七. (15分) 设 $\xi \sim N(2,4)$, $\eta \sim N(5,16)$, 且 ξ 与 η 相互独立。求 $\xi+\eta$ 的期望、方差和特征函数。

解: $\xi+\eta \sim N(7, 20)$

$E(\xi+\eta) = 7$.

$D(\xi+\eta) = 20$

$f_{\xi+\eta}(t) = e^{iat - \frac{1}{2}\sigma^2 t^2} = e^{7it - 10 t^2}$

$E(\xi+\eta) = E\xi + E\eta$

$D(\xi+\eta) = D\xi + D\eta$.

$f(t) = Ee^{itx}$ def

$f(t) = Ee^{it(\xi+\eta)} = Ee^{it\xi} \cdot e^{it\eta}$.

$= \phi_\eta(t) \cdot \phi_\xi(t)$

八. (10分) 证明"马尔可夫不等式": 设 η 是一随机变量, r 是一正整数, 则

$$P\{|\eta| \geq \varepsilon\} \leq \frac{E|\eta|^r}{\varepsilon^r} \qquad \forall \varepsilon > 0.$$

证明: $P\{|\eta| \geq \varepsilon\} = \int_{|x| \geq \varepsilon} dF_\eta(x)$

$= \int_{|x| \geq \varepsilon} \frac{|x|^r}{\varepsilon^r} dF_\eta(x)$

$\leq \frac{1}{\varepsilon^r} \int_{-\infty}^{+\infty} |x|^r dF_\eta(x)$

$= \frac{E|\eta|^r}{\varepsilon^r}$. $\square$