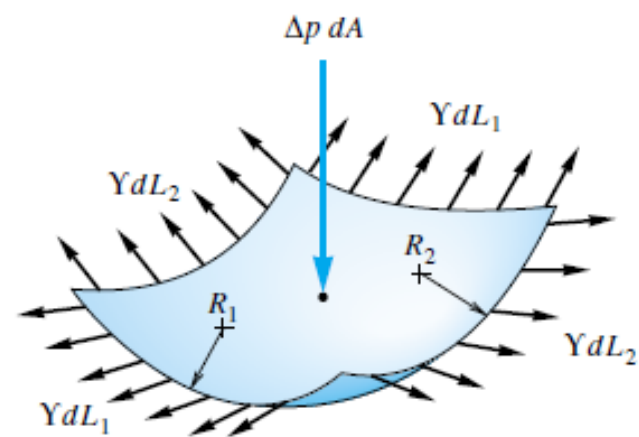
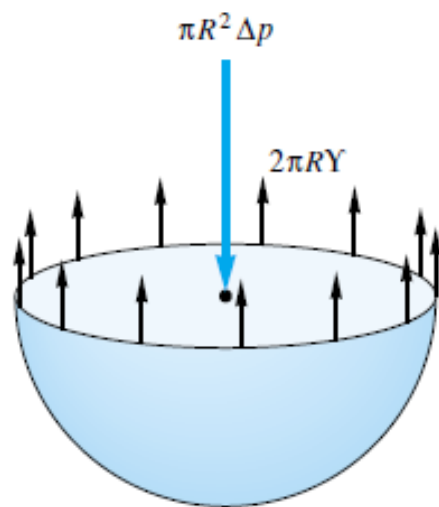
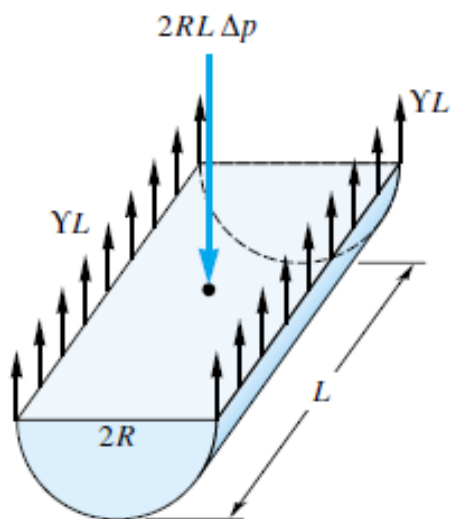
$$\text{y que } I = \frac{3}{10} m r_0^2 \Rightarrow \frac{3}{10} m r_0^2 \frac{d\omega}{dt} = - \frac{\pi \mu \omega r_0^4}{2h \sin \theta}$$

$$\Rightarrow \frac{d\omega}{\omega} = - \frac{5\pi \mu r_0^4}{3m r_0^2 h \sin \theta} dt \Rightarrow \omega(t) = \omega_0 \exp \left( - \frac{5\pi \mu r_0^2}{3m h \sin \theta} t \right)$$

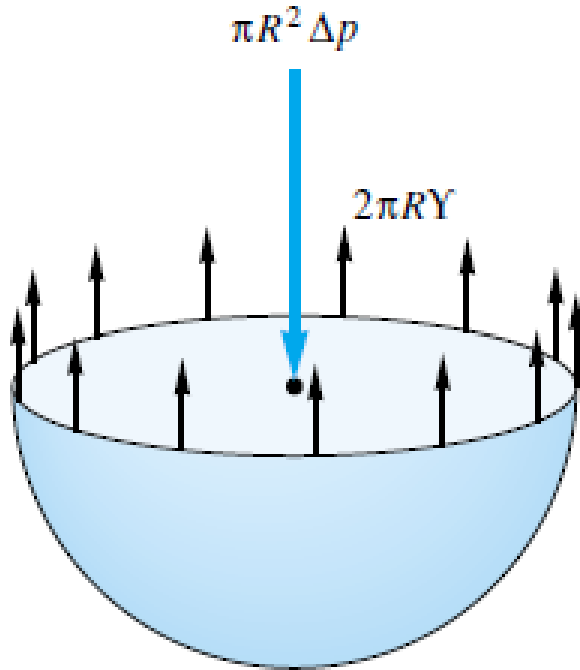
# Propiedades superficiales

- Un líquido cualquiera en contacto con un segundo líquido o gas forma una interface. Las partículas en la superficie tienen menos “vecinos” que las partículas del interior. Esto crea un efecto mecánico conocido como tensión superficial  $\gamma$ . Otra forma de definirla es la energía necesaria para estirar una superficie por unidad de superficie. Si un corte  $dL$  es hecho en una superficie interfacial entonces la fuerza que hay que vencer está dada por:

