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Feynman variance-to-mean in the context of passive neutron coincidence counting

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ABSTRACT

Passive Neutron Coincidence Counting (PNCC) based on shift register autocorrelation time analysis of the detected neutron pulse train is an important Nondestructive Assay (NDA) method. It is used extensively in the quantification of plutonium and other spontaneously fissile materials for purposes of nuclear materials accountancy. In addition to the totals count rate, which is also referred to as the singles, gross or trigger rate, a quantity known as the reals coincidence rate, also called the pairs or doubles, is obtained from the difference between the measured neutron multiplicities in two measurement gates triggered by the incoming events on the pulse train. The reals rate is a measure of the number of time correlated pairs present on the pulse train and this can be related to the fission rates (and hence material mass) since fissions emit neutrons in bursts which are also detected in characteristic clusters.

A closely related measurement objective is the determination of the reactivity of systems as they approach criticality. In this field an alternative autocorrelation signature is popular, the so called Feynman variance-to-mean technique which makes use of the multiplicity histogram formed the periodic, or clock-triggered opening of a coincidence gate.

Workers in these two application areas share common challenges and improvement opportunities but are often separated by tradition, problem focus and technical language. The purpose of this paper is to recognize the close link between the Feynman variance-to-mean metric and traditional PNCC using shift register logic applied to correlated pulse trains. We, show using relationships for the late-gate (or accidentals) histogram recorded using a multiplicity shift register, how the Feynman *Y*-statistic, defined as the excess variance-to-mean ratio, can be expressed in terms of the singles and doubles rates familiar to the safeguards and waste assay communities. These two specialisms now have a direct bridge between them and we anticipate fruitful cross fertilization, for example on assay algorithms, including corrections for measurement item perturbation factors, and on data acquisition systems.

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1. Introduction

The analysis of pulse trains from a neutron detector comprising a moderated array of ^{3}He-filled proportional counters as it responds to a source in which fission (either spontaneous fission or induced fission or both) is taking place is the essential element in performing neutron measurements to accurately determine the properties of Special Nuclear Material (SNM) bearing items. The fission reactions are considered to give rise to a prompt burst of neutrons which are then detected over a far longer period commensurate with the thermal lifetime of neutrons in the moderated detector assembly. Because neutrons from the same burst remain, on the average, correlated in time while they transport and get detected, the pulse train contains usable information about the number and length of the fission chains, which can be separated from the random events coming (mainly) from (α,n) reactions, which are usually also taking place in SNMs, and delayed neutrons following fission. The autocorrelated signature is usually obtained by some form of time interval analysis or time correlation analysis.

In this work we consider the similarity between the Feynman variance-to-mean approach [1] to time correlation analysis and traditional Passive Neutron Coincidence Counting (PNCC) with shift-register logic [2], which use neutron multiplicities measured in event triggered coincidence gates to obtain important physical parameters of an item. Usually, physical parameters of interest include the efficiency (ε), the α-ratio ($\alpha = S_\alpha/\overline{\nu}_s F_s$ where S_α is the rate of random neutrons in the item that are mostly due to (α,n) reactions, $\overline{\nu}_s$ is the average number of neutrons emitted from spontaneous fission and F_s is the spontaneous fission rate), the

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system leakage multiplication (M_L) and the effective ^{240}Pu mass (m_{240}). $F_s = m_{240} \cdot g$, where g is the specific spontaneous fission rate of ^{240}Pu.

Both Feynman analysis and PNCC use coincidence gates in what is commonly called time correlation analysis (TCA), in contrast to time interval analysis (TIA) which uses the distribution of times between subsequent neutrons to quantify the observed neutron distribution. The Feynman approach uses the measured multiplicity in a series of clock-triggered gates to calculate the Y-statistic (excess relative variance), which is then related to physical properties of the item. The relative variance, $Y = VMR - 1$, is the variance-to-mean ratio less one, the utility of which will become clear later. In the PNCC, the observed multiplicities (number distribution) in an early (reals plus accidentals) gate and a late (accidentals) gate are used to calculate the singles (S) and doubles (D) values which are then mathematically inverted using a simple physical model (the point model) to obtain properties of the item under study. The goal here is to demonstrate the close relationship between the Feynman and PNCC approaches, and to derive the formal relationship between the Y-value, S and D. The PNCC approach is the overwhelmingly dominant perspective of correlating counting experts in the safeguards domain while in the field of sub-criticality experiments for transport code and data benchmarking the Feynman technique has largely been favored. Our present discussion is limited to building a bridge between the PNCC and Feynman variance-to-mean conventions. However we are motivated by the potential to exploit ideas and tools developed in on domain to the other and vice versa. For example, various means of making corrections for dead-time losses exist for each approach. We shall not consider dead-time corrections here but note that having established the link the analyst is free to draw on the methods of both traditions to achieve best results.

In the following sections we shall explain neutron coincidence counting, the Feynman variance-to-mean method, the relation between the Feynman Y-parameter and the singles and doubles of coincidence counting, and how the Feynman Y statistic may be used to extract the leakage self-multiplication of an item. For completeness we summarize some key aspects of the point model used to interpret the data, and which underpins both time correlation analysis methods. Reconciling Feynman's seminal work and current coincidence counting view point, which is compactly described by Cifarelli and Hage [3], requires attention to several important aspects of how the point model is applied in the two cases. The first is in essence the treatment of a self sustaining critical assembly while the second is the treatment of a source driven highly sub-critical system. Another difference is the distinction which must be made between an internal detector and an external detector which introduces the difference between the efficiency per fission and the efficiency per neutron. A third important aspect is the difference between total (internal) multiplication and leakage (emerging neutron) multiplication in the various equations to be developed.

We shall consider how the correlated pulse train recorded from a single detector may be analyzed and then interpreted in terms of a simple physical model. We shall restrict ourselves to singles (totals or gross) and doubles (reals, coincidence or pairs) counting. Since we are considering a single pulse train, the analysis is a form of autocorrelation. There are many ways of performing autocorrelation, we shall consider just two shift register and Feynman-Y neutron correlation counting in the steady state. With multiple detectors other (cross correlation) possibilities not discussed here are possible. We shall consider how to extract the correlated information on the pulse train by these counting techniques, which are usually called time correlation analysis, TCA, methods (to distinguish them from methods that examine the data in the time domain using time interval analysis, TIA, techniques).

