Formula Sheet 3

- Jean's Mass: $M_{critical} = 18M_\odot\sqrt{\frac{T^3}{\rho}}$
  - $M_{critical}$ is the mass necessary for a cloud to condense.
  - $M_\odot$ is the mass of the Sun, $\approx 2 \times 10^{30}\ kg$.
  - T is the temperature of the gas cloud, measured in Kelvin.
  - ρ is the density of the gas cloud, measured in $\frac{particles}{cm^2}$.
- Scale Factor vs. Redshift: $\frac{a_{today}}{a_0} = 1 + z$
  - a_{today} is the scale factor today (usually set to 1).
  - a_0 is the scale factor at the time the light was emitted.
  - z is the redshift observed.
- Schwarzschild Radius: $r_{schwarzschild} = \frac{2GM}{c^2} \approx (3\ km)\frac{M}{M_\odot}$
  - $r_{schwarzschild}$ is the radius of the event horizon if not distorted by any other factors.
  - G is the Universal Gravitational Constant, $6.67 \times 10^{-11}\ Nm^2kg^{-2}$ (number changes depending on units).
  - M is the mass of the black hole.
  - c is the speed of light, $3 \times 10^8\ \frac{m}{s}$.
  - $M_\odot$ is the mass of the Sun, $\approx 2 \times 10^{30}\ kg$.

Formula Sheet 2

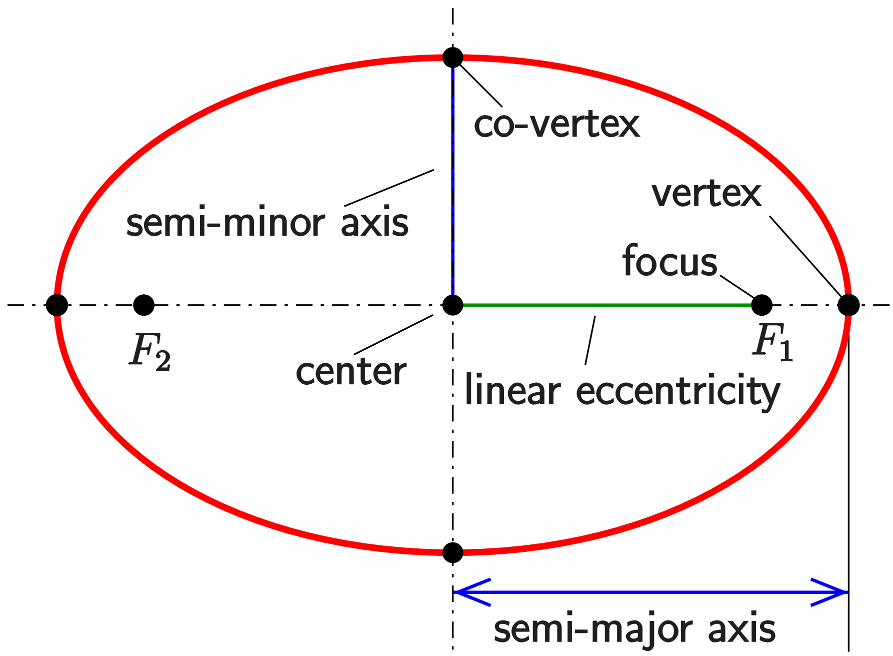
- Absolute vs. Apparent Magnitude: $M = m - 5\log(d) + 5$ or $m = M + 5\log(d) - 5$
  - M is the absolute magnitude
  - m is the apparent magnitude
  - d is the distance from the object to the observer measured in parsecs.
  - The logarithm is in base 10.
- Absolute Magnitude vs. Luminosity: $\frac{L_1}{L_2} = 10^{\frac{2}{5}(M_2 - M_1)}$
  - L_1, L_2 are the luminosities of object 1, object 2 respectively.
  - M_1, M_2 are the absolute magnitudes of object 1, object 2 respectively.
  - Note this is the exact same relation as Apparent Magnitude vs. Apparent Brightness.
- Apparent Brightness (Inverse Square Law): $F = \frac{L}{4\pi r^2}$
  - F is apparent brightness (flux). See Stefan-Boltzmann Law for Power generated per m^2.
  - L is the object's luminosity.
  - r is the distance between the object and the observer.
- Apparent Magnitude vs. Apparent Brightness: $\frac{F_1}{F_2} = 10^{\frac{2}{5}(m_2 - m_1)}$
  - F_1, F_2 are the apparent brightnesses of object 1, object 2 respectively.
  - m_1, m_2 are the apparent magnitudes of object 1, object 2 respectively.
  - Note this is the exact same relation as Absolute Magnitude vs. Luminosity.
- Change of Distance (Apparent Magnitude): $10^{\frac{2}{5}(m_1 - m_2)} = (\frac{d_1}{d_2})^2$
  - m_1 is the initial magnitude.
  - m_2 is the final magnitude.
  - d_1 is the initial distance. If $d_2 = 10$ pc, m_2 is absolute magnitude, then $d_1 = 10^{\frac{m-M+5}{5}}$
  - d_2 is the final distance. If final distance is 10 pc, simplifies to Absolute vs. Apparent Magnitude equation.

