

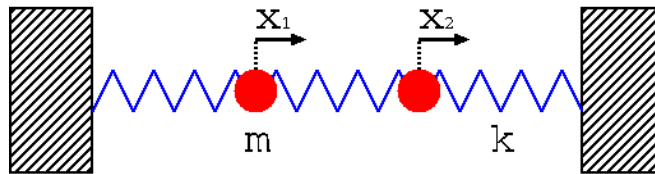
Physics and Finance

Exercise 1, due Wednesday August 11, 7:00

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1. Lagrange multiplier (3p): Consider the motion of two equal masses m that are held in place by three springs with spring constant k , subject to the constraint $3x_1 + x_2 = c$, where x_1 and x_2 are the displacements of the masses from their equilibrium position without the constraint taken into account.



- (a) Determine the Lagrangian for the system;
 - (b) Determine the equations of motion;
 - (c) Calculate the equilibrium positions with the constraint taken into account;
 - (d) Determine the eigenfrequency of the constrained system.
2. Portfolio selection (5p): Unexpectedly, you have inherited some money from a distant aunt that you want to invest in the stock market. Your aunt's testament, however, forces you to invest 30 % of your inheritance in Coca-Cola stocks. The other three quarters you split between Apple stocks and an index fund, based on a mix of SP500 companies. From the course webpage, please download the file `stocks.dat` with the daily closing values of Apple, SP500, and Coca-Cola for all trading days in the period (27.3.2018-27.3.2019). The file is a text file and contains three columns: the first one indicates the trading day t_d , the second contains the Apple data S_1 , the third the SP500 data S_2 , and the fourth contains the Coca-Cola data S_3 . A short MATLAB script to load this file `get_stocks.m` is also available from the webpage.

- (a) Plot the three stock values S versus the trading day to obtain an impression of how the stocks evolve.
- (b) Calculate the day-to-day returns $r_j = (S_{j+1} - S_j)/S_j$, plot the values and calculate their average. Reminder: scale the average daily returns with the trading days to obtain the annual returns.
- (c) Calculate the covariance matrix for the three stocks.
- (d) You want to use Markowitz's theory to partition the remainder of your inheritance in the other two stocks in order to obtain an annual return of 5 %, 10 %, and 15 %. You need, however, adapt the theory to account for the fixed investment in the Coca-Cola stocks. Note: just using $\sum_i w_i = 0.7$ does not work! Instead, introduce a third Lagrange multiplier to constrain w_3 to $x = 0.3$ and adjust the theory from Section 3.2 appropriately. Implement the adapted algorithm in MATLAB or Python or C or Fortran and discuss your results.

Note: We do not accept solutions based on financial toolboxes!

3. Stock evolution (2p): What is the probability that a stock with an annual return $\rho = 0.1$ and an annual volatility of $\sigma = 0.3$ after 2 years has a value between $2/5$ and $3/5$ of its present value? Note: numerical solutions are admissible.

Document your solutions well! That makes it easier to spot silly mistakes and give many points despite of them. It's for your own benefit! Moreover, please submit your program files along with your write-up for the homework.

You can submit electronic versions, preferably pdfs from Word or Latex, but you can also prepare handwritten solutions that you photograph with your phone. But then please combine all images to a single pdf file. It's a pain in the neck to do that for 20 or more students. We'll subtract 2 points, if we have to do it.

Please prepare individual solutions. We will not tolerate receiving essentially the same pdf from several students. Please play fair!

Otherwise, please feel free to collaborate and teach each other—It is called autocatalytic teaching. That's the best!

If you have any question at all, please ask. There is no penalty for asking! We'll frequently check my email and try to reply quickly. Note that many short questions are easier to handle than long lists of questions, which get filed away to be answered (much) later...