Explicit Electromagnetic Particle-In-Cell Simulation KEMPO1 & 2 :

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Software
for
Space Plasma Simulations

http://space.rish.kyoto-u.ac.jp/software/

References: TERRAPUB e-Library

Advanced Methods for Space Simulations (ISSS-7)

"One-dimensional Electromagnetic Particle Code: KEMPO1"

Edited by H. Usui and Y. Omura (2007)

Computer Space Plasma Physics:

Simulation Technique and Software (ISSS-4)

"KEMPO1: Technical Guide to one-dimensional electromagnetic particle code"

Edited by H. Matsumoto and Y. Omura (1993)

Computer Simulation of Space Plasmas (ISSS-1)

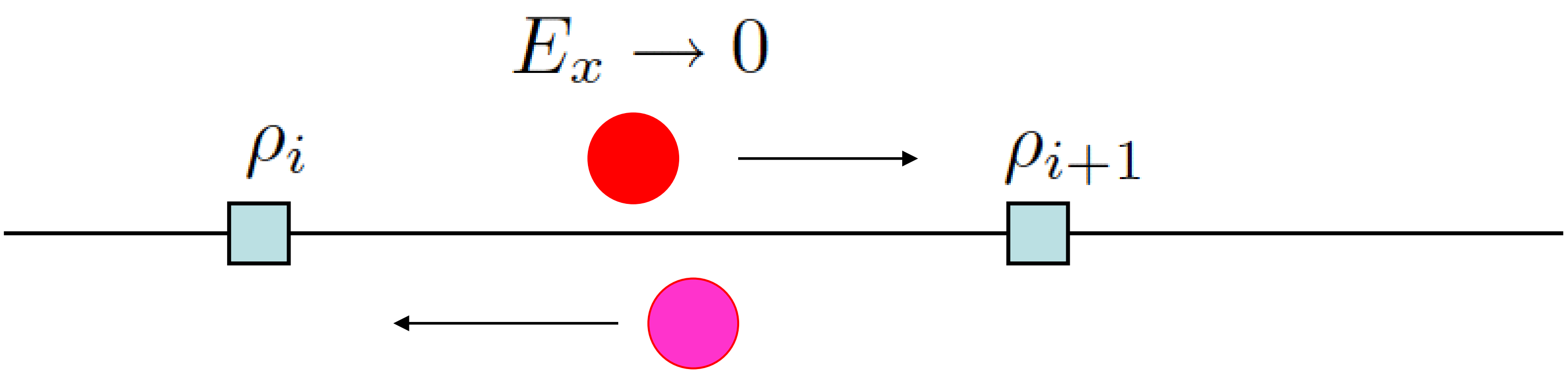
"Particle simulation of electromagnetic waves and its application to space plasmas"

Edited by H. Matsumoto and T. Sato (1985)

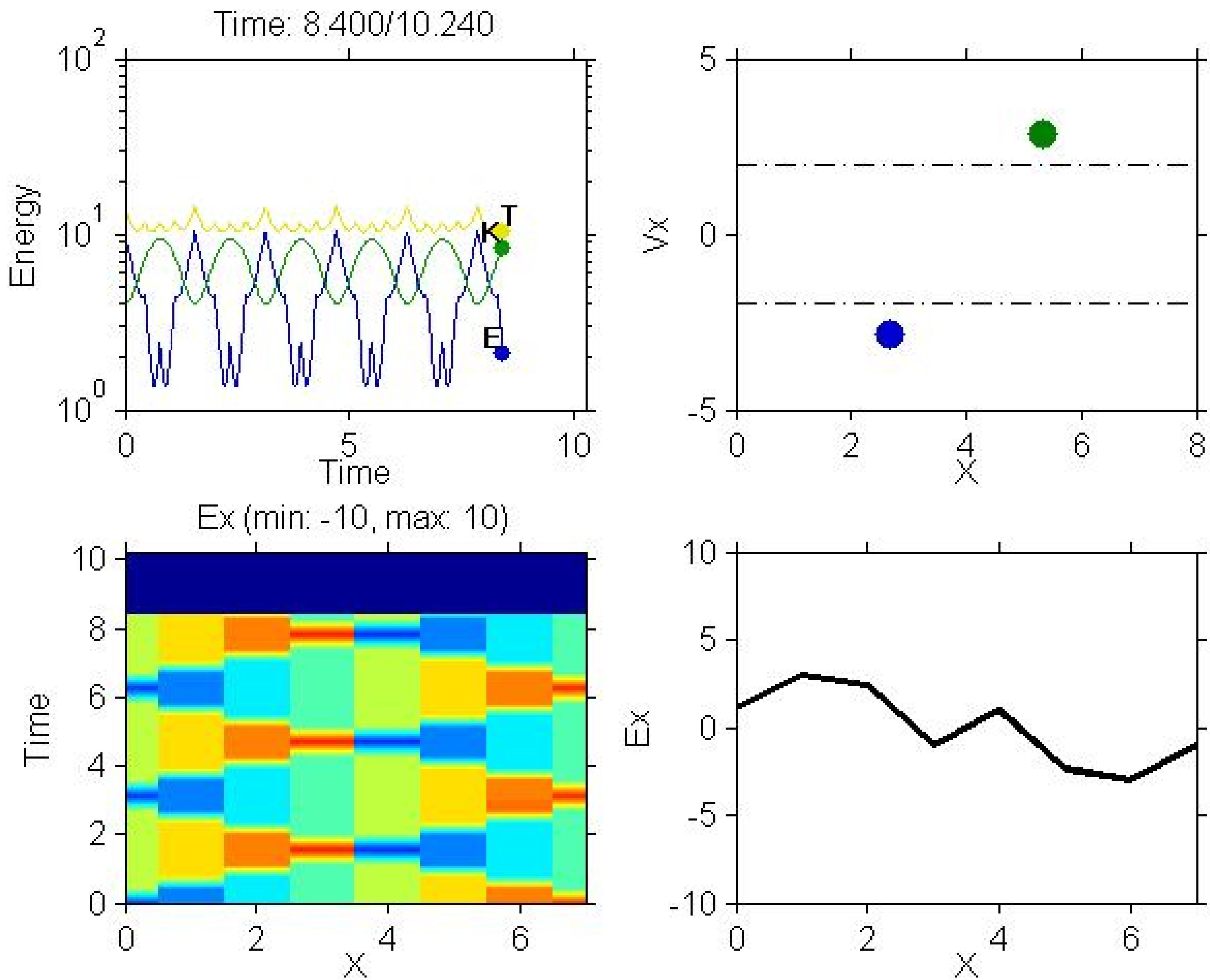
PIC code for Space Plasmas

- **Space Plasmas**: **Collisionless**
- **Particle-In-Cell Code**
- **Particles**: x(t), **v**(t)
- **Fields**: ***E***(t, X), ***B***(t, X)

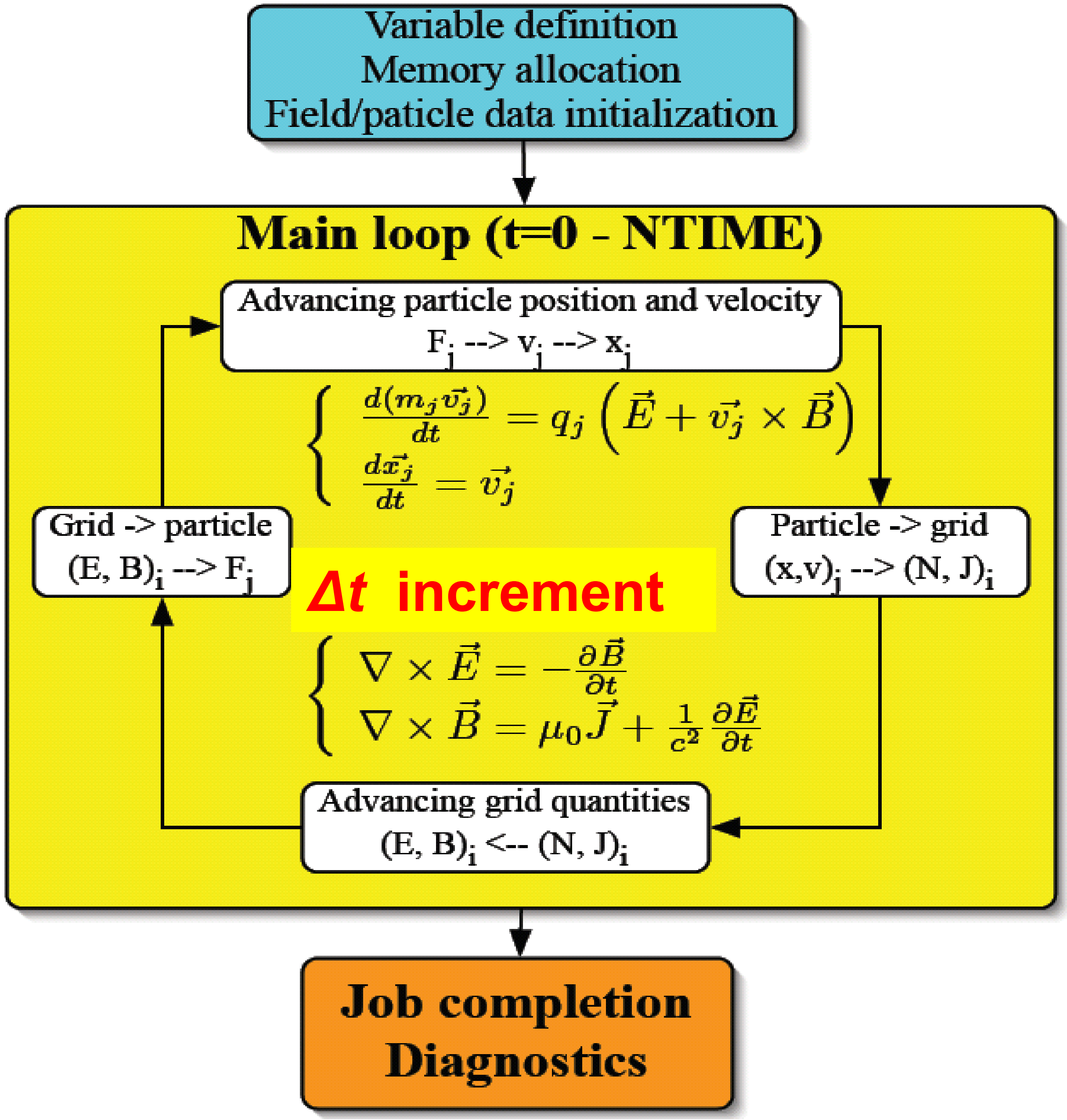
  E and ***B*** are defined on grid points, and calculated from ρ and ***J***. The electrostatic force between two particles in the same cell disappears.



KEMPO1



Basic procedure of PIC simulation code



Maxwell's Equations

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}$$

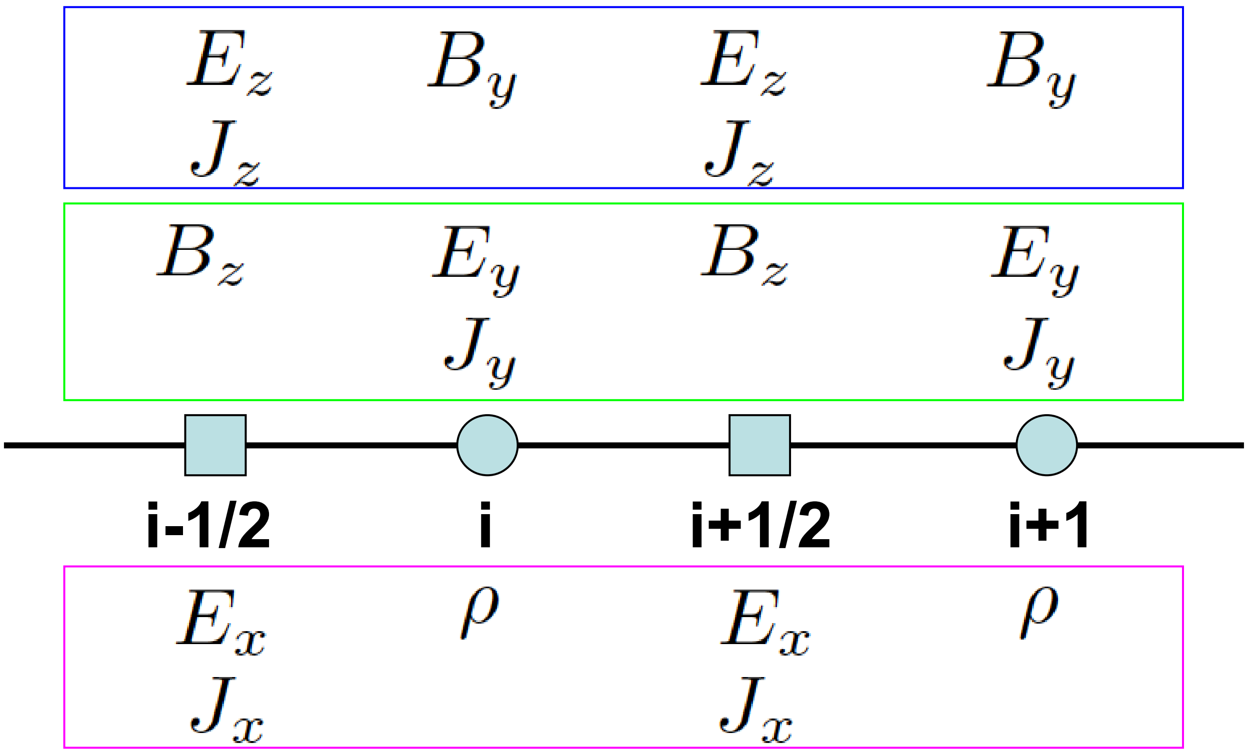
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_o}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

where $\varepsilon_0 \mu_0 = \dfrac{1}{c^2}$

Grid Assignment



$$\frac{\partial E_z}{\partial t} = c^2 \frac{\partial B_y}{\partial x} - J_z$$

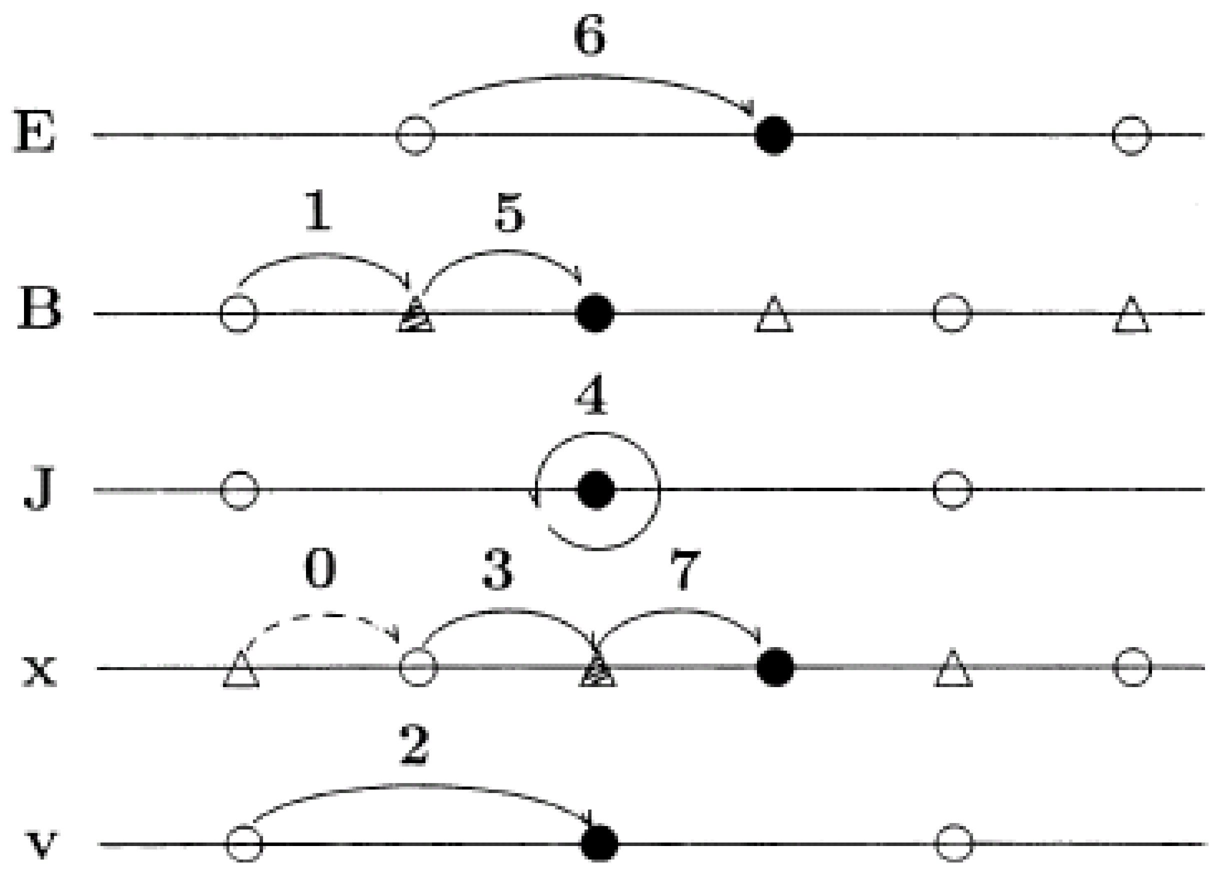
$$\frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x}$$

$$\frac{\partial E_y}{\partial t} = -c^2 \frac{\partial B_z}{\partial x} - J_y$$

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x}$$

$$\frac{\partial E_x}{\partial t} = -J_x$$

Time Step Chart



Centered Difference Scheme

$$E(X_i, t) = E_o \exp(ikX_i - i\omega t)$$

$$\frac{\partial E(X_i, t)}{\partial x} = \frac{E(X_i + \Delta x/2, t) - E(X_i - \Delta x/2, t)}{\Delta x}$$

$$= \frac{1}{\Delta x}\left[\exp(ik\Delta x/2) - \exp(-ik\Delta x/2)\right] E(X_i, t)$$

$$= i\frac{\sin(k\Delta x/2)}{\Delta x/2} E(X_i, t) = iKE(X_i, t)$$

$$k \longrightarrow K = \frac{\sin(k\Delta x/2)}{\Delta x/2}$$

$$\omega \longrightarrow \Omega = \frac{\sin(\omega\Delta t/2)}{\Delta t/2}$$

Courant Condition

Electromagnetic modes in vacuum

$$\omega^2 = c^2 k^2$$

Centered Difference Scheme in space and time

$$\Omega^2 = c^2 K^2 \qquad K = \frac{\sin(k\Delta x/2)}{\Delta x/2}$$

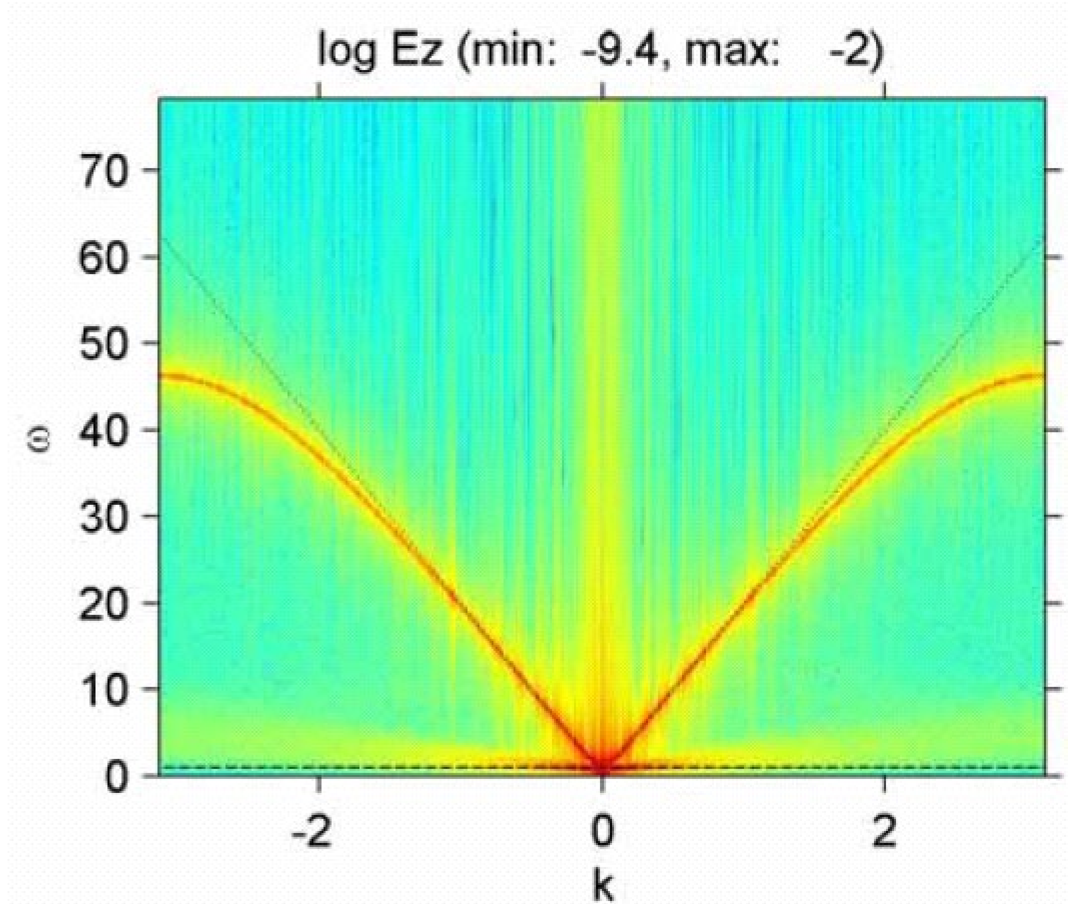
For $k = \dfrac{\pi}{\Delta x}$ we have $\sin(\dfrac{\omega \Delta t}{2}) = \dfrac{\Delta t}{\Delta x} c < 1$