- Radius vs. Luminosity and Temperature: $\frac{R_1}{R_2} = (\frac{T_2}{T_1})^2\sqrt{\frac{L_1}{L_2}}$
  - R_1, R_2 are the radii of object 1, object 2 respectively.
  - T_1, T_2 are the surface temperatures of object 1, object 2 respectively.
  - L_1, L_2 are the luminosities of object 1, object 2 respectively.
- Relativistic Length Contraction: $L_{obs} = \frac{L_{prop}}{\gamma}$
  - L_{obs} is the length measured by an observer moving relativistically.
  - L_{prop} is the length measured at rest.
  - γ is the Lorentz Factor: $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$
- Relativistic Mass Multiplication: $m_{obs} = m_{prop}\gamma$
  - m_{obs} is the mass measured by an observer moving relativistically.
  - m_{prop} is the mass measured at rest.
  - γ is the Lorentz Factor: $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$
- Relativistic Time Dilation: $t_{obs} = t_{prop}\gamma$
  - t_{obs} is the time measured by an observer moving relativistically.
  - t_{prop} is the time measured at rest.
  - γ is the Lorentz Factor: $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$
- Relativistic Velocity Addition: $v = \frac{v_1+v_2}{1+\frac{v_1 v_2}{c^2}}$
  - v is the final velocity.
  - v_1 is the velocity of object 1.
  - v_2 is the velocity of object 2.
  - c is the speed of light, $3 \times 10^8 \ \frac{m}{s}$.
- (Heisenberg's) Uncertainty Principle: $\Delta x \Delta p \geq \frac{h}{4\pi}$
  - Δx is the uncertainty in position.
  - Δp is the uncertainty in momentum.
  - h is Planck's constant, for Joules and seconds, $6.63 \times 10^{-34} \ Js$.

Formula Sheet 1

- Angular Momentum (circular motion, in $\frac{kgm^2}{s}$): $L = rmv_\perp$
  - L is the angular momentum.
  - r is the radius of the circle.
  - m is the mass of the moving object.
  - $v_\perp$ is the perpendicular component of velocity.
- Angular Resolution / Diffraction Limit (in arcseconds): $L = 2.5 \times 10^5 \frac{\lambda}{d_{\text{telescope}}}$
  - L is the diffraction limit.
  - λ is the wavelength of light targeted.
  - $d_{\text{telescope}}$ is the diameter of the telescope.
- Angular Size / Angular Separation (in degrees): $\theta = d_{\text{actual}} \times \frac{360°}{2\pi d_{\text{to Earth}}}$
  - d_{actual} is the diameter of object (size) or the distance between the two objects (separation)
  - $d_{\text{to Earth}}$ is the distance between the object and Earth.
- Arc Length (same units as radius): $s = \theta r$
  - s is the arc length.
  - θ is the angle swept by the arc, in radians. If in degrees, use $\frac{\theta\pi}{180}$.
  - r is the radius (or radial distance) of the object.
  - Notable use case is Eratosthenes' circumference of Earth approximation with known θ and s as well as approximating actual diameter of a far away object given angular size and distance from Earth.
- Center of Mass (of two bodies): $\frac{m_1 d_1 + m_2 d_2}{m_1 + m_2}$
  - m_1, m_2 are the masses of the two objects.
  - d_1, d_2 are the distances of the two objects.

