Tarea 2
Mecánica de Fluidos

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Problema 1. Adimensionalize las ecuaciones de Navier-Stokes utilizando las siguientes variables:

$$x' = \frac{x}{L}; \ \vec{V}' = \frac{\vec{V}}{U}; \ t' = \frac{tU}{L}; \ p' = \frac{p + \rho gz}{\mu U/L}$$

Solución 1. Primero escribimos las ecuaciones de Navier-Stokes,

$$\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) = -\frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) + \rho g_x$$

$$\rho \left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) = -\frac{\partial P}{\partial y} + \mu \left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right) + \rho g_y$$

$$\rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial P}{\partial z} + \mu \left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + \rho g_z$$

Para la primera ecuación se tiene,

- $\dfrac{\partial v_x}{\partial t} = \dfrac{\partial v_x}{\partial v'_x} \dfrac{\partial v'_x}{\partial t'} \dfrac{\partial t'}{\partial t} = \dfrac{U^2}{L} \dfrac{\partial v'_x}{\partial t'}$

- $\dfrac{\partial v_x}{\partial x} = \dfrac{\partial v_x}{\partial v'_x} \dfrac{\partial v'_x}{\partial x'} \dfrac{\partial x'}{\partial x} = \dfrac{U}{L} \dfrac{\partial v'_x}{\partial x'}$

- $\dfrac{\partial v_x}{\partial y} = \dfrac{\partial v_x}{\partial v'_x} \dfrac{\partial v'_x}{\partial y'} \dfrac{\partial y'}{\partial y} = \dfrac{U}{L} \dfrac{\partial v'_x}{\partial y'}$

- $\dfrac{\partial v_x}{\partial z} = \dfrac{\partial v_x}{\partial v'_x} \dfrac{\partial v'_x}{\partial z'} \dfrac{\partial z'}{\partial z} = \dfrac{U}{L} \dfrac{\partial v'_x}{\partial z'}$

- $\dfrac{\partial p}{\partial x} = \dfrac{\partial p}{\partial p'} \dfrac{\partial p'}{\partial x'} \dfrac{\partial x'}{\partial x} = \dfrac{\mu U}{L^2} \dfrac{\partial^2 p'}{\partial x'^2}$

- $\dfrac{\partial^2 v_x}{\partial x^2} = \dfrac{\partial}{\partial x} \left(\dfrac{\partial v_x}{\partial x} \right) = \dfrac{\partial}{\partial x'} \left(\dfrac{U}{L} \dfrac{\partial v'_x}{\partial x'} \right) \dfrac{\partial x'}{\partial x} = \dfrac{U}{L^2} \dfrac{\partial^2 v'_x}{\partial x'^2}$

- $\dfrac{\partial^2 v_x}{\partial y^2} = \dfrac{\partial}{\partial y} \left(\dfrac{\partial v_x}{\partial y} \right) = \dfrac{\partial}{\partial y'} \left(\dfrac{U}{L} \dfrac{\partial v'_x}{\partial y'} \right) \dfrac{\partial y'}{\partial y} = \dfrac{U}{L^2} \dfrac{\partial^2 v'_x}{\partial y'^2}$

- $\dfrac{\partial^2 v_x}{\partial z^2} = \dfrac{\partial}{\partial z} \left(\dfrac{\partial v_x}{\partial z} \right) = \dfrac{\partial}{\partial z'} \left(\dfrac{U}{L} \dfrac{\partial v'_x}{\partial z'} \right) \dfrac{\partial z'}{\partial z} = \dfrac{U}{L^2} \dfrac{\partial^2 v'_x}{\partial z'^2}$

