ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

# Lecture with Computer Exercises: Modelling and Simulating Social Systems with MATLAB

Project Report

**Insert Title Here**
**...**

Name 1 & Name 2

Zurich
May 2008

## Agreement for free-download

We hereby agree to make our source code for this project freely available for download from the web pages of the SOMS chair. Furthermore, we assure that all source code is written by ourselves and is not violating any copyright restrictions.

Name 1 Name 2

# Contents

# 1 Abstract

# 2 Individual contributions

# 3 Introduction and Motivations

## 3.1 What is Self-Organized Criticality?

Self-Organized Criticality involves 2 very important concepts: Self-Organization and Critical behavior. These 2 phenomenon arouse in various fields in very different forms.In certain class of systems, the dynamics can be very well explained by a few collective degrees of freedom. This dimensional reduction is termed as "Self Organization". It leads to evolution of global patterns in the system even though all the interactions are local. Let us take an example from the field of physics.Crystallization is the process of formation of crystals. It usually requires a small nucleus for the growth of the crystal. Once the nucleus is formed, crystal can grow on this nucleus. If we consider a particle which is going to crystallize on a given site, the choice of this site is decided only by the local environment seen by the particle. It does not depend on the particle on every other site on the crystal. But this local interaction leads to the formation of an ordered crystalline phase. Bird flocking is an example of self organization from the field of Biology.

On the other hand, there is another set of dynamical systems where the individual degree of freedom keep itself more or less in a stable balance and it is not possible to describe such a system by a few collective degrees of freedom. The key point in such systems is that the interdependence of various degrees of freedom make such systems very susceptible to noise. That's why, these systems can be called as "Critical". Critical state usually represent a point where 2 different phases occur simultaneously at the same time. A critical state has specific temporal and spatial signature. The power spectrum of the quantity representing the dynamics of the system is similar to that flicker noise. (More on flicker noise later!). The spatial signature of a critical state is "self-similarity" which is what we find for the case of fractals.

Self-Organized criticality combines the above 2 concepts. In it, the dynamics of a physical system converge to a critical point independent of the bestowed boundary conditions. This critical point commonly is not the equilibrium point in a traditional, physical sense. We usually say that the *critical point* is an attractor of the system dynamics. The concept is best understood by example, so we shall bring up a very educative one introduced first by [1]. Imagine a sand pile in 2 dimensions. One can discretisze this pile into cells, which are denoted with the indices $n$. Each of those cells has an amount of sand grains on it, which is directly corresponding to

the height $z_n$. Now we state the simple rule, that, if the difference in height between two neighboring cells becomes larger than a predefined limit, the higher cell will give grains to the lower cell (avalanche). Now every timestep, we drop a grain on cell one. Independent of the initial conditions, eventually, a straight slope will evolve. The critical point is reached if every difference between two neighboring cells is exactly the limit. What happens if we drop the next grain? Then the difference $z_1 - z_2 > p$ and one grain will drop on $z_2$. Subsequently, the difference $z_2 - z_3$ will be too big and the process repeats until it hits the last cell. We therefore call a state critical if a yet so small disturbance can propagate throughout the whole system. It is important to note here, that this critical state is *not* the equilibrium state of the system, since that would be a flat surface.

Self-Organized Criticality is the Concept that the dynamics of a physical system converge to a critical point independant of the bestowed boundary conditions. This critical point commonly is not the equilibrium point in a traditional, physical sense. The concept is best understood by example, so we shall bring up a very educative one introduced first by [**?** ] ). Imagine a sand pile in 2 dimensions. One can discretisze this pile into cells, which are denoted with the indices $n$. Each of those cells has an amount of sand grains on it, which is directly corresponding to the height $z_n$. Now we state the simple rule, that, if the difference in height between two neighboring cells becomes larger than a predefined limit, the higher cell will give grains to the lower cell (avalanche). Now every timestep, we drop a grain on cell one. Independant of the initial conditions, eventually, a straight slope will evolve. The critical point is reached if every difference between two neighboring cells is exactly the limit. What happens if we drop the next grain? Then the difference $z_1 - z_2 > p$ and one grain will drop on $z_2$. Subsequently, the difference $z_2 - z_3$ will be too big and the process repeats until it hits the last cell. We therefore call a state critical if a yet so small disturbance can propagate throughout the whole system. A further condition for SOC is the presence of the critical point, in the example, that would be the slope that should form in every experiment independant of the starting conditions. It is important to note here, that this critical state is *not* the equilibrium state of the system, since that would be a flat surface.

### 3.2 What is Self-Organized Criticality

Self-organized criticality (SOC) is a concept applied to spacially extended dynamic physical systems. It is often quite difficult to descripe the temporal and spatial evolution of such a system and a lot of research happens in that field. Usually, one uses a mean-field approach to such a problem, where the indidual coupled degrees of freedom are replaced by a field. This (external) field then acts on the DOF. SOC is a different approach to this problem. It is the concept, that a certain class of

dissipative, coupled dynamic systems converge to a critical point independantly of their underlying boundary and initial conditions. What are the properties of such a critical point? First of all, it is stable with respect to small disturbances. If this was not true, the system would not evolve to such a point. Note that this does not imply that such a point *exists*. If a certain point is unstable, the system commonly will not evolve to this point. The critical point, however, generally is not the physical equilibrium point of the system.

### 3.3 Flicker Noise

Flicker noise is something often mentioned when talking about dissipative coupled dynamical systems. Flicker Noise (or $1/f$-Noise) is the description for an often observed phenomenon of such systems; The observation, that the power spectrum $S(f)$ scales with $1/f$ or more generally, with $f^{-\beta}$. This implies that $S(f) \cdot f \propto 1$

### 3.4 Forest-Fire Model

A Forest Fire Model is basically a cellular automata. Each cell has three possible states:

1. Empty Site
2. Alive Tree
3. Burning Tree

There are four rules defined for each cell, which are executed simultaneously:

- A burning tree turns into an empty site
- A tree starts to burn if at least one of its neighbors are burning
- A tree will grow on an empty site randomly with probability $p$
- A tree will burn randomly with probability $f$

### 3.5 Forest-Fire Model (FFM) as an example of Self-Organized Criticality

A Forest Fire Model is basically a cellular automata. Each cell can have one of the following three states:

- 0: A green tree

- 1: Empty site
- 2: Fire

The dynamics of the system is defined though the following rules.They are applied to every step as we go from 1 time step to the next.

