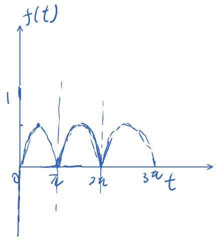
HW3

2021年3月9日 19:30

Section 6.4

6. $f(t)=\begin{cases} \sin(t) & 0<t<\pi \\ -\sin(t) & \pi<t<2\pi \\ \sin(t) & 2\pi<t<3\pi \\ 0 & 3\pi<t \end{cases}$

graph :



step function

plug in model

$f_1=\sin(t)$ $0=0$

$f_2=-\sin(t)$ $t_1=\pi$

$f_3=\sin(t)$ $t_2=2\pi$

$f_4=0$ $t_3=3\pi$

Then the step function is

$f(t)=f_1+u(t-t_1)[f_2(t)-f_1(t)]$
$+u(t-t_2)[f_3(t)-f_2(t)]$
$+u(t-t_3)[f_4(t)-f_3(t)]$

result:

$f(t)=\sin(t)-u(t-\pi)2\sin(t)$
$+2\sin(t)u(t-2\pi)$
$-\sin(t)u(t-3\pi)$

16. $x'+x=2+u(t-1)e^{-t}$ $X(0)=0$

then use the laplace transforms

$\mathcal{L}(x'+x)=sX(s)-\cancel{X(0)}+X(s)$ 0

$=[s+1]X(s)$

$\mathcal{L}[2+u(t-1)e^{-t}]=\frac{2}{s}+e^{-s}\cdot\mathcal{L}(e^{-t-1})$

$=\frac{2}{s}+e^{-s}\cdot[e^{-1}\cdot\frac{1}{s+1}]$

$=\frac{2}{s}+\frac{e^{-s-1}}{s+1}$

$$X(s)=\frac{2}{(s+1)s}+\frac{e^{-s}\cdot e^{-1}}{(s+1)^2}$$

$$\mathcal{L}^{-1}X(s)=2\mathcal{L}^{-1}\left(\frac{1}{s}-\frac{1}{s+1}\right)+\mathcal{L}^{-1}\left(e^{-1}\cdot e^{-s}\cdot\frac{1}{(s+1)^2}\right)$$

$$=2\cdot(1)-2e^{-t}+e^{-1}u(t-1)\cdot(t-1)\cdot e^{-(t-1)}$$

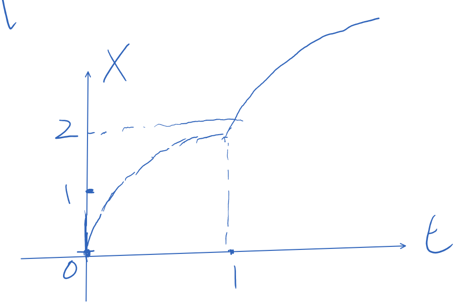
$e^{-1}\cdot(t-1)e^{-t+1}=f_2-f_1$

$f_2=e^{-t}(t-1)+2-2e^{-t}$

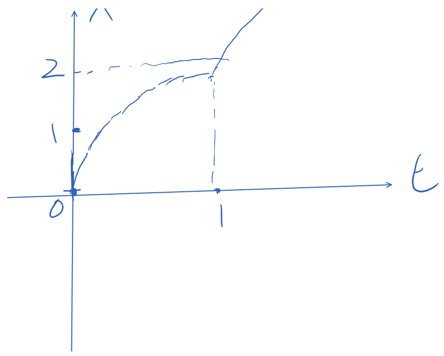
result

$$x=\begin{cases} 2-2e^{-t} & 0<t<1 \\ e^{-t}(t-1)+2-2e^{-t} & 1<t \end{cases}$$

graph :



graph :



18 : $x'' + 2x' + 5x = 2 + u(t-2)$

transform:

