Laboratory Report 2

Water Tank

MA-MECH-25-VZ

Advanced Control Engineering Laboratory

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Introduction

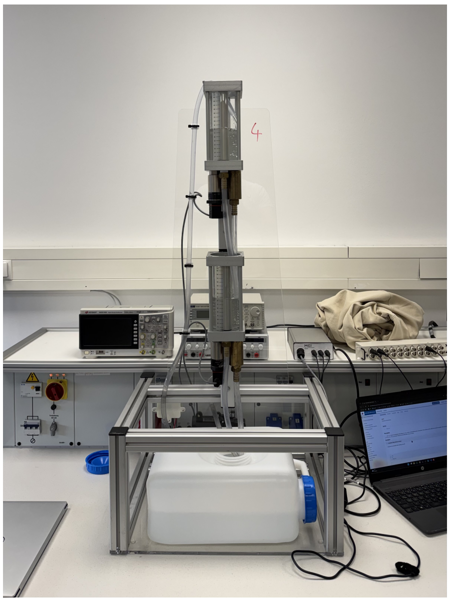

Figure 1.1: Water Tank laboratory system

This laboratory investigates the control of a nonlinear hydraulic two-tank system. Water is pumped from a basin into the upper reservoir and flows into the lower reservoir due to gravity, where the water level is measured and regulated by controlling the pump input.

The task focuses on modelling the nonlinear system dynamics, linearising the model around a suitable operating point and designing a control strategy capable of regulating the water level and enabling smooth transitions using trajectory planning and a flatness-based feed-forward approach. MATLAB and Simulink are used for modelling, simulation and controller design, as well as for the implementation of the control algorithm on the laboratory hardware via a National Instruments interface.

The report is structured as follows:

- Chapter 2 summarises the theoretical background.
- Chapter 3 outlines the laboratory setup and parameter identification.
- Chapter 4 presents the simulation models and control structure.
- Chapter 5 discusses the results.
- Chapter 6 concludes the report.

Chapter 2

Theoretical Background

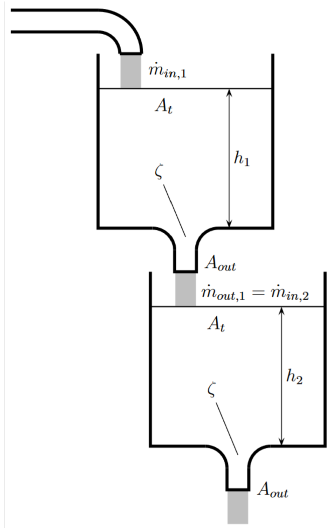

Figure 2.1: Schematic representation of the two-tank hydraulic system [1]

The water tank system is modeled as a nonlinear dynamic system consisting of two interconnected reservoirs and a pump-driven inflow. Water enters the upper reservoir and flows into the lower reservoir due to gravity. A detailed physical description of the system and the corresponding parameter definitions are provided in the laboratory assignment [1].

Assuming incompressible fluid behaviour, the nonlinear system dynamics can be expressed in state-space form as

$$\dot{x}(t) = f(x(t), u(t)), \qquad x = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \tag{2.1}$$

where the control input $u(t)$ represents the pump mass flow. The controlled output is defined as the water level of the lower reservoir,

$$y(t) = h_2(t).$$

2.1 State linearization

For controller design, the nonlinear model is linearized around a steady operating point, resulting in a local linear time-invariant approximation. In addition to feedback control, a flatness-based feed-forward approach is considered, exploiting the fact that the water level of the lower reservoir constitutes a flat output of the nonlinear system.

$$\dot{x}_1 = -a_1\sqrt{x_1} + \frac{1}{\rho A_t}u \tag{2.2}$$

$$\dot{x}_2 = a_1\sqrt{x_1} - a_2\sqrt{x_2} \tag{2.3}$$

where the parameters are defined as $a_i = \frac{A_{out,i}}{A_t}\sqrt{\frac{2g}{1+\zeta_i}}$ for $i = 1, 2$.

