# Lista 8

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## 1.

$$\begin{split} \operatorname{BT} \beta &= \forall \alpha. \left(\beta \to \alpha \to \alpha \to \alpha\right) \to \alpha \to \alpha \\ \operatorname{Node} &= \Lambda \beta. \, \lambda x^{\beta}. \, \lambda l^{\operatorname{BT} \beta}. \, \lambda r^{\operatorname{BT} \beta}. \, \Lambda \alpha. \, \lambda n^{\beta \to \alpha \to \alpha \to \alpha}. \, \lambda t^{\alpha}. \, n \, x \, (l \, \alpha \, n \, t) \, (r \, \alpha \, n \, t) \\ \operatorname{Tip} &= \Lambda \beta. \, \Lambda \alpha. \, \lambda n^{\beta \to \alpha \to \alpha \to \alpha}. \, \lambda t^{\alpha}. \, t \\ \operatorname{\_BTit} &= \Lambda \beta. \, \lambda b^{\operatorname{BT} \beta}. \, \Lambda \alpha. \, \lambda n^{\beta \to \alpha \to \alpha \to \alpha}. \, \lambda t^{\alpha}. \, b \, \alpha \, n \, t \end{split}$$

\_BTit 
$$\beta$$
 (Node  $\beta$   $x$   $l$   $r$ )  $\alpha$   $n$   $t$   $\to^*$   $n$   $x$  (\_BTit  $\beta$   $l$   $\alpha$   $n$   $t$ ) (\_BTit  $\beta$   $r$   $\alpha$   $n$   $t$ ) \_BTit  $\beta$  Tip  $\alpha$   $n$   $t$   $\to^*$   $t$ 

#### 4.

$$\frac{\overline{\Gamma, \forall \alpha. \varphi[\alpha] \to \gamma \vdash \forall \alpha. \varphi[\alpha] \to \gamma}}{\Gamma, \forall \alpha. \varphi[\alpha] \to \gamma \vdash \varphi[\alpha := \sigma] \to \gamma} \underbrace{\forall E} 
\frac{\Gamma \vdash \varphi[\alpha := \sigma]}{\Gamma, \forall \alpha. \varphi[\alpha] \to \gamma \vdash \varphi[\alpha := \sigma]} \underbrace{\forall W}_{T \vdash (\forall \alpha. \varphi[\alpha] \to \gamma) \to \gamma} \to I$$

$$\frac{\overline{\Gamma, \forall \alpha. \varphi[\alpha] \to \gamma \vdash \gamma}}{\Gamma \vdash (\forall \alpha. \varphi[\alpha] \to \gamma) \to \gamma} \underbrace{\forall I}$$

$$\frac{\Gamma_1 \vdash \forall \gamma. (\forall \alpha. \varphi[\alpha] \to \gamma) \to \gamma}{\Gamma_1 \vdash (\forall \alpha. \varphi[\alpha] \to \psi) \to \psi} \underbrace{\forall E} 
\frac{\overline{\Gamma_2, \varphi[\alpha] \vdash \psi}}{\overline{\Gamma_2 \vdash \varphi[\alpha] \to \psi} \to I} \underbrace{\forall I}$$

$$\frac{\Gamma_1 \vdash (\forall \alpha. \varphi[\alpha] \to \psi) \to \psi}{\Gamma_1 \vdash (\forall \alpha. \varphi[\alpha] \to \psi) \to \psi} \underbrace{\forall E} 
\frac{\overline{\Gamma_2, \varphi[\alpha] \vdash \psi}}{\Gamma_2 \vdash \varphi[\alpha] \to \psi} \to I$$