1. If a tree is burning at a given time step (2), we make it an empty site during the next time step.
2. If a site is empty (1), a tree can be grown on it with a probability p. This means that if we found out a site to be empty, we generate a random number between 0 and 1. If the number is less than p, we change the state of the cell to 0, otherwise it stays an empty site.
3. If a site is a green tree (0), we burn this tree in the next step if:
   - Either of its neighbors is a burning tree.
   - We generate a random number between 0 and 1, if it is less than f (lightening probability), we burn this tree.

It is important to note that the Forest Fire Model does not only apply to forest fires as its name would suggest. A whole class of problems follows the same basic rules and can be treated accordingly. An example would be the spreading of a disease, where one can directly transform the above rules and states.

This model describes an SOC state in the regime where the ratio of p and f is very large and p itself is very small. An important question is how to characterize a state as an SOC state. This is described below.

### 3.6 Motivation to study Forest Fire Model

Why is it important to study Forest Fire Model? Forest fire models apply to a wide range of problems and are therefore helpful in predicting the behavior of such systems. Self-Organized criticality is the concept that describes the dynamics of such models and the two fields are therefore closely connected. One has closely studied these effects to make better predictions about forest fires. The main problem was, that in most cases, even relatively small fires were extinguished by the authorities. Since that led to an *overcritical state*, the probability for a huge, devastating fire rose quickly. This has happened in the past and had a serious impact on the ecosystem of those areas. Since the problem of self-organized criticality has been understood, one has stopped to extinguish every little fire, therefore avoiding to reach an overcritical state at which the disturbance (a small fire) can propagate throughout the whole system

(large-scale fire). The same ideas can be applied to a variety of problems such as disease spreading or urban planning.

### 3.7 What do we want to know from the Forest Fore Model- The QUESTIONS

First, we would like to understand what are the quantitative characteristics of a Self Organized Critical state in the Forest-Fire model? We would like to have a well defined idea regarding when does the Forest Fire model manifest SOC behavior and why. We would like to understand what are the various phases seen in a Forest Fire model besides the Self Organized Critical phase. It will be important to know what are the parameters characterizing the phase transition in the Forest Fire Model. Some of the ambitious questions will be to understand the dependence of SOC of the Forest Fire model on the dimensionality of the Lattice or on the neighborhood selection on the grid.
It will also be interesting to find out if computational restrictions like finite size grids, periodic boundary conditions and cell updating procedures have a quantitative effect on the behavior of the model?

## 4 Description of the Model

### 4.1 Understanding the SOC regime of Forest Fire Model

For FFM to be an SOC, we need to ensure 2 things. One is that the probability of growing a tree should be much larger than the probability of lightening. We can intuitively think about the ratio of p and f as the number of trees growing between 2 lightening strokes.This condition ensures long range correlation in the system (which will lead to criticality) building out of purely local interactions (randomly growing trees). Another condition which we need to ensure is that the time needed to burn a large cluster should be much smaller than the time needed to grow a tree. This condition ensures invariance under a change of the length scale [2]. The 2 conditions can be written together as:

$$T_{burn}(cluster_{max}) << p^{-1} << f^{-1} \tag{1}$$

### 4.2 Cellular automata

A cellular automata is a very powerful way of simulating problems which are defined by a set of rules. It is usually simulated on a grid, but not necessarily

restricted to those geometrical constraints. The main characteristics are:

- Every grid point has a state
- Grid points change their state depending on the neighbor states according to a set of rules
- Random actions may be introduced

In the example of the FFM, every grid point has three states (empty, alive and burning) and four rules, the most obvious of which is change state to burning if you are alive and your neighbor is burning.

Cellular Automata can exhibit very complicated behavior when fed with very simple rules, which is the main reason why they are so powerful. There are similarities to agent-based models and networks, but it is not to be mistaken for one of these.

The second approach to grid-based updating is with a second grid that stores temporary information. [Insert Description here]

### 4.3 Critical Exponents

Usually, a critical point is associated with certain power law distributions of various physical quantities characterizing a critical state. Power laws are functions of the form $P(s) \propto s^{-\tau}$ . These functions are found in many data sets, such as the size distribution of cities in a certain area. Take any country, one will find one very large metropolis, a few large cities, many medium sized towns and a huge lot of small towns. Power laws have also prominently been found in data sets of earthquakes, where they apply almost perfectly, meaning that small earthquakes happen very often in respect to the frequency of big ones. Power laws imply that the frequency of a general data point is inversely proportional to its magnitude.

At a critical point, the power laws are characterized by critical exponents which are the indicators of the critical state. They usually depend only on the dimension of the system and the nature of interactions/ dynamics involved. These exponents are independent of the initial condition and the size of the system. These critical exponents can be calculated either by mean field approaches or through numerical simulations. The mean field approach becomes more and more exact in higher and higher dimensions.

In context to the FFM, it is proposed as a way of quantifying the frequency of Fires with respect to their cluster size.

### 4.4 Key Parameters in FFM SOC

To find out whether a basic FFM exhibits SOC behavior, we need to define a set of parameters which serves as indicators for the desired analysis.

- The time needed to burn down a forest cluster: This is heavily dependent on the form of the implementation. If we choose to implement it in a way like [reference needed], we will have an instantaneous fire which essentially takes no time to burn and just resets all the connected trees. If however, we choose the form of a visible fire, meaning that the ratio $f/p \ll 1$ is not 0 in limit and that it takes the fire one time step to advance a grid cell, then the time needed to burn a forest cluster is an important variable we need to extract from the simulation.
- Number distribution of the size of clusters: Here we first need to define what a cluster is. A cluster is a set of neighboring cells (Von Neumann neighborhood) all obtaining the same state. When we talk about forest clusters, we of course mean connected trees. In the implementation where the fire spreads infinitely fast, the forest cluster containing the igniter cell is the same as the burnt area. The size distribution therefore is a very good indicator for the fire size distribution.
- The mean number of forest clusters in a unit volume $n(s)$ , where s is the number of trees in a cluster.
- The radius of a cluster, it is defined as the root mean squared distance of all the elements of a cluster from the mean. It gives an estimate of the size of cluster.
- Number of fires per unit time step: This is a tuning parameter of the model.
- Correlation length.

We have tried to implement the model in a way that the measurement of these desired variables is possible with the least needed effort.

## 5 Implementation

As discussed before, there are two main implementation methods for the model: One way is to update the whole grid every time step, whereas the second method picks a cell randomly at every time step an only updates the chosen one. One of us worked on the first method and the other one worked on the second method.