The nonlinear state-space model is linearized at an operating point determined by the desired water level in the second reservoir. As per the task description, this is chosen as $\bar{x}_2 = 5\,\mathrm{cm}$. The equilibrium conditions $\dot{x} = 0$ yield:

$$\bar{x}_2 = 0.05\,\mathrm{m}, \quad \bar{x}_1 = \left(\frac{a_2}{a_1}\right)^2 \bar{x}_2, \quad \bar{u} = \rho A_t a_1\sqrt{\bar{x}_1} \tag{2.4}$$

The partial derivatives of the non-linear terms $f(x, u)$ evaluated at this operating point are:

$$\left.\frac{\partial f_1}{\partial x_1}\right|_{\bar{x}} = -\frac{a_1}{2\sqrt{\bar{x}_1}} \tag{2.5}$$

$$\left.\frac{\partial f_2}{\partial x_1}\right|_{\bar{x}} = \frac{a_1}{2\sqrt{\bar{x}_1}} \tag{2.6}$$

$$\left.\frac{\partial f_2}{\partial x_2}\right|_{\bar{x}} = -\frac{a_2}{2\sqrt{\bar{x}_2}} \tag{2.7}$$

Introducing deviations $\Delta\mathbf{x} = \mathbf{x} - \bar{\mathbf{x}}$ and $\Delta u = u - \bar{u}$, the linearized system $\Delta\dot{\mathbf{x}} = \mathbf{A}\Delta\mathbf{x} + \mathbf{B}\Delta u$ is given by:

$$\begin{bmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{a_1}{2\sqrt{\bar{x}_1}} & 0 \\ \frac{a_1}{2\sqrt{\bar{x}_1}} & -\frac{a_2}{2\sqrt{\bar{x}_2}} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{\rho A_t} \\ 0 \end{bmatrix} \Delta u \tag{2.8}$$

Chapter 3

Laboratory Setup and Parameter Identification

The laboratory setup consists of a two-tank hydraulic system equipped with a pump and pressure sensors for water level measurement. Water is pumped from a basin into the upper reservoir, while gravity-driven flow connects the upper and lower reservoirs. The water level of the lower reservoir is used as the controlled output.

Control and data acquisition are handled via National Instruments hardware. The system model and control structure are implemented in MATLAB and Simulink and deployed to the laboratory hardware using MATLAB Coder. Physical constraints such as pump voltage saturation and the maximum admissible reservoir heights impose strict bounds on system operation and must be considered during parameter identification and control design.

3.1 Parameter identification

3.1.1 Pump parameters

The pump provides the inlet mass flow to the upper reservoir and is driven by an input voltage. Based on the characteristic curve given in [1], the pump is approximated by a static linear model

$$K_{\mathrm{pump}} = \frac{\dot{m}}{(U_{\mathrm{in}} - U_0)} \tag{3.1}$$

where the gain K_{pump} and the voltage offset U_0 are determined by linear regression of the diagram, given in the laboratory script [1], depicted in figure 3.1.

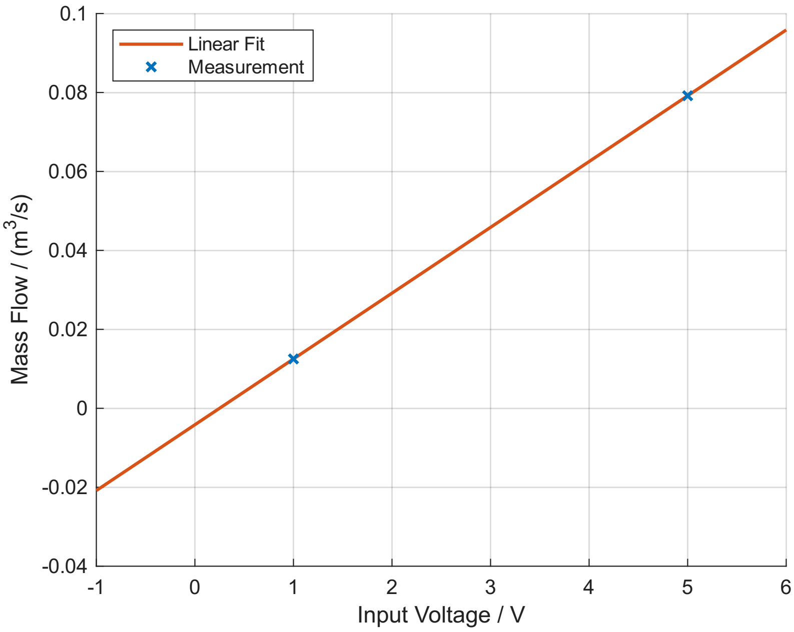

Figure 3.1: Pump characteristic curve and linear fit [1]

Linear regression yields $K_{\text{pump}} = 0.0167\,\mathrm{kg\,s^{-1}\,V^{-1}}$ and $U_0 = 0.25\,\mathrm{V}$.

