ESE 351 Spring 2024 Homework #6
Assigned Tuesday, March 26
Due Tuesday, April 4 – Submit in gradescope
Material to be covered in Quiz 3 Tuesday April 9

Read Chapter 5 in Oppenheim and Willsky. The '*Basic Problems with Answers*' are all good introductory problems, especially the first half.

Written problems to turn in (25 points each). Solve the problems below (a mix of custom problems and problems from the text). Acknowledgement to course AI Brian Sun for additional problems. Partial credit will be given: if you cannot solve a problem completely, describe your approach and the difficulty you are having.

For each problem, show all supporting work, e.g., consider the following items:

- A brief problem statement (rephrased in your own words when possible),
- your approach, including key expressions and variables, sketches when appropriate,
- the results of your solution and interpretation when possible.
- Include justification of any steps used to produce your results (results without justification will not receive full credit).

1. Compute the Fourier transform, $X(e^{j\omega})$, of each of the following signals. You may use properties of the Fourier transform, along with known transform pairs, but indicate the source of that information.
   a. $x[n] = 2^n \cos\left(\frac{\pi}{4}n\right) u[-n]$
   b. $x[n] = \sin\left(\frac{\pi}{4}n\right) + \cos(n)$
   c. $x[n] = \frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n} \cos\left(\frac{4\pi}{3}n\right)$
   d. A DT rectangular function (as in Homework 4, but now interpreted via the DTFT),

$$x[n] = \begin{cases} 1, & -N_1 \le n \le N_1 \\ 0, & \text{else} \end{cases}.$$

   For each of the following functions, use Matlab (with an N-point fft) to compute and plot the DTFT ($\left|X(e^{j\omega})\right|, \angle X(e^{j\omega})$, $\omega = k\frac{2\pi}{N}$, for $k = 0, 1, \dots, N-1$) and provide a brief interpretation of the results, including any symmetry.
      i. $x[n]$, with $N_1 = 3$, and $N = 16, 64$ and 256.
      ii. $x[n]$, with $N = 256$, and $N_1 = 2, 6$ and 10.
      iii. For $N_1 = 6, N = 256$.
         1. $x[n-3]$
         2. $x[n] - \frac{1}{2}$
         3. $\cos(\pi n)\, x[n]$

2. Determine the discrete-time signal, $x[n]$, corresponding to each of the following Fourier transforms.
   a. $X\left(e^{j\omega}\right) = 3 - 4e^{-j\omega} + 2e^{-j3\omega} + 5e^{-j5\omega}$
   b. $X\left(e^{j\omega}\right) = \cos(\omega)$
   c. $X\left(e^{j\omega}\right) = \sum_{k=-\infty}^{\infty} \cos\left(\frac{\pi}{2}k\right)\delta\left(\omega - \frac{\pi}{2}k\right)$
   d. $X\left(e^{j\omega}\right) = \begin{cases} 1, \frac{\pi}{4} \le |\omega| \le \frac{3\pi}{4} \\ 0,\ 0 \le |\omega| < \frac{\pi}{4}, \frac{3\pi}{4} \le |\omega| \le \pi \end{cases}$
   e. $X\left(e^{j\omega}\right) = \frac{1-\left(\frac{1}{2}\right)^4 e^{-j4\omega}}{1-\frac{1}{2}e^{-j\omega}}$
3. Use the Fourier transform properties to solve the following problems:
   a. Show that $h[n] = (n+1)a^n u[n], |a| < 1 \leftrightarrow H\left(e^{j\omega}\right) = \frac{1}{\left(1-ae^{-j\omega}\right)^2}$
   b. Find $\sum_{n=-\infty}^{\infty}|x[n]|^2$, or $\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|X\left(e^{j\omega}\right)\right|^2 d\omega$ (the energy in $x[n]$) for
      i. $x[n] = \left(\frac{1}{7}\right)^n u[n]$.
      ii. $x[n] = \frac{\sin\left(\frac{37\pi}{64}n\right)}{n}$.
   c. Text problem 5.27 parts (a) and (b) for signals (i) and (iv)
4. Solve these LTI input-output problems
   a. Text problem 5.30 parts (a) and (b) for $h[n]$ in (ii) and (iii)
   b. Text problem 5.33, parts (a) and (b)

5.27. **(a)** Let $x[n]$ be a discrete-time signal with Fourier transform $X(e^{j\omega})$, which is illustrated in Figure P5.27. Sketch the Fourier transform of

$$w[n] = x[n]p[n]$$

for each of the following signals $p[n]$:

