Ejercicio pendiente Clase 37

P2)

$$\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1}\,dx,$$

$$\begin{array}{l} x^4-2x^2+4x+1 : x^3-x^2-x+1 = x+1 \\ -\underline{(x^4-x^3-x^2+x)} \\ x^3-x^2+3x+1 \\ -\underline{(x^3-x^2-x+1)} \\ 4x \end{array}$$

Luego,

$$\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1}\,dx = \int\left\{x+1+\frac{4x}{x^3-x^2-x+1}\right\}dx$$

$$= \int\left\{x+1+\frac{4x}{(x-1)^2(x+1)}\right\}dx$$

$$= \int (x+1)\,dx + \int \frac{4x}{(x-1)^2(x+1)}\,dx$$

Luego,

$$\frac{4x}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)} = \frac{A(x-1)(x+1)+B(x+1)+C(x-1)^2}{(x-1)^2(x+1)}$$

$$= \frac{A(x^2-1)+B(x+1)+C(x^2-2x+1)}{(x-1)^2(x+1)}$$

$$= \frac{(A+C)x^2+(B-2C)x+(B+C-A)}{(x-1)^2(x+1)}$$

Luego,

$$\begin{array}{l} A+C=0 \\ B-2C=4 \\ B+C-A=0 \end{array} \Rightarrow \begin{cases} A=-C \\ B-2C=4 \\ B+C+C=0 \end{cases} \Rightarrow \begin{cases} A=-C \\ -2C-2C=4 \\ B=-2C \end{cases} \Rightarrow \begin{cases} A=1 \\ C=-1 \\ B=2 \end{cases}$$

$$\int (x+1)\,dx + \int \frac{4x}{(x-1)^2(x+1)}\,dx = \int (x+1)\,dx + \int \left(\frac{1}{(x-1)} + \frac{2}{(x-1)^2} - \frac{1}{x+1} \right) dx$$

$$= \frac{x^2}{2} + x + \int \frac{1}{x-1}\,dx + 2\int \frac{1}{(x-1)^2}\,dx - \int \frac{1}{x+1}\,dx$$

$$= \frac{x^2}{2} + x + \ln(|x-1|) - \ln(|x+1|) + 2\underbrace{\int \frac{1}{(x-1)^2}\,dx}_{(*)}$$

Luego,

$$\int \frac{1}{(x-1)^2}\,dx = \int \frac{1}{u^2}\,du = -\frac{1}{u} + C = -\frac{1}{x-1} + C$$

$u = x-1$
$du = dx$

Finalmente,

$$\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1}\,dx = \frac{x^2}{2} + x + \ln(|x-1|) - \ln(|x+1|) - 2\cdot\frac{1}{x-1} + C \,//$$

P3) $\displaystyle\int \frac{2x^2-x+4}{x^3+4x}\,dx = \int \frac{2x^2-x+4}{x(x^2+4)}\,dx$

Luego, $\displaystyle\frac{2x^2-x+4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4} = \frac{A(x^2+4)+x(Bx+C)}{x(x^2+4)} = \frac{(A+B)x^2+Cx+4A}{x(x^2+4)}$

$$\Rightarrow \begin{cases} A+B=2 \\ C=-1 \\ 4A=4 \end{cases} \Rightarrow \begin{cases} 1+B=2 \\ C=-1 \\ A=1 \end{cases} \Rightarrow \begin{cases} A=1 \\ B=1 \\ C=-1 \end{cases}$$

Luego,

$$\int \frac{2x^2-x+4}{x(x^2+4)}\,dx = \int\left(\frac{1}{x} + \frac{x-1}{x^2+4}\right)dx$$

$$= \underbrace{\int \frac{1}{x}\,dx}_{(1)} + \underbrace{\int \frac{x}{x^2+4}\,dx}_{(2)} + \underbrace{\int \frac{1}{x^2+4}\,dx}_{(3)}$$

(1): $\displaystyle\int \frac{1}{x}\,dx = \ln(|x|)+C$

(2): $\displaystyle\int \frac{x}{x^2+4}\,dx = \int \frac{1}{2}\,\frac{1}{u}\,du = \frac{1}{2}\ln(|u|)+C = \frac{1}{2}\ln(|x^2+4|)+C$

$u = x^2+4$
$du = 2x\,dx$

(3): $\displaystyle\int \frac{1}{x^2+4}\,dx = \int \frac{1}{4(\frac{1}{4}x^2+1)}\,dx = \frac{1}{4}\int \frac{1}{(\frac{1}{2}x)^2+1}\,dx = \frac{1}{4}\int \frac{2}{u^2+1}\,du = \frac{1}{2}\operatorname{Arctan}(u)+C$

$u = \frac{1}{2}x$
$du = \frac{1}{2}dx$

$\Rightarrow \frac{1}{2}\operatorname{Arctan}\left(\frac{1}{2}x\right)+C$

Finalmente, $\displaystyle\int \frac{2x^2-x+4}{x^3+4x}\,dx = \ln(|x|) + \frac{1}{2}\ln(|x^2+4|) + \frac{1}{2}\operatorname{Arctan}\left(\frac{1}{2}x\right)+C$ //