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Tutorial-3

① (i) $T=4 \quad \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

(a)

$$y(t) = \alpha_0 + a_1 e^{j\omega_0 t} + a_{-1} e^{-j\omega_0 t} = t + 2e^{j\pi/2 t} + 2e^{-j\pi/2 t} = t + 4\cos(\pi/2 t)$$

(ii) $$h(t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega t}\, d\omega = \left.\frac{e^{j\omega t}}{jt}\right]_{-\pi}^{\pi} \Rightarrow \frac{1}{2\pi} \cdot \frac{e^{j\omega t} - e^{-j\pi t}}{jt}$$

$$= e^{j\theta} - e^{-j\theta} = 2j\sin\theta \Rightarrow \frac{1}{2\pi} \cdot \frac{2j\sin(\pi t)}{jt} = \frac{1}{\pi}\,\frac{\sin(\pi t)}{t}$$

(b) $$\frac{d^2y}{dt^2} + 4\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

$\Rightarrow x(t) = e^{-2t}u(t)$

$$(j\omega)^2 Y(j\omega) + 4(j\omega)Y(j\omega) + 8Y(j\omega) = 2X(j\omega)$$

$$= Y(j\omega)\left[(j\omega)^2 + 4(j\omega) + 8\right] = 2X(j\omega)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2}{(j\omega)^2 + 6(j\omega) + 8} = \frac{2}{(j\omega+2)(j\omega+4)}$$

using practical fractions

$$\frac{2}{(s+2)^2(s+4)} = \frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{(s+4)}$$

$$2 = A(s+2)(s+4) + B(s+4) + C(s+2)^2$$

let $s = -2$

$$2 = A(0)(-2+4) + B(-2+4) + C(0)^2$$

$$2 = 2B$$

$$B = 1$$

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let $s = -4$

$$2 = A(-4+2)(0) + 1(0) + C(-4+2)^2$$

$$2 = 4C$$

$$C = \frac{1}{2}$$

let $s = 0$

$$2 = A(2)(4) + 1(4) + \frac{1}{2}(0+2)^2$$

$$2 = 8A + 4 + 2$$

$$2 - 6 = 8A$$

$$A = \frac{-1}{2}$$

$$Y(j\omega) = \left(-\frac{1}{2} \cdot \frac{1}{(s+2)} + \frac{1}{(s+2)^2} + \frac{1}{2} \cdot \frac{1}{(s+4)}\right)$$

$$y(t) = \left(-\frac{1}{2}e^{-2t} + te^{-2t} + \frac{1}{2}e^{-4t}\right) u(t)$$

② $\frac{dy(t)}{dt} + 2y(t) = x(t)$ $\quad x(t) = e^{t}u(t)$

$$j\omega Y(\omega) + 2Y(\omega) = X(\omega)$$

$$Y(\omega)[j\omega + 2] = \frac{1}{1+j\omega}$$

$$Y(\omega) = \frac{1}{(j\omega+2)(1+j\omega)}$$

using partial fraction

$$\frac{1}{(s+2)(s+1)} = \frac{A}{(s+2)} + \frac{B}{(s+1)}$$

$$1 = A(s+1) + B(s+2)$$

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let $s = -1$

$1 = A(\cdot 0) + B(-1+2)$

$1 = B$

let $s = -2$

$1 = A(-2+1) + B(-2+2)$

$1 = -A$

$A = -1$

$$\frac{-1}{(s+2)} + \frac{1}{(s+1)} = (e^{-t} - e^{-2t})\, u(t)$$

(3) $w(t) = e^{-4(t-2)}\, u(t-2) \qquad x(t) = \delta(t+2) + \delta(t-2)$

$$H(\omega) = \int_2^{\infty} e^{-4(t-2)}\, e^{-j\omega t}\, dt$$

$$\int_2^{\infty} e^{-4t - j\omega t}\, , e^{8}\, dt$$

$$= e^{8} \int_2^{\infty} e^{-(4+j\omega t)}\, dt \Rightarrow -\frac{e^{-(4+j\omega t)}}{4+j\omega t}\Bigg]_2^{\infty}$$

