

An abstract graphic consisting of several flowing, curved lines in shades of light blue and white, creating a sense of motion and fluidity. The lines are layered and overlap, giving it a three-dimensional feel.

Dynamic Car Dispatching and Pricing: Revenue and Fairness for Ridesharing

Zishuo Zhao, Xi Chen, Xuefeng Zhang, Yuan Zhou



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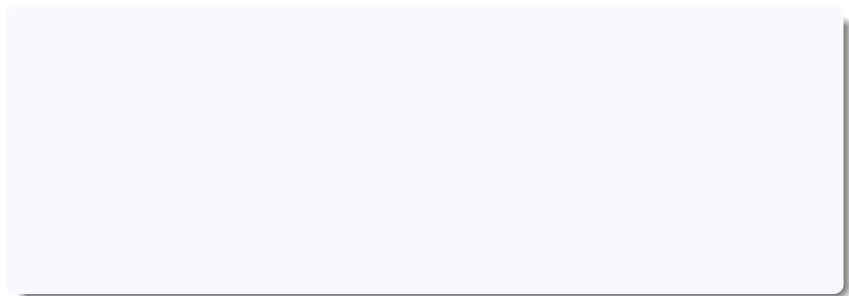
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 - ▶ Using over-simplified models (equidistant, specified graphs, etc)
 - ▶ No revenue or fairness guarantees. (Will companies use and will drivers and riders be satisfied?)



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- ▶ We propose a versatile generalized network flow model for the task, with theoretical guarantees on optimal revenue.
- ▶ We propose a novel two-phase pricing mechanism decoupling prices on drivers' and riders' sides to meet misaligned interests and ensure fairness.
- ▶ We consider the stochastic nature of ridesharing orders and perform online learning on incomplete information to balance the exploration-exploitation trade-off.

Phase 1: Max-Revenue Car Dispatching

Non-linear network flow model



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Basic example:

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- ▶ $r_{(A,B)}(3 + j) = 18 - 2j$, $j \geq 0$.

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- ▶ e_{cruise} : capacity $+\infty$, weight $-c = -2$.



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Theorem

With regularity condition, the Max-Revenue Car Dispatching can be solved with the maximum weighted flow problem.

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Hardness

Without the regularity constraint, Max-Revenue Car Dispatching is NP-hard.

- ▶ We can construct a MRCD to solve Set Cover.

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Within the same total budget, we can allocate rewards differently to drivers.

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- ▶ Envy-freeness: No driver feels the mechanism is more favorable to others than themselves.

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Lemma

Given a routing plan \mathcal{A} , a reward re-allocation $y : S^2 \rightarrow \mathbb{R}^{\geq 0}$ is fair if and only if there exists a *potential function* $P : S \rightarrow \mathbb{R}^{\geq 0}$ such that

- ▶ For any $s \in S$ where \mathcal{A} directs at least one driver to leave at state s (*terminal states*), it holds that $P(s) = 0$.
- ▶ $\forall (s, s') \in Q, y(s, s') - c(s, s') \leq P(s) - P(s')$.
- ▶ $\forall (s, s') \in Q : F(s, s') > 0, y(s, s') - c(s, s') = P(s) - P(s') \geq 0$.
- ▶ $\sum_{s \in S} P(s)(\deg_i(s) - \deg_o(s)) = \sum_{(s, s') \in Q} F(s, s')(p(s, s') - c(s, s'))$, where $\deg_i(s)$ and $\deg_o(s)$ are the number of drivers to enter and leave the platform at the state s respectively.

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Theorem

Let $S_{\#} \subseteq S$ denote the set of terminal states. If there exist $s_1 \in S \setminus S_{\#}$ and $s_2 \in S$ such that $F(s_1, s_2) > 0$ and $\mathcal{I} \geq \sum_{(s, s') \in Q} F(s, s') \cdot c(s, s')$ (\mathcal{I} is the total income collected from the riders), then there exists a fair reward allocation plan.

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- We use Quadratic Programming to find a min-square-distortion feasible scheme, if multiple ones exist.



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- ▶ We can still compute the edge reward function as *expected* rewards for k drivers dispatched on (s, s') , from the distribution $\mathcal{D}(s, s')$.
- ▶ We then perform Thompson Sampling to balance exploration and exploitation and update the estimated distributions.



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- ▶ We set the posterior distributions as the prior of day $t + 1$.

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- ▶ Offline: 23% increased revenue than fixed price on DiDi dataset.

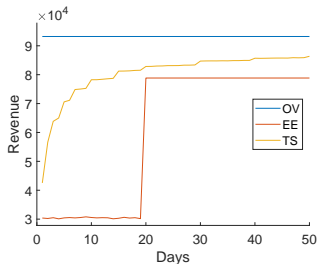
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- ▶ Online: Significantly lower regret than explore-and-exploit scheme.



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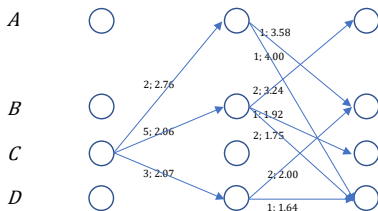
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12

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- $C \rightarrow A \rightarrow D$ has significantly higher reward than $C \rightarrow D \rightarrow D$.



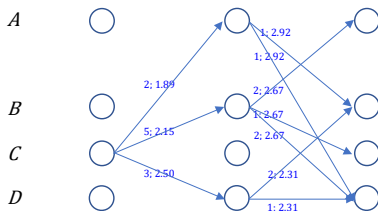
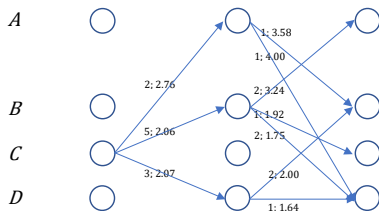
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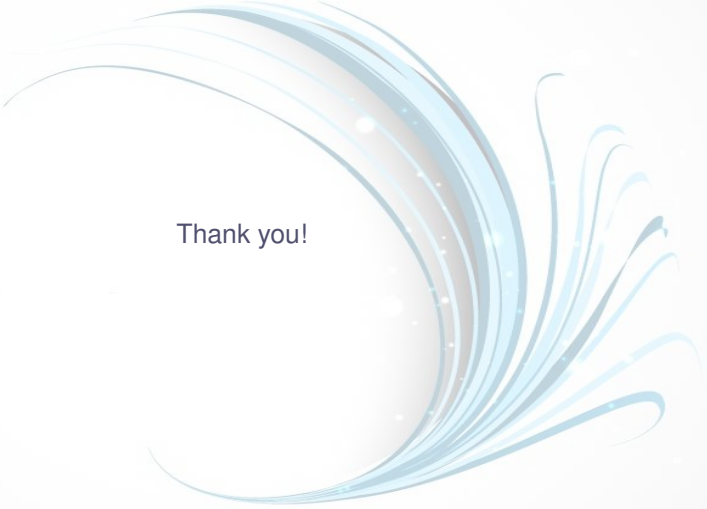
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- ▶ $C \rightarrow A \rightarrow D$ has significantly higher reward than $C \rightarrow D \rightarrow D$.
- ▶ After Phase 2: both 4.81.





Appreciation to Shiyuan Wang for discussion of relevant topics on mechanism design.



Thank you!