2. Background and the point model

Traditional passive neutron coincidence counting (PNCC) of so called reals or pairs is closely related to the Rossi-α method. Here $1/\alpha$ is the $1/e$ time constant describing the prompt neutron exponential growth or decay (depending in its sign) of the neutron population in a reactor [4–6]. Orndoff [7] explains the origins noting that in the early days of Los Alamos, prompt neutron behavior of near critical configurations was studied by injecting a short burst of neutrons from a cyclotron and observing the decay of the neutron population. He reports that Bruno Rossi first suggested that the mean lifetime of a neutron, which is extended by the number distribution of fission chains taking place in such sub-critical assemblies operated at sustained rates near critical (within a few % of prompt critical, say, so that control via delayed neutron emission was possible), can be obtained under suitable conditions by measuring the average time distribution of neutrons associated with a common ancestor. White [8] attributes Rossi's suggestion to the year 1944. This method exploits the fact that fission neutrons do not appear singly as would be the case for a random or Poisson source but in time correlated chains. Further, the multiplying assemblies are self-modulated in the sense that the mean time between the emission of neutrons is of the order of or longer than the prompt neutron period. These types of measurements are commonly referred to as Rossi-α measurement. The 1-Dimensional (1-D) Rossi-α distribution is an event triggered record of neutron detection. Every detected neutron begins a trace and the arrival time of subsequent events is binned in a histogram with fine resolution (bin width very much less than the characteristic $1/e$ time). We use the prefix 1-D to separate the approach from other time correlation methods where the time to the second event following the trigger, as an example, can also be plotted. The theory behind the Rossi-α experiments in terms of physical parameters of the assembly has its origins in the work of Feynman [9]. The Rossi-α method and its conceptual derivatives have a long history of application from the determination of prompt neutron time scales for various 'chain reactors' [10], to the determination of effective delayed neutron fractions [11]; while the time dependence (or what we might call the detection time probability density function following birth) that follows from the Rossi-α theory forms the basis of formulating a dead-time correction applicable to passive neutron coincidence (doubles) counting that makes use of shift-register time correlation counting logic [12]. In PNCC the item being measured is assumed to be in dynamic equilibrium, that is to say the neutron production rate is steady consistent with the underlying production mechanisms. There is no cyclotron pulse. However the internal neutron events themselves induce fission chains and so the item is self interrogating and the analogy can be drawn.

In traditional PNCC the 1-D Rossi-α curve is not created explicitly but is analyzed in hardware. Every detected event triggers the opening of two gates each of duration T_g. The first is the (R+A)-gate, opened after a short predelay, T_p, sufficient to allow the detector to recover to quiescent conditions. The second is the A-gate, opened after a long delay T_L, sufficient for any time correlations associated with the triggered event to have dissipated. Every event in the gate is scored as one pair, that is the trigger and an event in the gate form a pair and is scaled as a coincidence. The difference in the number of pairs accumulated in the (R+A)-gate and A-gate at the end of the assay is equal to the net number pairs corrected for accidental or chance coincidences subject to the finite gating structure imposed. In other words, in reals counting, we have simply formed the difference between the two regions of summation (areas) of the Rossi-α curve. The reals per trigger or more commonly the reals rate (D) directly is our measure of the strength of the time correlated signal. Because of the way the shift register logic just described works, with every event opening gates and the way in which pairs are defined and counted, the reals becomes a measure of the second reduced factorial moment of the pulse train as we shall see later.

Another way to analyze the pulse train for correlations is to form a simple count or number histogram. Suppose one periodically opens non-overlapping gates of width T_g and keeps a record of the number of times 0, 1, 2, etc. detected events are observed with the gate. The distribution of counts contains information about the degree of randomness and correlation on the pulse train. A simple index of dispersion is the variance-to-mean ratio (VMR). Since neutrons are detected in clusters due to the presence of fission events and fission chains in multiplying media, the VMR will be greater than the value of unity which would be the result for a purely random or Poissonian pulse train. When the VMR exceeds unity the dataset is said to be over-dispersed. In the present context $Y=(VMR-1)$ is referred to as the Feynman-Y statistic and is a measure of the excessive relative variance of the measurement results [13]. The application of the Feynman-Y statistic is described in a paper published in 1956 [1]. This paper refers to experimental work performed during 1944 on the first low power "water boiler" reactor, LOPO, at Los Alamos in which the second factorial moment of prompt neutron following fission was estimated. The water boiler was a spherical vessel, about a foot in diameter, containing a solution of uranium enriched in ^{235}U. The physical theory of how the Feynman-Y statistic is related to the properties of the multiplying assembly is elegantly summarized by de Hoffmann [14]. Interest in enumerating power-level and neutron-number fluctuations resulting from fission chains has been maintained in support of reactor safety and proposed accelerator driven sub-critical systems that could transmute long lived radioactive nuclear waste [15]. The theory underpinning the Feynman-Y formula consequently does not need to be repeated here in full. However, it is instructive to briefly discuss the theory, which from the point model perspective is the same as that used to develop the Rossi-α equation, in order to explain a couple of key differences in the way the safeguards nondestructive assay (NDA) community use multiplication and lifetime compared to the reactor noise analysis community.

Consider a one region isolated point-like sub-critical multiplying body in the one neutron-energy group (i.e., monoenergetic) approximation. Following Feynman and de Hoffman, we shall assume that there are no sources of passive neutrons present such as (α,n) or spontaneous fission. The population of neutrons at time t, $n(t)$, in a reactor such as the Los Alamos water boiler operating near critical following an instantaneous pulse (injection) of external neutrons changes according to the difference between the neutron gain and neutron loss mechanisms. Neutron gain is a consequence of induced fission while losses are a consequence of neutrons undergoing fission reactions, parasitic capture reactions and leaking from the system. For now we shall focus only on the prompt temporal behavior of the neutrons in that we shall assume neutron emission following fission is instantaneous with the fission reaction and that delayed neutron production can be ignored over the time scales of the prompt neutron mean life time. Let us call the overall probability of neutron loss per unit time $1/\tau_o$, where τ_o is the mean neutron lifetime of a neutron following its creation in the system. Similarly we introduce τ_f, the mean life time for fission so that $1/\tau_f$, is the probability per unit time that a neutron will undergo fission. In terms of material properties, the mean life time for fission is just the time it takes a neutron moving with speed s to travel a mean free path distance $1/\Sigma_f$ for fission, that is, we have $\tau_f=1/(s.\Sigma_f)$, where Σ_f is the macroscopic cross-section for fission. Similarly for capture losses we can write $\tau_c=1/(s.\Sigma_c)$. There is a dilemma in that these expressions are for infinite media when in fact we are dealing with a point-like system. However, for a point there is no spatial dimension. Fortunately for a reactor such as the water boiler close to critical the model works well because the neutron chains are long and the progeny flood the entire system giving it the appearance of an effective uniformity for the purposes of our initial discussion. The equation governing the rate of change of the neutron population may thus be written as:

