Laboratory Report 2

Water Tank

MA-MECH-25-VZ

Advanced Control Engineering Laboratory

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Introduction

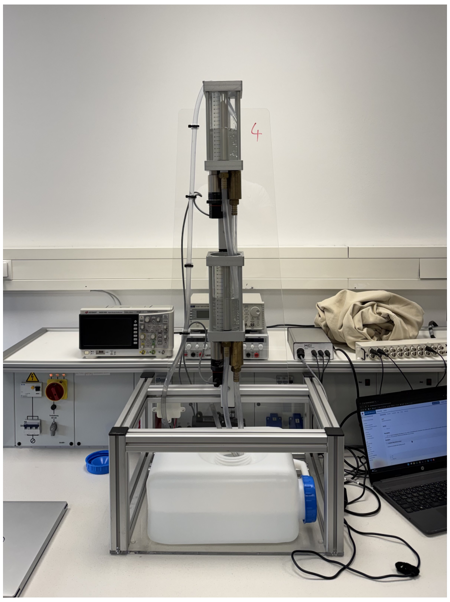

Figure 1.1: Water Tank laboratory system

This laboratory experiment investigates the control of a nonlinear hydraulic two-tank system. The objective is to regulate the water level of the lower reservoir by controlling the pump input. MATLAB and Simulink are used for modeling, controller design and simulation, as well as for the implementation of the control algorithm on the laboratory hardware via a National Instruments interface. In addition to feedback control, a flatness-based feed-forward approach is considered to enable smooth reference trajectory tracking.

The report is structured as follows:

- Chapter 2 summarizes the theoretical background.
- Chapter 3 describes the laboratory setup and parameter identification.
- Chapter 4 presents the simulation models and control design.
- Chapter 5 discusses the results.
- Chapter 6 concludes the report.

Chapter 2

Theoretical Background

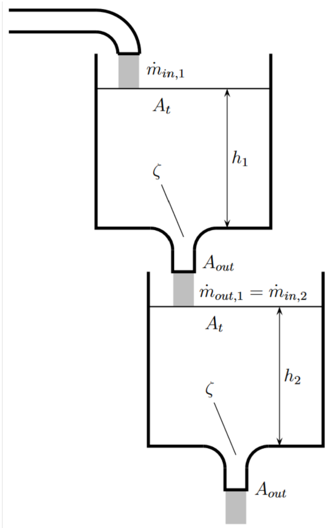

Figure 2.1: Schematic representation of the two-tank hydraulic system [1]

The water tank system is modeled as a nonlinear dynamic system consisting of two interconnected reservoirs and a pump-driven inflow. Water enters the upper reservoir and flows into the lower reservoir due to gravity. A detailed physical description of the system and the corresponding parameter definitions are provided in the laboratory assignment [1].

Assuming incompressible fluid behaviour, the nonlinear system dynamics can be expressed in state-space form as

$$\dot{x}(t) = f(x(t), u(t)), \qquad x = \begin{bmatrix} h_1 & h_2 \end{bmatrix}^\top, \tag{2.1}$$

where the control input $u(t)$ represents the pump mass flow. The controlled output is defined as the water level of the lower reservoir,

$$y(t) = h_2(t).$$

2.1 State linearization

For controller design, the nonlinear model is linearized around a steady operating point, resulting in a local linear time-invariant approximation. In addition to feedback control, a flatness-based feed-forward approach is considered, exploiting the fact that the water level of the lower reservoir constitutes a flat output of the nonlinear system.

$$\dot{x}_1 = -a_1\sqrt{x_1} + \frac{1}{\rho A_t}u \tag{2.2}$$

$$\dot{x}_2 = a_1\sqrt{x_1} - a_2\sqrt{x_2} \tag{2.3}$$

where the parameters are defined as $a_i = \frac{A_{out,i}}{A_t}\sqrt{\frac{2g}{1+\zeta_i}}$ for $i = 1, 2$.

