Advanced Control Engineering II: Water Tank

2. Laboratory

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Aim of the laboratory exercise

The learning goal for this lab is the mathematical modeling and state linearization of a hydraulic system and the design of an appropriate closed-loop controller. The system consists of two reservoirs where the outlet of the first is directed to the second reservoir. The outlet of the second reservoir ends in the basin. A pump can transfer water from the basin to the uppermost one. The aim of the laboratory is to control the water height of the lower reservoir by controlling the water flow through the pump. The controller to be designed for this purpose is to be implemented in Matlab/Simulink and auto-code generated to a NI-Hardware. The implementation can be used for both the test preparation (simulation) and directly on the test rig.

System description

The hydraulic system used in this laboratory consists of a water basin, a pump and two sequential water reservoirs (see Figure 1). The pump transfers the water from the basin to the uppermost reservoir (reservoir 1) where it exits through an output pipe at the bottom to reservoir 2. Each reservoir has an emergency drain if the liquid level is approaching the maximum height.

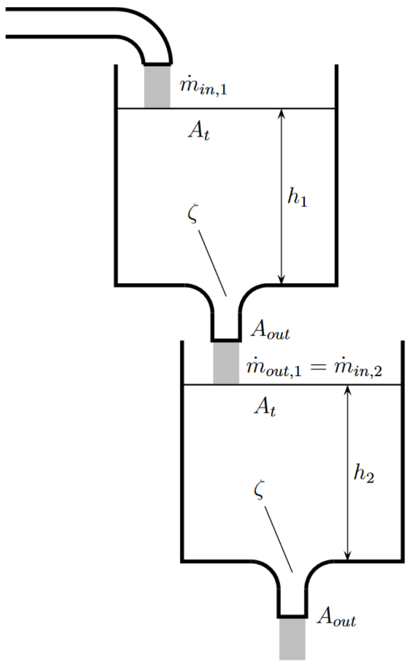


Fig. 1 – Schematics of the double tank problem

All parameters for the simulation at home are listed in the following table:

Name	Symbol	Value	Unit
Area tank	A_t	50	cm^2
Area outlet	A_{out}	25	mm^2
Pressure loss	ζ	0.05	-
Density of Water	ρ	1000	$\mathrm{kg\,m^{-3}}$
Gravity constant	g	9.81	$\mathrm{m\,s^{-2}}$
Height of emergency outlet	h_{max}	0.3	m
max Pump Voltage	u_{max}	5	V

Modeling of the reservoirs

As shown in Figure 1, the input of reservoir 1 is the inlet mass flow $\dot{m}_1$ and the output is the mass flow at the outlet $\dot{m}_2$.

The continuity equation of a container with in- and outlet is

$$\dot{m} = \dot{m}_{in} - \dot{m}_{out}\,.$$

The left hand side can be expressed by the change of fill level $\dot{m} = \rho\, A_t\, \dot{h}_1$. For the upper reservoir, the inlet mass flow is given by the pump, and the outlet mass flow follows as

$$\dot{m}_{out,1} = \rho\, A_{out}\, v_{out,1} = \rho\, A_{out}\sqrt{\frac{2\,g\,h_1}{1+\zeta}}$$

This yields for the upper reservoir

$$\dot{h}_1 = \frac{\dot{m}_{in,1}}{\rho\, A_t} - \frac{A_{out}}{A_t}\sqrt{\frac{2\,g\,h_1}{1+\zeta}}$$

Since the outlet of the upper is the inlet of the lower reservoir, the equation of the second reservoir can be analogously defined as above and is given by

$$\dot{h}_2 = \frac{A_{out}}{A_t}\sqrt{\frac{2\,g\,h_1}{1+\zeta}} - \frac{A_{out}}{A_t}\sqrt{\frac{2\,g\,h_2}{1+\zeta}}$$

To sum up, the system with the state vector $\boldsymbol{x} = [h_1,\ h_2]^T$ and the input $u = \dot{m}_{in,1}$ the non-linear state-space model is represented by

