ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Lecture with Computer Exercises: Modelling and Simulating Social Systems with MATLAB

Project Report

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Name 1 & Name 2

Zurich
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1 Abstract

2 Individual contributions

3 Introduction and Motivations

3.1 What is Self-Organized Criticality?

Self-Organized Criticality is the Concept that the dynamics of a physical system converge to a critical point independant of the bestowed boundary conditions. This critical point commonly is not the equilibrium point in a traditional, physical sense. The concept is best understood by example, so we shall bring up a very educative one introduced first by (P.Bak, C.Tang, K.Wiesenfeld, Phys. Rev. A 38, 1988). Imagine a sand pile in 2 dimensions. One can discretisze this pile into cells, which are denoted with the indices n. Each of those cells has an amount of sand grains on it, which is directly corresponding to the height z_n. Now we state the simple rule, that, if the difference in height between two neighboring cells becomes larger than a predefined limit, the higher cell will give grains to the lower cell (avalanche). Now every timestep, we drop a grain on cell one. Independant of the initial conditions, eventually, a straight slope will evolve. The critical point is reached if every difference between two neighboring cells is exactly the limit. What happens if we drop the next grain? Then the difference $z_1 - z_2 > p$ and one grain will drop on z_2. Subsequently, the difference $z_2 - z_3$ will be too big and the process repeats until it hits the last cell. We therefore call a state critical if a yet so small disturbance can propagate throughout the whole system. A further condition for SOC is the presence of the critical point, in the example, that would be the slope that should form in every experiment independant of the starting conditions. It is important to note here, that this critical state is *not* the equilibrium state of the system, since that would be a flat surface.

3.2 What is Self-Organized Criticality

Self-organized criticality (SOC) is a concept applied to spacially extended dynamic physical systems. It is often quite difficult to descripe the temporal and spatial evolution of such a system and a lot of research happens in that field. Usually, one uses a mean-field approach to such a problem, where the indidual coupled degrees of freedom are replaced by a field. This (external) field then acts on the DOF. SOC is a different approach to this problem. It is the concept, that a certain class of dissipative, coupled dynamic systems converge to a critical point independantly of their underlying boundary and initial conditions. What are the properties of such a

critical point? First of all, it is stable with respect to small disturbances. If this was not true, the system would not evolve to such a point. Note that this does not imply that such a point *exists*. If a certain point is unstable, the system commonly will not evolve to this point. The critical point, however, generally is not the physical equilibrium point of the system.

3.3 Flicker Noise

Flicker noise is something often mentioned when talking about dissipative coupled dynamical systems. Flicker Noise (or $1/f$-Noise) is the description for an often observed phenomenon of such systems; The observation, that the power spectrum $S(f)$ scales with $1/f$ or more generally, with $f^{-\beta}$. This implies that $S(f) \cdot f \propto 1$

3.4 Forest-Fire Model

A Forest Fire Model is basically a cellular automata. Each cell has three possible states:

1. Empty Site
2. Alive Tree
3. Burning Tree

There are four rules defined for each cell, which are executed simultaneously:

- A burning tree turns into an empty site
- A tree starts to burn if at least one of its neighbors are burning
- A tree will grow on an empty site randomly with probability p
- A tree will burn randomly with probability f

It is important to note that the Forest Fire Model does not only apply to forest fires as its name would suggest. A whole class of problems follows the same basic rules and can be treated accordingly. An example would be the spreading of a disease, where one can directly transform the above rules and states.

3.5 Power Laws

Power laws are functions of the form $P(s) \propto s^{-\tau}$. These functions are found in many data sets, such as the size distribution of cities in a certain area. Take any country, one will find one very large metropolis, a few large cities, many medium

sized towns and a huge lot of small towns. Power laws imply that the frequency of a general data point is inversely proportional to its magnitude.

In context to the FFM, it is proposed as a way of quantifying the frequency of Fires with respect to their cluster size.

Power laws have also prominently been found in data sets of earthquakes, where they apply almost perfectly, meaning that small earthquakes happen very often in respect to the frequency of big ones.

3.6 Key Parameters in SOC

To find out wether a basic FFM exhibits SOC behavior, we need to define a set of parameters which serves as indicators for the desired analysis.

