

$$f_{Ma}(v_{\parallel}, v_{\perp}) = \frac{1}{\pi^{3/2} v_{T\parallel a} v_{T\perp a}^2} \exp \left(-\frac{(v_{\parallel} - V_{Da})^2}{v_{T\parallel a}^2} - \frac{v_{\perp}^2}{v_{T\perp a}^2} \right)$$

$$v_{T\parallel a}^2 = \frac{2T_{\parallel a}}{m_a}, \quad v_{T\perp a}^2 = \frac{2T_{\perp a}}{m_a}$$

$$v_A^2 = \frac{B^2}{4\pi n_i m_i}$$

$$\beta_{\parallel a} = \frac{8\pi n_a T_{\parallel a}}{B^2}, \quad \beta_{\perp a} = \frac{8\pi n_a T_{\perp a}}{B^2}$$

$$v_{\parallel} = v_A u_{\parallel}, \quad v_{\perp} = v_A u_{\perp}, \quad U_{Da} = \frac{V_{Da}}{v_A}$$

$$\mu_a = \frac{m_i}{m_a}, \quad \tilde{q}_a = \frac{q_a}{q_i}, \quad \tilde{n}_a = \frac{n_a}{n_i}$$

$$f_{Ma}(u_{\parallel}, u_{\perp}) = \frac{1}{\pi^{3/2} v_{T\parallel a} v_{T\perp a}^2} \exp \left(-\frac{v_A^2 (u_{\parallel} - U_{Da})^2}{v_{T\parallel a}^2} - \frac{v_A^2 u_{\perp}^2}{v_{T\perp a}^2} \right)$$

$$\frac{v_{T\parallel a}^2}{v_A^2} = \frac{m_i}{m_a} \frac{n_i}{n_a} \frac{8\pi n_a T_{\parallel a}}{B^2} = \frac{m_i}{m_a} \frac{n_i}{n_a} \beta_{\parallel a} = \frac{\mu_a}{\tilde{n}_a} \beta_{\parallel a}, \quad v_{T\parallel a} = v_A \sqrt{\frac{\mu_a}{\tilde{n}_a}} \sqrt{\beta_{\parallel a}}$$

$$\frac{v_{T\perp a}^2}{v_A^2} = \frac{m_i}{m_a} \frac{n_i}{n_a} \frac{8\pi n_a T_{\perp a}}{B^2} = \frac{m_i}{m_a} \frac{n_i}{n_a} \beta_{\perp a} = \frac{\mu_a}{\tilde{n}_a} \beta_{\perp a}, \quad v_{T\perp a}^2 = v_A^2 \frac{\mu_a}{\tilde{n}_a} \beta_{\perp a}$$

$$f_{Ma}(u_{\parallel}, u_{\perp}) = \frac{1}{\pi^{3/2} v_{T\parallel a} v_{T\perp a}^2} \exp \left(-\frac{(u_{\parallel} - U_{Da})^2}{\frac{\mu_a}{\tilde{n}_a} \beta_{\parallel a}} - \frac{u_{\perp}^2}{\frac{\mu_a}{\tilde{n}_a} \beta_{\perp a}} \right)$$

$$= \frac{1}{\pi^{3/2} v_{T\parallel a} v_{T\perp a}^2} \exp \left(-\tilde{n}_a \frac{(u_{\parallel} - U_{Da})^2}{\mu_a \beta_{\parallel a}} - \frac{\tilde{n}_a u_{\perp}^2}{\mu_a \beta_{\perp a}} \right)$$

$$= \frac{1}{v_A^3 \pi^{3/2} \mu_a^{3/2} \sqrt{\beta_{\parallel a} \beta_{\perp a}}} \exp \left(-\tilde{n}_a \frac{(u_{\parallel} - U_{Da})^2}{\mu_a \beta_{\parallel a}} - \frac{\tilde{n}_a u_{\perp}^2}{\mu_a \beta_{\perp a}} \right)$$

