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Summary Sheet

To Merge in the Streamline

As part of the toll highway, barrier tolls frequently become the bottleneck of the high-speed traffic, which is a critical issue for transportation engineers. Due to the low processing efficiency of traditional tollbooths, we have to build more tollbooths than lanes. In our paper, we have designed a curved shape for the fan-in part after the toll gate, which can improve passing efficiency, lowers accident rate and save unnecessary costs.

Firstly, to find out the capacity of lanes, we computed the minimum distance between vehicles to ensure safety (as a function of speed). The idea of safety distance serves as the basis of the whole paper. Since most drivers keep this safety distance while driving, it can indicate the maximum density of vehicles on the highway. Then we are able to derive the maximum traffic throughput, i.e. the capacity, of each lane. By comparing this to the toll collecting efficiency of a tollbooth, we worked out the minimum *B/L* ratio for highways.

Secondly, to merge *B* lanes from tollbooths to *L* lanes of the highway, we estimated the passing efficiency without any guidance. For simplification we used the gridded-road model. The model divides the road into grids according to the safety distance, and for each period all vehicles try to move on to the next grid. Using an analogy with the information channel model, we did a rough estimation which can show the dramatic decreased passing efficiency in heavy traffic. So some guidance is in need.

To make the merging smoother, we introduced the trapezoid convergence model with a long trapezoid road where the number of lanes decreases one by one. In the gradual merging, we assume deadlocks no longer happen and as many vehicles as possible can move one grid for each iteration. The simulation shows that the passing efficiency keeps to ~97% even in saturated traffic, suggesting fairly good result in practice.

Thirdly, we noticed that traffic accelerates when flowing out of tollbooths, which leads to a gradual increase in safety distance, so the trapezoid should be deformed into a trumpet shape. On the other hand, our simulation shows that more congestions take place near the end of the convergence area, which also suggests a longer tail. In our trumpet-shaped convergence model, we carefully discussed the optimal shape taking efficiency, safety, fluency and cost into account, and specified the most suitable shape considering all aspects. In order to combine two sides of the road in a compact and elegant way, we designed a pattern to stagger tollbooths on different sides.

Finally, we reached the conclusion, contrary to the common belief, that wider areas do not necessarily mean more fluent traffic flow, and it is proper guidance that leads vehicles to merge efficiently and safely. We also discussed the variation of traffic situations and tollbooth types, proved that our models are applicable to a wide range of scenes, and got an extra finding that modern technology can greatly decrease the land cost, or even supersede the configuration of fan-out and fan-in areas.

Key words: safety distance, grid model, channel model, dynamic simulation, multi-target optimization

Contents

1 Introduction

1.1 Background

As a complementary source of road taxes, the toll highways have been used in many countries in recent years. In America, for example, there are 9500-km-long toll highways in 26 states up to 2013. Though merely accounting for 3.6% of all the highways, there are more toll highways in the planning. In 2013 the length of toll highways increased by 9% compared to 2011, and the number of vehicles in toll highways increased by 14% in 2015 compared to 2011. What's more, considering the traffic accident rate, the toll highways are safer with 0.5 times per hundred million vehicle miles accident rate than the roads free of charge with 1.47 times per hundred million vehicle miles. Therefore, a good toll plaza model is in need.

1.2 Problem Restatement

The problem that we need to solve in this paper is analyzed into three parts:

- Develop a model of a toll plaza to facilitate the barrier toll, consisting of the fan-out area before the barrier toll, the toll barrier itself, and the fan-in area after the toll barrier. Determine the shape, size, and merging pattern of the area following the toll barrier.
- Take the accident prevention, throughput (numbers of vehicles per hour passing the point where the end of the plaza joins the L outgoing traffic lanes), and cost (mainly land and road construction) into consideration.
- Determine the performance of our model in light and heavy traffic, and the performance as more autonomous vehicles are added to the traffic mix. Discuss how our model is affected by the proportions of conventional tollbooths, exact-change tollbooths, and electronic toll collection booths.

1.3 Existing Models Review

There are already some toll plaza models in use. One kind of model is the number of tollbooths is only a little more than the number of lanes, which will cause the queue in the entrance of the toll station. Another kind of model, with the shape of “D” on one side as the **Figure 1**, have a no ideal length of the toll station, which may cost more to build, and will cause the uneven traffic flow, which may lead to the traffic accidents.

Figure 1 one of the toll plaza model in our life

As we can see above, the present models can be improved in the efficiency of the toll plaza, the prevention of traffic accidents and the cost of the buildings.

1.4 Our Work

In our paper, we discuss the “barrier toll”, which collect tolls from motorists on Multi-lane divided limited-access toll highways. We will determine the shape, size, and merging pattern of the area following the toll barrier in which vehicles fan in from tollbooth egress lanes down to lanes of the highway. Our model improves in some aspects including accident prevention, throughput and cost. In the expansion, we will illustrate that our model can suffice for many other cases, such as the autonomous vehicles, the automated tollbooths, and the light or heavy traffic.

2.Terminology

2.1 Term

- Reaction distance: Reaction time is the period from the moment when the driver finds the emergency to the moment when the driver presses on the brake pedal. Reaction distance is the length that the vehicle travels during this period.
- Braking distance: Braking distance refers to the distance a vehicle will travel from the point when its brakes are fully applied to when it comes to a complete stop.

