# Data Representation

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CS/COE 0449
Introduction to Systems Software

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# Binary Encoding

Bits, Bytes, and Nybbles

# Positional Number Systems

- **The numbers we use are written positionally: the position of a digit within the number has a meaning.**

Most Significant       Least Significant

$$1{,}234 = \begin{array}{l} 1 \times 10^3 + \\ 2 \times 10^2 + \\ 3 \times 10^1 + \\ 4 \times 10^0 \end{array}$$

| 1000s | 100s | 10s | 1s |
|---|---|---|---|
| $10^3$ | $10^2$ | $10^1$ | $10^0$ |

- **How many digit symbols do we have in our number system?**
  - 10: 0, 1, 2, 3, 4, 5 ,6 ,7, 8, 9

# Ranges of Representation

- Suppose we have a 4-digit numeric display.
- What is the smallest number it can show?
- What is the biggest number it can show?
- How many *different* numbers can it show?
  - 9999 - 0 + 1 = 10,000
- What power of 10 is 10,000?
  - $10^4$
- With n digits:
  - We can **represent** $10^n$ numbers
  - The **largest number** is $10^n-1$



# Numeric Bases

- These 10s keep popping up… and for good reason
- We use a base-10 (decimal) numbering system
  - 10 different digits, and each place is a power of 10
- But we can use (almost) any number as a base!
- The most common bases when dealing with computers are base-2 (binary) and base-16 (hexadecimal)
- When dealing with multiple bases, you can write the base as a subscript to be explicit about it:

$$5_{10} = 101_2$$

# Let's make a base-2 system

- Given base B,
  - There are **B** digit symbols
  - Each place is worth $\mathbf{B^i}$, starting with $\mathbf{i = 0}$ on the right
  - Given **n** digits,
    - You can represent $\mathbf{B^n}$ numbers
    - The largest representable number is $\mathbf{B^n - 1}$
- So how about base-2?

# Binary (base-2)

- We call a **B**inary dig**IT** a ***bit*** – a single 1 or 0
- When we say an *n*-bit number, we mean one with *n* binary digits

MSB LSB

$$1001\ 0110 =$$

| $2^7$ | $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
|---|---|---|---|---|---|---|---|
| 128s | 64s | 32s | 16s | 8s | 4s | 2s | 1s |

**To convert binary to decimal:** ignore 0s, add up place values wherever you see a 1.

$1 \times 128 +$
$0 \times 64 +$
$0 \times 32 +$
$1 \times 16 +$
$0 \times 8 +$
$1 \times 4 +$
$1 \times 2 +$
$0 \times 1$

