Formulario 543214-2 Capítulos 1-4

Linealización

$\dot{x} = f(x,u,p) \quad y = h(x,u,p)$

$0 = f(x_0,u_0,p_0)$

$A = \frac{\partial f(x,u,p)}{\partial x}\Big|_{\substack{x=x_0\\u=u_0\\p=p_0}} \quad B = \frac{\partial f(x,u,p)}{\partial u}\Big|_{\substack{x=x_0\\u=u_0\\p=p_0}}$

$C = \frac{\partial h(x,u,p)}{\partial x}\Big|_{\substack{x=x_0\\u=u_0\\p=p_0}} \quad D = \frac{\partial h(x,u,p)}{\partial u}\Big|_{\substack{x=x_0\\u=u_0\\p=p_0}}$

$E = \frac{\partial f(x,u,p)}{\partial p}\Big|_{\substack{x=x_0\\u=u_0\\p=p_0}} \quad F = \frac{\partial h(x,u,p)}{\partial p}\Big|_{\substack{x=x_0\\u=u_0\\p=p_0}}$

$\dot{\Delta x} = A\Delta x + B\Delta u + E\Delta p$

$y = C\Delta x + D\Delta u + F\Delta p$

$\Delta x = x - x_0 \quad \Delta u = u - u_0 \quad \Delta p = p - p_0$

Propiedades de simetría

$x_p(t) = \frac{x(t)+x(-t)}{2}$

$x_i(t) = \frac{x(t)-x(-t)}{2}$

$x_p(t) = x_p(-t) \iff x_p(t)$ es par

$x_i(t) = -x_i(-t) \iff x_i(t)$ es impar

Transformaciones simples

$f(t) \to g(t) = \alpha f(at+b) + \beta$

$f(t) \to g(t) = \alpha f(t) + \beta$, transf. var. dependiente

$f(t) \to g(t) = f(at+b)$, transf. var. indepentiende

Convolución

$f(t) * g(t) = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau$

$f(t) * g(t) = g(t) * f(t)$

$f(t) * [g(t) + h(t)] = f(t) * g(t) + f(t) * h(t)$

$f(t) * [g(t) * h(t)] = [f(t) * g(t)] * h(t)$

$f(t) * \delta(t-t_0) = f(t-t_0)$

$f(t) * u(t) = \int_{-\infty}^{t} f(\tau)d\tau$

$f(kT) * g(kT) = \sum_{i=-\infty}^{\infty} f(kT-iT)g(iT)$

$f(kT) *_c g(kT) = \sum_{i=0}^{Q-1} f(kT-iT)g(iT)$

Transformada de Laplace

$\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t)e^{-st}dt = f(s)$

$f(t) = \delta(t)$	$f(s) = 1$
$f(t) = u(t)$	$f(s) = \frac{1}{s}, s > 0$
$f(t) = r(t)$	$f(s) = \frac{1}{s^2}, s > 0$
$f(t) = e^{-at}$	$f(s) = \frac{1}{s+a}, s > a$
$f(t) = sen(bt)$	$f(s) = \frac{b}{s^2+b^2}, s > 0$
$f(t) = \cos(bt)$	$f(s) = \frac{s}{s^2+b^2}, s > 0$
$f(t-a)$	$e^{-as}f(s)$, $f(t)$ soporte positivo
$e^{at}f(t)$	$f(s-a)$
f'	$\mathcal{L}\{f'\} = sf(s) - f(0)$
f''	$\mathcal{L}\{f''\} = s^2f(s) - sf(0) - f'(0)$
$f(t) * g(t)$	$f(s)g(s)$
$f(t)\vert_{t=0}$	$\lim_{s\to\infty} sf(s)$
$f(t)\vert_{t=\infty}$	$\lim_{s\to 0} sf(s)$

Transformada de Fourier

$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt = f_2(s)|_{s=j\omega} = f(\omega)$

$e^{\pm j\omega t} = cos(\omega t) \pm j sen(\omega t)$

$cos(\omega t) = (e^{j\omega t} + e^{-j\omega t})/2$

$sen(\omega t) = (e^{j\omega t} - e^{-j\omega t})/(2j)$

$\mathcal{F}\{\frac{e^{j\omega_0 t}}{2\pi}\} = \delta(\omega - \omega_0)$

$\mathcal{F}\{cos(\omega_0 t)\} = \pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)$

$g(t) = f(\omega)|_{\omega=t} \iff g(\omega) = 2\pi f(t)|_{t=-\omega}$

$\mathcal{F}\{f(t) * g(t)\} = f(\omega)g(\omega)$

$f(t)g(t) = f(\omega) * g(\omega)/(2\pi)$

$f(n) = \frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0} f(t)e^{-jn\omega_0 t}dt$

$= \frac{1}{T_0}\int_0^{T_0} f(t)e^{-jn\omega_0 t}dt = \frac{1}{T_0}f(\omega)|_{\omega=n\omega_0}$

$f(t) = \sum_{n=-\infty}^{\infty} f(n)e^{jn\omega_0 t}$, $f(t)$ periódica

$f(\Omega) = \sum_{k=-\infty}^{\infty} f(kT)e^{-j\Omega kT}$

$f(m) = \frac{1}{T_0/T}\sum_{k=0}^{T_0/T-1} f(kT)e^{-jm\Omega_0 kT}$

$= \frac{1}{N}\sum_{k=0}^{N-1} f(kT)e^{-jmk2\pi/N} = \frac{1}{N}f(\Omega)|_{\Omega=m\Omega_0}$

Transformada Z

$\mathcal{Z}\{f(kT)\} = \sum_{k=0}^{\infty} f(kT)z^{-k} = f(z)$

$f_2(z)|_{z=e^{j\Omega T}} = f(\Omega)$

$f(kT) = \delta(kT)$	$f(z) = 1$
$f(kT) = u(kT)$	$f(z) = \frac{1}{1-z^{-1}} = \frac{z}{z-1}, \vert z\vert > 1$
$f(kT) = r(kT)$	$f(z) = Tz^{-1}\frac{1}{(1-z^{-1})^2}, \vert z\vert > 1$
$f(kT) = e^{-kT}u(kT)$	$f(z) = \frac{1}{1-e^{-T}z^{-1}}, \vert z\vert > \vert e^{-T}\vert$
$f(kT) - f(kT-T)$	$(1-z^{-1})f(z)$
$y(kT+T)$	$z(y(z) - z^{-0}y(0))$
$y(kT+2T)$	$z^2(y(z) - z^{-1}y(T) - z^{-0}y(0))$
$f(kT) * g(kT)$	$f(z)g(z)$
$f(kT)\vert_{k=0}$	$\lim_{z\to\infty} f(z)$
$f(kT)\vert_{k=\infty}$	$\lim_{z\to 1}(1-z^{-1})f(z)$

Solución de ec. de estado

$\dot{x} = Ax + Bu$

$y = Cx + Du$

$\Phi(t) = e^{At} \quad \Phi(s) = (sI - A)^{-1}$

$y_h(t) = C\Phi(t)x_0$

$y_f(s) = C\Phi(s)Bu(s) + Du(s)$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, ad - bc \neq 0$

Solución de ec. de diferencias de estado

$x(kT+T) = Ax(kT) + Bu(kT)$

$y(kT) = Cx(kT) + Du(kT)$

$\Phi(kT) = A^k \quad \Phi(z) = (zI - A)^{-1}z$

$y_h(kT) = C\Phi(kT)x_0$

$y_f(z) = Cz^{-1}\Phi(z)Bu(z) + Du(z)$