

# The egalitarian effect of search engines

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## ABSTRACT

Search engines have become key media for our scientific, economic, and social activities by enabling people to access information on the Web in spite of its size and complexity. On the down side, search engines bias the traffic of users according to their page-ranking strategies, and some have argued that they create a vicious cycle that amplifies the dominance of established and already popular sites. We show that, contrary to these prior claims and our own intuition, the use of search engines actually has an egalitarian effect. We reconcile theoretical arguments with empirical evidence showing that the combination of retrieval by search engines and search behavior by users mitigates the attraction of popular pages, directing more traffic toward less popular sites, even in comparison to what would be expected from users randomly surfing the Web.

## Categories and Subject Descriptors

H.3.3 [Information Storage and Retrieval]: Information Search and Retrieval; H.3.4 [Information Storage and Retrieval]: Systems and Software—*Information networks*; H.3.5 [Information Storage and Retrieval]: Online Information Services—*Commercial, Web-based services*; H.5.4 [Information Interfaces and Presentation]: Hypertext/Hypermedia—*Navigation, user issues*; K.4.m [Computers and Society]: Miscellaneous

## General Terms

Measurement

## Keywords

Search engines, bias, popularity, traffic, PageRank, in-degree.

## 1. INTRODUCTION

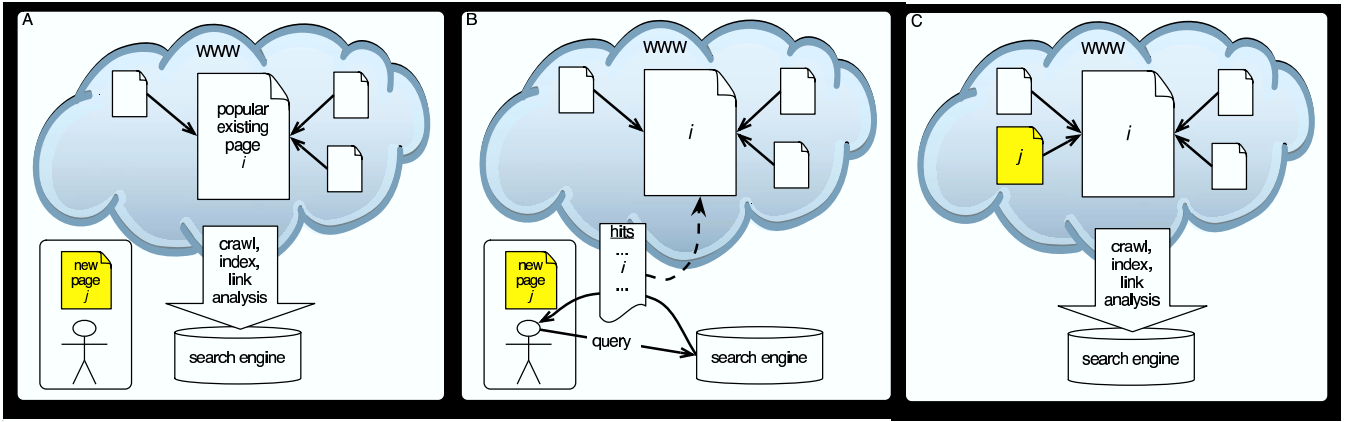
The crucial role of the Web as a communication medium and its unsupervised, self-organized development have triggered the intense interest of the scientific community. The topology of the Web as a complex, scale-free network is

now well characterized [2, 16, 8, 1, 17]. Several growth and navigation models have been proposed to explain the Web's emergent topological characteristics and their effect on users' surfing behavior [5, 18, 15, 28, 22, 23, 6]. As the size and complexity of the Web have increased, users have become reliant on search engines [19, 20], so that the paradigm of search is replacing that of navigation as the main interface between people and the Web [31, 29]. This leads to questions about the role of search engines in shaping the use and evolution of the Web.

One common belief is that the use of search engines biases traffic toward popular sites. This is at the origin of the vicious cycle illustrated in Fig. 1. Pages highly ranked by search engines are more likely to be discovered and consequently linked to by other pages. This in turn would further increase the popularity and raise the average rank of those pages. As popular pages become more and more popular, new pages are unlikely to be discovered [9]. Such a cycle would accelerate the rich-get-richer dynamics already observed in the Web's network structure and explained by preferential attachment and link copy models [5, 16, 18]. This presumed phenomenon, also known as *search engine bias*, *entrenchment effect*, or *googlearchy*, has been widely discussed in computer, social and political science [14, 24, 4, 13, 9, 26] and methods to counteract it are being proposed [10, 26].

In this paper we use both empirical and theoretical arguments to show that the bias of search engines is of the opposite nature, namely directing more traffic toward less popular pages compared to the case in which no search occurs and all traffic is generated by surfing hyperlinks. Our contributions are organized as follows:

- We develop a simple modeling framework in which one can quantify the amount of traffic that Web sites receive in the extreme cases in which users browse the Web by surfing random hyperlinks and in which users only visit pages returned by search engines in response to queries. The framework, introduced in Section 2, allows to make and compare predictions about how navigation and search steer traffic and thus bias the popularity of Web sites.



**Figure 1: Illustration of search engine bias.** A. Page  $i$  is “popular” in that it has many incoming links and high PageRank. A user creates a new page  $j$ . B. The user consults a search engine to find pages related to  $j$ . Since  $i$  is ranked highly by the search engine, it has a high probability of being returned to the user. C. The user, having discovered  $i$ , links to it from  $j$ . Thus  $i$  becomes even more popular from the search engine’s perspective.

- In Section 3 we provide a first empirical study of the traffic toward Web pages as a function of their in-degree. This particular relationship is the one that can directly validate the models in Section 2. As it turns out, both the surfing and searching models are surprisingly wrong; the bias in favor of popular pages seems to be mitigated, rather than enhanced, by the combination of search engines and users’ search behavior. This result contradicts prior assumptions about search engine bias.
- The unexpected empirical observation on traffic is explained in Section 4, where we take into consideration a previously neglected factor about search results, namely the distribution and composition of hit set size. This distribution, determined empirically from actual user queries, allows one to reconcile the searching model with the empirical data of Section 3. Using theoretical arguments and numerical simulations we show that the search model, revised to take queries into account, accurately predicts traffic trends confirming the egalitarian bias of search engines.