- Traffic volume: The number of vehicles passing through a certain cross section of the road per unit of time.
- Channel model: It can be expressed as follows. If the number of vehicles exceeds the capacity of the road at certain time, all vehicles will be blocked up and cannot move forward, otherwise all vehicles can move forward. When encountering conflict, all vehicles can only wait until the following two requirements is satisfied:

 •some cars make a concession to let others go first;

 •the number of vehicles attempted to move forward no more than the capacity of the road.

 Only then can vehicles move later.

2.2 Symbol

Symbol	Description	Unit
d	safe distance: the shortest distance between two adjacent cars which avoids crash when the front car stops suddenly	m
t_0	reaction time of the driver to stop his/her car when the front car suddenly stops	s
a	braking acceleration	$m \cdot s^{-2}$
l	average length of vehicles	m
t	a period of time	s
v	the speed of traffic flow	$m \cdot s^{-1}$
n	the number of vehicles passing by a certain cross section during t (a period of time)	1
P	the traffic volume of the section	$1 \cdot h^{-1}$
L	number of lanes that can be driven in each direction	1
W	the processing efficiency of the toll plaza	$1 \cdot h^{-1}$
W_1	the processing efficiency of the toll plaza using human-staffed tollbooths	$1 \cdot h^{-1}$
W_2	the processing efficiency of the toll plaza using automated tollbooths	$1 \cdot h^{-1}$
W_3	the processing efficiency of the toll plaza using electronic tollbooths	$1 \cdot h^{-1}$

B	the number of tollbooths in each direction	1
E	the expected rate of passing capacity from tollbooth to road	$1 \cdot h^{-1}$
η	passing efficiency: the average expected rate of not delaying	1
p	unwilling probability (*p*): the probability that a driver attempts to move forward and isn't willing to let others go first when encounters conflict willing probability(*1-p*): the opposite of unwilling probability. It is the probability that a driver attempts to let others go first when encounters conflict.	1
k	the probability that a grid is occupied by a vehicle	1
q	the probability that a vehicle comes out of an arbitrary tollbooth in Δt	1
x	$x=p\times q$. It is only a symbol to simplify the formula in the following essay.	1
Δt	$\Delta t=\frac{(d+l)}{v}$. It means the time it takes to drive through each lattice.	s
w_L	the width of a lane on highway	m
w_B	the total width of a lane in the tollbooth	m
H	the length of the trapezoid merging area	m

3.Model

3.1 Model 1: the traffic stream model based on safety distance

3.1.1 Assumption

In this model, we give two assumptions to simplify the question.

●The distance between adjacent vehicles are safe distance when in the traffic peak.

●The safe distance can be given by $d=\frac{v^2}{2a}+vt_0$. Because safe distance consists of reaction distance and braking distance. In this case, we assume that the car moves uniformly in the process of reaction and decelerates uniformly in the braking process. Generally, a is about $5m \cdot s^{-2}$ and t_0 is 1s or so.

3.1.2 The estimation of the number of tollbooths

First we only deal with the simplest condition when there is no congestion at the toll plaza. In order to ensure that vehicles don't block up, we should figure out the lower bound to the ratio of tollbooth and lane (B / L). We have already known that the safe distance can be given by the following equation.

$$d=\frac{v^2}{2a}+vt_0 \tag{1}$$

During a period of t, we consider the condition of a section of L lanes with n vehicles respectively in one direction. Suppose the average length of vehicles is l, thus we express the length of traffic flow as follows.

$$n(d+l)=Lvt \tag{2}$$

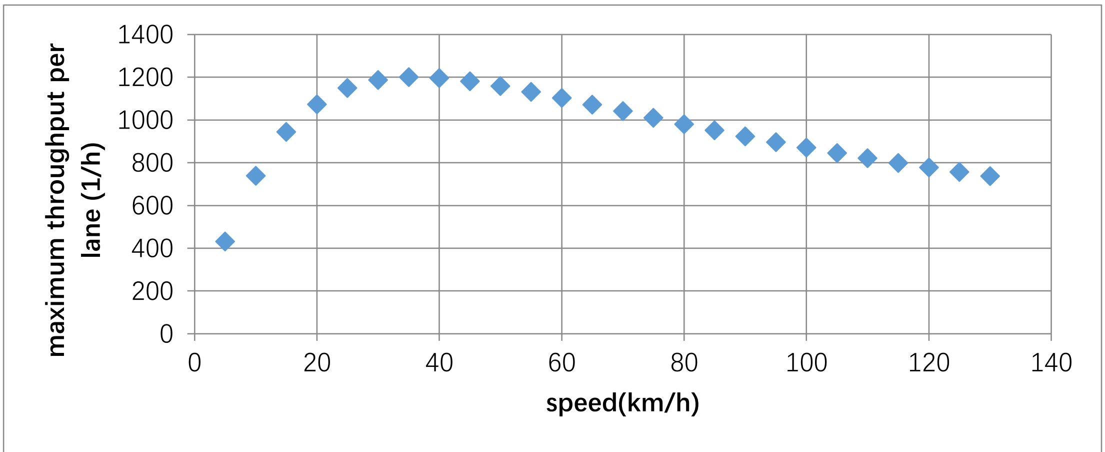
According to the definition, the traffic volume of the section (P) is the number of vehicles passing by a certain cross section per second. Thus, we figure out the traffic volume of the section (P) by the following equation.

$$P=\frac{n}{t}=\frac{Lv}{(d+l)}=\frac{Lv}{\frac{v^2}{2a}+vt_0+l} \tag{3}$$

Suppose the processing efficiency of every tollbooth is W. In order to avoid congestion at toll plaza, we give the following restriction.

$$WB \geq P \tag{4}$$

According to real situation, we let $a=5\mathrm{m}\cdot\mathrm{s}^{-2}$, $t_0=1\mathrm{s}$ and $l=10\mathrm{m}$ in the formula above. According to the formula (3), we draw the **Gragh1** to express the relationship between maximum throughput per lane and speed.