Courant Condition

$$\boxed{c\Delta t < \Delta x}$$

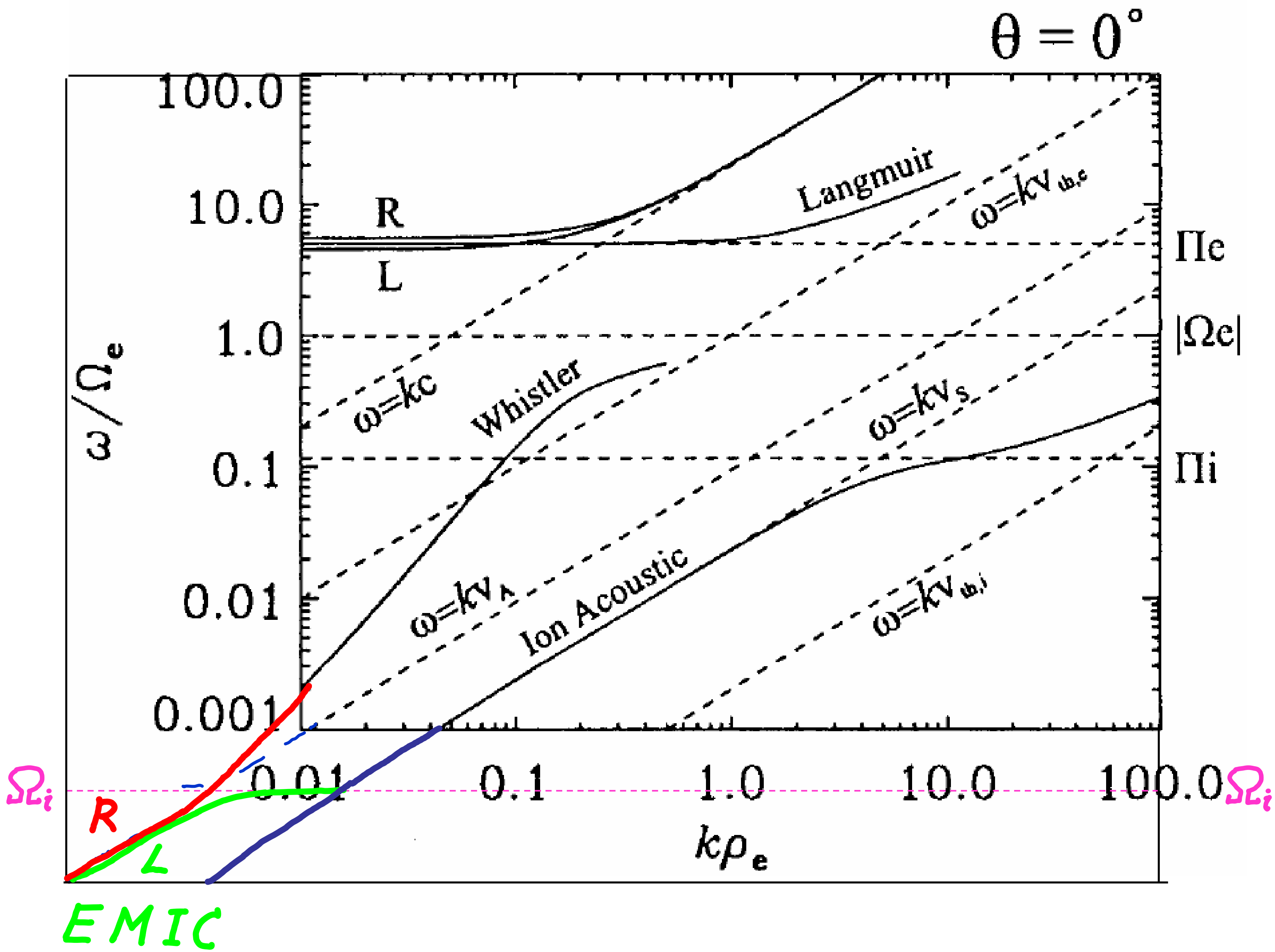
Dispersion Relation of Light Mode

$$\Omega^2 = c^2 K^2$$

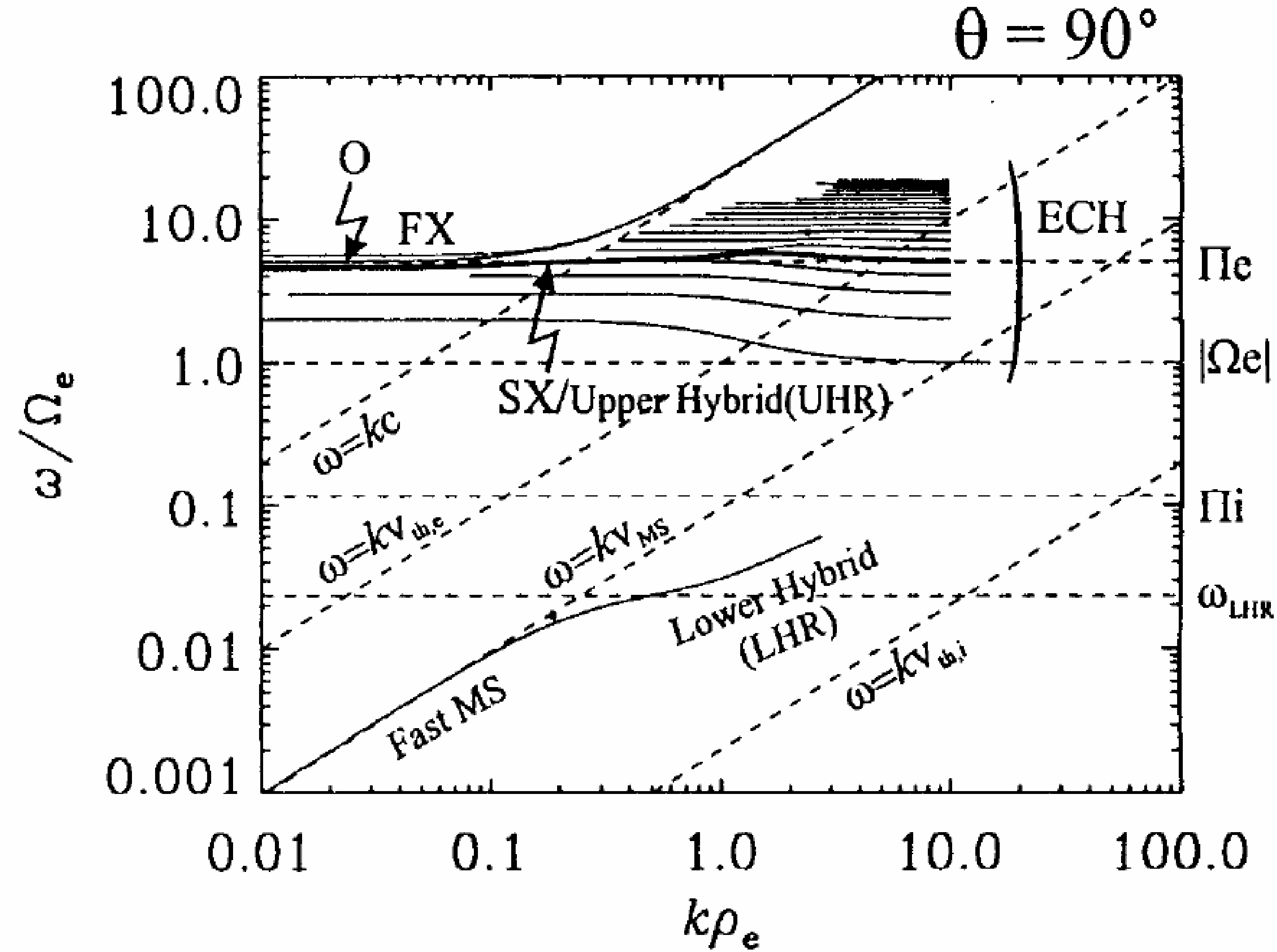


$$\Omega = \frac{\sin(\omega \Delta t/2)}{\Delta t/2}, \qquad K = \frac{\sin(k \Delta x/2)}{\Delta x/2}$$

Waves in Plasmas (Parallel Propagation)



Waves in Plasmas (Perpendicular Propagation)



Charge Density

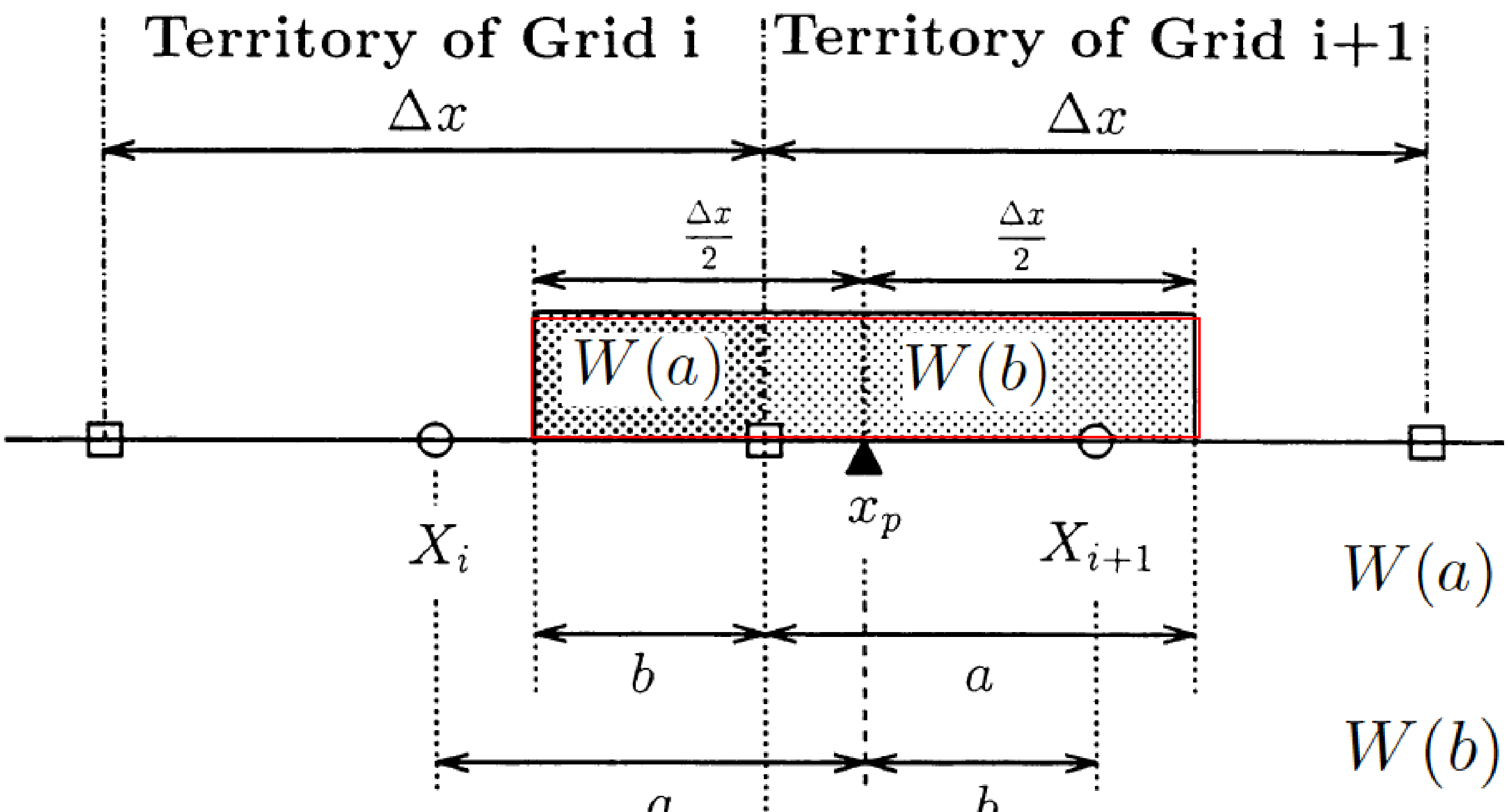
$$\rho_i = \frac{1}{\Delta x} \sum_{j}^{N_p} q_j W(x_j - X_i)$$

Shape Function

$$W(x) = 1 - \frac{|x|}{\Delta x}, \quad |x| \le \Delta x$$
$$= \quad 0, \quad |x| > \Delta x$$

Np: Number of Particles

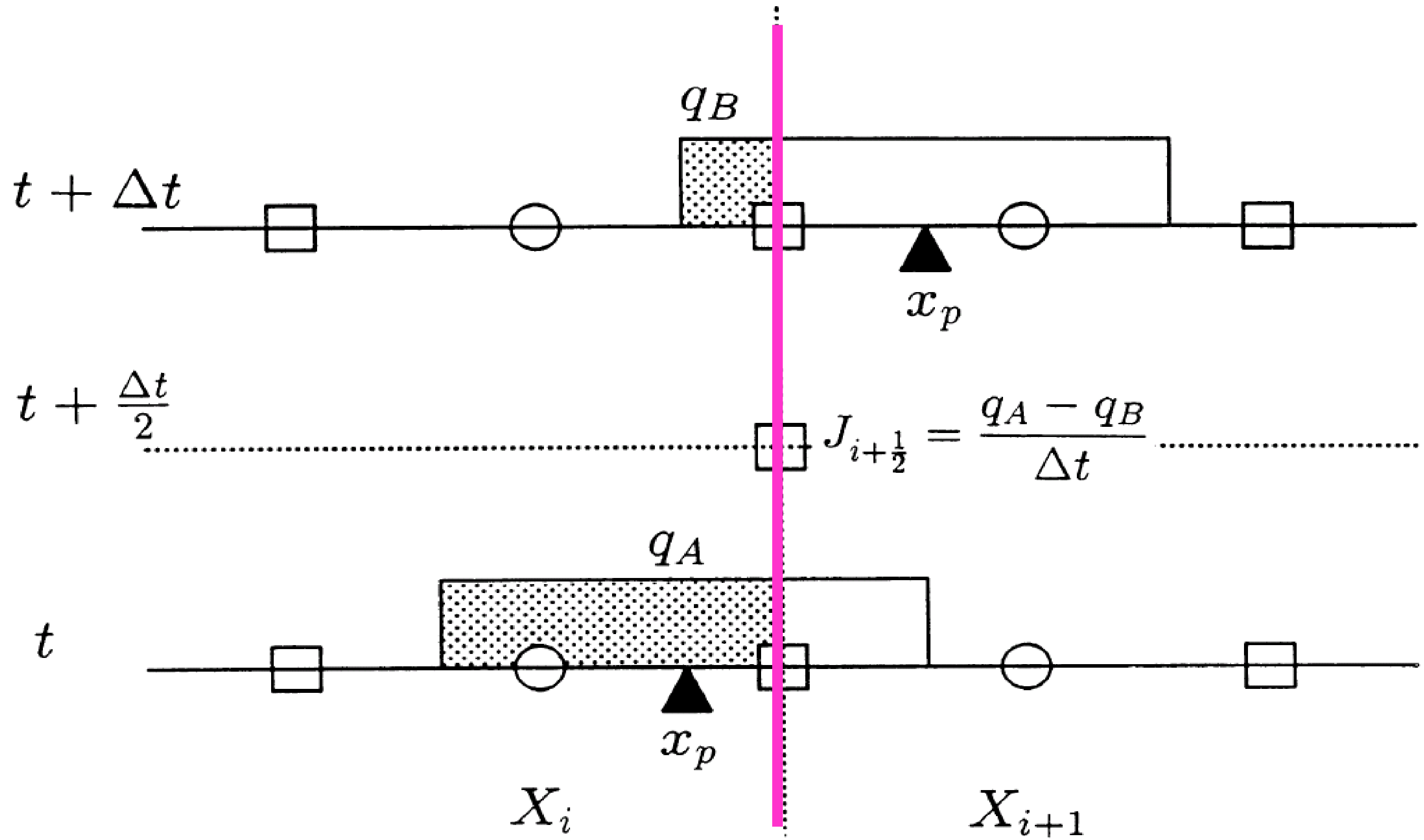
"fat particle"



