Lista 7

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1	2	3	4	5	6
+	+				+

1.

(a)

$$
\begin{aligned}
FV(x) &= \{x\} \\
FV(\lambda x : \tau.M) &= FV(M) \setminus \{x\} \\
FV(MN) &= FV(M) \cup FV(N) \\
FV(\Lambda\alpha.M) &= FV(M) \\
FV(M\tau) &= FV(M)
\end{aligned}
$$

$$
\begin{aligned}
FTV(x) &= \{\} \\
FTV(\lambda x : \tau.M) &= FTV(\tau) \cup FTV(M) \\
FTV(MN) &= FTV(M) \cup FTV(N) \\
FTV(\Lambda\alpha.M) &= FTV(M) \setminus \{\alpha\} \\
FTV(M\tau) &= FV(M) \cup FTV(\tau) \\
FTV(\alpha) &= \{\alpha\} \\
FTV(\tau \to \sigma) &= FTV(\tau) \cup FTV(\sigma) \\
FTV(\forall\alpha.\tau) &= FTV(\tau) \setminus \{\alpha\}
\end{aligned}
$$

(b)

$$
\begin{aligned}
BV(x) &= \{\} \\
BV(\lambda x : \tau.M) &= BV(M) \cup \{x\} \\
BV(MN) &= BV(M) \cup BV(N) \\
BV(\Lambda\alpha.M) &= BV(M) \\
BV(M\tau) &= BV(M)
\end{aligned}
$$

$$
\begin{aligned}
BTV(x) &= \{\} \\
BTV(\lambda x : \tau.M) &= BTV(\tau) \cup BTV(M) \\
BTV(MN) &= BTV(M) \cup BTV(N) \\
BTV(\Lambda\alpha.M) &= BTV(M) \setminus \{\alpha\} \\
BTV(M\tau) &= FV(M) \cup BTV(\tau) \\
BTV(\alpha) &= \{\} \\
BTV(\tau \to \sigma) &= BTV(\tau) \cup BTV(\sigma) \\
BTV(\forall\alpha.\tau) &= BTV(\tau) \cup \{\alpha\}
\end{aligned}
$$

(c)

$$y[x := N] = \begin{cases} N, & \text{gdy } y = x \\ y, & \text{w p.w.} \end{cases}$$

$$(\lambda y : \tau.M)[x := N] = \begin{cases} \lambda y : \tau.M, & \text{gdy } x = y \\ \lambda y : \tau.M[x := N] & \text{gdy } y \notin FV(N) \\ \lambda z : \tau.M[y := z][x := N] & \text{w p.w. } (z \text{ świeże}) \end{cases}$$

$$(M_1 M_2)[x := N] = M_1[x := N] M_2[x := N]$$

$$(\Lambda\alpha.M)[x := N] = \Lambda\alpha.M[x := N]$$

$$(M\tau)[x := N] = M[x := N]\tau$$

$$y[\alpha := \sigma] = y$$

$$(\lambda x : \tau.M)[\alpha := \sigma] = (\lambda x : \tau[\alpha := \sigma].M[\alpha := \sigma])$$

$$(M_1 M_2)[\alpha := \sigma] = M_1[\alpha := \sigma] M_2[\alpha := \sigma]$$

$$(\Lambda\beta.M)[\alpha := \sigma] = \begin{cases} \Lambda\beta.M, & \text{gdy } \alpha = \beta \\ \Lambda\beta.M[\alpha := \sigma], & \text{gdy } \beta \notin FTV(\sigma) \\ \Lambda\gamma.M[\beta := \gamma][\alpha := \sigma], & \text{w p.w. } (\gamma \text{ świeże}) \end{cases}$$

$$(M\tau)[\alpha := \sigma] = M[\alpha := \sigma]\tau[\alpha := \sigma]$$

$$\beta[\alpha := \sigma] = \begin{cases} \sigma, & \text{gdy } \alpha = \beta \\ ,\beta & \text{w p.w.} \end{cases}$$

$$(\tau_1 \to \tau_2)[\alpha := \sigma] = \tau_1[\alpha := \sigma] \to \tau_2[\alpha := \sigma]$$

$$(\forall\beta.\tau)[\alpha := \sigma] = \begin{cases} \forall\beta.\tau, & \text{gdy } \alpha = \beta \\ \forall\beta.\tau[\alpha := \sigma], & \text{gdy } \beta \notin FTV(\sigma) \\ \forall\gamma.\tau[\beta := \gamma][\alpha := \sigma], & \text{w p.w. } (\gamma \text{ świeże}) \end{cases}$$

2.

