ESE 441 FL 2024: Case Study 2

Due on: 12/09 by 5pm on Canvas
Title: Artificial Pancreas

1 High-level Instructions (Read Carefully)

This is a Case Study where there is no single 'solution.' The idea is to explore the scenario and justify your rationale and approach. You should try and invoke principles from class whenever possible, avoiding things that are totally heuristic. You should especially avoid bringing into play algorithms or methods that are extremely outside the scope of the class and that you cannot justify. This Case Study will be assessed on the basis of **clarity of thought and justification**, rather than the use of some special formula that cannot be explained.

Your engagement of each prompt may involve a combination of analysis and MATLAB numerical approaches.

Do not attempt to 'delegate' specific responsibilities, e.g., 'Person A was responsible for writing the report.' We will spot check participant's ability to explain their reports in class. Everyone involved in a group should be comfortable will all phases of the Case Study.

Please ensure that the work you submit is your own. Cases of academic dishonestly will be referred to the University academic integrity office.

2 Scenario

The glucose-insulin system is a regulatory mechanism in humans that maintains blood glucose levels within a range, typically 70–125 mg/dL. This regulation ensures consistent supply of energy to cells while avoiding harm from hyperglycemia (elevated glucose levels) or hypoglycemia (reduced glucose levels). Blood glucose primarily derives from carbohydrates, which are digested and absorbed into the bloodstream. Glucose is removed from the bloodstream through cellular uptake and metabolism, which relies on the presence of insulin.

Insulin is a hormone secreted by the pancreas in response to rising blood glucose levels (e.g., after a meal). It facilitates glucose uptake by muscle and fat cells, thereby reducing blood glucose levels. This mechanism, whereby rising glucose triggers insulin release, and insulin action reduces glucose levels, maintains homeostasis under normal conditions.

This regulation becomes impaired in diabetes. For example, in Type 1 Diabetes, the pancreas produces little to no insulin, necessitating external insulin delivery. Modern diabetes management often relies on medical technologies such as insulin pumps, which mimic or enhance the body's natural glucose-insulin regulation by delivering insulin based on real-time glucose measurements.

3 Simplified Mathematical Model

Glucose-insulin interaction can be modeled as a dynamical system with three key components: (i) glucose clearance, (ii) insulin action, and (iii) exogenous input, including meals and insulin injections. These dynamics are amenable to feedback control analysis and design.

In this case study, you will design a glucose-insulin delivery control control system to regulate blood glucose levels. **There is more than one way to approach this analysis and design.**

Below is a simple (sometimes known as a 'toy') model of glucose-insulin dynamics.

3.1 States and Parameters

- $G(t)$: Blood glucose concentration (mg/dL).

- $I(t)$: Plasma insulin concentration (mU/L).
- $X(t)$: Insulin action
- G_b: Baseline glucose concentration (mg/dL).
- I_b: Baseline plasma insulin concentration (mU/L).
- p_1, p_2, p_3: System parameters describing insulin sensitivity and glucose dynamics.
- $u(t)$: Exogenous insulin delivery rate (mU/min).

The model consists of the following differential equations:

Glucose Dynamics Describes how glucose levels in the blood change over time

$$\frac{dG(t)}{dt} = -p_1(G(t) - G_b) - X(t)G(t) + D(t),$$

where: p_1 is the rate of glucose decay (1/min), and $D(t)$ is the rate of glucose influx from meals (mg/dL/min).

Insulin Action Dynamics Models the pharmacokinetics of insulin delivery.

$$\frac{dX(t)}{dt} = -p_2X(t) + p_3(I(t) - I_b),$$

where p_2 is the rate of insulin action decay (1/min), and p_3 is the effectiveness of insulin in stimulating glucose uptake (1/min).

Plasma Insulin Dynamics

$$\frac{dI(t)}{dt} = -nI(t) + u(t),$$

where n is the insulin clearance rate (1/min) and $u(t)$ is the insulin delivery rate (control input, mU/min).

Parameter Values Assume that the typical patient has the following parameters, plus minus 15%.

- $p_1 = 0.03\,\text{min}^{-1}$
- $p_2 = 0.02\,\text{min}^{-1}$
- $p_3 = 0.01\,\text{min}^{-1}$
- $n = 0.1\,\text{min}^{-1}$
- $G_b = 100\,\text{mg/dL}$
- $I_b = 10\,\text{mU/L}$

Measurement Assume that you have a glucose sensor, such that the output of the model is $y(t) = G(t)$.

4 Control Objective

Your challenge is to specify a controller that produces $u(t)$ such that glucose levels $G(t)$ are regulated at a desired set point G_{ref}. For example, a typical setpoint might be $G_{\text{ref}} = 100\,\text{mg/dL}$. You would like this regulation to withstand meal disturbances with that occur as frequently as every 4 hours.

5 Case Study Requirements

You are required, at minimum, to engage and address on the following

1. Use a combination of state-space and frequency domain formulations within your modeling, analysis and design processes. Your report should clearly delineate how you are using these different formulations.
2. Explain any linearization that is performed.
3. Make a design choice as to how you will model for the meal disturbance signal $D(t)$. Your choice should balance analytical tractability and compatibility with your available methods, with physiological realism.
4. **Every** design choice should be justified and related to a class concept. *Strategies that are opaque as to the underlying principle, borne out of trial and error, or invoke mysterious resources outside of the class, will be summarily disregarded.*
5. You should engage the question of robustness, that is how does your design handle model uncertainty. That is, you design based on one set of parameters, but the actual model deviates from what was assumed.
6. You should use simulations to carefully study how well your design is performing within the provided scenario. Be sure to contrast your controlled model relative to the open-loop situation where there is no insulin input.
7. Comment on the feasibility of your design, and possible tradeoffs as it pertains to cost, ease of deployment, and efficacy.

6 Bonus Opportunity

The Case Study includes an opportunity to earn a 10% bonus by giving a 5 minute 'design pitch', accompanied by three slides, providing preliminary methods, results and conclusions, in class on 12/4. Slides need to be uploaded to canvas by 12/2, 5pm.

7 What to turn in (read carefully, has changed from Case Study 1)

1. A single PDF report, approximately 4-6 pages single spaced, 11pt Font, 1 inch margins, describing your approach and providing salient figures. 11 Pt Font, It is recommended to use the IEEE template. At this stage in your academic career, **there is no excuse for sloppy, poorly formatted documents**. **Margin violations, illegible captions, handwritten figures, inconsistent font size and paragraph formatting will have zero tolerance.**

   Best: Use LateX/Overleaf; Adequate: Word Desktop; Not recommended: Google Docs
2. Published MATLAB scripts.
3. Academic Integrity violations will result in a score of zero and a referral to the University academic integrity process. I expect a mature, professional approach to this case study.
4. Rubric:
   (a) Presentation quality: 20%
   (b) Thoroughness and consistency of argument with fundamental knowledge: 20%
   (c) Integration of both state space and frequency domain arguments: 20%
   (d) Numerical simulations cohesiveness and consistency with fundamental knowledge: 25%
   (e) Embracing the design spirit: 10%
   (f) Participation in pitch presentations: 5%
   (g) Bonus opportunity: +10% (5% Quality, 5% Completeness)