- The time needed to burn down a forest cluster: This is heavily dependant on the form of the implementation. If we choose to implement it in a way like [reference needed], we will have an instantaneous fire which essentially takes no time to burn and just resets all the connected trees. If however, we choose the form of a visible fire, meaning that the ratio $f/p \ll 1$ is not 0 in limit and that it takes the fire one timestep to advance a grid cell, then the time needed to burn a forest cluster is an important variable we need to extract from the simulation.
- Number distribution of the size of clusters: Here we first need to define what a cluster is. A cluster is a set of neighboring cells all obtaining the same state. When we talk about forest clusters, we of course mean connected trees. In the implementation where the fire spreads infinitly fast, the forest cluster containing the ignitor cell is the same as the burnt area. The size distribution therefore is a very good indicator for the fire size distribution.
- The mean number of forest clusters in a unit volume $n(s)$, where s is the number of trees in a cluster.
- Number of fires per unit time step: This is a tuning parameter of the model.
- Correlation length.

We have tried to implement the model in a way that the measurment of these desired variables is possible with the least needed effort.

3.7 Motivation

Why is it important to study these effects? As mentioned before, forest fire models apply to a wide range of problems and are therefore helpful in predicting the behaviour of such systems. Self-Organized criticality is the concept that describes the

dynamics of such models and the two fields are therefore closely connected. One has closely studied these effects to make better predictions about forest fires. The main problem was, that in most cases, even relatively small fires were extinguished by the authorities. Since that led to an overcritical state, the probability for a huge, devastating fire rose quickly. This has happended in the past and had a serios impact on the ecosystem of those areas. Since the problem of self-organized criticality has been understood, one has stopped to extinguish every little fire, therefore avoiding to reach an overcritical state at which the disturbance (a small fire) can propagate throughout the whole system (large-scale fire). The same ideas can be applied to a variety of problems such as disease spreading or urban planning.

4 Description of the Model

4.1 Cellular automata

A cellular automata is a very powerful way of simulating problems which are defined by a set of rules. It is usually simulated on a grid, but not necessarily restricted to those geometrical constraints. The main characteristics are:

- Every grid point has a state
- Grid points change their state depending on the neighbor states according to a set of rules
- Random actions may be introduced

In the example of the FFM, every grid point has three states (empty, alive and burning) and four rules, the most obvious of which is change state to burning if you are alive and your neighbor is burning.

Cellular Automata can exhibit very complicated behavior when fed with very simple rules, which is the main reason why they are so powerful. There are similarities to agent-based models and networks, but it is not to be mistaken for one of these.

The second approach to grid-based updating is with a second grid that stores temporary information. [Insert Description here]

5 Implementation

As discussed before, there are two main implementation methods for the model: One way is to update the whole grid every timestep, whereas the second method picks a cell randomly at every timestep an only updates the chosen one. Both versions store the Grid in a Matrix which is initialized as follows:

```
Grid=zeros(Grid_Size);
```

or if the grid should be filled by a degree of k:

```
Grid=floor(rand(Grid_Size)+k);
```

5.1 Grid Based Updating

The basic construct of the grid based updating implementation looks like this:

```
for i=1:t % Loop over all timesteps
    for j=1:size(Grid,1) % Loop over all y-coordinates
        for k=1:size(Grid,2) % Loop over all x-coordinates
            if Grid(j,k)==0 %If the grid point is empty
                //Grid(j,k)=1 with some probability p
            end
        end
    end
    for j=1:size(Grid,1) % Loop over all y-coordinates
        for k=1:size(Grid,2) % Loop over all x-coordinates
            //if Neighbor(j,k)==3 % If grid neighbor is burning
                Grid(j,k)=2 % Set current grid point on fire
            end
        end
    end
    for j=1:size(Grid,1) % Loop over all y-coordinates
        for k=1:size(Grid,2) % Loop over all x-coordinates
            if Grid(j,k)==3 % If Grid point is burning
                Grid(j,k)=0; % Turn it into an empty site
            end
            if Grid(j,k)==2 % If grid point is ignited
                Grid(j,k)=3; % Turn it into an empty site.
            end
        end
    end
end
```

Several things are to note here:

- Commands with // in front are not implemented exactly like that. Check the appendix for the complete source code.

