Planck's Quantum Hypothesis

In the year 1900, Max Planck (1858–1947) proposed a theory that was able to reproduce the graphs of Fig. 27–4. His theory, still accepted today, made a new and radical assumption: that the energy of the oscillations of atoms within molecules cannot have just any value; instead each has energy which is a multiple of a minimum value related to the frequency of oscillation by

$$E = hf.$$

Here h is a new constant, now called **Planck's constant**, whose value was estimated by Planck by fitting his formula for the blackbody radiation curve to experiment. The value accepted today is

$$h = 6.626 \times 10^{-34}\,\mathrm{J\cdot s}.$$

Planck's assumption suggests that the energy of any molecular vibration could be only a whole number multiple of hf:

$$E = nhf, \qquad n = 1, 2, 3, \cdots, \tag{27–3}$$

where n is called a **quantum number** ("quantum" means "discrete amount" as opposed to "continuous"). This idea is often called **Planck's quantum hypothesis**, although little attention was brought to this point at the time. In fact, it appears that Planck considered it more as a mathematical device to get the "right answer" rather than as an important discovery. Planck himself continued to seek a classical explanation for the introduction of h. The recognition that this was an important and radical innovation did not come until later, after about 1905 when others, particularly Einstein, entered the field.

The quantum hypothesis, Eq. 27–3, states that the energy of an oscillator can be $E = hf$, or $2hf$, or $3hf$, and so on, but there cannot be vibrations with energies between these values. That is, energy would not be a continuous quantity as had been believed for centuries; rather it is **quantized**—it exists only in discrete amounts. The smallest amount of energy possible (hf) is called the **quantum of energy**. Recall from Chapter 11 that the energy of an oscillation is proportional to the amplitude squared. Another way of expressing the quantum hypothesis is that not just any amplitude of vibration is possible. The possible values for the amplitude are related to the frequency f.

A simple analogy may help. Compare a ramp, on which a box can be placed at any height, to a flight of stairs on which the box can have only certain discrete amounts of potential energy, as shown in Fig. 27–5.

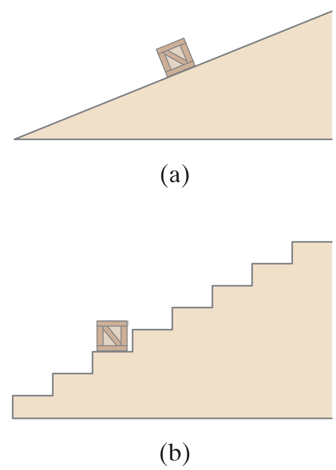

FIGURE 27–5 Ramp versus stair analogy. (a) On a ramp, a box can have continuous values of potential energy. (b) But on stairs, the box can have only discrete (quantized) values of energy.

27–3 Photon Theory of Light and the Photoelectric Effect

In 1905, the same year that he introduced the special theory of relativity, Einstein made a bold extension of the quantum idea by proposing a new theory of light. Planck's work had suggested that the vibrational energy of molecules in a radiating object is quantized with energy $E = nhf$, where n is an integer and f is the frequency of molecular vibration. Einstein argued that when light is emitted by a molecular oscillator, the molecule's vibrational energy of nhf must decrease by an amount hf (or by $2hf$, etc.) to another integer times hf, such as $(n-1)hf$. Then to conserve energy, the light ought to be emitted in packets, or *quanta*, each with an energy

Photon energy

$$E = hf, \tag{27–4}$$

where f is here the frequency of the emitted light. Again h is Planck's constant. Because all light ultimately comes from a radiating source, this idea suggests that *light is transmitted as tiny particles*, or **photons** as they are now called, as well as via the waves predicted by Maxwell's electromagnetic theory. The photon theory of light was also a radical departure from classical ideas. Einstein proposed a test of the quantum theory of light: quantitative measurements on the photoelectric effect.

