# Variation in site entropy explains differences in structure-sequence relationships of proteins

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#### Abstract

Recent work has shown that structural properties are capable of predicting site-specific sequence variability for a given protein. However, the strength and significance of these structure-sequence relations appear to vary widely among different proteins, with absolute correlation strengths ranging from 0.1 to 0.8. In the introduction and abstract need to better highlight the two papers (Yeh and Shah) that contradict on the surface and they do not. Hold off on Rate4Site for Now.

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#### 1 Introduction

Proteins are subject to a number of biophysical and functional constraints (Scherrer et al., 2012; Wilke and Drummond, 2010). These constraints result site-specific patterns of sequence variability within a protein. Recently several site-specific structural properties that can explain patterns of sequence variability in proteins have been identified. One of the earliest examples is Relative Site Accessibility (RSA). Franzosa and Xia (2009) identified RSA as the strong predictor of evolutionary rate and found that residues that are buried in the core of proteins tend to be more conserved than exposed residues close to the surface of the protein. In their analysis they considered the ability of both RSA and various definitions of residue packing density to predict evolutionary rate. They found that RSA and evolutionary rate shared a significant linear relationship. Afterwards, several other works (Ramsey et al., 2011; Scherrer et al., 2012) also found that RSA as a very significant predictor of evolutionary rate and found this linear relationship as well. However, these papers all have the same flaw. During the course of their analysis they binned the protein sites and average over all sites within a bin when looking at the trend of RSA. This process may have produce artifacts that account for this strong linear trend.

Recently, Yeh et al. (2014) performed a similar analysis on a series of enzyme monomer proteins and found that packing density, as defined by CN and WCN (Liao et al., 2005; Yeh et al., 2014; Huang et al., 2014), was the strongest determinant of site variability. A year later, Shahmoradi et al. (2014) also performed a site-wise analysis on a series of viral proteins. In this analysis they found that RSA had the strongest correlation with site variability as opposed to local packing density. Additionally the effect seen between CN and WCN was of a much smaller magnitude as compared to (Yeh et al., 2014). Therefore the relationship between local pacing density, RSA and site-specific measures of sequence variability is not well understood. Here we attempt to reconcile the work done in this area. We find that site variability is the primary determinant of the strength of structure-sequence relationships and some differences in previous work can be explained in terms of differing levels of site variability.

### 2 Materials and Methods

#### Structures, sequences, and measures of sequence properties

The results presented in this work are based on two datasets. The first is a dataset of 209 monomeric enzymes taken from Huang et al. (2014), originally from Yeh et al. (2014). The original dataset was comprised of 213 proteins but we removed four of the proteins (PDB IDS: 1BBS, 1BS0, 1DIN, 2HPL) that had did not have data at insertion sites. Briefly, these proteins are all enzyme monomers randomly picked from the Catalytic Site Atlas 2.2.11 (Porter et al., 2004) with protein sizes in the sample ranging from 95 to 1287 residues in length. For each structure we had a corresponding alignment of up to 300 homologous sequences. The second dataset was taken from Shahmoradi et al. (2014) and is comprised of nine viral proteins. The viral proteins range from 122 - 557 residues in length and each

structure is accompanied by a sequence alignment of up to 2362 homologous sequences. Sequence alignments for both datasets were constructed by aligning the amino-acid sequences using the alignment software MAFFT (Katoh et al., 2002, 2005), specifying the auto flag to select the optimal algorithm for the given data set. The alignments were then used to calculate the site-specific measures of sequence variability for each individual protein in both datasets. To do so, we relied on two independent methods of measuring sequence variability. First, we calculated the Shannon entropy  $(H_i)$  – the sequence entropy at each alignment column i:

$$H_i = -\sum_{i} P_{ij} \ln P_{ij} \tag{1}$$

where  $P_{ij}$  is the relative frequency of amino acid j at position i in the alignment. The sequence entropy is a measure of variability at each site. We also calculated a measure of site-specific evolutionary rate for each protein using software Rate4site. First the Maximum Likelihood phylogenetic trees were inferred with RAxML, using the LG substitution matrix and the CAT model of rate heterogeneity (Stamatakis, 2006, 2014). For each structure, we then used the respective sequence alignment and phylogenetic tree to infer site-specific substitution rates with Rate4Site using the empirical Bayesian method and the JTT model of sequence evolution (Mayrose et al., 2004).

