Laboratory Report 2

Water Tank

MA-MECH-25-VZ

Advanced Control Engineering Laboratory

Author(s)	Felix Raffl, Lenard Wild
Student ID(s)	2510620034, 2510620041
Supervisor	Dipl.-Ing. Dr. techn. Phillip Kronthaler
Cohort	MA-MECH-25-VZ
Group	MA-MECH-25-VZ-2B
Lecturer	Dipl.-Ing. Dr. techn. Phillip Kronthaler

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Chapter 1

Introduction

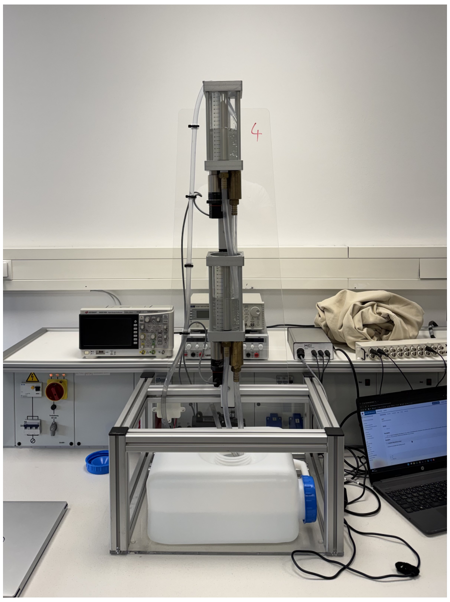

Figure 1.1: Water Tank laboratory system

This laboratory experiment investigates the control of a nonlinear hydraulic two-tank system. The objective is to regulate the water level of the lower reservoir by controlling the pump input. MATLAB and Simulink are used for modeling, controller design and simulation, as well as for the implementation of the control algorithm on the laboratory hardware via a National Instruments interface. In addition to feedback control, a flatness-based feed-forward approach is considered to enable smooth reference trajectory tracking.

The report is structured as follows:

- Chapter 2 summarizes the theoretical background.
- Chapter 3 describes the laboratory setup and parameter identification.
- Chapter 4 presents the simulation models and control design.
- Chapter 5 discusses the results.
- Chapter 6 concludes the report.

Chapter 2

Theoretical Background

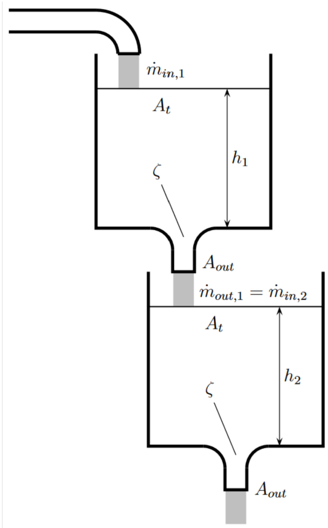

Figure 2.1: Schematic representation of the two-tank hydraulic system [1]

The water tank system is modeled as a nonlinear dynamic system consisting of two interconnected reservoirs and a pump-driven inflow. Water enters the upper reservoir and flows into the lower reservoir due to gravity. A detailed physical description of the system and the corresponding parameter definitions are provided in the laboratory assignment [1].

Assuming incompressible fluid behaviour, the nonlinear system dynamics can be expressed in state-space form as

$$\dot{x}(t) = f(x(t), u(t)), \qquad x = \begin{bmatrix} h_1 & h_2 \end{bmatrix}^\top, \tag{2.1}$$

where the control input $u(t)$ represents the pump mass flow. The controlled output is defined as the water level of the lower reservoir,

$$y(t) = h_2(t).$$

2.1 State linearization

For controller design, the nonlinear model is linearized around a steady operating point, resulting in a local linear time-invariant approximation. In addition to feedback control, a flatness-based feed-forward approach is considered, exploiting the fact that the water level of the lower reservoir constitutes a flat output of the nonlinear system.

$$\dot{x}_1 = -a_1\sqrt{x_1} + \frac{1}{\rho A_t}u \tag{2.2}$$

$$\dot{x}_2 = a_1\sqrt{x_1} - a_2\sqrt{x_2} \tag{2.3}$$

where the parameters are defined as $a_i = \frac{A_{out,i}}{A_t}\sqrt{\frac{2g}{1+\zeta_i}}$ for $i = 1, 2$.