$$F = \gamma dL$$



# Ejemplo: Presión en una gota



$$\pi R^2 \Delta P = 2\pi R \Upsilon$$

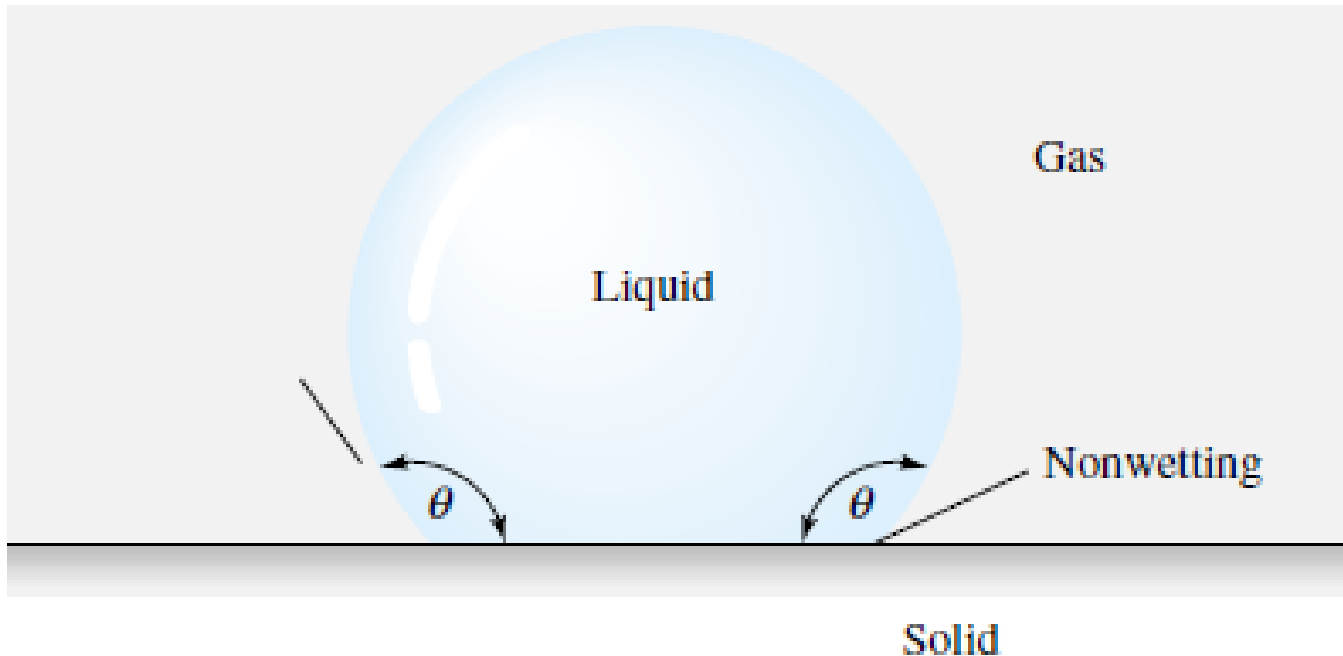
$$\Rightarrow \Delta P = \frac{2\Upsilon}{R}$$

# Ejemplo: Presión en una burbuja

$$\pi R^2 \Delta P = 2\pi R_{ext} \Upsilon + 2\pi R_{int} \Upsilon$$

$$\Rightarrow \Delta P = \frac{4\Upsilon}{R}$$

- Un segundo efecto superficial es el ángulo de contacto que aparece cuando una interface líquida (o líquida-gas) intercepta un sólido