$\mathcal{L}(x'' + 2x' + 5x)$

$= s^2 X(s) - s x(0) - x'(0) +$

$2s X(s) - 2x(0) +$

$5 X(s)$

$= (s^2 + 2s + 5) X(s)$

$\mathcal{L}(2 + u(t-2)] = \frac{2}{s} + \frac{e^{-2s}}{s}$

$= \frac{2 + e^{-2s}}{s}$

$X(s) = \frac{2 + e^{-2s}}{(s^2 + 2s + 5)s}$

$X(t) = \mathcal{L}^{-1}\left(\frac{2}{s(s^2+2s+5)} + \frac{e^{-2s}}{(s^2+2s+5)s} \right)$

$$\frac{2}{s(s^2+2s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+5}$$

$$2 = As^2 + 2As + 5A + Bs^2 + Cs$$

$$\begin{cases} A + B = 0 \\ 2A + C = 0 \\ 5A = 2 \end{cases}$$

$A = 0.4$

$B = -0.4$

$C = -0.8$

$\mathcal{L}^{-1}\left[\frac{0.4}{\ } , \frac{-0.4s - 0.4}{\ } + \frac{-0.4}{\ } \right]$

$$\mathcal{L}^{-1}\left[\frac{0.4}{s}+\frac{-0.4s-0.4}{(s+1)^2+2^2}+\frac{-0.4}{(s+1)^2+2^2}\right]$$

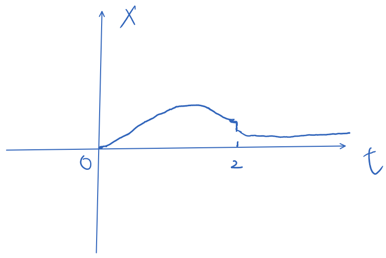
$$= 0.4 - 0.4\,e^{-t}\cdot\cos(2t) - 0.1\cdot e^{-t}\cdot\sin(2t)$$

$$g = \frac{e^{-2s}}{(s^2+2s+5)s} = \frac{1}{2}e^{-2s}\left[\frac{0.4}{s}+\frac{-0.4-0.4}{(s+1)^2+2^2}+\frac{-0.4}{(s+1)^2+2^2}\right]$$

$$\mathcal{L}^{-1}(g) = \frac{1}{2}u(t-2)\left[0.4 - 0.4\,e^{-t}\cdot\cos(2t) - 0.1\cdot e^{-t}\cdot\sin(2t)\right]$$

Conclusion:

$$x(t) = \begin{cases} 0.4 - 0.4\,e^{-t}\cdot\cos(2t) - 0.1\cdot e^{-t}\cdot\sin(2t) & 0<t<2 \\ \frac{1}{2}\left[0.4 - 0.4\,e^{-t}\cdot\cos(2t) - 0.1\cdot e^{-t}\cdot\sin(2t)\right] & 2<t \end{cases}$$



