Parâmetros de plasma e distribuições para diferentes espécies

Os parâmetros relevantes para a espécie de partículas a são n_a (densidade), T_a (temperatura), $T_{\parallel a}$ (temperatura paralela), $T_{\perp a}$ (temperatura perpendicular), κ_a (parâmetro capa),

$$
\begin{aligned}
v_{Ta} &= \sqrt{\frac{2T_a}{m_a}} && \text{(velocidade térmica)} \\
v_{Aa} &= \frac{B_0}{\sqrt{4\pi n_a m_a}} && \text{(velocidade de Alfvén)} \\
\theta_a &= \sqrt{\frac{\kappa_a - 3/2}{\kappa_a}} v_{Ta} && \text{(dispersão supertérmica de velocidades)} \\
\beta_a &= \frac{8\pi n_a T_a}{B_0^2} = \frac{v_{Ta}^2}{v_{Aa}^2} && \text{(plasma beta)}
\end{aligned}
$$

As principais distribuições de velocidades são:

$$
\begin{aligned}
f_{M,a}\left(v\right) &= \frac{e^{-v^2/v_{Ta}^2}}{\pi^{3/2} v_{Ta}^3} && \text{(Maxwelliana)} \\
f_{BM,a}\left(v_\parallel, v_\perp\right) &= \frac{e^{-\left(v_\parallel - v_{ba}\right)^2/v_{T\parallel a}^2} e^{-v_\perp^2/v_{T\perp a}^2}}{\pi^{3/2} v_{T\parallel a} v_{T\perp a}^2} && \text{(bi-Maxwelliana + drift)} \\
f_{\kappa,a}\left(v\right) &= \frac{1}{\pi^{3/2}\theta_a^3} \frac{\Gamma\left(\kappa_a + 1\right)}{\kappa_a^{3/2}\Gamma\left(\kappa_a - 1/2\right)} \left(1 + \frac{v^2}{\kappa_a \theta_a^2}\right)^{-(\kappa_a+1)} && \text{(standard kappa)} \\
f_{B\kappa,a}\left(v_\parallel, v_\perp\right) &= \frac{1}{\pi^{3/2}\theta_{\parallel a}\theta_{\perp a}^2} \frac{\Gamma\left(\kappa_a + 1\right)}{\kappa_a^{3/2}\Gamma\left(\kappa_a - 1/2\right)} \left(1 + \frac{\left(v_\parallel - v_{ba}\right)^2}{\kappa_a \theta_{\parallel a}^2} + \frac{v_\perp^2}{\kappa_a \theta_{\perp a}^2}\right)^{-(\kappa_a+1)} && \text{(bi-kappa + drift)}
\end{aligned}
$$

Note que todas as distribuições são normalizadas à unidade, i.e.,

$$
\int d^3v\, f_a\left(v_\parallel, v_\perp\right) = 1.
$$

Seja a espécie/população principal chamada de espécie-$*$. Então, temos n_*, T_*, v_{T*}, v_{A*}, β_*, etc.

Vamos normalizar as distribuições em termos de v_{A*}. Então, definindo $\boldsymbol{u} = \boldsymbol{v}/v_{A*}$, as distribuições normalizadas serão tais que, ainda,

$$
\int d^3u\, f_a\left(u_\parallel, u_\perp\right) = 1.
$$

Ou seja,

$$
d^3u\, f_a\left(u_\parallel, u_\perp\right) = d^3v\, f_a\left(v_\parallel, v_\perp\right) \Longrightarrow f_a\left(u_\parallel, u_\perp\right) = v_{A*}^3\, f_a\left(v_\parallel, v_\perp\right)\Big|_{\boldsymbol{v}=v_{A*}\boldsymbol{u}}.
$$

Desta forma, temos para a espécie-$*$,

$$\frac{v^2}{v_{T*}^2} = \frac{u^2}{v_{T*}^2/v_{A*}^2} = \frac{u^2}{\beta_*}, \quad v_{T*} = v_{A*}\beta_*^{1/2},$$

$$\frac{v^2}{\theta_*^2} = \frac{u^2}{\tilde{\theta}_*^2}, \quad \tilde{\theta}_* = \frac{\theta_*}{v_{A*}} = \sqrt{\frac{\kappa_* - 3/2}{\kappa_*}}\beta_*^{1/2}$$

$$f_{M,*}(u) = v_{A*}^3\, f_{M,*}(v_\parallel, v_\perp)\big|_{\boldsymbol{v}=v_{A*}\boldsymbol{u}} = \frac{n_a e^{-u^2/\beta_*}}{\pi^{3/2}\beta_*^{3/2}}$$

$$f_{BM,*}(u_\parallel, u_\perp) = \frac{e^{-\left(u_\parallel - u_{b*}\right)^2/\beta_{\parallel *}} e^{-u_\perp^2/\beta_{\perp *}}}{\pi^{3/2}\beta_{\parallel *}^{1/2}\beta_{\perp *}}$$