Para la segunda ecuación se tiene,

$$\bullet \quad \frac{\partial v_y}{\partial t} = \frac{\partial v_y}{\partial v_y'}\frac{\partial v_y'}{\partial t'}\frac{\partial t'}{\partial t} = \frac{U^2}{L}\frac{\partial v_y'}{\partial t'}$$

$$\bullet \quad \frac{\partial v_y}{\partial x} = \frac{\partial v_y}{\partial v_y'}\frac{\partial v_y'}{\partial x'}\frac{\partial x'}{\partial x} = \frac{U}{L}\frac{\partial v_y'}{\partial x'}$$

$$\bullet \quad \frac{\partial v_y}{\partial y} = \frac{\partial v_y}{\partial v_y'}\frac{\partial v_y'}{\partial y'}\frac{\partial y'}{\partial y} = \frac{U}{L}\frac{\partial v_y'}{\partial y'}$$

$$\bullet \quad \frac{\partial v_y}{\partial z} = \frac{\partial v_y}{\partial v_y'}\frac{\partial v_y'}{\partial z'}\frac{\partial z'}{\partial z} = \frac{U}{L}\frac{\partial v_y'}{\partial z'}$$

$$\bullet \quad \frac{\partial p}{\partial y} = \frac{\partial p}{\partial p'}\frac{\partial p'}{\partial y'}\frac{\partial y'}{\partial y} = \frac{\mu U}{L^2}\frac{\partial p'}{\partial y'}$$

$$\bullet \quad \frac{\partial^2 v_y}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial v_y}{\partial x}\right) = \frac{\partial}{\partial x'}\left(\frac{U}{L}\frac{\partial v_y'}{\partial x'}\right)\frac{\partial x'}{\partial x} = \frac{U}{L^2}\frac{\partial^2 v_y'}{\partial x'^2}$$

$$\bullet \quad \frac{\partial^2 v_y}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial v_y}{\partial y}\right) = \frac{\partial}{\partial y'}\left(\frac{U}{L}\frac{\partial v_y'}{\partial y'}\right)\frac{\partial y'}{\partial y} = \frac{U}{L^2}\frac{\partial^2 v_y'}{\partial y'^2}$$

$$\bullet \quad \frac{\partial^2 v_y}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial v_y}{\partial z}\right) = \frac{\partial}{\partial z'}\left(\frac{U}{L}\frac{\partial v_y'}{\partial z'}\right)\frac{\partial z'}{\partial z} = \frac{U}{L^2}\frac{\partial^2 v_y'}{\partial z'^2}$$

Para la tercera ecuación se tiene,

$$\bullet \quad \frac{\partial v_z}{\partial t} = \frac{\partial v_z}{\partial v_z'}\frac{\partial v_z'}{\partial t'}\frac{\partial t'}{\partial t} = \frac{U^2}{L}\frac{\partial v_z'}{\partial t'}$$

$$\bullet \quad \frac{\partial v_z}{\partial x} = \frac{\partial v_z}{\partial v_z'}\frac{\partial v_z'}{\partial x'}\frac{\partial x'}{\partial x} = \frac{U}{L}\frac{\partial v_z'}{\partial x'}$$

$$\bullet \quad \frac{\partial v_z}{\partial y} = \frac{\partial v_z}{\partial v_z'}\frac{\partial v_z'}{\partial y'}\frac{\partial y'}{\partial y} = \frac{U}{L}\frac{\partial v_z'}{\partial y'}$$

$$\bullet \quad \frac{\partial v_z}{\partial z} = \frac{\partial v_z}{\partial v_z'}\frac{\partial v_z'}{\partial z'}\frac{\partial z'}{\partial z} = \frac{U}{L}\frac{\partial v_z'}{\partial z'}$$

$$\bullet \quad \frac{\partial p}{\partial z} = \frac{\partial p}{\partial p'}\frac{\partial p'}{\partial z'}\frac{\partial z'}{\partial z} = \frac{\mu U}{L^2}\frac{\partial p'}{\partial z'}$$

$$\bullet \quad \frac{\partial^2 v_z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial v_z}{\partial x}\right) = \frac{\partial}{\partial x'}\left(\frac{U}{L}\frac{\partial v_z'}{\partial x'}\right)\frac{\partial x'}{\partial x} = \frac{U}{L^2}\frac{\partial^2 v_z'}{\partial x'^2}$$

$$\bullet \quad \frac{\partial^2 v_z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial v_z}{\partial y}\right) = \frac{\partial}{\partial y'}\left(\frac{U}{L}\frac{\partial v_z'}{\partial y'}\right)\frac{\partial y'}{\partial y} = \frac{U}{L^2}\frac{\partial^2 v_z'}{\partial y'^2}$$

$$\bullet \quad \frac{\partial^2 v_z}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial v_z}{\partial z}\right) = \frac{\partial}{\partial z'}\left(\frac{U}{L}\frac{\partial v_z'}{\partial z'}\right)\frac{\partial z'}{\partial z} = \frac{U}{L^2}\frac{\partial^2 v_z'}{\partial z'^2}$$