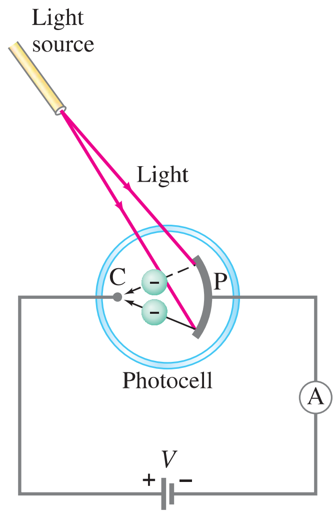

FIGURE 27–6 The photoelectric effect.

When light shines on a metal surface, electrons are found to be emitted from the surface. This effect is called the **photoelectric effect** and it occurs in many materials, but is most easily observed with metals. It can be observed using the apparatus shown in Fig. 27–6. A metal plate P and a smaller electrode C are placed inside an evacuated glass tube, called a **photocell**. The two electrodes are connected to an ammeter and a source of emf, as shown. When the photocell is in the dark, the ammeter reads zero. But when light of sufficiently high frequency illuminates the plate, the ammeter indicates a current flowing in the circuit. We explain completion of the circuit by imagining that electrons, ejected from the plate by the impinging light, flow across the tube from the plate to the "collector" C as indicated in Fig. 27–6.

That electrons should be emitted when light shines on a metal is consistent with the electromagnetic (EM) wave theory of light: the electric field of an EM wave could exert a force on electrons in the metal and eject some of them. Einstein pointed out, however, that the wave theory and the photon theory of light give very different predictions on the details of the photoelectric effect. For example, one thing that can be measured with the apparatus of Fig. 27–6 is the maximum kinetic energy $(\mathrm{KE}_{\max})$ of the emitted electrons. This can be done by using a variable voltage source and reversing the terminals so that electrode C is negative and P is positive. The electrons emitted from P will be repelled by the negative electrode, but if this reverse voltage is small enough, the fastest electrons will still reach C and there will be a current in the circuit. If the reversed voltage is increased, a point is reached where the current reaches zero—no electrons have sufficient kinetic energy to reach C. This is called the *stopping potential*, or *stopping voltage*, V_0, and from its measurement, $\mathrm{KE}_{\max}$ can be determined using conservation of energy (loss of kinetic energy = gain in potential energy):

$$\mathrm{KE}_{\max} = eV_0.$$

Now let us examine the details of the photoelectric effect from the point of view of the wave theory versus Einstein's particle theory.

First the wave theory, assuming monochromatic light. The two important properties of a light wave are its intensity and its frequency (or wavelength). When these two quantities are varied, the wave theory makes the following predictions:

Wave theory predictions

1. If the light intensity is increased, the number of electrons ejected and their maximum kinetic energy should be increased because the higher intensity means a greater electric field amplitude, and the greater electric field should eject electrons with higher speed.
2. The frequency of the light should not affect the kinetic energy of the ejected electrons. Only the intensity should affect $\mathrm{KE}_{\max}$.

The photon theory makes completely different predictions. First we note that in a monochromatic beam, all photons have the same energy $(= hf)$. Increasing the intensity of the light beam means increasing the number of photons in the beam, but does not affect the energy of each photon as long as the frequency is not changed. According to Einstein's theory, an electron is ejected from the metal by a collision with a single photon. In the process, all the photon energy is transferred to the electron and the photon ceases to exist. Since electrons are held in the metal by attractive forces, some minimum energy W_0 is required just to get an electron out through the surface. W_0 is called the **work function**, and is a few electron volts $(1\ \mathrm{eV} = 1.6 \times 10^{-19}\ \mathrm{J})$ for most metals. If the frequency f of the incoming light is so low that hf is less than W_0, then the photons will not have enough energy to eject any electrons at all. If $hf > W_0$, then electrons will be ejected and energy will be conserved in the process. That is, the input energy (of the photon), hf, will equal the outgoing kinetic energy KE of the electron plus the energy required to get it out of the metal, W:

$$hf = \mathrm{KE} + W. \qquad \textbf{(27–5a)}$$

The least tightly held electrons will be emitted with the most kinetic energy $(\mathrm{KE}_{\max})$, in which case W in this equation becomes the work function W_0, and KE becomes KE_{max}:

$$hf = \text{KE}_{\text{max}} + W_0. \qquad \text{[least bound electrons]} \qquad \textbf{(27–5b)}$$

Many electrons will require more energy than the bare minimum (W_0) to get out of the metal, and thus the kinetic energy of such electrons will be less than the maximum.

