$$f_{Ma}\left(v_{\parallel}, v_{\perp}\right) = \frac{1}{\pi^{3/2} v_{T\parallel a} v_{T\perp a}^2} \exp\left(-\frac{\left(v_{\parallel} - V_{Da}\right)^2}{v_{T\parallel a}^2} - \frac{v_{\perp}^2}{v_{T\perp a}^2}\right)$$

$$v_{T\parallel a}^2 = \frac{2T_{\parallel a}}{m_a} \quad v_{T\perp a}^2 = \frac{2T_{\perp a}}{m_a}$$

$$v_A^2 = \frac{B^2}{4\pi n_i m_i}$$

$$\beta_{\parallel a} = \frac{8\pi n_a T_{\parallel a}}{B^2} \quad \beta_{\perp a} = \frac{8\pi n_a T_{\perp a}}{B^2}$$

$$v_{\parallel} = v_A u_{\parallel} \quad v_{\perp} = v_A u_{\perp} \quad U_{Da} = \frac{V_{Da}}{v_A}$$

$$\mu_a = \frac{m_i}{m_a} \quad \tilde{q}_a = \frac{q_a}{q_i} \quad \tilde{n}_a = \frac{n_a}{n_i}$$

$$f_{Ma}\left(u_{\parallel}, u_{\perp}\right) = \frac{1}{\pi^{3/2} v_{T\parallel a} v_{T\perp a}^2} \exp\left(-\frac{v_A^2 \left(u_{\parallel} - U_{Da}\right)^2}{v_{T\parallel a}^2} - \frac{v_A^2 u_{\perp}^2}{v_{T\perp a}^2}\right)$$

$$\frac{v_{T\parallel a}^2}{v_A^2} = \frac{m_i}{m_a} \frac{n_i}{n_a} \frac{8\pi n_a T_{\parallel a}}{B^2} = \frac{m_i}{m_a} \frac{n_i}{n_a} \beta_{\parallel a} = \frac{\mu_a}{\tilde{n}_a} \beta_{\parallel a} \quad v_{T\parallel a} = v_A \sqrt{\frac{\mu_a}{\tilde{n}_a}} \sqrt{\beta_{\parallel a}}$$

$$\frac{v_{T\perp a}^2}{v_A^2} = \frac{m_i}{m_a} \frac{n_i}{n_a} \frac{8\pi n_a T_{\perp a}}{B^2} = \frac{m_i}{m_a} \frac{n_i}{n_a} \beta_{\perp a} = \frac{\mu_a}{\tilde{n}_a} \beta_{\perp a} \quad v_{T\perp a}^2 = v_A^2 \frac{\mu_a}{\tilde{n}_a} \beta_{\perp a}$$

$$f_{Ma}\left(u_{\parallel}, u_{\perp}\right) = \frac{1}{\pi^{3/2} v_{T\parallel a} v_{T\perp a}^2} \exp\left(-\frac{\left(u_{\parallel} - U_{Da}\right)^2}{\frac{\mu_a}{\tilde{n}_a} \beta_{\parallel a}} - \frac{u_{\perp}^2}{\frac{\mu_a}{\tilde{n}_a} \beta_{\perp a}}\right)$$

$$= \frac{1}{\pi^{3/2} v_{T\parallel a} v_{T\perp a}^2} \exp\left(-\tilde{n}_a \frac{\left(u_{\parallel} - U_{Da}\right)^2}{\mu_a \beta_{\parallel a}} - \frac{\tilde{n}_a u_{\perp}^2}{\mu_a \beta_{\perp a}}\right)$$

$$= \frac{1}{v_A^3} \frac{\tilde{n}_a^{3/2}}{\pi^{3/2} \mu_a^{3/2} \sqrt{\beta_{\parallel a}} \beta_{\perp a}} \exp\left(-\tilde{n}_a \frac{\left(u_{\parallel} - U_{Da}\right)^2}{\mu_a \beta_{\parallel a}} - \frac{\tilde{n}_a u_{\perp}^2}{\mu_a \beta_{\perp a}}\right)$$