## 2. MODELING THE VICIOUS CYCLE

For a quantitative definition of popularity we turn to the probability that a generic user clicks on a link leading to a specific page [10]. We will also refer to this quantity as the traffic to the same page.

### 2.1 Surfing model of traffic

In the absence of search engines, people would browse Web pages primarily by following hyperlinks. It is natural to assume that the amount of such surfing-generated traffic directed toward a given page is proportional to the number of links  $k$  pointing to it. The more the pages pointing to that page, the larger the probability that a randomly surfing user will discover it. Successful search engines, Google being the premier example [7], have modeled this effect in their

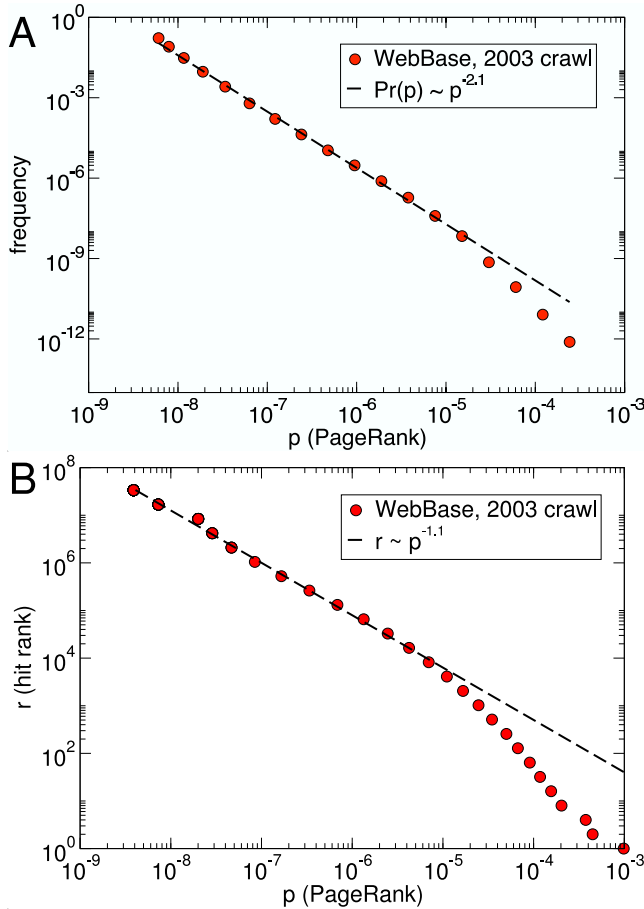
ranking functions to gauge page importance. The PageRank value  $p(i)$  of page  $i$  is defined as the probability that a random walker moving on the Web graph will visit  $i$  next, thereby estimating the page’s discovery probability according to the global structure of the Web. Experimental observations and theoretical results show that, with good approximation,  $p \sim k$  (see Appendix A). Therefore, in the surfing model where users only visit pages by following links, the traffic through a page is given by  $t \sim p \sim k$ .

### 2.2 Searching model of traffic

When navigation is mediated by search engines, to estimate the traffic directed toward a page, one must consider how search engines retrieve and rank results, as well as how people use these results. Following the seminal paper by Cho and Roy [9], this means that we need to find two relationships: (i) how the PageRank translates into the rank of a result page, and (ii) how the rank of a hit translates into the probability that the user clicks on the corresponding link thus visiting the page.

The first step is to determine the scaling relationship between PageRank (and equivalently in-degree as discussed above) and rank. Search engines employ many factors to rank pages. Such factors are typically query-dependent: whether the query terms appear in the title or body of a page, for example. They also use a global (query-independent) importance measure, such as PageRank, to judge the value of search hits. If we average across many user queries, we expect PageRank to determine the average rank  $r$  of each page within search results: the page with the largest  $p$  has average rank  $r \simeq 1$  and so on, in decreasing order of  $p$ .

Statistically,  $r$  and  $p$  have a non-linear relationship. There is an exact mathematical relationship between the value of a variable  $p$  and the rank of that value, assuming that a set of measures is described by a normalized histogram (or distribution)  $Pr(p)$ . The rank  $r$  is essentially the number of measures greater than  $p$ , i.e.,  $r = N \int_p^{p_{max}} Pr(x) dx$ , where  $p_{max}$  is the largest measure gathered and  $N$  the number of mea-



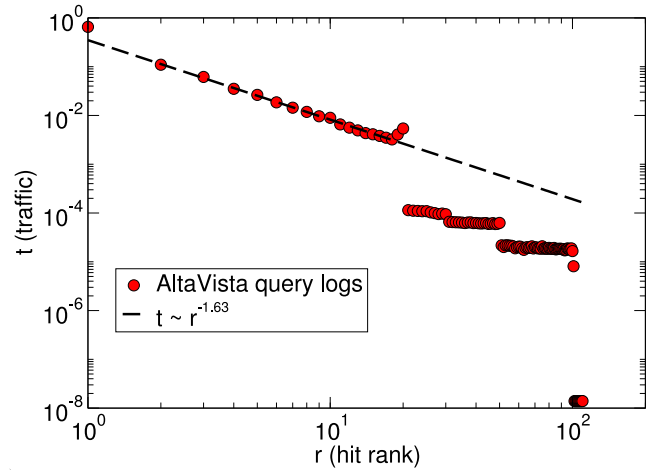
**Figure 2: A: Distribution of PageRank  $p$ :** the log-log plot shows a power law  $\text{Pr}(p) \sim p^{-2.1}$ . **B: Empirical relation between rank and PageRank:** the log-log plot shows a power law  $r \sim p^{-1.1}$ . Both plots are based on data from a WebBase 2003 crawl [30].

tures. Empirically we find that the distribution of PageRank is a power law  $p^{-\mu}$  with exponent  $\mu \approx 2.1$  (Fig. 2A). In general, when the variable  $p$  is distributed according to a power law with exponent  $-\mu$  and neglecting large  $N$  corrections one obtains:

$$r(p) \sim p^{-\beta} \quad (1)$$

where  $\beta = \mu - 1 \approx 1.1$ . Cho and Roy [9] derived the relation between  $p$  and  $r$  differently, by fitting the empirical curve of rank vs. PageRank obtained from a large WebBase crawl. Their fit returns a somewhat different value for the exponent  $\beta$  of  $3/2$ . To check this discrepancy we used Cho and Roy's method and fitted the empirical curve of rank vs. PageRank from our WebBase sample, confirming our estimate of  $\beta$  over three orders of magnitude (Fig. 2B).