$$W(a) = \frac{b}{\Delta x}$$

$$W(b) = \frac{a}{\Delta x}$$

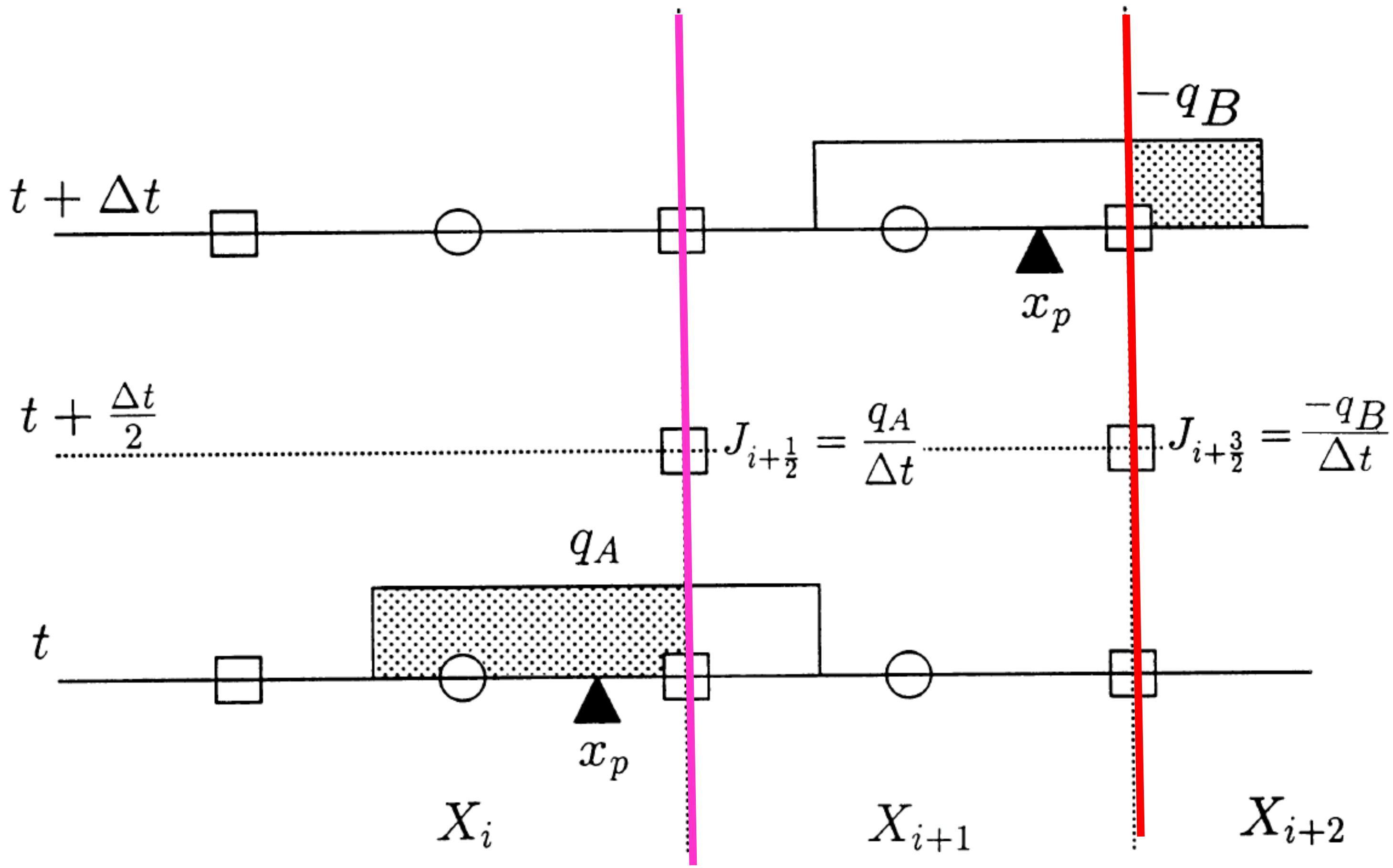
Current Density J_x: Case 1



$$q_A = q\frac{X_{i+1} - x_p(t)}{\Delta x}$$

$$q_B = q\frac{X_{i+1} - x_p(t + \Delta t)}{\Delta x}$$

Current Density J_x: Case 2



$$q_A = q\frac{X_{i+1} - x_p(t)}{\Delta x}$$

$$q_B = q\frac{X_{i+1} - x_p(t + \Delta t)}{\Delta x}$$

Program for Current Density Computation

$$J_{x,i+1/2}^{t+\Delta t/2} - J_{x,i-1/2}^{t+\Delta t/2} = -\frac{\Delta x}{\Delta t}(\rho_i^{t+\Delta t} - \rho_i^t)$$

```
for m = (n1+1):n2
%-- charge conversion method --
        qhs = qh * sign(vx(m));
        avx = abs(vx(m));
        x1 = x(m) + 2.0 -avx;
        x2 = x(m) + 2.0 +avx;
        i1 = floor(x1);
        i2 = floor(x2);
        ajx(i1) = ajx(i1) + (i2 - x1)*qhs;
        ajx(i2) = ajx(i2) + (x2 - i2)*qhs;
end
```

Only valid for particles satisfying $|v\Delta t| < \Delta x$

which is automatically satisfied in the relativistic code.

Cancellation of Uniform Current

$$\frac{\partial \boldsymbol{J}_u}{\partial t} = \frac{n_e e^2}{m_e} \boldsymbol{E}_u$$

$$\frac{\partial \boldsymbol{E}_u}{\partial t} = -\boldsymbol{J}_u$$

$$\boldsymbol{J}_u = \boldsymbol{J}_o \exp\left(i\omega_{pe} t\right) \qquad \boldsymbol{E}_u = \frac{i}{\omega_{pe}} \boldsymbol{J}_o \exp\left(i\omega_{pe} t\right)$$

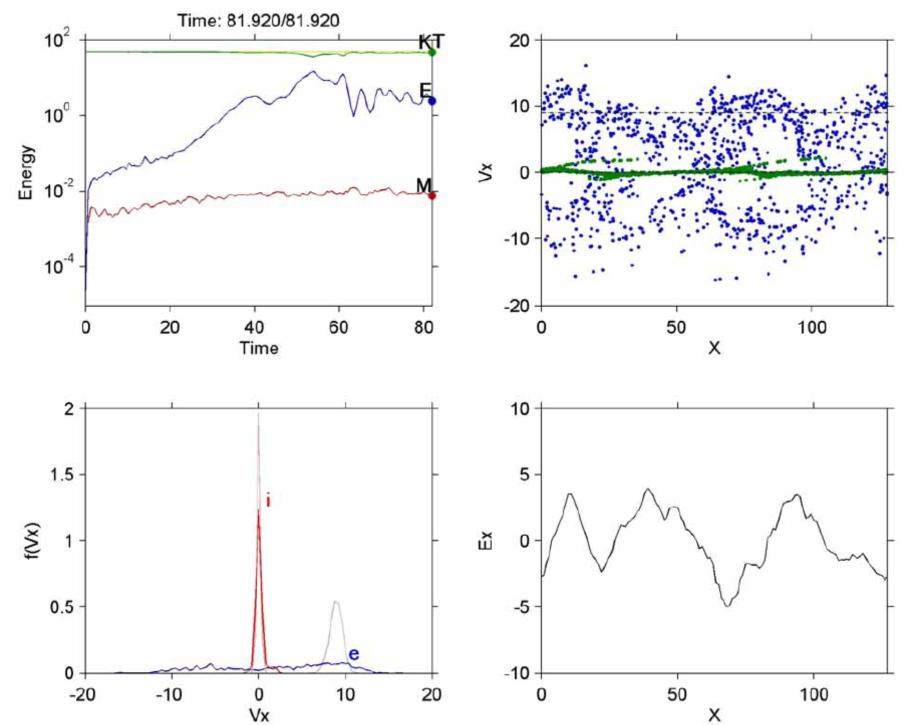
The uniform component must be subtracted.

$$\boldsymbol{J}_u = \frac{1}{N_x} \sum_{i=1}^{N_x} \boldsymbol{J}_i \qquad \boldsymbol{J}_{i,sub} = \boldsymbol{J}_i - \boldsymbol{J}_u$$

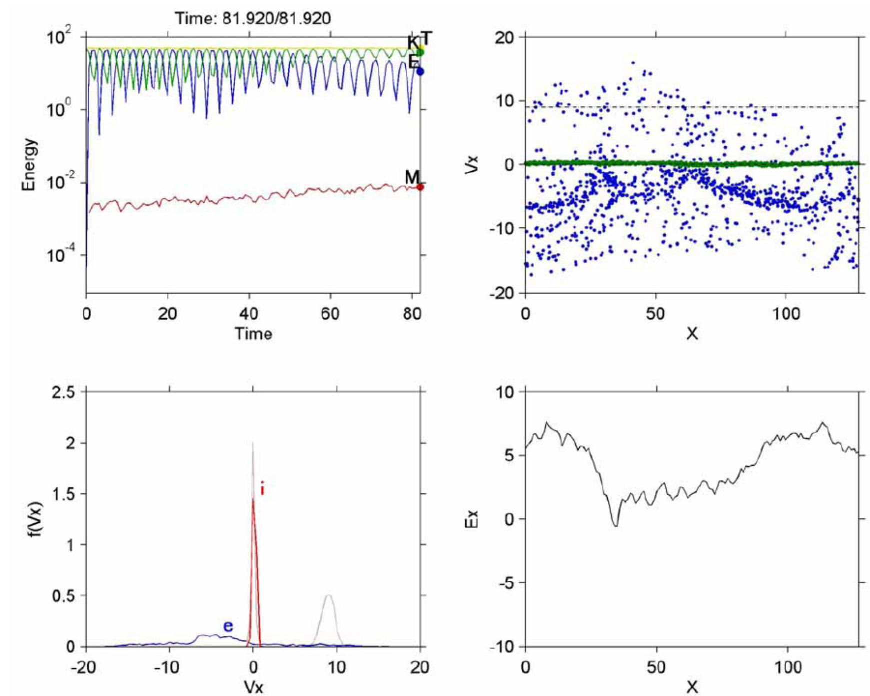
"Periodic systems are current-neutral." (V. Decyk)

KEMPO1

Without Uniform Current



With Uniform Current

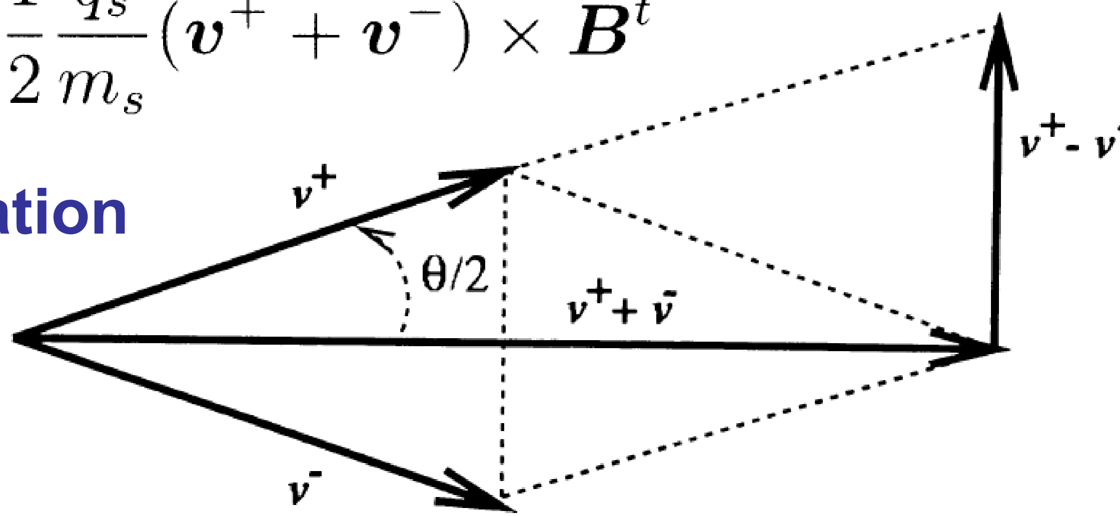


Buneman-Boris Method

$$\frac{\boldsymbol{v}^{t+\Delta t/2} - \boldsymbol{v}^{t-\Delta t/2}}{\Delta t} = \frac{q_s}{m_s}\left(\boldsymbol{E}^t + \frac{\boldsymbol{v}^{t+\Delta t/2} + \boldsymbol{v}^{t-\Delta/2}}{2} \times \boldsymbol{B}^t\right)$$

$$\boldsymbol{v}^- = \boldsymbol{v}^{t-\Delta t/2} + \frac{q_s}{m_s}\boldsymbol{E}^t\frac{\Delta t}{2} \qquad \boldsymbol{v}^+ = \boldsymbol{v}^{t+\Delta t/2} - \frac{q_s}{m_s}\boldsymbol{E}^t\frac{\Delta t}{2}$$

$$\frac{\boldsymbol{v}^+ - \boldsymbol{v}^-}{\Delta t} = \frac{1}{2}\frac{q_s}{m_s}(\boldsymbol{v}^+ + \boldsymbol{v}^-) \times \boldsymbol{B}^t$$



Kinetic Energy Conservation

$$\boxed{(\boldsymbol{v}^+)^2 = (\boldsymbol{v}^-)^2}$$

Small Phase Delay

$$\Omega_c = \frac{\tan^{-1}\omega_c\Delta t/2}{\Delta t/2}$$

$$\boxed{\Omega_c/\omega_c = 0.9967 \text{ with } \omega_c\Delta t = 0.2}$$

Relativistic Equation of Motion

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}(m\boldsymbol{v}) = q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right)}$$

$$m = \gamma m_0$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$\boldsymbol{u} = \frac{c}{\sqrt{c^2 - |\boldsymbol{v}|^2}}\boldsymbol{v}$$

$$\boldsymbol{B}_u = \frac{c}{\sqrt{c^2 + |\boldsymbol{u}|^2}}\boldsymbol{B}$$

$$\boxed{\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = \frac{q}{m_0}\left(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}_{\boldsymbol{u}}\right)}$$

$$\boldsymbol{v} = \frac{c}{\sqrt{c^2 + |\boldsymbol{u}|^2}}\boldsymbol{u}$$

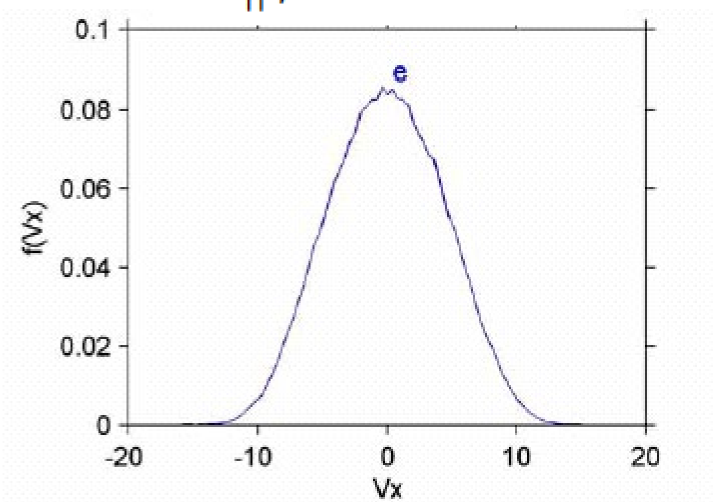
Initial Velocity Distribution Function

$$f(u_{\|}, u_{\perp}) \propto \exp\left(-\frac{(u_{\|} - V_{d\|})^2}{2V_{t\|}^2} - \frac{(u_{\perp} - V_{d\perp})^2}{2V_{t\perp}^2}\right)$$

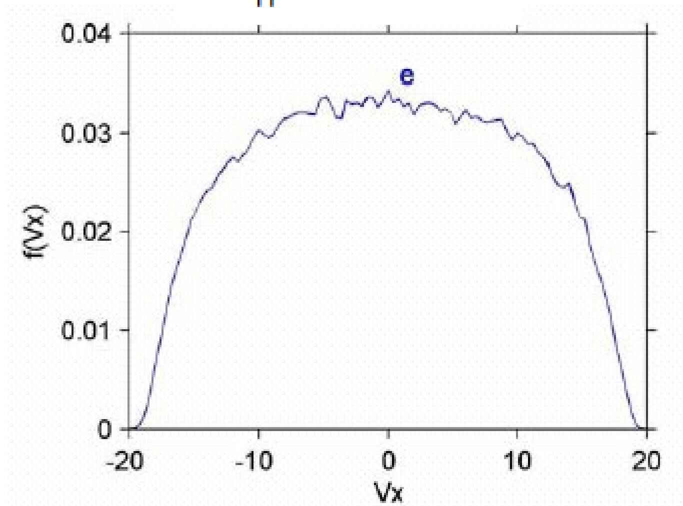
$$\boldsymbol{v} = \boldsymbol{u}/\gamma = \frac{c}{\sqrt{c^2 + u_x^2 + u_y^2 + u_z^2}}\boldsymbol{u}$$

$$V_{d\|} = V_{d\perp} = 0$$

$V_{t\|}/c = 0.25$



$V_{t\|}/c = 1$

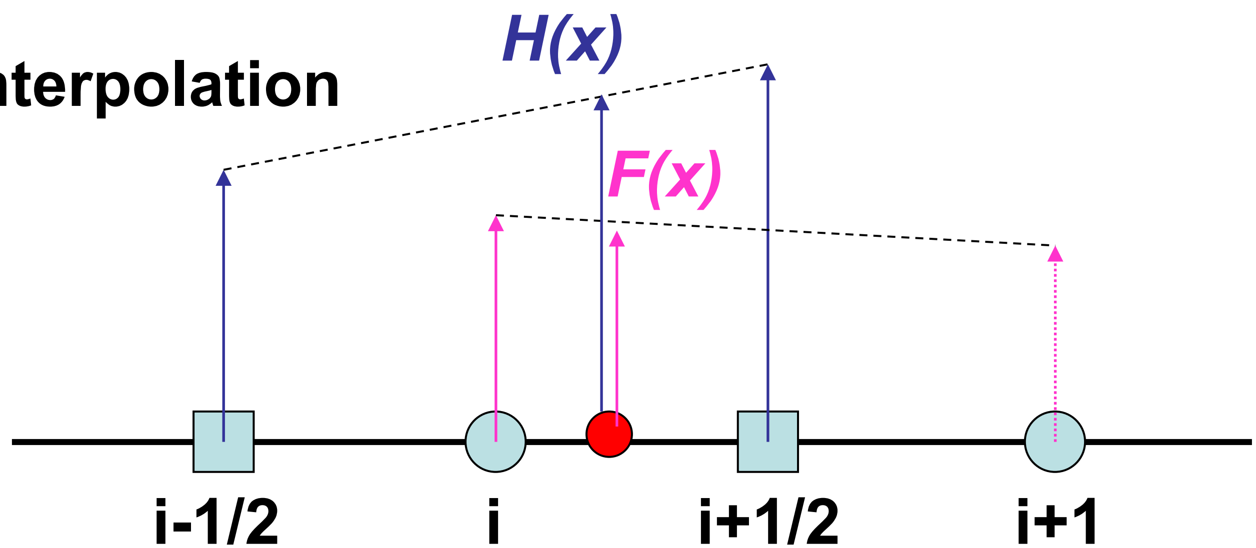


Field Interpolation to Particle Position

$$F(x) = \sum_{i=1}^{N_x} \boxed{F_i} W(x - X_i)$$

$$H(x) = \sum_{i=1}^{N_x} \boxed{H_{i+1/2}} W(x - X_{i+1/2})$$

Linear Interpolation



Electrostatic Self-force Cancellation

$$\rho_i = \frac{1}{\Delta x}\sum_{j}^{N_p} q_j \underline{W(x_j - X_{\boxed{i}})}$$

$$\frac{E_{x,i+1/2} - E_{x,i-1/2}}{\Delta x} = \frac{\rho_i}{\varepsilon_0}$$

$$E_x(x) = \sum_{i=1}^{N_x} E_{x,i+1/2}\underline{W(x - X_{\boxed{i+1/2}})}$$

Self-force

Relocation

$$E_{x,i} = \frac{E_{x,i-1/2} + E_{x,i+1/2}}{2}$$

$$E_x(x) = \sum_{i=1}^{N_x} E_{x,i}\underline{W(x - X_{\boxed{i}})}$$

No Self-force

Magnetostatic Self-force Cancellation

Magnetostatic equation (Ampere's Law) $\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}$

$$\boldsymbol{J}_{yz,\boxed{i+1/2}} = \frac{1}{\Delta x} \sum_{j}^{N_p} q_j \boldsymbol{v}_{yz} W(x_j - X_{\boxed{i+1/2}})$$

Source Relocation

$$J_{y,i} = \frac{J_{y,i-1/2} + J_{y,i+1/2}}{2}$$

$$\frac{B_{z,\boxed{i+1/2}} - B_{z,\boxed{i-1/2}}}{\Delta x} = -\mu_o J_{y,i}$$

Field Relocation

$$\frac{B_{y,i+1} - B_{y,i}}{\Delta x} = \mu_o J_{z,i+1/2}$$

$$B_{y,\boxed{i+1/2}} = \frac{B_{y,i} + B_{y,i+1}}{2}$$

Relative Unit System

We assume in KEMPO1

$$\varepsilon_0 = 1, \qquad \mu_0 = \frac{1}{c^2}$$

Reference Values for Space and Time

Speed of light: *c*

Characteristic frequency: ω max (ω_p , ω_c)

We choose Δt so that $\omega \Delta t << 2\pi$

We choose Δx so that $c\Delta t < \Delta x$

Charge Neutrality Condition

Number Density $$n_i = \frac{N_p}{N_x \Delta_x}$$

Nx: Number of Grid Points

Particle Charge $$q_i = \frac{\rho_i}{n_i} = \frac{\omega_{pi}^2}{(q_i/m_i)n_i}$$

q_i/m_i **is an arbitrary value.**

For simplicity, -1 for electrons.

