

The point types are:

2	2	2	2	2	2	2	2	2	1	1	1	0	0	0	0	0	0	0
4	3	3	3	3	3	3	3	3	3	3	3	1	1	0	0	0	0	0
4	3	3	3	3	3	3	3	3	3	3	3	3	3	1	1	0	0	0
4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	1	0	0
4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	1	0	0
4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	1	0
4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	1	0
4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	1
4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	7
6	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	8

The boundary condition on the curved edge is

$$\nabla T(x, y, t) \cdot n = -1 \quad \rightarrow \quad \frac{\partial T(x, y, t)}{\partial x} n_x + \frac{\partial T(x, y, t)}{\partial y} n_y = -1 \quad \rightarrow \quad \frac{\partial T_{j,k,i}}{\partial x} n_x + \frac{\partial T_{j,k,i}}{\partial y} n_y = -1$$

At points of type 1 using one sided differences this is:

$$\begin{aligned} \frac{n_x}{2h} (T_{j,k-2,i} - 4T_{j,k-1,i} + 3T_{j,k,i}) + \frac{n_y}{2h} (-3T_{j,k,i} + 4T_{j+1,k,i} - T_{j+2,k,i}) &= -1 \\ \frac{3n_x}{2h} T_{j,k,i} - \frac{3n_y}{2h} T_{j,k,i} &= -1 + \frac{n_x}{2h} (4T_{j,k-1,i} - T_{j,k-2,i}) + \frac{n_y}{2h} (T_{j+2,k,i} - 4T_{j+1,k,i}) \\ T_{j,k,i} &= \left( \frac{3n_x - 3n_y}{2h} \right)^{-1} \left( -1 + \frac{n_x}{2h} (4T_{j,k-1,i} - T_{j,k-2,i}) + \frac{n_y}{2h} (T_{j+2,k,i} - 4T_{j+1,k,i}) \right) \end{aligned}$$

At points of type 7 using one sided differences this is:

$$\begin{aligned} \frac{n_x}{2h} (T_{j,k-2,i} - 4T_{j,k-1,i} + 3T_{j,k,i}) + \frac{n_y}{2h} (-4T_{j,k,i} + 4T_{j+1,k,i}) &= -1 \\ \frac{3n_x}{2h} T_{j,k,i} - \frac{4n_y}{2h} T_{j,k,i} &= -1 + \frac{n_x}{2h} (4T_{j,k-1,i} - T_{j,k-2,i}) - \frac{n_y}{2h} T_{j+1,k,i} \\ T_{j,k,i} &= \left( \frac{3n_x - 4n_y}{2h} \right)^{-1} \left( -1 + \frac{n_x}{2h} (4T_{j,k-1,i} - T_{j,k-2,i}) - \frac{n_y}{2h} T_{j+1,k,i} \right) \end{aligned}$$

At points of type 8 using one sided differences this is:

$$\begin{aligned} \frac{n_x}{2h} (T_{j,k-2,i} - 4T_{j,k-1,i} + 3T_{j,k,i}) &= -1 \\ T_{j,k,i} &= -\frac{2h}{n_x} + 4T_{j,k-1,i} - T_{j,k-2,i} \end{aligned}$$

The 2 dimensional discretization of the wave equation on an equally spaced grid is:

$$u_{i,j}^{k+1} = 2u_{i,j}^k - u_{i,j}^{k-1} + r(u_{i+1,j}^k + u_{i-1,j}^k + u_{i,j+1}^k + u_{i,j-1}^k - 4u_{i,j}^k)$$

where  $h = \Delta x = \Delta y$  and

$$r = \left( \frac{H \Delta t^2}{\rho h^2} \right).$$

Substituting  $u(x, y, t_n) = g(\xi)^n e^{I\xi xy}$  where  $I = \sqrt{-1}$  yields:

$$g(\xi)^{k+1} e^{I\xi h^2 ij} = 2g(\xi)^k e^{I\xi h^2 ij} - g(\xi)^{k-1} e^{I\xi h^2 ij} + r g(\xi)^k \left( e^{I\xi h^2 (i+1)j} + e^{I\xi h^2 (i-1)j} + e^{I\xi h^2 i(j+1)} + e^{I\xi h^2 i(j-1)} - 4e^{I\xi h^2 ij} \right).$$

Dividing by  $g(\xi)^{k-1}$  on both sides yields:

$$g(\xi)^2 e^{I\xi h^2 ij} = 2g(\xi) e^{I\xi h^2 ij} - e^{I\xi h^2 ij} + r g(\xi) \left( e^{I\xi h^2 (i+1)j} + e^{I\xi h^2 (i-1)j} + e^{I\xi h^2 i(j+1)} + e^{I\xi h^2 i(j-1)} - 4e^{I\xi h^2 ij} \right).$$

Dividing by  $e^{I\xi h^2 ij}$  on both sides yields:

$$g(\xi)^2 = 2g(\xi) - 1 + r g(\xi) \left( e^{I\xi h^2} + e^{-I\xi h^2} + e^{I\xi h^2} + e^{-I\xi h^2} - 4 \right),$$

$$g(\xi)^2 = 2g(\xi) - 1 + r g(\xi) \left( 2e^{I\xi h^2} + 2e^{-I\xi h^2} - 4 \right),$$

$$0 = -1 + r g(\xi) \left( 4 \cos(\xi h^2) - 4 + \frac{2}{r} \right) - g(\xi)^2$$