#### Calculation of Structural Properties

In our analysis we used two types of measures of local packing density used in previous studies: Contact Number (CN) and Weighted Contact Number (WCN). For the purposes of our comparison between the two datasets of interest we used the CN and WCN for the enzyme proteins calculated in Huang et al. (2014) and the CN and WCN values for the viral proteins from Shahmoradi et al. (2014). In both of these works and in Yeh et al. (2014), WCN and CN are both defined the same. Contact Number is defined as the number of  $C_{\alpha}$  within a pre-redefined radius,  $r_0$ . In this case,  $r_0 = 13$  as in the previous papers. Weighed Contact Number for a residue, i, is defined as in (Liao et al 2005, Huang et al 2014), as:

$$WCN_i = \sum_{i \neq j}^{N} \frac{1}{r_{ij}^2} \tag{2}$$

where  $r_j$  is the length between the  $C_{\alpha}$  of residue *i* and residue *j* in a protein of length *N* (Yeh et al., 2014).

We used DSSP (Kabsch and Sander, 1983) to calculate the Accessible Surface Area (ASA) for each site. We then normalized the ASA for each site by the theoretical maximum solvent accessibility values of Tien et al. (2013) to obtain the Relative Solvent Accessibility (RSA) for all individual sites in all proteins.

All data and analysis scripts required to reproduce the work are publicly available to view and download at https://github.com/wilkelab/rate\_variability\_variation.

#### 3 Results

We calculate the strength between the structural properties and site variability by calculating the Spearman correlation,  $\rho$ , between the structural property (solvent exposure and local packing density) and site variability, as measured by either evolutionary rate or site entropy. On average the correlations between CN and entropy are larger in absolute magnitude in the enzyme proteins as compared to viral proteins. The average  $\rho$  between each structural property and each measure of site variability (entropy and evolutionary rates) can be seen in Table 1 and Table 2. The viral proteins experience much lower site variability. Figure 1 shows the correlations between CN and entropy for the virus and enzyme proteins. Even though on average the correlations in the enzyme proteins are larger as reported, by Yeh et al. (2014), the virus proteins still have correlation strengths that are comparable to some of the enzyme proteins with lower correlations.

The average site variability of a given protein does not seem to be a significant determinant of the strength of structural correlations (Figures 1A, 2A, 3A). However, when examining the variance of entropy there is a clear trend within the enzyme proteins. Proteins with a higher variance in site variability across the protein typically have have lower correlations (Figures 1B, 2B, 3B). If you extrapolate the trend from the enzyme proteins, the viral proteins follow a similar trend. The correlation between the  $\rho$  between RSA and entropy and the variance of entropy is positive. The correlation is negative between local packing density (both WCN and CN) and the variance of entropy is negative. Unlike with entropy, there is no relationship between variance of evolutionary rates at sites and any of the measured structural properties (Figures 4, 5). There also is a wide spread in the variance of the evolutionary rates across proteins for both the enzyme and viral proteins. Although both evolutionary rate (as measured by Rate4Site) and entropy are measures of site variability, these quantities are distinctly different. Rate4Site measures the rate at which a site changes over time whereas site entropy measure the absolute variation at a site. A site may have a high evolutionary rate if it changes between frequently between the same few amino acids but have low entropy. It appears that the rate at which a site evolves is not important for predicting the strength of structure-sequence relationships.

In order to further examine the relationship between entropy and the structure-sequence relationships, we used mean entropy and variance of entropy as predictors of structure-sequence relationships. Table 3 illustrates the the coefficients of venous linear models. In all three models that use the mean entropy as a predictor of structure-entropy correlations, the dataset is a significant predictor. For WCN-Entropy correlations, mean entropy is not a significant predictor. Although for CN-Entropy correlations, mean entropy is significant, the coefficient is 0.079 and therefore predictive power is low. The mean site variability of a protein as measured by entropy is not indicative of whether packing density can be used to predict site variation. Mean Entropy is also not a significant predictor for the strength of the relationship between RSA and Entropy. For all linear models where mean entropy and dataset were used to predict structure-sequence correlations, dataset was a significant predictor. This means that enzyme and virus proteins have different correlations when used mean entropy as a predictor. This agrees with differences seen in the previous works by Yeh et al. (2014) and Shahmoradi et al. (2014).

However, when looking at the variance in entropy there are some stark differences. For

all structural predictors (i.e., CN, WCN and RSA), the variance in entropy at sites within a protein is a significant predictor of the strength of structure-sequence relations. For packing density, proteins with a larger variance in site-wise entropy have lower correlations. Proteins with a higher variance in entropy tend to have stronger RSA-Entropy correlations. When using variance and dataset as predictors of structure-sequence correlations, dataset was not a significant predictor in any model. When looking at the variance of entropy, proteins within both datasets at in a similar fashion. Overall trends seen between the variance of site variation and the strength of structure-sequence relationships is preserved across both datasets.