The nonlinear state-space model is linearized at an operating point determined by the desired water level in the second reservoir. As per the task description, this is chosen as $\bar{x}_2 = 5\,\mathrm{cm}$. The equilibrium conditions $\dot{x} = 0$ yield:

$$\bar{x}_2 = 0.05\,\mathrm{m}, \quad \bar{x}_1 = \left(\frac{a_2}{a_1}\right)^2 \bar{x}_2, \quad \bar{u} = \rho A_t a_1 \sqrt{\bar{x}_1} \tag{2.4}$$

The partial derivatives of the non-linear terms $f(x, u)$ evaluated at this operating point are:

$$\left.\frac{\partial f_1}{\partial x_1}\right|_{\bar{x}} = -\frac{a_1}{2\sqrt{\bar{x}_1}} \tag{2.5}$$

$$\left.\frac{\partial f_2}{\partial x_1}\right|_{\bar{x}} = \frac{a_1}{2\sqrt{\bar{x}_1}} \tag{2.6}$$

$$\left.\frac{\partial f_2}{\partial x_2}\right|_{\bar{x}} = -\frac{a_2}{2\sqrt{\bar{x}_2}} \tag{2.7}$$

Introducing deviations $\Delta\mathbf{x} = \mathbf{x} - \bar{\mathbf{x}}$ and $\Delta u = u - \bar{u}$, the linearized system $\Delta\dot{\mathbf{x}} = \mathbf{A}\Delta\mathbf{x} + \mathbf{B}\Delta u$ is given by:

$$\begin{bmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{a_1}{2\sqrt{\bar{x}_1}} & 0 \\ \frac{a_1}{2\sqrt{\bar{x}_1}} & -\frac{a_2}{2\sqrt{\bar{x}_2}} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{\rho A_t} \\ 0 \end{bmatrix} \Delta u \tag{2.8}$$

Chapter 3

Laboratory Setup and Parameter Identification

3.1 Parameter identification

3.1.1 Pump parameter identification

The pump generates the inlet mass flow to the upper reservoir and is driven by an input voltage. Based on the characteristic curve provided in the laboratory script [1], the pump is approximated as a static linear system:

$$K_{\text{pump}} = \frac{\dot{m}}{(U_{\text{in}} - U_0)} \tag{3.1}$$

where the gain K_{pump} and the voltage offset U_0 are determined by linear regression of the diagram, given in the laboratory script [1], depicted in figure 3.1.

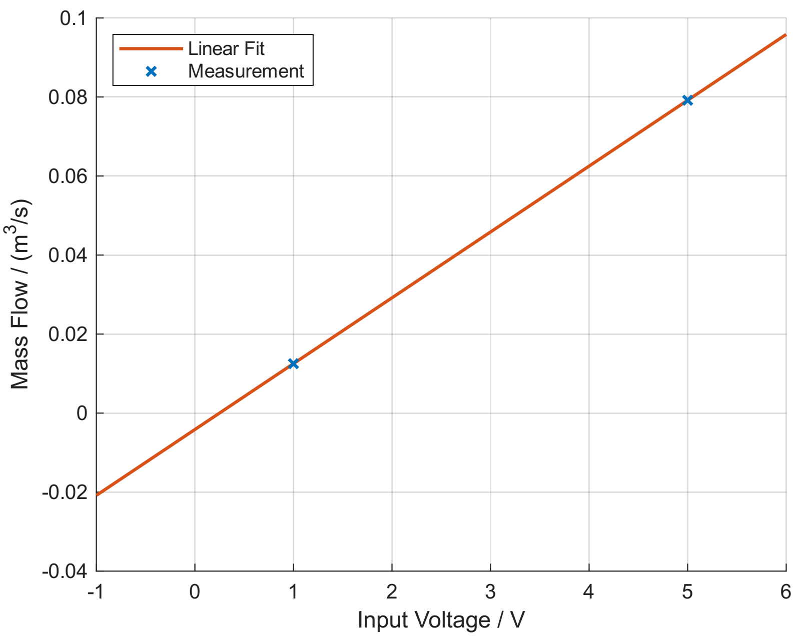

Figure 3.1: Diagram of the linear pump relationship given the laboratory script [1]