$$f_{\kappa,*}(u) = \frac{1}{\pi^{3/2}\tilde{\theta}_*^3}\frac{\Gamma(\kappa_* + 1)}{\kappa_*^{3/2}\Gamma(\kappa_* - 1/2)}\left(1 + \frac{u^2}{\kappa_*\tilde{\theta}_*^2}\right)^{-(\kappa_*+1)}$$

$$f_{B\kappa,*}(u_\parallel, u_\perp) = \frac{1}{\pi^{3/2}\tilde{\theta}_{\parallel *}\tilde{\theta}_{\perp *}^2}\frac{\Gamma(\kappa_* + 1)}{\kappa_*^{3/2}\Gamma(\kappa_* - 1/2)}\left(1 + \frac{\left(u_\parallel - u_{b*}\right)^2}{\kappa_*\tilde{\theta}_{\parallel *}^2} + \frac{u_\perp^2}{\kappa_*\theta_{\perp *}^2}\right)^{-(\kappa_*+1)}$$

Já para as outras espécies/populações $(a \neq *)$ temos:

$$\frac{v^2}{v_{Ta}^2} = \frac{u^2}{v_{Ta}^2/v_{A*}^2} \Longrightarrow \frac{v_{Ta}^2}{v_{A*}^2} = \frac{2T_a}{m_a}\frac{4\pi n_* m_*}{B_0^2} = \frac{8\pi n_a T_a}{B_0^2}\frac{n_*}{n_a}\frac{m_*}{m_a} = \frac{n_*}{n_a}\frac{m_*}{m_a}\beta_a \Longrightarrow \frac{v_{Ta}}{v_{A*}} = \left(\frac{n_*}{n_a}\right)^{1/2}\left(\frac{m_*}{m_a}\right)^{1/2}\beta_a^{1/2}$$

$$\frac{v^2}{\theta_a^2} = \frac{u^2}{\theta_a^2/v_{A*}^2}, \quad \frac{\theta_a}{v_{A*}} = \sqrt{\frac{\kappa_a - 3/2}{\kappa_a}}\frac{v_{Ta}}{v_{A*}} = \left(\frac{n_*}{n_a}\right)^{1/2}\left(\frac{m_*}{m_a}\right)^{1/2}\tilde{\theta}_a \quad \tilde{\theta}_a = \sqrt{\frac{\kappa_a - 3/2}{\kappa_a}}\beta_a^{1/2}$$

Então, as distribuições normalizadas ficam

$$f_{M,a}(u) = \frac{(n_a/n_*)^{3/2}}{\pi^{3/2}(m_*/m_a)^{3/2}\beta_a^{3/2}}\exp\left(-\frac{n_a}{n_*}\frac{u^2}{\frac{m_*}{m_a}\beta_a}\right)$$

$$f_{BM,a}(u_\parallel, u_\perp) = \frac{(n_a/n_*)^{3/2}}{\pi^{3/2}(m_*/m_a)^{3/2}\beta_{\parallel a}^{1/2}\beta_{\perp a}}\exp\left(-\left(\frac{n_a}{n_*}\right)\frac{\left(u_\parallel - u_b\right)^2}{\left(\frac{m_*}{m_a}\right)\beta_{\parallel a}}\right)\exp\left(-\left(\frac{n_a}{n_*}\right)\frac{u_\perp^2}{\left(\frac{m_*}{m_a}\right)\beta_{\perp a}}\right)$$

$$f_{\kappa,a}(u) = \frac{(n_a/n_*)^{3/2}}{\pi^{3/2}(m_*/m_a)^{3/2}\tilde{\theta}_a^3}\frac{\Gamma(\kappa_a + 1)}{\kappa_a^{3/2}\Gamma(\kappa_a - 1/2)}\left(1 + \frac{n_a}{n_*}\frac{u^2}{\kappa_a\frac{m_*}{m_a}\tilde{\theta}_a^2}\right)^{-(\kappa_a+1)}$$

$$f_{B\kappa,a}(u_\parallel, u_\perp) = \frac{(n_a/n_*)^{3/2}}{\pi^{3/2}(m_*/m_a)^{3/2}\tilde{\theta}_{\parallel a}\tilde{\theta}_{\perp a}^2}\frac{\Gamma(\kappa_a + 1)}{\kappa_a^{3/2}\Gamma(\kappa_a - 1/2)}\left(1 + \left(\frac{n_a}{n_*}\right)\frac{\left(u_\parallel - u_{ba}\right)^2}{\kappa_a\left(\frac{m_*}{m_a}\right)\tilde{\theta}_{\parallel a}^2} + \left(\frac{n_a}{n_*}\right)\frac{u_\perp^2}{\kappa_a\left(\frac{m_*}{m_a}\right)\tilde{\theta}_{\perp a}^2}\right)^{-(\kappa_a+1)}$$

sendo

$$u_{ba} = \frac{v_{ba}}{v_{A*}}.$$