The second step, still following ref. [9], is to approximate the traffic to a given page by the probability that when the page is returned by a search engine, the user will click on its link. We expect the traffic  $t$  to a page to be a decreasing function of its rank  $r$ . Lempel and Moran [21] reported a non-linear relation  $t \sim r^{-\alpha}$ , confirmed by our analysis using query logs from AltaVista as shown in Fig. 3.



**Figure 3: Scaling relationship between click probability  $t$  and hit rank  $r$ :** the log-log plot shows a power law  $t \sim r^{-1.63}$  (data from a sample of 7 million queries submitted to AltaVista between September 28 and October 3, 2001).

Note that the rank plotted on the x-axis of Fig. 3 does not refer exactly to the absolute position of a hit  $i$  in the list of hits, but rather to the rank of the result page where the link to  $i$  appears. Search engines display query results in pages containing a fixed number of hits (usually 10). Assuming that each result page contains 10 items, as in the AltaVista queries we examined, all hits from the first to the tenth will appear in the first result page and the corresponding click probabilities will be cumulated, giving the leftmost point in the plot. The same is done for the hits from the 11<sup>th</sup> to the 20<sup>th</sup>, from the 21<sup>st</sup> to the 30<sup>th</sup>, and so on. In lack of better information we consider result pages instead of single hits, implicitly assuming that within each result page the probability to click on a link is independent of its position. This assumption is reasonable, although there can still be a gradient between the top and the bottom hits, as people usually read the list starting from the top.

The sudden drop near the 21<sup>st</sup> result page in Fig. 3 is due to the way AltaVista operated during the summer 2001, when they decided to limit the list of results to 200 pages per query (displayed in 20 result pages). We therefore limited the analysis to the first 20 data points, which can be fitted quite well by a simple power law relation between the probability  $t$  that a user clicks on a hit and the rank  $r_p$  of the result page where this hit is displayed:

$$t \sim r_p^{-\alpha} \quad (2)$$

with exponent  $\alpha = 1.63 \pm 0.05$ . The fit exponent obtained by Cho and Roy was  $3/2$ , which is close to our estimate.

In our calculations we took into account the grouping of the hits in result pages, consistently with the empirical result of Fig. 3. However we noticed that if one replaces in Eq. 2 the rank  $r_p$  of the result page with the absolute rank  $r$  of the individual hits, the final results do not change appreciably. Therefore to simplify the discussion we shall assume from now on that

$$t \sim r^{-\alpha}. \quad (3)$$

The rapid decrease of  $t$  with the rank  $r$  of the hit clearly indicates that users focus with larger probability on the top results.

We are now ready to express the traffic as a function of page in-degree  $k$  using the general scaling relation  $t \sim k^\gamma$ . In the pure surfing model,  $\gamma = 1$ ; in the searching model, we take advantage of the relations between  $t$  and  $r$ , between  $r$  and  $p$ , and between  $p$  and  $k$  to obtain

$$t \sim r^{-\alpha} \sim (p^{-\beta})^{-\alpha} = p^{\alpha\beta} \sim k^{\alpha\beta} \quad (4)$$

and therefore  $\gamma = \alpha\beta$ , ranging between  $\gamma \approx 1.8$  (according to our measures  $\alpha \approx 1.63$ ,  $\beta \approx 1.1$ ) and 2.25 (according to estimates by others [21, 9]).

In all cases, the searching model leads to a value  $\gamma > 1$ . This superlinear behavior implies that the common use of search engines will bias traffic toward already popular sites. This is at the origin of the vicious cycle illustrated in Fig. 1. Pages highly ranked by search engines are more likely to be discovered (as compared to pure surfing) and consequently linked to by other pages. This in turn would further increase their PageRank and raise the average rank of those pages. Popular pages become more and more popular, while new pages are unlikely to be discovered [9]. Such a cycle would accelerate the rich-get-richer dynamics already observed in the Web's network structure [5, 16, 18]. This presumed phenomenon has been dubbed *search engine bias* or *entrenchment effect* and has been recently brought to the attention of the technical Web community [4, 9, 26], and methods to counteract it have been proposed [10, 26]. There are also notable social and political implications to such a *googlearchy* [14, 24, 13].

### 3. EMPIRICAL DATA

To determine whether such a vicious cycle really exists, let us consider the empirical data. Given a Web page, its in-degree is the number of links pointing to it, which can be easily estimated using a search engine such as Google or Yahoo [12, 32]. Traffic is the fraction of all user clicks in some period of time that lead to the page; this quantity, also known as *view popularity* [10], can be estimated using the Alexa Traffic Rankings service, which monitors the sites viewed by users of its toolbar [3]. We used the Yahoo and Alexa services to estimate in-degree and traffic for a total of 28,164 Web pages. Of these, 26,124 were randomly selected using Yahoo's random page service. The remaining 2,040 pages were selected among the sites that Alexa reports as the ones with highest traffic. The resulting density plot is shown in Fig. 4A.

