1 Exercise I.1

Using proof by induction, prove the following statement (assume n is a natural number). (1)

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Let's assume that the statements is true for n = p,

$$\sum_{i=1}^{p} i^2 = \frac{p(p+1)(2p+1)}{6}$$

Next we will show that it is also true for the next case n = p+1,

$$\sum_{i=1}^{p+1} i^2 = \frac{(p+1)(p+2)(2p+3)}{6}$$

First we calculate and simplify the left hand side (LHS) of the p+1 case,

$$LHS_{p+1} = \sum_{i=1}^{p+1} i^2$$
$$= LHS_p + (p+1)^2$$
$$= RHS_p + (p+1)(p+1)$$
$$= \frac{p(p+1)(2p+1)}{6} + (p+1)(p+1)$$
$$= \frac{2p^3 + 3p^2 + p}{6} + p^2 + 2p + 1$$
$$\frac{2p^3 + 9p^2 + 13p + 6}{6}$$

Then we do the same to the right hand side (RHS) of the same n = p+1 case,

$$RHS_{p+1} = \frac{(p+1)(p+2)(2p+3)}{6}$$
$$= \frac{(p^2 + 3p + 2)(2p+3)}{6}$$
$$= \frac{2p^3 + 3p^2 + 6p^2 + 9p + 4p + 6}{6}$$
$$= \frac{2p^3 + 9p^2 + 13p + 6}{6}$$

The right hand side equals the left hand side,
Q.E.D.

2 Exercise I.1

Using proof by induction, prove the following statement (assume n is a natural number). (2)

$$\sum_{j=1}^{n}(2j-1) = n^2$$

Let's assume that the statements is true for n = p,

$$\sum_{j=1}^{p}(2j-1) = p^2$$

Next we will show that it is also true for the next case n = p+1,

$$\sum_{j=1}^{p+1}(2j-1) = (p+1)^2$$

First we calculate and simplify the left hand side (LHS) of the p+1 case,

$$\begin{aligned} LHS_{p+1} &= \sum_{j=1}^{p+1}(2j-1) \\ &= LHS_p + 2p + 1 \\ &= RHS_p + 2p + 1 \\ &= p^2 + 2p + 1 \end{aligned}$$

Then we do the same to the right hand side (RHS) of the same n = p+1 case,

$$\begin{aligned} RHS_{p+1} &= (p+1)^2 \\ &= p^2 + 2p + 1 \end{aligned}$$

The right hand side equals the left hand side,
Q.E.D.