From these considerations, the photon theory makes the following predictions:

Photon theory predictions

1. An increase in intensity of the light beam means more photons are incident, so more electrons will be ejected; but since the energy of each photon is not changed, the maximum kinetic energy of electrons is not changed by an increase in intensity.
2. If the frequency of the light is increased, the maximum kinetic energy of the electrons increases linearly, according to Eq. 27–5b. That is,
$$\text{KE}_{\text{max}} = hf - W_0.$$
This relationship is plotted in Fig. 27–7.
3. If the frequency f is less than the "cutoff" frequency f_0, where $hf_0 = W_0$, no electrons will be ejected, no matter how great the intensity of the light.

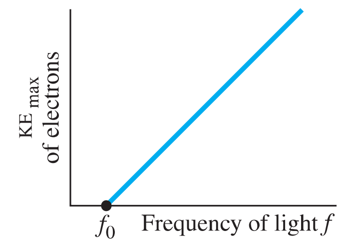

FIGURE 27–7 Photoelectric effect: the maximum kinetic energy of ejected electrons increases linearly with the frequency of incident light. No electrons are emitted if $f < f_0$.

These predictions of the photon theory are very different from the predictions of the wave theory. In 1913–1914, careful experiments were carried out by R. A. Millikan. The results were fully in agreement with Einstein's photon theory.

One other aspect of the photoelectric effect also confirmed the photon theory. If extremely low light intensity is used, the wave theory predicts a time delay before electron emission so that an electron can absorb enough energy to exceed the work function. The photon theory predicts no such delay—it only takes one photon (if its frequency is high enough) to eject an electron—and experiments showed no delay. This too confirmed Einstein's photon theory.

EXAMPLE 27–3 **Photon energy.** Calculate the energy of a photon of blue light, $\lambda = 450\,\text{nm}$ in air (or vacuum).

APPROACH The photon has energy $E = hf$ (Eq. 27–4) where $f = c/\lambda$ (Eq. 22–4).

SOLUTION Since $f = c/\lambda$, we have

$$E = hf = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\,\text{J}\cdot\text{s})(3.00 \times 10^{8}\,\text{m/s})}{(4.5 \times 10^{-7}\,\text{m})} = 4.4 \times 10^{-19}\,\text{J},$$

or $(4.4 \times 10^{-19}\,\text{J})/(1.60 \times 10^{-19}\,\text{J/eV}) = 2.8\,\text{eV}$. (See definition of eV in Section 17–4, $1\,\text{eV} = 1.60 \times 10^{-19}\,\text{J}$.)

EXAMPLE 27–4 **ESTIMATE** **Photons from a lightbulb.** Estimate how many visible light photons a 100-W lightbulb emits per second. Assume the bulb has a typical efficiency of about 3% (that is, 97% of the energy goes to heat).

APPROACH Let's assume an average wavelength in the middle of the visible spectrum, $\lambda \approx 500\,\text{nm}$. The energy of each photon is $E = hf = hc/\lambda$. Only 3% of the 100-W power is emitted as visible light, or $3\,\text{W} = 3\,\text{J/s}$. The number of photons emitted per second equals the light output of 3 J/s divided by the energy of each photon.

SOLUTION The energy emitted in one second ($= 3\,\text{J}$) is $E = Nhf$ where N is the number of photons emitted per second and $f = c/\lambda$. Hence

$$N = \frac{E}{hf} = \frac{E\lambda}{hc} = \frac{(3\,\text{J})(500 \times 10^{-9}\,\text{m})}{(6.63 \times 10^{-34}\,\text{J}\cdot\text{s})(3.00 \times 10^{8}\,\text{m/s})} \approx 8 \times 10^{18}$$

per second, or almost 10^{19} photons emitted per second, an enormous number.

