

ESE319 – EnMath B

FINAL PROJECT

Spring 2024



(Concept art by Scifi Shipyards)

0 Welcome to Starfleet

Congratulations on your assignment to the USS *Cerritos*, ensign! We're excited to see how your unique skills can be used for the good of the ship and its crew. It's a tradition on this vessel to give new crewmembers their choice of first assignment, and this document serves as a guide to the tasks on offer. Please read through **all** of it before reporting to your first duty shift. Here's how the process works:

- 📌 You'll select **one** of our six projects (from the following sections numbered 1 through 6), each associated with a division of Starfleet, and work on it independently.
- 📌 When you complete your project, you'll write a short report about your approach and results and deliver it to your superior officer via Gradescope. (Yes, we're still using it in the 24th century.) Details on this are provided in section 7, and your report is due promptly by

1:00 PM CDT on 6 May 2024.

- 📌 Your superior officer will give you feedback on your efforts following the rubric in section 8 shortly after the due date above.

Don't forget: it's strongly recommended to review **all** the available projects and the report requirements before deciding which option is best for you.

1 Unstoppable

Tribbles are small, furry creatures that are seemingly harmless. However, they reproduce at an alarming rate and can quickly overwhelm the local environment when left to their own devices. Starfleet would like to know more about tribbles, but they don't trust us enough to give us one of our own. Instead, they've just provided a really complicated equation, which they claim models the volume fraction of tribbles within a given space based on the time elapsed without outside interference. (Or it's just a practical joke; sometimes it's hard to tell with these people.) Either way, we've been ordered to work on this problem. This doesn't have to be your assignment, but we know you can handle it if you'd like to give it a try!

Loosely-related Song: "Unstoppable" by Sia

Loosely-related *Trek* Episode: *The Original Series* Season 2, Ep. 15, "The Trouble with Tribbles"

Project 1 Description

Project Name: Trials & Tribble-ations

Division: Sciences

Technical Objectives: For a volume fraction φ of tribbles at t hours elapsed given by

$$12t^2(3t+1)^2 \frac{d^2\varphi}{dt^2} + 7t \frac{d\varphi}{dt} - 2\varphi = 0,$$

complete the following:

1. Propose a solution technique and justify your choice.
2. List the minimal interval of convergence (IoC) based on your solution technique **and** explain what it means in this context.
3. Develop a general solution for the volume fraction of tribbles $\varphi(t)$.

Creative Objective: In the event Starfleet lets us have a tribble, we'll need a name for our new pet. Actually, it'd be better to think of more than one name for when it inevitably reproduces. Provide a list of at least eight (8) pet tribble names so that we won't have to come up with them on the fly before being overrun...

2 The Space Between

Between space and subspace lies an interfold layer. Not much is known about it except for it's unstable and communications through it are challenging. However, there's a theory that may prove useful: mathematicians within the Federation have proposed that an expansion based on Chebyshev polynomials may allow for stable, prolonged communications with objects in the interfold layer. If true, this would open up a new era in the exploration of the cosmos. The comms officer on the ship is a (concerningly) huge fan of Tom Paris, and so he wants the *Cerritos* to be at the forefront of this research because Paris flew through the interfold layer that one time. We'd really appreciate it if someone could look into this, if for no other reason than to get him to shut up about it. Oh, and it could revolutionize the field or whatever.

Loosely-related Song: "The Space Between" by Dave Matthews Band

Loosely-related *Star Trek* Episode: *Voyager* Season 3, Ep. 22, "Real Life"

Project 2 Description

Project Name: Interfold Layer Communications

Division: Operations

Technical Objectives:

1. Show that **all** of the conditions of the Sturm-Liouville (S-L) Theorem are met by Chebyshev's problem:

$$(1 - x^2) y'' - xy' + n^2 y = 0 \text{ where } y(-1) \text{ and } y(1) \text{ are both finite.}$$

Hints: You may set $\lambda = n^2$, and it may help to divide everything by $\sqrt{1 - x^2}$.

2. Solutions to Chebyshev's problem using the substitution $x = \cos \theta$ are given by the Chebyshev polynomials: $T_n(\cos \theta) = \cos(n\theta)$ for $n = 0, 1, 2, \dots$. Using these, construct a general formula for a Fourier-Chebyshev (F-C) series representation of any function $f(\cos \theta)$.
3. Using your F-C series from the second objective, create an F-C series expansion of the function $f(x) = x^2$ as a proof of concept for this communication strategy.

Creative Objective: Starfleet wants to be prepared in the event of encountering intelligent life in the interfold layer. Draft a friendly welcome message to be broadcast to an unknown alien species using this new technology. Due to bandwidth constraints, your message can be no more than fifty (50) words long.