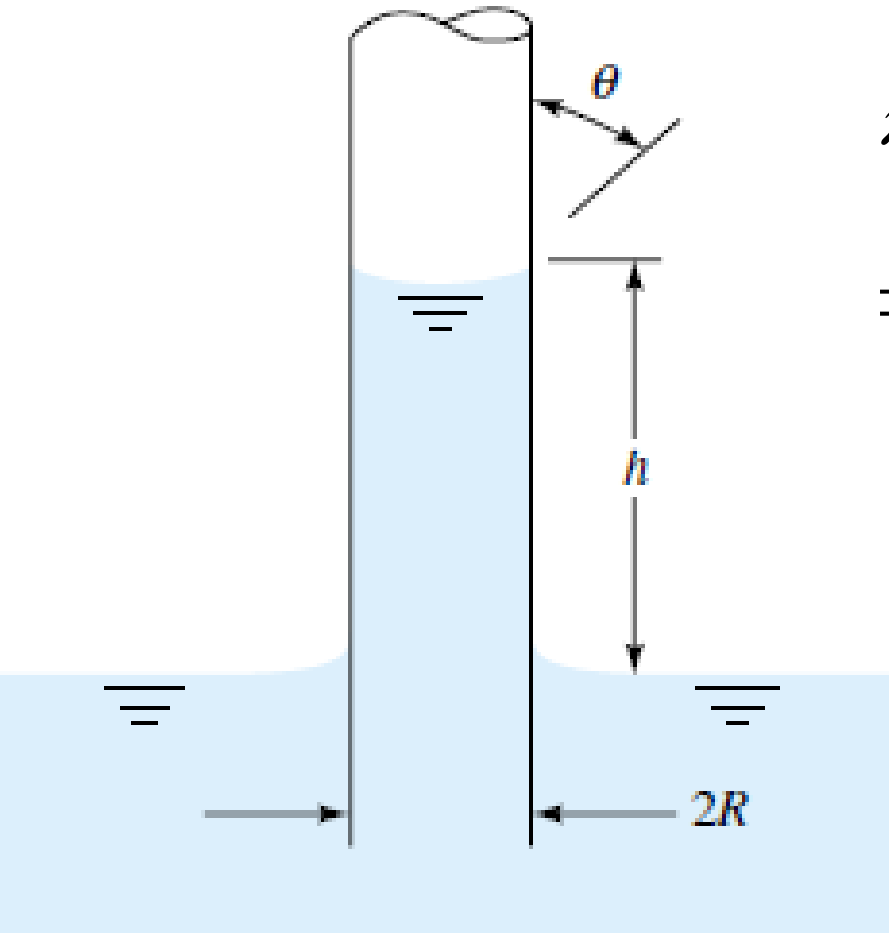


$\theta > 90^\circ$  no moja

$\theta < 90^\circ$  si moja



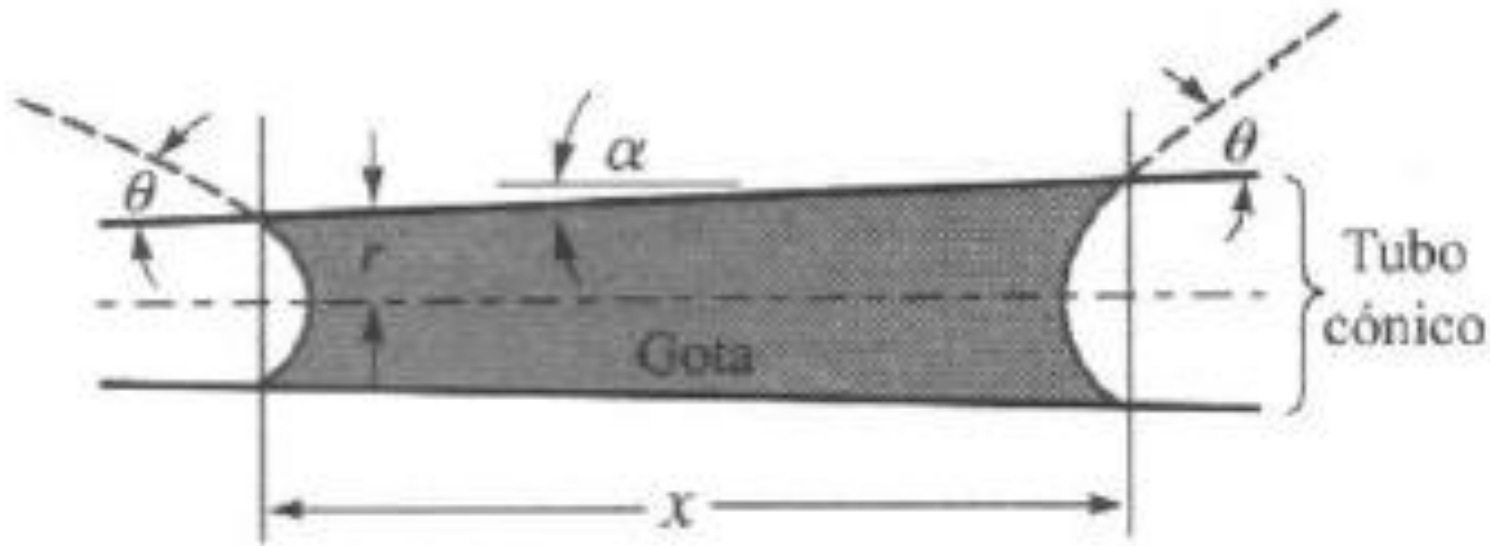
# Ejemplo: Ascenso capilar



$$\pi R^2 h \rho g = 2\pi R \Upsilon \cos \theta$$

$$\Rightarrow h = 2 \frac{\Upsilon \cos \theta}{\rho g R}$$

Ejemplo: El tubo cónico tiene su eje horizontal y contiene una gota alargada de líquido. Encuentre la fuerza que tiende a mover la gota hacia la derecha.



$$F = 2\pi\Upsilon(r + x \tan \alpha) \cos(\theta + \alpha) - 2\pi\Upsilon r \cos(\theta - \alpha)$$