Sharing Power Storage for Undeveloped Countries

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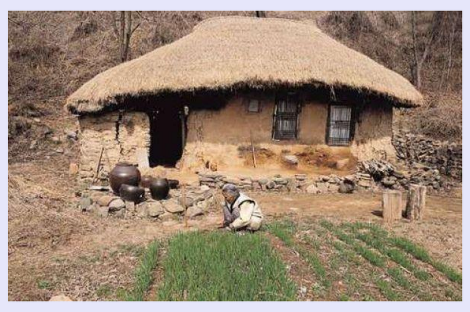
Background

- Undeveloped countries do not have 24-hour power
- Users purchase/rent batteries for light-up
- Shared batteries for risk pooling & better efficiency





Specification of the problem

In an undeveloped country where the electricity is only available for about one hour in a day, the organization decides to set up a storage system to provide full-day power to users.

As the grid is on only for a short time in a day, the organization fully charges the battery during this time, and discharges it to provide power to users for the rest of the day.

The total energy provided to users within the day cannot exceed the capacity, and if the capacity is depleted, a cost proportional to the shortage will be imposed on the society.

This research discusses about strategies for deciding on the capacity to minimize the total social cost and charging users in a fair way to cover the construction and running cost of the organization.

Challenges

- Optimal capacity?
  Trade off between efficiency and stability
  Solution: Investigate users' demand distribution
- Limited information
  Users do not even know their demands!
  Solution: Do power-on experiments for data
- Optimal sample size?
  Larger sample, more sampling cost but less error
  Solution: High-level estimation based on statistics
- How to charge users?
  Solution: Compromise between efficiency and fairness

Setting up

- Experimentally power $m \ll n$ users among n for demand data
- Decide on optimal capacity and fully build the system
- Charge users in a proper standard

Model of Costs

- System capacity: C
- Amortized daily cost of storage: $s \cdot C$, proportional to C
- User i's daily demand: $X_i \sim N(\mu_i, \sigma_i^2)$, independent
- $\sum X_i < C$: sparing cost rate s
- $\sum X_i > C$: shortage penalty rate $\gamma > s$
- $\sum X_i \sim N(\sum \mu_i, \sum \sigma_i^2)$
- Optimal capacity:

$$C_* = \sum \mu_i + K_1 \sqrt{\sum \sigma_i^2}$$

- Risk-reserving ratio K_1 s.t. (from SPP)

$$Q(K_1) = \frac{s}{s+\gamma}$$

  in which $Q(\cdot)$ is the gaussian tail probability function
- Basic capacity:

$$C_b = \sum \mu_i$$

- Risk-reserving capacity:

$$C_r = K_1 \sqrt{\sum \sigma_i^2}$$

Sample Setting

- Sampling cost:

$$c_{smp} = \Theta(m)$$

- Sampling deviation of $nE[\mu_i]$ and $nE[\sigma_i^2]$:

$$\delta_1^{(1)} = \Theta\left(\frac{n}{\sqrt{m}}\right)$$

- Error between expected sums and actual sums:

$$\delta_1^{(2)} = \Theta(\sqrt{n})$$

- Sampling error of parameters:

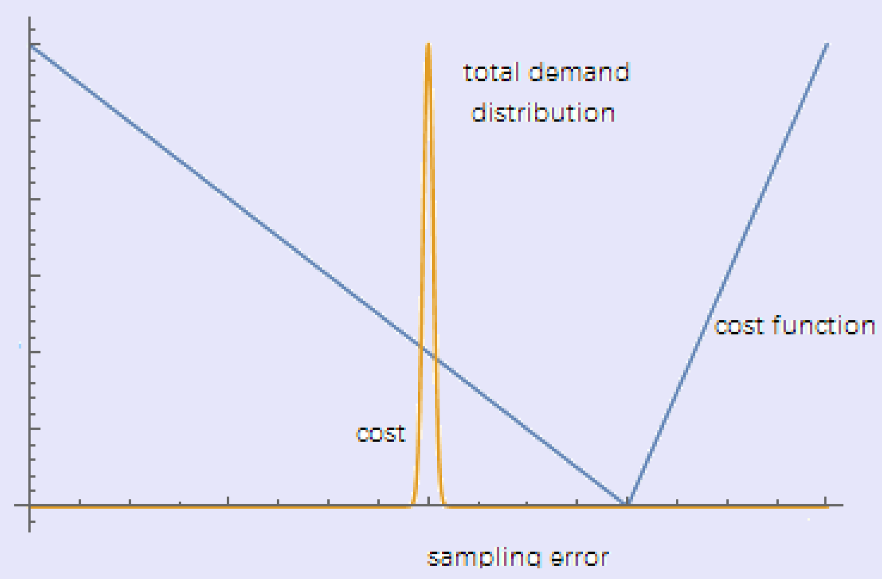
$$\begin{aligned}\delta_1 &= \delta_1^{(1)} + \delta_1^{(2)} \\ &= \Theta\left(\frac{n}{\sqrt{m}}\right)\end{aligned}$$

- Demand fluctuation:

$$\delta_2 = \Theta(\sqrt{n})$$

- For $n \to \infty, m = o(n)$, $\delta_1 \gg \delta_2$
- Error cost: c_{err}
- When $\delta_1 \gg \delta_2$:

$$c_{err} \sim \delta_1$$



The optimal sample size:

$$m_{opt} = \Theta(n^{2/3})$$

- When $\delta_1 \ll \delta_2$ (large $E[\sigma_i^2]$, small n)

$$c_{err} \sim \delta_1^2$$
$$m_{opt} \sim n$$

Payment Policy

- Assume that the organization do not want to make profit, only charge users to cover the cost.
- Only calculating the additional cost of the system, electricity cost is charged as usual.
- Negative externality of user i: w_i s.t.

$$\frac{\partial w_i}{\partial \mu_i} = s$$
$$\frac{\partial w_i}{\partial \sigma_i} = K_1 s \frac{\sigma_i}{\sqrt{nE[\sigma_i^2]}}$$

$$\Longrightarrow \quad w_i = \left(\mu_i + K_1 \frac{\sigma_i^2}{2\sqrt{nE[\sigma_i^2]}}\right) s$$

- Problem:

$$\begin{aligned}\sum w_i &= s \cdot \left(C_b + \frac{1}{2} C_r\right) \\ &< s \cdot C_*\end{aligned}$$

  —Due to scale effect: more users, lower marginal cost
  —Cannot cover the cost!
- Alternative 1:

$$w_i^{(1)} = \left(\mu_i + K_1 \frac{\sigma_i^2}{\sqrt{nE[\sigma_i^2]}}\right) s$$

  —Respecting the cost structure of the system
  —Realizing fairness of outcome
- Alternative 2:

$$w_i^{(2)} = \left(\mu_i + K_1 \frac{\sigma_i \sqrt{nE[\sigma_i^2]}}{nE[\sigma_i]}\right) s$$

  —Equivalence under the Veil of Ignorance
  —Realizing fairness of opportunity
- Conflicts between "fairness"
  If I exactly double up my power consumption, should I pay more than double?
  —Yes, because I am bringing more than double cost to the organization.
  —No, the extra cost is not my fault, it comes from some disadvantage of the system!
- When $n \to \infty$, as the portion of C_r diminishes with $\frac{1}{\sqrt{n}}$, user i's payment in all policies converges to $\mu_i \cdot s$
  When we share more, there will be not only more efficiency, but also fewer conflicts!