$$\Rightarrow e^{8}\left[0 - \left(-\frac{e^{-8}\, e^{-j2\omega}}{4+j\omega}\right)\right]$$

$$H(\omega) = \frac{e^{-j2\omega}}{4+j\omega}$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} \Rightarrow Y(\omega) = H(\omega) \cdot X(\omega)$$

$$= \frac{e^{-j2\omega}}{4+j\omega} \cdot (e^{j2\omega} + e^{-j2\omega})$$

$$= \frac{e^{-j2\omega}(e^{-j2\omega} + e^{-j2\omega})}{4+j\omega} \Rightarrow$$

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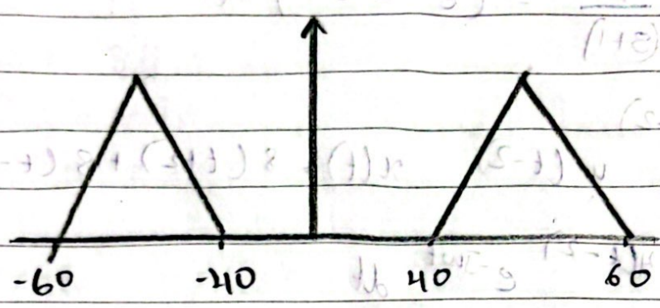
$$\frac{e^{-j2\omega t-j2\omega}+e^{j2\omega+j2\omega}}{4j\omega} \Rightarrow \frac{t+e^{-j4\omega}}{4j\omega}$$

(4) i) multiply $\cos(50t)$

$$\cos(50t)=\frac{1}{2}\left(e^{j50t}-e^{-j50t}\right)$$

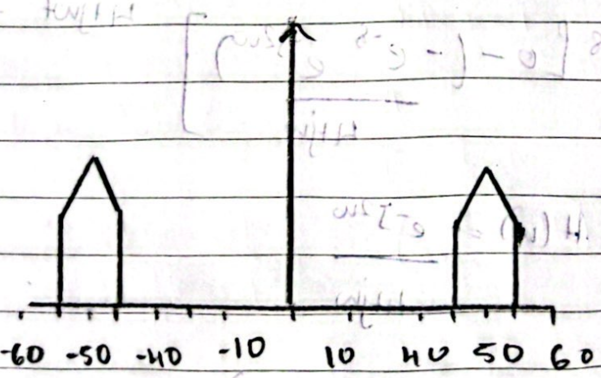
$$x(t)\cdot\cos(50t)=\frac{1}{}x(t)e^{j50t}-\frac{1}{}x(t)e^{j50t}$$

$$x_1(\omega)=\frac{1}{2}\left[x(\omega-50)+x(\omega+50)\right]$$

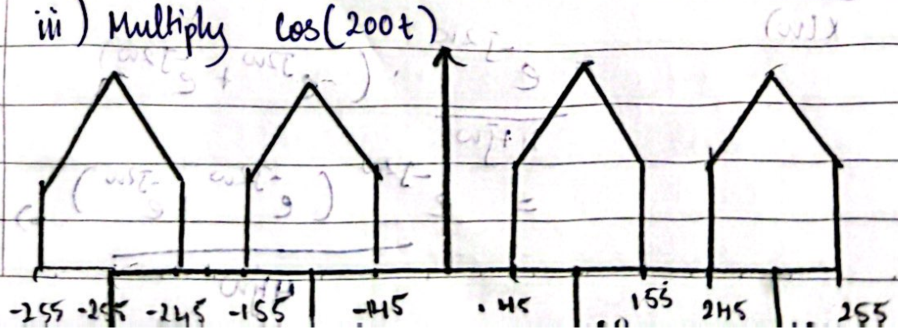


ii) Filter $H_1(\omega)$

$+50 \rightarrow H_1$ keeps $[45,55]$
$-50 \rightarrow H_1$ keeps $[-55,45]$



iii) Multiply $\cos(200t)$



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iv) Filter $H_2(\omega)$