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NEUTRON MULTIPLICATION MEASUREMENTS USING MOMENTS OF THE NEUTRON COUNTING DISTRIBUTION

A.A. ROBBA, E.J. DOWDY and H.F. ATWATER

Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

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We demonstrate an improved technique for determining the multiplication of highly subcritical systems that uses the moments of the counting distribution from a neutron detector. In the investigation, the multiplications of each of 2, 3, and 4 kg parts of a plutonium sphere were measured and compared with values calculated using three-dimensional Monte Carlo techniques. The measured and calculated neutron multiplications agree within the statistical uncertainties of the measurements and calculations.

1. Introduction

We have developed an improved technique for determining the multiplication of highly subscritical systems. The technique analyzes the moments of the counting distribution from a neutron detector and includes an improved treatment of the deadtime losses of the neutron counting system. To demonstrate the technique, we measured three plutonium metal parts from the Los Alamos Thor * [1] assembly in a large, polyethylene-moderated well counter. The signals from the ^{3}He tubes in the well counter were handled by standard proportional counter electronics: high-voltage supply, preamplifier, amplifier, and discriminator. Detection events were counted in a scaler/timer interfaced to a Data General microNOVA computer that acquired the count distribution, calculated the moments, and performed the analysis necessary to determine the neutron multiplication.

2. Theory

We consider a system in which neutrons are produced by (α, n) reactions, spontaneous fission and neutron induced fission [2]. We use the following forms for $P(t)$ and $P_f(t)$; the probability that a neutron born at time 0 produces a pulse in dt about t and the probability that a neutron born at time 0 produces a fission in dt about t, respectively

$$P(t)\mathrm{d}t = \epsilon\beta\,\mathrm{e}^{-\beta t}\mathrm{d}t, \tag{1}$$

$$P_f(t)\mathrm{d}t = \frac{k_p\beta}{\bar{\nu}_1}\mathrm{e}^{-\alpha t}\mathrm{d}t,$$

where:

ϵ = detector efficiency (assumed independent of neutron origin),
β = inverse neutron lifetime,
α = Rossi-alpha = $(1 - k_p)\beta$,

* Thor is a fast-neutron critical assembly operated at the Los Alamos National Laboratory Critical Experiments Facilities. Thor consists of a spherical delta-phase plutonium metal core centered in an equilateral cylinder of metallic thorium. In the present work, only the plutonium core pieces were used.

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k_p = prompt neutron multiplication factor,
$\bar{\nu}_1$ = mean number of neutrons per induced fission.
Let $P_n(t_1, t_2, \cdots, t_n)\mathrm{d}t_1\, \mathrm{d}t_2 \cdots \mathrm{d}t_n$ be the probability of detecting n pulses occurring in $\mathrm{d}t_1$ about t_1, $\mathrm{d}t_2$ about t_2, $\cdots$ and $\mathrm{d}t_n$ about t_n, then

$$P_1(t_1) = \epsilon S, \tag{2}$$

$$P_2(t_1, t_2) = \epsilon^2 S^2 + \epsilon^2 \overline{\nu(\nu-1)}F \frac{\beta\, \mathrm{e}^{-\alpha(t_2-t_1)}}{2(1-k_p)},$$

$$P_3(t_1, t_2, t_3) = \epsilon^3 S^3 + \frac{\epsilon^3 S\overline{\nu(\nu-1)}F\alpha}{2(1-k_p)^2}\left[\mathrm{e}^{-\alpha(t_3-t_1)} + \mathrm{e}^{-\alpha(t_3-t_2)} + \mathrm{e}^{-\alpha(t_2-t_1)}\right]$$

$$+ \frac{\epsilon^3 \overline{\nu(\nu-1)(\nu-2)}F\alpha}{3(1-k_p)^3}\mathrm{e}^{-\alpha(t_3+t_2-2t_1)} + \frac{\epsilon^3 \overline{\nu(\nu-1)}Fk_p\overline{\nu_1(\nu_1-1)}\alpha^2}{2\bar{\nu}_1(1-k_p)^4}\mathrm{e}^{-\alpha(t_3-t_1)}$$

where:
S = total source strength = $(R + \bar{\nu}_0 F_0 + \bar{\nu}_1 F_1)$,
R = uncorrelated (α, n) source strength,
$\bar{\nu}_0 F_0$ = spontaneous fission neutron source strength,
$\bar{\nu}_1 F_1$ = neutron induced fission neutron source strength,
ν_0 = mean number of neutrons per spontaneous fission,
$\bar{\nu}_1$ = mean number of neutrons per induced fission,
F_0 = spontaneous fission rate,
F_1 = induced fission rate;
and we have used the shorthand