EXERCISE B A beam contains infrared light of a single wavelength, 1000 nm, and monochromatic UV at 100 nm, both of the same intensity. Are there more 100-nm photons or more 1000-nm photons?

EXAMPLE 27–5 **Photoelectron speed and energy.** What is the kinetic energy and the speed of an electron ejected from a sodium surface whose work function is $W_0 = 2.28\ \text{eV}$ when illuminated by light of wavelength (*a*) 410 nm, (*b*) 550 nm?

APPROACH We first find the energy of the photons $(E = hf = hc/\lambda)$. If the energy is greater than W_0, then electrons will be ejected with varying amounts of KE, with a maximum of $\text{KE}_{\text{max}} = hf - W_0$.

SOLUTION (*a*) For $\lambda = 410\ \text{nm}$,

$$hf = \frac{hc}{\lambda} = 4.85 \times 10^{-19}\ \text{J} \qquad \text{or} \qquad 3.03\ \text{eV}.$$

The maximum kinetic energy an electron can have is given by Eq. 27–5b, $\text{KE}_{\text{max}} = 3.03\ \text{eV} - 2.28\ \text{eV} = 0.75\ \text{eV}$, or $(0.75\ \text{eV})(1.60 \times 10^{-19}\ \text{J/eV}) = 1.2 \times 10^{-19}\ \text{J}$. Since $\text{KE} = \frac{1}{2}mv^2$ where $m = 9.1 \times 10^{-31}\ \text{kg}$,

$$v_{\text{max}} = \sqrt{\frac{2\text{KE}}{m}} = 5.1 \times 10^5\ \text{m/s}.$$

Most ejected electrons will have less KE and less speed than these maximum values.

(*b*) For $\lambda = 550\ \text{nm}$, $hf = hc/\lambda = 3.61 \times 10^{-19}\ \text{J} = 2.26\ \text{eV}$. Since this photon energy is less than the work function, no electrons are ejected.

NOTE In (*a*) we used the nonrelativistic equation for kinetic energy. If v had turned out to be more than about $0.1c$, our calculation would have been inaccurate by more than a percent or so, and we would probably prefer to redo it using the relativistic form (Eq. 26–5).

EXERCISE C Determine the lowest frequency and the longest wavelength needed to emit electrons from sodium.

By converting units, we can show that the energy of a photon in electron volts, when given the wavelength λ in nm, is

$$E\ (\text{eV}) = \frac{1.240 \times 10^3\ \text{eV}\cdot\text{nm}}{\lambda\ (\text{nm})}. \qquad \text{[photon energy in eV]}$$

Applications of the Photoelectric Effect

The photoelectric effect, besides playing an important historical role in confirming the photon theory of light, also has many practical applications. Burglar alarms and automatic doors often make use of the photocell circuit of Fig. 27–6. When a person interrupts the beam of light, the sudden drop in current in the circuit activates a switch—often a solenoid—which operates a bell or opens the door. UV or IR light is sometimes used in burglar alarms because of its invisibility. Many smoke detectors use the photoelectric effect to detect tiny amounts of smoke that interrupt the flow of light and so alter the electric current. Photographic light meters use this circuit as well. Photocells are used in many other devices, such as absorption spectrophotometers, to measure light intensity. One type of film sound track is a variably shaded narrow section at the side of the film, Fig. 27–8. Light passing through the film is thus "modulated," and the output electrical signal of the photocell detector follows the frequencies on the sound track. For many applications today, the vacuum-tube photocell of Fig. 27–6 has been replaced by a semiconductor device known as a **photodiode** (Section 29–9). In these semiconductors, the absorption of a photon liberates a bound electron so it can move freely, which changes the conductivity of the material and the current through a photodiode is altered.

FIGURE 27–8 Optical sound track on movie film. In the projector, light from a small source (different from that for the picture) passes through the sound track on the moving film.