Computer Vision and Image Analysis **WS 14/15**

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Solution of assignment 4

(Submission: 11. Nov)

Task 4.1: *the Fourier transform with pen and paper* [ ] **out of 10 Points**

$$f(x,y) = \begin{cases} 1, & \text{if } 0 \leq x \leq \frac{1}{2} \text{ and } -\frac{1}{4} \leq y \leq \frac{1}{4}, \\ 0, & \text{otherwise} \end{cases}$$

$$\hat{f}(w_x, w_y) = \int_{\mathbb{R}} \int_{\mathbb{R}} f(x,y) \exp\left(-2\pi i(w_x x + w_y y)\right) \mathrm{d}x \, \mathrm{d}y$$

Due to the case distinction of $f(x,y)$, we can limit the integrals and replace the function with 1. Then we simplify the integrals:

$$\hat{f}(w_x, w_y) = \int_{-\frac{1}{4}}^{\frac{1}{4}} \int_{0}^{\frac{1}{2}} 1 \exp\left(-2\pi i(w_x x + w_y y)\right) \mathrm{d}x \, \mathrm{d}y$$

$$\hat{f}(w_x, w_y) = \int_{-\frac{1}{4}}^{\frac{1}{4}} \left(\int_{0}^{\frac{1}{2}} \exp\left(-2\pi i(w_x x + w_y y)\right) \mathrm{d}x \right) \mathrm{d}y$$

$$\hat{f}(w_x, w_y) = \int_{-\frac{1}{4}}^{\frac{1}{4}} \left(\frac{i \exp\left(-i\pi(w_x + 2w_y y)\right)}{2\pi w_x} - \frac{i \exp\left(-i\pi w_x - i\pi(w_x + 2w_y y)\right)}{2\pi w_x} \right) \mathrm{d}y$$

$$\hat{f}(w_x, w_y) = \frac{\sin\left(\frac{\pi w_y}{2}\right) \left(\sin\left(\pi w_x\right) + i\cos\left(\pi w_x\right) - i\right)}{2\pi^2 w_x w_y}$$

We use this formular and get following results and also amplitude as $\log|\hat{f}|$ and phase as $\arg \hat{f}$:

	result	$\log\lvert\hat{f}\rvert$	$\arg \hat{f}$
$\hat{f}(1,0)$	$-\frac{i}{2\pi}$	$\log \frac{1}{2\pi} = -1,838$	$-\frac{\pi}{2}$
$\hat{f}(0,1)$	$\frac{1}{2\pi}$	$\log \frac{1}{2\pi} = -1,838$	0
$\hat{f}(-1,0)$	$\frac{i}{2\pi}$	$\log \frac{1}{2\pi} = -1,838$	$\frac{\pi}{2}$
$\hat{f}(0,-1)$	$\frac{1}{2\pi}$	$\log \frac{1}{2\pi} = -1,838$	0

Task 4.2: *the Fourier transform in Matlab* [ ] **out of 15 Points**

(b) The amount of shifted pixels depends on the size of the kernel.

(c) It would apply noise because the values of the outer regions of the Fourier transformation of the central difference kernel are a lot higher compared to the derivate of the Gaussian kernel, which means, that some unimportant frequencies are pushed.

Task 4.3: *hybrid images* [ ] **out of 5 Points**