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, u) = \begin{pmatrix} -a_1\sqrt{x_1} \\ a_1\sqrt{x_1} - a_2\sqrt{x_2} \end{pmatrix} + \begin{pmatrix} \frac{1}{\rho\, A_t} \\ 0 \end{pmatrix} u$$

with $a_i = \frac{A_{out,i}}{A_t}\sqrt{\frac{2\,g}{1+\zeta_i}}$, $i = 1, 2$ for each reservoir, as they may not be exactly the same, and the output equation

$$y = g(\boldsymbol{x}, u) = x_2\,.$$

Task: Compute pump model

Use the data depicted in Figure 2 to find the transfer function of the pump

$$P(s) = \frac{\dot{m}}{U_{in} - U_0} = K_{pump},$$

where $\dot{m}$ (in $\mathrm{kg\,s^{-1}}$) is the mass flow, U_{in} (in V) is the input voltage of the pump and U_0 is the offset of the voltage. Assume that the pump is a linear system and neglect the missing data close to the origin of the graph.

Task: Compute pressure sensor model

Each of the reservoirs is equipped with a pressure sensor that measures the pressure, which is proportional to the water level. As only the water level of reservoir 2 has to be controlled, only one sensor is required to be modeled. Figure 3 represents the sensor characteristics as a relation between the water level h and the output voltage u_{out}. Assume that the pressure sensor is a linear system.
Note: This is not relevant for the simulation, but for the implementation in the lab.