$$\frac{dn(t)}{dt}=-\frac{n(t)}{\tau_o}+\frac{n(t)}{\tau_f}\times\nu_{I1}=\frac{k-1}{\tau_o}\times n(t)=\frac{1}{M_T\times\tau_0}\times n(t)=\frac{1}{\tau}\times n(t). \tag{1}$$

where ν_{I1} is the mean number of prompt neutrons emitted following induced fission and $k=(\tau_o/\tau_f).\nu_{I1}<1$ for a sub-critical system, is the familiar prompt neutron reproduction factor which is also commonly referred to as the multiplication factor, $(k-1)$ is the excess number of neutrons at the end of each generation, each generation taking on the average a time τ_o to run its course. $M_T=1/(k-1)$ is the total multiplication factor which is the limiting geometric sum of all generations $\lim_{r\to\infty}1+k+k^2+k^3\ldots k^r$. We have introduced the parameter $\tau=M_T.\tau_o$ as the effective $1/e$ (mean) time constant of the system which is extended over the value τ_o by the factor M_T. Orndoff and Johnstone [16] provide a salutary worked example showing that for compact assemblies of SNM (such as highly enriched uranium (HEU) or weapons grade plutonium (WGPu)), for example simple bare and reflected critical metallic assemblies of the kind used at Los Alamos National Laboratory to obtain basic information about fast-neutron behavior [17,18], the value of τ_o is quite small, of the order of 20 ns. It is only because M_T is very high, about 132 in the worked example at delayed critical, that $\tau=M_T\times\tau_o$ could be measured directly at all for such assemblies. In the realm of safeguards NDA M_T rarely exceeds a few (perhaps 4)—an example of being a fresh reactor fuel assembly. In this case even the product $M_T.\tau_o$ remains small (for dry systems) in comparison with other influences and we usually speak in terms of environmental and detector temporal characteristics (instead of time constants describing the behavior of neutrons inside the object), since they often utterly dominate.

However, continuing with our analysis, solving for $n(t)$ with the boundary condition $n(0)=1$ corresponding to a single neutron present initially gives the $p(t)$ describing the average neutron population due to an individual neutron present in the system a time t prior as:

$$p(t)=e^{-t/\tau}. \tag{2}$$

For a subcritical system we see that the neutron population decays away exponentially. Note that in the one group point model all neutrons are equivalent at all times, that is there is no dependence on future behavior from the past, and so, an important simplification is that $p(t)$ applies to all neutrons present in the system at any and all arbitrary times. The behavior of a fission burst which releases several neutrons into the system can then be constructed from this basic behavioral information [7].

In our rate of change equation (Eq. (1)) we have neglected delayed neutrons. Delayed neutrons are vital to reactor control [19] but as we will now show are unimportant for our present development. Following fission a small fractions ($\sim$1%) of the fission neutrons get emitted long ($>$ms) after the prompt ($<$ns) neutrons evaporated from the fission fragments as part of the almost instantaneous deexcitation processes. These delayed neutrons are delayed in time by the radioactive decay of the so called delayed neutron precursor nuclides in the fission debris. It is conventional to represent the delayed neutron production by a few (3 −1 2) groups with effective periods of about 0.2 s to ~80 s [20]. For systems very close to prompt critical the neutron population (power) doubling period can therefore be controlled, by exploiting the slow evolution of the delayed neutron component, and set to the orders of hours or longer. Commercial power nuclear reactors are essentially steady state devices although other feedback mechanisms such as temperature changes and changing isotopic compositions also need to be included into any meaningful model. But the point we wish to make is that close to criticality, once stimulated, a sub-critical system can be viewed, at least on the timescale of interest for reactor noise (or short term fluctuation) measurements, to be continually producing neutrons by induced fission which then disappear to fission, parasitic capture and leakage. That is why we, following de Hoffman, were able to begin our discussion assuming no spontaneous fission of (α,n) neutrons to be present since they are not needed at criticality to maintain the neutron number density. It is a convenient simplification in the case of the water boiler but potentially confusing when one wishes to apply the Feynman-Y analysis, for example, to highly sub-critical conditions. We shall deal with this explicitly later in our discussion.

In the realm of NDA of spontaneously fissile materials, such as Cf, Pu or Cm, we often find ourselves in a very different regime, namely the total multiplication factor k is very close to zero rather than within a few % of unity and the total multiplication factor M_T is consequently rather close to unity. In this case our rate of change expression (Eq. (1)) must be modified by four additional neutron production terms: spontaneous fission neutron production at a rate of $n_S \times \nu_{S1}$, (α,n) neutron production at a rate of $n_\alpha.1$, delayed neutron production from spontaneous fission and delayed neutron production from induced fission. Instead of the delayed neutron population decaying exponentially as predicted by $p(t)$, therefore, a state of dynamic equilibrium is attained with $n(t)$ being constant on the average. Once again it is the short term fluctuations in $n(t)$ with which we are concerned with and which form the basis of time correlation and time interval analyses.

To support the definition of total multiplication it is instructive to evaluate the total number of neutrons per initial neutron that are created in the system. To do this we must add to the initial neutron the extra neutrons that come from fission. The extra neutrons represent the chance that the initial neutron will undergo fission. Since dt/τ_f is the probability that a neutron will undergo fission in the incremental time interval dt, $p(t)$ the expectation of a neutron survival, and on average ν_{I1} prompt neutron will be born, the total number of neutrons per initial neutron is given by the following balance equation:

$$1+\tau \times \int_{t=0}^{\infty} \frac{e^{-t/\tau}}{\tau} \times \frac{dt}{\tau_f} \times \nu_{I1} = 1+\frac{\tau}{\tau_f} \times \nu_{I1} = 1+M_T \times \frac{\tau_o}{\tau_f} \times \nu_{I1} = 1+\frac{k}{1-k} = \frac{1}{1-k} = M_T. \tag{3}$$

Thus we arrive again at our earlier result, that a single neutron is boosted by a factor of M_T. The extra neutrons came from fission and so we immediately see that the average number of fissions per initial neutron is given by:

$$\text{Induced fissions per neutron} = \frac{(M_T-1)}{\nu_{I1}} = p_f \times M_T. \tag{4}$$

From the above operations it is also apparent that:

$$\frac{k}{1-k} = (M_T-1) = -\frac{1}{\rho}. \tag{5}$$

where $\rho=(k-1)/k$ is the so called reactivity.