Gragh1 maximum throughput per lane - speed relationship

We can learn from the graph that for every single lane, the inequality $P/L \leq 1000$

is suitable for most situations. But we should consider that *P/L* rises to 1200 if the traffic is usually very busy in the road. Here in normal situations, we consider *P/L* =1000. Thus we can know the relationship between the number of lanes that can be driven in each direction (L) and the number of tollbooths in each direction (B) as follows:

$$B \geq \frac{1000}{W} L \tag{5}$$

According to the real situation, the number of lanes that can be driven in each direction (L) is 3 and the processing efficiency of the toll plaza (W) is 400 (usually W is 400~480, here we use the low limit to ensure traffic fluency). Thus we can get the following inequality:

$$B \geq 8 \tag{6}$$

3.2 Model 2: the trapezoid convergence model

3.2.1 Assumption

In this model, we make three assumptions to simplify the question.

●Vehicles move with a uniform speed in the trapezoidal merging lane.

●Unwilling probability and willing probability is the same for every driver.

●Divide the road into grids as **Figure2** and then we assume that every grid has the equal probability to be occupied by vehicles. The probability of having vehicles at any exits of tollbooth at every Δt, denoted as q, is given by the equation:

$$q = k \times \frac{L}{B} \tag{7}$$

And according to the model 1, we assume that the length of one grid is $d + l$, where d is safe distance and l is average length of vehicles.

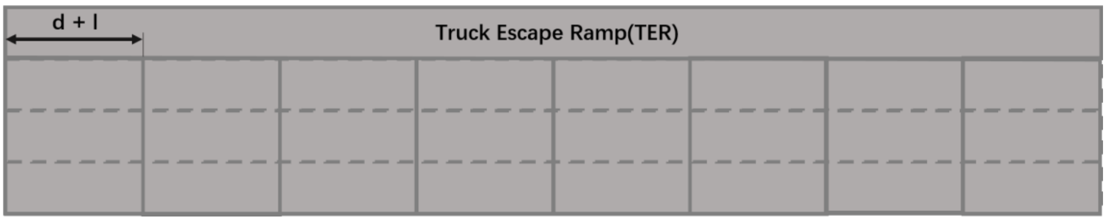


Figure 2 the road divided into grids in our model 2

3.2.2 Passing efficiency without guidance

From our model 1 we have known that if $B \geq 2.5L$, we can ensure that vehicles will not result in congestion at the entrance of toll plaza. But if there are enough tollbooths, we should ensure that the vehicle will not be congested at the exit of toll plaza so as to minimize the probability of congestion. We design a trapezoidal road-convergence model. In order to ensure the same probability for every vehicle driving back to the lane, we design to use guiding lines to replace obvious lane lines, to guide vehicles converge in order.

Firstly, we consider the expected rate of passing capacity from tollbooth to road (E) without the trapezoidal model. In this case, roads will directly merge into L lanes. Here we use channel model (which we have illustrated in part 2) to figure out E.

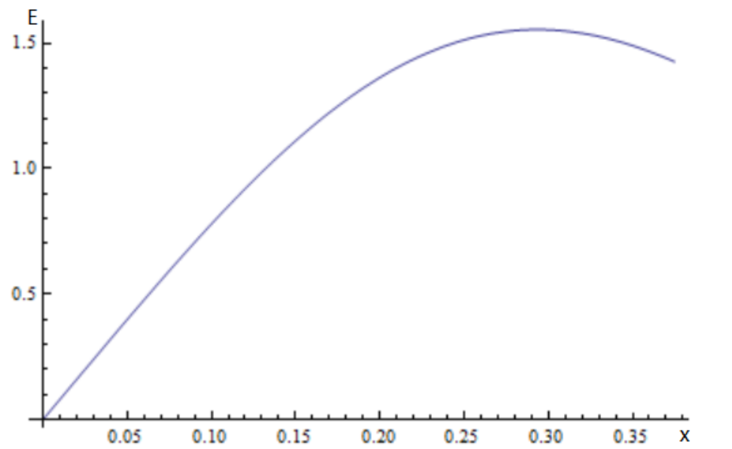
Thus the expected rate of passing capacity from tollbooth to road (E) is given as follows:

In this case, the probability of having vehicles pass through for every tollbooth is given as follows.

$$x = pq \tag{8}$$

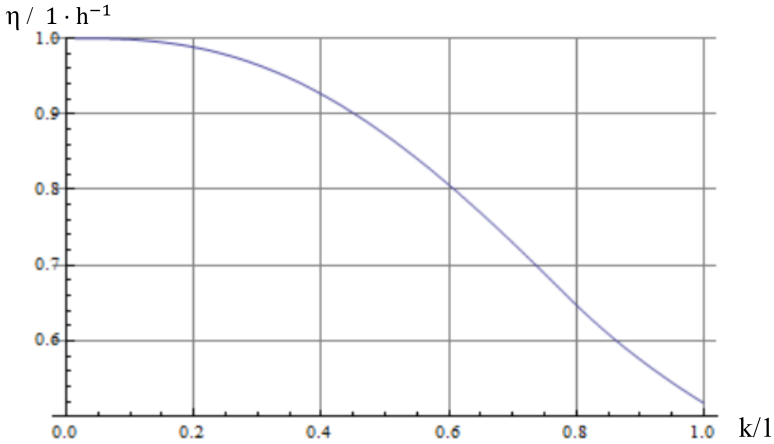
$$E = \sum_{n=1}^{L} C_B^n n x^n (1-x)^{B-n} \tag{9}$$

According to the model 1, L is 3 and B is 8. Taking different values for q and p, and then we draw the **Graph2** to express the relationship between E and x.