$$\overline{\nu(\nu-1)}F = \overline{\nu_0(\nu_0-1)}F_0 + \overline{\nu_1(\nu_1-1)}F_1,$$

$$\overline{\nu(\nu-1)(\nu-2)}F = \overline{\nu_0(\nu_0-1)(\nu_0-2)}F_0 + \overline{\nu_1(\nu_1-1)(\nu_1-2)}F_1.$$

If the detection system deadtime after a pulse is τ, then the expected number of counts in a time channel of width T_0 is

$$\bar{C} = \int_0^{T_0}\mathrm{d}t_1 P_1(t_1)\exp\left[-\int_{t_1-\tau}^{t_1} W_1(t_1, t_2)\mathrm{d}t_2\right], \tag{3}$$

where $W_1(t_1, t_2)\mathrm{d}t_2$ is the conditional probability that given a pulse at t_1, there is also a pulse in $\mathrm{d}t_2$ about t_2, and is given by the relation

$$P_2(t_1, t_2) = P_1(t_1)W_1(t_1, t_2).$$

For small deadtime, we make the approximation

$$\exp\left[-\int_{t_1-\tau}^{t_1} W_1(t_1, t_2)\mathrm{d}t_2\right] \cong 1 - \int_{t_1-\tau}^{t_1} W_1(t_1, t_2)\mathrm{d}t_2,$$

and obtain:

$$\bar{C} = \int_0^{T_0-\tau}\mathrm{d}t_1\left[P_1(t_1) - \int_{t_1-\tau}^{t_1}\mathrm{d}t_2 P_2(t_1, t_2)\right]. \tag{4}$$

Similarly:

$$\frac{\overline{C(C-1)}}{2} = \int_0^{T_0-\tau}\mathrm{d}t_1\int_{t_1+\tau}^{T_0}\mathrm{d}t_2 P_2(t_1, t_2)\exp\left\{-\left[\int_{t_1-\tau}^{t_1}\mathrm{d}t_3 W_2 + \int_{t_2-\tau}^{t_2}\mathrm{d}t_3 W_2\right]\right\}. \tag{5}$$

where $W_2(t_1, t_2, t_3)\mathrm{d}t_3$ is the conditional probability that, given a pulse pair at t_1, t_2, there is also a pulse in $\mathrm{d}t_3$ about t_3 and is given by the relation:

$$P_3(t_1, t_2, t_3) = P_2(t_1, t_2) W_2(t_1, t_2, t_3).$$

Then using an approximation similar to that used above for the case of small deadtime, we obtain:

$$\frac{\overline{C(C-1)}}{2} = \int_0^{T_0-\tau}\mathrm{d}t_1\int_{t_1+\tau}^{T_0}\mathrm{d}t_2\left[P_2(t_1, t_2) - \int_{t_1-\tau}^{t_1} P_3(t_1, t_2, t_3)\mathrm{d}t_3 - \int_{t_2-\tau}^{t_2} P_3(t_1, t_2, t_3)\mathrm{d}t_1\right]. \tag{6}$$

Finally, we obtain:

$$\bar{C} = \epsilon S T_0\left[1 - \epsilon\tau\left\{S + \frac{\overline{\nu(\nu-1)}F\beta}{2S(1-k_p)}\right\}\right], \tag{7}$$

and

$$\overline{C(C-1)} = a_1 T^2 + 2a_2\,\mathrm{e}^{-\alpha\tau}\frac{T}{\alpha}\left[1 - \frac{1-\mathrm{e}^{-\alpha T}}{\alpha T}\right] + a_3\,\mathrm{e}^{-2\alpha\tau}\frac{T}{\alpha}\left[1 - \frac{1-\mathrm{e}^{-2\alpha T}}{2\alpha T}\right], \tag{8}$$

where:

$$T = T_0 - \tau,$$

$$a_1 = \epsilon^2 S^2 - 2\epsilon^3 S^3\tau - \frac{\epsilon^3 S\overline{\nu(\nu-1)}F\alpha\tau}{(1-k_p)^2},$$

