Data Representation II

3

CS/COE 0449
Introduction to Systems Software

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(with content borrowed from Vinicius Petrucci and Jarrett Billingsley)

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Bit Manipulation

Flippin' Switches

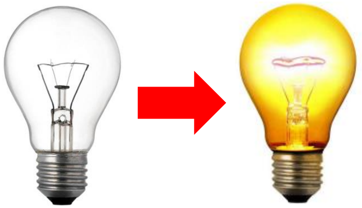
What are "bitwise" operations?

- The "numbers" we use on computers aren't *really* numbers right?
- It's often useful to treat them instead as a pattern of bits.
- Bitwise operations treat a value as a pattern of bits.

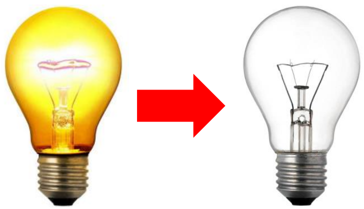
1 0 0 0

The simplest operation: NOT (logical negation)

- If the light is off, turn it on.



- If the light is on, turn it off.



A	Q
0	1
1	0

- We can summarize this in a truth table.
- We write NOT as ~A, or ¬A, or $\overline{\mathbf{A}}$

- In C, the NOT operation is the “!” operator

Applying NOT to a whole bunch of bits

- If we use the not instruction (~ in C), this is what happens:

~	0	0	1	1	1	0	1	0
=	1	1	0	0	0	1	0	1

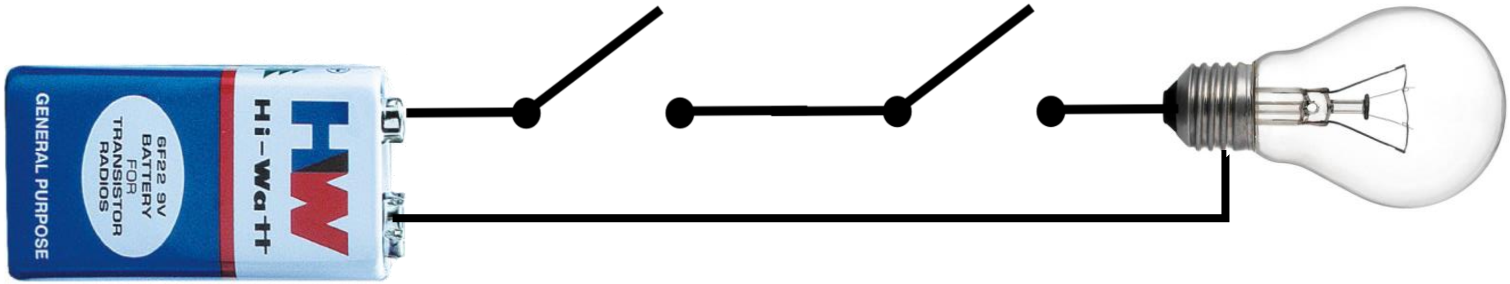
we did 8 **independent** NOT operations

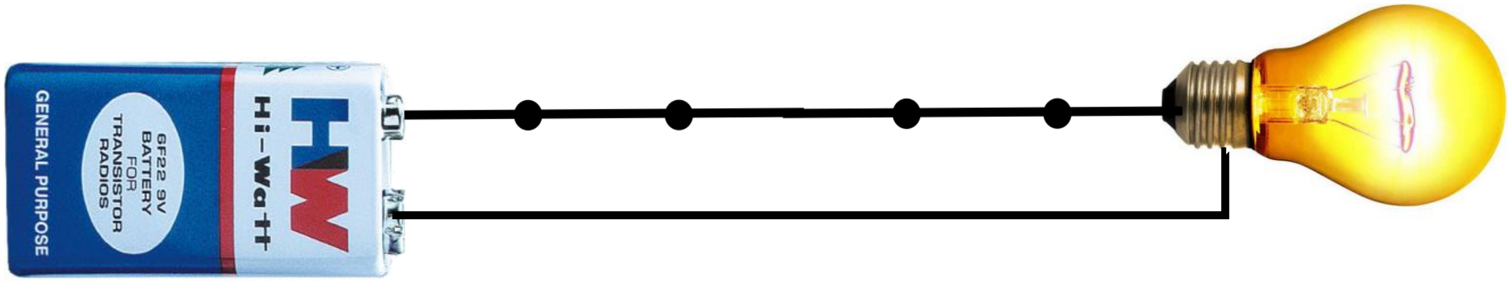
That's it.