Graph 2 E-x curve

Taking different probability of having vehicles on the grid-divided road (k), we draw the **Graph3** to express the relationship between η (passing efficiency: the average expected rate of passing capacity from tollbooth to road compared to the ideal condition) and k. Here η is the maximum value for every fixed k.



Graph 3 η-k curve

Form the **Graph2** and **Graph3**, we can learn that when k is rather small, E is higher with p getting closer to 1. That means vehicles can pass through without making a concession. When in a traffic peak, k is 1 and q is 0.375 (substituting k=1 into equation (8)). Thus, from **Graph2**, the maximum of E is about 1.5 under the condition that drivers can make decisions according to the road condition. In this case, η is only about 50%. Therefore, a more ordered mechanics of merging is needed.

3.2.3 The effect of trapezoid's guidance

While we notice that directly merging B tollbooths into L lanes may cause jams in heavy traffic, we can design a long trapezoid to make the merging softer, improving the efficiency. As many accidents happen in congestion, this can also reduce the accident rate.

In our simplified model, we assume that on the merging road, each time vehicles move forward $d + l$, the width decreases by a lane, so the number of slots decreases by one, as in the **Figure 3**.

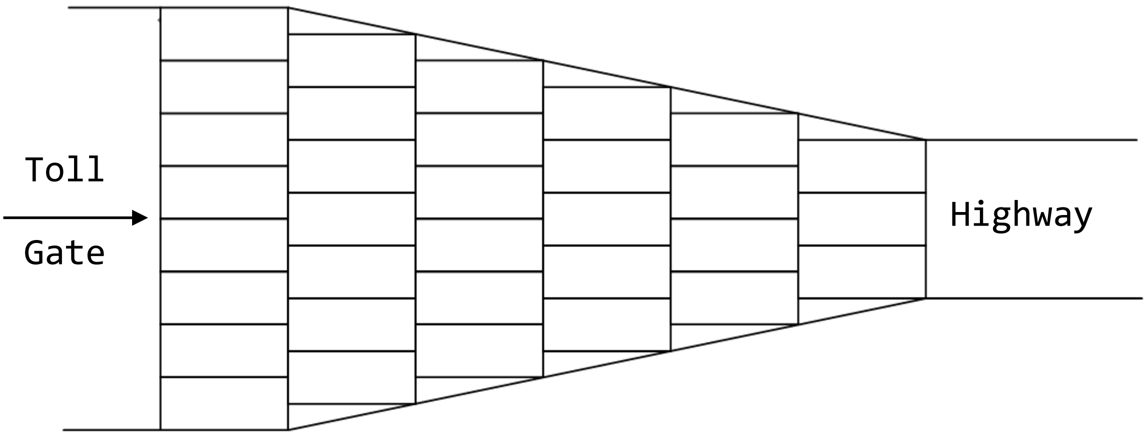


Figure 3 a long trapezoid merging road

We define $C_i(L \leq i \leq B)$ as corresponding columns with i slots, and $x_i(t)$ the number of vehicles in C_i at time t. In each Δt, the expected number of vehicles out of the toll gate is $kL = qB$, so the probability that there is a vehicle in an arbitrary slot of C_B is q. We assume they are independent, then $x_B(t)$ obeys the binomial distribution $B(B, q)$.

As the propagation of traffic stream is more fluent than in (2.1), we can assume that “deadlocks” do not happen. That is to say, the traffic can move forward in order. In each Δt, vehicles move ahead in this way:

Vehicles in C_L leave the trapezoid first. Then for $i = L + 1, \ldots, B,$ if there are at most $i - 1 - C_{i-1}$ vehicles in C_i, they will find their way into C_{i-1} without quarrel, and if there are more, as many vehicles in C_i as possible will move on to C_{i-1}, and the rest will remain in C_i. Finally, cars running out of the toll gate will enter C_B. If there are any vehicle already stuck in C_B, this vehicle is counted as a “delay” and removed from the system, indicating one following car cannot get out in time and thus the toll gate becomes a bottleneck of the highway. In fact, the number of delays is overestimated because the stuck vehicle may not be in the tollbooth where another vehicle is about to get out.

To check the efficiency, we wrote a program and simulated the traffic condition with k close to 1, with the parameters $L = 3, B = 8$ computed from Model 1. For each k, we simulate for 10006 rounds (because it takes a vehicle at least $6\Delta t$ to get out of the trapezoid). Results of simulations are shown in the **Figure4**.

