ME422: Finite Elements 1

Homework Set #1

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CM 570

The point types are:

2	2	2	2	2	2	2	2	2	1	1	1	0	0	0	0	0	0	0
4	3	3	3	3	3	3	3	3	3	3	3	1	1	0	0	0	0	0
4	3	3	3	3	3	3	3	3	3	3	3	3	3	1	1	0	0	0
4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	1	0	0
4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	1	0	0
4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	1	0
4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	1	0
4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	1
4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	7
6	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	8

The boundary condition on the curved edge is

$$\nabla T(x,y,t)\cdot n = -1 \quad \rightarrow \quad \frac{\partial T(x,y,t)}{\partial x}n_x + \frac{\partial T(x,y,t)}{\partial y}n_y = -1 \quad \rightarrow \quad \frac{\partial T_{j,k,i}}{\partial x}n_x + \frac{\partial T_{j,k,i}}{\partial y}n_y = -1$$

At points of type 1 using one sided differences this is:

$$\frac{n_x}{2h}\left(T_{j,k-2,i} - 4T_{j,k-1,i} + 3T_{j,k,i}\right) + \frac{n_y}{2h}\left(-3T_{j,k,i} + 4T_{j+1,k,i} - T_{j+2,k,i}\right) = -1$$

$$\frac{3n_x}{2h}T_{j,k,i} - \frac{3n_y}{2h}T_{j,k,i} = -1 + \frac{n_x}{2h}(4T_{j,k-1,i} - T_{j,k-2,i}) + \frac{n_y}{2h}(T_{j+2,k,i} - 4_{j+1,k,i})$$

$$T_{j,k,i} = \left(\frac{3n_x - 3n_y}{2h}\right)^{-1}\Big(-1 + \frac{n_x}{2h}(4T_{j,k-1,i} - T_{j,k-2,i}) + \frac{n_y}{2h}(T_{j+2,k,i} - 4_{j+1,k,i})\Big)$$

At points of type 7 using one sided differences this is:

$$\frac{n_x}{2h}\left(T_{j,k-2,i} - 4T_{j,k-1,i} + 3T_{j,k,i}\right) + \frac{n_y}{2h}\left(-4T_{j,k,i} + 4T_{j+1,k,i}\right) = -1$$

$$\frac{3n_x}{2h}T_{j,k,i} - \frac{4n_y}{2h}T_{j,k,i} = -1 + \frac{n_x}{2h}(4T_{j,k-1,i} - T_{j,k-2,i}) - \frac{n_y}{2h}T_{j+1,k,i}$$

$$T_{j,k,i} = \left(\frac{3n_x - 4n_y}{2h}\right)^{-1}\Big(-1 + \frac{n_x}{2h}(4T_{j,k-1,i} - T_{j,k-2,i}) - \frac{n_y}{2h}T_{j+1,k,i}\Big)$$

At points of type 8 using one sided differences this is:

$$\frac{n_x}{2h}\left(T_{j,k-2,i} - 4T_{j,k-1,i} + 3T_{j,k,i}\right) = -1$$

$$T_{j,k,i} = -\frac{2h}{n_x} + 4T_{j,k-1,i} - T_{j,k-2,i}$$