0.0625, 0.01 or 1/1836 for protons.

$$\sum_i \rho_i = \sum_i q_i n_i = \boxed{\sum_i \frac{\omega_{pi}^2}{q_i/m_i} = 0}$$

Enhanced Thermal Fluctuation

Electrostatic Field Energy Density

$$F_E = \frac{T}{2}\int_{-\infty}^{\infty} \frac{1}{1+k^2\lambda_D^2}\frac{dk}{2\pi} = \frac{T}{4\lambda_D}$$

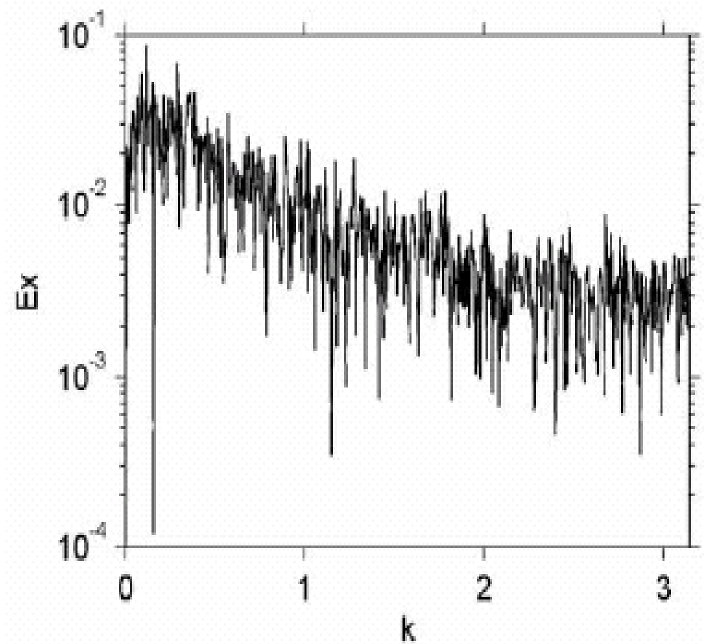
where $T\,(\equiv mV_t^2)$

Thermal Energy Density

$$T_E = \frac{1}{2}nT$$

Normalized Thermal Fluctuation

$$\frac{F_E}{T_E} = \frac{1}{2}\frac{1}{n\lambda_D} = \frac{1}{2}\boxed{\frac{N_x}{N_p}}\,\boxed{\frac{\Delta x}{\lambda_D}}$$



[*Ueda et al., Comp. Phys. Comm.* 1994]

Debye Length / Grid Spacing

Density Perturbation with a small potential $\delta\phi$

$$n \sim n_0 \exp(-\frac{q\delta\phi}{T})$$

Inserting into Poisson's Equation,

$$\frac{\partial^2 \delta\phi}{\partial x^2} \sim \frac{q^2 n}{\varepsilon_o T}\delta\phi = \frac{\omega_p^2}{V_t^2}\delta\phi = \frac{1}{\lambda_D^2}\delta\phi$$

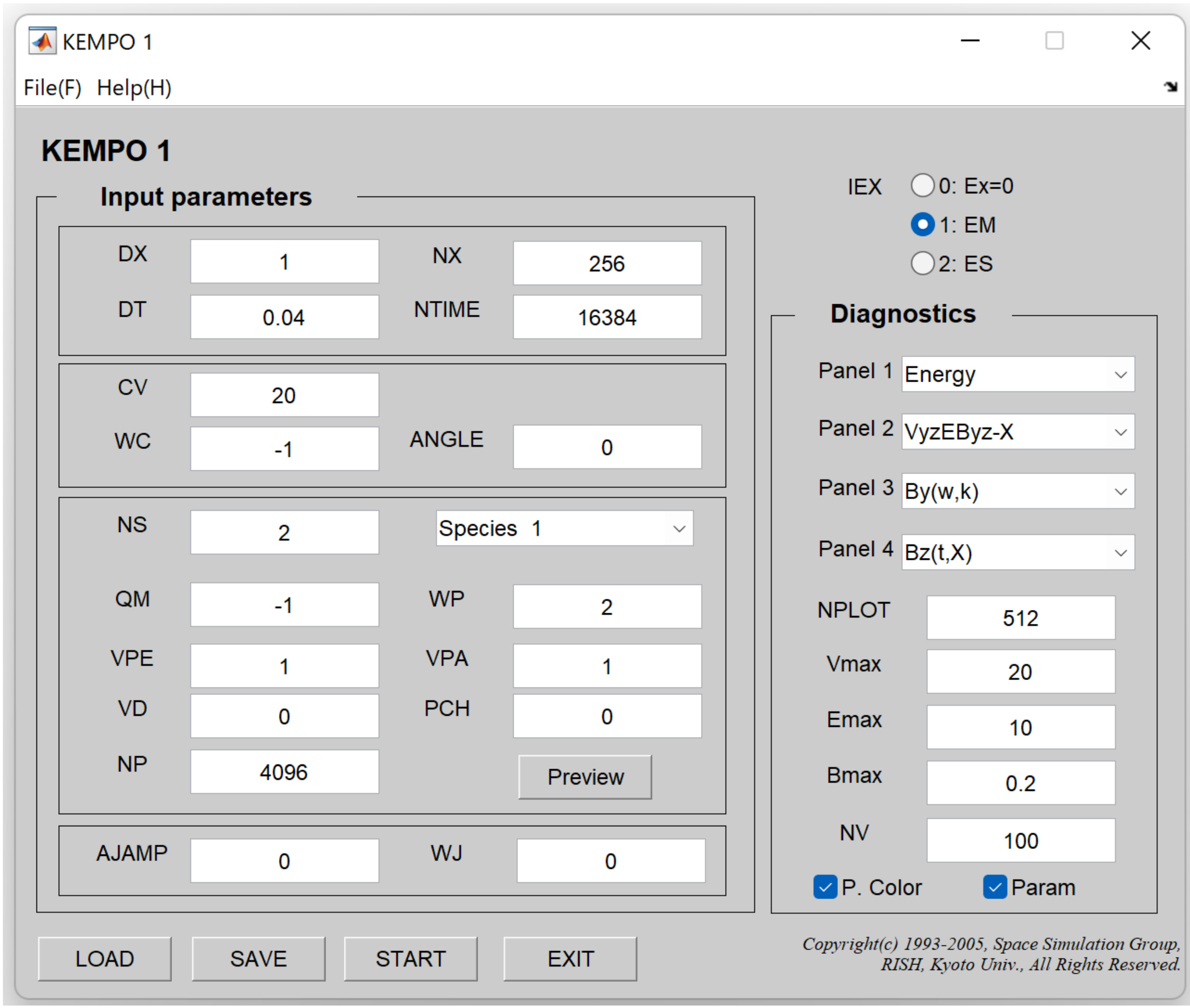
Fourier transformation

$$(k^2 - \frac{1}{\lambda_D^2})\delta\phi_k \sim 0 \qquad |k| \sim \frac{1}{\lambda_D}$$

Replacing *k* with *K*

$$|\sin(k\Delta x/2))| \sim \frac{\Delta x}{2\lambda_D} < 1 \qquad \boxed{\Delta x < 2\lambda_D}$$

GUI for Input Parameters

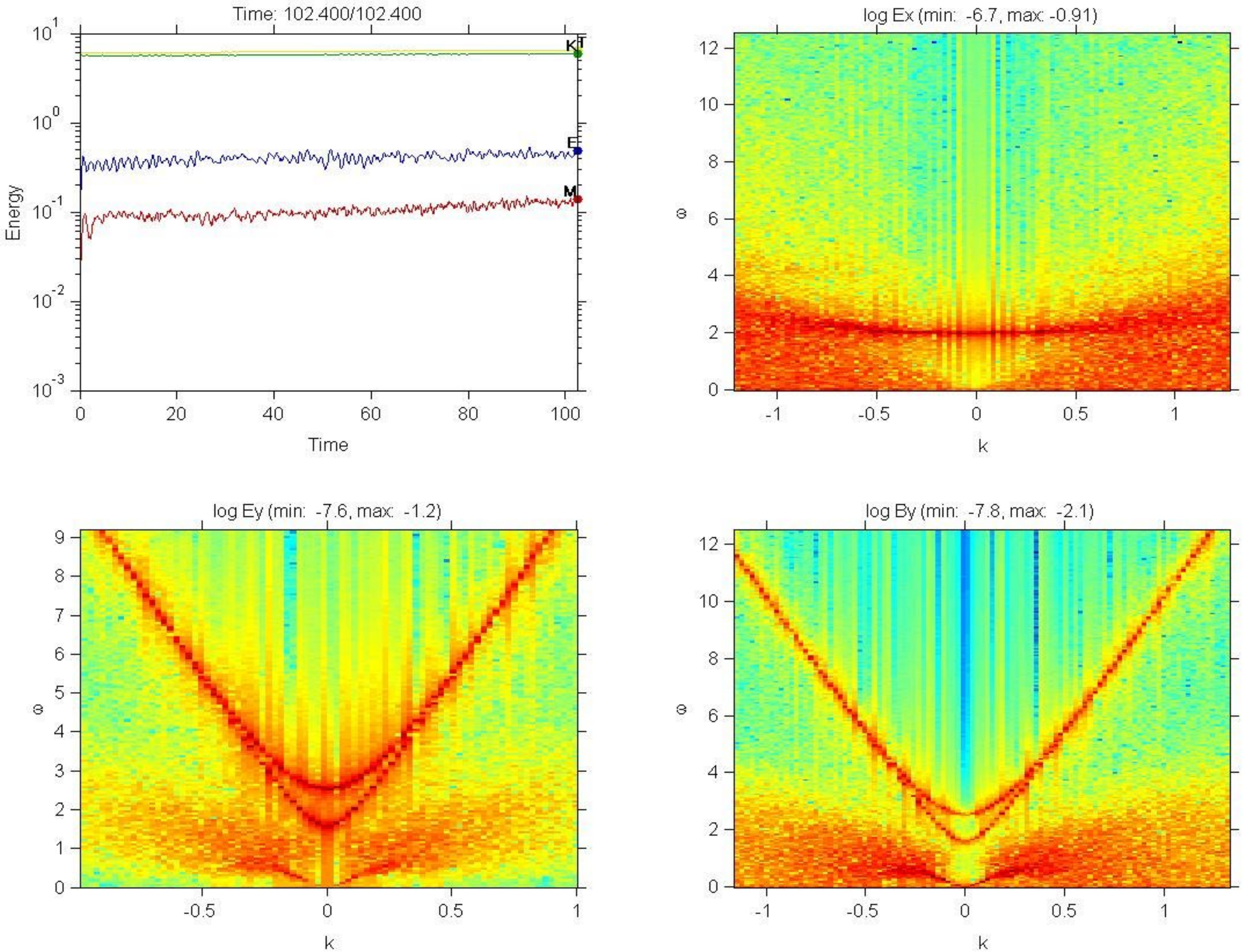


Renormalization

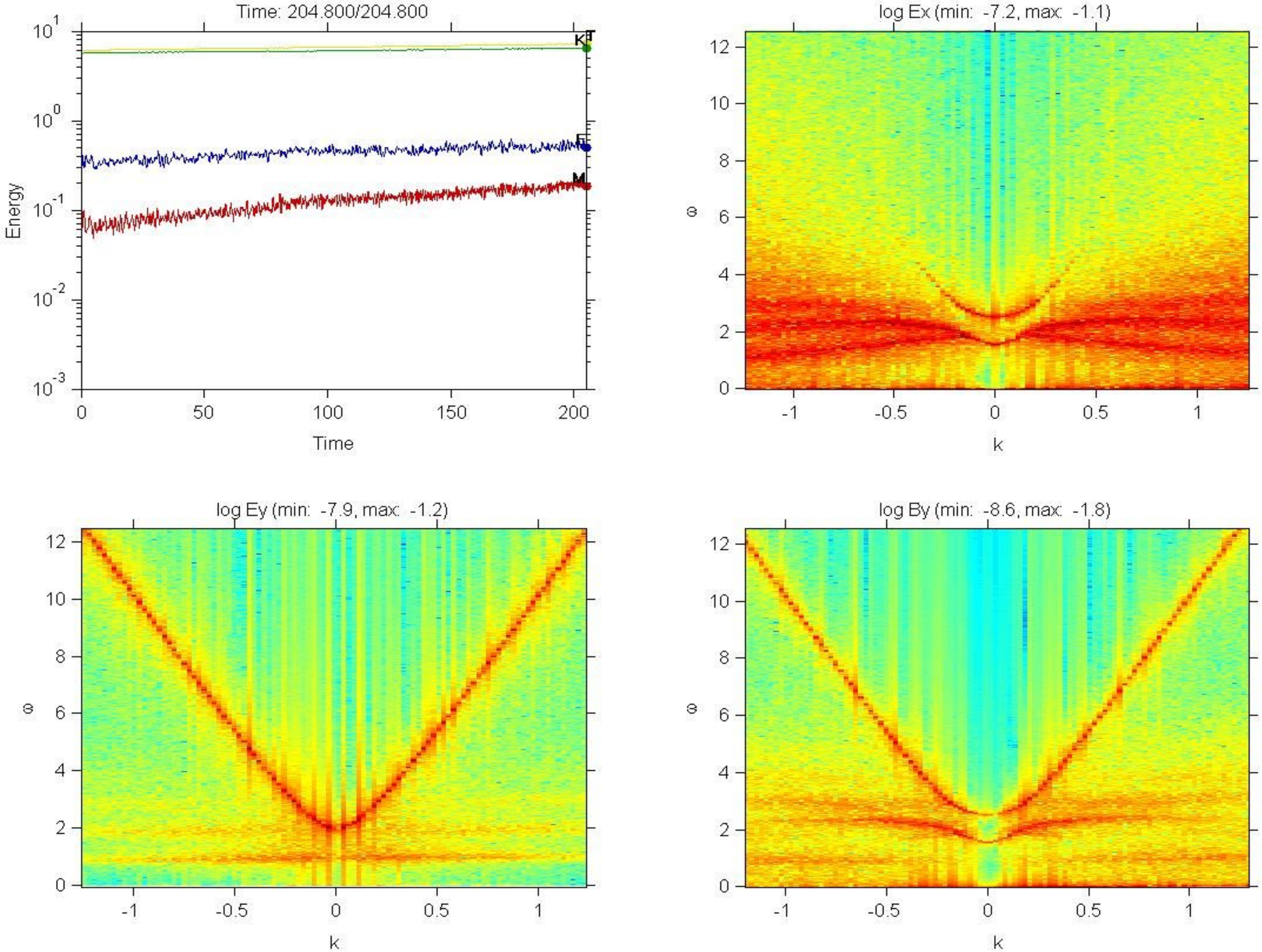
Simulation Unit System ← **Relative Unit System**

distance	$x_S = (1/\Delta x) x_R$
time	$t_S = (2/\Delta t) t_R$
velocity	$v_S = (\Delta t/2)(1/\Delta x) v_R$
electric field	$E_S = (\Delta t/2)^2 (1/\Delta x) E_R$
magnetic field	$B_S = (\Delta t/2) B_R$
charge density	$\rho_S = (\Delta t/2)^2 \rho_R$
current density	$J_S = (\Delta t/2)^3 (1/\Delta x) J_R$
energy density	$\sigma_S = (\Delta t/2)^4 (1/\Delta x)^2 \sigma_R$
number density	$n_S = \Delta x n_R$
charge	$q_S = (\Delta t/2)^2 (1/\Delta x) q_R$
mass	$m_S = (\Delta t/2)^2 (1/\Delta x) m_R$