The nonlinear state-space model is linearized at an operating point determined by the desired water level in the second reservoir. As per the task description, this is chosen as $\bar{x}_2 = 5\,\mathrm{cm}$. The equilibrium conditions $\dot{x} = 0$ yield:

$$\bar{x}_2 = 0.05\,\mathrm{m}, \quad \bar{x}_1 = \left(\frac{a_2}{a_1}\right)^2 \bar{x}_2, \quad \bar{u} = \rho A_t a_1\sqrt{\bar{x}_1} \tag{2.4}$$

The partial derivatives of the non-linear terms $f(x, u)$ evaluated at this operating point are:

$$\left.\frac{\partial f_1}{\partial x_1}\right|_{\bar{x}} = -\frac{a_1}{2\sqrt{\bar{x}_1}} \tag{2.5}$$

$$\left.\frac{\partial f_2}{\partial x_1}\right|_{\bar{x}} = \frac{a_1}{2\sqrt{\bar{x}_1}} \tag{2.6}$$

$$\left.\frac{\partial f_2}{\partial x_2}\right|_{\bar{x}} = -\frac{a_2}{2\sqrt{\bar{x}_2}} \tag{2.7}$$

Introducing deviations $\Delta\mathbf{x} = \mathbf{x} - \bar{\mathbf{x}}$ and $\Delta u = u - \bar{u}$, the linearized system $\Delta\dot{\mathbf{x}} = \mathbf{A}\Delta\mathbf{x} + \mathbf{B}\Delta u$ is given by:

$$\begin{bmatrix}\Delta\dot{x}_1 \\ \Delta\dot{x}_2\end{bmatrix} = \begin{bmatrix}-\frac{a_1}{2\sqrt{\bar{x}_1}} & 0 \\ \frac{a_1}{2\sqrt{\bar{x}_1}} & -\frac{a_2}{2\sqrt{\bar{x}_2}}\end{bmatrix}\begin{bmatrix}\Delta x_1 \\ \Delta x_2\end{bmatrix} + \begin{bmatrix}\frac{1}{\rho A_t} \\ 0\end{bmatrix}\Delta u \tag{2.8}$$

Chapter 3

Laboratory Setup and Parameter Identification

3.1 Parameter identification

3.1.1 Pump parameter identification

The pump generates the inlet mass flow to the upper reservoir and is driven by an input voltage. Based on the characteristic curve provided in the laboratory script [1], the pump is approximated as a static linear system

$$K_{\mathrm{pump}} = \frac{\dot{m}}{(U_{\mathrm{in}} - U_0)} \tag{3.1}$$

where the gain K_{pump} and the voltage offset U_0 are determined by linear regression of the diagram, given in the laboratory script [1], depicted in 3.1.

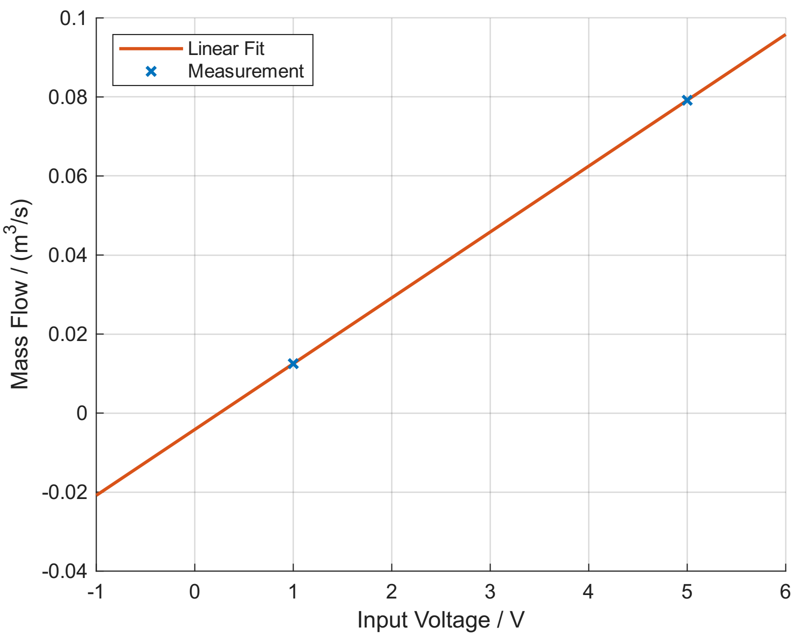

Figure 3.1: Transition from $0.1\,\mathrm{m}$ to $0.4\,\mathrm{m}$ using the Simulink-Simulation and the non-linear model of the system

