

Attempt all questions

a) What is proposition?

(2Marks)

*A proposition is the building block of logic which is a sentence that is either true or false but is not both. If a proposition is true, then we say its truth value is true, and if a proposition is false, we say its truth value is false.*

b) Given that p and q are propositions construct the truth table of; (4 Marks)

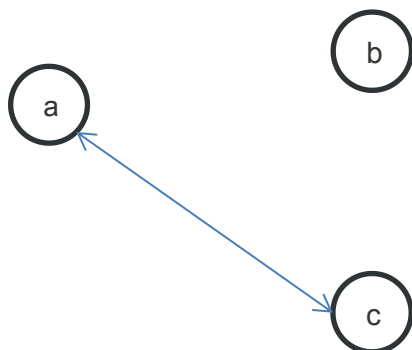
i.  $p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

ii.  $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

c) Draw the graph represented by the adjacency matrix  $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ . (3 Marks)



d) According to information obtained from mathematics department regarding three mathematics units done by 100 students, those who are doing calculus are 45, those doing discrete are 49 and those doing statistics are 38. Those doing calculus and discrete are 17, those doing calculus and statistics are 12 and those doing the three units are 5. Use Venn diagram to find the number of students doing discrete and statistics but not calculus.

(7 Marks)

$$N(U) = 100$$

$$N(C) = 45$$

$$N(D) = 49$$

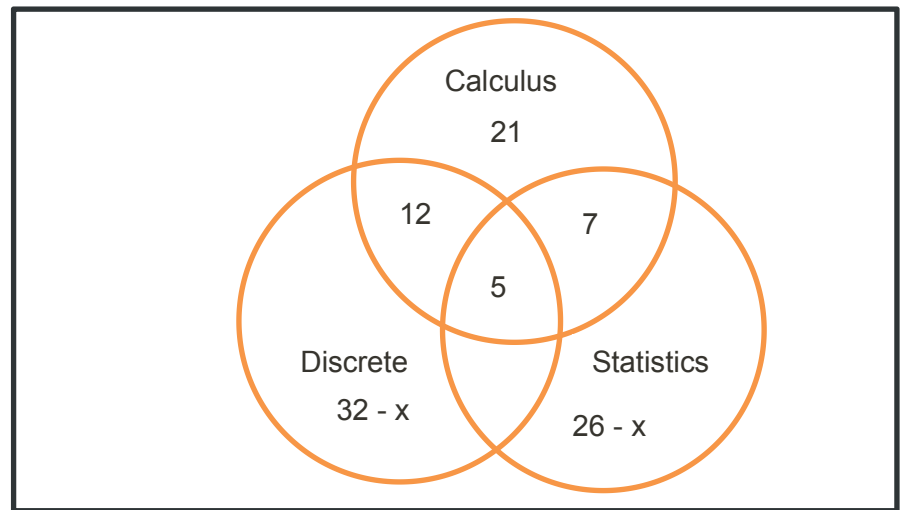
$$N(S) = 38$$

$$N(C \cap D) = 17$$

$$N(C \cap S) = 12$$

$$N(C \cap S) = ?$$

$$N[(C \cap D) \cap S] = 5$$



$$N[C' \cap D \cap S] \cap S = N(D \cap S) - n(C \cap S \cap D) = ?$$

$$? = x$$

$$45 - (7 + 12 + 5) = 21$$

$$32 - x = 49 - (12 + x + 5)$$

$$26 - x = 38 - (7 + x + 5)$$

$$n(U) = 100$$

$$103 - x = 21 + 32 - x + 26 - x + 7 + 12 + x + 5$$

$$x = 103 - 100$$

$$x = 3$$

$$\text{Therefore } N[C' \cap D \cap S] \cap S = 3$$

e) In how many ways can one select 7 member committee from 10 distinct persons if only three persons qualify to be chairperson? (4 Marks)

*One chairperson can be elected from three candidates in three ways.*

*We have to choose 6 from the rest of 9 people.*

$$\text{It can be done } C_6^9 = C_3^9 = 9 \times 8 \times 7 / 1 \times 2 \times 3$$

$$= 504/6$$

$$= 84$$

Taking three opportunities of the choice of the chairperson into account, we have

$$3 \times 84 = 252 \text{ ways.}$$

- f) Prove that the conditional proposition  $p \rightarrow q$  and its contrapositive  $\neg q \rightarrow \neg p$  are logically equivalent using the truth table. (6 Marks)

p	q	$P \rightarrow q$	$\neg q \rightarrow \neg p$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Each row of  $(\neg q \rightarrow \neg p)$  is identical to the corresponding row of  $p \rightarrow q$ . Therefore, conditional proposition is logically equivalent to its contrapositive.

- g) Prove by mathematical induction the formula;

$$1^3 + 2^3 + 3^3 + 4^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4} \quad (4\text{marks})$$

**Basis Step**

$$n=1 \quad 1=1$$

$$1^3 = \frac{1^2(1+1)^2}{4}$$

$$1 = \frac{1(2)^2}{4} = 4/4 = 1$$

**Induction Step**

Assuming  $n = k$

$$1^3 + 2^3 + 4^3 + \dots + k^3 = \frac{k^2(k+1)^2}{4}$$

Showing True  $n = k+1$

$$1^3 + 2^3 + 4^3 + \dots + k^3 + k+1 = \frac{(k+1)^2((k+1)+1)^2}{4}$$

$$\frac{k^2(k+1)^2}{4} + k+1 = \frac{(k+1)^2(k+2)^2}{4}$$

$$k^2(k+1)2 + 4(k+1) = (k+1)2(k+2)2$$

$$k^4 + 2k^3 + k^2 + 4k + 4 = k^4 + 2k^3 + 4k + 4 + 1$$