3.1.2 Water tank parameters

The hydraulic outflow behaviour of the reservoirs is described by effective flow coefficients derived from the nonlinear model. These parameters are identified under steady-state operating conditions, where the water levels remain constant and the inflow equals the outflow. System operation is constrained by the admissible pump voltage and the physical height limits of the reservoirs.

Under steady-state conditions, the outflow coefficient of the upper reservoir is obtained as

$$a_1 = \frac{b\,K_{\text{pump}}(U_{\text{in}} - U_0)}{\sqrt{h_1}}, \tag{3.2}$$

with

$$b = \frac{1}{\rho A_t} = \frac{1}{1000\,\mathrm{kg/m^3} \times 50 \times 10^{-4}\,\mathrm{m^2}} = 0.2\,\mathrm{m\,kg^{-1}}. \tag{3.3}$$

Evaluating this expression using multiple steady-state measurements yields

$$a_1 = 0.0574\,\mathrm{m^{1/2}/s}. \tag{3.4}$$

The outflow coefficient of the lower reservoir follows from the steady-state relation between both tank levels,

$$a_2 = a_1 \frac{\sqrt{h_1}}{\sqrt{h_2}}, \tag{3.5}$$

resulting in

$$a_2 = 0.0524\,\mathrm{m}^{1/2}/\mathrm{s}. \tag{3.6}$$

3.1.3 Sensor parameters

Water levels in the reservoirs are measured using pressure sensors. Owing to the proportional relation between hydrostatic pressure and water column height, the sensor behaviour is modeled as a linear static mapping between output voltage and water level.

Sensor gain and voltage offset are identified from steady-state measurements by linear regression of the calibration data shown in Figure 3.2.

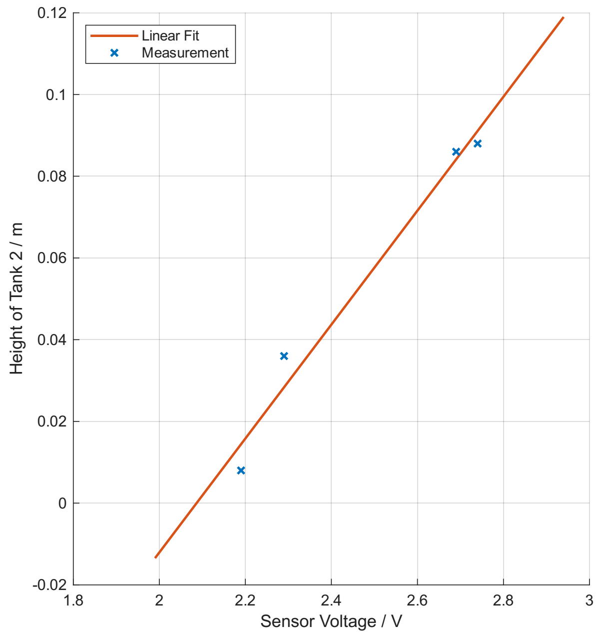

Figure 3.2: Pressure sensor characteristic and linear fit

For the pressure sensor of the second reservoir, the identified parameters are $K_{\text{sens}} = 0.1394\,\mathrm{m\,V^{-1}}$ and $U_{0,\text{sens}} = 2.0866\,\mathrm{V}$.

Chapter 4

Simulation

Figure 4.1 shows the complete Simulink model used for simulation of the closed-loop water tank system. The nonlinear plant is implemented according to the state equations derived in Chapter 2. The states correspond to the water levels of the upper and lower reservoirs and are obtained by numerical integration of the level dynamics.