### 5.1 Grid Based Updating

The algorithm for grid initialization looks like:

```
Grid(N,N)=zeros(N,N)
rand_grid=rand(N,N) % we generate a grid of random numbers
m=1;
while m<=size(Grid,1) % Loop over all y-coordinates
n=1;
while n<=size(Grid,2) % Loop over all x-coordinates
if rand_grid(m,n)<k %If the random number for a site is less than k,
%we make the site empty, else it has a tree
Grid(j,k)=1    % chossing different k will lead to different
%initial state of the grid
end
end
end
```

Basically, we initialize the grid with trees and empty sites distributed randomly on the grid. The updating of the grid based on the simple rules described above can be done in 2 different ways. One is when we make a copy of the current grid ($Grid_{copy}$) and then update the current grid using this $grid_copy$. The other is that we use the current grid (which is being updated) as the one which also responsible for updating. The 2nd procedure require us to pick the sites randomly and update them, but it will be hard to keep track about the sites which have been updated at a time step. In our simulations, we follow the first procedure whose algorithm is described below:

```
it=1:
while it<=Iter % Loop over all time steps
    Forest_grid_temp=Forest_grid;  %Forest_grid_temp is the copy
    %which is used to update the grid at the next time step.
% we need to take care that the neighbors of every cell are correct
%as based on the periodic boundary condition.

%This has been handled by making the size of the copy as 2 units
%larger in each of the dimensions

    Copy the first column of Forest_grid to last column of Forest_grid_copy
    Copy the last column of Forest_grid to first column of Forest_grid_copy
```

```
    Copy the first row of Forest_grid to last row of Forest_grid_copy
    Copy the last row of Forest_grid to first row of Forest_grid_copy
    m=2;  %looping over the Forest_grid and its copy,
    while m<N
        n=2;
        while n<N
            if Forest_grid_temp(m,n)==2
                Forest_grid(m,n)=1;
              %a fire becomes an empty site

            elseif Forest_grid_temp(m,n)==1
                emp=emp+1;
                if rand(1,1)<p
                    Forest_grid(m,n)=0;
                end
              %an empty site becomes a tree if a generated random no. is <p
            else
                if (rand(1,1)<f || Forest_grid_temp(m-1,n)==2 ||
                Forest_grid_temp(m+1,n)==2 || Forest_grid_temp(m,n-1)==2 ||
                 Forest_grid_temp(m,n+1)==2)
                     Forest_grid(m,n)=2;
                %if any of the neighbours of the current site is burning or a
                % random number is generated which is greater than lightening
                %probability, we burn the tree.
                end
            end
            n=n+1;
        end
        m=m+1;
    end
    it=it+1
end
```

One of the important reasons why we made another model is that the algorithm is quite slow because of all the loops and if-statements,we are using.

The next important thing which we need to find out is indexing uniquely all the clusters of trees. The algorithm is described below:

```
Index=3;
m=1;  %looping over the Forest_grid and its copy,
```

```
while m<N
   n=1;
   while n<N
    if Forest_grid(m,n) is an un-indexed tree
    we search all its neighbors to find the smallest index
    if index is found,
    Forest_grid(m,n)=index;
    We also update the index of all the nearest
    trees with this index
    if index is not found,
    Forest_grid(m,n)=Index;
    We also update the index of all the nearest
    trees with this Index
    Index=Index+1;
    end
end
end
end
%The above loop will be sufficient if we have absorbing boundary conditions.
%Periodic boundary condition requires that we should check all the indexing
%again
m=1;  %looping over the Forest_grid and its copy,
while m<N
   n=1;
   while n<N
    if Forest_grid(m,n) is a tree
    we search all its neighbors to find the smallest index
    If the index of any of the neighbors or the Forest_grid(m,n)
    is different than this smallest index, we replace all the trees
    of that index with this smallest index
end
end
end

for i=1:t % Loop over all timesteps
    for j=1:size(Grid,1) % Loop over all y-coordinates
        for k=1:size(Grid,2) % Loop over all x-coordinates
            if Grid(j,k)==0 %If the grid point is empty
                //Grid(j,k)=1 with some probability p
```

```
            end
        end
    end
    for j=1:size(Grid,1) % Loop over all y-coordinates
        for k=1:size(Grid,2) % Loop over all x-coordinates
            //if Neighbor(j,k)==3 % If grid neighbor is burning
                Grid(j,k)=2 % Set current grid point on fire
            end
        end
    end
    for j=1:size(Grid,1) % Loop over all y-coordinates
        for k=1:size(Grid,2) % Loop over all x-coordinates
            if Grid(j,k)==3 % If Grid point is burning
                Grid(j,k)=0; % Turn it into an empty site
            end
            if Grid(j,k)==2 % If grid point is ignited
                Grid(j,k)=3; % Turn it into an empty site.
            end
        end
    end
end
```

Several things are to note here:

- Commands with // in front are not implemented exactly like that. Check the appendix for the complete source code.
- We need several spatial loops in order to successfully suppress updating fragments like infinite fire propagation speed in updating direction.
- For the same reason, we need a differention between newly ignited (state 2) and burning (state 3) trees.
- The Algorithm is quite slow because of all the loops and if-statements.

Once the indexing is done, we can simply count the number of trees corresponding to a given index and can find out the number of number of clusters of a given size (where by size, we mean the number of trees in the cluster).

Calculation of the radius of a cluster is little subtle. Naively, it just the root-mean squared distance of all the elements from the cluster mean. The formula can be simplified as:

$$R = \sqrt{\frac{1}{N}\sum x_i^2 - \frac{1}{N}(\sum x_i)^2} \tag{2}$$

This would have worked well if we have absorbing boundary conditions, but since our boundary is periodic; there can be possible a cluster which is going over the boundary. In that case, above formula can lead to a cluster mean which is outside the cluster. In such a situation, we need to rotate our grid to calculate the cluster radius properly and efficiently. The underlying idea is very simple. If you are scanning through different columns (rows) from left to right (top to bottom), you should get an element of a cluster in every column since the starting column for that cluster. In case, there is a break, it implies that you have cluster going though the edges, otherwise it cant be a cluster. Another important property of a cluster going through their boundary is that it should have a member in the leftmost (topmost) and rightmost (down most) column (row). We exploit these in the following algorithm:

```
%Column shift
define variables F and ml for every cluster index
initialize F and ml with zero for every cluster
m=1
while m<=N  %going through all the columns
n=1;
while n<=N  %going through all the rows of a given column
if the current element is a tree,store the index of current
element in variable cl
if F(cl)==0
%this is the first column where an element of this cluster appears.
if m~=1
F(cl)=3;
%this implies that this cluster cannot be connected via the boundary
else
F(cl)=1;
ml(cl)=m;
  end
elseif F(cl)==1
if m~=ml(cl)+1
%this is the case when we have atleast one column in between where no
%element of the given cluster is found and this can happen only if
%the cluster passes through the boundary.
  F(cl)=2;
end
ml(cl)=m;
```

```
end
end
end
%The column shift needed for a given cluster can be calculated as:
if F(cl)==2
shift_col(cl)=N+1-ml(cl)
else
no shift
end

%An exactly similar procedure is followed to calculate the row shift for every cl
%index where we go though all the columns for a gicen row and very all the rows.
```