$$a_2 = \frac{\epsilon^3\overline{\nu(\nu-1)}F\alpha}{2(1-k_p)^2} - \frac{2\epsilon^3\tau S\overline{\nu(\nu-1)}F\alpha}{(1-k_p)^2} - \frac{\epsilon^3\tau\overline{\nu(\nu-1)(\nu-2)}F\alpha^2}{3(1-k_p)^3} - \frac{\epsilon^3\overline{\nu(\nu-1)}Fk_p\overline{\nu_1(\nu_1-1)}\alpha^2}{\bar{\nu}_1(1-k_p)^4},$$

$$a_3 = -\frac{\epsilon^3\tau\overline{\nu(\nu-1)(\nu-2)}F\alpha^2}{3(1-k_p)^3}.$$

Using the approximations:

$$\left(1 - \frac{T^2}{T_0^2}\right) \cong \frac{2\tau}{T_0} \text{ and } a_1 = \frac{\bar{C}^2}{T_0^2},$$

we obtain:

$$\overline{C^2} - \bar{C}^2 - \bar{C} - \frac{2\tau}{T_0}\bar{C}^2 = \frac{\epsilon^2\left[\overline{\nu_0(\nu_0-1)}F_0 + \overline{\nu_1(\nu_1-1)}F_1\right]}{(1-k_p)^2}\,Tg(\alpha T)\,\mathrm{e}^{-\alpha\tau}(1-4\tau h), \tag{9}$$

where $g(x) = 1 - (1-\mathrm{e}^{-x})/x$.

$$h = \epsilon S + \frac{\epsilon\beta}{6}\frac{\overline{\nu(\nu-1)(\nu-2)}}{\overline{\nu(\nu-1)}}\left[1 + \mathrm{e}^{-\alpha\tau}\frac{g(2\alpha T)}{2g(\alpha T)}\right] + \frac{\beta\epsilon k_p\overline{\nu_1(\nu_1-1)}}{2\bar{\nu}_1(1-k_p)}$$

$$\cong \epsilon S + \frac{\epsilon\beta k_p\overline{\nu_1(\nu_1-1)}}{2\bar{\nu}_1(1-k_p)}.$$

The approximation results from neglecting the term due to counting of triples from a fission event.

The first and second moments of the counting distribution are $\bar{C}$ and $\overline{C^2}$, respectively.

The variable α that appears in the right-hand-side factors $g(\alpha T)$ and $e^{\alpha t}$ provides the time behavior of the neutron population seen by the detector. For a detector embedded in or closely coupled to a multiplying system, this variable should be the Rossi α of the system. In the present measurements, however, the fast time behaviour of the plutonium metal parts can be treated as instantaneous relative to the 60.7 μs decay time in the detector. A cadmium liner in the well of the detector prevents neutrons thermalized in the detector from returning to the multiplying assembly and prolonging its time behavior. Thus, as seen by the detector, neutrons enter in a burst of essentially zero width and decay with the time constant of the detector. In what follows, therefore, we will replace α by β, the decay constant of the detector.

In our derivation, we considered only a single interrogating source, which we represent as a distributed spontaneous fission source. If, in fact, it is an (α, n) source, it can be treated as a pseudo spontaneous fission source with $\nu_0 = \nu_0^2 = 1$ and with F_0 interpreted as the (α, n) production rate.

Recognizing that

$$\frac{\nu_1 F_1}{\nu_0 F_0} = \frac{k_p}{1 - k_p} = M - 1,$$

where M is the prompt multiplication, and defining

$$Y = \frac{\overline{C^2}}{\bar{C}} - \bar{C} - 1, \tag{10}$$

and

$$Y_c = \frac{Y + (2\tau/T_0)\bar{C}}{1 - 4\tau h}, \tag{11}$$

we can write

$$Y_c = \epsilon M\left[D_0 + D_1(M-1)\right]\frac{T}{T_0}g(\beta T)\,e^{-\beta\tau}, \tag{12}$$

where

$$D_0 = \frac{\overline{\nu_0(\nu_0 - 1)}}{\bar{\nu}_0}, \quad D_1 = \frac{\overline{\nu_1(\nu_1 - 1)}}{\bar{\nu}_1},$$

and

$$h = \frac{\bar{C}}{T_0} + \frac{\epsilon\beta D_1(M-1)}{2}. \tag{13}$$

If a similar measurement is now made using the same detector but with a ^{252}Cf source replacing the unknown whose multiplication is to be determined, we can obtain

$$Y_c^{Cf} = \epsilon D_0^{Cf}\frac{T}{T_0}g(\beta T)\,e^{-\beta\tau},$$

where the superscript Cf indicates values for ^{252}Cf; we set $D_1^{Cf} = 0$ and $M = 1$ in this equation because the probability of induced fission in the ^{252}Cf source is extremely low due to its small mass. Combining the two measurements, we obtain

$$M = \frac{D_1 - D_0}{2D_1} + \left[\left(\frac{D_1 - D_0}{2D_1}\right)^2 + \frac{D_0^{Cf}Y_c}{D_1 Y_c^{Cf}}\right]^{1/2}. \tag{14}$$

