Lecture 1

Coulomb's law (Review)

Lecture Overview

Coulomb's law was developed by a French physicist called Charles Coulomb around 1780. The law is used to predict electric forces between any two charged bodies. It is introduced in BPY 1103 Electricity magnetism 1 taught in first year. This course reviews the same law with main focus on the limitations of the law.

Coulomb's law ignored the geometry aspect of object. All figures were reduced to point charges. Regardless of the shape and size of the charged object, the law only consider the total amount of charge to be concentrated at a single point.

Karl Fredrick Gauss (1777 – 18551) study of charges took care of the object's geometry. Gauss's law related electric field at point to the amount of charge enclose by a given surface. This made it easy to calculate the charge distribution for different geometrical solids.

This section will assist to review the relationship between Electric force, Electric field and the potential energy.

By the end of this lecture, you should be able to:

i) State and apply Coulomb's law to calculate electric force of attraction and repulsion.

ii) Define and calculate electric field intensity.

iii) Define electric potential.

iv) Solve problems related to electric fields.

Coulomb's law

This law predicts the amount of electric force experienced between any two charged bodies and it gives the direction of the force.

Coulomb's law states that the force between two charged particles is directly proportional to the product of the amount of charges accumulated on these particles and inversely proportional to the square of the distance between them. The force act along a straight line joining the centers of these particles.

$F \, \alpha \, \frac{Q_a \, Q_b}{r^2}$ Given Q_a and Q_b as the quantities of charges and **r** as the distance separating the two charged bodies.

The distance between point ***a*** and ***b*** can also be denoted as r_{ab}^2.

$$F \, \alpha \, \frac{Q_a \, Q_b}{r_{ab}^2} \quad \rightarrow F = \boldsymbol{K} \frac{Q_a \, Q_b}{r_{ab}^2}$$

Given ***K*** is the coefficient of proportionality whose value depends on the medium in which the charges interact.

The value of $K = \frac{1}{4\pi \, \varepsilon_0}$ for a vacuum or dry air. The symbol, ε is a Greek letter epsilon.

Where symbol, ε_0 (epsilon with a subscript 0) represent the permittivity of a vacuum (or dry air).

This is an experimental value, $\varepsilon_0 = \; 8.85 \, x \, 10^{-12} \; {C^2}/{Nm^2}$.

$$K = \frac{1}{4 \, \pi \, (8.85 \, x \, 10^{-12})} = \frac{1}{1.112 \, x \, 10^{-10}} = 8.99 \, x \, 10^9 \; {Nm^2}/{C^2}$$

Example 1.1

Calculate the attractive force between an electron and a proton which they are 1 cm apart and they are interacting in a vacuum.

Solution

$$F = \boldsymbol{K} \frac{Q_a \, Q_b}{r_{ab}^2} = \left(8.99 \, x \, 10^9 \; {Nm^2}/{C^2}\right) \frac{(+1.6 \, x \, 10^{-19} \; C)(-1.6 \, x \, 10^{-19} \; C)}{1 \, x \, 10^{-2}} = -\, 2.30 \, x \, 10^{-26} \; N$$

The negative sign in the answer indicate the force is negative.

If the interaction of charges is taking place in a medium other than vacuum or air, the value of K is given by; $K = \frac{1}{4\pi\varepsilon_0\varepsilon_r}$ where ε_r is known as relative permittivity. Take note of the subscript *r* which indicate relative permittivity.

$\varepsilon_r = \frac{force\ in\ vacum}{force\ in\ the\ medium}$, a ratio of force in vacuum to the force in any other medium between the same pair of charges separated by the same distance.

Permittivity, ε of any other insulating substance is given by $\varepsilon = \varepsilon_0 \varepsilon_r$ where, ε_r gives the relative permittivity of the substance. Therefore, $K = \frac{1}{4\pi\varepsilon}$ of any material that is not a vacuum. **Permittivity** is a material medium property that affects the Coulomb force between two point charges when they interact in that medium.

Relative permittivity ε_r also known as dielectric constant. Some books use symbol k for dielectric constant.

This coulomb law is only valid for point charges only.