Luego reemplazando lo anterior en las ecuaciones de Navier-Stokes para un fluido Newtoniano y haciendo algunas manipulaciones algebraicas, se tiene que,

$$\frac{\partial v'_x}{\partial t'} + v'_x \frac{\partial v'_x}{\partial x'} + v'_y \frac{v'_x}{\partial y'} + v'_z \frac{\partial v'_x}{\partial z'} = \frac{\mu}{UL\rho} \left(-\frac{\partial p'}{\partial x'} + \frac{\partial^2 v'_x}{\partial x'^2} + \frac{\partial^2 v'_x}{\partial y'^2} + \frac{\partial^2 v'_x}{\partial z'^2} \right) + \frac{g_x}{U^2}$$

$$\frac{\partial v'_y}{\partial t'} + v'_x \frac{\partial v'_y}{\partial x'} + v'_y \frac{v'_y}{\partial y'} + v'_z \frac{\partial v'_y}{\partial z'} = \frac{\mu}{UL\rho} \left(-\frac{\partial p'}{\partial y'} + \frac{\partial^2 v'_y}{\partial x'^2} + \frac{\partial^2 v'_y}{\partial y'^2} + \frac{\partial^2 v'_y}{\partial z'^2} \right) + \frac{g_y}{U^2}$$

$$\frac{\partial v'_z}{\partial t'} + v'_x \frac{\partial v'_z}{\partial x'} + v'_y \frac{v'_z}{\partial y'} + v'_z \frac{\partial v'_z}{\partial z'} = \frac{\mu}{UL\rho} \left(-\frac{\partial p'}{\partial z'} + \frac{\partial^2 v'_z}{\partial x'^2} + \frac{\partial^2 v'_z}{\partial y'^2} + \frac{\partial^2 v'_z}{\partial z'^2} \right) + \frac{g_z}{U^2}$$

Luego notamos que, $Re = \frac{UL\rho}{\mu}$, lo que se puede reemplazar en las ecuaciones,

$$\frac{\partial v'_x}{\partial t'} + v'_x \frac{\partial v'_x}{\partial x'} + v'_y \frac{v'_x}{\partial y'} + v'_z \frac{\partial v'_x}{\partial z'} = \frac{1}{\mathrm{Re}} \left(-\frac{\partial p'}{\partial x'} + \frac{\partial^2 v'_x}{\partial x'^2} + \frac{\partial^2 v'_x}{\partial y'^2} + \frac{\partial^2 v'_x}{\partial z'^2} \right) + \frac{g_x}{U^2}$$

$$\frac{\partial v'_y}{\partial t'} + v'_x \frac{\partial v'_y}{\partial x'} + v'_y \frac{v'_y}{\partial y'} + v'_z \frac{\partial v'_y}{\partial z'} = \frac{1}{\mathrm{Re}} \left(-\frac{\partial p'}{\partial y'} + \frac{\partial^2 v'_y}{\partial x'^2} + \frac{\partial^2 v'_y}{\partial y'^2} + \frac{\partial^2 v'_y}{\partial z'^2} \right) + \frac{g_y}{U^2}$$

$$\frac{\partial v'_z}{\partial t'} + v'_x \frac{\partial v'_z}{\partial x'} + v'_y \frac{v'_z}{\partial y'} + v'_z \frac{\partial v'_z}{\partial z'} = \frac{1}{\mathrm{Re}} \left(-\frac{\partial p'}{\partial z'} + \frac{\partial^2 v'_z}{\partial x'^2} + \frac{\partial^2 v'_z}{\partial y'^2} + \frac{\partial^2 v'_z}{\partial z'^2} \right) + \frac{g_z}{U^2}$$

Problema 2. En las ecuaciones determinadas en (1), analice el límite cuando $\rho U L/\mu \longrightarrow 0$. Las ecuaciones encontradas se llaman de Stokes o de "flujo reptante" (en estas ecuaciones se desprecian los términos convectivos por sobre los viscosos)