To ensure the robustness of our analysis, we collected our data twice at a distance of two months. While there were differences in the numbers (for example Yahoo increased the size of its index significantly in the meanwhile), there were no differences in the scaling relations. We also collected in-degree data using Google [12], again yielding different numbers but the same trend. The in-degree measures exclude links from the same site. For example, to find the in-degree for <http://informatics.indiana.edu/>, we would submit the query "link:<http://informatics.indiana.edu/> -site:informatics.indiana.edu". Note that the in-degree data provided by search engines is only an estimate of the true number. First, a search engine can only know of links from pages that it has crawled and indexed. Second, for per-

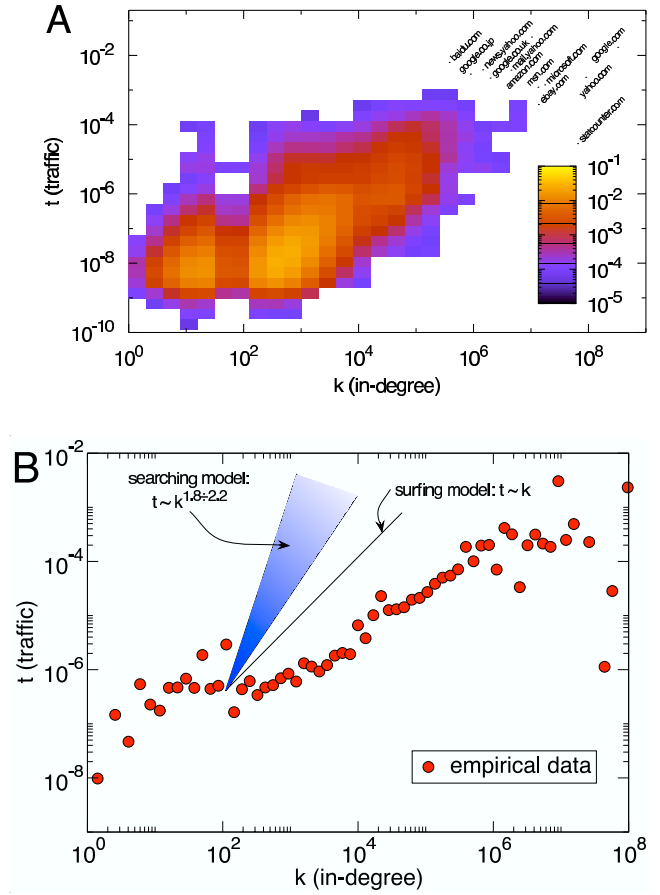


Figure 4: A. Density plot of traffic versus in-degree for a sample of 28,164 Web sites. Colors represent the fraction of sites in each log-size bin, on a logarithmic color scale. A few sites with highest in-degree and/or traffic are highlighted. The source of in-degree data is Yahoo [32]; using Google [12] yields the same trend. Traffic is measured as the fraction of all page views in a three-month period, according to Alexa data [3]. B. Relationship between average traffic and in-degree obtained with logarithmic binning of in-degree. The power-law predictions of the surfing and searching models discussed in the text are also shown.

formance reasons, the algorithms counting inlinks use various unpublished approximations based on sampling.

Traffic is measured as page views per million in a three-month period. Alexa collects and aggregates historical traffic data from millions of Alexa Toolbar users. Page views measure the number of pages viewed by these users. Multiple page views of the same page made by the same user on the same day are counted only once. Our measure of traffic  $t$  corresponds to Alexa's count, divided by  $10^6$  to express the fraction of all the page views by toolbar users go to a particular site. Since traffic data is only available for Web sites rather than single pages, we correlate the traffic of a site with the in-degree of its main page. For example, suppose that we want the traffic for <http://informatics.indiana.edu/>. Alexa reports the 3-month average traffic of the domain [informatics.indiana.edu](http://informatics.indiana.edu/)

as 9.1 page views per million. Further, Alexa reports that 2% of the page views in this domain goes to the `informatics.indiana.edu` subdomain. Thus we reach the estimate of 0.182 page views per million.

To derive a scaling relation, we average traffic along logarithmic bins for in-degree, as shown in Fig. 4B. Surprisingly, both the searching and surfing models fail to match the observed scaling, which is not modeled well by a power law. Contrary to our expectation, the scaling relation is sublinear, suggesting that search engines actually have an *egalitarian* effect, directing more traffic than expected to less popular sites — those having lower PageRank and fewer links to them. Search engines thus have the effect of counteracting the skewed distribution of links in the Web, directing some traffic toward sites that users would never visit otherwise. This result is at odds with the previous theoretical discussion; in order to understand the empirical data, we need to include a neglected but basic feature of the Web: the semantic match between queries and page content.