$$\int d^3 v \, f_{Ma}\left(v_{\parallel}, v_{\perp}\right) = 1 \implies \int d^3 u \, v_A^3 f_{Ma}\left(u_{\parallel}, u_{\perp}\right) = 1$$

$$\tilde{f}_{Ma}\left(u_{\parallel}, u_{\perp}\right) = v_A^3 f_{Ma}\left(u_{\parallel}, u_{\perp}\right) \implies \boxed{\tilde{f}_{Ma}\left(u_{\parallel}, u_{\perp}\right) = \frac{\tilde{n}_a^{3/2}}{\pi^{3/2} \mu_a^{3/2} \sqrt{\beta_{\parallel a}} \beta_{\perp a}} \exp\left(-\tilde{n}_a \frac{\left(u_{\parallel} - U_{Da}\right)^2}{\mu_a \beta_{\parallel a}} - \frac{\tilde{n}_a u_{\perp}^2}{\mu_a \beta_{\perp a}}\right)}$$

Kappa:

$$f_{\kappa a}\left(v_{\parallel}, v_{\perp}\right) = \frac{1}{\pi^{3/2} \kappa_a^{3/2} \theta_{\parallel a} \theta_{\perp a}^2} \frac{\Gamma\left(\kappa_a + 1\right)}{\Gamma\left(\kappa_a - 1/2\right)} \left(1 + \frac{\left(v_{\parallel} - V_{Da}\right)^2}{\kappa_a \theta_{\parallel a}^2} + \frac{v_{\perp}^2}{\kappa_a \theta_{\perp a}^2}\right)^{-(\kappa_a + 1)}$$

$$\theta_{\parallel a}^2 = \left(1 - \frac{3}{2\kappa_a}\right) v_{T\parallel a}^2 \quad \theta_{\perp a}^2 = \left(1 - \frac{3}{2\kappa_a}\right) v_{T\perp a}^2$$

$$\theta_{\parallel a}^2 = v_A^2 \tilde{\theta}_{\parallel a}^2 \quad \theta_{\perp a}^2 = v_A^2 \tilde{\theta}_{\perp a}^2$$

$$\tilde{\theta}_{\parallel a}^2 = \left(1 - \frac{3}{2\kappa_a}\right) \frac{\mu_a}{\tilde{n}_a} \beta_{\parallel a} \quad \tilde{\theta}_{\perp a}^2 = \left(1 - \frac{3}{2\kappa_a}\right) \frac{\mu_a}{\tilde{n}_a} \beta_{\perp a}$$

$$f_{\kappa a}\left(u_{\parallel}, u_{\perp}\right) = \frac{1}{v_A^3 \pi^{3/2} \kappa_a^{3/2} \tilde{\theta}_{\parallel a} \tilde{\theta}_{\perp a}^2} \frac{\Gamma\left(\kappa_a + 1\right)}{\Gamma\left(\kappa_a - 1/2\right)} \left(1 + \frac{\left(u_{\parallel} - U_{Da}\right)^2}{\kappa_a \tilde{\theta}_{\parallel a}^2} + \frac{u_{\perp}^2}{\kappa_a \tilde{\theta}_{\perp a}^2}\right)^{-(\kappa_a + 1)}$$

$$\tilde{f}_{\kappa a}\left(u_{\parallel}, u_{\perp}\right) = v_A^3 f_{\kappa a}\left(u_{\parallel}, u_{\perp}\right)$$

$$\boxed{\tilde{f}_{\kappa a}\left(u_{\parallel}, u_{\perp}\right) = \frac{1}{\pi^{3/2} \kappa_a^{3/2} \tilde{\theta}_{\parallel a} \tilde{\theta}_{\perp a}^2} \frac{\Gamma\left(\kappa_a + 1\right)}{\Gamma\left(\kappa_a - 1/2\right)} \left(1 + \frac{\left(u_{\parallel} - U_{Da}\right)^2}{\kappa_a \tilde{\theta}_{\parallel a}^2} + \frac{u_{\perp}^2}{\kappa_a \tilde{\theta}_{\perp a}^2}\right)^{-(\kappa_a + 1)}}$$