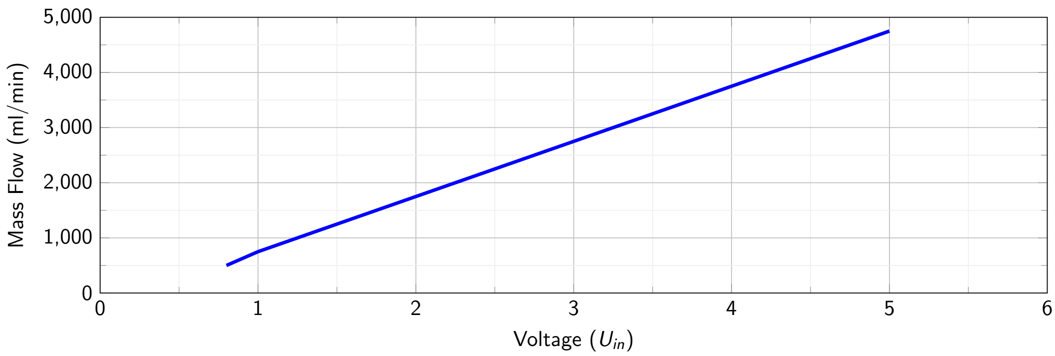


Fig. 2 – Linear relation between volumetric flow rate and pump input voltage

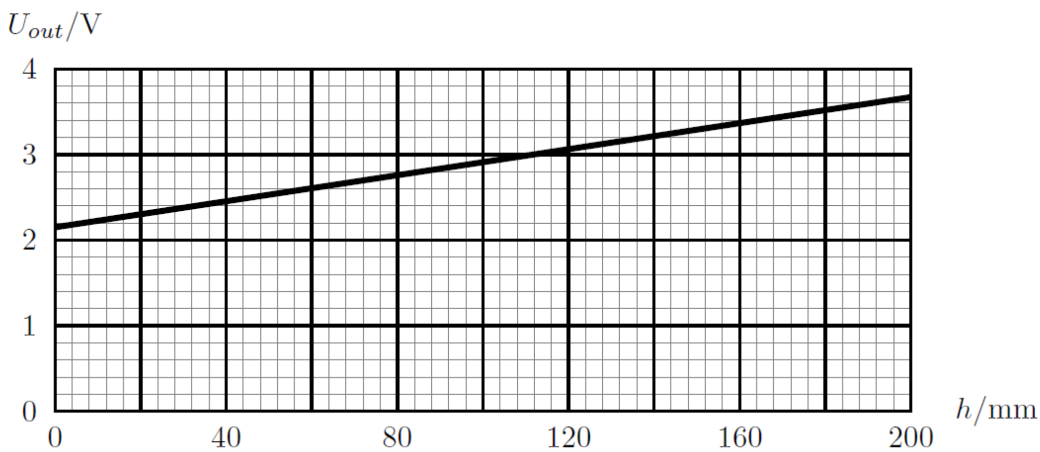


Fig. 3 – Pressure sensor characteristics

Control Design

Task: Linearize the non-linear state-space model

Linearize the non-linear state-space model in order to enable a more straightforward PID controller design. Choose an appropriate operating point (e.g. $\bar{h}_2 = 5$ cm). Calculate the transfer function of the LTI-system.

Task: PID control design

Use the linearized model and design a PID-controller with the Matlab-tool "pidTuner" for controlling the water level in the second reservoir. Choose the parameters wisely, since the system has constrains in the reservoir water level as well as the pump input voltage.

Task: Simulate closed loop system

Simulate your closed-loop system in Simulink by using the **non-linear state-space model** as plant. Use parameters defined in a script to easily modify all parameters in the lab.

- Add a **saturation** at the voltage input.
- Add a **noise** at the measured signal (Band-Limited White Noise) and use a gain-block to get a max. Amplitude of the noise of approx. 1 cm.
- Use a **output limitation** at the integrator, since the water level can never get below zero and over the emergency outlet height (use therefore 30 cm in the simulations).

Repeat the PID control design, if troubles occur due to the saturation or the signal noise. Use a PT1-filter, if necessary, to reduce the output noise (e.g. with $\tau_y = 1$ s).
Hint: You did a linearization, so be aware of adding the operating point at the input. Furthermore, the initial states of the integrator can be set correctly. Before starting controlling the system, check whether the steady state of the operating point is really a steady state. If not, fix the problems.

Task: Flatness based feed forward

With a little experience, it is obvious that the height h_2 of reservoir 2 is a flat output of the nonlinear system - explain why.
Calculate the input by means of the flat output. How often must a trajectory be continuously differentiable, so that it can be used as reference for the flat output?
Calculate a polynomial transition (as introduced in the exercises) for the flat output and calculate the reference trajectory as well as the feed-forward signal. Choose the step size and the transition time wisely. Add your trajectory to the simulation and observe the improvement.

Task: Prepare for lab

Read the instructions on what to do in the lab, because time in the lab is short.

- How to calibrate the pressure sensor?
- How to get the system constant $a = \frac{A_{out}}{A_t}\sqrt{\frac{2\,g}{1+\zeta}}$ by measurements? Note that ρ and A_t are well known and can be used if necessary.
- How to setup the NI-Hardware $\rightarrow$ Read the "how to" on the next pages

Implementation in the lab

Implement the designed control system on the real hardware using Simulink external mode (with Simulink Coder), similar to Figure 4. The transfer function of the pump and the sensor gain can be used from your preparation. However, the offset of the pressure sensor as well as signal gain needs to be calibrated precisely with 3 decimal places before implementing a controller.

Since the pressure sensor provides a digital signal, this output must first be converted into a corresponding height value. Similarly, if your controller uses mass flow as the model input, you may need to convert its output into a voltage, as the pump expects a voltage.