The superscript U (for unknown) added to Y_c indicates a measurement made using the assembly whose multiplication is to be determined. In eq. (14), Y_c^U is a function of M and, thus, iterations are required. However, except for a very large multiplication or very small interrogating source strength, it is a weak function of M and the iterations converge rapidly.

Three parameters remain to be determined to obtain a solution; namely, the detector deadtime τ, the detector decay constant β, and the detector efficiency ϵ. The detector deadtime is determined from a measurement of Y_c made using an uncorrelated neutron source such as F(α, n). It is known that for such an uncorrelated source, the deadtime-corrected value of the variance-to-mean ratio $(1 + Y_c)$ is identically 1. Thus, from such a measurement and eq. (11), the deadtime can be readily determined. For the counters and electronics used in these measurements, this procedure yielded a deadtime of 1.046 μs. The decay constant can be determined by measuring Y_c^{Cf} for a range of values of T and fitting this data to the expression

$$Y_c^{Cf} = A\left(1 - \frac{1 - e^{-\beta T}}{\beta T}\right)\frac{T}{T_0}, \tag{15}$$

where A and β are both fitted constants. This procedure was followed for the well counter; the results (fig. 1) provide a decay constant of 0.0165 μs^{-1}. The efficiency can then be determined from eq. (13), because all other terms in this equation are now known. Using this method, we obtained an efficiency for detecting ^{252}Cf spontaneous fission neutrons of 4.43% for the well counter used in these measurements.

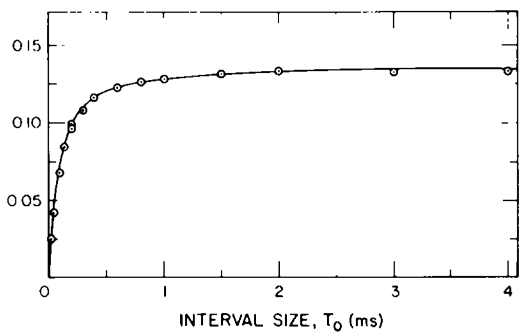

Fig. 1. Fit of Y_c to measured ^{252}Cf data.

3. Experimental and calculational procedure

3.1. Thor parts

To demonstrate the technique, we measured and calculated the multiplication of each of the three major plutonium parts that make up the core of the Thor assembly. These parts, a center segment and two polar caps, approximate a 10.59 cm diam. sphere when assembled. The composition of each part is 93.89 wt% of ^{239}Pu, 5.10 wt% of ^{240}Pu, and 1.01 wt% of Ga. The plutonium masses of the top, center, and bottom parts are 3193, 4086, and 2174 g, respectively. We chose these parts to measure because of their significant multiplication and immediate availability. Moreover, the uncomplicated shapes of the parts made their geometry easy to specify for the Monte Carlo calculations.

3.2. Well counter

The polyethylene-moderated well counter used for the measurements appears in fig. 2. This detector has twenty 4 atm ^{3}He proportional counters equally spaced on a 27.9 cm diam. circle. The counters are 2.54 cm in diam. by 61.0 cm long with an active length of 50.8 cm. Aluminum cylinders (10.2 cm long) are located above and below the counter well to flatten the axial response of the detector. The entire well volume and the two alumium cylinders are enclosed in 0.127 cm of cadmium to minimize the thermal neutron return