### 5.2 Random Based Updating

In a random based updating implementation, at the beginning of every time step, one random cell is chosen. It then checks for the rules of the model and acts accordingly. In order to see fires, we need to have instantaneous burning, which is actually permitted by the model. However, it changes the dynamics of the system drastically. But more about that later. The following steps are taken for each timestep:

1. Choose a single cell
2. If cell is empty, grow a tree with probability $p$
3. If cell is a tree, with probabiltiy $f$, burn it and the connected cluster

In Matlab, this looks something like this:

```
for i=1:t % Loop over all timesteps
(j,k)=ceil(Grid_Size*rand(1,2)); % Choose a random cell
if  Grid(j,k)==0 % If the cell is empty
\\With some probability p set G(j,k)==1;
end
if Grid(j,k)==1 % If the cell is alive
\\with some probability f execute burn(j,k);
end
end
```

This implementation of the forest fire model has several advantages:

- There are no problems due to updating processes
- Much shorter implementation
- Easier to extract data

The last point is to be explained more extensively: Every time a tree ignites, the whole connected cluster is burned down. Since the whole process happens in a single timestep, there are no big issues.

### 5.2.1 keeping track of the clusters

Keeping track of the different clusters was more difficult than we thought at first, but we managed to find an elegant solution. It works by keeping a second grid which stores all the cluster indices. This has several advantages:

- if a new tree grows, we can just look at the neighbor cluster index to determine the current cell index
- if a tree burns, we simply delete all the trees with the same index as the burned tree.

**Problems and Solutions** Problems arose quickly. Here is a short summary of all the major problems and the corresponding solutions:

**Storing the indices** The problem here was to find a way to store all possible cluster indices and their availability. Since the maximum number of clusters can be easily found to be $Nc_{max} = \frac{G^2}{2}$ (imagine a perfect chessboard-layout filling half of the grid; if one more is added, they combine into a bigger cluster), we already knew the size of the vector. The final format was a Matrix with $Nc_{max}$ Columns and 2 rows. The first row would simply store all the possible indices from 1 to $Nc_{max}$ and the second row would store the corresponding binary information about the availability.

**growing a tree between two clusters** It is simple to determine the cluster index of a new tree if there is just one neighbor. The big problem arises if there are more than two neighbors with *different* cluster indices. The solution to this problem was that we set the index as the lower of both neighbors and changed the whole cluster with the higher index to the lower one. This was relatively

easy to do by using the matlab built-in function —ismember— which searches for all entries with a certain value and returns a matrix containing only that information.

### 5.3 The *neighbor* function

The *neighbor* function is the same for both implementations. It takes the current cell coordinates $(j, k)$ and returns a vector containing the coordinates of its direct neighbors. It works like this:

```
Set all neighbors to standard
// Ex: West_X=j-1; West_Y=k;
Treat Special cases
// Ex: if j==1
// West_X=size(Grid,1);
```

Note that the *neighbor* function creates periodic boundary conditions.

### 5.4 Saving the data

Because we were doing a lot of parameter sweeps and thus generating a lot of data, it was crucial to implement an automatic data saving procedure. This would store all the relevant information of a single simultation in a custom matlab struct and save the important figures. We also created a custom naming concept which allows for quick search for the desired data. All simulation results are named after their input parameters:

## 6 Simulation Results and Discussion

We will first present to understand the traditional model of Forest fire. Here we can consider 4 different regimes.

- small p ($< 0.001$) and small $\frac{p}{f}$ ($\approx 1$)
- large p ($< 0.001$) and large $\frac{p}{f}$ ($\approx 10^4$)
- large p ($< 0.001$) and small $\frac{p}{f}$ ($\approx 1$)
- small p ($< 0.001$) and large $\frac{p}{f}$ ($\approx 10^4$)

Let us first try to understand what we expect intuitively for these cases. In the first case, we will have growth rate of trees to be very small and comparable rate of fires.(The smaller values of usually make the time scale of variation of the system very large.) As a result, we don't expect long-range correlations for the system to build up.

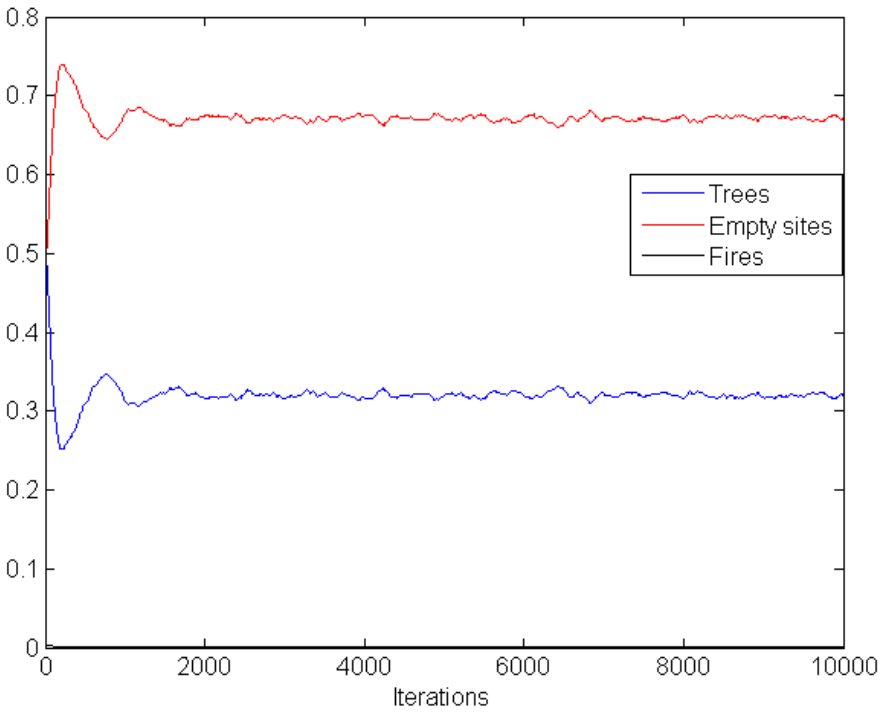


Figure 1: $N = 500,\ \ p = 0.001,\ \frac{p}{f} = 10$(Case1)

In the second case, system will try to build long correlations since the tree growth rate is large. But since p is large, it won't be possible to make the system critical because before a large cluster is burnt by the fire, trees will start growing again in the cluster and the system won't really be in a critical condition.

