Use of Criticality Eigenvalue Simulations for Subcritical Benchmark Evaluations

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INTRODUCTION

Subcritical multiplication experiments and simulations are important for a variety of applications including nonproliferation, safeguards, and criticality safety monitoring. Recently, subcritical experiments have been evaluated for inclusion in the International Criticality Safety Benchmark Evaluation Process (ICSBEP) [1-2]. In addition, new subcritical experiments have recently been designed [3]. In these experiments, list-mode data (a list of each absorption in the neutron detector) is recorded and neutron noise analysis is used to determine the leakage multiplication (M_L) of each configuration.

One downside to the simulation of these types of experiments is that they are computationally expensive. This is because analog Monte Carlo with no variance reduction must be performed in order to maintain the time accuracy required in the measurement analysis.

This work demonstrates that criticality eigenvalue simulations can be used to estimate sensitivities and uncertainties for three parameters: singles count rate (R_1), doubles count rate (R_2), and leakage multiplication (M_L). This will provide a useful aid both in the design and evaluation of new subcritical measurements.

CURRENT APPROACH

Two recent subcritical experiments were evaluated in ICSBEP. Both experiments included the BeRP ball, a cladded α-phase plutonium sphere; the first evaluation from 2014 used nickel reflectors [1] and the second evaluation used tungsten reflectors [2]. Both experiments used the same He-3 based NPOD systems to acquire list-mode data [4]. A separate talk detailing the tungsten experiment is being presented at this ANS meeting [5].

In both evaluations, three benchmark parameters are evaluated. The detector singles count rate (R_1), is simply the count rate in the detector system. The doubles count rate (R_2) is the rate in the detector system in which two neutrons from the same fission chain are detected. Last, leakage multiplication (M_L) is the number of neutrons escaping a system per starter neutron.

Simulations for subcritical experiments must be run in a different manner than critical experiments to allow for multiplicity analysis. MCNP®6 [6] enables the user to directly run fixed-source problems in which the user can obtain list-mode data from the generated PTRAC files. Scripts to read the PTRAC file to develop list-mode data have recently been developed [7]. In order to perform multiplicity analysis on the simulated data, the simulations must be run in analog Monte Carlo with no variance reduction.

List-mode simulations can be computationally expensive. Table I compares the simulation time to produce list-mode data versus the time to perform a criticality eigenvalue (KCODE) calculation given the same input file. For this table, two adjustments were performed. The first adjustment was performed on all simulations to give the time required on a single processor (because the number of processors was not constant). The second adjustment modified the KCODE simulation times so that both would result in the same statistical error. It can be seen in Table I that the total time for all KCODE simulations is roughly 30 times faster than the list-mode simulations.

Table I. Comparison of simulation times (number of hours if performed on a single processor) for the BeRP/W evaluation.

Case	List-mode	KCODE
0	480	0.5
1	480	1.5
2	640	5
3	640	11
4	800	21
5	800	35
6	800	46
7	880	58
Sum	5520	178

The sensitivity and uncertainty analysis of experimental parameters is performed in a similar fashion to that of critical experiments in ICSBEP evaluations. For each experimental parameter x (such as fissionable material radius), a perturbation to the reference model is made (equal to Px). This perturbation is simulated and one can then calculate the sensitivity to experimental parameter x:

$$S_{k,x} = \frac{k_P - k_R}{P_x} \quad (1)$$

Here $S_{k,x}$ is the sensitivity of the benchmark parameter k (either R_1, R_2, or M_L) due to the experimental parameter perturbation P_x (such as the amount that the plutonium

radius was perturbed). The subscripts P and R are for "perturbed" and "reference". Note that the amount which the experimental parameter is perturbed (P_x) must be large enough to produce a response in k greater than the statistical uncertainty of the simulation but must be small enough such that linearity assumptions remain valid. The uncertainty due to the experimental parameter x is:

$$\delta k_x = u_x S_{k,x} \tag{2}$$

where u_x is the uncertainty in experimental parameter x. This is identical to the approach used in critical experiments, except there k_{eff} is always used for the benchmark parameter k. More information on calculation of uncertainties for critical experiments can be found in the ICSBEP uncertainty guide [8].

