七. (10分) 设ξ_1, ξ_2, ξ_3是相对独立的且服从$N(0,1)$的随机变量，且令$\eta = \max\{\xi_1, \xi_2, \xi_3\}$。求$\eta$的分布的密度函数$p_\eta(x)$。

解: $x \sim \xi_1(x)$ $\quad p_1(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ $\quad p(x,y,z) = p_1(x)p_2(y)p_3(z)$

$y \sim \xi_2(y)$ $\quad p_2(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}$ $\quad F_1(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt.$

$z \sim \xi_3(z)$ $\quad p_3(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$

$u \sim \eta(u).$

~~$P\{\eta=u\} = 3P\{\xi_1 \le u, \xi_2 < u, \xi_3 = u\}$~~

~~$= 3F^2(u) \cdot p_3(u)$~~

~~$= \int_{-\infty}^{u}$~~

$P\{\eta = u\} = P\{\xi_1 \le u, \xi_2 \le u, \xi_3 \le u\}$

$= P\{\xi_1 \le u\} P\{\xi_2 \le u\} P\{\xi_3 \le u\}$

$= F_1^3(u)$

$\Rightarrow p_\eta(u) = (F_1^3(u))'$

$= 3F_1^2(u) F_1'(u)$

$p_\eta(u) = 3F^2(x) F'(x)$

$= 3\left(\int_{-\infty}^{x} \varphi(x) dx\right)^2 \varphi(x)$

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八. (10分) 设二维随机向量(X,Y)有联合密度函数

$$p(x,y) = \begin{cases} \frac{e^{-x/y} e^{-y}}{y} & \text{if } 0 < x < \infty, 0 < y < \infty, \\ 0 & \text{if otherwise.} \end{cases}$$

计算条件概率$P(X > 1 | Y = y)$。

解: $P(X>1 | Y=y)$

$= 1 - P(X \le 1 | Y=y)$

$P(X \le 1 | Y=y)$

$= \int_0^1 \frac{p(x,y)}{p_2(y)} dx$

$= \int_0^1 \frac{e^{-x/y}}{y} dx$

$= -e^{-\frac{x}{y}} \Big|_0^1$

$= 1 - e^{-\frac{1}{y}}$

$p_1(x) = \int_{-\infty}^{+\infty} p(x,y) dy$

$= \int_0^{+\infty} \frac{e^{-x/y} \cdot e^{-y}}{y} dy$

$p_2(y) = \int_{-\infty}^{+\infty} \frac{e^{-\frac{x}{y}} \cdot e^{-y}}{y} dx$

$= \left(-e^{-y} \cdot e^{-\frac{x}{y}}\right)\Big|_0^{+\infty}$

$= e^{-y}$

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