1) $f \in \mathcal{C}^2$, $\dfrac{\partial^2 f}{\partial r^2}(r) + \dfrac{1}{r}\dfrac{\partial f}{\partial r}(r) = 0$, $r \neq 0$.

$$u(x,y) = f\left(\sqrt{x^2+y^2}\right), \quad \Delta u = 0.$$

$$\begin{tikzcd} & u \arrow[d, dash] & \\ & f \arrow[d, dash] & \\ & r \arrow[dl, dash] \arrow[dr, dash] & \\ x & & y \end{tikzcd}$$

Sea $r(x,y) = \sqrt{x^2+y^2}$. Por regla de la cadena.

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

$$\frac{\partial u}{\partial x} = \frac{\partial f}{\partial r} \cdot \frac{\partial r}{\partial x} = \frac{\partial f}{\partial r} \cdot \frac{x}{\sqrt{x^2+y^2}}.$$

$$\frac{\partial u}{\partial y} = \frac{\partial f}{\partial r} \cdot \frac{\partial r}{\partial y} = \frac{\partial f}{\partial r} \cdot \frac{y}{\sqrt{x^2+y^2}}.$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial r}\cdot\frac{\partial r}{\partial x}\right).$$

$$= \frac{\partial^2 f}{\partial r^2}\cdot\frac{\partial r}{\partial x}\cdot\frac{\partial r}{\partial x} + \frac{\partial f}{\partial r}\cdot\frac{\partial^2 r}{\partial x^2}.$$

$$= \frac{\partial^2 f}{\partial r^2}\cdot\frac{x^2}{x^2+y^2} + \frac{\partial f}{\partial r}\cdot\frac{\sqrt{x^2+y^2} - x\frac{1}{\cancel{2}\sqrt{x^2+y^2}}\cdot\cancel{2}x}{x^2+y^2}$$

$$= \frac{\partial^2 f}{\partial r^2}\cdot\frac{x^2}{x^2+y^2} + \frac{y^2}{(x^2+y^2)^{3/2}}\cdot\frac{\partial f}{\partial r}$$

$$\begin{tikzcd}[column sep=small] & \partial u \arrow[d, dash] & \\ & \partial f \arrow[d, dash] & \\ & r \arrow[dl, dash] \arrow[dr, dash] & \\ x & & y \end{tikzcd}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial r}\cdot\frac{\partial r}{\partial y}\right)$$

$$= \frac{\partial^2 f}{\partial r^2}\left(\frac{\partial r}{\partial y}\cdot\frac{\partial r}{\partial y}\right) + \frac{\partial f}{\partial r}\cdot\frac{\partial^2 r}{\partial y^2}.$$

$$= \frac{\partial^2 f}{\partial r^2}\cdot\frac{y^2}{x^2+y^2} + \frac{\partial f}{\partial r}\,\frac{x^2}{(x^2+y^2)^{\frac{3}{2}}}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \cancel{\frac{\partial^2 f}{\partial r^2} \frac{x^2}{x^2+y^2}} + \frac{y^2}{(x^2+y^2)^{3/2}} \frac{\partial f}{\partial r} + \cancel{\frac{\partial^2 f}{\partial r} \frac{y^2}{x^2+y^2}} + \frac{x^2}{(x^2+y^2)^{3/2}} \frac{\partial f}{\partial r}$$

$$= \frac{\partial^2 f}{\partial r^2} + \frac{1}{\sqrt{(x^2+y^2)}} \frac{\partial f}{\partial r}.$$

$$= \frac{\partial^2 f}{\partial r^2} + \frac{1}{r} \frac{\partial f}{\partial r}.$$

$$\Rightarrow \Delta u = 0.$$

2.) 1. $f:[a,b] \longrightarrow \mathbb{R}$, continue en $[a,b]$.

$$F(x) = \int_0^x f(t)\,dt.$$

$$, \quad f(x) = \frac{dF(x)}{dx}. \quad , \quad F(x) = \int_0^{\alpha(x)} f(t)\,dx \implies f(x) = f(\alpha(x)) \cdot {}'\alpha(x).$$

2. $$\int_a^b f(x)\,dx = F(b) - F(a).$$

2.2). $$f(u,v,w) = \int_u^v g(x,w)\,dx.$$

$$\frac{\partial f}{\partial w}(u,v,w) = \int_u^v \frac{\partial g}{\partial w}(x,w)\,dx.$$ Leibnitz.