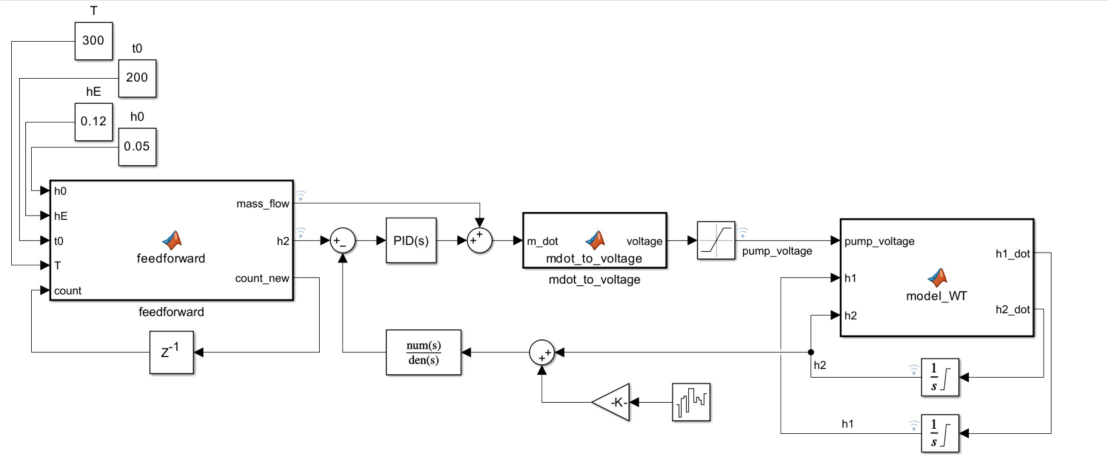

Figure 4.1: Schematic of the Simulink simulation model

The control structure combines a feedback controller with a feed-forward term. A reference mass flow is generated by the feed-forward block based on the desired water level trajectory, while disturbances and modeling inaccuracies are compensated by the feedback controller. The resulting control signal is converted into a pump voltage using the identified actuator model and applied to the nonlinear plant.

Non-ideal effects are included to reflect realistic operating conditions. The pump input voltage is limited by a saturation block to respect actuator constraints. Measurement noise is added to the water level signal using band-limited white noise. Furthermore, the water levels are constrained to remain within the physically admissible range of the laboratory setup.

Chapter 5

Results and Interpretation

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Figure 5.1 shows the simulated height response of the Ball-in-Tube system using the nonlinear model and the parameters identified in Chapters 2 and 3. A seventh-order polynomial trajectory is used to generate smooth transitions between the lower height of $0.1\,\mathrm{m}$ and the upper height of $0.4\,\mathrm{m}$. The downward motion is obtained by reversing the boundary conditions of the trajectory.

The simulated height closely follows the reference trajectory during both the upward and downward transitions. The response is smooth and does not exhibit overshoot, while only minor deviations occur due to the injected measurement noise. Overall, the results indicate that the PID controller and the flatness-based feed-forward provide satisfactory tracking performance in simulation. These results serve as a reference for evaluating the behavior of the control strategy on the real system.

