Computer Vision and Image Analysis
Assignment Sheet 4 - 11.11.2014

Next exercise group: 14.11.2014

- Deadline for exercise: **20.11.2014, 18:00**, hard deadline.
  Please submit solutions either on our Ilias web page (preferred method), or via e-Mail in a single ZIP-Archive to `ole.johannsen@uni-konstanz.de`.
- You may work in groups of up to three students, make sure all participants are clearly mentioned or assigned to the submission in Ilias.

Exercise 4.1 (the Fourier transform with pen and paper, 10 points)

Computing Fourier coefficients with pen and paper is not something you want to do on a regular basis, but should have done once to fall in love with the Matlab function `fft2`. It also helps in understanding what all the integrals mean.

Consider the function

$$f(x,y) = \begin{cases} 1 & \text{if } 0 \leq x \leq \frac{1}{2} \text{ and } -\frac{1}{4} \leq y \leq \frac{1}{4}, \\ 0 & \text{otherwise.} \end{cases}$$

Compute the Fourier coefficients $\hat{f}(1,0)$, $\hat{f}(0,1)$, $\hat{f}(-1,0)$, $\hat{f}(0,-1)$ using the transformation formula

$$\hat{f}(\omega_x, \omega_y) = \int_{\mathbb{R}} \int_{\mathbb{R}} f(x,y) \exp\big(-2\pi i(\omega_x x + \omega_y y)\big) \, \mathrm{d}x \, \mathrm{d}y.$$

Remember that the results will be complex numbers. You may use the symmetry rules you have learned in the lecture, but may wish to compute all four manually to verify that these rules are indeed correct. What are amplitude and phase for the corresponding elementary waves?

Hints: Don't panic. From the definition of f, the integrals over $\mathbb{R}$ are turned into integrals over finite intervals. Remember to treat i like any other constant, and try to solve the integrals like you learned in basic calculus. Use Euler's formula for exp. For the double integrals, you can just start with the innermost integral first, or you may exchange the order of integration.

Exercise 4.2 (the Fourier transform in Matlab, 5+5+5 points)

(a) The additional material provided with this exercise includes a script `fourier.m` to get you started, which shows how to deal with the Fourier transform in Matlab. In particular, what it does is

- Compute the Fourier coefficients,
- Visualize amplitude and phase like in the lecture,
- Set all Fourier coefficients in the upper right and lower left quadrant to zero,
- Compute the backwards transform.

Familiarize yourself with the code and try to understand what is going on. Afterwards, modify it so that all Fourier coefficients **below** a certain frequency are set to zero, and show the result.

Note: In case you are interested or need some more examples, there is a second script `lebecca_and_rukas.m`, you can guess what it does.

(b) Implement Gaussian convolution using the Fourier transform. Receipe:

- Create a Gaussian kernel (try something large, say, standard deviation $\sigma = 10$).
- Compute Fourier transforms of the image and the kernel, make sure to use the same target grid size.
- Multiply the transforms point-wise.
- Transform the result back.

Try to make sense of the result. Why is it shifted, and by what amount? What kind of boundary conditions were used? Can you implement a version where the 'repeating' boundary conditions are used instead? Compare speed to your previous implementation of convolution.

(c) Visualize the Fourier transform of a central difference kernel, and explain why convolution with this kernel amplifies noise. Compare to the Fourier transforms of derivative of Gaussian kernels with various standard deviations.

Exercise 4.3 (hybrid images, 5 points)

Write a Matlab script to create a hybrid image!

Note: This exercise is for fun, and does not help much with understanding the core material in the lecture - only start it when you have thoroughly worked on the first two. Any points awarded count as a bonus.

Receipe:

- Take two images.
- Remove or supress the high frequencies in one of the image (e.g. by filtering with a Gaussian, or manipulating Fourier coefficients directly).
- Remove or supress the low frequencies in the second image (e.g. by subtracting a version of the image filtered with a Gaussian, or manipulating Fourier coefficients directly).
- Add the results together (maybe a weighted average is necessary).

Some trial and error is probably required to get the filter parameters right. Bonus cookies if the result looks artistically pleasing and works well at the scale of a projector in a seminar room (I guess this can only be found out by experiment). To get color images, by the way, you can work on the three channels (red, green, blue) separately.