Returning to our rate equation (Eq. (1)) we note that the mean probability per unit time that a neutron gets removed from the population is simply the sum of all competing channels, thus:

$$\frac{1}{\tau_o} = \frac{1}{\tau_f} + \frac{1}{\tau_c} + \frac{1}{\tau_L}. \tag{6}$$

Rearranging we obtain:

$$1 = \frac{\tau_o}{\tau_f} + \frac{\tau_o}{\tau_c} + \frac{\tau_o}{\tau_L} = p_f + p_c + p_L. \tag{7}$$

where p_f,p_c and p_L, are the probabilities for a neuron to undergo fission, parasitic capture and leakage, respectively. The calculation of p_f,$p_c$andp_L from the first principles for actual items requires knowledge of both the materials, that is the interaction cross-sections, but also the size and geometry of the item [21]. In actual items they will also be functions of position and energy distribution. In the point model they are simple, single valued, constants.

We can now look at multiplication another way; in terms of probability, and the point model simply becomes a way of writing down the fate of a neutron in terms of probabilities for this simple physical model. Note the probability approach in itself does not demand a pure exponential behavior—it is only concerned about outcomes considered over all time. In the NDA domain the source term kinematics are fast compared to the transport and detection of neutron in the external detector and so it is common to consider transport in the measurement item as taking place practically instantaneously and for the detector to dictate the observed time behavior. In terms of probabilities we note that we can rewrite k as $k = p_f \times \nu_{I1}$ so that:

$$M_T = \frac{1}{1-p_f \times \nu_{I1}} \tag{8}$$

and

$$M_T - 1 = \frac{p_f \times \nu_{I1}}{1-p_f \times \nu_{I1}}. \tag{9}$$

We may re-arrange Eq. (9) to obtain an important relationship for $p_f \times M_T$ that occurs in both the Feynman-Y and neutron coincidence counter interpretational expressions, since it relates to the signal amplification that takes place inside multiplying items. This is:

$$\frac{M_T-1}{\nu_{I1}} = \frac{p_f}{1-p_f \times \nu_{I1}} = p_f \times M_T. \tag{10}$$

So far we have been concerned with the population and temporal behavior of prompt fission neutrons inside the item, that is the behavior that could be tracked if an ideal non perturbing detector probe were placed inside the item, for example a small fission chamber detector placed inside a large reactor core. However, suppose we have an external detector which may be considered, by a combination of distance and shielding to be decoupled from the item under observation. In this case we do not respond directly to the internal neutron population but to the neutrons that leak from it. In the point model the probability of leakage is just p_L and so the concept of leakage multiplication (M_L) emerges naturally from our discussion, M_L being defined by:

$$M_L = M_T \times p_L = M_T \times (1-p_f-p_c) = \frac{1-p_f-p_c}{1-p_f \times \nu_{I1}}. \tag{11}$$

When a non perturbing internal detector is used to sample the neutron population the use of M_T is natural. However, when we use a detector external to the item we find it natural to work in terms of M_L exclusively since we do not wish to carry both M_T and M_L in expressions which are already underdetermined in terms of experimental observables. Although we cannot obtain a perfect expression for $p_f.M_T$ in terms of M_L, for most traditional practical applications of safeguards interest, for example bulk containers of dry PuO_2 or MOX fuel, a good approximation can be had by noting the following:

$$\frac{M_L-1}{\nu_{I1}-1}=\left[p_f-\frac{p_c/p_f}{\nu_{I1}-1}\right]\times M_T \approx p_f \times M_T. \quad (12)$$

where the approximate numerical equality holds provided $(p_c/p_f) \ll 1$ since $(\nu_{I1}-1)>1$.

2. Coincidence counting

In the PNCC, the observed singles counting rate and doubles counting rate obtained from signal-triggered coincidence gates are corrected for background (and if needed, dead-time losses) to yield estimates for the true singles rate (S) and the true rate of correlated doubles (D) coming from the item [2]. The number of singles and doubles counts can be obtained from the pulse train directly in shift register module hardware using up/down counters to make the necessary computations. However, for our discussion we shall assume that a multiplicity shift register (MSR) is employed so that full event histograms are recorded. Two histograms are accumulated during the data acquisition period, corresponding to the number of times each of the triggered measurement gates contained 0, 1, 2, etc. events. Reduced factorial moments are calculated from these histograms. Provided certain simplifying assumptions hold, collectively referred to as the point model assumptions, closed-form expressions for S and D can be obtained, in terms of the physical properties of the item and the detector. The expressions may be inverted to obtain estimates of the main physical parameters of interest. Usually in PNCC, the primary goal is to determine the effective ^{240}Pu mass (m_{240}).

To recap, in shift register analysis every neutron detection triggers a pair of coincidence gates each of duration T_g, commensurate with the effective average lifetime (die-away time), τ, of thermal neutrons in the moderated ^{3}He-based detector system. The first gate is called the "reals plus accidentals" gate, or (R+A)-gate, and is opened following a short pre-delay, T_p, typically much larger than the effective system dead-time per event but much shorter than T_g, following the signal-trigger. The gate duration is selected to ensure that there is a high probability that any detected neutrons that are correlated in time with the trigger neutron fall within the first gate. The second gate is called the "accidentals" gate, or A-gate, and is opened a long time ($T_L \gg \tau$) after the trigger so that any correlation between the pulses and the counts falling in the A-gate has dissipated.

For each signal-triggered (R+A)-gate containing i neutrons, the histogram bin, $N(i)$, is incremented by one. In the following this will be called the Signal Triggered Inspection (*STI*) multiplicity histogram. Each neutron detection also leads to the opening of the A-gate and the histogram bin, $B(i)$, corresponding to the occupancy number, i, is incremented by one. The delay between the (R+A)-gate and the A-gate (T_L) is long enough to ensure that no genuine correlations with the triggering event are present. The A-gate can therefore be considered to be a random sample of the pulse train and the corresponding histogram will be referred to as the Random Triggered Inspection (*RTI*) multiplicity histogram.

In traditional PNCC with shift register logic, the combined results from the (R+A)-gate and the A-gate are used to derive expressions for the singles and doubles. The *RTI* histogram can be thought of as the complete background for the *STI* for doubles counting although it still contains correlated information since it may contain neutrons originating from the same neutron burst in the item. For convenience we shall assume here that the number of (R+A)- and A-gates are equal as would be the case for conventional shift register acquisition [2], where every detected neutron opens both an (R+A)-gate and an A-gate. However, the following expressions can be easily modified by applying appropriate normalization to the histogram for the case where the numbers of gates openings differ as would be the case when the A-histogram is accumulated by using a high rate periodic trigger to sample the pulse train with higher statistical efficiency. In general high rate periodic triggering to form the A-histogram is recommended because improved precision is obtained.