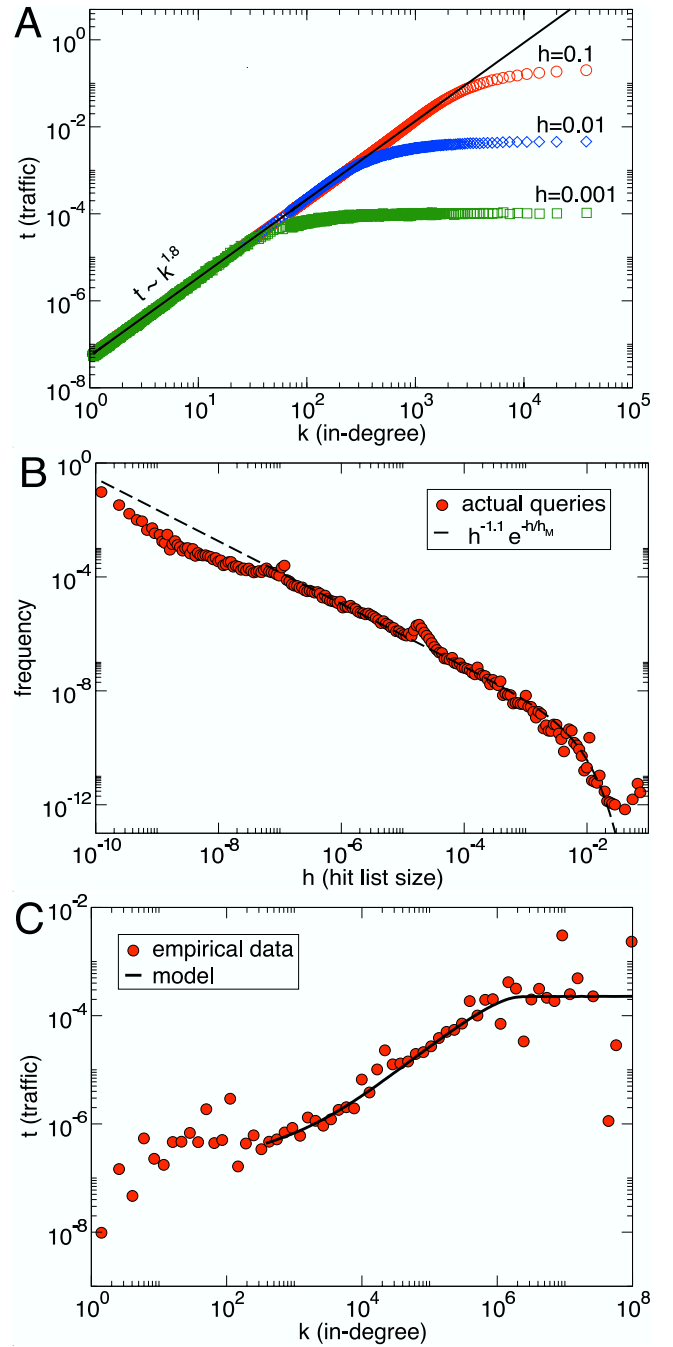
#### 4. QUERIES AND HIT SET SIZE

In the previous theoretical estimate of traffic as driven by search engines, we considered the global rank of a page, computed across *all* pages indexed by the search engine. However, any given query typically returns only a small number of pages compared to the total number indexed by the search engine. The size of the “hit” set and the nature of the query introduce a significant bias in the sampling process. If only a small fraction of pages are returned in response to a query, their rank within the set is not representative of their global rank as induced, say, by PageRank.

Let us assume that all query result lists derive from a Bernoulli process such that the number of hits relevant to each query is on average  $hN$  where  $h$  is the relative hit set size. In Appendix B we show that this assumption leads to an alteration in the relationship between traffic and in-degree. To illustrate this effect, Fig. 5A shows how the click probability changes with  $h$ . The result  $t \sim k^\gamma$  (or  $t \sim r^{-\alpha}$ , cf. Fig. 3) only holds in the limit case  $h \rightarrow 1$ . Since the size of the hit sets is not fixed, but depends on user queries, we measured the distribution of hit set sizes for actual AltaVista queries as shown in Fig. 5B, yielding  $\text{Pr}(h) \sim h^{-\delta}$ , with  $\delta \approx 1.1$  over seven orders of magnitude. The exponential cutoff in the distribution of  $h$  is due to the maximum size  $h_M$  of actual hit lists corresponding to non-noise terms, and thus can be disregarded for our analysis.

The traffic behavior is therefore a convolution of the different curves reported in Fig. 5A, weighted by  $\text{Pr}(h)$ . The final relation between traffic and degree can thus be obtained by numerical techniques (see Appendix B) and, strikingly, the resulting behavior reproduces the empirical data over four orders of magnitude, including the peculiar saturation observed for high-traffic sites (Fig. 5C). Most importantly, the theoretical behavior predicts a traffic increase for pages with increasing in-degree that is noticeably slower than the predictions of both the surfing and searching models. In other words, the combination of search engines, the semantic attributes of queries, and users’ own behavior mitigates the rich-get-richer dynamics of the Web, providing low-degree pages with increased visibility.

Of course, actual Web traffic is a combination of both surfing and searching behaviors. Users rely on search engines heavily, but also navigate from page to page through static



**Figure 5:** A. Scaling relationship between traffic and in-degree when each page has a fixed probability  $h$  of being returned in response to a query. The curves (not normalized for visualization purposes) are obtained by simulating the process  $t[r(k), h]$  (see Appendix B). B. Distribution of relative hit set size  $h$  for 200,000 actual user queries from AltaVista logs. The hit set size data were obtained from Google [12]. Frequencies are normalized by logarithmic bin size. The log-log plot shows a power law with an exponential cutoff. C. Scaling between traffic and in-degree obtained by simulating 4.5 million queries with a realistic distribution of hit set size on a one-million node network. Empirical data from Fig. 4B.

links as they explore the neighborhoods of pages returned in response to search queries [29]. It would be easy to model a mix of our revised searching model (taking into account the more realistic distribution of hit set sizes) with the random surfing behavior. The resulting mixture model would yield a prediction somewhere between the linear scaling  $t \sim k$  of the surfing model (cf. Fig. 4B) and the sublinear scaling of our searching model (cf. Fig. 5C). The final curve would be sublinear and still in agreement with the empirical traffic data.

## 5. DISCUSSION AND OUTLOOK

Our heavy reliance on search engines as a means of coping with the Web's size and growth does affect how we discover, link to, and visit pages. However, in spite of the rich-get-richer dynamics implicitly contained in the use of link analysis to rank search hits, the net effect of search engines on traffic appears to produce an egalitarian effect, smearing out the traffic attraction of high-degree pages. Our empirical data clearly shows a sublinear scaling relation between referral traffic from search engines and page in-degree. This seems to be in agreement with the observation that search engines lead users to visiting about 20% more pages than surfing alone [29]. Such an effect may be understood within a theoretical model of information retrieval that considers the users' clicking behavior and the heavy-tailed distribution observed for the number of query hits.

This result has relevant conceptual and practical consequences. It suggests that, contrary to intuition and prior hypotheses, the use of search engines contributes to a more level playing field, in which new Web sites have a greater chance of being discovered and thus of acquiring links and popularity — as long as they are about specific topics that match the interests of users as expressed through their search queries.

Such a finding is particularly relevant for the design of realistic models for Web growth. The connection between the popularity of a page and its acquisition of new links has led to the well-known rich-get-richer growth paradigm that explains many of the observed topological features of the Web. The present findings, however, show that several non-linear mechanisms involving search engine algorithms and user behavior regulate the popularity of pages. This calls for a new theoretical framework that considers more of the various behavioral and semantic issues that shape the evolution of the Web. How such a framework may yield coherent models that still agree with the Web's observed topological properties is a difficult and important theoretical challenge.