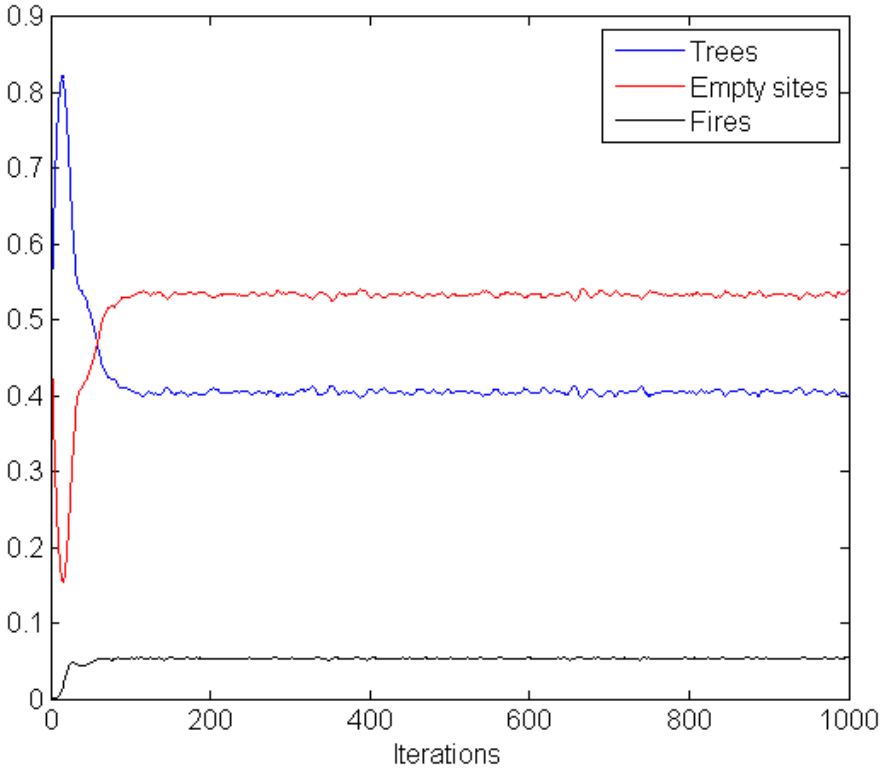


Figure 2: $N = 500,\ \ p = 0.1,\ \frac{p}{f} = 1000$(Case2)

The 3rd case takes the worst part of the above 2 cases. The large tree growth rate don't let the system to become critical. The small $\frac{p}{f}$ wont let the formation of long range correlation in the system and avoid from making the system being critical.

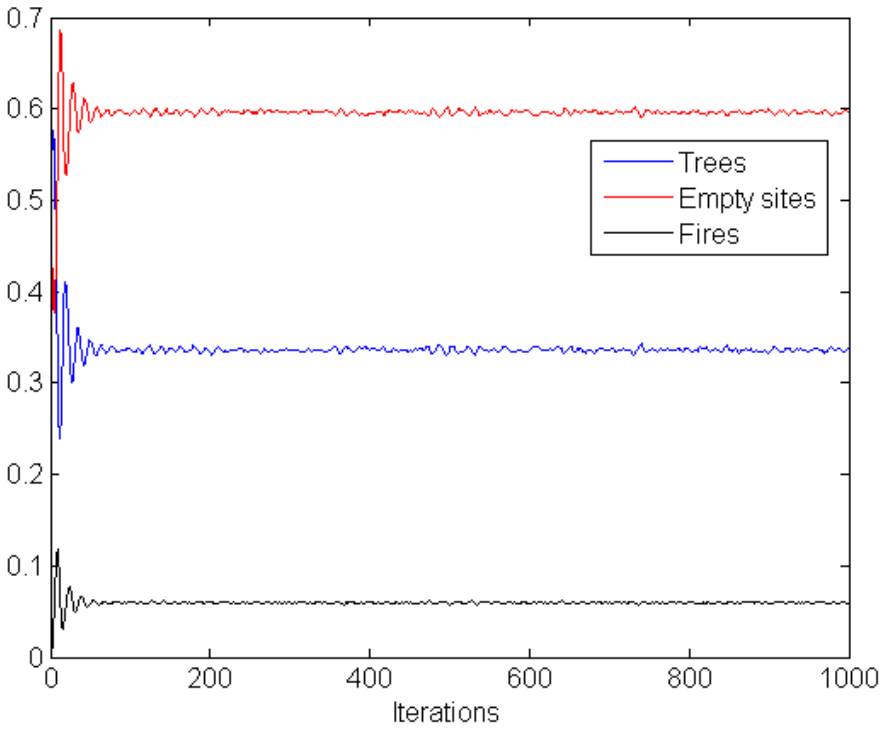


Figure 3: $N = 500,\ \ p = 0.1,\ \frac{p}{f} = 10$(Case3)

In conclusion, these 3 cases are some sort of equilibrium populations where system never becomes critical.

The final case is the one we are interested in. Since $\frac{p}{f}$ is large, large correlations can be built in the system. Since p is small, formation of large correlations make the system critical.