$$\frac{\partial f}{\partial u} = \frac{\partial}{\partial u}\left(\int_0^{v} \rho(x,w)\,dx - \int_0^{u} \rho(x,w)\,dx\right).$$

$$= \frac{\partial}{\partial u}\int_0^{v} \rho(x,w)\,dx - \frac{\partial}{\partial u}\int_0^{u} \rho(x,w)\,dx.$$

$$= \cancel{\rho(u,w)\cdot\frac{\partial v}{\partial u}}^{\,0} - \rho(u,w)\cdot\frac{\partial u}{\partial u}.$$

$$= -\rho(u,w).$$

3. $n(t) = f(\rho_1(t), \rho_2(t), t) = \int_{\rho_1(t)}^{\rho_2(t)} \rho(t,x)\,dx$

¿$n'(t)$?

$$\frac{d}{dt} h(t) = \frac{\partial}{\partial t} f(g_1, g_2, t)$$

$$= \frac{\partial f}{\partial g_1} \cdot \frac{\partial g_1}{\partial t} + \frac{\partial f}{\partial g_2} \cdot \frac{\partial g_2}{\partial t} + \frac{\partial f}{\partial t}$$

$$= -g(t, g_1) g_1' + g(t, g_2) g_2' + \frac{\partial}{\partial t} \int_{g_1(t)}^{g_2(t)} g(t,x)\,dx$$

$$= -g(t, g_1) g_1' + g(t, g_2) g_2' + \int_{g_1(t)}^{g_2(t)} \frac{\partial}{\partial t} g(t,x)\,dx$$

$$\begin{tikzcd}[arrows=dash]
 & h \arrow[d] & \\
 & f \arrow[dl] \arrow[d] \arrow[dr] & \\
g_1 \arrow[d] & g_2 \arrow[d] & t \\
t & t &
\end{tikzcd}$$

3) $f(x,y) = x^3 + y^3 - x^2 y + xy - 1.$

Polinomio de taylor en $P(-1,2)$.

taylor centrado en (x_0, y_0)

maclaurin centrado $(0,0)$.

$$T(x,y)\Big|_{(-1,2)} = f(-1,2) + \frac{1}{1!}\left[\frac{\partial f}{\partial x}(-1,2)(x+1) + \frac{\partial f}{\partial y}(-1,2)(y-2)\right].$$

$$\left(\left(x+1, y-2\right) \cdot \left(\frac{\partial f}{\partial x}(-1,2), \frac{\partial f}{\partial y}(-1,2)\right)\right)$$

$\underline{h = x + x_0}$

$$+ \frac{1}{2!}\left[\frac{\partial^2 f}{\partial x^2}(-1,2)(x+1)^2 + 2\frac{\partial^2 f}{\partial x \partial y}(-1,2) \cdot (x+1)(y-2)\right.$$

$$\left. + \frac{\partial^2 f}{\partial y^2}(-1,2)(y-2)^2\right] + \frac{1}{3!}\left[\frac{\partial^3 f}{\partial x^3}(-1,2)(x+1)^3 + \right.$$

$$+ 3\frac{\partial^3 f}{\partial x^2 \partial y}(-1,2)(x+1)^2(y-2) + \frac{3\partial^3 f}{\partial y^2 \partial x}(-1,2)(x+1)(y-2)^2$$

$$+ \frac{\partial^3 f}{\partial y^3}(-1,2)(y-2)^3 \Big]$$

$$\frac{\partial f}{\partial x} = 3x^2 - 2xy + y$$

$$\frac{\partial f}{\partial y} = 3y^2 - x^2 + x$$

$$\frac{\partial^2 f}{\partial x^2} = 6x - 2y$$

$$\frac{\partial^2 f}{\partial x \partial y} = -2x + 1 = \frac{\partial^2 f}{\partial y \partial x}$$

$$\frac{\partial^2 f}{\partial y^2} = 6y$$

$$\frac{\partial^3 f}{\partial x^3} = 6 \qquad \frac{\partial^3 f}{\partial x^2 \partial y} = -2 \qquad T(x,y) = f(x,y) + \sum_{k=1}^{m} \frac{(h \cdot \nabla f)^k}{k!}(x,y).$$

$$\frac{\partial^3 f}{\partial x^2 \partial y} = -2 \qquad \frac{\partial^3 f}{\partial x \partial y^2} = 0 \qquad h = x - x_0$$

$$\frac{\partial^3 f}{\partial y^2 \partial x} = 0 \qquad \frac{\partial^3 f}{\partial y^3} = 6.$$

$$T(x,y) = 2 + 9(x+1) + 10(y-2) + \frac{1}{2}\left(-10(x+1)^2 + 6(x+1)(y-2) + 12(y-2)^2\right) + \frac{1}{6}\left(6(x+1)^3 - 6(x+1)^2(y-2) + 0(x+1)(y-2)^2 + 6(y-2)^3\right).$$