Example 1.2

Calculate the permittivity of a material whose relative permittivity is 50 times that of dry air.

Solution

$$\varepsilon = \varepsilon_0 \varepsilon_r = 8.85\ x\ 10^{-12}\ \mathrm{C}^2/_{Nm^2}\ x\ 50 = 4.424\ x\ 10^{-10}\ \mathrm{C}^2/_{Nm^2}$$

Example 1.3

Two charges $Q_1 = -3.0\ X\ 10^{-7}\mathrm{C}\ and\ Q_2 = 4.0\ X\ 10^{-7}\mathrm{C}$ are placed 14 m apart. If the space between the two charges is filled with a material of permittivity ε that is 20 times that of air, determine the force experienced by the negative charges.

Solution

$$F = \frac{KQ_1Q_2}{r^2\varepsilon_r} = \frac{\left(8.99\ x\ 10^9\ Nm^2/_{C^2}\right) x(-3.0\ x\ 10^{-7}\ C)(4.0x\ 10^{-7}\ C)}{20\ \ (14\ \mathrm{m})^{\ 2}} = -\ 2.75\ x\ 10^7\ N$$

Example 1.4

Calculate the value of two equal charges. If they repel each other with a force of 0.1 N when they are situated 50 cm apart in a vacuum.

Solution

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r_{ab}^2} \qquad Q_1 = Q_2 \quad \therefore\ Q_1 Q_2 = Q^2$$

$$0.1\ N = \frac{(8.99\ x\ 10^9)\ Q^2}{(0.5)^{\ 2}}$$

$$Q = \sqrt{\frac{0.1\ X\ (0.5)^2}{8.99\ X\ 10^9}} \quad = 1.7\ X\ 10^{-6}\ \mathrm{C} = \ 1.7\ X\ 10^{-6}\ \mu\mathrm{C}$$

Activity 1.1

a) Two charged particles Q_1 and Q_2 are positioned on a cartesina plane at $x_1 =$ 5.1 cm and $x_2 = -$ 3.4 cm respectively. Given that $Q_1 = +6.0\ \mu C$ and $Q_1 = -5.0\ \mu C$, Calculate the magnitude of force experienced by Q_2 if Q_1 is fixed. (assume both are ineracting in dry air)

b) Two charges are attracted by a force of 25 N when separated by 10 cm. What is the force between the charges when the distance between them is 50 cm?

Electric force is a vector quantity hence it also has modulus and direction. Directions are normally in unit vectors. The coulomb law discussed so far is silent about the direction.

It only give the magnitude such that, $|F| = \frac{Q_a Q_b}{r^2}$

Force as a vector is given as, $\vec{\boldsymbol{F}} = \frac{Q_a Q_b}{|r|^2} \cdot \hat{r} = \vec{\boldsymbol{F}}_x + \vec{\boldsymbol{F}}_y$ (components form)

Given that $\hat{r}$ is a unit vector in the direction of force. We obtain unit vector by the ratio of the vector and its modulus, $\hat{r} = \frac{\vec{\boldsymbol{r}}}{|r|} \rightarrow u\ nitvector = \frac{(vector\ \vec{\boldsymbol{r}})}{(modulus\ of\ vector\ |r|)}$

Example 1.5

Two charged particles Q_1 and Q_2 are positioned on a cartesina plane at $Q_1(3,4)$ and $Q_2 = (9,12)$ respectively. The value of $Q_1 = +\,3.37\ \mu C$ and $Q_2 = -\,3.37\ \mu C$ Calculate;

a) The column vector, $\vec{\boldsymbol{r}}$ between Q_1 and Q_2.

b) The modulus of the postion vector $|r|$

c) Unit vector $\hat{r}$

d) If the two charged particles are interacting in the vacuum, calculate the force between the two in vector form.