only 8 bits shown cause 32 bits on a slide is too much

Let's add some switches

- There are two switches in a row connecting the light to the battery.
- How do we make it light up?





AND (Logical product)

- AND is a binary (two-operand) operation.
- It can be written a number of ways:

  A&B A∧B A·B AB

- If we use the and instruction (& in C):

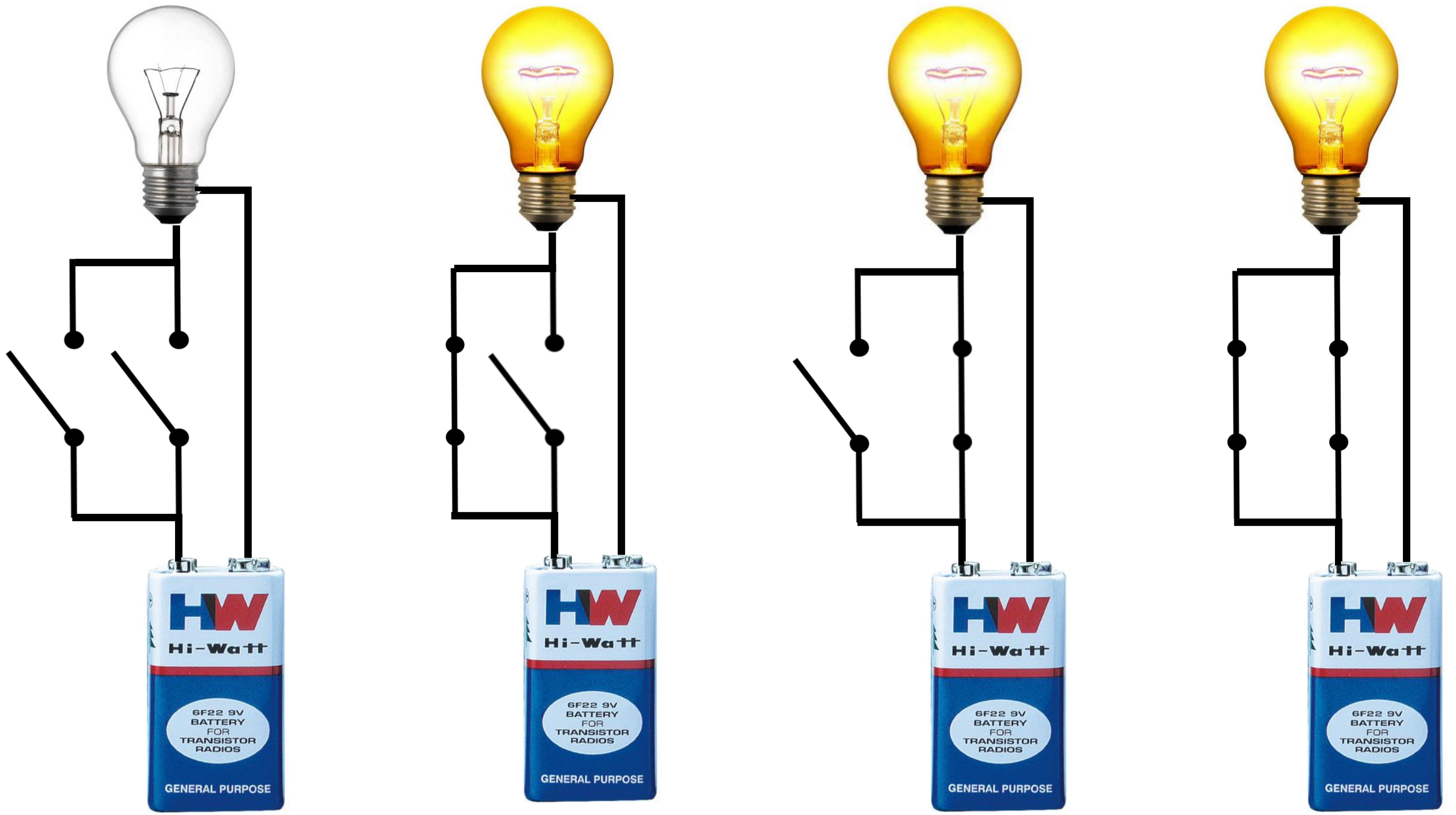
	1	1	1	1	0	0	0	0
&	0	0	1	1	1	0	1	0
=	0	0	1	1	0	0	0	0

