# 1 The Koch Snowflake

The *Koch snowflake*, one of the first fractals, is based on work by the Swedish mathematician Helge von Koch [1]. It is what we get if we start with an equilateral triangle

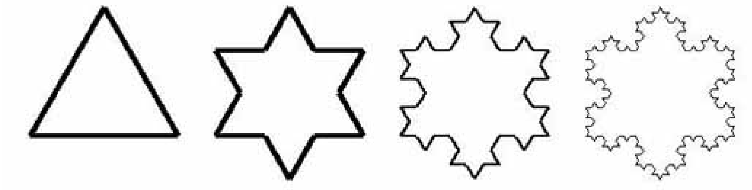

Figure 1: The initial equilateral triangle and the refinement of the Koch snowflake after one, two, and three iterations.

and repeat the following an infinite number of times:

> *Divide all line segments into three segments of equal length. Then draw, for each middle line segment, an equilateral triangle that has the middle segment as its base and points outward. Finally, remove all middle segments*

Figure 1 shows the first iterations in the constructions.

**Theorem 1.**

*Proof.* $\Delta$ $N_i$ $L_i$ $i$

$$N_n = \begin{cases} 3 & \text{if } n = 0 \text{ (i.e. before), and} \\ & \text{otherwise,} \end{cases}$$

which solves to

$$\cdot 4^n, \tag{1}$$

while

$$\frac{L_{n-3}}{3^3} \cdots \tag{2}$$

From Eqs. 1 and 2, the total length

$$\left(\frac{4}{3}\right)^n$$

$4/3 > 1$ $n \to \infty$ $\square$

*Proof.* Eq. 1, can be simplified to

$$T_n = . \tag{3}$$

$a_n$ of each such triangle, with the exception of the area of by Eqs. 3 and **??**, the area of all added triangles

All in all,

$$\begin{aligned} A_n &= a_0 + \sum_{k=1}^{n} b_k \\ &= a_0 \left( \right). \end{aligned}$$

Now, since

$$\lim_{n\to\infty},$$

it follows that, i.e. the Koch snowflake has finite area. □

# References


[1] Helge von Koch. *Sur une courbe continue sans tangente, obtenue par une construction géométrique élémentaire.*, Arkiv för matematik, astronomi och fysik, Kungliga Vetenskapsakademien. **1**, 681-702, 1904.