Linear Auto-Calibration for Ground Plane Motion

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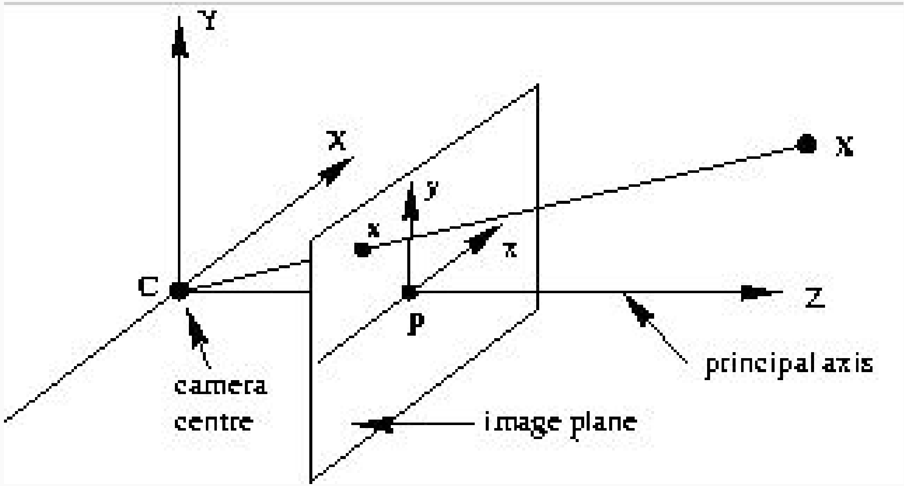
The Camera Matrix

The camera matrix describes how a particular camera transforms 3D points into a 2D image.

When you can list this matrix, we call the camera calibrated. Calibration allows you to reconcile your camera image with the 3D world.

Optical zoom changes the values in this matrix.



$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

Auto Calibration

We don’t always remember to calibrate our cameras beforehand....

Projective Geometry

In a Euclidean space, parallel lines never intersect.

A Projective space includes an additional plane where parallel lines intersect, called the *plane at infinity*.

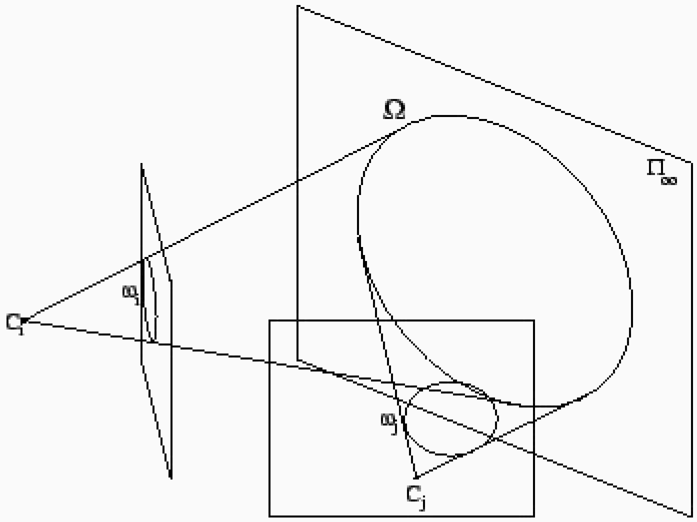
Importantly, the plane at infinity is invariant to translation. (which is why the moon appears to stand still, even when you move around)

The Image of the Absolute Conic (IAC)

On the plane at infinity, there is an important structure called the *absolute conic*.

Within a 2D camera image of the 3D world, the projection of absolute conic is called the Image of the Absolute Conic, or IAC.

The IAC has an important property: it is invariant to rotation as a set - rotate your camera and the IAC remains in place (though the location of the IAC within the AC might change)



IAC Calibration

A camera projection completely constrains its IAC.

Thus, if we can describe the IAC, we can recover the camera calibration parameters.

Furthermore, because of its translation and rotation invariance, we can move the camera around while trying to figure out the same IAC.

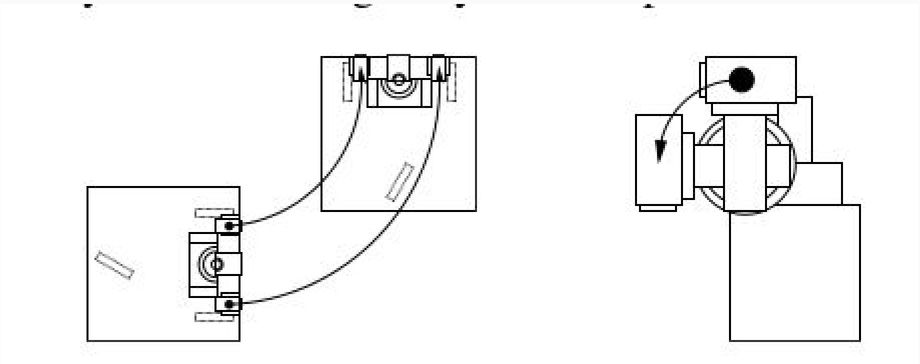
Planar Motion

Planar motion is motion in which the rotation component is perpendicular to a scene plane.