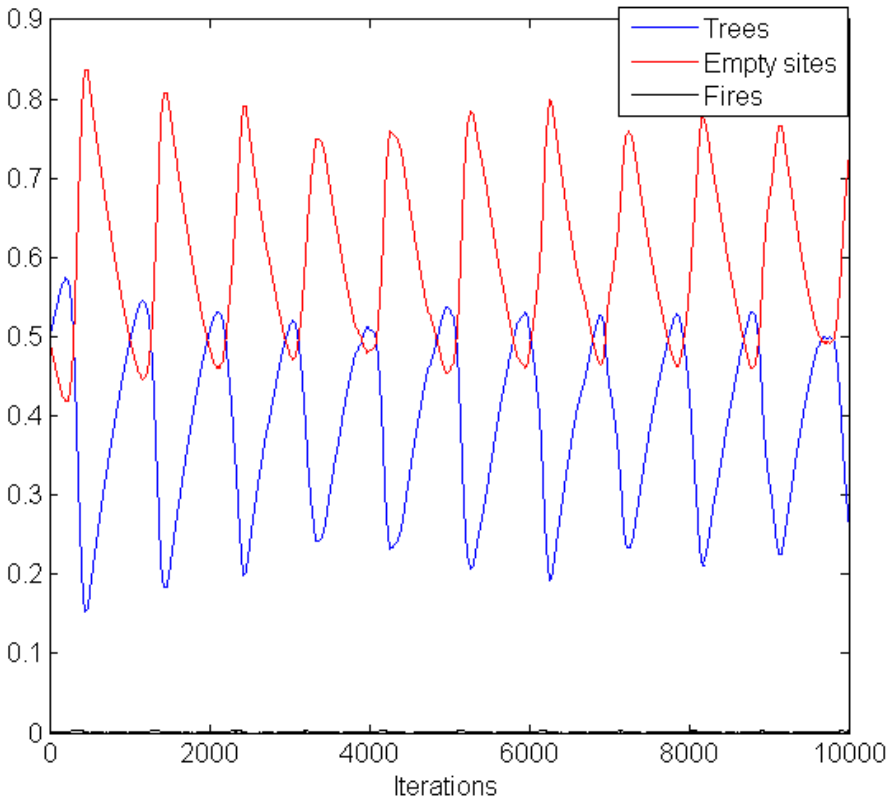


Figure 4: $N = 500,\ \ p = 0.001,\ \frac{p}{f} = 1000$(Case4)

From the way we model our system (we are considering case 4 above), it is very difficult to find out whether for a given set of parameters the time required to burn the biggest cluster in system is much smaller than the time needed to grow a tree. But there is a very simple qualitative way to see this through the variation of various fractions as the system evolves. When an event of lightening happens in the system, the fraction of empty sites grows quickly. But shortly after that when the cluster is fully burnt, the trees again start growing and the fraction of empty sites start to decrease again. If the slope of the slope of increase of the fraction of empty sites when the fire is burning the cluster is much larger than the slope of decrease of fraction of empty sites when the trees start growing, it implies that the cluster burning process is almost instantaneous in comparison to the tree growth. Hence, in the 4th case we expect the system to be in SOC state. In the following figure, we show the case

when the system is critical in the sense that long range correlations are forming (a fire is burning big cluster) but the fire is not instantaneous in comparison to the tree growth rate.

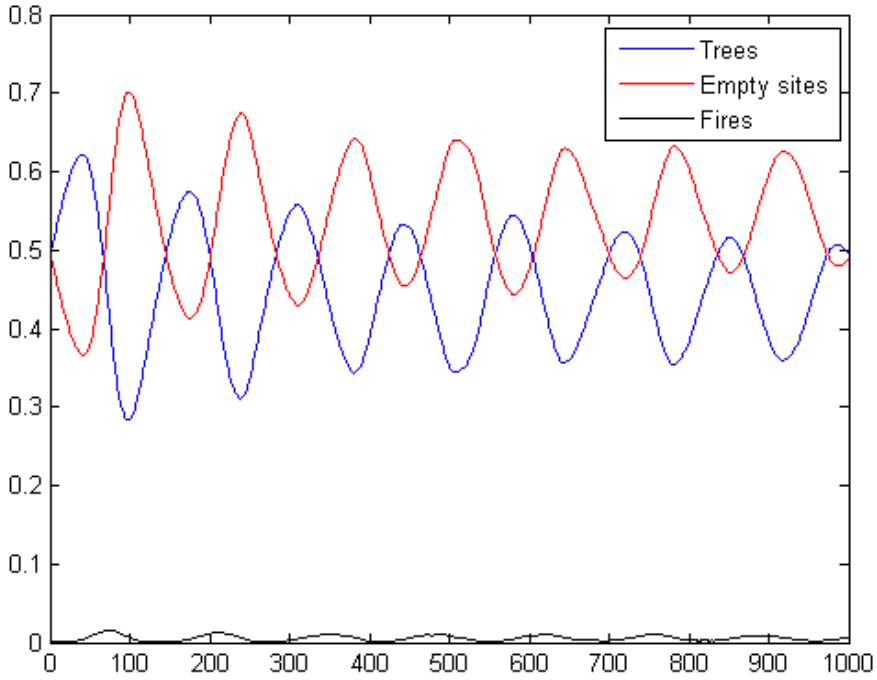


Figure 5: $N = 500,\ \ p = 0.01,\ \frac{p}{f} = 1000$(Case4)

An important point to make here is how the size of the grid affects this self-organized critical state. We see that if the grid is big, we will be having larger long range correlations. It will take larger time for a fire to burn a cluster and hence the regime where the cluster burning is almost instantaneous as compared to tree growth will require lower values of p. This has been shown in the following figure by considering a much smaller grid size as for the case 4 but same parameters.

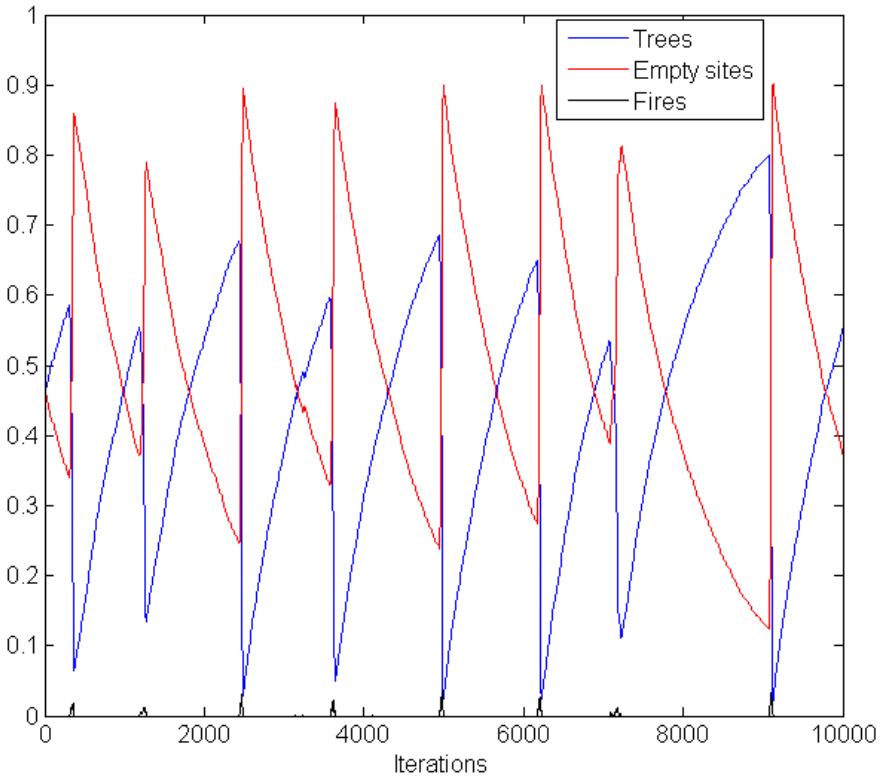


Figure 6: $N = 50,\ \ p = 0.001,\ \frac{p}{f} = 1000$(Case4)