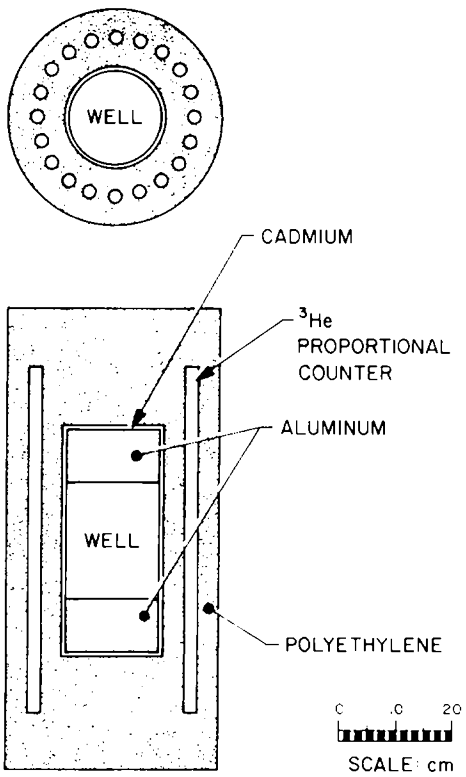

Fig. 2. Polyethylene-moderated well counter.

from the moderator. The measured efficiency of the detector for ^{252}Cf neutrons was found to be 4.43%. Although a detector with high efficiency is desirable, the well counter is clearly more elaborate than what is required for our measurements. We selected it because it was adequate and immediately available.

3.3. Multiplication calculations

We used the MCNP Monte Carlo code [3] to calculate steady-state prompt-neutron multiplication in the separate Thor parts for both bare and reflected (well counter) configurations. The MCNP code is a general purpose code for neutron, photon, or coupled neutron–photon transport that is capable of calculating eigenvalues for criticality problems. The code treats an arbitrary three-dimensional configuration of materials in geometric cells bounded by first and second degree surfaces. Pointwise cross-section data are used. All neutron reactions in a given cross-section evaluation, such as ENDF/B-V, are included in the cross-section library. Thermal-neutron reactions are treated by both the free gas and by molecular binding models.

Using the MCNP code, we defined a calculational model that includes a complete material and geometrical representation of the plutonium Thor parts and the well counter. The calculations treat neutrons that result from ^{240}Pu spontaneous fissions in the plutonium parts as uniformly distributed and isotropically emitted; their starting energies are sampled from the fission spectrum. Each source neutron and any of its progeny are followed through all scattering, absorption, or leakage collisions until the neutrons are terminated. The code calculates a sufficient number of neutron histories to yield a subcritical effective prompt-neutron multiplication factor k_p that is then used to calculate the multiplication M from

$$M = \frac{1}{1 - k_p}.$$

4. Results and discussion

A comparison of the measured and calculated multiplication for each of the three Thor parts appears in table 1. For each measurement, we used a 200 μs interval size (T_0) and analyzed 5×10^6 intervals. The calculated multiplication of the Thor parts, in both a bare configuration and reflected by the well counter, are given in table 1. Although the cadmium liner of the well counter stops most thermal neutrons, it will permit neutrons above the 0.4 eV cadmium cut-off to be reflected back into the well where they cause additional fissions in the plutonium and increase the multiplication. Because the measured multiplication is clearly that of the plutonium Thor part reflected by the well counter and because our principal objective was to test the method, we compared the measurements with the reflected calculations. For all three parts our measurements agree with the calculations within approximately 1–3%. Statistical uncertainty in the calculations and measurements was approximately 2% and 0.6%, respectively. From the calculations, we determined that the effect of the well counter reflection on multiplication is about 5%. Even though the masses of the top and center parts differ by almost 900 g, they have approximately the same multiplication because of the top segment's more compact shape, that is, smaller surface-to-volume ratio.

Our technique requires a close match between the efficiency for detecting neutrons from the unknown assembly and the efficiency for detecting neutrons from the ^{252}Cf standard. In our measurements, we did nothing to improve this match as the leakage from the Thor parts was high and the spectrum of leakage neutrons was close to that of a fission spectrum. We can easily envision situations where more effort must be put into matching these efficiencies. For example, the placement of a substantial moderator around a multiplying assembly would have to be matched by placing a similar amount of moderator around the ^{252}Cf standard because both the leakage and the leakage spectrum can be strongly affected by such an external moderator.

Table 1
Thor multiplication.

Thor part	Mass (g)	Calculated multiplication [a]	Measured multiplication	Ratio [b]	Calculated multiplication [c]
Top	3193	2.26	2.197	0.972	2.19
Center	4086	2.28	2.236	0.981	2.16
Bottom	2174	1.89	1.866	0.987	1.83

[a] In the well counter.
[b] Ratio of measured value to that calculated in the well counter.
[c] In vacuum.

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