- We need several spatial loops in order to successfully suppress updating fragments like infinite fire propagation speed in updating direction.
- For the same reason, we need a differention between newly ignited (state 2) and burning (state 3) trees.
- The Algorithm is quite slow because of all the loops and if-statements.

5.2 Random Based Updating

In a random based updating implementation, at the beginning of every time step, one random cell is chosen. It then checks for the rules of the model and acts accordingly. In order to see fires, we need to have instantaneous burning, which is actually permitted by the model. However, it changes the dynamics of the system drastically. But more about that later. The following steps are taken for each timestep:

1. Choose a single cell
2. If cell is empty, grow a tree with probability p
3. If cell is a tree, with probabiltiy f, burn it and the connected cluster

In Matlab, this looks something like this:

```
for i=1:t % Loop over all timesteps
(j,k)=ceil(Grid_Size*rand(1,2)); % Choose a random cell
if  Grid(j,k)==0 % If the cell is empty
\\With some probability p set G(j,k)==1;
end
if Grid(j,k)==1 % If the cell is alive
\\with some probability f execute burn(j,k);
end
end
```

This implementation of the forest fire model has several advantages:

- There are no problems due to updating processes
- Much shorter implementation
- Easier to extract data

The last point is to be explained more extensively: Every time a tree ignites, the whole connected cluster is burned down. Since the whole process happens in a single timestep, we were able to use a recursive function. This function would search every neighboring cell and pass the function on to it if it is alive, just after burning itself. Now with this function structure, we were easily able to retrieve much wanted data such as total function calls (Fire size) or recursion depth (Cluster size). Problems with this implementation will be discussed later.

5.2.1 The "burn" function

The burn function makes the magic happen for the whole program. It basically takes the current grid cell coordinates and the recursion depth as input. Here is what it does with that:

1. Set the current cell as 0 (empty)
2. Add one to the recursion level
3. Retrieve the direct neighbor coordinates from the *neighbor* function
4. check every neighbor for being alive. If so, call the *burn* function with the neighbor coordinates.
5. Add the returned values to the current values (trees burned)
6. If no neighbor is alive, return trees burned as one.

In other words: This function propagates throughout the system until it reaches a grid cell which has no alive neighbors. It then goes back to the last grid cell where it did have another possibility to go to and chooses that one. This process repeats until there are no trees left in the cluster. Each time the function goes back on its path, it accumulates all the trees burned by that branch. In the end, that gives us a very clear picture about how many trees were burned by evaluating the initial call.

Problems with the *burn* function The big problem with this function was, that it would exceed the available stack memory pretty fast. This would always happen if the grid is relatively full. What then happens is that there is no boundary to the cluster, since we are using periodic boundary conditions. The function would propagate throughout the whole system in one single branch. This causes *MATLAB* to crash gracefully and tell the user that one should set the recursion limit to a higher value. What we learned from that, is, that a normal computer crashes at about 2'500 recursions. Now this means that we would not be able to simulate grids of size 25×25 or more which is not acceptable.

Solution The solution obviously was to limit the recursion depth of the function. This was relatively easy to implement, we just had to add 1 every time the function was called. There was, however, a payoff. Since the Function did not have "full" freedom anymore, it could happen that it did not burn the whole cluster. The fraction was in the range of 0.99, but still, it was not quite the optimum.

Improvements were made in these ways:

- The order in which the neighbors were checked was turned into random. This effectively resulted in the "drunkards walk" and proved to be a little more effective.
- The recursion depth was discretized into directions, which made it also more effective.

5.3 Diagnostics

Since we wanted to determine some properties of the simulation, we had to implement diagnostics. These were the properties measured:

Number of alive trees The Number of trees alive is relatively easy to measure in the random based updating implementation. This comes from the property that there are only two states, 0 (empty) and 1 (alive). It therefore is sufficient to execute

```
N=sum(sum(Grid));
```

This value also allows us to compute other properties very quickly, like the mean number of alive trees in a simulation or the number of empty sites. In the grid based updating implementation, this was implemented with a loop and a counting variable. The Loop would simply check every entry of the grid for being 1 (alive) and subsequently add 1 to the counting variable.