It should be noted that the BeRP/W evaluation has approximately 30 different experimental uncertainty parameters (x). It is easy to see from Table I, the computational time required to perform the complete sensitivity/uncertainty analysis for such a benchmark is very large. Some of these parameters require even longer times than those presented in Table I to reduce the statistical uncertainties to help ensure that the change is larger than the statistical uncertainties.

When designing an experiment (subcritical or critical), it is desired to estimate uncertainties prior to execution of the experiment. It can easily be seen, however, that this is currently not practical for subcritical measurements, given the large computational resources required. This work presents an alternate approach for experimental design.

PROPOSED APPROACH

The leakage multiplication (M_L) parameter is related to the multiplication factor (k_{eff}) of a system using the following basic reactor physics equations:

$$k_{eff} = \frac{k_p}{1-\beta_{eff}} \tag{3}$$

$$k_p = 1 - \frac{1}{M_T} \tag{4}$$

$$M_T = \frac{M_L \bar{\nu} - 1 - \alpha}{\bar{\nu} - 1 - \alpha} \tag{5}$$

where k_p is the prompt multiplication factor (due to prompt neutrons only), β_{eff} is the effective delayed neutron fraction, M_T is the total neutron multiplication (as opposed to leakage multiplication), $\bar{\nu}$ is the average number of neutrons created per fission, and α is the capture cross-section divided by the fission cross-section of the fissile material.

The parameters β_{eff}, $\bar{\nu}$, and α mostly depend upon the fissionable material (in this case Pu-239) and therefore will not vary much between configurations. For this reason, it is very easy to approximate M_L given k_{eff} values. It is therefore reasonable to suspect that perhaps one could approximate the uncertainty in leakage multiplication from a criticality eigenvalue calculation.

The measurement approach in the BeRP/Ni and BeRP/W evaluations is based upon the Hage-Cifarelli formalism [9] of the Feynman Variance-to-Mean method [10]. If one assumes that no neutrons are produced from (α,n) interactions, then the equations for singles and doubles count rate reduce to:

$$R_1 = \varepsilon b_{11} F_S \tag{6}$$

$$R_2 = \varepsilon^2 b_{21} F_S \tag{7}$$

with

$$b_{11} = M_L \bar{\nu}_{S(1)}$$

$$b_{21} = M_L^2 \left[\bar{\nu}_{S(2)} + \frac{M_L - 1}{\bar{\nu}_{I(1)} - 1} \bar{\nu}_{S(1)} \bar{\nu}_{I(2)} \right]$$

where ε is the absolute detector efficiency and F_S is the spontaneous fission rate of the system, The terms $\bar{\nu}_{S(1)}$, $\bar{\nu}_{S(2)}$, $\bar{\nu}_{I(1)}$, and $\bar{\nu}_{I(2)}$ are the first and second factorial moments of the P_ν distribution where S refers to the isotope producing spontaneous fission neutrons and I refers to the isotope undergoing induced fission. These P_ν moments are defined in Ref. 9. Note that $\bar{\nu}_{I(1)}$ (the first moment for the induced isotope) is equal to $\bar{\nu}$ in equation 5.

All of the $\bar{\nu}$ parameters are fairly constant amongst all configurations with the same fissionable material. The spontaneous fission rate is known for the BeRP ball [1] and is constant regardless of the configuration. The only configuration-dependent variable in Eq. 6-7, therefore, is detector efficiency. The detection efficiency has two components: the solid angle and the intrinsic efficiency. The solid angle is constant amongst all configurations (since they had the same detector setup); it is known that for a bare system, the NPOD efficiency is ~0.01 at 50 cm. The intrinsic efficiency is configuration dependent (due to the change in the energy of neutrons reaching the He-3 tubes due to absorption in the reflector). This can, however, easily be estimated (a model of the detector is not required to approximate this).

Given that one can approximate all of the parameters in Eq. 6-7, it is reasonable to suspect that one could also approximate uncertainties in singles and doubles count rates from criticality eigenvalue simulations; note that to accomplish this, one first uses k_{eff} from the KCODE

simulation to determine M_L using Eq. 3-5, then that M_L is plugged into Eq. 6-7 to solve for R_1 and R_2. Table II shows the parameters which are used in Eq. 3-7; an explanation of why the values in Table II were selected will be given in the presentation.