Solution

a) $\vec{\boldsymbol{r}} = \binom{9}{12} - \binom{3}{4} = \binom{6}{8}$

b) $|r| = \sqrt{6^2 + 8^2} = 10$

c) $\hat{r} = \frac{\vec{\boldsymbol{r}}}{|r|} = \frac{6i+8j}{10}\ or\ (0.6i + 0.8j)$

d) $\vec{\boldsymbol{F}} = \frac{Q_a\,Q_b}{|r|^2} \cdot \hat{r} = \frac{(+\,3.37\ \mu C)(-\,3.37\ \mu C)}{10^2} \cdot (0.6i + 0.8j)$

$$F = -\left(6.8\ x\ 10^{-14}\right)i - \left(9.8\ x\ 10^{-14}\right)j$$

The negative signs indicate Q_1 is fixed and Q_2 is attracted towards Q_1.

Activity 1.2

a) Two charged particles Q_1 and Q_2 are positioned on a cartesina plane at $Q_1(1,4)$ and $Q_2 = (4,9)$ respectively. The value of $Q_1 = +\,4.0\ \mu C$ and $Q_2 = -\,3.0\ \mu C$ Calculate;

i. The column vector, $\vec{\boldsymbol{r}}$ between Q_1 and Q_2.

ii. The modulus of the postion vector $|r|$

iii. Unit vector $\hat{r}$

iv. If the two charged particles are interacting in the vacuum, calculate the force between the two in vector form. (assume both are interacting in dry air)

Unit vector $\hat{r}$ can also be obtained by $\hat{r} = \cos\theta\, i + \sin\theta\, j$.

Example 1.6

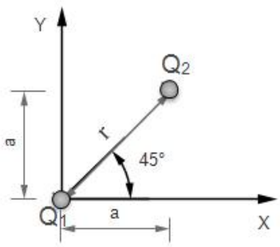
Two charges are positioned such that Q_2 is $30°$ from Q_1 a distance of 0.4 m. Calculate the unit vector $\hat{r}$ in the direction of Q_2 from Q_1.

Solution

$\hat{r} = \cos\theta\, i + \sin\theta\, j = \cos 30°\, i + \sin 30°\, j = 0.866\, i + 0.5\, j$

Activity 1.3

Two charges arranged as shown in the figure below.



Calculate the force on charge Q_1 due to Q_2. Take $Q_1 = 6.0\,\mu C$, $Q_2 = 3.0\,\mu C$ and $a = 2.0\, x\, 10^{-2}\, m$. (assume both are interacting in dry air)

Electric Field

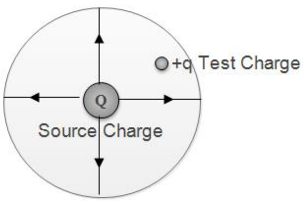
This is a region or space surrounding a charged object where the effect of an electric force is experienced. This region is marked using electric field lines.

Electric field lines are paths along which a unit positive charge will follow if free to do so. The direction of lines starts from the positively charged surfaces and sinks (ends) into negatively charged surfaces.

Electric Field Intensity, $\vec{\mathrm{E}}$

The strength of a field is described as a force in newton that is experienced by a unit positively test charge when it is brought to the space surrounding the charge. It is a vector quantity with both size and modulus. The charge generating the field called a source charge.

Field intensity, $\vec{\mathrm{E}} = \frac{\boldsymbol{Force}\ (N)}{\boldsymbol{Charge}\ (C)}$ The SI unit is Newtom per Coulomb, $\frac{N}{C}$.



Considering that the force is electric and is obtained using Coulombs' law,

$$\vec{\mathbf{E}} = \frac{\boldsymbol{Force}\ (\boldsymbol{N})}{\boldsymbol{Charge}\ (\boldsymbol{C})} = \frac{1}{4\pi\varepsilon_0}\frac{Q\cancel{q}}{r^2} \cdot \frac{1}{\cancel{q}}$$

The test charge, q in the equation cancel out and we have;

$|\boldsymbol{E}| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$, Only the modulus. $\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \cdot \hat{r} = E_x + E_y$, As a vector.

The $\vec{\mathbf{E}}$ **field intensity** depend on the medium where the source charge is located because of permittivity, the amount of charge and the distance from the source charge. The intensity of $\vec{\mathbf{E}}$ **field intensity** reduces with square distance – inverse square distance.