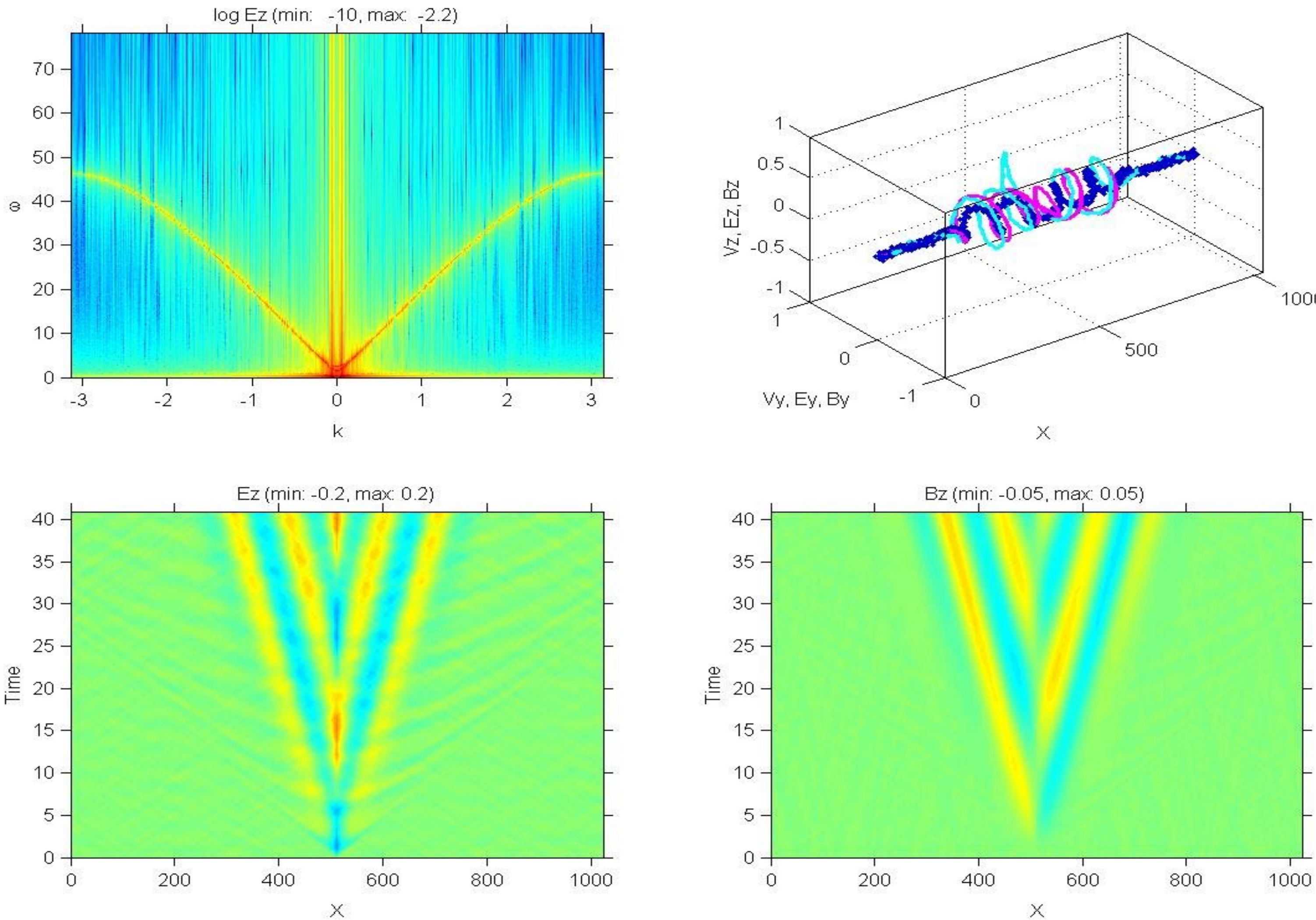
Normal Modes in the Parallel Direction



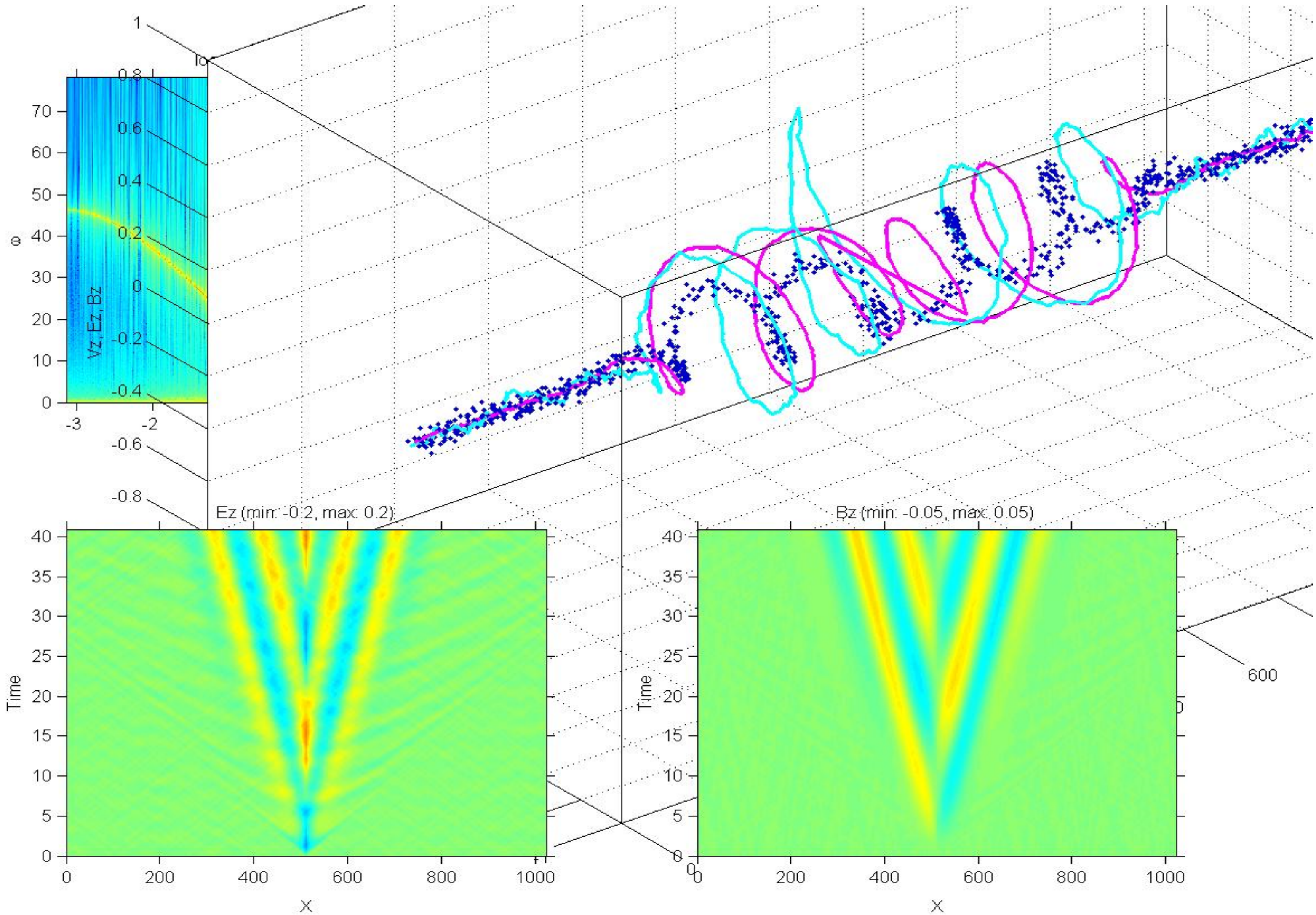
Normal Modes in the Perpendicular Direction



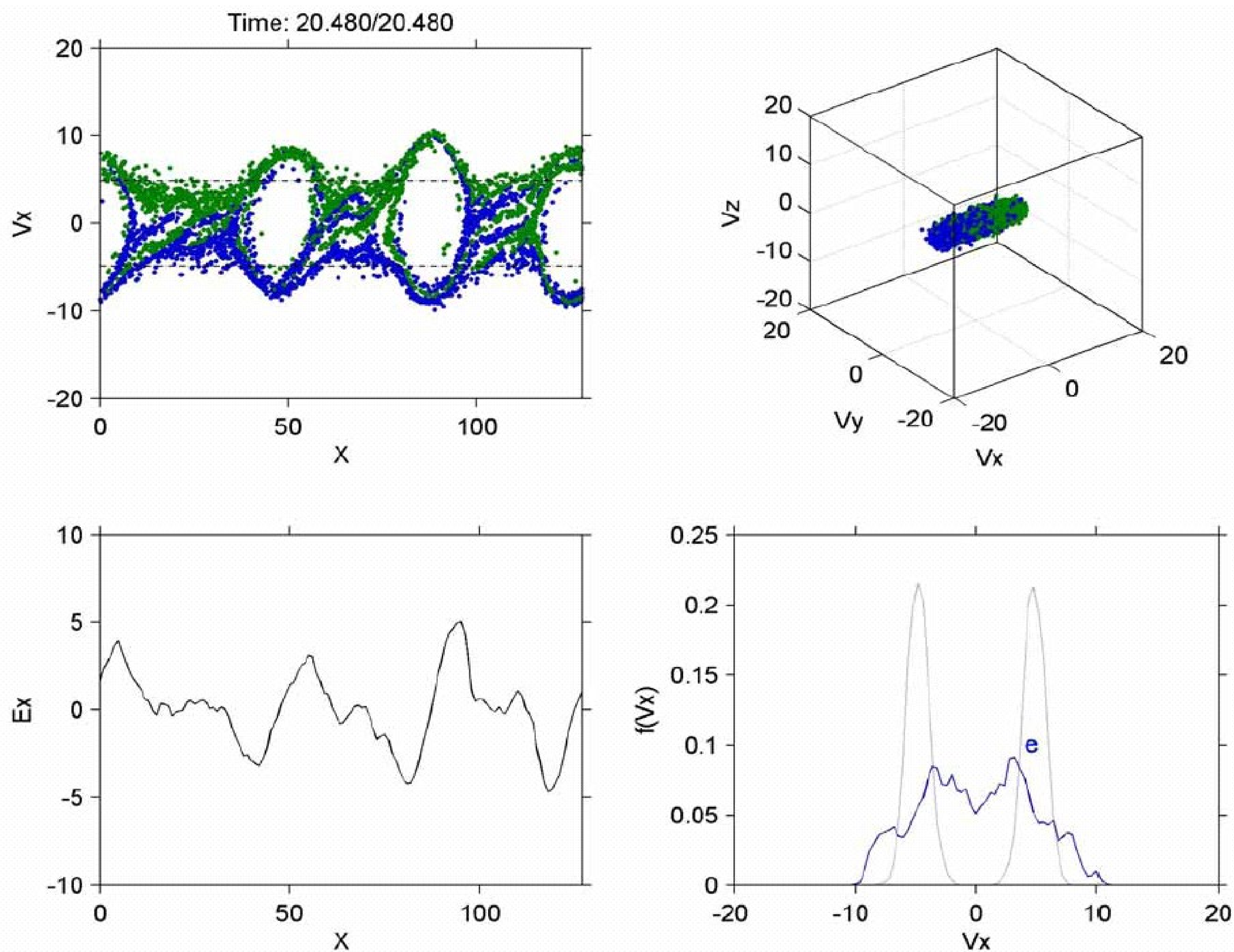
Whistler Mode Wave Radiation



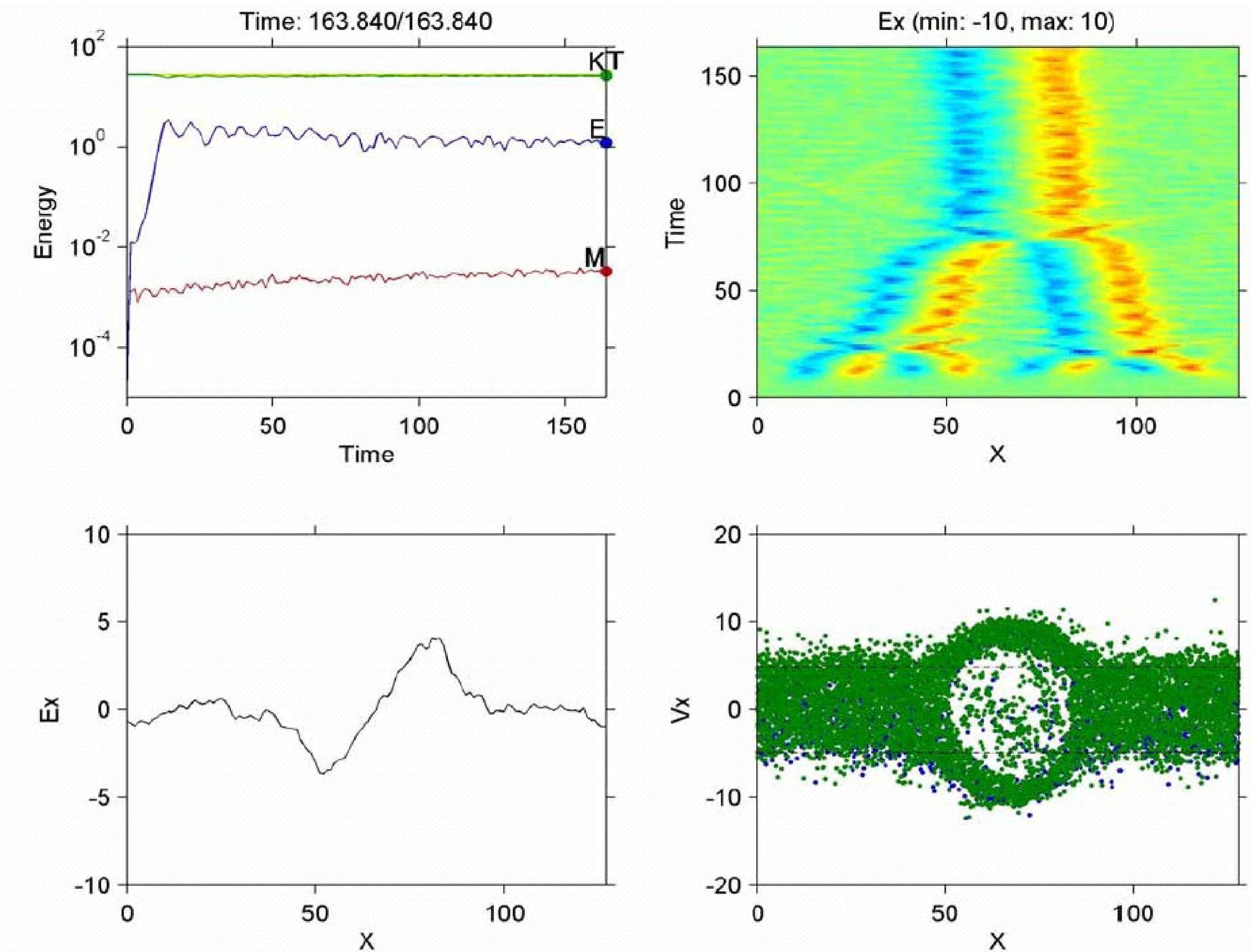
Circularly Polarized Wave



Electron Two-stream Instability

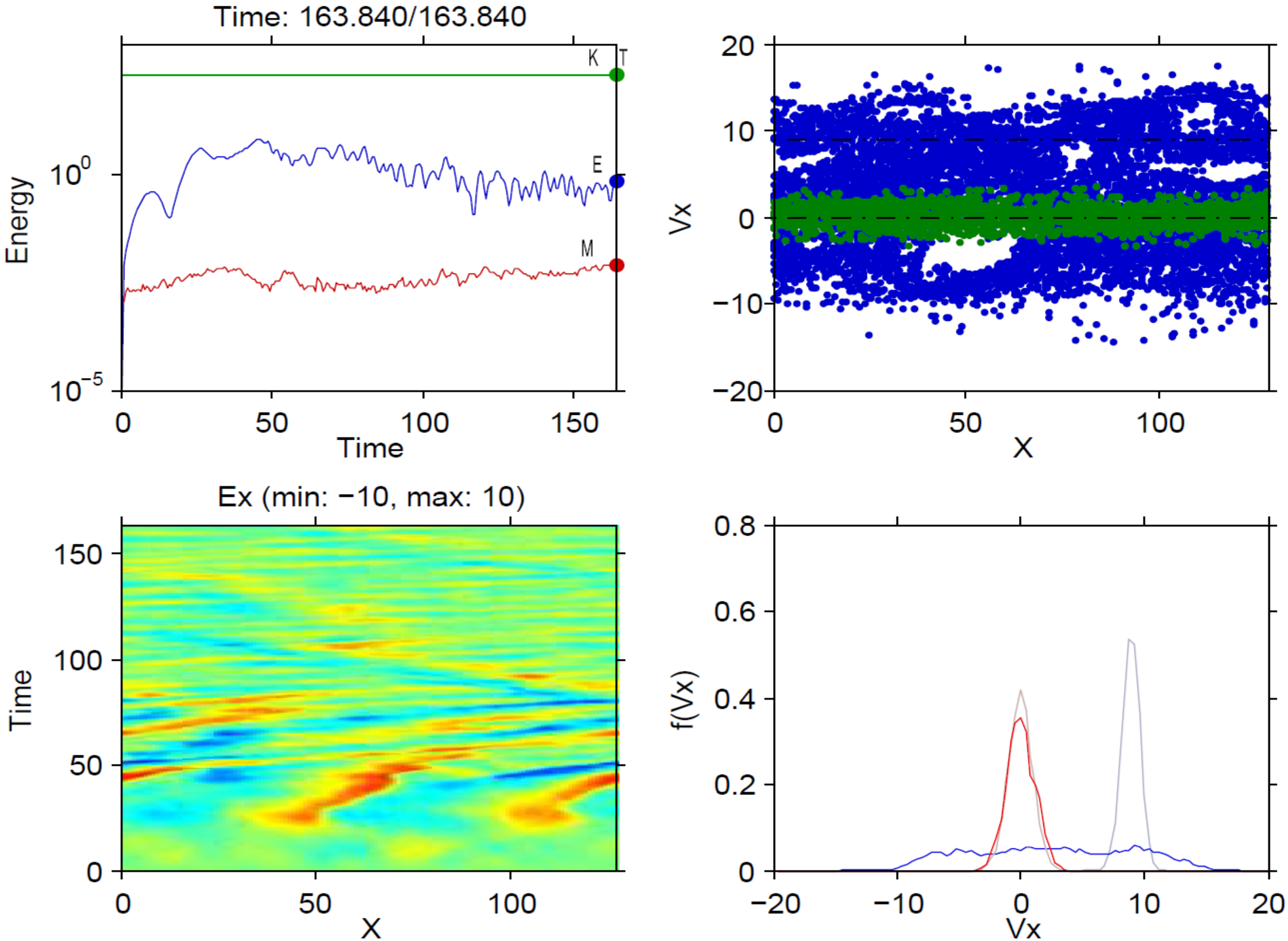


Electrostatic Solitary Waves



[*Omura et al., JGR,* 1996]

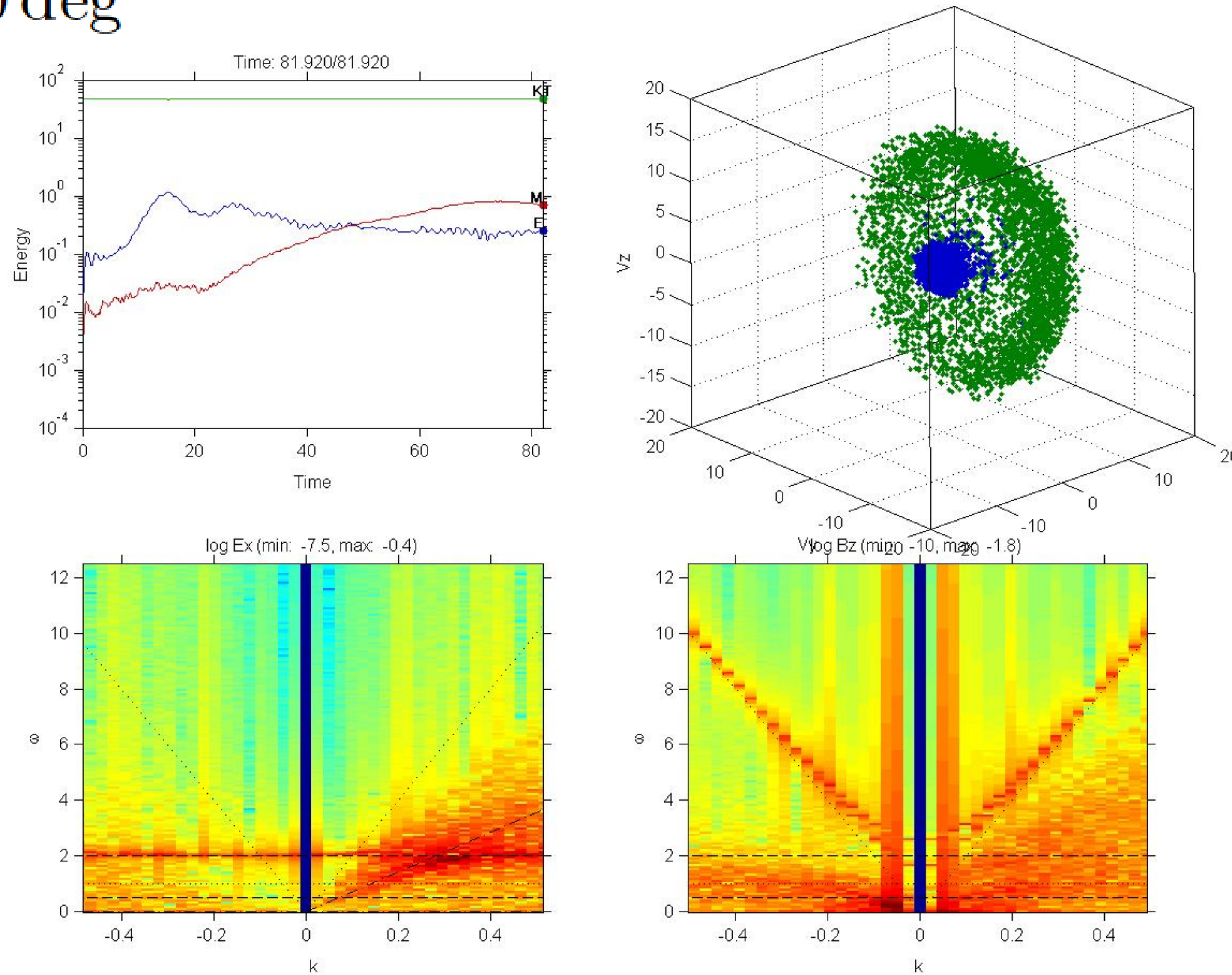
Buneman Instability



[e.g., *Omura et al., JGR*, 2003]

