Dynamic Car Dispatching and Pricing: Revenue and Fairness for Ridesharing

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  - Using over-simplified models (equidistant, specified graphs, etc)
  - No revenue or fairness guarantees. (Will companies use and will drivers and riders be satisfied?)

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- We propose a versatile generalized network flow model for the task, with theoretical guarantees on optimal revenue.
- We propose a novel two-phase pricing mechanism decoupling prices on drivers' and riders' sides to meet misaligned interests and ensure fairness.
- We consider the stochastic nature of ridesharing orders and perform online learning on incomplete information to balance the exploration-exploitation trade-off.

Phase 1: Max-Revenue Car Dispatching
Non-linear network flow model

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- $r_{(A,B)}(3 + j) = 18 - 2j$, $j \geq 0$.

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- e_{cruise}: capacity $+\infty$, weight $-c = -2$.

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Hardness

Without the regularity constraint, Max-Revenue Car Dispatching is NP-hard.

- We can construct a MRCD to solve Set Cover.

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- Envy-freeness: No driver feels the mechanism is more favorable to others than themselves.

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Lemma

Given a routing plan $\mathcal{A}$, a reward re-allocation $y : S^2 \to \mathbb{R}^{\geq 0}$ is fair if and only if there exists a *potential function* $P : S \to \mathbb{R}^{\geq 0}$ such that

- For any $s \in S$ where $\mathcal{A}$ directs at least one driver to leave at state s (*terminal states*), it holds that $P(s) = 0$.
- $\forall (s, s') \in Q$, $y(s, s') - c(s, s') \leq P(s) - P(s')$.
- $\forall (s, s') \in Q : F(s, s') > 0$, $y(s, s') - c(s, s') = P(s) - P(s') \geq 0$.
- $\sum_{s \in S} P(s)(\deg_i(s) - \deg_o(s)) = \sum_{(s,s') \in Q} F(s, s')(p(s, s') - c(s, s'))$, where $\deg_i(s)$ and $\deg_o(s)$ are the number of drivers to enter and leave the platform at the state s respectively.

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Theorem

Let $S_{\#} \subseteq S$ denote the set of terminal states. If there exist $s_1 \in S \setminus S_{\#}$ and $s_2 \in S$ such that $F(s_1, s_2) > 0$ and $\mathcal{I} \geq \sum_{(s,s') \in Q} F(s,s') \cdot c(s,s')$ ($\mathcal{I}$ is the total income collected from the riders), then there exists a fair reward allocation plan.

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- We use Quadratic Programming to find a min-square-distortion feasible scheme, if multiple ones exist.

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Stochastic demand setting

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- We can still compute the edge reward function as *expected* rewards for k drivers dispatched on (s, s'), from the distribution $\mathcal{D}(s, s')$.
- We then perform Thompson Sampling to balance exploration and exploitation and update the estimated distributions.

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- We set the posterior distributions as the prior of day $t + 1$.

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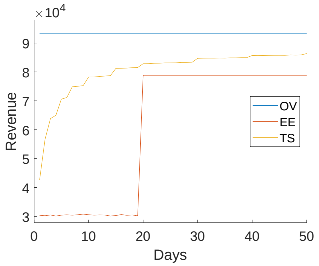
- Online: Significantly lower regret than explore-and-exploit scheme.



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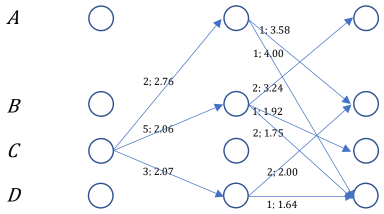

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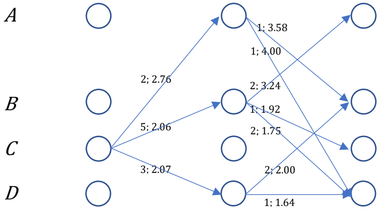
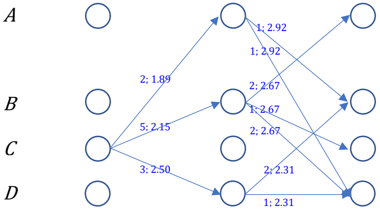
- After Phase 2: both 4.81.





Acknowledgement

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Thank you!