$$\int d^3v f_{Ma}(v_{\parallel}, v_{\perp}) = 1 \implies \int d^3u v_A^3 f_{Ma}(u_{\parallel}, u_{\perp}) = 1$$

$$\tilde{f}_{Ma}(u_{\parallel}, u_{\perp}) = v_A^3 f_{Ma}(u_{\parallel}, u_{\perp}) \implies \tilde{f}_{Ma}(u_{\parallel}, u_{\perp}) = \frac{\tilde{n}_a^{3/2}}{\pi^{3/2} \mu_a^{3/2} \sqrt{\beta_{\parallel a} \beta_{\perp a}}} \exp \left(-\tilde{n}_a \frac{(u_{\parallel} - U_{Da})^2}{\mu_a \beta_{\parallel a}} - \frac{\tilde{n}_a u_{\perp}^2}{\mu_a \beta_{\perp a}} \right)$$

Kappa:

$$f_{\kappa a}(v_{\parallel}, v_{\perp}) = \frac{1}{\pi^{3/2} \kappa_a^{3/2} \theta_{\parallel a}^2 \theta_{\perp a}^2} \frac{\Gamma(\kappa_a + 1)}{\Gamma(\kappa_a - 1/2)} \left(1 + \frac{(v_{\parallel} - V_{Da})^2}{\kappa_a \theta_{\parallel a}^2} + \frac{v_{\perp}^2}{\kappa_a \theta_{\perp a}^2} \right)^{-(\kappa_a + 1)}$$

$$\theta_{\parallel a}^2 = \left(1 - \frac{3}{2\kappa_a} \right) v_{T\parallel a}^2 \quad \theta_{\perp a}^2 = \left(1 - \frac{3}{2\kappa_a} \right) v_{T\perp a}^2$$

$$\theta_{\parallel a}^2 = v_A^2 \tilde{\theta}_{\parallel a}^2 \quad \theta_{\perp a}^2 = v_A^2 \tilde{\theta}_{\perp a}^2$$

$$\tilde{\theta}_{\parallel a}^2 = \left(1 - \frac{3}{2\kappa_a} \right) \frac{\mu_a}{\tilde{n}_a} \beta_{\parallel a} \quad \tilde{\theta}_{\perp a}^2 = \left(1 - \frac{3}{2\kappa_a} \right) \frac{\mu_a}{\tilde{n}_a} \beta_{\perp a}$$

$$f_{\kappa a}(u_{\parallel}, u_{\perp}) = \frac{1}{v_A^3 \pi^{3/2} \kappa_a^{3/2} \tilde{\theta}_{\parallel a}^2 \tilde{\theta}_{\perp a}^2} \frac{\Gamma(\kappa_a + 1)}{\Gamma(\kappa_a - 1/2)} \left(1 + \frac{(u_{\parallel} - U_{Da})^2}{\kappa_a \tilde{\theta}_{\parallel a}^2} + \frac{u_{\perp}^2}{\kappa_a \tilde{\theta}_{\perp a}^2} \right)^{-(\kappa_a + 1)}$$

$$\tilde{f}_{\kappa a}(u_{\parallel}, u_{\perp}) = v_A^3 f_{\kappa a}(u_{\parallel}, u_{\perp})$$

$$\tilde{f}_{\kappa a}(u_{\parallel}, u_{\perp}) = \frac{1}{\pi^{3/2} \kappa_a^{3/2} \tilde{\theta}_{\parallel a}^2 \tilde{\theta}_{\perp a}^2} \frac{\Gamma(\kappa_a + 1)}{\Gamma(\kappa_a - 1/2)} \left(1 + \frac{(u_{\parallel} - U_{Da})^2}{\kappa_a \tilde{\theta}_{\parallel a}^2} + \frac{u_{\perp}^2}{\kappa_a \tilde{\theta}_{\perp a}^2} \right)^{-(\kappa_a + 1)}$$