In the PNCC the singles rate is defined as the mean neutron detection rate. The doubles rate is defined as the rate of correlated pairs of neutrons that are observed inside the inspection gates triggered by neutron detections. The higher-order correlations (triples (T), quadruples (Q)) are defined similarly although we shall not consider them here. The expressions for the correlated rates in terms of the measured histogram data as well as the item and detector parameters are given by Cifarelli and Hage [3] and the basic expressions will be outlined below.

We will introduce terms $m_b(i)$ and $m_n(i)$ to represent the ith reduced factorial moments formed over the normalized B- and N-distributions, respectively.

$$m_n(i)=\frac{1}{N_g}\sum_{x=i}^{x_{max}}\binom{x}{i}N(x) \quad (13)$$

$$m_b(i)=\frac{1}{N_g}\sum_{x=i}^{x_{max}}\binom{x}{i}B(x) \quad (14)$$

where $\binom{x}{i}=\frac{x!}{(x-i)!i!}$ is the binomial coefficient and ! denotes the factorial operation.

For shift register analysis, the number of measurement gates, N_g, is equal to the total number of neutrons detected (N_T) so that $N_g{=}N_T{=}S\times T_M$, where T_M is the duration of the data acquisition, based on the normalization condition, $m_b(0){=}m_n(0){=}1$. The upper limit of the summations corresponds to the number of the highest histogram populated.

The correlated multiplicities are written in terms of the reduced factorial moments of the net and background distributions according to Cifarelli and Hage [3] as:

$$S=S\times m_n(0) \quad (15)$$

$$D=S\times(m_n(1)-m_b(1)) \quad (16)$$

We shall limit our discussion to the singles and doubles rates since these are needed for expressing the Y-statistic from Feynman analysis. Expressions for triples, quadruples (Quads) and pentuples (Pents) may be found elsewhere [22].

In the terminology of Cifarelli and Hage [3], the observed multiplets rates (S, D, etc.) are functions of the moments (R_m) of the true correlated neutron distribution and the gate utilization factors (f_2, f_3, etc.) such that $S{=}R_0/T_M$, and $D{=}f_2R_1/T_M$, where T_M is the measurement time. The gate utilization factors correct for the fact that not all the correlations on the pulse train are counted because of the finite coincidence gating structure (T_p and T_g). Truly correlated neutrons are those that are emitted from a single fission chain. The moments R_m are functions of the properties of the item and also detector parameters such as efficiency, but not of the coincidence gate structure. If the detector can be considered to be neutronically isolated from the item and to dominate the temporal behavior, then the gate factors may be treated as constants, characteristic of the system.

In the case of a single pure exponential neutron capture time distribution with average lifetime τ, the signal triggered gate utilization factor f_2 for doubles counting is [23,24]:

$$f_2 \equiv f_d = e^{-T_p/\tau}(1-e^{-T_g/\tau}) \tag{17}$$

The fully (background and dead-time) corrected experimentally observed S, and D rates are related to the properties of the items and the neutron detector through algebraic relationships derived from the 1-energy group, prompt induced fission, point model [25,26]. The assumptions of this model are those we reviewed earlier.

$$S = \frac{R_0}{T_M} = F_S \varepsilon \left(\frac{\overline{\nu}_S}{1}\right)(1+\alpha)M_L \tag{18}$$

$$D = f_2 \frac{R_1}{T_M} = F_S \varepsilon^2 \left(\frac{\overline{\nu}_{S2}}{2}\right) f_d {M_L}^2 \left[1+(M_L-1)(1+\alpha)\left[\frac{\overline{\nu}_{S1}}{(\overline{\nu}_I-1)}\right] \times \left[\frac{\overline{\nu}_{I2}}{\overline{\nu}_{S2}}\right]\right] \tag{19}$$

The spontaneous fission rate $F_s = m_{240} \cdot g$ is a function of the effective ^{240}Pu mass m_{240} and the specific spontaneous fission rate (g) of ^{240}Pu; M_L represents the system leakage multiplication; ε the efficiency and α the ratio of random neutrons, mostly from (α,n) reactions, to the number of neutrons emitted from spontaneous fission. The values ν_{Si} are the ith factorial moments of the spontaneous fission prompt neutron emission distribution, $P_S(\nu)$:

$$\overline{\nu}_{Si} = \sum_{\nu=i}^{\nu_{max}} \frac{\nu!}{(\nu-i)!} P_S(\nu) \tag{20}$$

where ν_{max} is the highest value of ν observed from a spontaneous fission. The induced fission emission moments (ν_{Ii}) are defined similarly. For brevity we will use the notation $\overline{\nu}_S = \overline{\nu}_{S1}$ and $\overline{\nu}_I = \overline{\nu}_{I1}$. The leakage multiplication, M_L, is the average number of neutrons that escape from the item (including induced fission neutrons) per each starter neutron in the item (see Eq. (11)). We have seen how from simple neutron balance considerations how the total and leakage multiplication factors are related:

$$M_L = \frac{1-p_F-p_c}{1-k} = p_L M_T \tag{21}$$

where $p_L = 1-p_F-p_c$.

The values of the S and D can be understood conceptually as follows. The singles value can be written as:

$$S = \varepsilon(F_S\overline{\nu}_S + S_\alpha)M_L \tag{22}$$

which is clearly interpreted as the total number of starter neutrons times the leakage multiplication. The doubles value can be written as:

$$D = \varepsilon^2 f_2 {M_L}^2 [S_f\overline{\nu}_{S2} + S_f\overline{\nu}_S p_F \overline{\nu}_{I2} M_T + S_\alpha p_{F\alpha}\overline{\nu}_{I2} M_T] \tag{23}$$

where the first term represents the contribution from doublets from spontaneous fission, the second term represents doublets from fission chains induced by spontaneous fission and the third term represents doublets from fission chains induced by (α,n) neutrons.

It was shown in [27] that the correlated multiplets (S, D, etc.) can also be obtained from the observed multiplets of the (R+A)-gate or the A-gate alone. Additionally, if the pulse train is acquired using list mode data acquisition, where each individual pulse arrival time is recorded and stored, an off-line analysis in software can be accomplished using any choice of gating structure.