#### 4 Discussion

#### SOME DISCUSSION.....

Need a section about: Rate4Site is completely different from the entropy variation and we do not know why. Explain the difference.

### 5 Acknowledgements

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## **Figures**

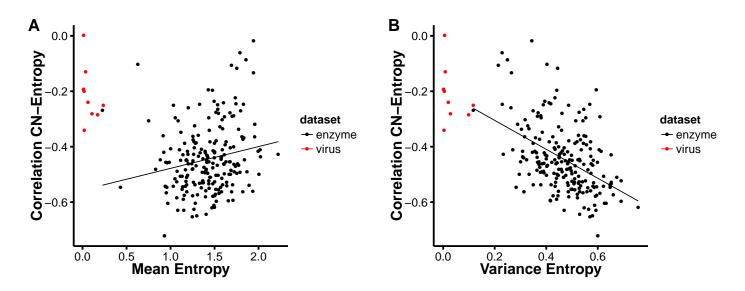


Figure 1: Correlations between the Contact Number (CN) and Entropy. For each protein we calculate the Spearman Correlation coefficient between CN and entropy at each site within the protein. A) Comparison between the correlation coefficients and mean entropy of site in each protein. B) Comparison between correlation coefficients and the variance of entropy of each protein. The enzyme proteins are colored in black and the virus proteins are visualized in red.

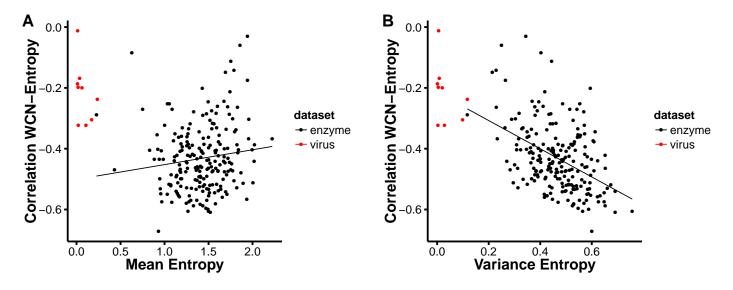


Figure 2: Correlations between the Weighted Contact Number (WCN) and Entropy. A) Comparison between the correlation coefficients and mean entropy of site in each protein. B) Comparison between correlation coefficients and the variance of entropy of each protein. The enzyme proteins are colored in black and the virus proteins are visualized in red.

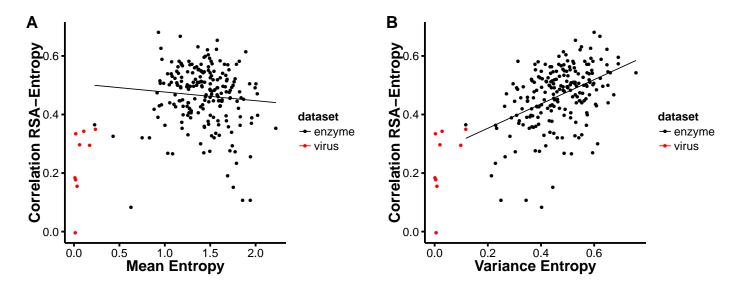


Figure 3: Correlations between Relative Solvent Accessibility (RSA) and Entropy. A) Comparison between the correlation coefficients and mean entropy of site in each protein. B) Comparison between correlation coefficients and the variance of entropy of each protein. The enzyme proteins are colored in black and the virus proteins are visualized in red.

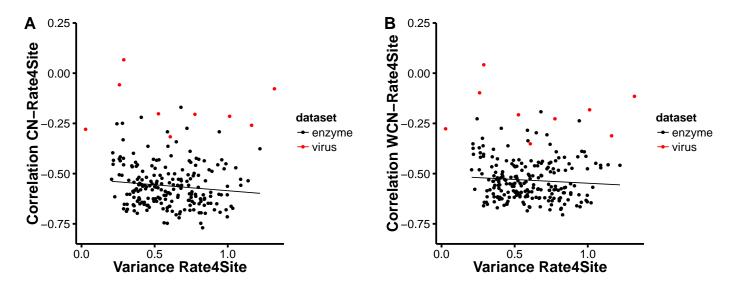


Figure 4: Comparison of Rate4Site correlations and the variance of Rate4Site at sites. A) Comparison between the correlation coefficients between CN and Rate4Site and variance of Rate4Site of site in each protein. B) Comparison between correlation coefficients between WCN and Rate4Site and the variance of entropy of each protein. The enzyme proteins are colored in black and the virus proteins are visualized in red.

