$E\xi = \int_1^2 x \cdot 2(x-1)\, dx$

$= 2x^2 - 2x\, dx \quad \frac{2}{3}x^3 - x^2 \Big|_1^2 = \frac{2}{3}(8-1) - 3$

$\cancel{\frac{16}{3} - \frac{4}{3}} \quad \frac{14}{3} - \frac{9}{3} = \frac{5}{3} \quad \frac{16}{3} - \frac{2}{3}$

三. (15分) 设随机变量 ξ 具有密度函数

4.1 和 4.2 ✓

$$p(x) = \begin{cases} 2(x-1) & 1 < x < 2; \\ 0 & 其它. \end{cases}$$

求 ξ 的期望与方差.

$E\xi = \int_{-\infty}^{+\infty} x p(x) dx = \int_1^2 x \cdot 2(x-1)\, dx = \int_1^2 (2x^2 - 2x)\, dx = \frac{2}{3}x^3 - x^2 \Big|_1^2$

$= \frac{16}{3} - 4 - \frac{2}{3} + 1$

$= \frac{5}{3} \quad \frac{14}{3} - \frac{9}{3}$

$D\xi = E(\xi - E\xi)$

$= E\xi^2 - (E\xi)^2$

$D\xi = E\xi^2 - (E\xi)^2 = \int_{-\infty}^{+\infty} x^2 p(x) dx - \frac{25}{9}$

$= \int_1^2 (2x^3 - 2x^2)\, dx - \frac{25}{9}$

$E\xi^2 = \int_0^{\infty}$

$= \left(\frac{1}{2}x^4 - \frac{2}{3}x^3\right)\Big|_1^2 - \frac{25}{9} = \frac{1}{18}$

四. (15 分) 设 ξ 的分布函数为 $F(x)$, 且其数学期望存在. 求证

$$E\xi = \int_0^{+\infty} (1 - F(x))dx - \int_{-\infty}^0 F(x)dx$$

$\int_1^2 2x^2(x-1)$

$2x^3 - 2x^2$

$\frac{2x^4}{4} - \frac{2x^3}{3} \Big|_1^2$

$\frac{15}{2} - \frac{14}{3}$

$= \frac{45 - 28}{6} = \frac{17}{6}$

$2 \times 3 \times 3$

$- \frac{25}{9} \quad 9 \times 3$

$= \frac{51 - 50}{18} = \frac{1}{8} \quad \int_{\infty}$

证明: $E\xi = \int_{-\infty}^{+\infty} \cancel{f(x)dx}\; x\, dF(x)$

$= \int_{-\infty}^0 x\, dF(x) + \int_0^{+\infty} x\, dF(x)$

$= \int_{-\infty}^0 x\, dF(x) - \int_0^{+\infty} x\, d(1 - F(x))$

$= xF(x)\Big|_{-\infty}^0 - \int_{-\infty}^0 F(x)dx - x(1 - F(x))\Big|_0^{+\infty}$

$+ \int_0^{+\infty} (1 - F(x))\, dx$

$F(-\infty) = 0$
$F(+\infty) = 1$

$x = -t$

又 $F(-\infty) = 0$. $F(+\infty) = 1$

所以. $E\xi = \int_0^{+\infty} (1 - F(x))dx - \int_{-\infty}^0 F(x)dx$

$= \int_{-\infty}^{\infty} x\, dF(x) = \int_0^{\infty} + \int_{-\infty}^0 x\, dF(x)$

$= \int_{\infty}^0 (-t)\, dF(-t)$

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$E\xi = \int_0^{\infty} (1 - F(x)) \cdot dx - \int_{\infty}^0 F(x)\, dx$

$F(-t) = 1 - F(t)$

$- \int_{\infty}^0 t\, d(1 - F(t))$

$= x F(x) \Big|_0^{\infty} \quad \int_0^{\infty} F(x)\, dx + \int_0^{\infty} F(x)\, dx$

$- t(1 - F(t))\Big|_{\infty}^0 - \int_{\infty}^0 \cancel{dF(t)} = \int_0^{\infty} dx$