3 Hot in Herre

The *Cerritos* has a few guest quarters, and recently a Betazoid ambassador moved into quarters previously occupied by a Vulcan diplomat. The new occupant is very uncomfortable in this room because it's much too warm for her liking, even after she adjusted the climate controls to their chilliest setting. There aren't any other (nice) rooms available, and none of the other guests wants to trade for the super-hot room. Engineering suspects the electro-plasma system (EPS) is to blame, and that adding a convection cooling system to the EPS conduit adjacent to the ambassador's room will resolve the issue. If you take on this project, you'll create a model for an EPS conduit with the new cooling technology attached to one end. We're aware that the Robin BCs necessary to model heat convection aren't covered at Starfleet Academy, but we're confident in your ability to learn on the fly!

Loosely-related Song: "Hot in Herre" by Nelly

Loosely-related *Star Trek* Episode: *Enterprise* Season 2, Ep. 4, "Dead Stop"

Project 3 Description

Project Name: EPS Conduit Cooling

Division: Engineering

Technical Objectives:

1. Find the eigenvalues λ_n for the BVP below.

$$\alpha u_{xx} = u_t \text{ on } 0 < x < L, t > 0$$

$$\text{with BCs } u(0, t) = 0 \text{ and } u_x(L, t) = -\frac{h}{\alpha}u(L, t) \text{ and IC } u(x, 0) = f(x)$$

Hint: Solutions to $cx + \tan Lx = 0$ may be approximated using $x \approx \frac{2n-1}{2} \frac{\pi}{L}$.

2. Using your eigenvalues from the first objective, write the general solution $u(x, t)$ to the BVP above, including the integral formula(s) for the coefficient(s).

Hint: The derivation on page 5 of the Unit 2 notes may be a helpful resource.

3. Using your solution from the second objective, provide a simplified solution $u(x, t)$ with the values below to model the EPS conduit adjacent to the ambassador's quarters.

$$L = 7 \text{ m} \quad \alpha = 0.15 \frac{\text{mm}^2}{\text{s}} \quad h = 600 \frac{\text{W}}{\text{m}^2\text{K}} \quad f(x) = 50^\circ\text{C}$$

Creative Objective: Propose and describe at least two (2) activities that the ambassador can do while the work on her room is completed. These must take place somewhere on the ship that isn't her room. Our goal is to keep her happy and distracted so the problem can be quickly fixed.

4 Compliance

Some mysterious officers in dark, foreboding uniforms showed up last week. They want work done quickly and off the record, but nobody on board wants to do it for fear of being disappeared. In addition to all the secrecy, Starfleet wants a nice graphical representation of the results because, and I quote, “all that math nonsense is for nerds.” We decided at that point it was best to stop asking questions. This project is all yours if you want it! (Starfleet assumes no responsibility past that point.)

Loosely-related Song: “Compliance” by Muse

Loosely-related *Trek* Ep.: *Deep Space Nine* Season 7, Ep. 16, “Inter Arma Enim Silent Leges”

Project 4 Description

Project Name: Top-Secret Number-Crunching

Division: Section 31

Technical Objectives: A mysterious object in the shape of a perfect sphere has some unknown quantity u that is distributed axisymmetrically following the equation $\nabla^2 u = 0$ throughout its interior. The quantity’s distribution on the outer surface of a sphere of diameter D is given by $u\left(\frac{D}{2}, \theta\right) = \frac{1}{6}K_0\left(\cos\frac{\theta}{2}\right)$. Based on this information...

1. Write a simplified expression for the steady-state solution $u(r, \theta)$ within a sphere of diameter D . Do **not** evaluate any integral(s).
2. List the first **six** non-zero terms in your solution from the first objective for a sphere of diameter $D = 4$ cm. Round to four sig figs as necessary.
3. Generate an accurate plot of your solution from the second objective showing the concentrations across the central circular cross-section of the sphere.

Note: Calculator and/or computer assistance is permitted and highly recommended throughout this project, but don’t forget to fully document your work.

Creative Objective: Describe what you believe the mysterious sphere to be **and** why you think Starfleet intelligence is interested in it.

5 Elevator Music

The denizens of Subos IV are trying to build their own space elevator and failing spectacularly. We've been called in to help.

The main issue they described is cable failure when the elevator is in motion. A stress analysis should do the trick, but these cables are *really* long. It's also possible the material they're using isn't ideal, so we should probably recommend something else just to cover our bases. This shouldn't take long, right?