The linear regression of the pump diagram in 3.1 returns following values for the corresponding parameters: $K_{\mathrm{pump}} = 0.0167\,\mathrm{kg\,s^{-1}\,V^{-1}}$ and $U_0 = 0.25\,\mathrm{V}$

3.1.2 Water tank parameter identification

The hydraulic outflow behaviour of the reservoirs is characterized by an effective flow coefficient, which is identified experimentally using steady-state measurements. System operation is constrained by the maximum admissible pump voltage and by the physical limits of the reservoir heights. a_1 can be calculated by:

$$a_1 = \frac{b \cdot k_{\text{pump}}(v_{\text{input}} - v_0)}{\sqrt{h_1}} \tag{3.2}$$

where b is:

$$b = \frac{1}{\rho A_t} = \frac{1}{1000\,\mathrm{kg/m^3} \times 50 \times 10^{-4}\,\mathrm{m^2}} = 0.2\,\mathrm{m\,kg^{-1}} \tag{3.3}$$

and v_{input} and h_1 are vectors, therefore the mean has to be taken. Resulting in:

$$a_{1,\text{final}} = \text{mean}(a_1) = 0.0574\,\mathrm{m^{1/2}/s} \tag{3.4}$$

a_2 can be calculated by:

$$a_2 = a_1 \cdot \frac{\sqrt{h_1}}{\sqrt{h_2}} \tag{3.5}$$

where h_1 and h_2 are vectors, therefore the mean has to be taken. Resulting in:

$$a_{2,\text{final}} = \text{mean}(a_2) = 0.0524\,\mathrm{m^{1/2}/s} \tag{3.6}$$

3.2 Sensor calibration

The water level in each reservoir is measured by a pressure sensor. Since hydrostatic pressure is proportional to the water column height, a linear sensor model is assumed. Calibration is performed by recording the sensor output voltage at known water heights as depicted in figure 3.2 and identifying the corresponding gain and offset parameters using linear regression.

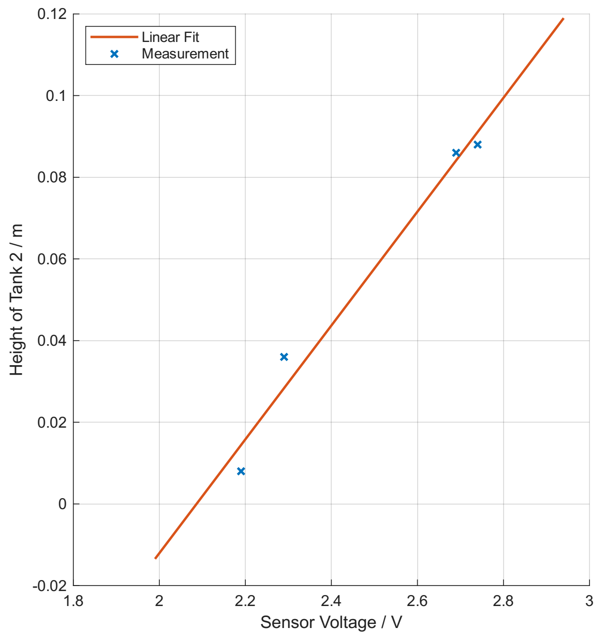

Figure 3.2: Diagram of the linear fit of the measurements to determine the sensor gain and offset

The linear regression of calibration of the sensor of the second water tank, which is depicted in diagram 3.2 returns following values for the corresponding parameters: $K_{\text{sens}} = 0.1394\,\mathrm{m}\,\mathrm{V}^{-1}$ and $U_{0,\text{sens}} = 2.0866\,\mathrm{V}$

