

2.

$$D(x) = Ae^{-\sqrt[3]{x}}$$

a)

$$\int_0^{\infty} D(x') dx' = 1$$

$$\int_0^{\infty} Ae^{-\sqrt[3]{x'}} dx' = 1 \implies \int_0^{\infty} e^{-\sqrt[3]{x'}} dx' = \frac{1}{A}$$

Make variable separation $u = \sqrt[3]{x'}$

When $x \rightarrow \infty$, $u \rightarrow \infty$

When $x \rightarrow 0$, $u \rightarrow 0$

$$\frac{du}{dx'} = \frac{1}{3}x'^{-2/3} \implies dx' = 3u^2 du$$

Use eq 7.20

$$\int Ae^{-u} 3u^2 du = 3A\Gamma(3) \implies A = \frac{1}{3\Gamma(3)}$$

b)

We can make the similar variable sub again and then use eq.7.23.

$$C(x) = \int_0^x D(x') dx' = \frac{1}{3\Gamma(3)} \int_0^x e^{-\sqrt[3]{x'}} dx' , \quad \frac{u}{2} = \sqrt[3]{x'} \quad dx' = 3\frac{u^2}{4} d\frac{u}{2}$$

$$\implies \frac{1}{8\Gamma(3)} \int_0^{u^{3/8}} e^{-u/2} 3u^2 du = \int_0^{u^{3/8}} \frac{1}{\Gamma(6/2)2^{(n/2)}} e^{-u} u^{6/2-1} du = P(6/2, x/2)$$

Where P is the incomplete gamma function.

2c)d)

By using built in functions gamma and gammainc, the following was obtained |