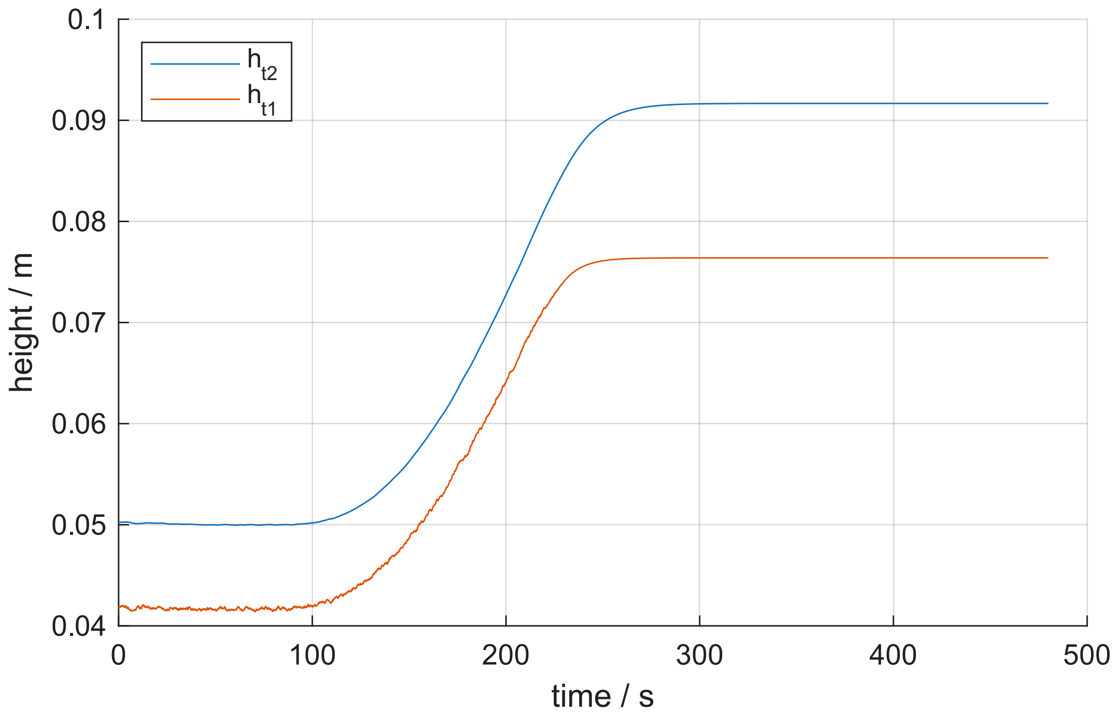

Figure 5.1: Transition from $0.1\,\mathrm{m}$ to $0.4\,\mathrm{m}$ using the Simulink-Simulation and the non-linear model of the system

Figure 5.2 shows the measured height response of the Ball-in-Tube system for the same reference trajectory used in the simulation. In contrast to the idealized simulation result, the real system exhibits small tracking deviations during the transition phases and a slight steady-state offset during the plateau.

The upward and downward motions remain smooth, indicating that the trajectory planning and feed-forward approach are effective also on the real hardware. However, measurement noise, actuator limitations, and unmodeled dynamics lead to a reduced tracking accuracy compared to the simulation. In particular, the finite fan dynamics and input saturation influence the transient response.

Overall, the real-system results confirm the validity of the control concept derived in simulation, while highlighting the impact of practical non-idealities that are not fully captured by the model.

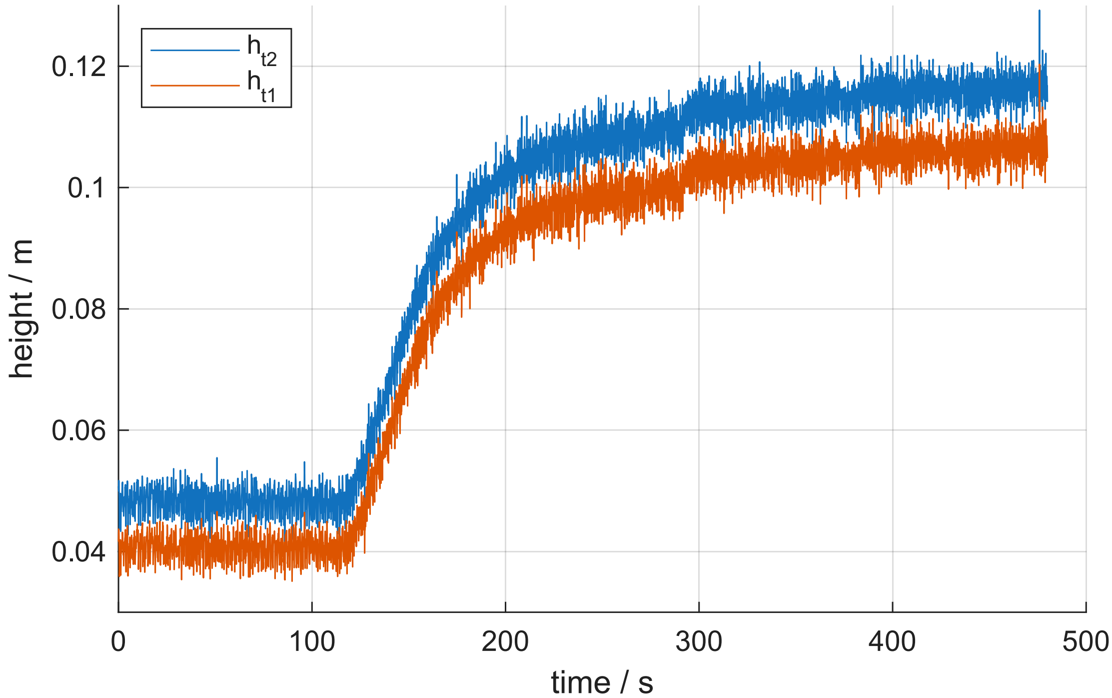

Figure 5.2: 3D-Modell einer Offline-Simulation, Bildquelle: Visual Components GmbH

Chapter 6

Conclusion

- Summary of achieved objectives
- Key findings from modeling, control design, and simulations
- Evaluation of controller performance and limitations
- Identified challenges and implemented solutions
- Possible improvements and future work

Bibliography

[1] P. Kronthaler, *Advanced Control Engineering I – Water Tank Laboratory*, Laboratory script, 2025.