Finally, the present results provide a first quantitative estimate of, and prediction for, the popularity and traffic generated by Web pages. This estimate promises to become an important tool to be exploited in the optimization of marketing campaigns, the generation of traffic forecasts, and the design of future search engines.

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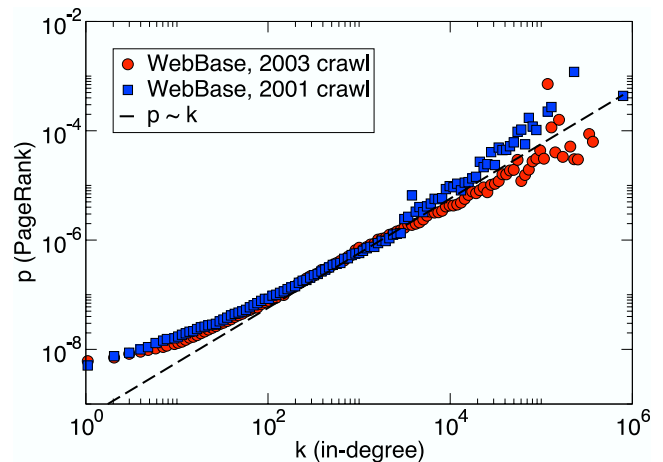
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## APPENDIX

### A. RELATIONSHIP BETWEEN IN-DEGREE AND PAGERANK

Let us inspect the scaling relationship between in-degree  $k$  and PageRank  $p$ . In our calculations of PageRank we used a damping factor 0.85, as in the original version of the algorithm [7] and in many successive studies. Our numerical analysis of the PageRank for the Web graph was performed on two samples produced by crawls made in 2001 and 2003 by the WebBase collaboration at Stanford [30]. The graphs are quite large: the former crawl has 80,571,247 pages and 752,527,660 links; the latter has 49,296,313 pages and 1,185,396,953 links.

In Fig. 6, in order to reduce fluctuations, we averaged the PageRank values over logarithmic bins of the degree. The data points mostly fall on a power law curve for both samples, with  $p$  increasing with  $k$ . The correlation coefficients



**Figure 6: PageRank as a function of in-degree for two samples of the Web taken in 2001 and 2003 [30].**

of the two sets of data, before binning, are 0.54 and 0.48 for the 2001 and 2003 crawl, respectively, as found for the Web domain of the University of Notre Dame [25], but in disagreement with the results of an analysis on the domain of Brown University and the WT10g Web snapshot [27]. The estimated exponents of the power law fits for the two curves are  $1.1 \pm 0.1$  (2001) and  $0.9 \pm 0.1$  (2003). As shown in Fig. 6, the two estimates are compatible with a simple linear relation between PageRank and in-degree. A linear scaling relation between  $p$  and  $k$  is also consistent with the observation that both have the same distribution. As it turns out,  $p$  and  $k$  are both distributed according to a power law with estimated exponent  $-2.1 \pm 0.1$ , in agreement with other estimates [27, 11, 8]. We assume, therefore, that PageRank and in-degree are, on average, proportional for large values.

### B. SIMULATION OF SEARCH-DRIVEN WEB TRAFFIC

When a user submits a query to a search engine, the latter will select all pages deemed relevant from its index and display the corresponding links ranked according to a combination of query-dependent factors, such as the similarity between the terms in the query and those in the page title, and query-independent prestige factors such as PageRank. Here we focus on PageRank as the main global ranking factor, assuming that query-dependent factors are averaged out across queries. The number of hit results depends on the query and it is in general much smaller than the total number of pages indexed by the search engine.

Let us start from the relation between click probability and rank in Eq. 3. If all  $N$  pages in the index were listed in each query, as implicitly assumed in ref. [9], the probability for the page with the smallest PageRank to be clicked would be  $N^\alpha$  ( $\alpha \approx 1.63$  in our study) times smaller than the probability to click on the page with the largest PageRank. If instead both pages ranked first and  $N^{\text{th}}$  appear among the  $n$  hits of a realistic query (with  $n \ll N$ ), they would still occupy the first and the last positions of the hit list, but the ratio of their click probabilities would be much smaller than before, i.e.  $n^\alpha$ . This leads to a redistribution of the clicking probability in favor of the less “popular” pages, which are

then visited much more often than one would expect at first glance. To quantify this effect, we must first distinguish between the *global* rank induced by PageRank across all Web pages and the *query-dependent* rank among the hits returned by the search engine in response to a particular query. Let us rank all  $N$  pages in decreasing order of PageRank, such that the global rank is  $R = 1$  for the page with the largest PageRank, followed by  $R = 2$  and so on.

Let us assume for the moment that all query result lists derive from a Bernoulli process with success probability  $h$  (i.e., the number of hits relevant to each query is on average  $hN$ ). The assumption that each page can appear in the hit list with the same probability  $h$  is in general not true, as there are pages that are more likely to be relevant than others, depending on their size, intrinsic appeal, and so on. If one introduces a fitness parameter to modulate the probability for a page to be relevant with respect to a generic query, the results would be identical as long as the fitness is not correlated with the PageRank of the page. In what follows we then stick to the simple assumption of equiprobability.