we did 8 **independent** AND operations

A	B	Q
0	0	0
0	1	0
1	0	0
1	1	1

"Switching" things up ;))))))))))))))))))))

- NOW how can we make it light up?



OR ("Logical" sum...?)

- We might say "and/or" in English.
- It can be written a number of ways:

  A|B   A∨B   A+B

- If we use the or instruction (| in C):

	1	1	1	1	0	0	0	0
|	0	0	1	1	1	0	1	0
=	1	1	1	1	1	0	1	0

We did 8 **independent** OR operations.

A	B	Q
0	0	0
0	1	1
1	0	1
1	1	1

Bit shifting

- Besides AND, OR, and NOT, we can move bits around, too.

`1 1 0 0 1 1 1 1` if we shift these bits **left by 1…**

`1 1 0 0 1 1 1 1 0` we stick a **0** at the bottom

`1 1 0 0 1 1 1 1 0 0` again!

`1 1 0 0 1 1 1 1 0 0 0` AGAIN!

`1 1 0 0 1 1 1 1 0 0 0 0` **AGAIN!!!!**

Left-shifting in C/Java (animated)

- C (and Java) use the << operator for left shift

```
B = A << 4; // B = A shifted left 4 bits
```

If the bottom 4 bits of the result are now 0s…

- …what happened to the *top* 4 bits?

`0011 0000 0000 1111 1100 1101 1100 1111`

the bit bucket is not a real place

it's a programmer joke ok

in the UK they might say the “Bit Bin”

bc that’s their word for trash

>_> >_> >_> ☺

- We can shift right, too

```
0 0 1 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 0 1 1 0 1 1 1 0 0 1 1 1 1
0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 0 1 1 0 1 1 1 0 0 1 1 1
0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 0 1 1 0 1 1 1 0 0 1 1
0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 0 1 1 0 1 1 1 0 0 1
0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 0 1 1 0 1 1 1 0 0
```

- C/Java use >>, this in MIPS is the **srl** (**S**hift **R**ight **L**ogical) instruction

see what I mean about 32 bits on a slide

Q: What happens when we shift a negative number to the right?

Shift Right (Logical)

- We can shift right, too (srl in MIPS)

```
 1 1 0 0 1 1 1 1
0 1 1 0 0 1 1 1 1
0 0 1 1 0 0 1 1 1
0 0 0 1 1 0 0 1 1
0 0 0 0 1 1 0 0 1
```

if we shift these bits **right by 1…**

we stick a **0** at the top

again!

AGAIN!

Wait… what if this was a negative number?

Shift Right (Arithmetic)

- We can shift right with sign-extension, too (MIPS: sra)

`1 1 0 0 1 1 1 1`

if we shift these bits **right by 1…**

`1 1 1 0 0 1 1 1 1`

we copy the 1 at the top (or 0, if MSB was a 0)

`1 1 1 1 0 0 1 1 1`

again!

`1 1 1 1 1 0 0 1 1`

AGAIN!