Loosely-related Song: "Elevator Music" by Beck

Loosely-related *Star Trek* Episode: *Lower Decks* Season 3, Ep. 2, "The Least Dangerous Game"

Project 5 Description

Project Name: A Big Lift

Division: Engineering

Technical Objectives:

1. Propose a material for the space elevator's cables to be made out of **and** provide a reasonable value for its Young's modulus E . (These can be factual **or** fictional.)
2. Find the steady-state stresses in an axisymmetric infinite cylinder $\sigma(r, z)$ (which models the elevator's cables) by solving the following BVP:

$$\nabla^2 \sigma = 0, \quad r < c, \quad |z| < \infty, \quad \text{BCs } \sigma_r(0, z) = 0 \text{ and } \sigma(c, z) = \begin{cases} t_0 & |z| < L \\ 0 & |z| \geq L \end{cases}.$$

3. Using your responses to the first two objectives and the values below as necessary, write a simplified solution $\sigma(r, z)$. Do **not** evaluate any integral(s).

$$L = 8 \text{ m} \qquad c = 50 \text{ cm} \qquad t_0 = 30 \text{ MPa}$$

Creative Objective: Recommend a song (including its title **and** artist, and that isn't the song listed above) to be played on loop within the space elevator's passenger cabin **and** describe why you selected that song.

6 Making It Up As I Go

We get it: you joined Starfleet to chart your own course! You are welcome to pursue your own project instead. To do this, send a custom project proposal to the head of ensign wrangling, Kevin Brennan (a.k.a. Mr. B), **no later than 5 PM CDT on Tuesday, 30 April 2024**. Your project must have a creative objective and three technical objectives; Mr. B can help you with elements of your project design if you'd like. Your project **must** be approved in advance by Mr. B in order to be accepted for credit.

Loosely-related Song: “Making It Up As I Go” by Kylie Morgan

Loosely-related *Star Trek* Episode: *Strange New Worlds* Season 2, Ep. 9, “Subspace Rhapsody”

Project 6 Description

Project Name: _____

Division: Training

Technical Objectives:

1. _____
2. _____
3. _____

Creative Objective: _____

7 Report Requirements

Once you've selected and attempted your project, you will write a report about your work. This report must meet these requirements for full credit:

- 📌 The report must be typed and predominantly in English.
- 📌 The report must be at least two (2) pages long and no longer than five (5) pages. The cover page and any appendixes are **not** included in the report's page count.
- 📌 The body text must be in a readable 11- or 12-point font with 1.5 or double line spacing and one inch (1") margins.
- 📌 The report must include a cover page, introduction, and conclusion. Other organizational elements, including a table of contents, are **not** required.
- 📌 The report must state which project you're attempting and include content that meets the technical **and** creative objectives of that project.
- 📌 The report should only include mathematical statements that are relevant and of major significance (*e.g.*, governing equations, key definitions, and simplified final solutions). It should **not** include irrelevant information or minor elements (*e.g.*, steps within a solution process). Your supervisor cares more about the big picture than any particular detail.
- 📌 All supporting calculations, screenshots, code, *etc.*, **must** be included as appendixes to the report. Appendixes are not subject to the formatting rules of the report itself and may be handwritten.
- 📌 All outside resources are permitted on this project; however, your report is an individual submission and **must** be your own original work. Sources may be cited, but if they are a reference list must be included in the report. Any report suspected of plagiarism will be reported to the academic integrity committee with a recommended penalty of a score of zero on this project and, in severe cases, a failing grade in this course.
- 📌 Your report and appendix(es) must be uploaded to Gradescope as a single submission no later than **1:00 PM CDT on 6 May 2024**. Late submissions will be subject to penalty of **four points plus one point for each hour elapsed since the deadline** before its submission.

If you'd like help getting started on your report, optional Word and \LaTeX templates that meet the formatting requirements above are available on Canvas. Creativity in your presentation and written work is encouraged but not required.

8 Grading Rubric

	0 pts	1 pt	2 pts	3 pts	4 pts
Report Formatting¹	Incorrectly formatted	Correctly formatted	–	–	–
Report Elements²	One or more missing	All required ones included	–	–	–
Report Language	Unclear or inappropriate	Overly technical	Clear and appropriate	–	–
Technical Objective 1	Not attempted	Attempted but incomplete	Completed with major error(s)	Completed with minor error(s)	Successfully completed
Technical Objective 2	Not attempted	Attempted but incomplete	Completed with major error(s)	Completed with minor error(s)	Successfully completed
Technical Objective 3	Not attempted	Attempted but incomplete	Completed with major error(s)	Completed with minor error(s)	Successfully completed
Supporting Appendix(es)	Not included	Included but incomplete ³	Included and complete	–	–
Creative Objective	Not attempted	Attempted but incomplete	Successfully completed	Outstanding submission ⁴	–

¹ The formatting requirements can be found in section 7; they are the top three items in the list.

² The required elements relevant to this rubric item are a cover page, an introduction, a summary of results from all attempted objectives, and a conclusion.

³ “Incomplete” appendixes are ones which do not fully document and support the results shown in the report.

⁴ This is potential **bonus** credit to be applied at the instructor’s discretion.

Post general questions about this project on Piazza, or contact Mr. B with individual concerns.

Live Long and Prosper