Solución 2. Del problema 1, despejamos nuestra solución

$$\left(-\frac{\partial p'}{\partial z'} + \frac{\partial^2 v'_z}{\partial x'^2} + \frac{\partial^2 v'_z}{\partial y'^2} + \frac{\partial^2 v'_z}{\partial z'^2}\right) = \frac{UL\rho}{\mu}\left(\frac{\partial v'_x}{\partial t'} + v'_x\frac{\partial v'_x}{\partial x'} + v'_y\frac{v'_x}{\partial y'} + v'_z\frac{\partial v'_x}{\partial z'} - \frac{g_x}{U}\right) \longrightarrow 0$$

$$\left(-\frac{\partial p'}{\partial y'} + \frac{\partial^2 v'_y}{\partial x'^2} + \frac{\partial^2 v'_y}{\partial y'^2} + \frac{\partial^2 v'_y}{\partial z'^2}\right) = \frac{UL\rho}{\mu}\left(\frac{\partial v'_y}{\partial t'} + v'_x\frac{\partial v'_y}{\partial x'} + v'_y\frac{v'_y}{\partial y'} + v'_z\frac{\partial v'_y}{\partial z'} - \frac{g_y}{U}\right) \longrightarrow 0$$

$$\left(-\frac{\partial p'}{\partial z'} + \frac{\partial^2 v'_z}{\partial x'^2} + \frac{\partial^2 v'_z}{\partial y'^2} + \frac{\partial^2 v'_z}{\partial z'^2}\right) = \frac{UL\rho}{\mu}\left(\frac{\partial v'_z}{\partial t'} + v'_x\frac{\partial v'_z}{\partial x'} + v'_y\frac{v'_z}{\partial y'} + v'_z\frac{\partial v'_z}{\partial z'} - \frac{g_z}{U}\right) \longrightarrow 0$$

entonces

$$-\frac{\partial p'}{\partial x'} + \frac{\partial^2 v'_x}{\partial x'^2} + \frac{\partial^2 v'_x}{\partial y'^2} + \frac{\partial^2 v'_x}{\partial z'^2} = 0$$

$$-\frac{\partial p'}{\partial y'} + \frac{\partial^2 v'_y}{\partial x'^2} + \frac{\partial^2 v'_y}{\partial y'^2} + \frac{\partial^2 v'_y}{\partial z'^2} = 0$$

$$-\frac{\partial p'}{\partial z'} + \frac{\partial^2 v'_z}{\partial x'^2} + \frac{\partial^2 v'_z}{\partial y'^2} + \frac{\partial^2 v'_z}{\partial z'^2} = 0$$

despejando, obtenemos

$$\frac{\partial p'}{\partial x'} = \frac{\partial^2 v'_x}{\partial x'^2} + \frac{\partial^2 v'_x}{\partial y'^2} + \frac{\partial^2 v'_x}{\partial z'^2} = \Delta v'_x$$

$$\frac{\partial p'}{\partial y'} = \frac{\partial^2 v'_y}{\partial x'^2} + \frac{\partial^2 v'_y}{\partial y'^2} + \frac{\partial^2 v'_y}{\partial z'^2} = \Delta v'_y$$

$$\frac{\partial p'}{\partial z'} = \frac{\partial^2 v'_z}{\partial x'^2} + \frac{\partial^2 v'_z}{\partial y'^2} + \frac{\partial^2 v'_z}{\partial z'^2} = \Delta v'_z$$

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Problema 3. Utilizando las ecuaciones de Stokes, demuestre que la velocidad para el fluido entre el bloque y la pared móvil está dada por:

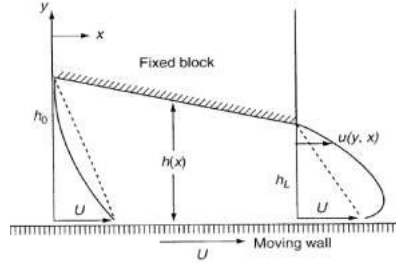$$u(x,y) = \frac{1}{2\mu}\frac{\partial p}{\partial x}y\,(y - h(x)) + U\left(1 - \frac{y}{h(x)}\right)$$

Figura 1: ilustración del problema 3.