Electrostatic and Whistler Instabilities

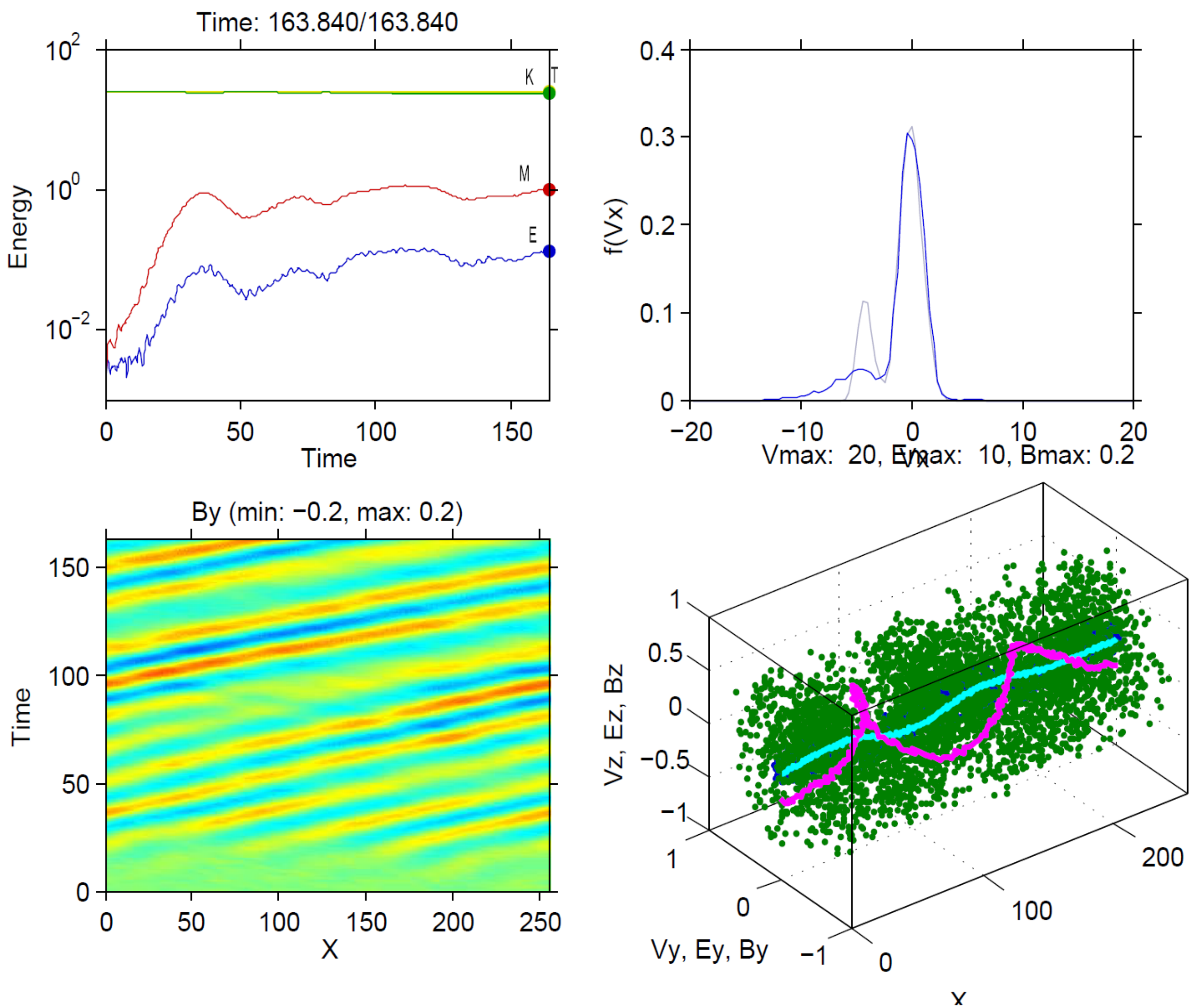
$$\theta = 0\,\mathrm{deg}$$



[*Omura and Matsumoto, JGR,* 1987]

Electron Beam Whistler Instability

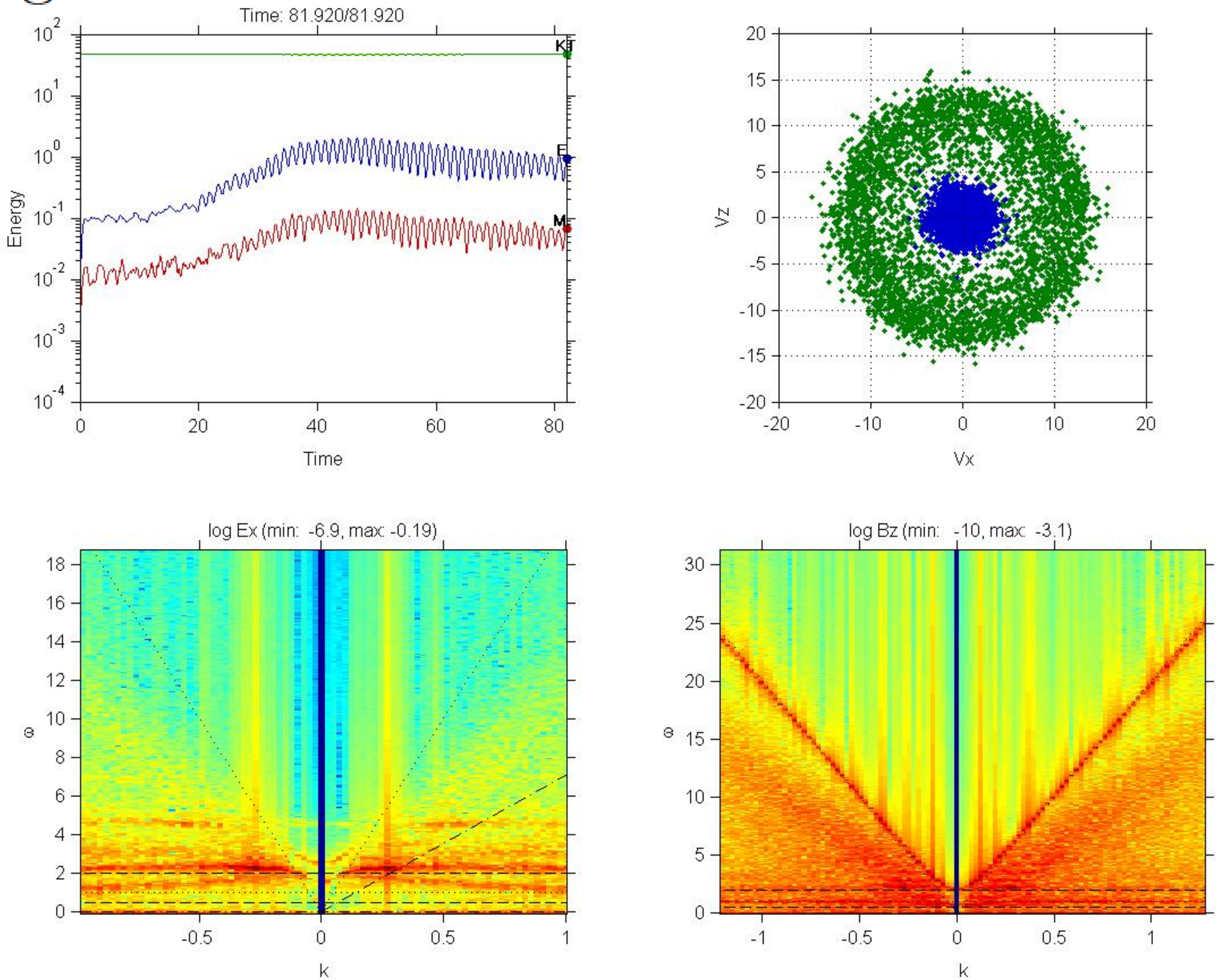
Ex = 0



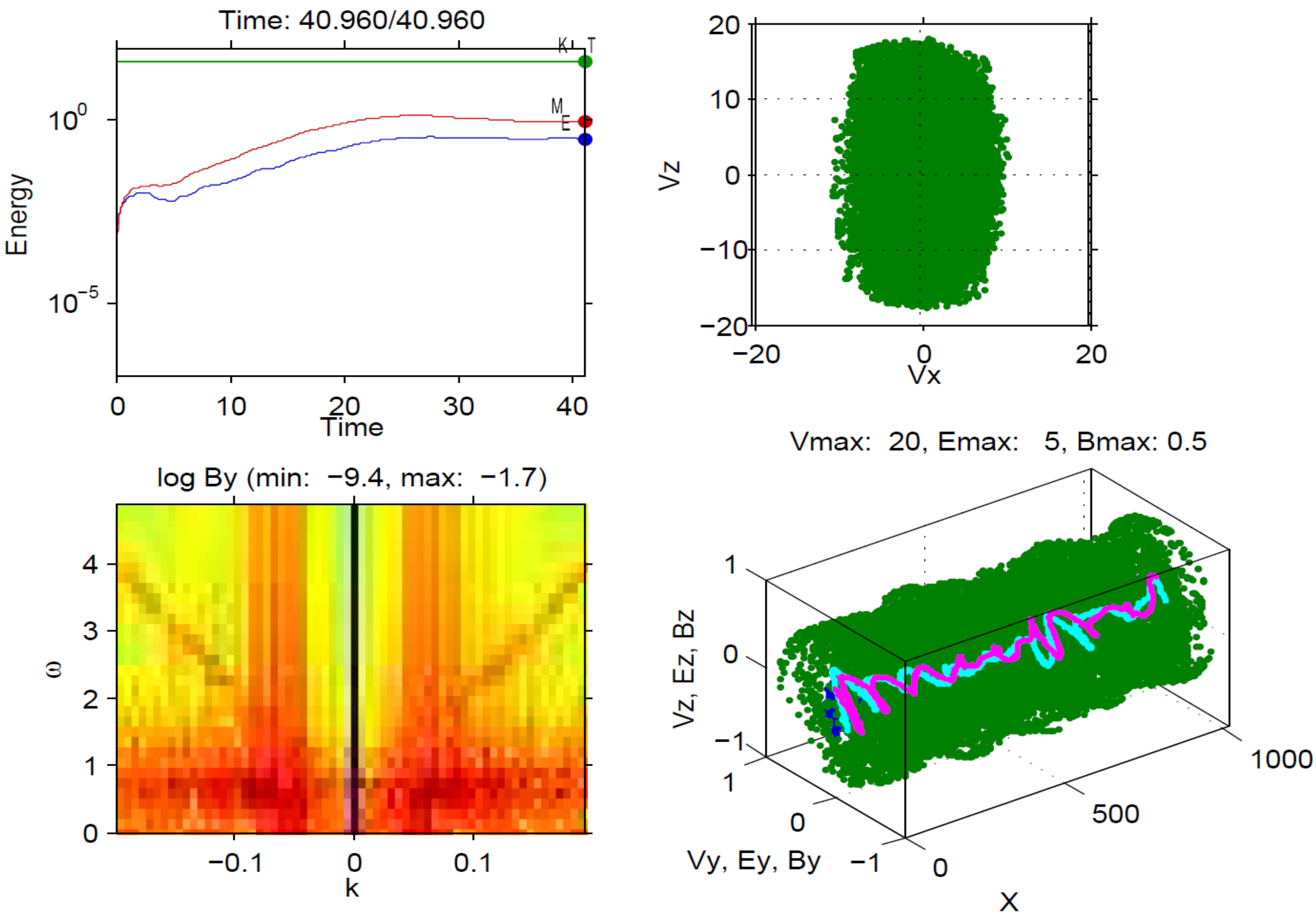
[*Matsumoto and Yasuda, Phys. Fluid*, 1976]

Electrostatic Cyclotron Wave Instability

$\theta = 90\,\mathrm{deg}$



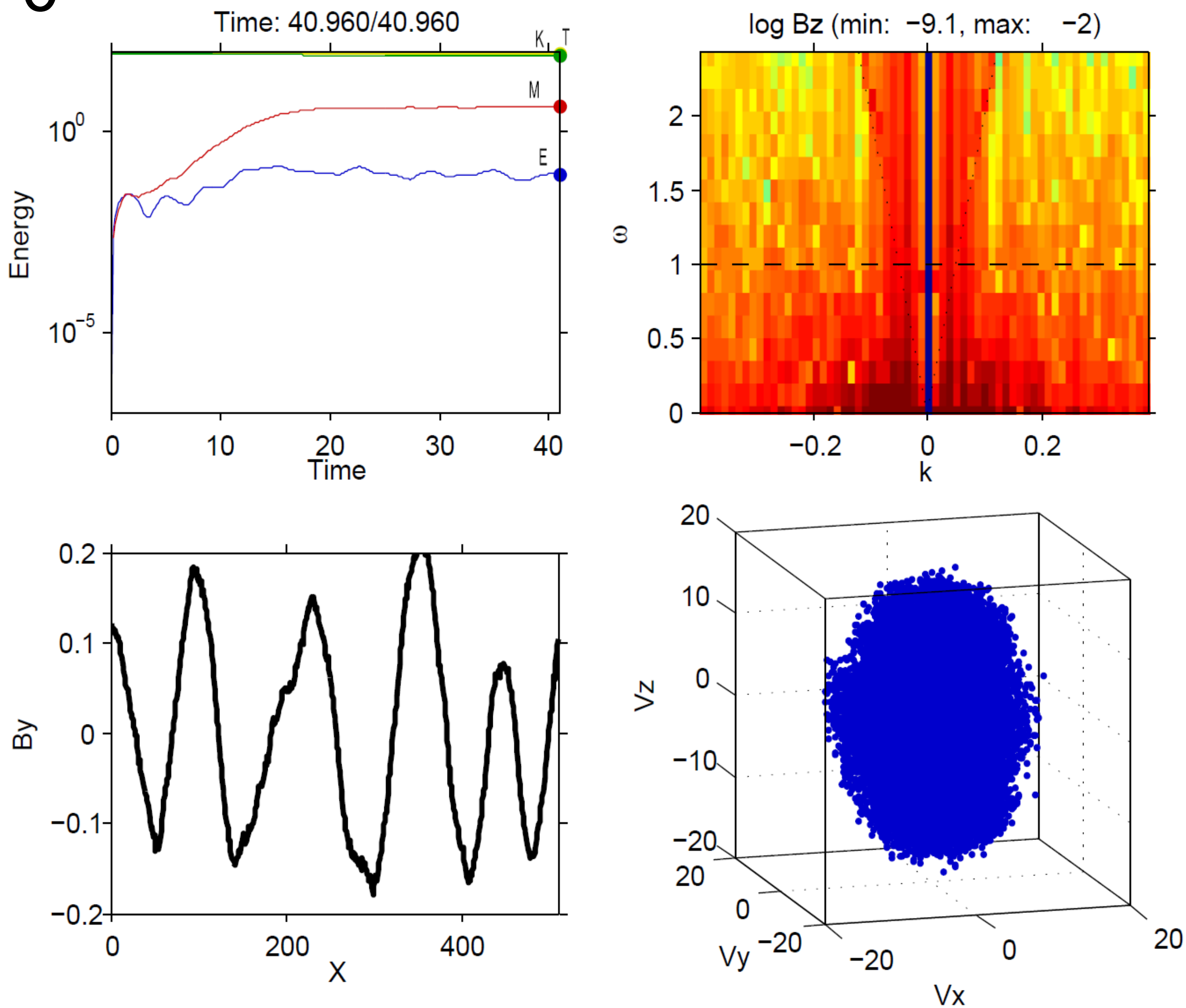
Relativistic Ring Beam Instability (Cyclotron Maser Instability)



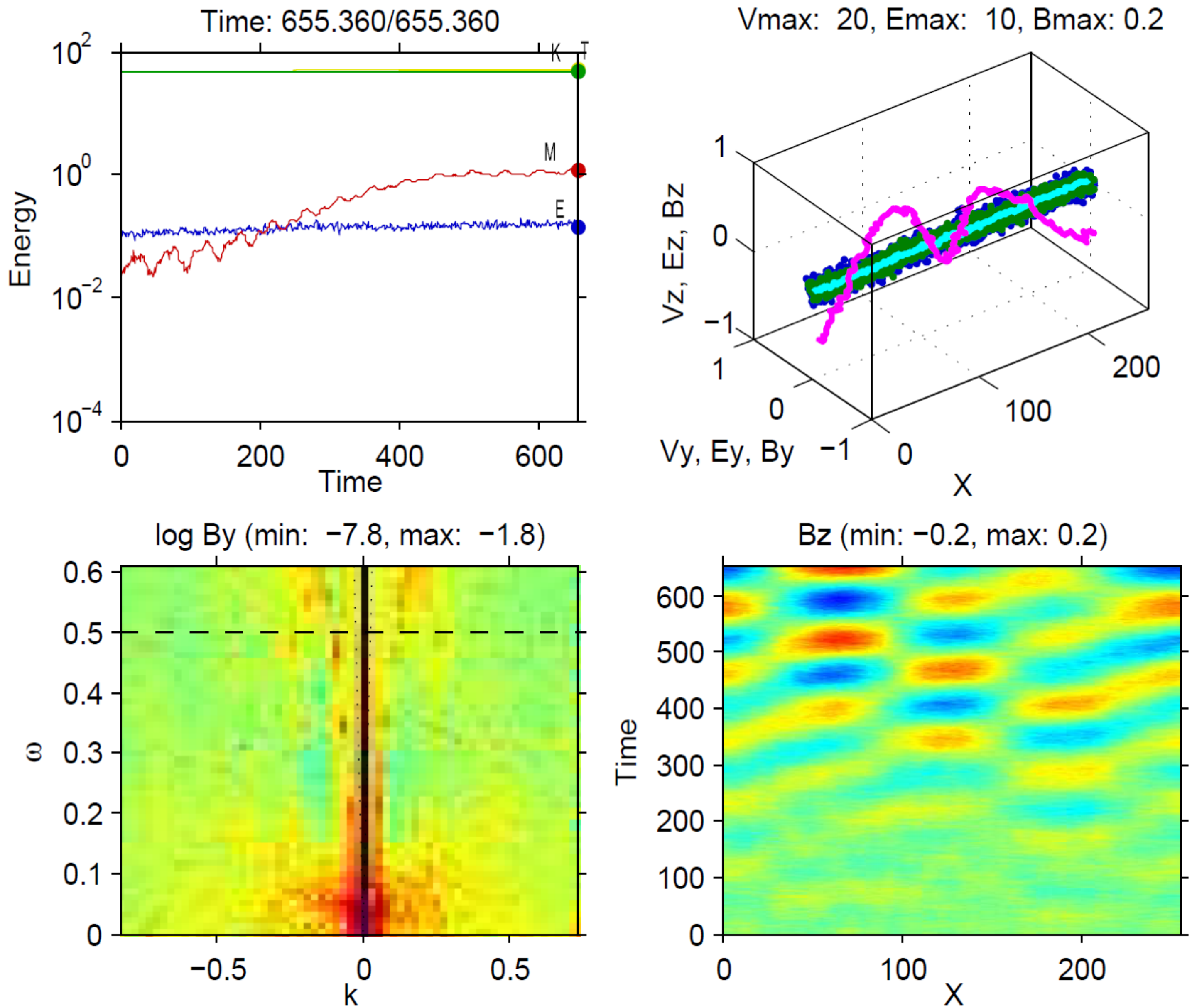
[cf.*Lee et al., Phys. Plasmas*, 2012]

Weibel Instability

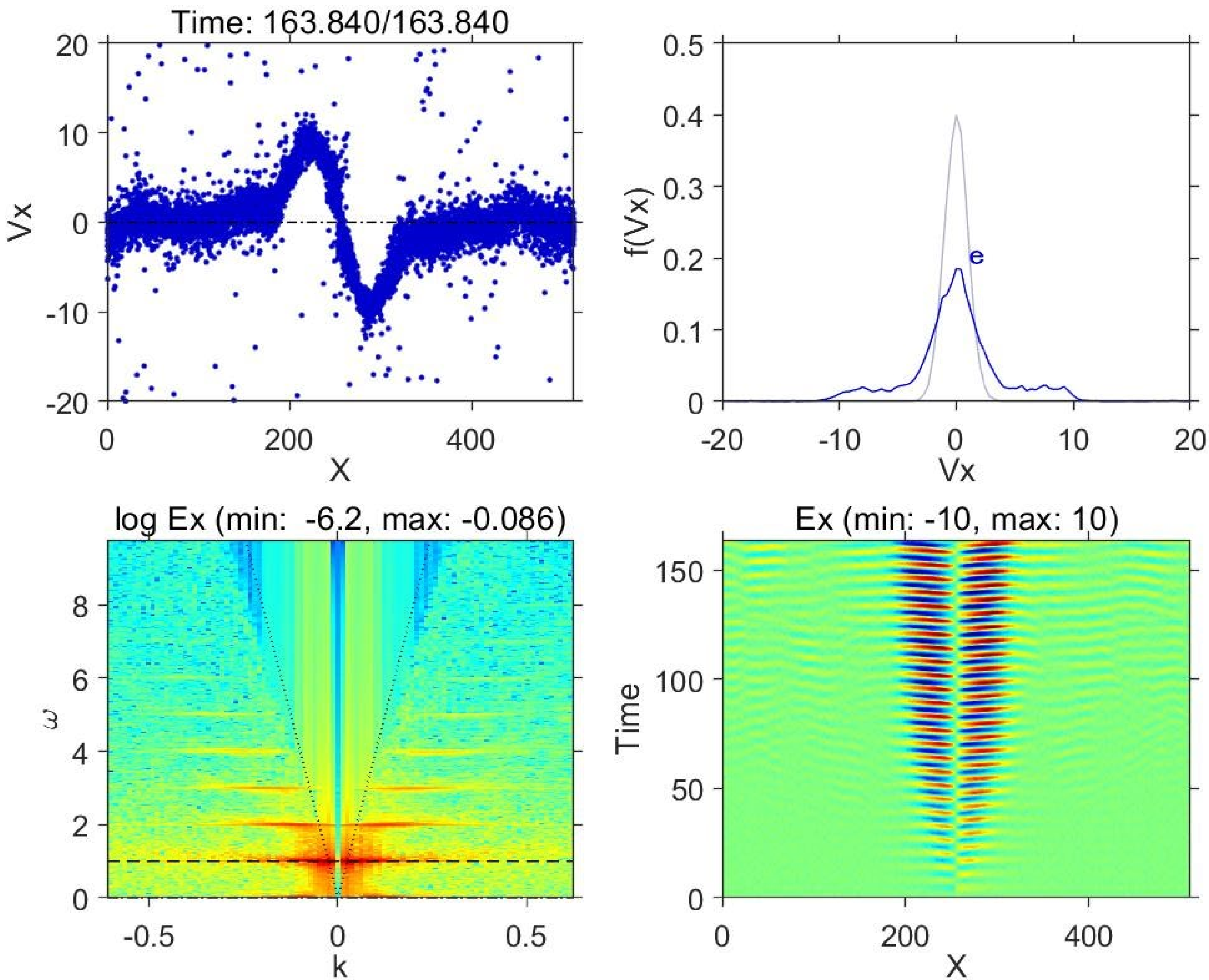
$B_0 = 0$



EMIC Wave Instability



Harmonic Langmuir Waves



[*cf. Yoon et al., Phys. Plasmas,* 2003]

KEMPO2

References

H. Matsumoto and Y. Omura, Particle Simulations of Electromagnetic Waves and its Applications to Space Plasmas, Computer Simulations of Space Plasmas, edited by H. Matsumoto and T. Sato, Terra Pub. and Reidel Co., pages 43-102, 1984.