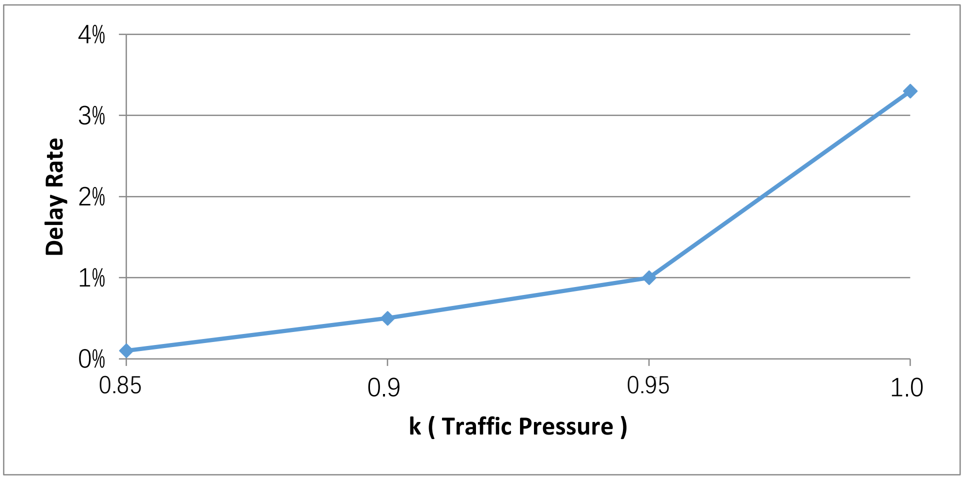
```
Round #10005
3: 3
4: 3
5: 5
6: 1
7: 2
8: 1

Round #10006
3: 3
4: 4
5: 2
6: 2
7: 1
8: 3

passed: 29384, delayed: 1005
efficiency: 96.7%
```

```
Round #10005
3: 3
4: 4
5: 4
6: 3
7: 3
8: 3

Round #10006
3: 3
4: 4
5: 4
6: 3
7: 3
8: 3

passed: 25521, delayed: 26
efficiency: 99.9%
```

```
Round #10005
3: 3
4: 4
5: 5
6: 1
7: 0
8: 3

Round #10006
3: 3
4: 4
5: 3
6: 0
7: 3
8: 5

passed: 27249, delayed: 124
efficiency: 99.5%
```

```
Round #10005
3: 3
4: 3
5: 4
6: 4
7: 2
8: 3

Round #10006
3: 3
4: 4
5: 4
6: 2
7: 3
8: 1

passed: 28410, delayed: 284
efficiency: 99.0%
```

Figure 4 results of simulations



Graph 4 the relation between delay rate and traffic pressure

The result in the **Graph 4** shows that for $k = 1$ the delay rate is only (less than) 3.3%, and for $k = 0.85$, a scenario of fairly heavy traffic, the delay can be practically ignored.

This design improves the efficiency in merging dramatically. That is mainly because the fluctuation of traffic in short time may cause disorder especially in a sudden merging, but during a long period, the average throughput can never exceed the capacity of the outward highway----because vehicles has just entered the toll gate from the “identical” way, since they have come in, they can surely get out! Therefore, as we provided a buffer to lead them fan in orderly, the merging problem can surely be solved. And this is one possible way.

In practice, we can make the trapezoid even longer, with width decrease by a lane every several $(l + d)$’s. In this way, there are more space for vehicles to change lanes, and the enlarged buffer can produce a more fluent traffic stream out from the toll gate.

3.3 Model 3: the trumpet-shaped convergence model

3.3.1 A variation of the trapezoid

Our Model 2 work well in guiding the traffic outwards from the toll gate to fan in the highway, but the trapezoid shape is designed on the basis that v, the speed of traffic stream, keeps constant during the convergence. In practice, when vehicles run out of the toll gate and set off for the highway, they tend to speed up. According to Model 1, the safety distance d keeps going up, so to keep the model more suitable, the trapezoid should be deformed to have longer tails.

Besides, as $q = k\frac{L}{B} \le \frac{L}{B}$, in general vehicles should not often get crowded at the beginning of the trapezoid. When close to the end, though, if k is close to 1, the traffic tends to be heavy, but compared to relatively higher speed, they have less space to fan in, so either congestion or accidents may happen more frequently.

To check this, we also ran the program to simulate the traffic. Every time vehicles in C_{i+1} cannot move into C_i completely, we report a “crowd” in C_i. Still assuming $(L, B) = (3,8)$, for $k = 0.9$ and $k = 1.0$, the result is as following in the **Figure5**:

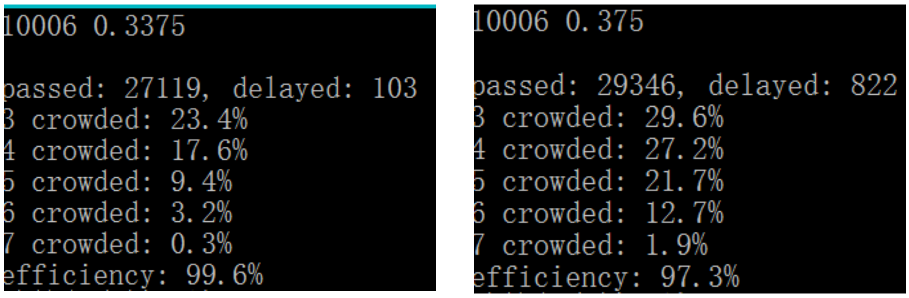


Figure 5 results of simulations

We noticed that for $k = 0.9$, crowds mainly appear in C_3 and C_4, and even for $k = 1.0$, rarely is C_7 crowded. Both issues above suggest that for both efficiency and safety, we should leave more space for C_i's with smaller i.