Now, we plot the radius distribution and the frequency of clusters as a function of number of trees in the cluster. Thes eare plotted in the figures below.

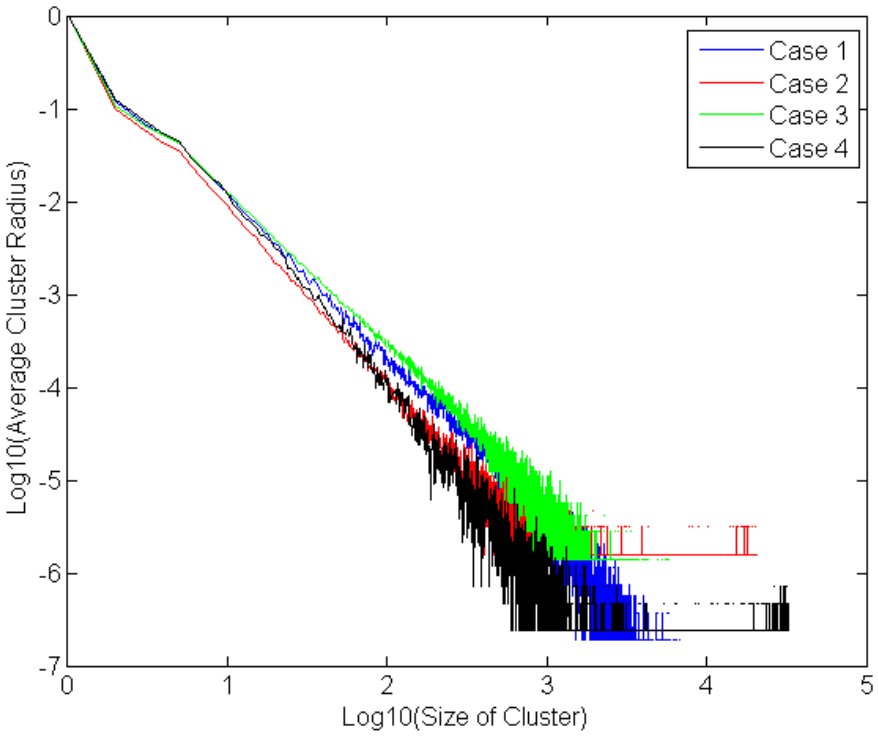


Figure 7: $N = 200$, Frequency Distribution

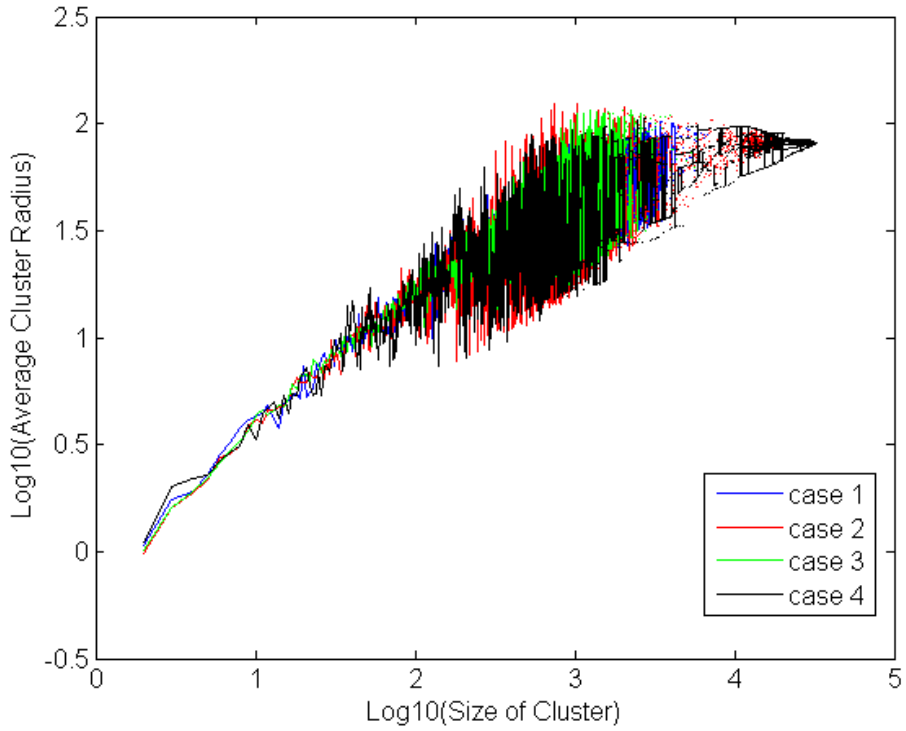


Figure 8: $N = 200$, Radius Distribution

I don't see any difference in the 4 cases. I hope the other method will show us something. I will write some more discussion here.