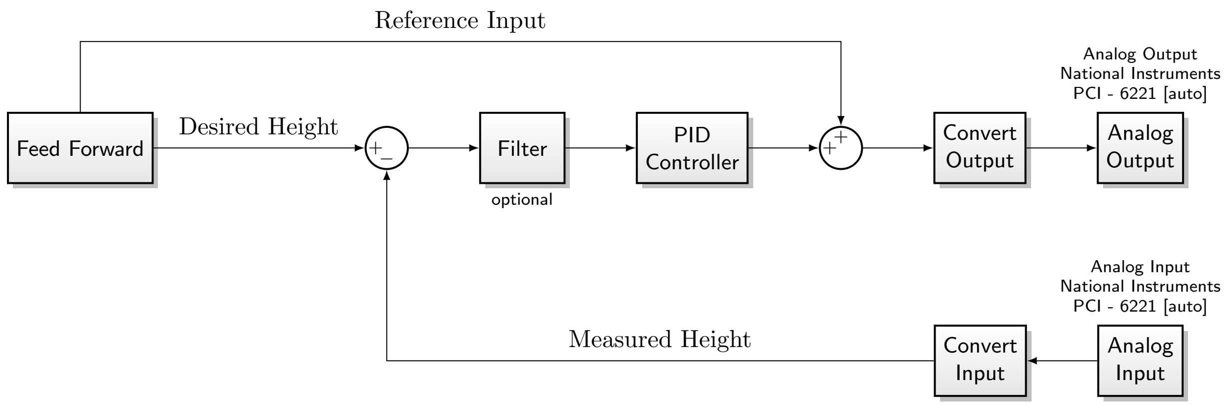
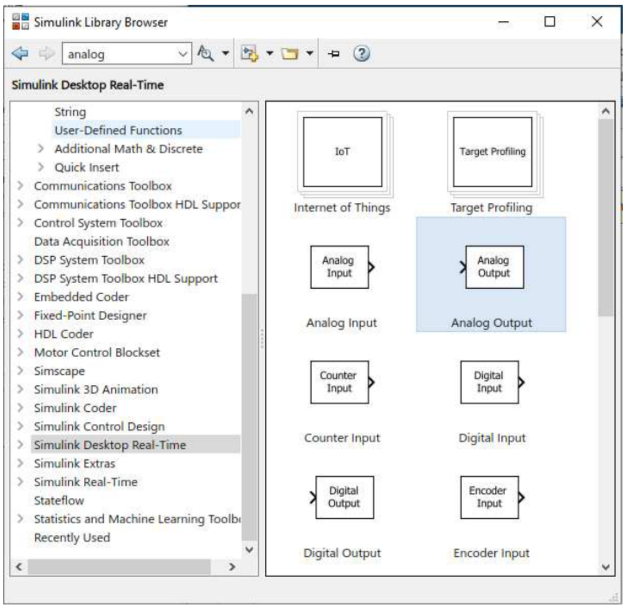
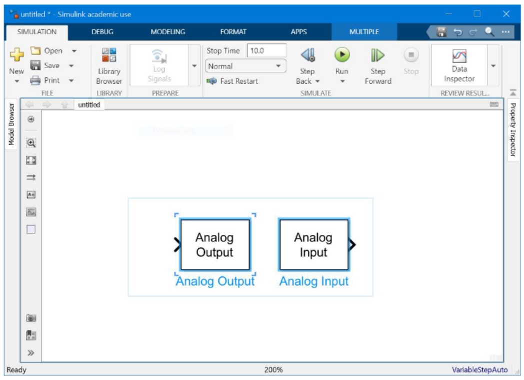