3.3.2 Detailed discussion on the optimal shape

Naturally we can think of exponential decay, as

$$w(x) = Bw_B \left(\frac{Lw_L}{Bw_B}\right)^{x/H}. \tag{10}$$

In real world, because toll booths between lanes take up some space, $w_B > w_L$. Therefore, there exists considerably a part of the area with $w(x) > Bw_L$, where no merging happen, i.e. C_B is far longer than needed. As we know, every meter of the merging road costs much, we should decrease the length of such a part, i.e. make the width decrease more rapidly at small x. So we altered the formula into

$$w(x) = Bw_B \left(\frac{Lw_L}{Bw_B}\right)^{(x/H)^\alpha} \tag{11}$$

with $0 < \alpha < 1$. The smaller α is, the more rapidly the width falls.

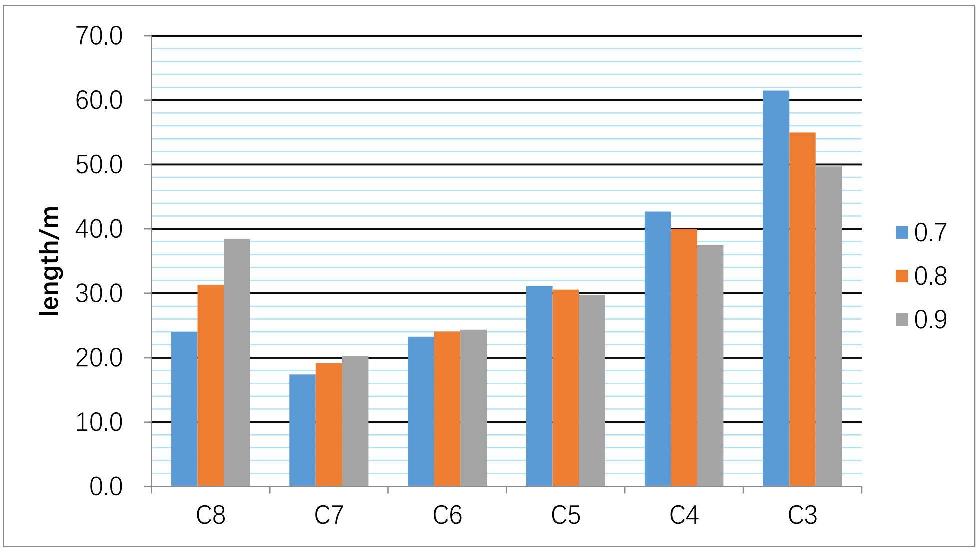
We still assume $(L, B) = (3,8)$, and from road data we get $w_L = 3.75m$, $w_B = 4.27m{\sim}5.79m$(dependent on types of vehicles). For simplification, we let $w_B = 5m$.

Now we try to determine a suitable H. We roughly assume the typical speed $\hat{v} = 30km/h$ during the acceleration. From Model 1, we get a typical lattice length $\widehat{d+l} \approx 25m$ with a w_L decrease in width. So the approximated length

$$\hat{H} = \frac{Bw_B - Lw_L}{\frac{w_L}{\widehat{d+l}}} \approx 192m. \tag{12}$$

So we just set $H = 200m$.

For $\alpha = 0.7, 0.8, 0.9$, the length of each C_i is computed as **Graph5**:



Graph 5 the length of different sections when taking different α

For $\alpha = 0.9, C_8$ takes nearly 20% of the length, and for $\alpha = 0.7$, safety speed in C_7 is lower than 20km/h, a typical speed of slow driving. Therefore, we choose

$$\alpha = 0.8$$

Then, the convergence part on one side is shaped as the **Figure6**:

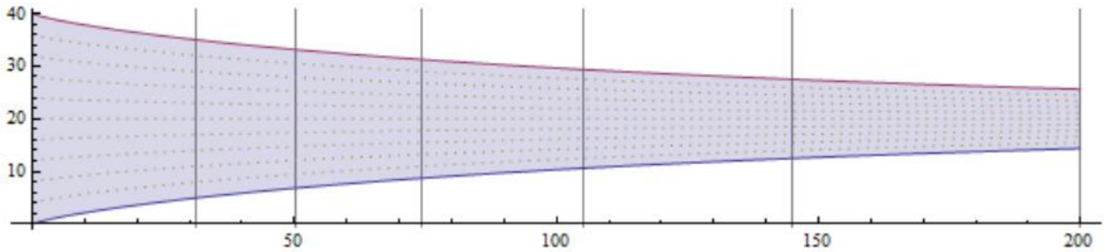


Figure 6 the shape of the convergence part when α is 0.8

C_3 and C_4 takes up a large part, fitting the demands of $k = 0.9$ well. According to Model 2, vehicles are allowed to accelerate from 20km/h to at most 60km/h along the way.

3.3.3 A compact configuration for both sides

Up to here, we have decided on the shape and merging pattern on either side, but if we simply construct the toll gate on both sides together, the maximum total width will be at least $2Bw_B$, or 80m on this particular case. As the fan-in area is curve shaped, this simple configuration leads to a sparse utilization of land, resulting in unnecessary costs.

As it takes a longer road to fan in than fan out, we can utilize the central symmetric configuration for a compact space usage as the **Figure 7**. Even in the aspect of aesthetics, it can also show the beauty of fluency.

