Dynamic Car Dispatching and Pricing: Revenue and Fairness for Ridesharing Platforms

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  - Myopic, only considering current orders but not incoming ones. (mostly as combinatorial optimization algorithms)
  - Only computing centralized plans, without ensuring drivers would be willing to follow the plan.
  - Using over-simplified models (equidistant, specified graphs, etc)
  - No revenue or fairness guarantees. (Will companies use and will drivers and riders be satisfied?)

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- We propose a novel two-phase pricing mechanism decoupling prices on drivers' and riders' sides to meet misaligned interests and ensure fairness.
- We consider the stochastic nature of ridesharing orders and perform online learning on incomplete information to balance the exploration-exploitation trade-off.

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- Each edge corresponding to a spatiotemporal path.

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- $r_{(A,B)}(3 + j) = 18 - 2j$, $j \geq 0$.

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- e_{cruise}: capacity $+\infty$, weight $-c = -2$.

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Hardness

Without the regularity constraint, Max-Revenue Car Dispatching is NP-hard.

- We can construct a MRCD to solve Set Cover.

NP-Hardness: Proof Sketch

- Construct an arc with $r(x+1) = A \cdot [x \geq k] + C$:
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- Construct a virtual arc (multiple paths in parallel) with
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  If there are N items and the optimal total reward is $N - m$, the minimum cover number is m.

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- Q.E.D.

Phase 2: Fair Reward Re-allocation to Drivers

Issue: Taxi Drivers’ Dilemma

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- Can we raise prices of cross-river trips in rush hour?
- Riders: "It is slow already, and you ask me to pay even more?"
- How to resolve misaligned incentives via pricing?

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- Subgame-perfectness: No driver is incentivized to deviate.
- Envy-freeness: No driver feels the mechanism is more favorable to others than themselves.

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Potential method

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Lemma

Given a routing plan $\mathcal{A}$, a reward re-allocation $y : S^2 \to \mathbb{R}^{\geq 0}$ is fair if and only if there exists a *potential function* $P : S \to \mathbb{R}^{\geq 0}$ such that

- For any $s \in S$ where $\mathcal{A}$ directs at least one driver to leave at state s (*terminal states*), it holds that $P(s) = 0$.
- $\forall (s, s') \in Q$, $y(s, s') - c(s, s') \leq P(s) - P(s')$.
- $\forall (s, s') \in Q : F(s, s') > 0$, $y(s, s') - c(s, s') = P(s) - P(s') \geq 0$.
- $\sum_{s \in S} P(s)(\deg_i(s) - \deg_o(s)) = \sum_{(s,s') \in Q} F(s, s')(p(s, s') - c(s, s'))$, where $\deg_i(s)$ and $\deg_o(s)$ are the number of drivers to enter and leave the platform at the state s respectively.

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Theorem

Let $S_{\#} \subseteq S$ denote the set of terminal states. If there exist $s_1 \in S \setminus S_{\#}$ and $s_2 \in S$ such that $F(s_1, s_2) > 0$ and $\mathcal{I} \geq \sum_{(s,s') \in Q} F(s, s') \cdot c(s, s')$ ($\mathcal{I}$ is the total income collected from the riders), then there exists a fair reward allocation plan.

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- Proof sketch: Topological Ordering
- We use Quadratic Programming to find a min-square-distortion feasible scheme, if multiple ones exist.

Stochastic demand setting: we only know the distribution of latent orders $\{\mathcal{D}(s, s')\}$ rather than exact ones.

- Setting:
  —Platform sets price p for an edge;
  —Riders accept the offer iff valuation is at least p *(qualified)*;
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- We can still compute the edge reward function as *expected* rewards for k drivers dispatched on (s, s'), from the distribution $\mathcal{D}(s, s')$.
- We then perform Thompson Sampling to balance exploration and exploitation and update the estimated distributions.

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- The probability mass function $\mathcal{P}(\cdot; s, s', p) : \mathbb{N} \to \mathbb{R}$ of the number of qualified orders:
  $\mathcal{P}(i; s, s', p) = \sum_{j=0}^{\infty} b(i, j; \Pr[v_t \geq p]) \Pr[x_{s,s'} = j]$, where
  $b(k, n; P) = \binom{n}{k} P^k (1 - P)^{n-k}$ computes the binomial distribution.

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  $b(k, n; P) = \binom{n}{k} P^k (1 - P)^{n-k}$ computes the binomial distribution.
- Let $\tilde{u}(n; s, s', p)$ be the number of the fulfilled orders; its expectation: $\mathbb{E}[\tilde{u}(n; s, s', p)] = \sum_{i=0}^{\infty} \mathcal{P}(i; s, s', p) \min\{i, n\}$.

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- The expected revenue at price p and n drivers:
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- The expected revenue function at optimal price:
  $r(n; e_{s,s'}^{(\mathrm{w})}) = \max_{p \in \mathbb{R}^{\geq 0}} \{\mathcal{R}(n, p; s, s')\}$

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- We set the posterior distributions as the prior of day $t + 1$.

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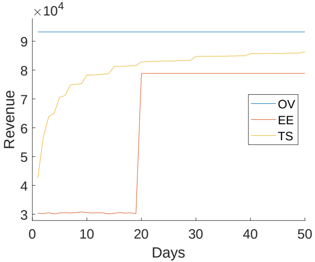
- Online: Significantly lower regret than explore-and-exploit scheme.



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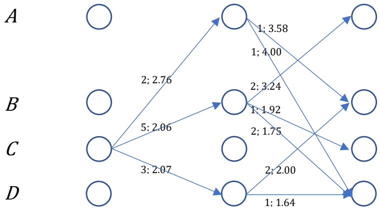

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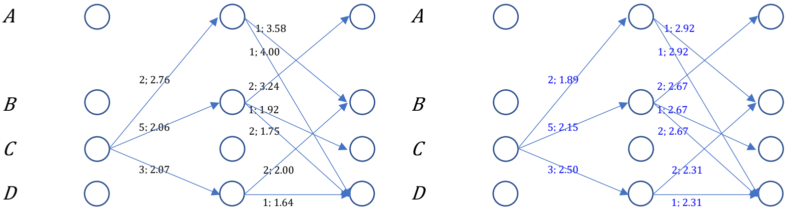
- After Phase 2: both 4.81.



Acknowledgement

- Thank Shiyuan Wang for discussion of relevant topics on mechanism design.

Q/A