Lista 4

Wiktor Kuchta (nr indeksu 315599)

21 marca 2023

1	2	3	4	5	6	7	8
+	+	+	+	+	+	+	

1.

$$\begin{aligned}
\neg &= \lambda b.\mathsf{if}\ b\ \mathsf{false}\ \mathsf{true} \\
\wedge &= \lambda bc.\mathsf{if}\ b\ c\ \mathsf{false} \\
\vee &= \lambda bc.\mathsf{if}\ b\ \mathsf{true}\ c \\
\rightarrow &= \lambda bc.\mathsf{if}\ b\ c\ \mathsf{true}
\end{aligned}$$

2.

$$\begin{aligned}
\mathsf{0} &= \lambda fx.x \\
\mathsf{suc} &= \lambda nfx.f(nfx) \\
\mathsf{Iter} &= \lambda nfa.nfa
\end{aligned}$$

$$\begin{aligned}
\mathsf{Iter}\ \mathsf{0}\ M\ N &\equiv (\lambda nfa.nfa)(\lambda fx.x)\ M\ N \\
&\to (\lambda fa.(\lambda fx.x)\ f\ a)\ M\ N \\
&\to (\lambda a.(\lambda fx.x)\ M\ a)\ N \\
&\to (\lambda fx.x)\ M\ N \\
&\to (\lambda x.x)\ N \\
&\to N
\end{aligned}$$

$$
\begin{aligned}
\mathsf{Iter}\ (\mathsf{suc}\ n)\ M\ N &\equiv (\lambda nfa.nfa)\ ((\lambda nfx.f(nfx))\ n)\ M\ N \\
&\to (\lambda nfa.nfa)\ (\lambda fx.f(nfx))\ M\ N \\
&\to^3 (\lambda fx.f(nfx))\ M\ N \\
&\to (\lambda x.M(nMx))\ N \\
&\to M\ (n\ M\ N) \\
&\leftarrow^3 M\ ((\lambda nfa.nfa)\ n\ M\ N) \\
&\equiv M\ (\mathsf{Iter}\ n\ M\ N)
\end{aligned}
$$

3.

$$G = \lambda yf.f(yf)$$

$$\forall F.\, MF = F(MF) \iff \forall F.\, MF = GMF \iff M = GM$$

W pierwszym przejściu korzystamy z $GMF = F(MF)$ oraz przechodniości, a w drugim η-redukcji (za F bierzemy zmienną) w prawo i kompatybilności z aplikacją w lewo.

4.

$$M(SI)F = SI(M(SI))F = IF(M(SI)F) = F(M(SI)F)$$

5.

$$
\begin{aligned}
Y^1 = Y(SI) = (\lambda x.SI(xx))(\lambda x.SI(xx)) &= (\lambda x.\lambda z.Iz(xxz))(\lambda x.\lambda z.Iz(xxz)) \\
&= (\lambda xz.z(xxz))(\lambda xz.z(xxz) = \Theta
\end{aligned}
$$

6.

```
newtype Self a = Fold { unfold :: Self a -> a }

a :: Self ((a -> b) -> a) -> (a -> b) -> b
a x y = y (unfold x x y)

theta :: (a -> a) -> a
theta = a (Fold a)
```

7.

$$
\begin{aligned}
h(\bar{x}) &= add(U_3^2(\bar{x}), U_3^3(\bar{x})) \\
mult(0, n) &= U_1^1(n) \\
mult(S(m), n) &= h(m, mult(m, n), n)
\end{aligned}
$$

$$
\begin{aligned}
h(\bar{x}) &= mult(U_3^2(\bar{x}), U_3^3(\bar{x})) \\
flippow(0, m) &= S(Z(m)) \\
flippow(S(n), m) &= h(n, flippow(n, m), m) \\
pow(\bar{x}) &= flippow(U_2^2(\bar{x}), U_2^1(\bar{x}))
\end{aligned}
$$

$$
\begin{aligned}
sucfst(\bar{x}) &= S(U_2^1(\bar{x})) \\
h(\bar{x}) &= mult(sucfst(\bar{x}), U_2^2(x)) \\
fact(0) &= 1 \\
fact(S(n)) &= h(n, fact(n))
\end{aligned}
$$