- Most likely the strong dual topologized by uniform convergence on bounded subsets of W. (F. Trèves, TVS', Dist, Kernels).

- Also see Lassner - Lassner "On the continuity of entropy".

Related to topology discussed here:

"Uniform physical topologies"

- $\beta = \beta(\mathcal{A}, \mathcal{G})$ Strong topology on $\mathcal{A}$ wrt functionals ~ states.

- $\beta^* = \beta(\mathcal{G}, \mathcal{A})$ Strong topology in $\mathcal{G}$.

β^*: $q_S(p) = \sup_{A \in S} |p(A)|$ where S is any weakly-bounded subset of $\mathcal{A}$. $|\langle Ax, y \rangle| \leq M$. "Expectation values converging on weakly bounded subsets"

$\mathcal{L}^+(\mathcal{D}\otimes \ell_{fin}(\mathbb{N}))$, β topology is the strong topology on $\mathcal{L}^+(\mathcal{D})$.

$$q_S(A) = \sup_{x,y\in S} |\langle Ax, y\rangle|$$

- S weakly bounded?

<u>M_∞-convexity:</u>

Set-up:

- $\mathcal{A}$ - closed Op^*-algebra on $\mathcal{D}$ $\quad$ $\mathcal{A}$ C^*-alg

- M_∞ - finitely supported (infinite) matrices.

- $\mathcal{A} \otimes M_\infty$ is an Op^*-algebra on $\mathcal{D} \otimes l_c \longrightarrow l^2$?

- Take closure of $\mathcal{A} \otimes M_\infty$ on the TVS completion of $\mathcal{D} \otimes l_c$ wrt the corresponding β topology.

- Define β topology on $\mathcal{A} \otimes M_\infty$ by seminorms

$$\beta_S(a \otimes M) = \sup_{p \in S} |tr((a \otimes M)p)|,$$

S is ~~w-bounded~~ subsets of $\mathcal{D}_1(\mathcal{D} \otimes l_c)$

Trace class on $\overline{\mathcal{L}^+(\mathcal{D} \otimes l_c)}$ such that

- ApB, Ap^*B are trace class
- property ii) ?

• Cts linear maps $\mathcal{L}(A \otimes M_\infty)$ wrt this β topology form the analogue of completely bounded maps

$\Phi : \mathcal{B}(\mathcal{H}) \longrightarrow \mathcal{B}(\mathcal{H})$ completely bounded ; $\sup_n \|\Phi_n\| < \infty$.

• Cts wrt β topology

for all β_s, $\exists M$ and $\beta_{s_1}, \ldots, \beta_{s_k}$; $\forall v \in \overline{\mathcal{D} \otimes \ell_c}$

$$\beta_s(Tv) \leq M \cdot (\beta_{s_1}(v) + \cdots\cdots + \beta_{s_k}(v))$$

• Channel $\Phi : \mathcal{A} \longrightarrow \mathcal{L}^+(\mathcal{D})$, we can write

$\Phi_\infty : \mathcal{A} \otimes M_\infty \longrightarrow \mathcal{L}^+(\mathcal{D}) \otimes M_\infty$ by

$$\Phi_\infty(a \otimes M) = \Phi(a) \otimes M$$

• Φ β cts ; $p_s(a) = \sup_{p \in s} |tr(ap)|$. Then given β_s on Φ_∞, then

$\beta_s(V) \leq M \cdot (\beta_{s_1}(V) + \ldots + \beta_{s_k}(V))$ (WTS).