Let us calculate the probability  $\Pr(R, r, N, n, h)$  that the page with global rank  $R$  has rank  $r$  within a list of  $n$  hits. This is the probability  $p_{r-1}^{R-1}$  to select  $r-1$  pages from the set  $\{1 \dots R-1\}$ :

$$\begin{aligned} p_{r-1}^{R-1} &= h^{r-1}(1-h)^{R-1-(r-1)} \binom{R-1}{r-1} \\ &= h^{r-1}(1-h)^{R-r} \binom{R-1}{r-1} \end{aligned} \quad (5)$$

times the probability  $p_{n-r}^{N-R}$  to select  $n-r$  pages from the set  $\{R+1 \dots N\}$ , times the probability  $h$  to select page  $R$ . So we obtain:

$$\begin{aligned} \Pr(R, r, N, n, h) &= p_{r-1}^{R-1} p_{n-r}^{N-R} h \\ &= h^n (1-h)^{N-n} \binom{R-1}{r-1} \binom{N-R}{n-r}. \end{aligned} \quad (6)$$

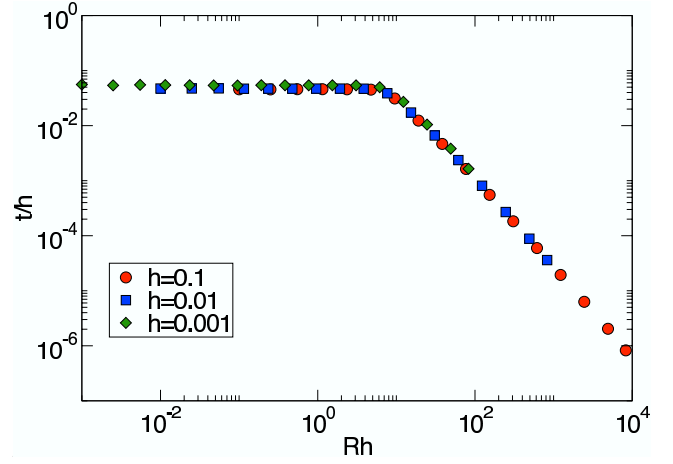
If page  $R$  has rank  $r$  in a list of  $n$  hits, the probability of being clicked will be

$$t(R, r, N, n, h) = \frac{r^{-\alpha}}{\sum_{m=1}^n m^{-\alpha}} \Pr(R, r, N, n, h) \quad (7)$$

where the denominator ensures the proper normalization of the click probability within the hit list. What remains to be done is to sum over the possible ranks  $r$  of page  $R$  in the hit list ( $r \in 1 \dots n$ ) and over all possible hit set sizes ( $n \in 1 \dots N$ ). The final result for the probability  $t(R, N, h)$  of the  $R$ -th page to be clicked is:

$$\begin{aligned} t(R, N, h) &= \sum_{n=1}^N \sum_{r=1}^n \frac{r^{-\alpha}}{\sum_{m=1}^n m^{-\alpha}} h^n (1-h)^{N-n} \cdot \\ &\quad \cdot \binom{R-1}{r-1} \binom{N-R}{n-r}. \end{aligned} \quad (8)$$

From Eq. 8 we can see that if  $h = 1$ , which corresponds to a list with all  $N$  pages, one recovers Eq. 3, as expected. For  $h < 1$ , however, it is not possible to derive a close expression for  $t(R, N, h)$ , so one has to calculate the binomials and perform the sums numerically. This can be easily done, but the time required to perform the calculation increases dramatically with  $N$ , so that it is not realistic to push the computation beyond  $N = 10^4$ . For this reason, instead of



**Figure 7: Scaling of  $t(R, N, h)/h$  with the variable  $Rh$ . The three curves refer to a sample of  $N = 10^5$  pages.**

carrying on an exact calculation, we performed Monte Carlo simulations of the process leading to Eq. 8.

In each simulation we produce a large number of hit lists, where every list is formed by picking each page of the sample with probability  $h$ . At the beginning of the simulation we initialize all entries of the array  $t(R, N, h) = 0$ . Once a hit list is completed, we add to the entries of  $t(R, N, h)$ , corresponding to the pages of the hit list, the click probability as given by Eq. 3 (with the proper normalization). With this Monte Carlo method we simulated systems with up to  $N = 10^6$  items. To eliminate fluctuations we averaged the click probability in logarithmic bins, as already done for the experimental data.

We found that the function  $t(R, N, h)$  obeys a simple scaling law:

$$t(R, N, h) = h F(Rh) A(N) \quad (9)$$

where  $F(Rh)$  has the following form:

$$F(Rh) \sim \begin{cases} \text{const} & \text{if } h \leq Rh \leq 1 \\ (Rh)^{-\alpha} & \text{if } Rh \geq 1. \end{cases} \quad (10)$$

An immediate implication of Eq. 9 is that if one plots  $t(R, N, h)/h$  as a function of  $Rh$ , for  $N$  fixed, one obtains the same curve  $F(Rh)A(N)$ , independently of the value of  $h$  (Fig. 7).

The decreasing part of the curve  $t(R, N, h)$ , for  $Rh > 1$  i.e.  $R > 1/h$ , is the same as in the case when  $h = 1$  (Eq. 3). This means that the finite size of the hit list affects only the top-ranked  $1/h$  pages. The effect is thus strongest when the fraction  $h$  is small, i.e., for specific queries that return few hits. The striking feature of Eq. 10 is the plateau for all pages between the first and the  $1/h$ -th. This implies that the difference in the values of PageRank among the top  $1/h$  pages does not produce a difference in the probability of clicking on those pages. For  $h = 1/N$ , which would correspond to lists containing on average a single hit, each of the  $N$  pages would have the same probability of being clicked, regardless of their PageRank. This is not surprising, as we assumed that all pages have the same probability to appear in a hit list.

So far we assumed that the number of query results is



drawn from a binomial distribution with a mean of  $hN$  hits. On the other hand, we know that real queries generate a broad range of possible hit set sizes, going from lists with only a single result to lists containing tens of millions of results. If the size of the hit list is distributed according to some function  $S(h, N)$ , one would need to convolute  $t(R, N, h)$  with  $S(h, N)$  to get the corresponding click probability:

$$t_S(R, N) = \int_{h_m}^{h_M} S(h, N) t(R, N, h) dh \quad (11)$$

where  $h_m$  and  $h_M$  are the minimal and maximal fraction of pages in a list, respectively. We stress that if there is a maximal hit list size  $h_M < 1$ , each curve  $t(R, N, h)$  of the overlap will have a flat portion going from the first to the  $1/h_M$ -th page, so in the set of pages ranked between 1 and  $1/h_M$  the click probability will be flat, independently of the distribution function  $S(h, N)$ .