3. Feynman variance-to-mean

The Feynman measurement method involves superposing on the detected neutron pulse train a series of non-overlapping clock-triggered gates of duration T_g and forming the histogram, $C(i)$, of the number of times the number of neutron detections in the gate is equal to $i = 1,2,3$, etc. The number of gates opened, N_g is given by:

$$N_g = \sum_{x=1}^{x_{max}} C(i) \tag{24}$$

and the normalized distribution is given by $C(i)/N_g$. The Feynman variance-to-mean method makes use of the parameter Y defined as excess of the ratio of the variance in the number of neutron counts to the average number of neutron counts compared to unity. To recap, $Y = VMR-1$ and the variance-to-mean ratio, VMR, would take on the value of unity for a pure Poisson (i.e., random) neutron source [1]. $VMR-1$ is therefore described as an 'excess' as it is positive for a correlated neutron pulse train. Note that the variance over the C-distribution, $\sigma_i^2 = \langle (i-\langle i \rangle)^2 \rangle = \langle i^2 \rangle - \langle i \rangle^2$ where we have introduced the bracket notation s to denote the mean (expectation) value of the quantity enclosed. Therefore we can write:

$$Y = \frac{\sigma_i^2}{\langle i \rangle} - 1 = \frac{\langle i^2 \rangle - \langle i \rangle^2}{\langle i \rangle} - 1 = \frac{1}{\langle i \rangle} \times [\langle i \times (i-1) \rangle - \langle i \rangle^2] \tag{25}$$

Note that $\langle i \rangle = m_c(1)$ and $\langle i(i-1) \rangle = 2 \cdot m_c(2)$ where the moments of the C-distribution have been defined analogously to $m_n(i)$ and $m_b(i)$. Thus, the Y-statistic can be written in terms of the reduced second factorial moments introduced earlier as:

$$Y = \frac{1}{m_c(1)} \times \left[2m_c(2) - (m_c(1))^2\right] = 2\frac{m_c(2)}{m_c(1)} - m_c(1) \tag{26}$$

In reactivity measurements, it is also common to use the one-group point reactor model to write the moments of the C-distribution in terms of physical quantities. These are given by:

$$m_c(1) = \in S_T T_g \tag{27}$$

and

$$m_c(2) = \frac{1}{2} \in^2 S_T^2 {T_g}^2 + \frac{1}{2} \in^2 T_g \nu_2 \frac{1}{(1-k)^2}[1 - \frac{1-e^{-\alpha_R T_g}}{\alpha_R T_g}] \tag{28}$$

where $S_T = S_\alpha + \overline{\nu}_S F_S + \overline{\nu}_I F_I$ is the total source of neutron singlets in the item, $\nu_2 = \overline{\nu}_{s2} F_S + \overline{\nu}_{I2} F_I$ is the total source of correlated neutron doublets in the item, and where F_I is the induced fission source. Eqs. (27) and (28) are a consequence of the Rossi-α probability distribution [1,5–7,9,14].

Recall that the Feynman analysis was developed for criticality studies which utilize different detector systems than is typical of NDA by PNCC (for example the assay of sealed plutonium waste drums). For criticality studies the item and the detector are highly coupled, the detector is treated as non-perturbing probe inside the item which samples the neutron field, and as a result the efficiency value utilized (ε) in Feynman analysis includes, in a detection sense, the probability of leakage from the item, p_L. In PNCC, these values are treated separately so in recasting the expressions into a common notation we need to make the transformation $\varepsilon\in = p_L\varepsilon$. In addition, the Feynman approach uses the Rossi-α parameter ($\alpha_R = \lambda(1-k)$; $1/\alpha_R = \tau \cdot M_T$) to describe the temporal distribution of correlated neutrons inside the item (e.g., only just sub-critical reactor core):

$$\mathcal{R}(t) = \alpha_R e^{-\alpha_R t} \tag{29}$$

where $\mathcal{R}(t)$ is the response function of the item plus detector (assumed to follow the neutron flux perfectly). The factor of $1-k$ takes into account the decreasing number of neutrons in each subsequent generation of a subcritical assembly. In the context of criticality measurements, $\tau = 1/\lambda$ is the mean lifetime of a neutron in the item plus detector assembly. In PNCC, the mean lifetime in the detector head is normally much much longer than the lifetime in the item. We see how the lifetime gets extended in a multiplying medium for many practical scenarios this effect is washed out by temporal response of the detector and so is unimportant.

For highly multiplying and reflective systems this effect can not only be observed, it is also informative. But the traditional perspective in PNCC is that the α-value that matters is that of the detector. In addition, PNCC uses the superfission approximation which assumes that all neutrons are emitted from a given fission chain in the same instant. These neutrons hit the detector head together where they are thermalized and begin to get detected. The capture time distribution in the detector describes the time distribution of the sampled neutrons (predominantly thermal neutrons) in the moderated detector head. In the context of PNCC assumptions one would write, $\alpha_R \approx \lambda$ where $\tau = 1/\lambda$ is the average lifetime of a neutron in the detector head alone, since what happens in the item has little impact on the overall temporal behavior. It is like the time behavior within the item is a delta-function impulse that stimulated the detector response function. Therefore, with the appropriate modifications to make it applicable to typical safeguards coincidence counting which has a neutronically isolated detector outside of the item under study, the equation for the second order reduced factorial moment, Eq. (28), of the C-distribution becomes:

$$m_c(2) = \frac{1}{2}\varepsilon^2 p_L^2 S_T^2 T_g{}^2 + \frac{1}{2}\varepsilon^2 p_L^2 T_g \nu_2 M_T^2 w_2 \tag{30}$$

where the doubles gate utilization factor for a random gate, w_2, has been inserted. For an exponential die-away profile, w_2 becomes [23]:

$$w_2 = 1 - \frac{1 - e^{-T_g/\tau}}{T_g/\tau} \tag{31}$$

It is possible to understand the moments of the C-distribution conceptually as follows. First, we can utilize the induced fission source, F_I, given by:

$$F_I = \overline{\nu}_S F_S p_F M_T + S_\alpha p_F M_T = \overline{\nu}_S F_S p_F M_T(\alpha+1) \tag{32}$$

to find the following useful relations:

$$S_T = S_\alpha + \overline{\nu}_S F_S + \overline{\nu}_I F_I = \overline{\nu}_S F_S M_T(\alpha+1) \tag{33}$$

$$\nu_2 = \overline{\nu}_{s2} F_S + \overline{\nu}_{I2} F_I = \overline{\nu}_{s2} F_S + \overline{\nu}_S F_S p_F \overline{\nu}_{I2} M_T + S_\alpha p_F \overline{\nu}_{I2} M_T \tag{34}$$

The first moment can then be written as:

$$m_c(1) = \varepsilon M_L T_g(\overline{\nu}_S F_S + S_\alpha) \tag{35}$$

which is to be interpreted as the normalized number of detected neutrons adjusted for the total amount of time covered by the measurement gates.