- Doppler Effect: $\frac{\lambda_{\text{observed}}-\lambda_{\text{actual}}}{\lambda_{\text{actual}}} = \frac{v_{\text{rad}}}{c}$
  - $\lambda_{\text{observed}}$ is wavelength observed from the object.
  - λ_{actual} is the wavelength that the object actually emits (found from measurements on Earth).
  - v_{rad} is the radial component (the component moving towards or away from us).
  - c is the speed of light, $3 \times 10^8\ \frac{m}{s}$
- Ellipse Eccentricity: $e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{c}{a}$

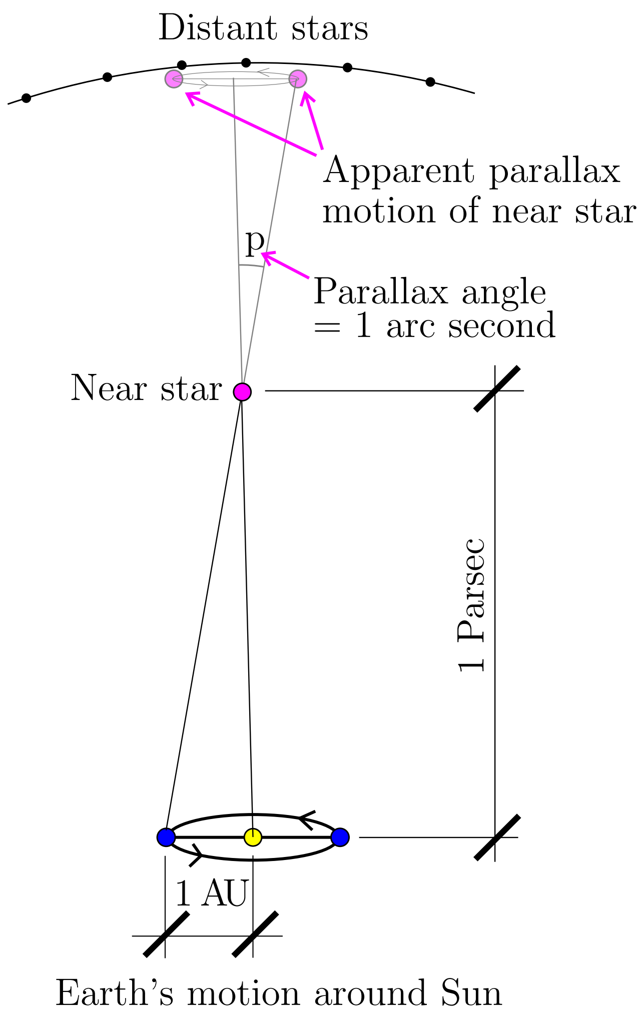


  - e is the eccentricity of the object.
  - a is the length of the semi-major axis (think of it as the longer radius).
  - b is the length of the semi-minor axis (think of it as the shorter radius).
  - c is the distance from the center to a focus. Can be calculated with $c = ae$