Para efectos de cálculo omita la acción de la gravedad y suponga que $h_0 - h_L << L$. Note que el fluido se mueve por efecto de la pared y la presión.

Solución 3. sea $u = cte$

Por la ecuación Navier-Stokes

$$\rho\left(\cancel{\frac{\partial v_x}{\partial t}}^{0} + v_x\cancel{\frac{\partial v_x}{\partial x}}^{0} + v_y\cancel{\frac{\partial v_x}{\partial y}}^{0} + v_z\cancel{\frac{\partial v_x}{\partial z}}^{0}\right) = -\frac{\partial P}{\partial x} + \mu\left(\cancel{\frac{\partial^2 v_x}{\partial x^2}}^{0} + \frac{\partial^2 v_x}{\partial y^2} + \cancel{\frac{\partial^2 v_x}{\partial z^2}}^{0}\right) + \cancel{\rho g_x}^{0}$$

$$x : \frac{\partial p_x}{\partial x} = \frac{u\partial^2 u(x,y)}{\partial y^2} \tag{1}$$

$$y : \frac{\partial p_y}{\partial y} = 0 \tag{2}$$

$$z : \frac{\partial p_z}{\partial z} = 0 \tag{3}$$

despejamos la ecuación (1)

$$\frac{\partial^2 u(x,y)}{\partial y} = \frac{\partial p_x}{\partial x}\frac{1}{u}$$

luego integrando , obtenemos la siguiente ecuación

$$\frac{\partial u(x,y)}{\partial y} = \frac{\partial p}{\partial x}\cdot\frac{1}{u}y + c$$

resolviendo la EDP

$$u(x.y) = \frac{\partial_x}{\partial x}\cdot\frac{1}{u}\cdot\frac{h(x)^2}{2} + c_1 y + c_2$$

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aplicando condiciones de contorno,
(a)   y=0   u(x,y)=U
(b)   y=h(x)   u(x,y)=0

con (a)

$$c_2 = U$$

con (b)

$$\frac{\partial p}{\partial x} \cdot \frac{1}{u} \cdot \frac{h(x)^2}{2} + c_1 h(x) + U = 0$$

luego,

$$c_1 = \frac{-U}{h(x)} - \frac{\partial p}{\partial x} \cdot \frac{1}{u} \cdot \frac{h(X)}{2}$$

ahora reemplazamos c_1 y c_2

$$u(x,y) = \frac{1}{2u} \cdot \frac{\partial p}{\partial x} \cdot (y^2 - h(x)y) + U\left(1 - \frac{y}{h(x)}\right)$$

Problema 4. Note que la distribución de presión $p(x)$ debe satisfacer la ecuación de continuidad

$$\int_0^h \frac{\partial u}{\partial x} dy = -\int_0^h \frac{\partial v}{\partial y} dy$$

pero las velocidades verticales son nulas, luego

$$\int_0^h \frac{\partial v}{\partial y} dy = v(h) - v(0) = 0$$

Utilizando el punto 3.- y las expresiones integrales anteriores demuestre que:

$$\frac{\partial}{\partial x}\left[h^3 \frac{\partial p}{\partial x}\right] = 6\mu U \frac{\partial h}{\partial x}$$

Solución 4.

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(\frac{1}{2u}\frac{\partial p}{\partial x}\left(y^2 - yh(x)\right) + U\left(1 - \frac{y}{h(x)}\right)\right)$$

$$\frac{\partial u}{\partial x} = \frac{y^2}{2u}\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\right) - \frac{y}{2u}\frac{\partial h(x)}{\partial x} - U\cancelto{0}{\frac{\partial}{\partial x}} - Uy\frac{\partial}{\partial x}\left(\frac{1}{f(x)}\right)$$

integrando ambas partes , obtenemos

$$\int_0^h \frac{\partial u}{\partial x} dy = \int_0^h \frac{y^2}{2u}\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\right) dy - \int_0^h \frac{y}{2u}\frac{\partial h(x)}{\partial x} dy - \int_0^h Uy\frac{\partial}{\partial x}\left(\frac{1}{f(x)}\right) dy$$