The second moment can be written and interpreted as follows:

$$\begin{aligned} m_c(2) = \varepsilon^2 M_L{}^2 \frac{1}{2}\Big[& \ldots w_2 T_g F_S \nu_{S2} && \text{Correlated doublets from spontaneous fission neutrons} \\ & + w_2 T_g F_S \nu_S \nu_{I2} p_F M_T && \text{Correlated doublets from fission chain induced by spontaneous fission} \\ & + (T_g \nu_S F_S)^2 && \text{Pairs of singlets from spontaneous fission} \\ & + w_2 T_g S_\alpha p_F \nu_{I2} M_T && \text{Correlated doublets from fission chain induced by } (\alpha,\text{n}) \text{ neutrons} \\ & + (T_g S_\alpha)^2 && \text{Pairs of } (\alpha,\text{n}) \text{ neutron singlets} \\ & + 2T_g{}^2 F_S \nu_S S_\alpha \Big] && \text{Pairs with one } (\alpha,\text{n}) \text{ singlet and one spontaneous fission singlet} \end{aligned} \tag{36}$$

Substituting the reduced moments into the equation for the Y-statistic one obtains:

$$Y = \varepsilon w_2 \frac{\nu_2}{S_T} M_L M_T \tag{37}$$

utilizing the substitutions in Eqs. (33), (34), Y can be written in terms of fundamental physical properties as follows:

$$Y = \varepsilon w_2 M_L \frac{\overline{\nu}_{S2} F_S + \overline{\nu}_S F_S p_F \overline{\nu}_{I2} M_T(\alpha+1)}{\overline{\nu}_S F_S(\alpha+1)} \tag{38}$$

4. Relationship between *Y*-value, singles and doubles

In the present work we are concerned with the formal relationship between the results obtained with traditional shift register analysis for PNCC and the parallel Feynman approach which introduced correlation counting as a NDA technique. First, it is important to understand that the *RTI* is (statistically) equivalent, apart from for the number of gate openings and the embedded covariance, to the Feynman histogram that results from a periodic clock-triggered gate of equal duration ($C(i) = B(i)$). This is true because the neutron bursts occur randomly in time and either a random gate or a clock-triggered gate sample the pulse train at random times with respect to the neutron detections from each neutron burst. As discussed in [27], the distributions measure the same thing regardless of the use of overlapping or non-overlapping gates. Overlapping gates oversample the pulse trains, which introduces correlations (covariances) in the uncertainty values but should not bias the expectation values of the results. These can be quantified by splitting the measurement time into a sequence of shorter cycles which can be subjected to standard statistical analysis. We have already noted that acquiring the A-histogram using a high frequency clock (10 times, say, greater than the highest anticipated event rate) is a means to improve overall precision. It is far preferable to using contiguous gates which was the original implementation of the Feynman approach.

In this work, to demonstrate the relationship between Y, S and D, we assume that the number of clock-triggered gates is equal to the total number of neutrons, $N_g = N_T$. In that case, we can state that the normalized moments of the clock-triggered and background distributions are also equal.

$$m_c(i) = m_b(i) \tag{39}$$

We may rewrite our expression for Y in terms of the reduced factorial moments of the normalized $B(i)$-histogram as follows:

$$Y = \frac{1}{m_b(1)}[2m_b(2) - m_b(1)^2] \tag{40}$$

It is now desirable to write the background correlated moments ($m_b(i)$) in terms of the singles and doubles. It was shown in [27] that using the same approach as in traditional PNCC, the correlated rates can be expressed in terms only of the *RTI* histogram as:

$$S = \frac{m_b(1)}{T_g} \tag{41}$$

$$D = \frac{1}{x_2 T_g}\left[m_b(2) - \frac{1}{2}m_b(1)^2\right] \tag{42}$$

In these expressions the parameter $x_2 = w_2/f_2$ is the ratio between the *RTI* and *STI* gate utilization factors. Eqs. (41), (42) can be inverted to write the background moments in terms of S and D as follows:

$$m_b(1) = S \times T_g \tag{43}$$

$$m_b(2) = x_2 D T_g + \frac{1}{2} S^2 T_g{}^2 \tag{44}$$

It is possible to verify through direct substitution of the S and D values into Eqs. (43), (44) that the moments of the background distribution calculated here are identical to the moments of the C-distribution developed for the Feynman approach.

The Y-factor can now be determined in terms of the familiar shift register S and D rates.

$$Y = \frac{1}{S \times T_g} \times \left[2\left[x_2 D T_g + \frac{1}{2} S^2 {T_g}^2\right] - (S \times T_g)^2\right] = 2x_2 \frac{D}{S} \tag{45}$$

We immediately appreciate that the first moment of the normalized B-histogram and the Feynman Y parameter offer alternative ways of obtaining the singles and doubles rates. To proceed we have:

$$D = \frac{1}{2} \frac{f_2}{w_2} SY \tag{46}$$

It is now possible to write the Y-factor in terms of the true correlation multiplicity moments from Cifarelli and Hage [3]:

$$Y = 2w_2 \frac{R_1}{R_0} \tag{47}$$

Finally, substituting the expressions for singles and doubles in terms of the physical properties of the item from PNCC theory one obtains:

$$Y = \varepsilon w_2 M_L \frac{\overline{\upsilon}_{S2}\left[1 + (M_L - 1)(1+\alpha)\frac{\overline{\upsilon}_{I2}}{\overline{\upsilon}_I - 1}\frac{\overline{\upsilon}_s}{\overline{\upsilon}_{s2}}\right]}{\overline{\upsilon}_S(1+\alpha)} \tag{48}$$

Recognizing that $(M_L - 1)/(\overline{\upsilon}_I - 1) = p_F M_T$ (from our earlier discussion), we obtain:

$$Y = \varepsilon w_2 M_L \frac{\overline{\upsilon}_{s2}\left[1 + p_F M_T(1+\alpha)\overline{\upsilon}_{I2}(\overline{\upsilon}_s/\overline{\upsilon}_{s2})\right]}{\overline{\upsilon}_s(\alpha+1)} \tag{49}$$

Distributing $\overline{\upsilon}_{s2}$ and multiplying the numerator and denominator by F_S yields:

$$Y = \varepsilon w_2 M_L \frac{\overline{\upsilon}_{s2} F_S + \overline{\upsilon}_s F_S p_F \overline{\upsilon}_{I2} M_T(\alpha+1)}{\overline{\upsilon}_s F_S(\alpha+1)} \tag{50}$$

which is equivalent to the value of Y derived directly in the Feynman approach when one takes care of the differences in the definition of detector efficiency and neutron lifetime between the original Feynman and traditional PNCC treatments, as we have taken pains to explain.