- Energy: $E_{\text{total}} = KE + PE$
  - E_{total} is the total energy of the object.
  - KE is the kinetic energy of the object, calculated with $KE = \frac{1}{2}mv^2$
    * m is the mass of the object.
    * v is the velocity of the object.
  - PE is the potential energy of the object, usually gravitational potential energy. Can be calculated with $PE = -\frac{GM_1M_2}{r}$ or on the surface of Earth, mgh.
    * G is the Universal Gravitational Constant, $6.67 \times 10^{-11}\ Nm^2kg^{-2}$ (number changes depending on units).
    * M_1, M_2 are the mass of the two bodies.
    * r is the distance between the two bodies.
    * m is the mass of the object on Earth.
    * g is the acceleration due gravity on Earth, $g \approx -9.8\frac{m}{s^2}$.
    * h is the height of the object from the surface.
- Escape Velocity (usually in $\frac{m}{s}$ or $\frac{km}{s}$): $v_{esc} = \sqrt{\frac{2GM}{R}}$
  - v_{esc} is the velocity needed to escape from the orbit of a body.
  - G is the Universal Gravitational Constant, $6.67 \times 10^{-11}\ Nm^2kg^{-2}$ (number changes depending on units).
  - M is the mass of the object you're escaping from.
  - R is your distance from the center of the object.
- Kepler's Third Law: $P^2 \propto a^3$; Newton's Version: $P^2 = \frac{4\pi^2}{G(M_1+M_2)}a^3$
  - P is the orbital period (measure of time)
  - a is the semi-major axis of the orbit (measure of distance)
  - G is the Universal Gravitational Constant, $6.67 \times 10^{-11}\ Nm^2kg^{-2}$ (number changes depending on units).
  - M_1, M_2 are the masses of the objects, usually one is a star and the other is a planet (but not necessarily). Newton's version can be re-arranged to solve for mass: $M_1 + M_2 = \frac{4\pi^2}{GP^2}a^3$.

- Law of Universal Gravitation (in N): $F_g = \frac{GM_1M_2}{r^2}$
  - F_g is the force of gravity that the objects exert on each other.
  - G is the Universal Gravitational Constant, 6.67×10^{-11} Nm^2kg^{-2} (number changes depending on units).
  - M_1, M_2 are the masses of the two objects.
  - r is the distance between the two objects.
- Mass-Energy Equivalence (usually in J): $E = mc^2$
  - E is the energy associated with the mass.
  - m is the mass being converted into energy.
  - c is the speed of light, 3×10^8 $\frac{m}{s}$.
- Momentum (in $\frac{kgm}{s} or Ns$): $p = mv$
  - p is the momentum.
  - m is the mass of the object.
  - v is the velocity of the object.
  - Useful context: calculating conservation of momentum (set initial and final momentum equal to each other and solve for a variable).
- Motion: x, s, v, a and t
  - x is the displacement of the object (has both magnitude and direction).
  - s is the speed of the object, calculated with $s = \frac{\|\Delta x\|}{\Delta t}$. Does not have direction.
  - v is the velocity of the object (has both magnitude and direction). Calculated with $v = \frac{\Delta x}{\Delta t}$.
  - a is the acceleration of the object (has both magnitude and direction). Calculated with $a = \frac{\Delta v}{\Delta t}$
  - t is time, Δt is the change in time.
- Newton's Second Law (in N): $F = ma = \frac{dp}{dt}$
  - F is the net force exerted on the object.
  - m is the mass of the object.
  - a is the acceleration associated with the net force.
  - $\frac{dp}{dt}$ is the derivative of momentum with respect to time. Think of it as the change in momentum over time. When $F = 0$, momentum is constant.

- Stefan-Boltzmann Law: Power per Square Meter = σT^4
  - σ is the Stefan-Boltzmann Constant, $5.7 \times 10^{-8}\ Wm^{-2}K^{-4}$
  - T is the temperature of the object, in Kelvin.
- Stellar Parallax: $d_{\text{to object}} = \frac{r}{p}$