The linear regression of the pump diagram in 3.1 returns following values for the corresponding parameters: $K_{\mathrm{pump}} = 0.0167\,\mathrm{kg\,s^{-1}\,V^{-1}}$ and $U_0 = 0.25\,\mathrm{V}$

3.1.2 Water tank parameter identification

3.2 Sensor calibration

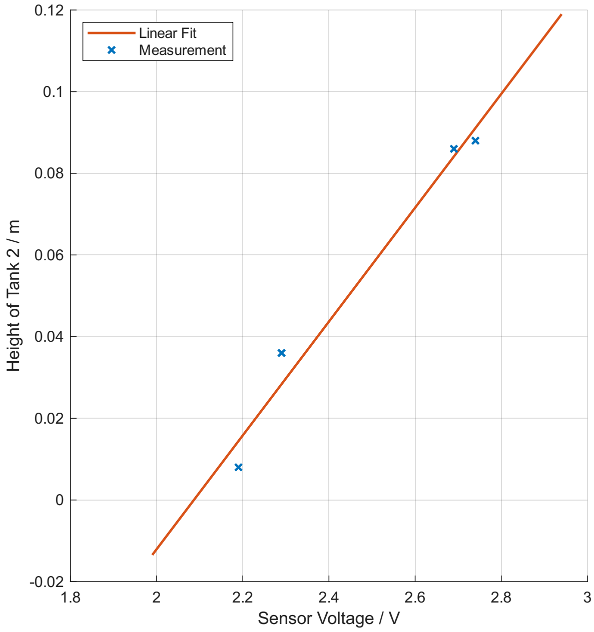

Figure 3.2: Transition from $0.1\,\mathrm{m}$ to $0.4\,\mathrm{m}$ using the Simulink-Simulation and the non-linear model of the system

The laboratory setup is used for experimental parameter identification of the hydraulic two-tank system and comprises a pump supplying the upper reservoir as well as pressure sensors for water level measurement.

where the gain K_{pump} and the voltage offset U_0 are determined by linear regression of the measured data.

Pressure sensor calibration The water level in each reservoir is measured by a pressure sensor. Since hydrostatic pressure is proportional to the water column height, a linear sensor model is assumed. Calibration is performed by recording the sensor output voltage at known water heights and identifying the corresponding gain and offset parameters using linear regression.

Hydraulic parameters and constraints The hydraulic outflow behaviour of the reservoirs is characterized by an effective flow coefficient, which is identified experimentally using steady-state measurements. System operation is constrained by the maximum admissible pump voltage and by the physical limits of the reservoir heights.

Chapter 4

Simulation

- Implementation of the nonlinear plant model in Simulink
- Integration of pump and sensor models
- Linearized model implementation for controller design
- PID controller tuning using `pidTuner`
- Inclusion of actuator saturation and output limitations
- Noise modeling and optional output filtering
- Implementation of flatness-based feed-forward control

Chapter 5

Results and Interpretation

- Closed-loop simulation results for reference tracking
- Comparison between linear and nonlinear model behavior
- Influence of saturation and measurement noise
- Performance improvement using feed-forward control
- Comparison of simulation results with laboratory measurements (if available)
- Discussion of stability, accuracy, and robustness

Chapter 6

Conclusion

- Summary of achieved objectives
- Key findings from modeling, control design, and simulations
- Evaluation of controller performance and limitations
- Identified challenges and implemented solutions
- Possible improvements and future work

Bibliography

[1] P. Kronthaler, *Advanced Control Engineering I – Water Tank Laboratory*, Laboratory script, 2025.