$= 150_{10}$

# Bits, Bytes, Nybbles, and Words

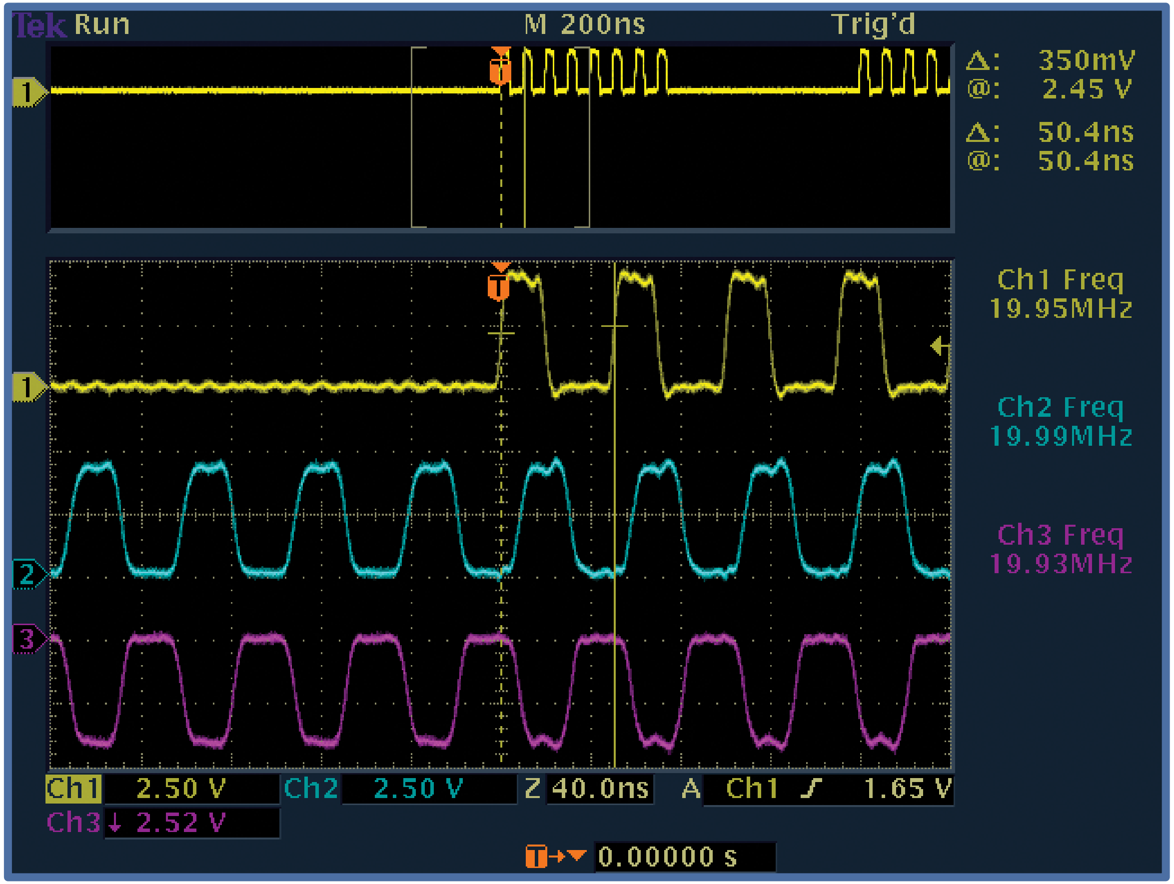
- A ***bit*** is one binary digit, and its unit is lowercase b.
- A ***byte*** is an 8-bit value, and its unit is UPPERCASE B.
  - This is why your 30 mega*bit* (Mb/s) internet connection can only give you at most 3.75 mega*bytes* (MB) per second!
- A ***nybble*** (awww!) is 4 bits – half of a byte.
  - Corresponds nicely to a single hex digit.
- A ***word*** is the "most comfortable size" of number for a CPU.
- When we say "32-bit CPU," we mean its *word* size is 32 bits.
  - This means it can, for example, add two 32-bit numbers at once.
- BUT WATCH OUT:
  - Some things (Windows, x86) use **word** to mean **16 bits** and **double word** (or **dword**) to mean **32 bits.**

# Why binary? *Whynary?*

- Because it's the easiest thing to implement!
- Basic arithmetic is a bit easier.
- So, everything on a computer is represented in binary.
    - **everything.**
    - ***<u>EVERYTHING.</u>***
    - ***01000101 01010110 01000101 01010010 01011001 01010100 01001000 01001001 01001110 01000111 00101110***
        - *("EVERYTHING.")*

# Binary Representation

- Computers translate electrical signals to either 0 or 1.
- It is relatively easy to devise electronics that operate this way.
- In reality, there is no such thing as "binary" so we often have to approximate and mitigate error.




Oscilloscope visualization of several digital wires. From @computerfact on Twitter.

# Integer Encoding

Casting is Not Just a Witch or Wizard Thing

# Hexadecimal

- Binary numbers can get really long, quickly.
  - **$3{,}927{,}664_{10}$ = $11\ 1011\ 1110\ 1110\ 0111\ 0000_2$**
- But nice "round" numbers in binary look arbitrary in decimal.
  - **$1000000000000000_2$ = $32{,}768_{10}$**
- This is because 10 is not a power of 2!
- We could use base-4, base-8, base-16, base-32, etc.
  - Base-4 is not much terser than binary
    - e.g. **$3{,}927{,}664_{10}$ = $120\ 3331\ 2323\ 0000_4$**
  - Base-32 would require 32 digit symbols. Yeesh.
    - They do, oddly, have their place… but not really in this context.
  - **Base-8** and **base-16** look promising!