• $\mathcal{S}$-weakly bounded subsets of $\mathcal{D}_1(\mathcal{D} \otimes \ell_c) \overset{?}{=} \mathcal{D}_1(\mathcal{D}) \otimes \mathcal{D}_1(\ell_c)$, then

Let $S = S^A \otimes S^\infty$, then $\beta_s(a \otimes M) = p_{s^A}(a) \cdot p_{s^\infty}(M)$

$p_{s^A}(\Phi a) \leq C \cdot (p_{s_1^A}(a) + \ldots + p_{s_k^A}(a))$

$\beta_s(\Phi_\infty(a \otimes M)) = p_{s^A}(\Phi a) \cdot p_{s^\infty}(M)$

Assume Φ cts $\longrightarrow$ $\Phi \otimes 1$ cts on $\underbrace{A \otimes M_\infty}$

• β prenorms

• $\mathcal{L}^+(W) \subseteq \mathcal{L}_B(W, W_b')$ –
$\mathcal{D}$ $\mathcal{D}, \mathcal{D}_b'$

W_b', $B \subseteq W$ bounded,
V nhood of 0 in $\mathbb{C}$
$U_{B,V} = \{u \in W' \mid u(B) \subseteq V\}$.

$$\mathcal{L}^+(D) \rightsquigarrow M_\infty \otimes \mathcal{L}^+(D)$$

$|u(b)| < \varepsilon$ $\forall b \in B$.

$$\mathcal{L}_b\left(M_\infty \otimes \mathcal{L}^+(D), M_\infty \otimes \mathcal{L}^+(D)\right) \overset{?}{=}: CB(\mathcal{L}^+(D), \mathcal{L}^+(D))$$

A, Op^*-alg on D, D complete.

$$A = \bigcup_{n=1}^{\infty} A_n \qquad A_n = \{x \in A \mid \exists \lambda > 0 \ \ |\langle x\xi, \xi\rangle| \le \lambda \langle a_n \xi, \xi\rangle \ \forall \xi \in D\}$$

$a_n \in A^+$ $a_n \le a_{n+1}$ $\rho_n(x) = \inf\{\lambda \mid$ holds$\}$.

ρ-topology

ρ_n norm on A_n A inductive limit of normed spaces.

Under certain conditions on D, ρ-topology $= \beta$-topology.

$$A \subset Bil(D, D) \cong (D \hat{\otimes} \overline{D})' \qquad A = \left((D \hat{\otimes} \overline{D}) / A_\perp\right)'$$

W^{-}

$S(\mathbb{R}^n)$

- **Metrizable:** $\sum p_{\alpha,\beta}(f) = \sum \sup_{x \in \mathbb{R}^n} |x^\alpha \cdot (\partial_x)^\beta f|$

- $\rho = \beta$ on $\mathcal{A}$. $\quad$ ($A_n \otimes M_\infty$ norm $\quad \bigcup_n A_n \otimes M_\infty = \underline{A \otimes M_\infty}$)

$$\overline{\quad} \ ||\,?$$
$$\beta$$

- $\underline{\mathcal{A} \otimes M_\infty}$ domain β and μ,

-

ℓ^2, $\{p_m\}_{m\infty}$ complete $\quad \forall M$

$\psi_n \to \psi$ if $\|M\psi_m - M\psi_n\| \to 0$ implies

$\{\psi_n\} \longrightarrow \psi \in \ell^2$

- $|\psi_n(k) - \psi_m(k)| \to 0$

$\psi(k) = \lim_n \psi_n(k)$

- $\begin{cases} \text{Heinrichs, Topological tensor products of} \\ \text{Unbounded on Fréchet domains} \end{cases}$
- Inoue, modular structure.

$\psi_\alpha \otimes f_\alpha$ Cauchy

$$p_{a\otimes m}(\psi \otimes f) = \|(a\otimes m)\cdot \psi \otimes f\|$$
$$= \|a\psi\| \cdot \|mf\|$$

$$\|(a\otimes m)\psi_i \otimes f_i - (a\otimes m)\psi_j \otimes f_j\|$$

$$\hookrightarrow \mathcal{L}_b(A \otimes M_\infty)$$

$\Phi \circ \Omega$, 1

$\Phi\circ\Omega \in \mathcal{U}_\varepsilon \ni 1.$

$\mathcal{U}_\varepsilon$,