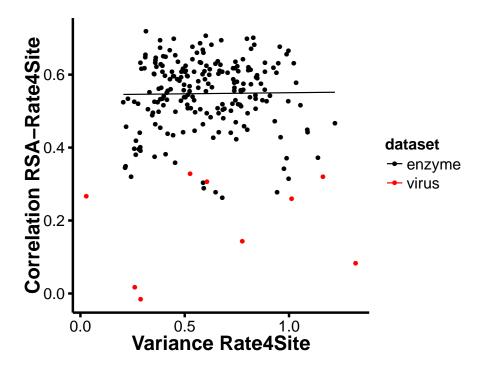


Figure 5: Comparison of the correlations between RSA and Rate4Site and the variance of Rate4Site at sites. The enzyme proteins are colored in black and the virus proteins are visualized in red.

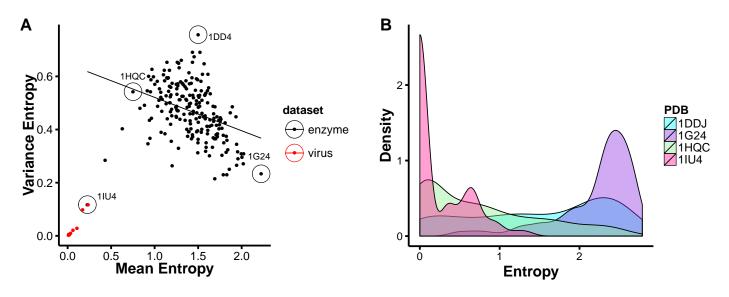


Figure 6: Comparison of the mean entropy and the variance of entropy for individual proteins. A) Variance in entropy at sites compared against overall mean entropy for proteins. Four proteins of interest are highlighted by open circles. The enzyme proteins are colored in black and the virus proteins are colored red B) Distributions of site-wise entropy values for four enzyme proteins.

Dataset	Mean $\rho$	Mean $\rho$	Mean $\rho$
	Entropy-CN	Entropy-	Entropy-
		WCN	RSA
Enzyme	-0.445	-0.432	0.464
Virus	-0.213	-0.217	0.237

Table 1: Averages of the Spearman Correlation Coefficients between Structural Properties and Entropy. The structural properties analyzed are solvent accessibility as calculated by RSA and packing density as calculated by CN and WCN.

Dataset	Mean $\rho$	Mean $\rho$	Mean $\rho$
	Rate4Site-CN	Rate4Site-	Rate4Site-
		WCN	RSA
Enzyme	-0.445	-0.432	0.464
Virus	-0.213	-0.217	0.237

Table 2: Averages of the Spearman Correlation Coefficients between Structural Properties and Evolutionary Rate. Evolutionary rate is calculated with Rate4Site. The structural properties analyzed are solvent accessibility as calculated by RSA and packing density as calculated by CN and WCN.

Model	Coefficient x	Coefficient y	Coefficient z
CN-Entropy $\rho$ = Mean	0.079**	0.382***	-0.583
Entropy + Dataset +			
Dataset*Mean Entropy			
WCN-Entropy $\rho = \text{Mean}$	0.049	0.323***	-0.568
Entropy + Dataset +			
Dataset*Mean Entropy			
RSA-Entropy $\rho$ = Mean	-0.0300	-0.336***	0.903*
Entropy + Dataset +			
Dataset*Mean Entropy			
CN-Entropy $\rho$ = Variance	-0.522***	0.013	-0.265
Entropy + Dataset +			
Dataset*Variance Entropy			
WCN-Entropy $\rho$ = Vari-	-0.464***	0.023	-0.325
ance Entropy + Dataset +			
Dataset*Variance Entropy			
RSA-Entropy $\rho$ = Vari-	0.415***	-0.076	0.927
ance Entropy + Dataset +			
Dataset*Variance Entropy			

Table 3: Linear models predicting structural correlations with various quantities. Coefficient x represents the coefficient of the predictor, coefficient x is the coefficient of the second predictor and coefficient z is the coefficient of the third predictor in each model. \*\*\* means p-value is less than 0.001. \*\* means p-value is less than 0.05.