| Hex | Decimal | Binary |
|---|---|---|
| 0 | 0 | 0000 |
| 1 | 1 | 0001 |
| 2 | 2 | 0010 |
| 3 | 3 | 0011 |
| 4 | 4 | 0100 |
| 5 | 5 | 0101 |
| 6 | 6 | 0110 |
| 7 | 7 | 0111 |
| 8 | 8 | 1000 |
| 9 | 9 | 1001 |
| A | 10 | 1010 |
| B | 11 | 1011 |
| C | 12 | 1100 |
| D | 13 | 1101 |
| E | 14 | 1110 |
| F | 15 | 1111 |

# Hexadecimal or “hex” (base-16)

- Digit symbols after 9 are A-F, meaning 10-15 respectively.
- Usually we call one hexadecimal digit a *hex digit*. No fancy name :(

$$
\begin{array}{cccccccc}
0 & 0 & 3 & B & E & E & 7 & 0 \\
16^7 & 16^6 & 16^5 & 16^4 & 16^3 & 16^2 & 16^1 & 16^0
\end{array}
=
$$

$$
\begin{aligned}
0 \times 16^7 &+ \\
0 \times 16^6 &+ \\
3 \times 16^5 &+ \\
11 \times 16^4 &+ \\
14 \times 16^3 &+ \\
14 \times 16^2 &+ \\
7 \times 16^1 &+ \\
0 \times 16^0 &=
\end{aligned}
$$

$$3{,}927{,}664_{10}$$

**To convert hex to decimal:** use a dang calculator lol

# Binary to Hex

(animated)

0100 1100 1010 0010 0000 0010 0110 0001

4 C A 2 0 2 6 1

0x4CA20261

32-bits! (Not so bad…)

**Q**: Create a random binary string and practice!

# Signed Numbers (sign-magnitude)

- Seems like a good time to think about "negative" values.
  - These are numbers that have nothing good to say.

- Binary numbers have bits which are either 0 or 1.
  - Well, yeah…

- So what if we used one bit to designate "positive" or "negative"
  - Called **sign-magnitude** encoding:

$$1\underbrace{0100010}\ = \ -\underbrace{34}$$

$$0\underbrace{0010110}\ = \ \underbrace{22}\ \texttt{(normal)}$$

# Signed Numbers (problems)

$$\underbrace{\color{red}{1}0000000}_{} = \underbrace{\color{red}{-}0}_{}$$

$$\underbrace{\color{red}{0}0000000}_{} = \underbrace{0}_{}$$

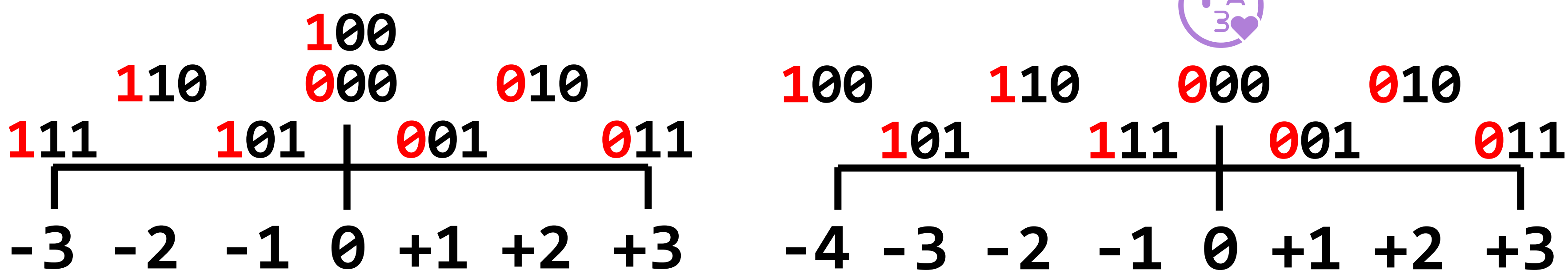
- **Waaaaait a second.**
  - What is negative zero???