`1 1 1 1 1 1 0 0 1`

AGAIN!!!!!! (It's still negative!)

Is there a sla instruction?

Huh... that's weird

- Let's start with a value like 5 and shift left and see what happens:

Binary	Decimal
`101`	5
`1010`	10
`10100`	20
`101000`	40
`1010000`	80

Why is this happening

Well uh... what if I gave you

49018853

How do you multiply that by 10?

by 100?

by 100000?

Something **very similar** is happening here

$a << n == a * 2^n$

- **Shifting left by *n* is the same as multiplying by 2^n**
  - You probably learned this as "moving the decimal point"
    - And moving the decimal point *right* is like shifting the digits *left*

- **Shifting is fast and easy on most CPUs.**
  - Way faster than multiplication in any case.
  - (It's not a great reason to do it when you're using C though)

- **Hey… if shifting *left* is the same as multiplying…**

a >> n == a / 2^n, ish

- You got it
- Shifting right by n is like dividing by 2^n
  - ***sort of.***

- What's 101_2 shifted right by 1?
  - 10_2, which is 2…
    - It's like doing **integer** (or **flooring**) division

- Generally, compilers are smart enough that you just multiply/divide
  - It’s confusing to shift just to optimize performance.
  - It’s good to not be clever until it is proven that you need to be.

C Bitwise Operations: Summary

C code	Description	MIPS instruction
`x | y`	`or`	`or  x, x, y`
`x & y`	`and`	`and x, x, y`
`x ^ y`	`xor`	`xor x, x, y`
`!x`	`not`	`seq x, x, $0` (“seqz”)
`~x`	`complement (negate)`	`nor x, x, $0` (“not”)
`x << y`	`left-shift logical`	`sll x, x, y`
`x >> y`	`right-shift logical`	`srl x, x, y`

When x is signed (most of the time…):

`x >> y`	`right-shift arithmetic`	`sra x, x, y`

Fractional Encoding

Every Time I Teach Floats I Want Some Root Beer

Fixing the point

- If we want to represent decimal places, one way of doing so is by assuming that the lowest *n* digits are the decimal places.

\$12.34	1234
+\$10.81	+1081
\$23.15	2315

this is called **fixed-point representation**

A rising tide

- A 16.16 fixed-point number looks like this:

0011 0000 0101 1010.1000 0000 1111 1111

binary point

the largest (signed) value we can represent is +32767.999

the smallest fraction we can represent is 1/65536

But if we let the binary point **float around...**

0011.0000 0101 1010 1000 0000 1111 1111

...we can get much higher **accuracy** near 0...

0011 0000 0101 1010 1000 0000.1111 1111

...and trade off accuracy for **range** further away from 0.

IEEE 754

- Established in 1985, updated as recently as 2008.
- Standard for floating-point representation and arithmetic that virtually every CPU now uses.
- Floating-point representation is based around scientific notation:

$$
\begin{aligned}
1348 &= +1.348 \times 10^{+3} \\
-0.0039 &= -3.9 \times 10^{-3} \\
-1440000 &= -1.44 \times 10^{+6}
\end{aligned}
$$

sign significand exponent

Binary Scientific Notation

- scientific notation works equally well in any other base!
  - (below uses base-10 exponents for clarity)

$$+1001\ 0101 = +1.001\ 0101 \times 2^{+7}$$

$$-0.001\ 010 = -1.010 \times 2^{-3}$$

$$-1001\ 0000\ 0000\ 0000 = -1.001 \times 2^{+15}$$

what do you notice
about the digit before
the **binary** point?