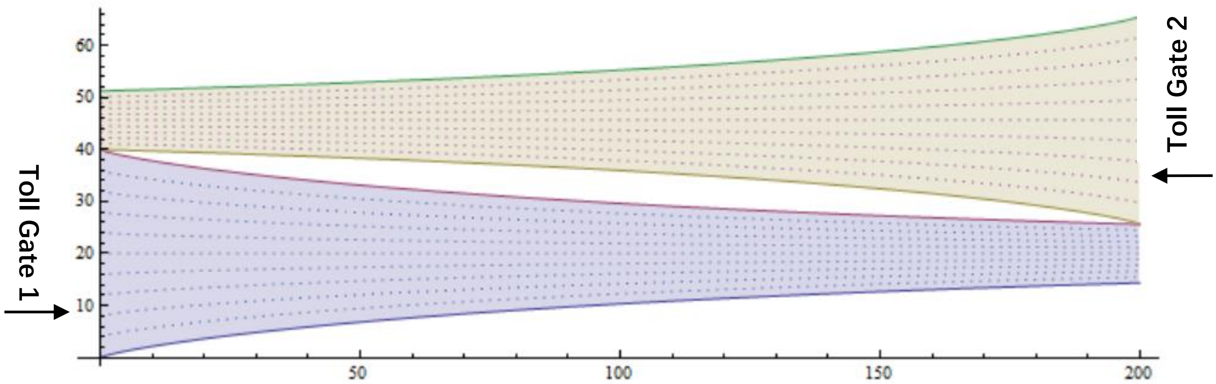


Figure 7 the shape of the convergence part that utilize central symmetric configuration

By calculation, the area of merging parts is $8412m^2$, and the unused area in the middle is $919m^2$, for a total of $9331m^2$.

4 Sensitivity Analysis

4.1 Determine the performance of our model in light and heavy traffic

According to model 1, we make the following conclusion: in general cases where traffic can move at a high speed (≥70km/h), the ratio of the traffic volume of the section (*P*) and number of lanes that can be driven in each direction (*L*) should satisfy P/L≤1066. That means in real situation, if we make *l* equal to 3 and *B* equal to 8, we can ensure that vehicles will not be blocked up in the toll plaza.

But for the highway where *P* is so large that the speed of traffic flow is usually very low, the value of *P/L* actually can reach to 1200. In this case, we should make *B/L* equal to 3 and make *B* equal to 9. This change slightly increases the length of the merging lane, but the calculating methods presented in our model still can be applicable to it.

4.2 Determine the performance when affected by the different kinds of tollbooth

Generally, there are three kinds of tollbooth: conventional (human-staffed) tollbooths, exact-change (automated) tollbooths, and electronic toll collection booths.

We discuss the influence of different kinds of tollbooths in two ways: the cost of equipment and operation and the processing efficiency of the toll plaza (which actually also affects the area of the toll plaza).

For the cost of equipment, in the long run, labor costs cannot be ignored and the cost of equipment will be lower. Thus in this aspect, the use of automated tollbooths and electronic toll collection booths can reduce cost compared to conventional tollbooths.

For the processing efficiency of the toll plaza, we discuss as follows.

When the toll plaza uses automated tollbooths only, the processing efficiency of the toll plaza (*W*) is 720[2]. Thus according to equation(5) from model 1($\mathrm{B} \geq \frac{1000}{\mathrm{W}}\mathrm{L}$), we can make *B* equal to 5 to ensure fluency at the entrance of the toll plaza. Then according to the equation (12) in model 3, we can figure out the following answer:

$$\hat{H} = \frac{Bw_B - Lw_L}{\frac{w_L}{\widehat{d+l}}} \approx 100\mathrm{m}$$

where B = 5, L = 3, $w_B = 5\mathrm{m}$, $w_L = 3.75\mathrm{m}$, $\widehat{d+l} = 25\mathrm{m}$.

Thus we can learn that the length of merging lane is reduced from 200 meters to 100 meters. Then we figure out the size of the area *A* (approximate according to the shape of trapezoid) as the following equation:

$$A \approx (BW_B + LW_L)\ H \approx 3600\mathrm{m}^2$$

We can see that the use of automated tollbooths can significantly reduce the cost. And the calculating methods presented in our model also can be applicable to this circumstance.

When the toll plaza uses electronic toll collection booths only, then the value of W will be higher, and the value of B can be smaller. When *W* reach to 1000 (for general highways) or 1200 (for the highway that are often congested), we can meet the need to make *B* equal to *L*. In this case, fan-out and fan-in are no longer required.

When the toll plaza uses more than two kinds of tollbooth, then we can figure out $\overline{W}$ according to the proportion. And then we can figure out the value of *B* according to model 1.

4.3 Determine the performance when affected by autonomous vehicles

Without the guidance in our Models 2 and 3, in the initial case of Channel Model, autonomous vehicles can communicate with one another and pass in order, greatly improve the efficiency from 50% to nearly 100%. But in our Models 2 and 3, we have already achieved ~97% passing efficiency without autonomous vehicles, so autonomous vehicles will not significantly influence the outcome of our model.

5 Strengths and weaknesses

5.1 Strengths

(1) Fitting the reality:

Our traffic throughput estimation based on safety distances agrees with drivers' psychology, and it can also explain the phenomenon that city traffic typically moves at about 30km/h (because this speed corresponds to the maximum throughput in heavy traffic). So Model 1 describes the pattern of traffic flows in a simple yet effective way.

(2) Multi-target optimization:

Our Models 2&3 have given an effective way to reduce congestion, both improving the passing efficiency and reducing accident rate.

(3) Compact and economic:

In our Model 3, a further analysis has specified a shape to make optimal use of area, also reducing the land cost.

