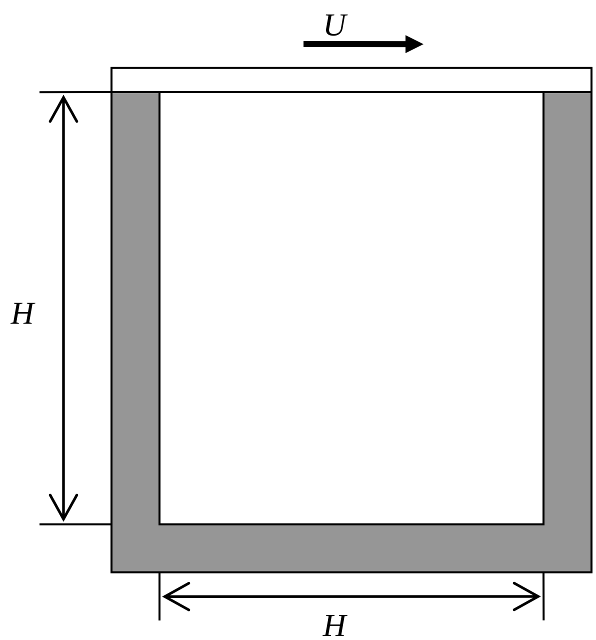
Homework 4

Problem 1

A two-dimensional cavity is filled with an incompressible Newtonian fluid. The fluid is driven by the lid moving with a constant velocity *U*. This problem is widely used as a benchmark to validate CFD models due to its simple geometry but nontrivial flow solution.



In this assignment you will numerically solve the cavity flow using the artificial compressibility method and find the steady state solution.

1. Solve the above problem using the artificial compressibility method. Describe the essential steps in physical terms, explaining why it can be used to model an incompressible fluid. Detail the computational grid you chose as well as the spatial and temporal discretization. State your boundary conditions and how they were implemented.

2. Consider a square cavity. In this problem the Reynolds number, defined by $\mathrm{Re} = UH/\nu$, characterizes the flow patterns. Compute the steady state solutions for both Re = 100 and Re = 400. State your steady state criteria (it should use all points in the domain). Plot the steady state streamlines. Plot the x component of velocity along the vertical centerline. Plot the y component of velocity along the horizontal centerline. Compare your results to those found in the literature. [1]

3. Describe the main features of the flow. How does the flow change with Re?

4. Discuss the stability criteria of the method. How was your time step chosen?

5. For Re = 100 and Re = 400, qualitatively show your two velocity profiles are converging to the literature values by considering three different grids with the grid resolution doubled each time.

6. Numerically compute the spatial convergence rate of your scheme. Detail how you computed the spatial convergence rate (hint: use the previous three grids, see Ferziger). Does it match what is expected based on your discretization?

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[1] See for example, Ghia, U. K. N. G., Kirti N. Ghia, and C. T. Shin. "High-Re solutions for incompressible flow using the Navier-Stokes equations and a multigrid method." *Journal of computational physics* 48.3 (1982): 387-411.