Chapter 4

Simulation

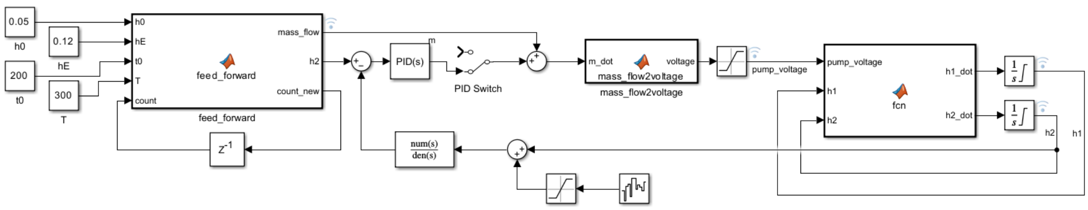

Figure 4.1: Schematic of the Simulink-Simulation

Chapter 5

Results and Interpretation

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Figure 5.1 shows the simulated height response of the Ball-in-Tube system using the nonlinear model and the parameters identified in Chapters 2 and 3. A seventh-order polynomial trajectory is used to generate smooth transitions between the lower height of $0.1\,\mathrm{m}$ and the upper height of $0.4\,\mathrm{m}$. The downward motion is obtained by reversing the boundary conditions of the trajectory.

The simulated height closely follows the reference trajectory during both the upward and downward transitions. The response is smooth and does not exhibit overshoot, while only minor deviations occur due to the injected measurement noise. Overall, the results indicate that the PID controller and the flatness-based feed-forward provide satisfactory tracking performance in simulation. These results serve as a reference for evaluating the behavior of the control strategy on the real system.

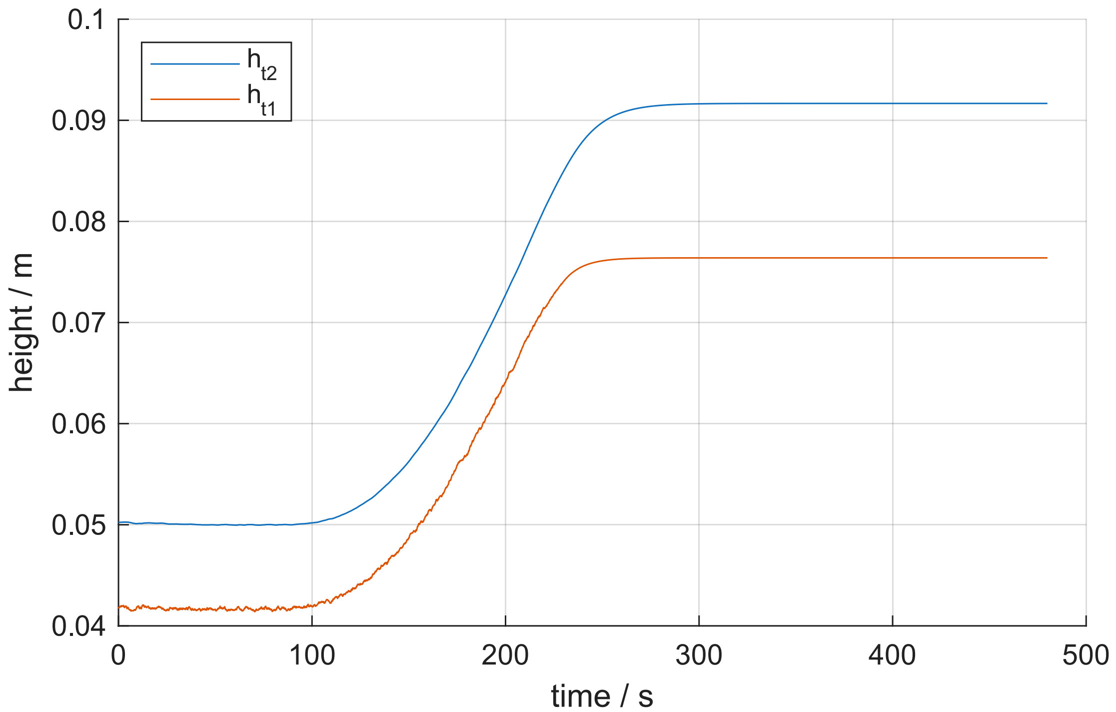

Figure 5.1: Transition from $0.1\,\mathrm{m}$ to $0.4\,\mathrm{m}$ using the Simulink-Simulation and the non-linear model of the system