(i) $p[n] = \cos \pi n$

(ii) $p[n] = \cos(\pi n/2)$

(iii) $p[n] = \sin(\pi n/2)$

(iv) $p[n] = \sum_{k=-\infty}^{\infty} \delta[n-2k]$

(v) $p[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$

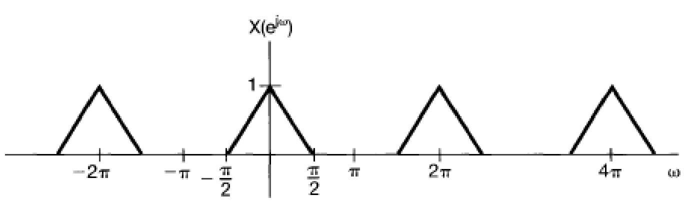

Fig P5.27

(b) Suppose that the signal $w[n]$ of part (a) is applied as the input to an LTI system with unit sample response

$$h[n] = \frac{\sin(\pi n/2)}{\pi n}.$$

Determine the output $y[n]$ for each of the choices of $p[n]$ in part (a).

5.30. In Chapter 4, we indicated that the continuous-time LTI system with impulse response

$$h(t) = \frac{W}{\pi}\operatorname{sinc}\left(\frac{Wt}{\pi}\right) = \frac{\sin Wt}{\pi t}$$

plays a very important role in LTI system analysis. The same is true of the discrete-time LTI system with impulse response

$$h[n] = \frac{W}{\pi}\operatorname{sinc}\left(\frac{Wn}{\pi}\right) = \frac{\sin Wn}{\pi n}.$$

(a) Determine and sketch the frequency response for the system with impulse response $h[n]$.

(b) Consider the signal

$$x[n] = \sin\left(\frac{\pi n}{8}\right) - 2\cos\left(\frac{\pi n}{4}\right).$$

Suppose that this signal is the input to LTI systems with the following impulse responses. Determine the output in each case.

(i) $h[n] = \frac{\sin(\pi n/6)}{\pi n}$

(ii) $h[n] = \frac{\sin(\pi n/6)}{\pi n} + \frac{\sin(\pi n/2)}{\pi n}$

(iii) $h[n] = \frac{\sin(\pi n/6)\sin(\pi n/3)}{\pi^2 n^2}$

(iv) $h[n] = \frac{\sin(\pi n/6)\sin(\pi n/3)}{\pi n}$

(c) Consider an LTI system with unit sample response

$$h[n] = \frac{\sin(\pi n/3)}{\pi n}.$$

Determine the output for each of the following inputs:

(i) $x[n] =$ the square wave depicted in Figure P5.30

(ii) $x[n] = \sum_{k=-\infty}^{\infty} \delta[n - 8k]$

(iii) $x[n] = (-1)^n$ times the square wave depicted in Figure P5.30

(iv) $x[n] = \delta[n + 1] + \delta[n - 1]$

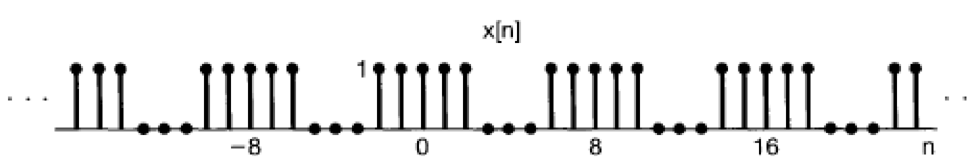

Fig P5.30

5.33. Consider a causal LTI system described by the difference equation

$$y[n] + \frac{1}{2}y[n-1] = x[n].$$

(a) Determine the frequency response $H(e^{j\omega})$ of this system.
(b) What is the response of the system to the following inputs?
(i) $x[n] = (\frac{1}{2})^n u[n]$
(ii) $x[n] = (-\frac{1}{2})^n u[n]$
(iii) $x[n] = \delta[n] + \frac{1}{2}\delta[n-1]$
(iv) $x[n] = \delta[n] - \frac{1}{2}\delta[n-1]$
(c) Find the response to the inputs with the following Fourier transforms:
(i) $X(e^{j\omega}) = \frac{1-\frac{1}{4}e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}}$
(ii) $X(e^{j\omega}) = \frac{1+\frac{1}{2}e^{-j\omega}}{1-\frac{1}{4}e^{-j\omega}}$
(iii) $X(e^{j\omega}) = \frac{1}{(1-\frac{1}{4}e^{-j\omega})(1+\frac{1}{2}e^{-j\omega})}$
(iv) $X(e^{j\omega}) = 1 + 2e^{-3j\omega}$