With a UAV flying over level ground, planar motion comes in two flavors:

1. Constant height motion

2. Single axis rotation (sit n' spin)



The Theory

Consider a camera imaging a scene plane (e.g. the ground plane) while undergoing rotation and translation.

The relationship between the scene plane within the camera images before and after movement can be described via a 3x3 homography, x' = Hx.

Importantly, if the rotation axis of the camera is normal to the scene plane, the complex eigenvectors of the homography, H, lie on the IAC

H = homography of the scene plane between the two images
K = camera matrix
M = camera motion

$$H = KMK^{-1}$$
$$H = K(R + d^{-1}tn^T)K^{-1}$$

R has one real eigenvector v, and two complex conjugate eigenvectors e and e^*, which lie on the IAC.

When the rotation axis and scene plane are perpendicular, $n = v$.

$$Me = (R + d^{-1}tn^T)e$$
$$Me = (R + d^{-1}tv^T)e$$
$$Me = Re + d^{-1}tv^Te$$

v and e are orthogonal, thus

$$\mathrm{Me} = \mathrm{Re} + d^{-1}t \cdot 0 = \mathrm{Re} = \lambda e$$

Therefore, e is an eigenvector of M, and

$$H(Ke) = KMK^{-1}Ke$$
$$H(Ke) = KMe$$
$$H(Ke) = K\lambda e$$
$$H(Ke) = \lambda(Ke)$$

Ke is an eigenvector of H, (as is Ke^*)

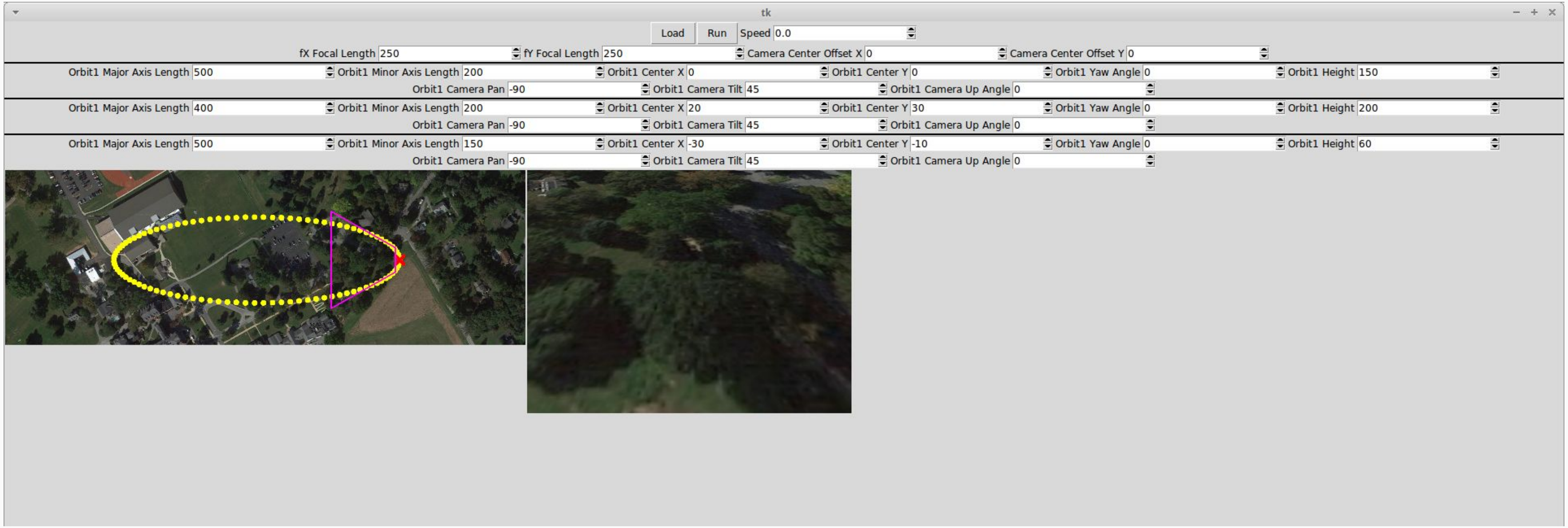
Thus, the complex eigenvectors of H are the images of the eigenvectors of R and lie on the IAC.

IAC Calibration

Thus, by undergoing a planar motion and capturing the homography that describes the imaging of the ground plane, we can determine two points on the IAC.

The IAC, like all non-degenerate 2-D conic sections, has 5 degrees of freedom. Therefore, if we use ground-place homographies from at least 3 distinct planar motions, we can construct the IAC and retrieve the camera calibration.

UAV Simulation



https://github.com/wildweasel/UAVautocal