Trees burned in a fire This was quite difficult to measure in the grid updating version. The big problem was that there was a possibility of multiple fires happening at the same time. It is therefore not possible to just loop over the whole grid and count the burning trees. One needs to track single fires and only add the trees burned by this incident. The other big problem is, that there is also the possibility of trees growing at the edge of the cluster that is on fire. This means that it is not possible to just search for the entire connected cluster, since its size can (and does!) vary during the fire. In the random updating version, this was a lot easier, since the fires would spread instantaneously. Since we were using a recursive function to burn the

cluster, we could just pass the trees burned by each sub-branch of the function back to the branch head (initial function call). There are also no issues of multiple fires happening simultaneously and changing cluster sizes.

Cluster size distribution For long simulation times, the cluster size distribution approaches the fire size distribution if the fires spread instantaneously.

5.4 The *neighbor* function

The *neighbor* function is the same for both implementations. It takes the current cell coordinates (j, k) and returns a vector containing the coordinates of its direct neighbors. It works like this:

```
Set all neighbors to standard
// Ex: West_X=j-1; West_Y=k;
Treat Special cases
// Ex: if j==1
// West_X=size(Grid,1);
```

Note that the *neighbor* function creates periodic boundary conditions.

5.4.1 Fourier Analysis

Since there is a lot of "noise" in the data (especially the tree population data), we decided to do a fourier analysis on it. Having a finite data set, we were able to use the MATLAB©built-in function *fft* to perform a fast fourier transform. The main objective was to reduce the noise content in the signal and to filter out the necessary parts. Since we only looked at data analysis, there were no constraints as to the causality of the desired filter. We implemented a simple low pass filter with a cutoff frequency of $\frac{1}{n \cdot T_s}$. Now the sampling time T_s obviously was set to 1, the data update speed. The parameter n, used for discretee fourier transforms, was the data size. What we used to determine was if the frequency of the excitation of the system would have any measurable impact on the frequency response of the system.

6 Simulation Results and Discussion

The first step in the simulation was to see if we could reproduce some of the results found in the references. For starters, we wanted to see if the Formula $\overline{s} \propto \Theta^{-1}$ would hold. This shold be explained: We define the factor $\frac{f}{p}$ as Θ and the mean number

of trees burned by one fire as $\overline{s}$. Then the change in the Number of trees present in the system for some large time t is given as

$$\Delta N = p \cdot t - f \cdot \overline{s} \cdot t$$

If we reach a critical point, then this difference should be zero for large times. Then we can write

$$p \cdot t = f \cdot \overline{s} \cdot t \text{ or } p = f \cdot \overline{s}$$

Since we defined $\Theta = f/p$, we can now write

$$\overline{s} \approx \Theta^{-1}$$

What does it tell us if we find this to be true? It tells us, that the system dynamics have a stationary point. However, since we average over quite a large time, the result is not a strong one, but merely a check if the simulations work right. In a first attempt to recreate this result, it did not work. We plotted the curve $\overline{s} \cdot \Theta$ for values of very small to very large Θ (which shold be constant), and observed a small, but relevant slope with quite big differences for small values of Θ. Now there comes a problem with the implementation. Since we only plant a tree with probability p if the site is *empty* and only set a site on fire if the site has an alive tree (and no actions are taken if these requirements are not fulfilled), the above equation has to be rewritten in terms of the probabilities:

$$\Delta N = \frac{(G - N)}{G} \cdot p \cdot t - \frac{N}{G} \cdot f \cdot \overline{s} \cdot t$$

where G is the total Number of Grid Points. For a stationary point, this ΔN again has to be 0, therefore we can write

$$(G - N) \cdot p = N \cdot f \cdot \overline{s}$$

This means that

$$\Theta \cdot \overline{s} = \frac{G - N}{N}$$

should hold. Now $\frac{G-N}{N}$ is indeed a constant, which should not change. However, it is implicitly dependant on Θ, since this has a direct impact on the mean number N of alive trees in the system. Imagine for example a very small f, meaning that fires are very rare. Then this constant factor is very small because N approaches G. For larger values of Θ, N will become smaller and therefore, the factor $\frac{G-N}{N}$ will become larger. That was the result that we obtained from the simulation. It therefore suggests that the simulation is working as expected.

7 Summary and Outlook

8 References