$$\begin{aligned}
\mathsf{if}\ \mathsf{true}\ \rho\ M\ N &= (\lambda b^{\mathsf{Bool}}.\, \Lambda\alpha.\, \lambda u^{\alpha}.\, \lambda v^{\alpha}.\, b\ \alpha\ u\ v)\ \mathsf{true}\ \rho\ M\ N \\
&\to (\Lambda\alpha.\, \lambda u^{\alpha}.\, \lambda v^{\alpha}.\, \mathsf{true}\ \alpha\ u\ v)\ \rho\ M\ N \\
&\to (\lambda u^{\rho}.\, \lambda v^{\rho}.\, \mathsf{true}\ \rho\ u\ v)\ M\ N \\
&\to (\lambda v^{\rho}.\, \mathsf{true}\ \rho\ M\ v)\ N \\
&\to \mathsf{true}\ \rho\ M\ N \\
&\equiv (\Lambda\alpha.\, \lambda x^{\alpha}.\, \lambda y^{\alpha}.\, x)\ \rho\ M\ N \\
&\to (\lambda x^{\rho}.\, \lambda y^{\rho}.\, x)\ M\ N \\
&\to (\lambda y^{\rho}.\, M)\ N \to M
\end{aligned}$$

$$\begin{aligned}
\mathsf{if}\ \mathsf{false}\ \rho\ M\ N &= (\lambda b^{\mathsf{Bool}}.\, \Lambda\alpha.\, \lambda u^{\alpha}.\, \lambda v^{\alpha}.\, b\ \alpha\ u\ v)\ \mathsf{false}\ \rho\ M\ N \\
&\to (\Lambda\alpha.\, \lambda u^{\alpha}.\, \lambda v^{\alpha}.\, \mathsf{false}\ \alpha\ u\ v)\ \rho\ M\ N \\
&\to (\lambda u^{\rho}.\, \lambda v^{\rho}.\, \mathsf{false}\ \rho\ u\ v)\ M\ N \\
&\to (\lambda v^{\rho}.\, \mathsf{false}\ \rho\ M\ v)\ N \\
&\to \mathsf{false}\ \rho\ M\ N \\
&\equiv (\Lambda\alpha.\, \lambda x^{\alpha}.\, \lambda y^{\alpha}.\, y)\ \rho\ M\ N \\
&\to (\lambda x^{\rho}.\, \lambda y^{\rho}.\, y)\ M\ N \\
&\to (\lambda y^{\rho}.\, y)\ N \to N
\end{aligned}$$

3.

4.

5.

6.

$$\dfrac{\dfrac{\dfrac{\dfrac{x : \forall\alpha.\,\alpha\to\alpha \vdash x : \forall\alpha.\,\alpha\to\alpha}{x : \forall\alpha.\,\alpha\to\alpha \vdash x : (\beta\to\beta)\to\beta\to\beta}\,\forall E \qquad \dfrac{x : \forall\alpha.\,\alpha\to\alpha \vdash x : \forall\alpha.\,\alpha\to\alpha}{x : \forall\alpha.\,\alpha\to\alpha \vdash x : \beta\to\beta}\,\forall E}{x : \forall\alpha.\,\alpha\to\alpha \vdash x\,x : \beta\to\beta}\,\to E}{\vdash \lambda x.\, x\,x : (\forall\alpha.\,\alpha\to\alpha)\to\beta\to\beta}\,\to I}{\vdash \lambda x.\, x\,x : \forall\beta.\,(\forall\alpha.\,\alpha\to\alpha)\to\beta\to\beta}\,\forall I$$

Niech $B\alpha = \alpha\to\alpha\to\alpha$, $B\beta = \beta\to\beta\to\beta$.

$$\dfrac{\dfrac{\dfrac{\dfrac{x : \forall\alpha.\,B\alpha \vdash x : \forall\alpha.\,B\alpha}{x : \forall\alpha.\,B\alpha \vdash x : B\beta\to B\beta\to B\beta}\,\forall E \qquad \dfrac{x : \forall\alpha.\,B\alpha \vdash x : \forall\alpha.\,B\alpha}{x : \forall\alpha.\,B\alpha \vdash x : B\beta}\,\forall E}{x : \forall\alpha.\,B\alpha \vdash x\,x : B\beta\to B\beta}\,\to E}{\vdash \lambda x.\, x\,x : (\forall\alpha.\,B\alpha)\to B\beta\to B\beta}\,\to I}{\vdash \lambda x.\, x\,x : \forall\beta.\,(\forall\alpha.\,B\alpha)\to B\beta\to B\beta}\,\forall I$$