- **This encoding allows two different zeros.**
  - This means we can represent how many different values (8-bit)?
    - 2^8 – 1 (minus the one redundant value) = 255 (-127 … 0 … 127)

- **Sign-magnitude is a little naïve… let's try a different approach…**

# Signed Numbers (2's Complement)

- This one, I promise, is juuuuust right.
  - But it's a little strange!
- We'll just make SURE there is only one zero:




Signed Magnitude

2's Complement

- So, we flip the bits… and add one.
  - Adding one makes sure our -0 is used for -1 instead!
- Sure, it's a little lopsided, but, hey, we get an extra number.
  - But, hmm, but -4 **doesn't have a valid positive number**.
    - That's the trade-off, but it's for the best.

# Signed Numbers (2's Complement)

- Let's look some examples:

11010100 = -00101011 = -(43+1) = -44

00100110 = 00100110 = 38

00000000 = 00000000 = 0

11111111 = -00000000 = -(0+1) = -1

- If the MSB is 1: Flip! Add one!
- Otherwise: Do nothing! It's the same!

# Signed Numbers (2's Complement)

- What happens when we add zeros to a positive number:

`00100110 = 38`

`000`

`10100110 = ?`

`-(01011001+1) = ?`

`-01011010 = -90`

- What happe :

`10100110 = -90`

`1111111110100110 =`

`-0000000001011001 = -90`

Dang that's cool!

# Can I Get an Extension?

- Sometimes you need to *widen* a number with fewer bits to more
- ***zero extension*** is easy: put 0s at the beginning.

$1001_2$ → *to 8 bits* → $0000\ 1001_2$

- But there are also signed numbers… what about those?
  - The **top bit (MSB)** of signed numbers determines **the sign (+/-)**
- ***sign extension*** puts *copies of the sign bit* at the beginning

$1001_2$ → *to 8 bits* → $1111\ 1001_2$

$0010_2$ → *to 8 bits* → $0000\ 0010_2$

**Q**: What happens when you sign extend the largest unsigned value?

# Integer Ranges

- **Recall:**
  - The range of an unsigned integer is 0 to $2^n - 1$
  - Q: Why do we subtract 1?

- **What is the range of a 2's complement number?**
  - Consider the sign bit, how many positive integers?
  - Consider, now, the negative integers.
  - Remember 0.

$$-2^{n-1} \text{ to } 2^{n-1} - 1$$

**Q**: What if you needed a range with more negatives than positives?

# Integers in C

- **C allows for variables to be declared as either signed or unsigned.**
  - Remember: "signed" does not mean "negative" just that it *can* be negative.

- **An unsigned integer variable has a range from 0 to $2^n - 1$**
- **And signed integers are usually 2's complement: $2^{n-1}$ to $2^{n-1} - 1$**
  - Where "n" is determined by the variable's size in bits.

- **Integer Types: (signed by default, their sizes are arbitrary!!)**
  - `char` — `unsigned char` — `8 bits (byte)`
  - `short int` — `unsigned short int` — `16 bits (half-word)`
  - `int` — `unsigned int` — `32 bits (word)`
  - `long int` — `unsigned long int` — `64 bits (double-word)`

- **Usually no strong reason to use anything other than (un)signed int.**

**Q**: What is the range of a `signed char`?

# Integers in C: Limits

- **Since sizes of integers are technically arbitrary…**
  - They are usually based on the underlying architecture.

- **… C provides standard library constants defining the ranges.**
  - https://pubs.opengroup.org/onlinepubs/009695399/basedefs/limits.h.html

```c
#include <limits.h>           // Provides INT_MAX etc
#include <stdio.h>            // Provides printf

int main() {
  printf("%d ", INT_MAX);     // Print the maximum signed int
  printf("%u\n", UINT_MAX);   // Print the maximum unsigned int
  return 0;
}               // Output: 2147483647 4294967295
```

**Q**: Experiment with using “%d” for both. What is the result?

# Casting

- **C lets you move a value from an unsigned integer variable to a signed integer variable. (and vice versa)**

- **However, this is not always valid! Yet, it will do it anyway.**
  - The binary value is the same, ***its interpretation is not***!
    - This is called ***coercion***, and this is a relatively simple case of it.
  - Since it ignores obvious invalid operations this is sometimes referred to as “weak” typing.
  - The strong/weak terminology has had very fragile definitions over the years and are arguably useless in our context. Let’s ignore them.

- **Moving values between different types is called *casting***
  - Which sounds magical and it sometimes is.

**Q**: What happens when we cast the smallest signed int to unsigned?