Table II. Values used for all configurations.

Parameter	Value
β_{eff}	0.00202
α	0.033
ε	0.010 +/- 0.003
F_S	130267
$\overline{\nu}_{S(1)}$	2.154 +/- 0.005
$\overline{\nu}_{S(2)}$	1.894 +/- 0.015
$\overline{\nu}_{I(1)}$	3.182 +/- 0.010
$\overline{\nu}_{I(2)}$	4.098 +/- 0.011

RESULTS

The results for the reference BeRP/W configurations are shown in Fig. 1.

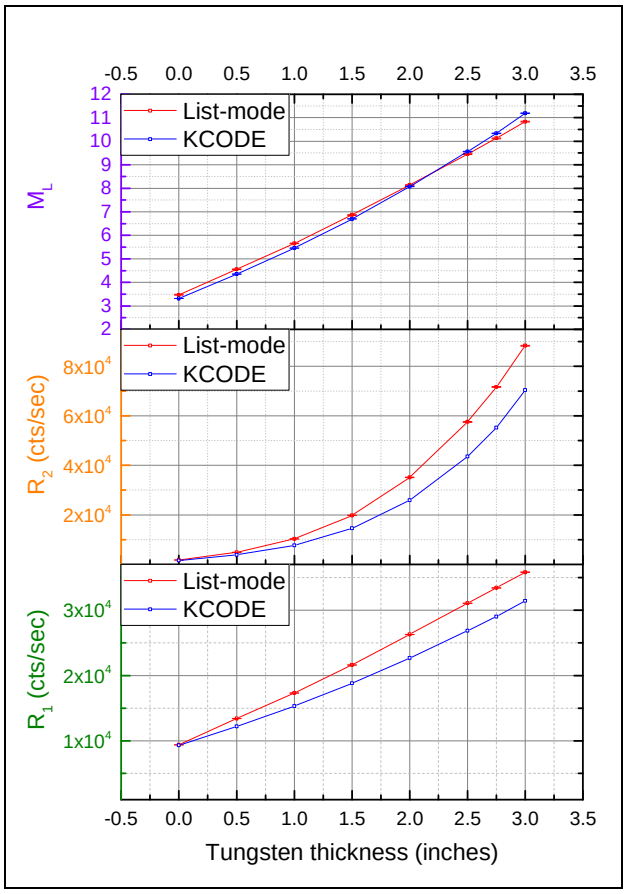

Fig. 1. Reference BeRP/W configuration results.

Both curves in Fig. 1 are simulated data using MCNP®6. The curves labeled "list-mode" were calculated using the list-mode capability (the same method used in the BeRP/Ni evaluation). The curve labeled "KCODE" are the results of solving Eq. 3-7 given simulated k_{eff} values and the information in Table II. It is not surprising that the two curves compare better for M_L than for R_1 and R_2. This is because more assumptions are used to determine these parameters (it is known, for instance, that the efficiency is not exactly 0.010 for all of these configurations).