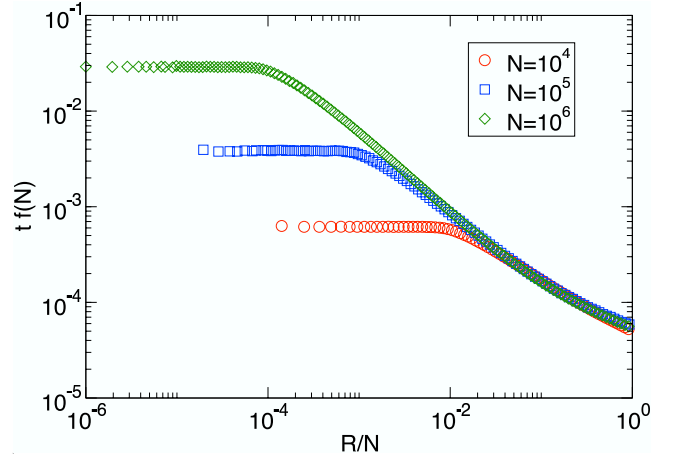
We obtained the hit list size distribution from a log of 200,000 actual queries submitted to AltaVista in 2001 (Fig. 5B). The data can be reasonably well fitted by a power law with an exponential cutoff due to the finite size of the AltaVista index. The exponent of the power law is  $\delta \approx 1.1$ . In our Monte Carlo simulations we neglected the exponential cutoff, and used the simple power law

$$S(h, N) = B(N) h^{-\delta} \quad (12)$$

where the normalization constant  $B(N)$  is just a function of  $N$ . The cutoff would affect only the part of the distribution  $S(h, N)$  corresponding to the largest values of  $h$ , influencing a limited portion of the curve  $t_S(R, N)$  and the click probability of the very top pages (cf. the scaling relation of Eq. 10). As there are no real queries that return hit lists containing all pages,<sup>1</sup> we have that  $h_M < 1$ . To estimate  $h_M$  we divided the largest observed number of Google hits in our collection of AltaVista queries (approximately  $6.6 \times 10^8$ ) by the total number of pages reportedly indexed by Google (approximately  $8 \times 10^9$  as of this writing), yielding  $h_M \approx 0.1$ . The top-ranked  $1/h_M \approx 10$  sites will have the same probability to be clicked. We then expect a flattening of the portion of  $t_S(R, N)$  corresponding to the pages with the highest PageRank/in-degree. This flattening seems consistent with the pattern observed in the real data (Fig. 5C).

As to the full shape of the curve  $t_S(R, N)$  for the Web, we performed a simulation for a set of  $N = 10^6$  pages. We used  $h_m = 1/N$ , as there are hit lists with a few or even a single result. The size of our sample is still very far from the total number of pages of the Web, so in principle we could not match the curve derived from the simulation with the pattern of the real data. However, the theoretical curves obey a simple scaling relation, as we can see in Fig. 8. It is indeed possible to prove that  $t_S(R, N)$  is a function of the ‘normalized’ rank  $R/N$  (and of  $N$ ) and not of the absolute rank  $R$ . On a log-log scale, this means that by properly shifting curves obtained for different  $N$  values along the  $x$  and  $y$  axes it is possible to make them overlap, exactly as we see in Fig. 8. This allows us to safely extrapolate to the limit of much larger  $N$ , and to lay the curve derived by our

<sup>1</sup>The policy of all search engines is to display at most 1000 hits, and we took this into account in our simulations. This does not mean that  $h \leq 1000/N$ ; the search engine scans all its database and can report millions of hits, but it will finally display only the top 1000.



**Figure 8: Scaling of  $t_S(R, N)$  for  $N = 10^4, 10^5, 10^6$ . The click probability  $t$  is multiplied for each curve by a number  $f(N)$  that depends only on  $N$ . In the limit  $N \rightarrow \infty$ ,  $f(N) \rightarrow N$ .**

simulation on the empirical data (as we did in Fig. 5C). The argument is rather simple, and is based on the *ansatz* of Eq. 9 for the function  $t(R, N, h)$  and the power law form of the distribution  $S(h, N)$  (Eq. 12). If we perform the convolution of Eq. 11, we have

$$t_S(R, N) = \int_{1/N}^{h_M} S(h, N) h A(N) F(Rh) dh, \quad (13)$$

where we explicitly set  $h_m = 1/N$  and  $F(Rh)$  is the universal function of Eq. 10. By plugging the explicit expression of  $S(h, N)$  from Eq. 12 into Eq. 13 and performing the simple change of variable  $z = hN$  within the integral we obtain

$$t_S(R, N) = \frac{A(N)B(N)}{N^{2-\delta}} \int_1^{h_M N} z^{1-\delta} F\left(\frac{R}{N} z\right) dz. \quad (14)$$

The upper integration limit can be safely set to infinity because  $h_M N$  is very large. The integral in Eq. 14 thus becomes a function of the ratio  $R/N$ . The additional explicit dependence on  $N$ , expressed by the term outside the integral, consists in a simple multiplicative factor  $f(N)$  that does not affect the shape of the curve (cf. Fig. 8).

We finally remark that the expression  $t_S(R, N)$  that we derived by simulation represents the relation between the click probability and the global rank of a page as determined by the value of its PageRank. For a comparison with the empirical data of Fig. 5C we need a relation between click probability and in-degree. We can relate rank to in-degree by means of Eq. 1 between rank and PageRank and by exploiting the proportionality between PageRank and in-degree discussed earlier.

However both Eq. 1 and the proportionality between  $p$  and  $k$  are not rigorous, but only hold in the asymptotic regime of low rank/large in-degree. If it were feasible to simulate queries on a Web graph with  $O(10^{10})$  nodes, the theoretical curve in Fig. 5C would extend over the entire range of the  $x$ -axis. In this case the low- $k$  part of the curve would have to be adjusted to account for the flattening observed in Fig. 6, which displays the relation between PageRank and in-degree. The leftmost part of this curve is quite flat for

over one order of magnitude, giving a plausible explanation for the flat pattern of the low- $k$  data in Fig. 5C.