debido a que $\int_0^h \frac{\partial u}{\partial x} dy = 0$

$$\frac{h^3}{6u}\frac{\partial}{\partial x}\left(\frac{\partial p}{\partial x}\right) - \frac{h^2}{4u}\frac{\partial h}{\partial x} - \frac{Uh^2}{2}\frac{\partial}{\partial x}\left(\frac{1}{h}\right) = 0$$

$$\Longrightarrow \frac{h^3}{6u}\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\right) = \frac{h^3}{4u}\frac{\partial}{\partial x} + \frac{U\partial h}{2\partial x}$$

$$\Longrightarrow \frac{\partial}{\partial x}\left(\frac{h^3\partial p}{\partial x}\right) = \frac{3h^3\partial}{2\partial x} + 3u\frac{U\partial h}{\partial x}$$

Problema 5. Determine el perfil de presión si $h(x) = h_0 - \frac{h_0 - h_L}{L}x$ y $p(0) = p(L) = p_\infty$. Para simplificar los cálculos defina $\alpha = \frac{h_L - h_0}{L}$

Solución 5. Se tiene la siguiente EDP con condiciones de contorno,

$$\frac{\partial}{\partial x}\left[h^3(x)\frac{\partial p}{\partial x}\right] = 6\mu U \frac{\partial h}{\partial x}$$

$$p(0) = p(L) = p_\infty$$

Aplicando el T.F.C, se obtiene,

$$h^3(x)\frac{\partial p}{\partial x} = 6\mu U h(x) + C_1$$

Despejamos y obtenemos,

$$\frac{\partial p}{\partial x} = \frac{6\mu U h(x)}{h^3(x)} + \frac{C_1}{h^3(x)}$$

$$\frac{\partial p}{\partial x} = 6\mu U h^{-2}(x) + \frac{C_1}{h^3(x)}$$

Aplicamos nuevamente el T.F.C.

$$p(x) + C = 6\mu U \int h^{-2}(x)dx + C_1 \int h^{-3}(x)dx$$

$$p(x) = \frac{-6\mu U}{h(x)} - \frac{C_1}{2h^2(x)} + C_2$$

$$p(x) = -\frac{6\mu U}{h(x)} - \frac{C_1}{2h^2(x)} + C_2$$

Luego aplicando las condiciones de contorno,

$$p(0) = -\frac{6\mu U}{h_0} - \frac{C_1}{2h_0^2} + C_2 = p_\infty \tag{4}$$

$$p(L) = -\frac{6\mu U}{h_L} - \frac{C_1}{2h_L^2} + C_2 = p_\infty \tag{5}$$

Luego resolvemos el sistema de ecuaciones.
Igualamos (4) con (5),

$$\frac{-6\mu U}{h_0(x)} - \frac{C_1}{2h_0^2} + \cancelto{0}{C_2} = \frac{-6\mu U}{h_L(x)} - \frac{C_1}{2h_L^2} + \cancelto{0}{C_2}$$

$$\Longrightarrow C_1 = \frac{6\mu U\left(\frac{1}{h_L} - \frac{1}{h_0}\right)}{\left(\frac{1}{2h_0^2} - \frac{1}{2h_L^2}\right)}$$

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Luego reemplazamos en (4) y obtenemos,

$$C_2 = \frac{6\mu U}{h_0} + \frac{6\mu U \left(\frac{1}{h_L} - \frac{1}{h_0}\right)}{2h_0^2 \left(\frac{1}{2h_0^2} - \frac{1}{2h_L^2}\right)} + p_\infty$$

Así, el perfil de presión queda,

$$p(x) = -\frac{6\mu U}{h(x)} - \frac{3\mu U \left(\frac{1}{h_L} - \frac{1}{h_0}\right)}{h^2(x) \left(\frac{1}{2h_0^2} - \frac{1}{2h_L^2}\right)} + \frac{6\mu U}{h_0} + \frac{3\mu U \left(\frac{1}{h_L} - \frac{1}{h_0}\right)}{h_0^2 \left(\frac{1}{2h_0^2} - \frac{1}{2h_L^2}\right)} + p_\infty$$

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