$$(+/-)1.f \times 2^{exp}$$

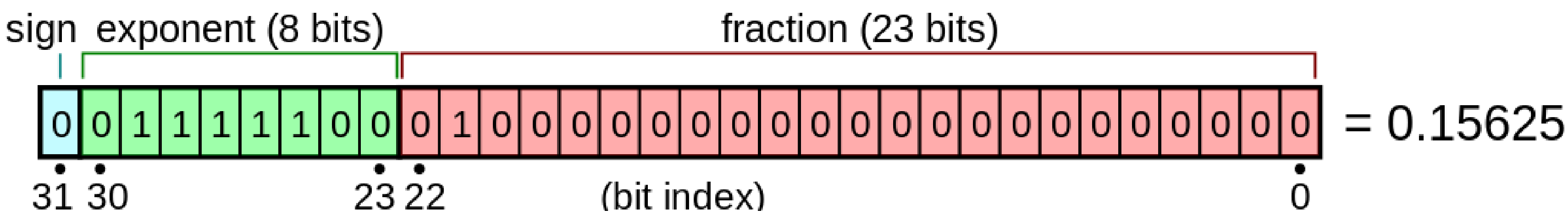
f – fraction

1.f – significand

exp – exponent

IEEE 754 Single-precision

- Known as float in C/C++/Java etc., 32-bit float format
- 1 bit for sign, 8 bits for the exponent, 23 bits for the *fraction*



- Tradeoff:
  - More accuracy ⍰ More fraction bits
  - More range ⍰ More exponent bits
- Every design has tradeoffs ¯_(ツ)_/¯
  - Welcome to Systems!

illustration from user Stannered on Wikimedia Commons

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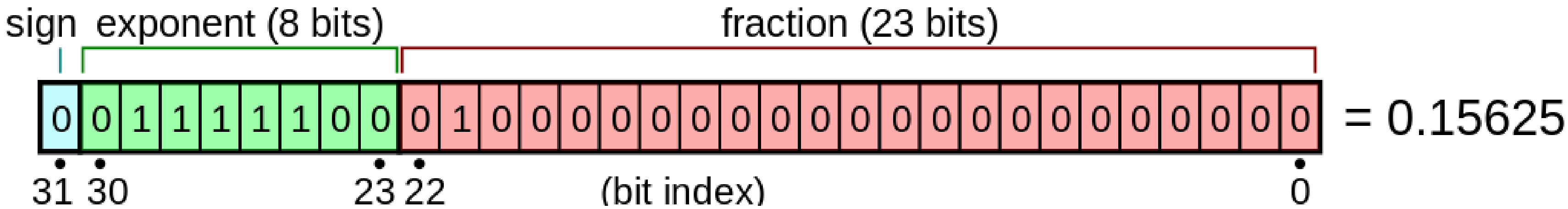


- The fraction field only stores the digits after the binary point
- The 1 before the binary point is implicit!
  - This is called normalized representation
  - In effect this gives us a 24-bit significand
  - The only number with a 0 before the binary point is 0!
- The significand of floating-point numbers is in sign-magnitude!
  - Do you remember the downside(s)?

illustration from user Stannered on Wikimedia Commons

The exponent field

- **The exponent field is 8 bits, and can hold positive or negative exponents, but... it doesn't use S-M, 1's, or 2's complement.**
- **It uses something called biased notation.**
  - biased representation = signed number + *bias constant*
  - **single-precision** floats use a bias constant of **127**

```
       -127 => 0
Signed  -10 => 117   Biased
         34 => 161
```

- **The exponent can range from -126 to +127 (1 to 254 biased)**
  - 0 and 255 are reserved!
- **Why'd they do this?**
  - so you can sort floats with integer comparisons??

Binary Scientific Notation (revisited)

- Our previous numbers are actually

$$+1.001\ 0101 \times 2^{+7} = (-1)^0 \times 1.001\ 0101 \times 2^{134-127}$$
$$-1.010 \times 2^{-3} = (-1)^1 \times 1.010 \times 2^{124-127}$$
$$-1.001 \times 2^{+15} = (-1)^1 \times 1.001 \times 2^{142-127}$$

$$(-1)^s \times 1.f \times 2^{exp-127}$$

s – sign

f – fraction

exp – biased exponent

Encoding an integer as a float

- You have an integer, like 2471 = $0000\ 1001\ 1010\ 0111_2$
  1. put it in scientific notation
     - $1.001\ 1010\ 0111_2 \times 2^{+11}$
  2. get the exponent field by adding the bias constant
     - $11 + 127 = 138 = 10001010_2$
  3. copy the bits **after the binary point** into the fraction field