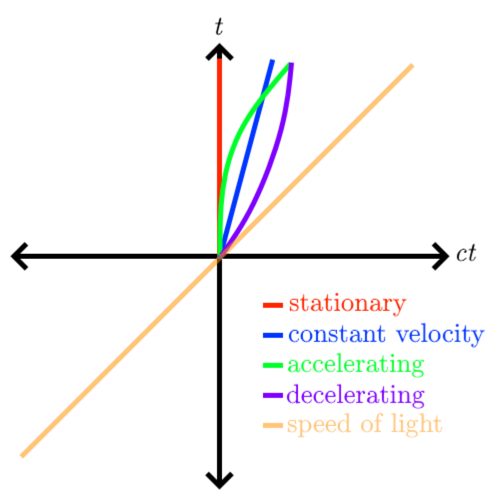


  - $d_{\text{to object}}$ is the distance from the Earth to the observed body.
  - r is the amount of distance the observer moves in order to observe parallax. In the context of Earth, this is $1\ AU = 1.5 \times 10^{11}\ m$.
  - p is the parallax angle, or the angle the surroundings change at.
- Torque (in Nm): $\tau = F_{\perp} r$
  - τ is Torque.
  - $F_{\perp}$ is the perpendicular component of the Force.
  - r is the radius of the circular motion.

- (Electromagnetic) Wave Properties: $\lambda f = c$, $E = hf$
  - λ is the wavelength of the wave. Can be calculated with $\lambda = \frac{c}{f}$.
  - f is the frequency of the wave. Can be calculated with $f = \frac{c}{\lambda}$.
  - c is the speed of light, $3 \times 10^8 \ \frac{m}{s}$.
  - E is the energy of the wave.
  - h is Planck's constant, for Joules and seconds, $6.63 \times 10^{-34} \ Js$.
- Wien's Law: $\lambda_{\text{peak}} = \frac{b}{T}$
  - λ_{peak} is the wavelength that the object radiates with highest relative intensity.
  - b is Wien's Displacement Constant, equal to $2.9 \times 10^{-3} \ mK$ or $2.9 \times 10^6 \ nmK$
  - T is the temperature of the object.

Additonal Useful Info

- Luminosity Classes
  - Give information about the star's radius.
  - I - Supergiant
  - II - Bright Giant
  - III - Giant
  - IV - Subgiant
  - V - Main Sequence Star
  - VI, VII - Subdwarf, White Dwarf
- Spectral Types
  - Give information about the star's surface temperature. In order of hottest to coolest:
  - O, B, A, F, G, A, K, M
  - Types also have subclasses numbered 0-9. Within each class, 0 subclass stars are hottest and 9 subclass stars are the coolest. For example, B0 is hotter than B1, but A0 is cooler than B9.
- Spacetime Diagrams
  - Tell you how a an object moves through spacetime.
  - x-axis is position, y-axis is time. Usually in light seconds and seconds as their respective units.
  - Stationary objects will be a vertical line, objects at a constant velocity move in a straight line, and accelerating objects will move in a curved line.
  - Light is usually a 45° line or $-45°$. Any slope less than that is impossible (moving faster than light).



- Stellar Classification
  - Brown Dwarfs: $\leq 0.08 M_\odot$. Not really stars (degeneracy pressure prevents fusion)
  - Low-Mass Stars: $0.08 M_\odot < M \leq 2 M_\odot$.
  - Intermediate Mass Stars: $2 M_\odot < M \leq 8 M_\odot$.
  - High Mass Stars: $> 8 M_\odot$.
  - Upper limit: $\approx 150 M_\odot$.
- Hubble's Law: Galaxies are moving away from us at $H_0 \approx 70 \frac{km/s}{pc}$.
  - Find the distance an object is at given velocity v with $d = \frac{v}{H_0}$
  - Find the age of the universe with $H(t)^{-1} \approx 13.8\ Ga$.
- Distance Ladder
  - Find the closest objects using radio technologies.
  - Find the distance of items within a parsecs using Stellar Parallax.
  - Find the distance of further objects using Cepheids as a standard candle.
  - The furthest objects are found using Supernova Type Ia (white dwarf supernovae) as a standard candle.

- H-R Diagrams
  - Plots the temperature and spectral class on the horizontal axis. Hotter stars are further to the left.
  - Plots the luminosity on the vertical axis. More luminous stars are higher.
  - Main sequence stars that fuse hydrogen are all generally shown on a diagonal line.
  - Protostars and giant stars are to the top and right of their respective main sequence stars.