Figure 5.2 shows the measured height response of the Ball-in-Tube system for the same reference trajectory used in the simulation. In contrast to the idealized simulation result, the real system exhibits small tracking deviations during the transition phases and a slight steady-state offset during the plateau.

The upward and downward motions remain smooth, indicating that the trajectory planning and feed-forward approach are effective also on the real hardware. However, measurement noise, actuator limitations, and unmodeled dynamics lead to a reduced tracking accuracy compared to the simulation. In particular, the finite fan dynamics and input saturation influence the transient response.

Overall, the real-system results confirm the validity of the control concept derived in simulation, while highlighting the impact of practical non-idealities that are not fully captured by the model.

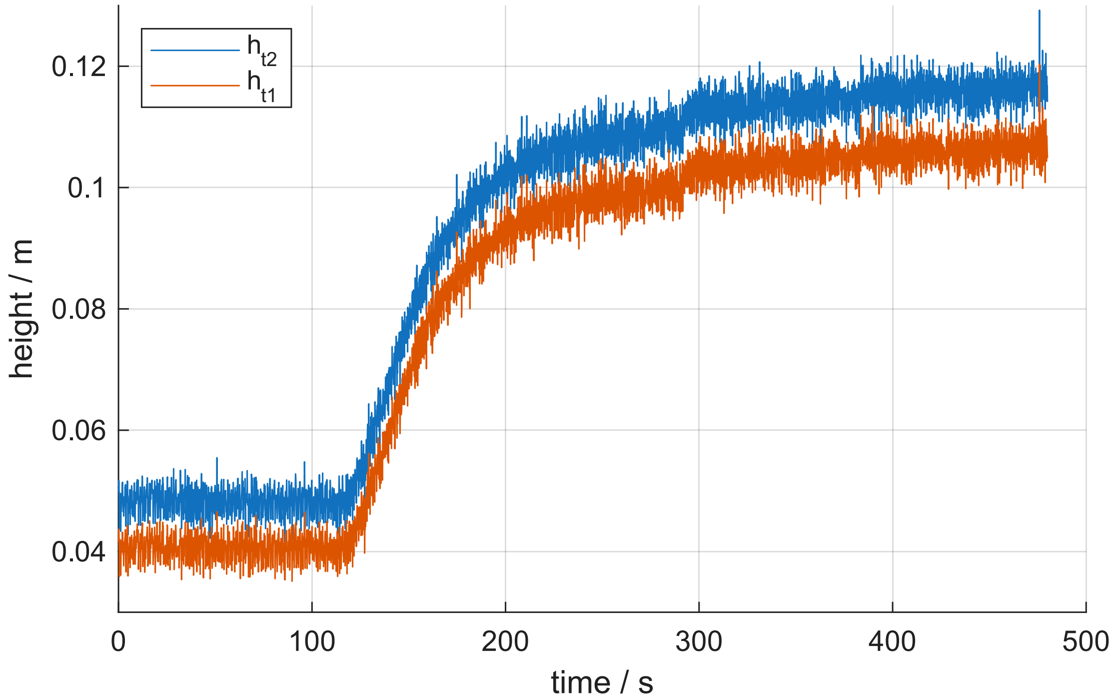

Figure 5.2: 3D-Modell einer Offline-Simulation, Bildquelle: Visual Components GmbH

Chapter 6

Conclusion

- Summary of achieved objectives
- Key findings from modeling, control design, and simulations
- Evaluation of controller performance and limitations
- Identified challenges and implemented solutions
- Possible improvements and future work

Bibliography

[1] P. Kronthaler, *Advanced Control Engineering I – Water Tank Laboratory*, Laboratory script, 2025.