Section 6.5

2. $F(s) = G(s)H(s)$

$F(s) = \frac{1}{s}\cdot\frac{1}{s+3}$

$\mathcal{L}F(s) = g(t) * h(s)$

$g(t) = \mathcal{L}^{-1}\left(\frac{1}{s}\right) = 1$

$h(t) = \mathcal{L}^{-1}\left(\frac{1}{s+3}\right) = e^{-3t}$

$$f(t) = \int_0^t 1\cdot e^{-3[t-\tau]}\,d\tau$$

$$= \int_0^t e^{-3t}\cdot e^{+3\tau}\,d\tau$$

$$= e^{-3t}\cdot\left[\frac{e^{+3\tau}}{3}\right]_0^t$$

$$= \cancel{e^{-3t}}\,\cancel{\frac{e^{+3t}}{3}} - e^{-3t}$$

$$= \frac{1}{3} - e^{-3t}$$

6. $F(s) = H(s)\cdot G(s)$

$F(s) = 2\frac{s}{s^2+1}\cdot\frac{1}{s^2+1}$

$h = \mathcal{L}^{-1}H(s) = 2\cdot\frac{s}{s^2+1} = 2\cos(t)$

$g = \mathcal{L}^{-1}G(s) = \frac{1}{s^2+1} = \sin(t)$

$\mathcal{L}^{-1}F(s) = h(t) * G(t)$

$$f = \int_0^t 2\cos(\tau)\cdot\sin(t-\tau)\,d\tau$$

$$f = 2\int_0^t \cos(\tau) \cdot \sin(\tau - t)\, d\tau$$

$$f = 2\int_0^t \cos(\tau) \cdot [\sin(\tau)\sin(t) - \cos(\tau)\cos(t)]\, d\tau$$

$$f = 2\int_0^t \sin(t) \cdot \frac{\sin(2\tau)}{2} - \frac{1}{2}(1+\cos 2\tau) \cdot \cos(t)\,]\, d\tau$$

$$= 2 \cdot \left[\sin(t) \cdot \frac{-\cos(2\tau)}{4}\right]_0^t - \cos(t)\left[\tau + \frac{\sin 2\tau}{2}\right]_0^t$$

$$= 2 \cdot \sin(t) \cdot \left[\frac{-\cos(2t)}{4} + 1\right] - \cos(t)\left[t + \frac{\sin 2t}{2} - 0 - 0\right]$$

$$= \frac{-\sin(t)\cos(2t)}{2} + 2\sin(t) - \cos(t) \cdot t - \frac{\sin(2t) \cdot \cos t}{2}$$

$$f(t) = -\frac{1}{2}\sin(3t) + 2\sin(t) - t\cos(t)$$

---

8. $x'' + 4x' + 4x = 3\delta(t-2)$

$\mathcal{L}^{-1}[x'' + 4x' + 4x]$

$= s^2 X(s) - s x(0) - x'(0) +$

$4s X(s) - 4x(0) +$ plug $\begin{cases} x(0) = 1 \\ x'(0) = -1 \end{cases}$

$4 X(s)$

$= X(s)[s^2 + 4s + 4] - s + 1 - 4$

$\mathcal{L}^{-1}[3\delta(t-2)]$

$= 3 \cdot e^{-2s}$

$$X(s) = \frac{3e^{-2s} + s + 3}{(s+2)^2}$$

$$\mathcal{L}^{-1}X(s) = \mathcal{L}^{-1}\left[3e^{-2s} \cdot \frac{1}{(s+2)^2} + \frac{s+2}{(s+2)^2} + \frac{1}{(s+2)^2}\right]$$

$$x(t) = 3u(t-2) \cdot f(t-2) + e^{-2t} + te^{-2t}$$

$\Downarrow$

$\tau e^{-2\tau}$

$\tau = t-2$

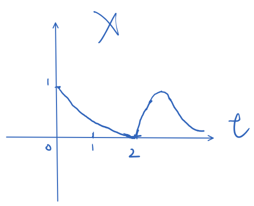
$$x(t) = u(t-2) \cdot 3 \cdot (t-2) \cdot e^{-2(t-2)} + \underbrace{e^{-2t} + te^{-2t}}_{f_1}$$

Conclusion:

$$x(t) = \begin{cases} e^{-2t} + te^{-2t} & 0 < t < 2 \\ e^{-2t} + te^{-2t} + 3 \cdot (t-2)e^{-2(t-2)} & 2 < t \end{cases}$$

$f_2 - f_1 = 3 \cdot (t-2) \cdot e^{-2(t-2)}$

sketch:



Index of comments

3.1 0.2

3.2 sin(2(t-2))

3.3 cos(2(t-2))

3.4 e^{-(t-2)}

3.5 0.2

3.6 e^{-(t-2)}

3.7 No heaviside turns this part of the function off. There's no need to rewrite in piecewise form, leaving it as a single function with heavisides is sufficient and less mistake prone.