Example 1.7

Calculate the magnitude of the electric filed strength at a point $5\ x\ 10^{-8}\ m$ from an electron, which is in a vacuum.

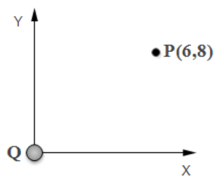
Solution

$$|\boldsymbol{E}| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \frac{1}{4\pi(8.85\ x\ 10^{-12}\ \mathrm{C}^2/_{Nm^2})} \cdot \frac{1.6\ x\ 10^{-19}\ C}{(5\ x\ 10^{-8}\ m)^2} = 5.75\ x\ 10^5\ {}^{N}/_{C}$$

The electric field in the solution above is very silent about the direction. The field lines points towards the electron because it is positively charged.

Example 1.8

A point charge $Q = -12\ nC$ is place on the Cartesian plane as shown below.



Calculate the $\vec{\mathbf{E}}$ **field strength** at a point P (6, 8) on the diagram.

Solution

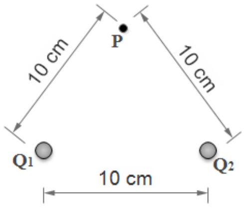
$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|r|^2} \cdot \hat{r} \quad \text{Position vector } \vec{r} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} = 6\,i + 8\,j \quad |r| = \sqrt{6^2 + 8^2} = 10$$

Therefore, $\hat{r} = \frac{6\,i+8\,j}{10} = 0.6\,i + 0.8\,j$

$$\vec{\mathbf{E}} = \frac{1}{4\pi\left(8.85\ x\ 10^{-12}\ \mathrm{C}^2/_{Nm^2}\right)} \cdot \frac{-12\ nC}{10^2} \cdot (0.6\,i + 0.8\,j) = -\,0.65\,i - 0.86\,j$$

Activity 1.4

Two point charges $Q_1 = Q_1 = -12\,nC$ are place as shown below.

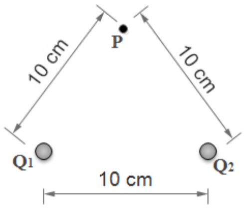


Calculate $|\boldsymbol{E}|$ **and** $\vec{\mathrm{E}}$ at point P due to source charges Q_1 and Q_1. (Assume the charges are in the vacuum)

Both components of the field are negative.

Activity 1.4

Two point charges $Q_1 = Q_1 = -12\,nC$ are place as shown below.



Calculate $|\boldsymbol{E}|$ **and** $\vec{\mathrm{E}}$ at point P due to source charges Q_1 and Q_1. (Assume the charges are in the vacuum)

Electric Potential, V

This is the work done on a unit charge to move it within an existing electric field. This is a scalar quantity. The work done on a charge is equivalent to energy it gains or losses for changing position within a field. (It is equivalent to mechanical Potential energy gained by a mass when it moves along pull of gravity)

The symbol used is V. $\boldsymbol{Electric\ potential, V} = \frac{word\ done\ (J)}{Quantity\ of\ charge\ (C)}$

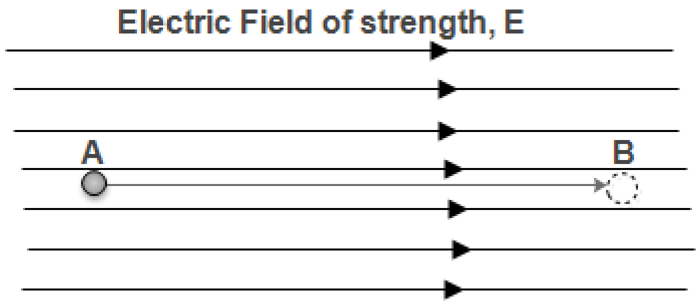
$1\,Volt = 1^{J}/_{C}$

$$\Delta V = \frac{\Delta w}{Q} = \frac{Force\ x\ Distance}{Amou\ ntof\ charge} = \frac{F\ x\ D}{Q}$$

Recall from the definition of electric field, $F = Q\,\vec{E}$. $\therefore\ \Delta V = \frac{Q\,\vec{E}\,D}{Q} = \vec{E}\,D$

This implies that $\vec{E} = \frac{\Delta V}{D}$, giving rise to another SI unit of electric field strength i.e. Newton/meter ($^{N}/_{m}$)

The figure below shows an electric field marked by straight field lines.