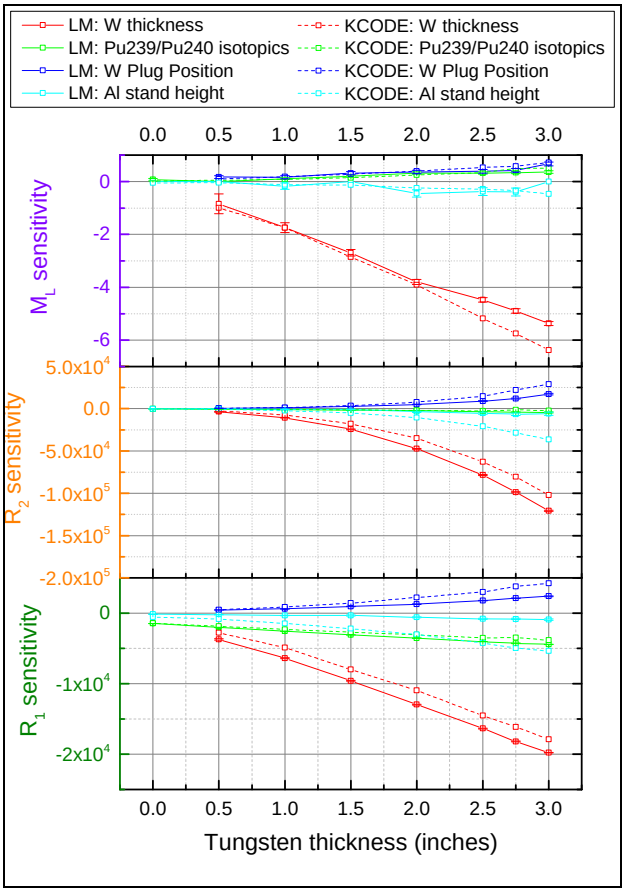

Fig. 2. Sensitivity results for four experimental parameters.

Five parameters were selected so compare the sensitivity and uncertainty analysis: plutonium sphere radius, Pu-239/Pu-240 isotopics, tungsten thickness, tungsten plug position, and Al stand height. The first four parameters were selected because they are large contributors to the total uncertainty for M_L than for R_1 and R_2 [2]. The uncertainty on the Al stand height was selected as a parameter known to have a negligible uncertainty on all parameters.

The sensitivity results are shown in Fig. 2. It can be seen that the solid (list-mode, labelled "LM") and dashed (KCODE) curves compare well.

The uncertainty results are shown in Fig. 3. It can be seen that the solid (list-mode) and dashed (KCODE) curves compare well. It can also be seen that the Al stand height uncertainty is negligible for M_L, R_1, and R_2 as expected.

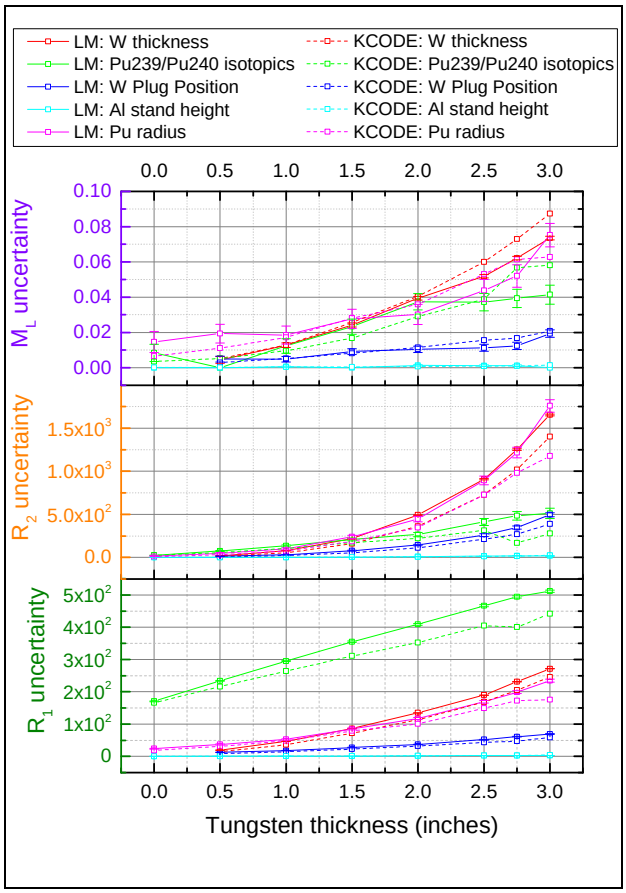

Fig. 3. Uncertainty results five experimental parameters.

CONCLUSIONS AND APPLICATION

It has been shown that one can use criticality eigenvalue calculations to approximate sensitivities and uncertainties for singles count rate (R_1), doubles count rate (R_2), and leakage multiplication (M_L). Note that the authors are not suggesting that criticality eigenvalue simulations should be used instead of list-mode simulations for benchmark evaluations. We are suggesting, however, that this approach could be very valuable in the experiment design phase. In addition, this method could also perhaps be used to confirm whether or not certain parameters are negligible. The benefit of using criticality eigenvalue calculations is that they are much faster (roughly 30 times faster for our simulations).

ACKNOWLEDGEMENT

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