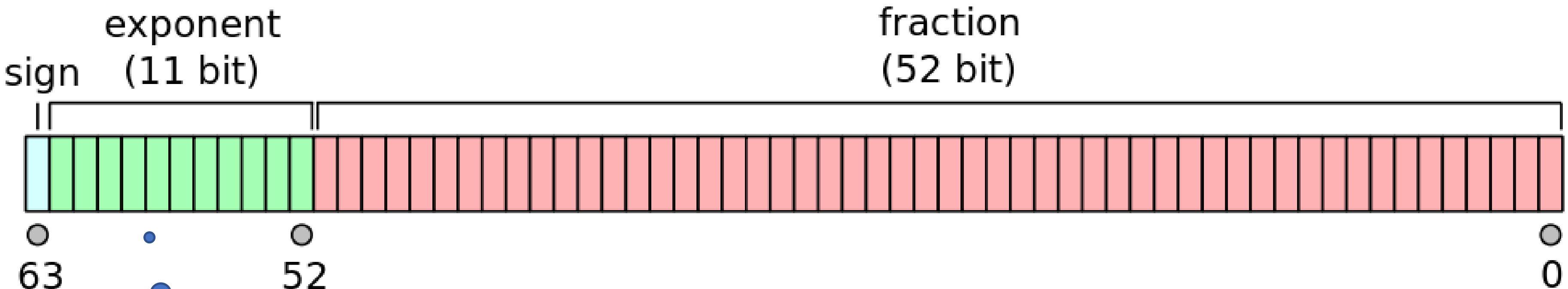
s	exponent	fraction
0	10001010	00110100111000000...000

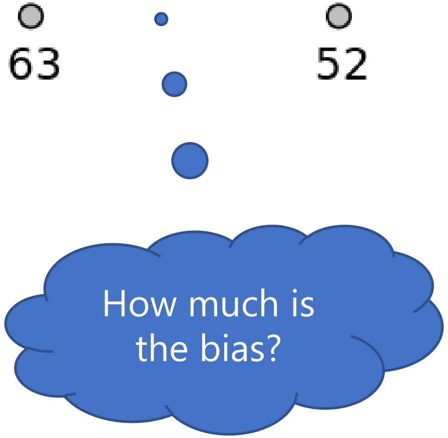
positive

start at the **left** side!

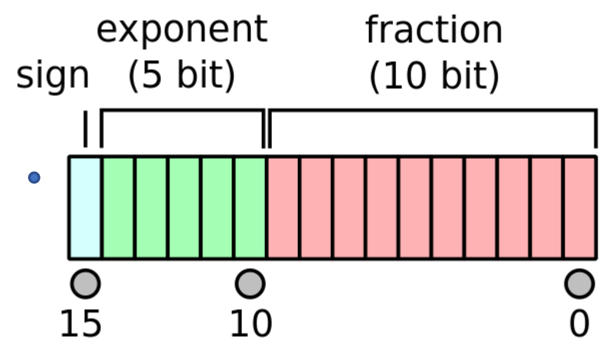
Other formats

- The most common other format is double-precision (C/C++/Java double), which uses an 11-bit exponent and 52-bit fraction





- GPUs have driven the creation of a half-precision 16-bit floating-point format. it's adorable



both illustrations from user Codekaizen on Wikimedia Commons

This could be a whole unit itself...

- Floating-point arithmetic is COMPLEX STUFF.

- But it's not super useful to know unless you're either:
  - Doing lots of high-precision numerical programming, or
  - Implementing floating-point arithmetic yourself.

- However...
  - It's good to have an understanding of *why* limitations exist.
  - It's good to have an *appreciation* of how complex this is... and how much better things are now than they were in the 1970s and 1980s!
  - It's good to know things do not behave as expected when using float and double!!