(4) Creative design:

With our models, we have designed a new shape of merging roads. The curve-shaped convergence area differs from ordinary thoughts and shows a compact and artistic structure.

5.2 Weaknesses

(1) Simplification Assumptions:

The grid model for roads and fan-in simulation may be oversimplified, and we mainly deal with "traffic streams", thus a more detailed simulation in the scope of individual vehicles can be implemented for a more accurate model.

(2) Accidents:

Although we can try our best to prevent accidents, they do happen anyway, but we have not considered their influence to the traffic flow.

(3) Rough cost analysis:

For lack of data, we only considered land area in the cost analysis. Although area is a dominant part of total cost, there exist other aspects.

6 Reference

[1] ***Design of highway toll station***. available at *20th January, 2017* at http://www.ixueshu.com/document/e4dc9dd8da07e7f4318947a18e7f9386.html.

[2] ***American highway toll station***. available at *20th January, 2017* at http://na.eastday.com/know/life/u1ai18719_K14.html

[3] ***the source of Figure 1***. available at *20th January, 2017* at http://gx.sina.com.cn/news/gx/2016-07-31/detail-ifxunyya2848793.shtml

[4] ***Technical Standard of Highway Engineering***. available at *23th January, 2017* at http://www.doc88.com/p-6931966204326.html

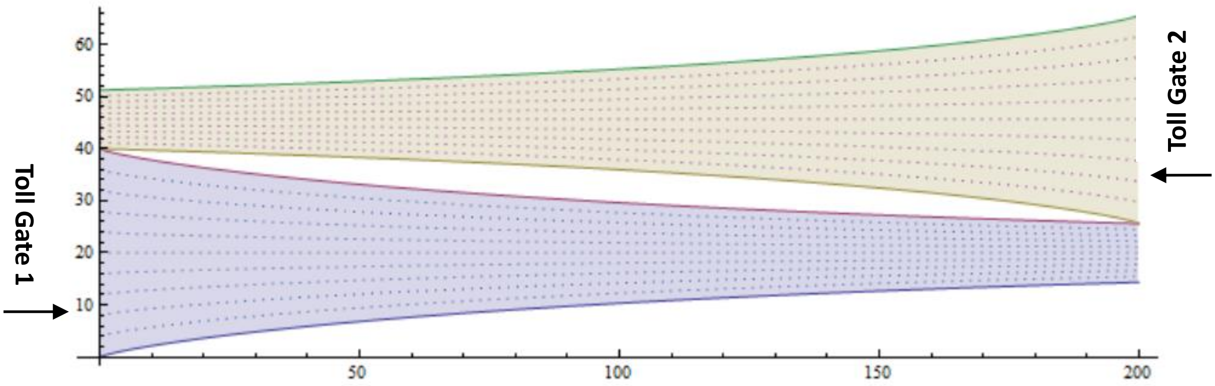
7 Appendix:A Letter to the New Jersey Turnpike Authority

To New Jersey Turnpike Authority,

In response to your questions about the design of the fan-in area, I am writing to tell you about our findings and our design of the optimal merging pattern.

As we know, the processing efficiency of conventional tollbooths is limited, so to prevent the toll gate from becoming a bottleneck of traffic, we should build more tollbooths than lanes. According to our calculation, the number of tollbooths should be at least 2.5 times the number of lanes. For a normal highway with 3 lanes on either side, the number of tollbooths on either side should be no less than 8. If the highway is often congested with overwhelming traffic, 9 tollbooths will be better.

Because all vehicles coming out of the toll gate actually come from the same highway, they never go beyond the capacity of the highway. Nevertheless, we still need proper guidance to lead the traffic merge in order, both to improve efficiency and to prevent accidents. For the case where 8 lanes are merging into 3, we should build a smooth trumpet-shaped merging area that is 200 meters long, 40 meters wide at the beginning, 19.3 meters wide long at the middle, and 11.25 meters wide at the end. For a compact use of space, we can stagger tollbooths of two sides in different locations, as the following figure shows:



It might be amazing to find that sparing some area in the middle results in an improvement of passing efficiency, but this is exactly the case. In fact, we thought of the trapezoid shape earlier, but when we noticed that the traffic accelerates along the merging area and found out that more congestions may happen near the end, we decided to compress the beginning part and allow the traffic flow to merge smoothly in the long tail. It can also decrease the length and save land cost! We have also tried many types of trumpet shapes, and this design is the optimal as far as we are concerned.

In addition, with the development of modern technology, new types of tollbooths, e.g. automated tollbooths or electronic toll collection (ETC) tollbooths are available currently. Since their efficiency are improved, they can reduce the necessary number of tollbooths, thus save a considerably amount of land cost. For example, if all conventional tollbooths are replaced by automated ones, only 5 tollbooths are needed, reducing the fan-in length to 100 meters and area by 60%. As ETC tollbooths has even higher efficiency, the number, length and area can be further decreased. When the technology allows the processing efficiency to match the road capacity, those fan-in and fan-out areas are no longer needed.

Frankly speaking, the introduction of new technology will add to the budget, but they can save the land cost, which is a huge part of the total cost, by an amazing ratio. We can also notice that in the long run, the cost of work force keeps rising, which cannot be ignored, and that the improvement of toll collecting efficiency is also the improvement of drivers' convenience. So as long as within your budget, our advice is to adopt the modern technology, and build a compact, safe, fluent and convenient toll gate.