If a positive charge is free to move and placed at point **A**, the field will push it towards **B**. The field force is working on the charge. This is positive work done by the field but the charged particle loses electrical energy. $-\Delta V = \frac{\Delta w}{Q}$

If an external agent (e.g the battery) pushes the charged particle against the **E** field force, the agent does negative work and the charged particle gains potential energy. $+\,\Delta V = \frac{\Delta w}{Q}$

This correspond to gain and loss of mechanical potential energy when a mass changes position within a gravitation field. Mass moving upward goes against the field and it gains potential energy. Mass dropping freely due to pull of gravity losses potential energy.

Example 1.9

Calculate the electric potential change, ΔV in energy when a unit charge moves through a distance of 10 mm within a constant field intensity 480 $^{V}/_{m}$.

Solution

$\Delta V = \vec{E}\,(\Delta D)$, where ΔD is change in distance.

$\Delta V = 480\ ^{V}/_{m}\ x\ 10^{-2}\ m = 4.8\ V$

Example 1.10

A proton moves in the direction of a constant field through a distance of 50 cm. The electric field strength given as $1.5\ x\ 10^{7}\ ^{V}/_{m}$. Determine;

a) The force on the proton.

b) The potential energy lost by the proton.

c) The work done on the proton.

Solution

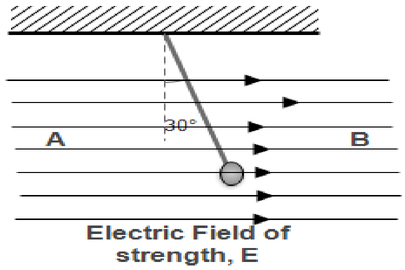
a) $F \ = \ Q\ E = 1.602\ x\ 10^{-19}\ C\ x\ 1.5\ x\ 10^{7}\ {}^{N}/_{C}\ = 2.4\ x\ 10^{-12}\ N$

b) $\Delta V = \ \vec{E}\ (\Delta l) = \ \ 1.5\ x\ 10^{7}\ {}^{V}/_{m}\ \ x\ 0.5\ m = 7.5\ x\ 10^{6}\ V$

c) $\Delta W = F\ (\Delta l) = \ 2.4\ x\ 10^{-12}\ N\ x\ 0.5\ m = 1.2\ x\ 10^{12}\ J$

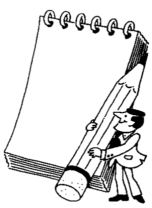
Activity 1.5

A uniform electric field points in the direction shown below.



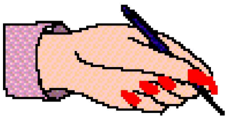
A small metal ball charged to – 2 mC and hangs at an angle 30° from a string of negligible mass. Calculate the magnitude of electric field intensity, **E** given that the string tension is 0.1 N.

Now take a break and reflect on some of the issues we have discussed today. After your break, answer the following questions:



You have come to the end of lecture one. In this lecture you have learnt the following:

- Electric force is calculated using Coulomb’s law.
- Electric force diminishes with square distance.
- Electric field intensity is the equivalent of gravitation strength on earth.
- Charged particles gain and lose potential when they change position within a field.
- Work done is gained as electrical potential.



a) State the Coulombs law. (2 marks)

b) Two charges are located on the positive x-axis of a coordinate system. Charge $q_1 = 200\mu C$ is 2 cm from the origin and charge $q_2 = 300\mu C$ is 4 cm from the origin.

i. Calculate the force exerted on the two charges (3 marks)

ii. Calculate the total force exerted by these two charges on $q_3 = 300\mu C$ located at the origin (3 marks)

E. M Purcell and D. J. Morin (2013) *Electricity and Magnetism* 3rd Edition. Cambridge University Press, UK. Pg 1- 16.

Young and Freedman, University Physics with Modern Physics 13th Edition. Pg 687 – 724.