T. Umeda, Y. Omura, T. Tominaga, and H. Matsumoto, A new charge conservation method in electromagnetic particle-in-cell simulations, Computer Physics Communications, Vol.156, No.1, pp.73-85, 2003.

Two-Dimensional Simulation System (x, y)

$$\frac{\partial}{\partial z} = 0 \qquad \nabla = \boldsymbol{e}_x \frac{\partial}{\partial x} + \boldsymbol{e}_y \frac{\partial}{\partial y}$$

$$\boldsymbol{E} = \boldsymbol{E}_{xy} + \boldsymbol{E}_z$$

$$\boldsymbol{B} = \boldsymbol{B}_{xy} + \boldsymbol{B}_z$$

$$\boldsymbol{J} = \boldsymbol{J}_{xy} + \boldsymbol{J}_z$$

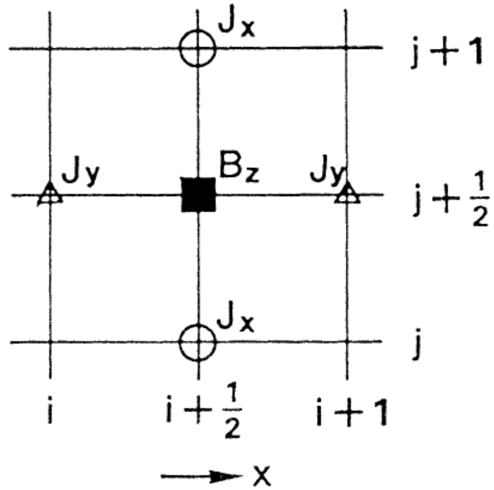
Courant Condition

$$\Omega = \frac{\sin(\omega \Delta t/2)}{\Delta t/2} \qquad K_x = \frac{\sin(k_x \Delta x/2)}{\Delta x/2} \qquad K_y = \frac{\sin(k_y \Delta y/2)}{\Delta y/2}$$

$$\Omega^2 = c^2 (K_x^2 + K_y^2) \qquad \sqrt{2} c \Delta t < \Delta x$$

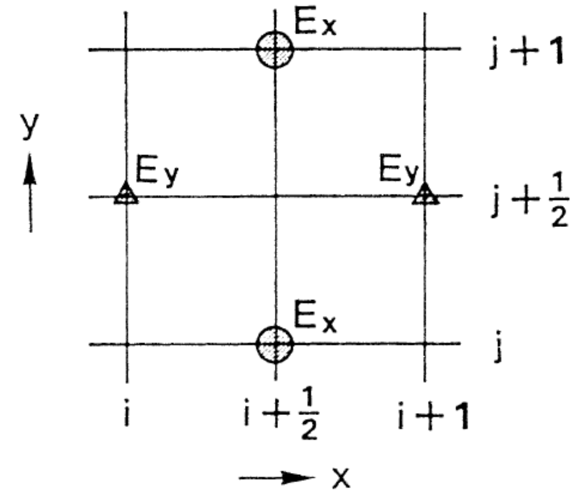
$$\frac{\partial \boldsymbol{B}_z}{\partial t} = -\nabla \times \boldsymbol{E}_{xy}$$

t=(n-1/2)Δt



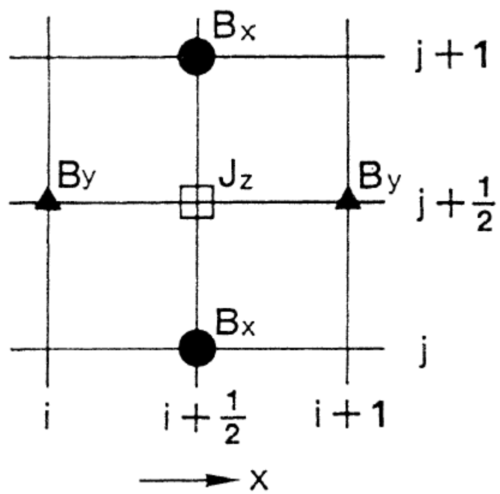
$$\frac{\partial \boldsymbol{E}_{xy}}{\partial t} = c^2\nabla \times \boldsymbol{B}_z - \boldsymbol{J}_{xy}$$

t=nΔt



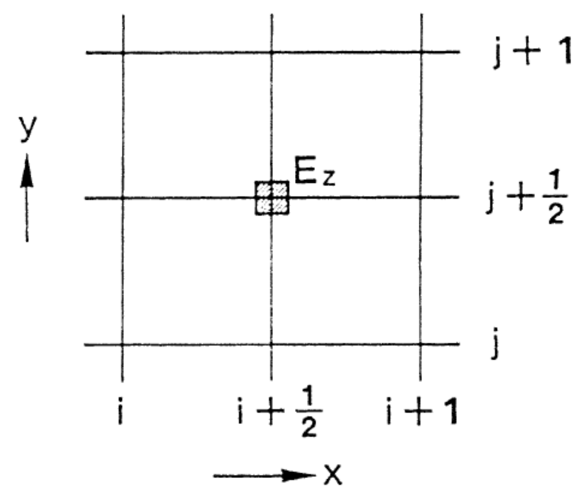
$$\frac{\partial \boldsymbol{B}_{xy}}{\partial t} = -\nabla \times \boldsymbol{E}_z$$

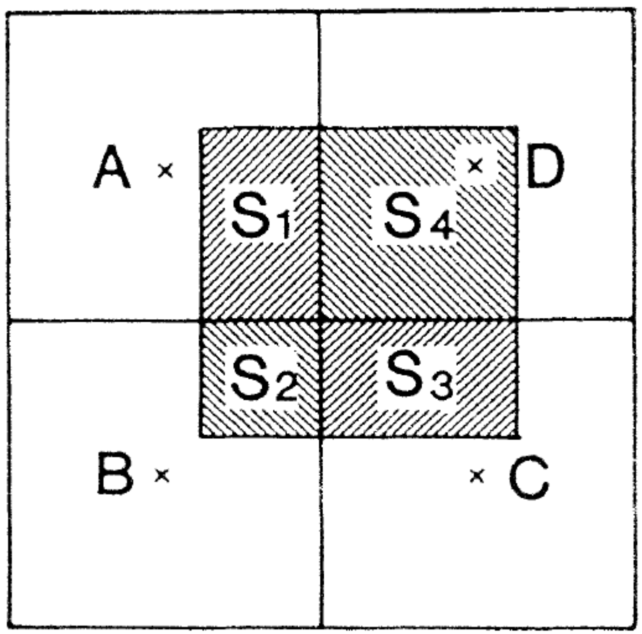
t=(n-1/2)Δt



$$\frac{\partial \boldsymbol{E}_z}{\partial t} = c^2\nabla \times \boldsymbol{B}_{xy} - \boldsymbol{J}_z$$

t=nΔt





	charge	current
A:	$\frac{S_1}{S} q_{s,i}^{(S)}$,	$\frac{S_1}{S} q_{s,i}^{(S)} \underset{\sim}{v}_{s,i}$
B:	$\frac{S_2}{S} q_{s,i}^{(S)}$,	$\frac{S_2}{S} q_{s,i}^{(S)} \underset{\sim}{v}_{s,i}$
C:	$\frac{S_3}{S} q_{s,i}^{(S)}$,	$\frac{S_3}{S} q_{s,i}^{(S)} \underset{\sim}{v}_{s,i}$
D:	$\frac{S_4}{S} q_{s,i}^{(S)}$,	$\frac{S_4}{S} q_{s,i}^{(S)} \underset{\sim}{v}_{s,i}$

Where $S = S_1 + S_2 + S_3 + S_4$

Relocations

Source Relocation

$$Jx_{i+1/2,j} = \frac{1}{2}(Jx_{i,j} + Jx_{i+1,j})$$

$$Jy_{i,j+1/2} = \frac{1}{2}(Jy_{i,j} + Jy_{i,j+1})$$

$$Jz_{i+1/2,j+1/2} = \frac{1}{4}(Jz_{i,j} + Jz_{i+1,j} + Jz_{i,j+1} + Jz_{i+1,j+1})$$

Field Relocation (E & B)

$$PEx_{i,j} = \frac{1}{2}(Ex_{i-1/2,j} + Ex_{i+1/2,j})$$

$$PEy_{i,j} = \frac{1}{2}(Ey_{i,j-1/2} + Ey_{i,j+1/2})$$

$$PEz_{i,j} = \frac{1}{4}(Ez_{i-1/2,j-1/2} + Ez_{i+1/2,j-1/2} + Ez_{i-1/2,j+1/2} + Ez_{i+1/2,j+1/2})$$

Solution of Electrostatic Field (Correction of Electric Field)

$$\frac{\partial \rho}{\partial t} + \boxed{\nabla \cdot \boldsymbol{J}} = 0$$

$$\nabla \cdot (\boldsymbol{E} + \boldsymbol{E}_c) = \rho \qquad \rho_c = \rho - \nabla \cdot \boldsymbol{E}$$

$$\boldsymbol{E}_c = -\nabla \psi \qquad \nabla \cdot \boldsymbol{E}_c = \rho_c$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\rho_c$$

$$\frac{\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{(\Delta x)^2} + \frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{(\Delta y)^2} = -\rho_{c,i,j}$$

$$\tilde{\rho}_{kx,ky} = \mathrm{FFT}\left\{\rho_c(x,y)\right\} \qquad K_x = \frac{\sin\left(k_x \Delta x/2\right)}{\Delta x/2} \qquad K_y = \frac{\sin\left(k_y \Delta y/2\right)}{\Delta y/2}$$

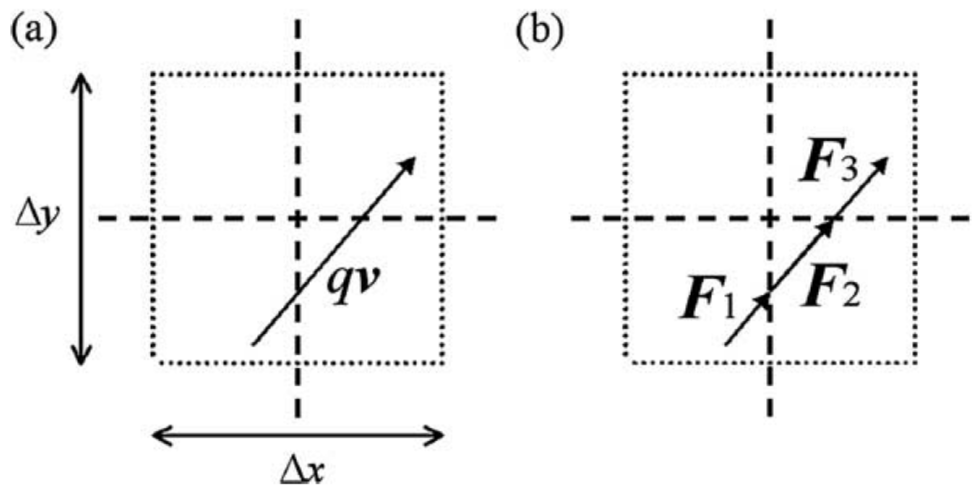
$$(K_x^2 + K_y^2)\tilde{\psi}_{kx,ky} = \tilde{\rho}_{kx,ky} \qquad \tilde{\psi}_{kx,ky} = \frac{\tilde{\rho}_{kx,ky}}{(K_x^2 + K_y^2)}$$

$$\psi(x,y) = \mathrm{IFFT}\left\{\tilde{\psi}_{kx,ky}\right\}$$

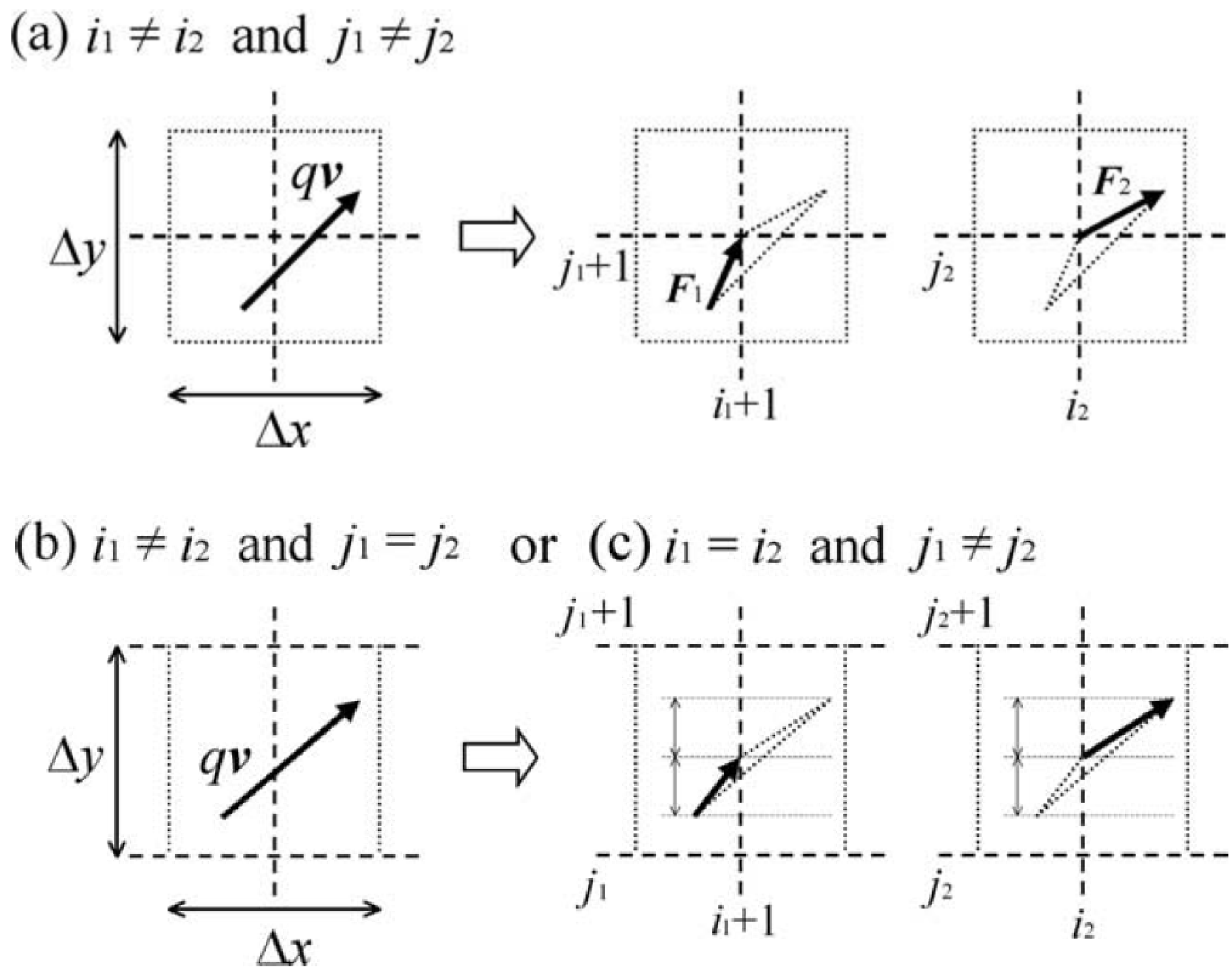
$$\boldsymbol{E} = \boldsymbol{E} - \nabla \psi$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \boldsymbol{J} = 0$$

$$\frac{J_x^{t+\Delta t/2}(i+\frac{1}{2}, j) - J_x^{t+\Delta t/2}(i-\frac{1}{2}, j)}{\Delta x} + \frac{J_y^{t+\frac{\Delta t}{2}}(i, j+\frac{1}{2}) - J_y^{t+\Delta t/2}(i, j-\frac{1}{2})}{\Delta y}$$
$$= \frac{\rho^t(i, j) - \rho^{t+\Delta t}(i, j)}{\Delta t}.$$



Zigzag Method (current2)



[*Umeda et al., CPC,* 2003]

KEMPO2: Main Program

Method 1:
Electrostatic Field Correction Method

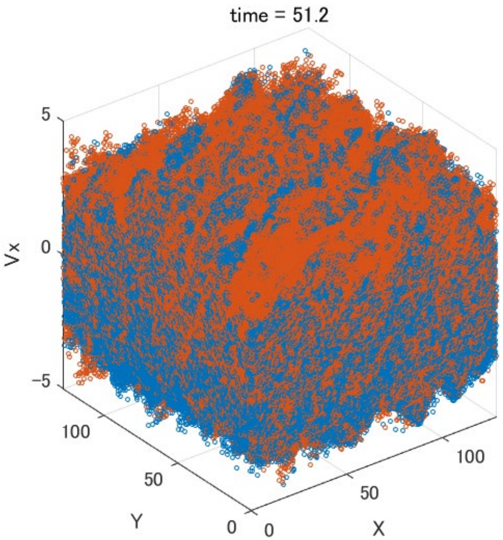
```
charge;
potential;
for itime = 1:ntime
    jtime = jtime +1;
    bfield;
    rvelocity;
    position;
    current;
    position;
    bfield;
    efield;
    charge;
    potential;
    diagnostics;
end
```

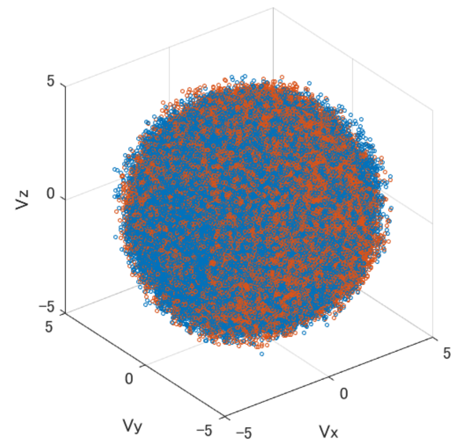
Method 2:
Charge Conservation (zigzag)

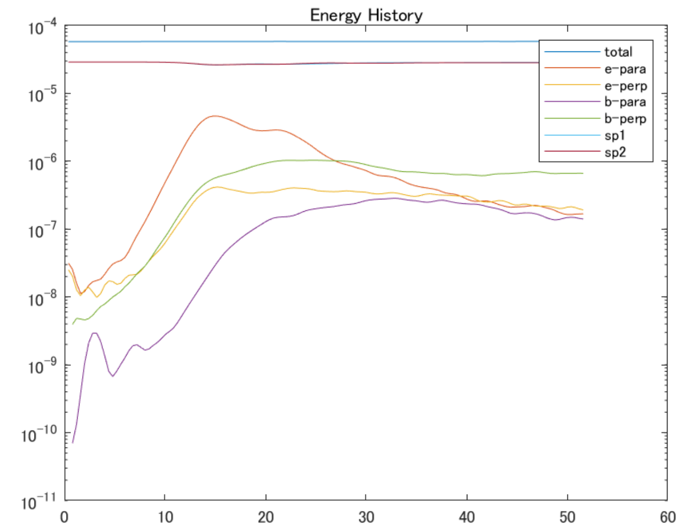
```
charge;
potential;
for itime = 1:ntime
    jtime = jtime +1;
    bfield;
    rvelocity;
    position;
    current2;
    position;
    bfield;
    efield;
    diagnostics;
end
```

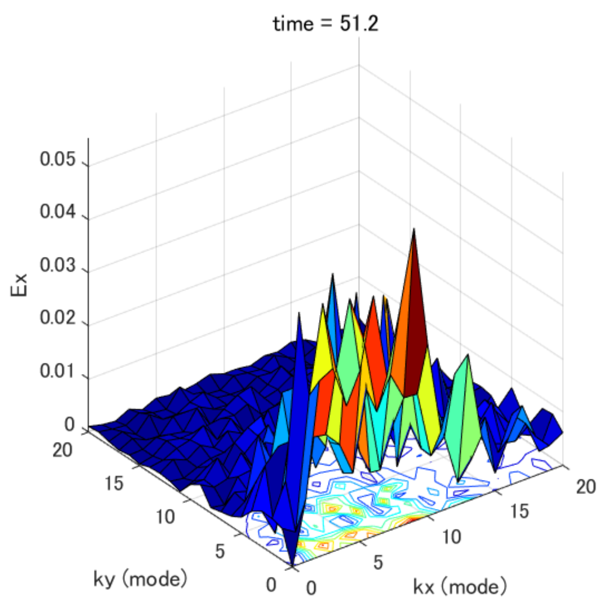
% Computation times for Methods 1 and 2 are nearly the same.