Fig. 4 – A sketch for a possible implementation in Simulink

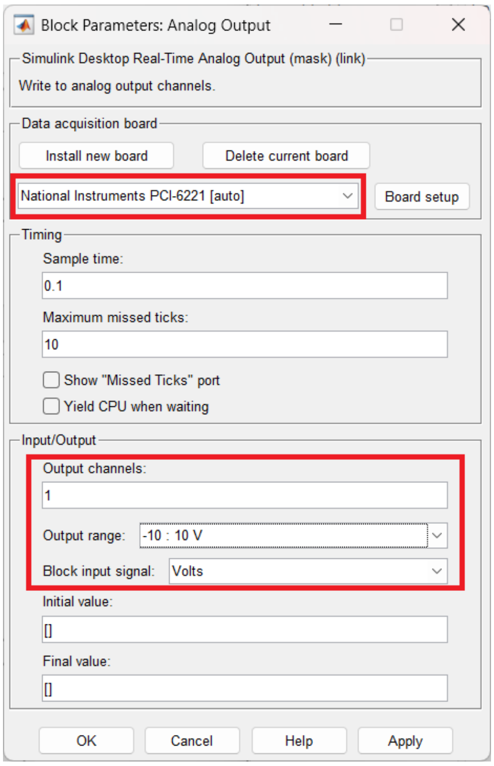
How to use NI-hardware

a) In order to access the I/O's you first need to place the correspondent blocks into your Simulink model. To this end in the Library Browser go to Simulink Desktop Real-Time and drag out an Analog Input and Analog Input.



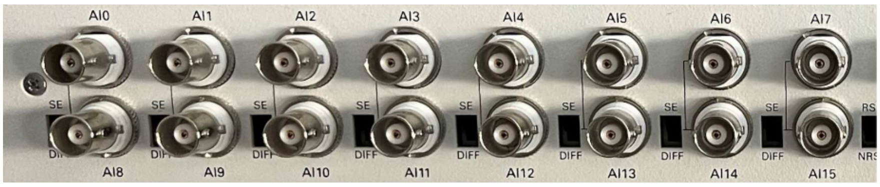


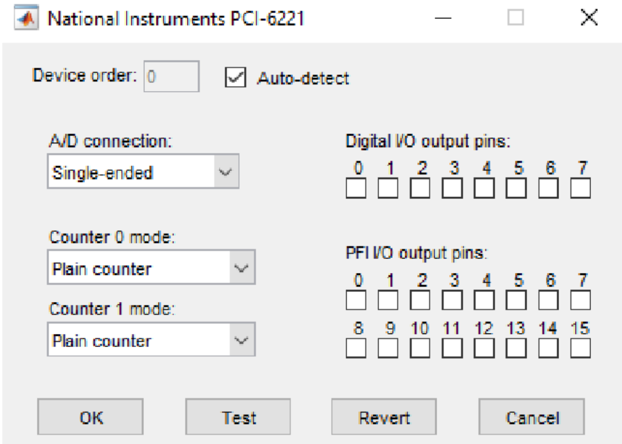
b) Select the installed National Instruments PCI-6221 cards in the setting of the IO-blocks



- Input/Output channels: 1
- Input/Output range: -10 to 10 V
- Block output signal: Volts

c) In Board setup the selected hardware can be tested. Click the respective button and wait for the green arrow to appear. The IO pins correspond to the respective connectors on the NI board:





Start your controller and investigate the performance regarding fidelity, stability and disturbance reaction.

Report

The report shall follow the usual MCI Guidelines adopted for the documentation of laboratory activities and shall be as short as possible, although including all the relevant information. The following is a proposed structure of the content of the report that is commonly adopted either in scientific papers as well as thesis and that can be easily customized and adapted to the specific laboratory activity:

1. Very brief definition of the learning target of the laboratory experiment and description of the structure of the document
2. Summary of theoretical aspects addressed both in preparatory work as well as in class activities (among others: state linearization, parameter identification, trajectory planning, ...)
3. Description of laboratory setup and parameter identification.
4. Simulations (Remark: this chapter contains mainly the block diagrams and relevant comments related to the implemented simulations)
5. Results and interpretation (Remark: in this chapter the measurements shall be compared with results out of the simulations possibly showing simulation and measurements on the same plots or with side-by-side figures)
6. Conclusions: summarize the main achievements and problems/solutions encountered during the laboratory experience

Please mark on the cover page at least the following information:

- Title of the activity
- Date of the lab
- Group members

Create a .zip-file containing the report in .pdf-format and a folder with simulations and measurements. Only this .zip-file has to be submitted in digital form on SAKAI Assignments (please upload one copy per group only) within the date reported on the announcements on SAKAI.