5. Using the *Y*-factor to determine multiplication

We now derive the leakage multiplication in terms of the Feynman-Y statistic. The Y-statistic can be written as:

$$Y = \varepsilon w_2 M_L \left[\frac{\overline{\upsilon}_{s2}}{\overline{\upsilon}_s(\alpha+1)} + \frac{\overline{\upsilon}_{I2}}{\overline{\upsilon}_I - 1}(M_L - 1)\right] \tag{51}$$

Rearranging terms, an equation that is parabolic in the leakage multiplicity, M_L, is obtained:

$$\frac{Y}{\varepsilon w_2} \frac{\overline{\upsilon}_I - 1}{\overline{\upsilon}_{I2}} = {M_L}^2 + \left[\frac{\overline{\upsilon}_{s2}(\overline{\upsilon}_I - 1)}{\overline{\upsilon}_s \overline{\upsilon}_{I2}(\alpha+1)} - 1\right] M_L \tag{52}$$

Using the quadratic formula yields the solution:

$$M_L = \frac{1}{2}\left[\left(1 - \frac{\overline{\upsilon}_{s2}(\overline{\upsilon}_I - 1)}{\overline{\upsilon}_s \overline{\upsilon}_{I2}(\alpha+1)}\right) \pm \sqrt{\left(1 - \frac{\overline{\upsilon}_{s2}(\overline{\upsilon}_I - 1)}{\overline{\upsilon}_s \overline{\upsilon}_{I2}(\alpha+1)}\right)^2 + 4\frac{(\overline{\upsilon}_I - 1)}{\varepsilon w_2 \overline{\upsilon}_{I2}} Y}\right] \tag{53}$$

6. Discussion

We have formally recognized and exploited the fact that the accidentals histogram acquired during multiplicity shift register analysis of a neutron pulse train records the correlations from fission in an equivalent way to the Feynman $Y = VMR - 1$ sampling method. We have shown the formal linkage and cast the results in a way which will be familiar to workers familiar only with MSR analysis. The development requires careful understanding and use of terms, which although given similar names in the two correlated neutron counting traditions have different physical meanings; efficiency for example. The demonstrated similarity of the two approaches at a fundamental level is not surprising since both are rooted in the same point-like reactor model assumptions and exploit time correlation (coincidence gate) analysis. Because of this formal linkage it is important to appreciate that developments or advances made in one field, such as how to apply corrections for dead-time losses, can be carried over directly to benefit the other. Other potential areas of interest include how calibrations are performed, what quantitative algorithms work best, how data is corrected for measurement perturbations such as matrix effects inside the item, and what form improved data acquisitions systems may take. These topics we believe will yield fruitful cross fertilization in the future.

The first moment of the A- (or *RTI*-) histogram yields information on the mean event rate, i.e., singles. The second moment (which enters also into both, the calculation of the variance of the distribution (which is a measure of the dispersion or width of the distribution) and of the second reduced factorial moment of the distribution) provides information on the correlated pairs present on the pulse train. When an MSR module is used to acquire time correlated analysis data, as opposed to a list mode data acquisition system, it is common for a single gate width to be used to extract correlated rates. The same gate width may not be ideal for use with the *RTI* only analysis [28]. Using list mode data post processing in software lifts this limitation, since the same pulse train may be analyzed using different gate widths T_g without significant penalty (the cost of the computer run time). Thus the same observables can be extracted using either standard MSR approach or *RTI* histograms. Initial indications are that with coincidence gate widths tailored to the particular approach, when using fast accidental sampling, comparable precision can be achieved working with either singles and doubles or singles and Feynman-Y [27,28]. The ideal instrument would allow Feynman-Y as a function of gate width to be determined so that the extrapolation to infinite gate can be made effectively allowing item specific gate utilization factors to be obtained.

In the PNCC context standard methods exist for the inversion of the physics expressions for S, D (and triples (T) in case of multiplicity counting) in order to extract quantities of interest such as such as spontaneous fission mass, leakage self-multiplication, the (α,n)-to-(SF,n) ratio, and detection efficiency under various supporting assumptions [2,29]. It is this framework that lends analytical power to the approach. Using a Monte Carlo simulation in the inversion step is generally not done, even when details about the item such as geometry are known independently fairly well, because of the long computational times required. We note that recently Mattingly and Varley [30] have implemented a technique to synthesize the Feynman-Y statistic as a function of gate width using deterministic neutron transport methods. This allows calculations to be made essentially instantaneously breaking the necessity to represent the item in terms of the point reactor model. This could prove an important step.

7. Conclusions

Neutron multiplicity counting using shift register logic is an established technique in widespread use, particularly by the safeguards and waste management communities, for the assay of spontaneously fissile materials. In the field of criticality experiment analysis the Feynman-Y statistic has long been used. In this paper we have built a bridge between these two communities by showing the underlying similarity of the two approaches, and in particular, how the same multiplet information can be extracted and expressed in a self-consistent way. We have concentrated on establishing in a clear manner using common terms the theoretical basis for the two approaches so that practitioners can fully understand and appreciate that the shift register coincidence counting and Feynman-Y methods extract the same fundamental information from the pulse train.

As the use of list mode data acquisition becomes more common place in both of these application areas we can anticipate that a given experimental record will increasingly be subjected to a wide variety of both time interval analysis (not discussed in this paper) and time correlation analysis techniques and not just the two presented implemented in isolation as is typically done today, and the power of using diverse approaches will be especially valuable for detector characterization and special investigations. This will further encourage an ongoing and fruitful exchange of ideas and tools between our respective disciplines, hopefully also in ways additional to those we have previously touched upon. At high rates however list mode files quickly become impractically big, and off-line data processing can be more time consuming that the time taken to acquire the raw data itself. Thus, there will remain, at least in the foreseeable future, a clear need to obtain data in a form that can deliver compact data files together with immediate quantitative results without overly sacrificing information content. Multi-input time synchronized multiplicity shift registers operating with various predelays and various coincidence gates, and implementing fast accidentals sampling, are one potentially attractive approach, and these can provide item specific temporal information over and above the Feynman-Y and higher order multiplet analysis options from an accidentals histogram with a fixed gate. When combined with the potential for essentially instantaneous inversion based on deterministic transport models, which overcome the restrictions of treating the item like a point, new opportunities for improved nondestructive assay are openned up. The traditional challenges of achieving adequate statistical precision and making dead time corrections accurately remain and demand renewed attention to detector design as well as algorithm development and numerical simulation. We and other groups are working in these areas too.

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