### 6.1 Tree population

*note that in this section,* $\Theta$ *is defined as* $p/f$*.*

A first step in the analysis of the results is to have a close look a the tree population during the simulation. Here are two examples:

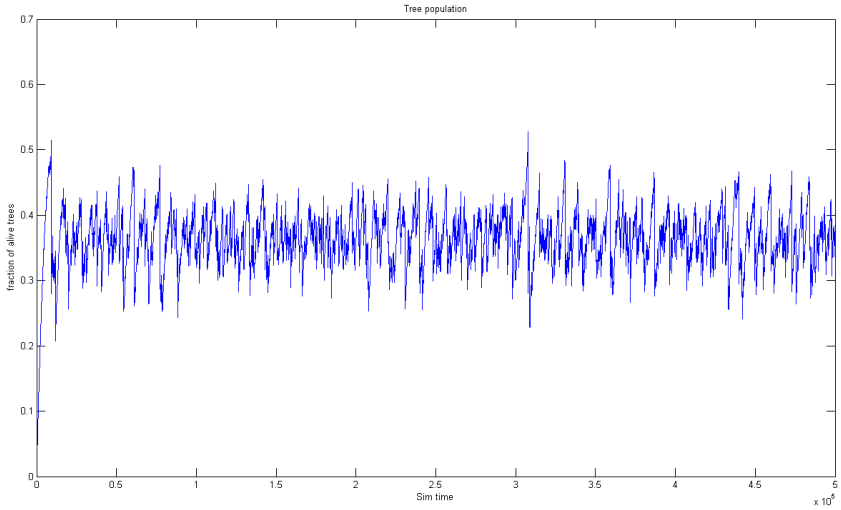


Figure 9: $N = 100$, $\Theta = 100$, Simulation time 500'000

What can we learn from these plots? Well, in the first figure, Theta is equal to $10^3$, which is a moderatly large number. Yet, the plot of the tree fraction looks like random noise.We are not quite sure what we should interpret from such a plot. At $\Theta = 100$, the plot looks a lot more stable, more like an actual *equilibrium* point. Re-running the simulation with the same $\Theta$ but changed $f$ yields the same, consistent picture; after a short transient period to overcome the initial condition, the plot approximates a straight line. Now we know that the SOC state is not generally an equilibrium state and further insight into the properties of the system can only be achieved by analysis of other properties.

### 6.2 Cluster Radius

The Cluster Radius is defined as the root mean of all the distances of all cells of a cluster to their common center. This property should, given an SOC behaviour of the system, exhibit a power-law-behaviour that would be described as $S(r) = r^{-\beta}$ where$S(r)$ is the number of occurrences of clusters with radius r. In theory, what this means is, if you plot a double-logarithmic histogram of all cluster radii, it should give us a straight line with slope $\beta$. Of course, we expected a little "noise", given that our simulation was not infinite in time and space, but it should have approached it pretty well. Here is one of those double-logarithmic histograms:

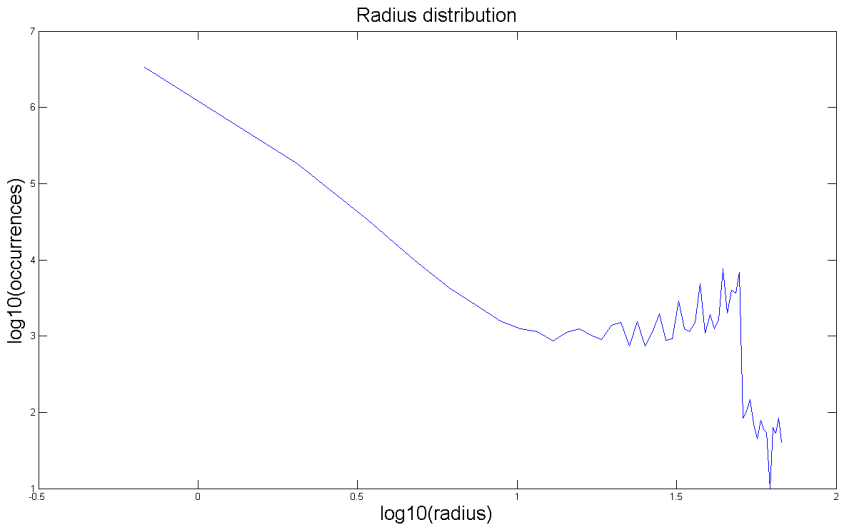


Figure 10: Cluster radius distribution ($N = 100$, $\Theta = 1000$, Sim Time 500'000)

**The Problem** that we see here is one that's closely related to the periodic boundary conditions. Imagine an empty grid with a small cluster that stretches from one side *over the boundary* to the other side. Its center will be in the middle, but its radius will be much larger than it actually is. We guess that this causes the tilt that one can observe at medium sized clusters. We are currently working on a way to get around that problem.

### 6.3 Cluster Size

The cluster size is another variable that we can easily measure. Here is a log-log-histogram of a simulation:

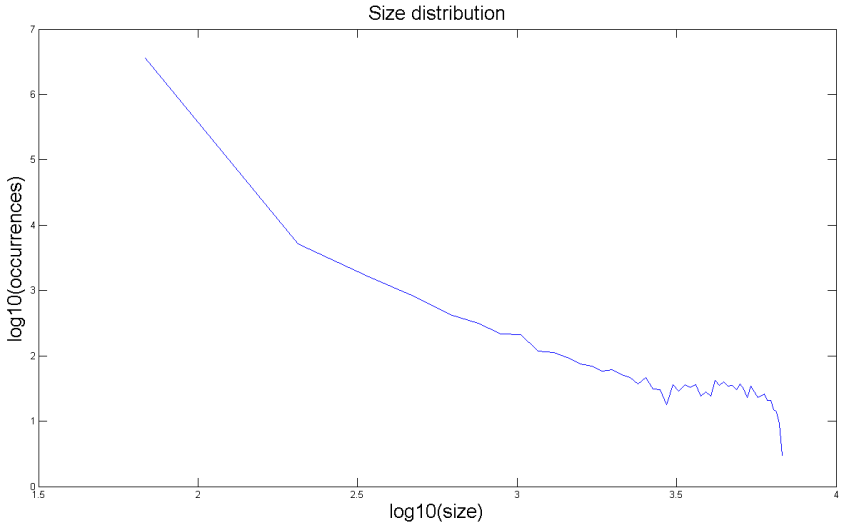


Figure 11: Cluster size distribution ($N = 100$, $\Theta = 1000$, Sim Time 500'000 )

# 7 Open Questions

We are wondering if it is "correct" to implement the forest fire in such a way with zero fire burning time like stated in [**?** ]. We have thus implemented two separate models, one of which uses finite burning speeds, while the other one (described in this text) uses infinite burning speed. Looking at the different timescales, it is obvious that the spreading of a fire should be very fast, but is it correct to use this in the limit of instantaneous fires?

# 8 Summary and Outlook

# References

[1] Per Bak, Chao Tang, and Kurt Wiesenfeld. Self-organized criticality. *Phys. Rev. A*, 38:364–374, Jul 1988.

[2] Siegfried Clar, Barbara Drossel, and Franz Schwabl. Forest fires and other examples of self-organized criticality. *Journal of Physics: Condensed Matter*, 8(37):6803, 1996.