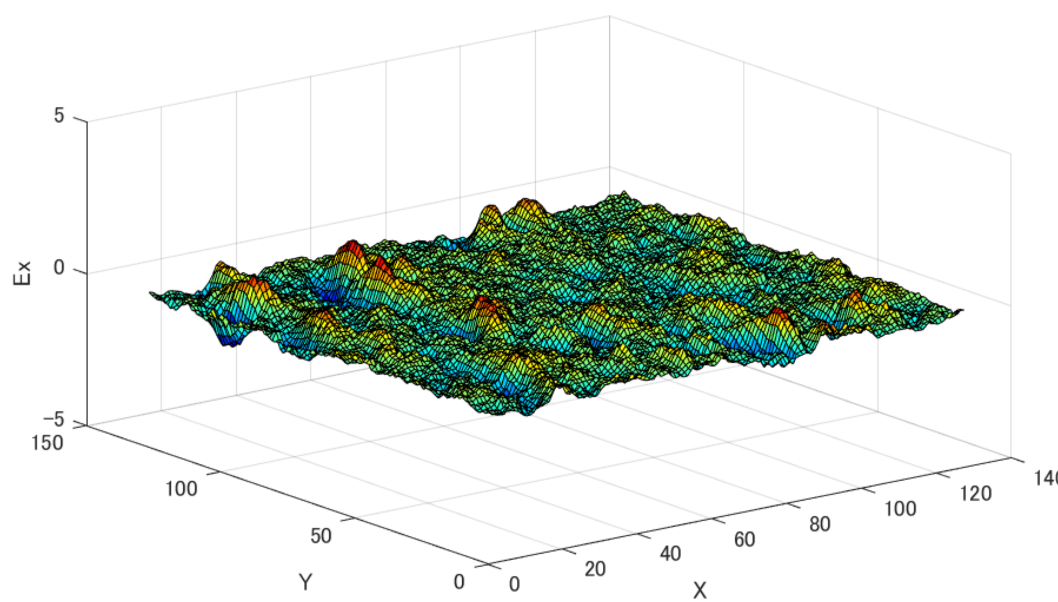
ElectronTwo-stream Instability





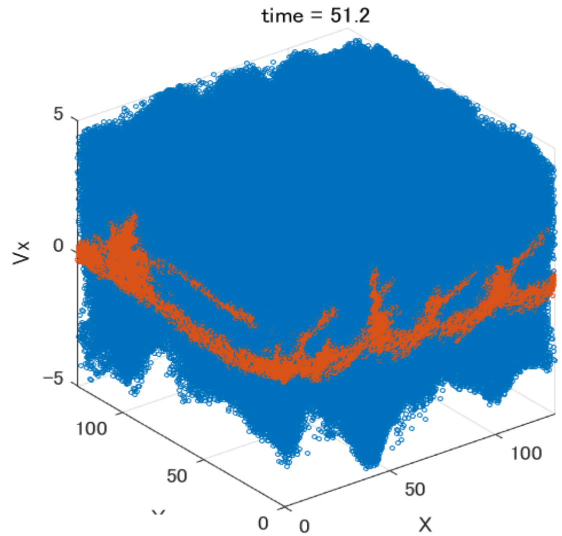


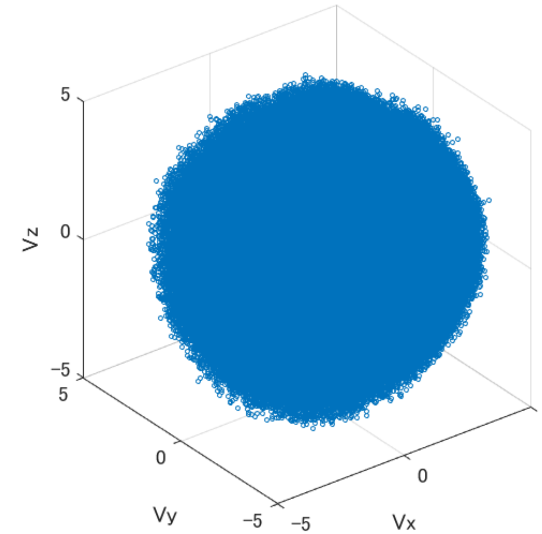


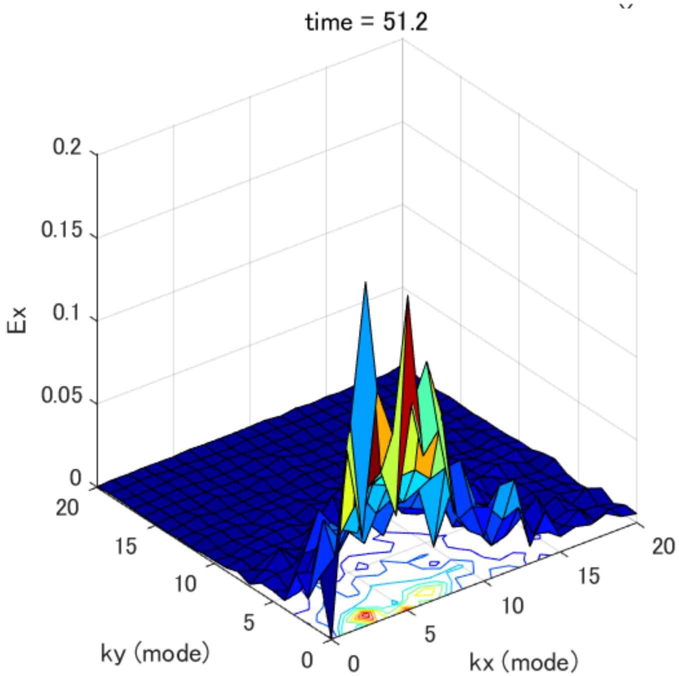


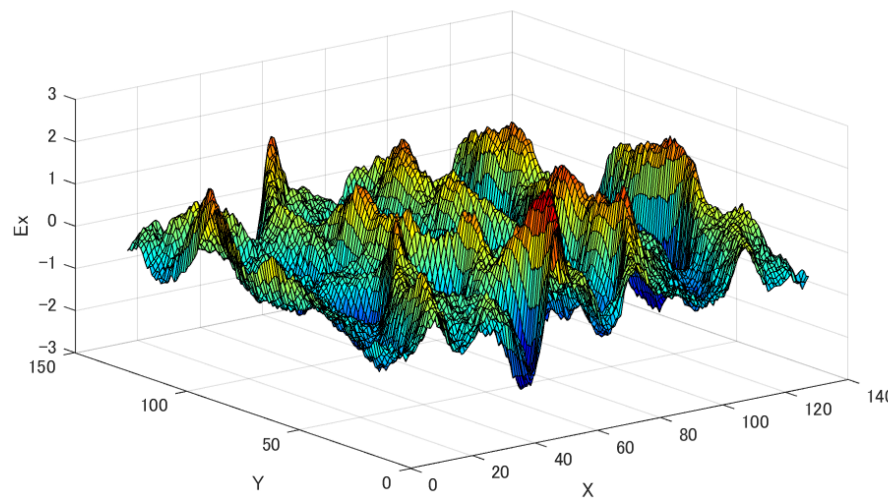
Buneman Instability

$m_i / m_e = 25$

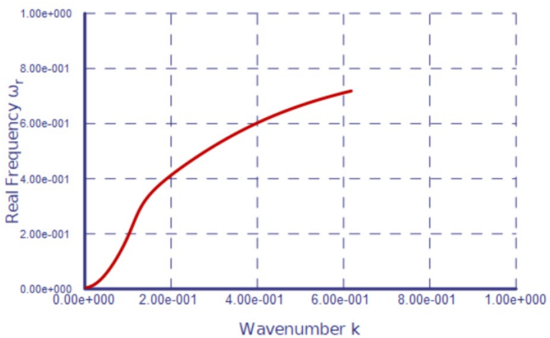


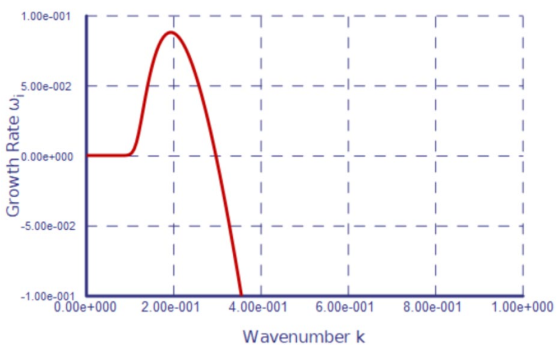


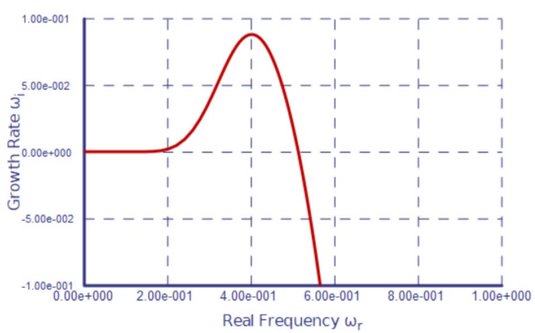


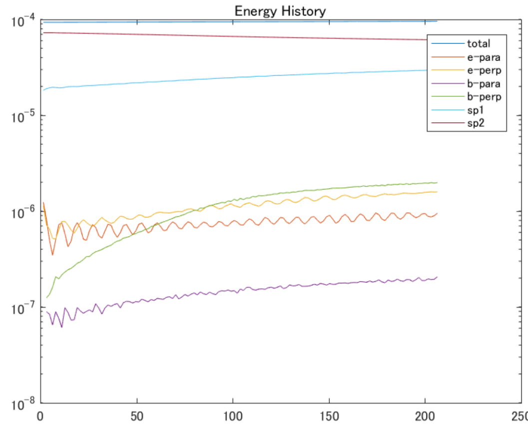


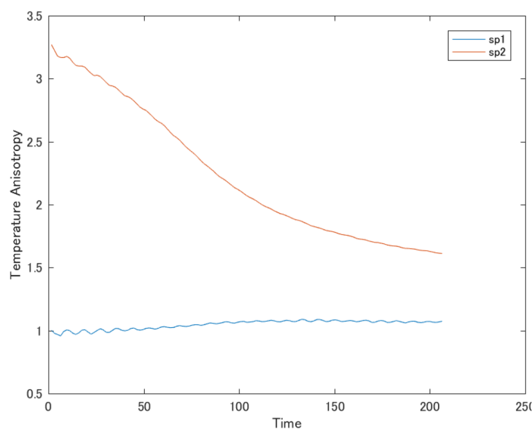
Whistler Mode Instability
Driven by Temperature Anisotropy







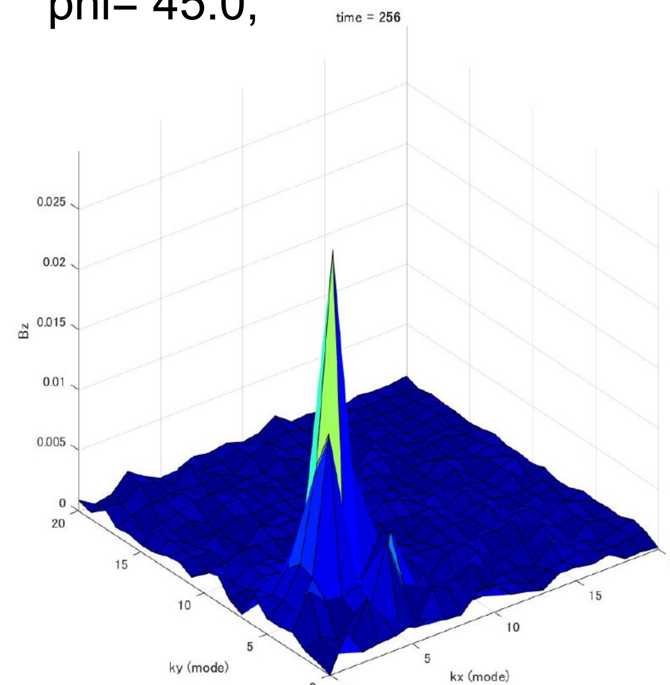




% direction of the static magnetic magnetic field

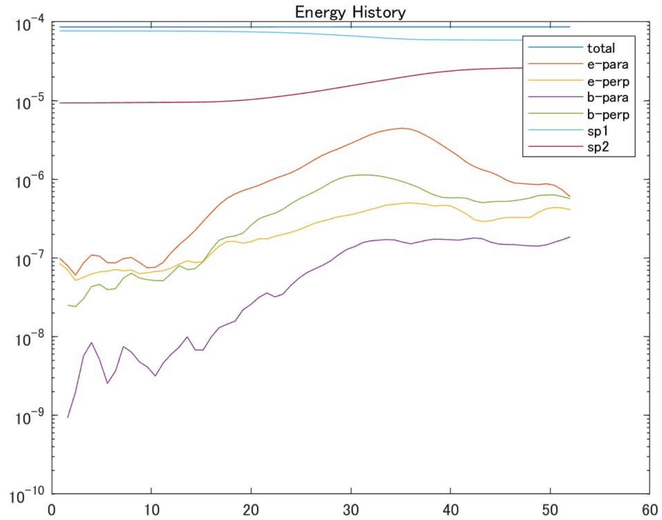
theta= 90.0;

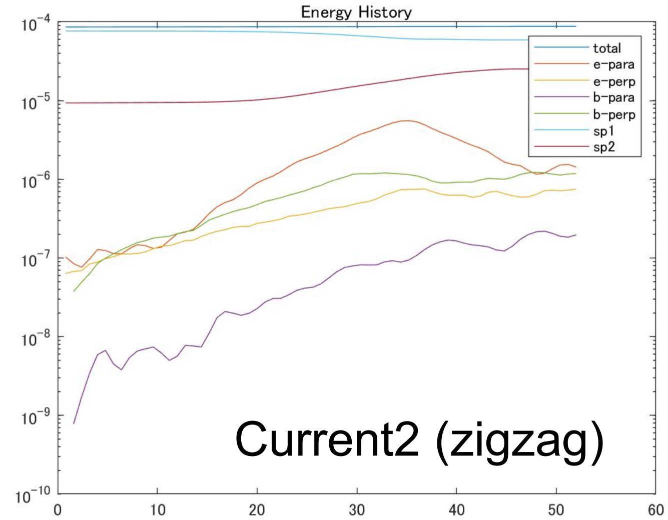
phi= 45.0;



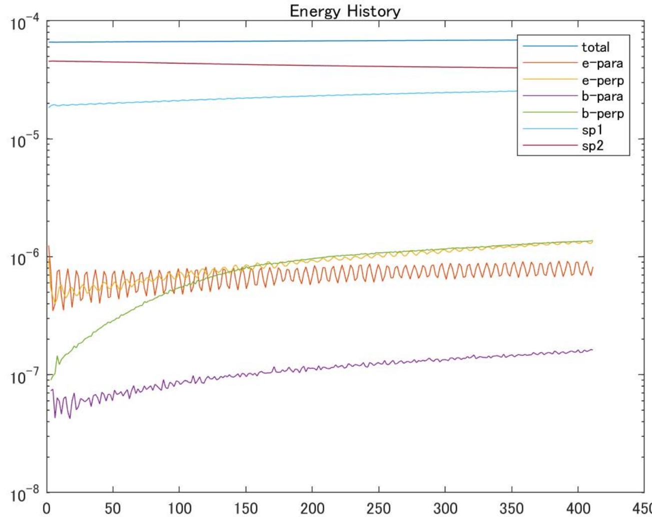

```
vs = sum(vx(m).^2 + vy(m).^2 + vz(m).^2);
vspara = sum(((vx(m)*bx0+vy(m)*by0+vz(m)*bz0)/b0).^2);
%Temperature Anisotropy  T_perp/T_para
   At(it,k) = 0.5*(vs/vspara -1);
```

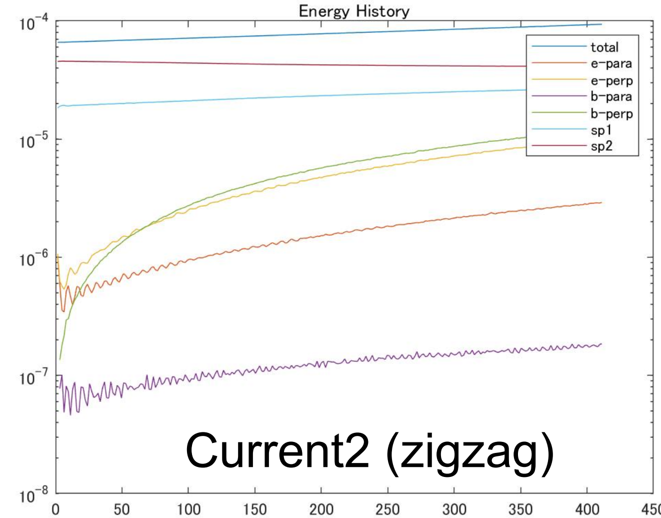
Buneman Instability





Whistler Instability





Comment on Zigzag Method

The Zigzag method for computation of the current density is slightly faster than the electrostatic correction method with charge density computation. It can enhance the electromagnetic fluctuations because of the non-physical trajectories of particles assumed to simplify the computation of the current.