Wind accretion in a circular binary system

William J. Henney[1]★
[1] *Instituto de Radioastronomía y Astrofísica, Universidad Nacional Autónoma de México, Apartado Postal 3-72, 58090 Morelia, Michoacán, Mexico*

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT
Commentary on Tejeda & Toalá (2025).

Key words: Binary stars – stellar winds – stellar accretion

1 INTRODUCTION

2 BINARY SYSTEM AND WIND PARAMETERS

Consider a binary system in which the secondary star with mass M_2 accretes from the wind of the primary star with mass M_1. The orbit is assumed to be circular with separation r. The orbital speed of the secondary in the rest frame of the primary is v_o. Kepler's laws gives the orbital period T as

$$T = 2\pi G\left(M_1 + M_2\right)/v_o^3 = 2\pi r/v_0. \tag{1}$$

The isotropic stellar wind from the primary has mass-loss rate $\dot{M}_w$ and hypersonic terminal velocity v_w. The undisturbed wind density ρ_w at the position of the secondary is therefore

$$\rho_w = \dot{M}_w/4\pi r^2 v_w. \tag{2}$$

The orbital velocity is purely tangential and thus perpendicular to the purely radial wind velocity. The relative speed between the wind and the secondary star is therefore

$$v_r = \left(v_w^2 + v_o^2\right)^{1/2}. \tag{3}$$

The system is characterized by two dimensionless parameters:

$$\text{Mass ratio:} \quad q \equiv M_2/\left(M_1 + M_2\right), \tag{4}$$

$$\text{Velocity ratio:} \quad w \equiv v_w/v_o. \tag{5}$$

3 BONDI–HOYLE–LITTLETON (BHL) ACCRETION

Following Hoyle & Lyttleton (1939); Bondi & Hoyle (1944), the mass accretion rate is

$$\dot{M}_{acc} = \pi r_B^2\, v_r\, \rho_w, \tag{6}$$

where

$$r_B = 2GM_2/v_r^2 \tag{7}$$

is the accretion radius. From equations (1, 3, 4, 5) we find that the ratio of the accretion radius to the orbital radius is

$$r_B/r = 2q/\left(1 + w^2\right). \tag{8}$$

We define a dimensionless accretion efficiency as the fraction of the stellar wind that is captured by the secondary:

$$\eta_B \equiv \dot{M}_{acc}/\dot{M}_w, \tag{9}$$

which from equations (2, 6) yields

$$\eta_B = \frac{1}{4}\left(\frac{v_r}{v_w}\right)\left(\frac{r_B}{r}\right)^2. \tag{10}$$

The boost of the relative wind speed due to the orbital motion is

$$v_r/v_w = w^{-1}\left(1 + w^2\right)^{1/2}, \tag{11}$$

which combine with equations (10, 8) implies

$$\eta_B = \frac{q^2}{w\left(1 + w^2\right)^{3/2}}. \tag{12}$$

The BHL analysis assumes that the wind density and velocity vector are constant over the entire accretion capture zone of radius r_B. For accretion from a wind, this is only true in the limit that $r_B \ll r$.

4 A PROBLEMATIC GEOMETRIC CORRECTION

Tejeda & Toalá (2025) point out an issue with equation (12) when the wind velocity is much smaller than the orbital velocity ($w \ll 1$): in the limit $w \to 0$ then $\eta_B \to q^2/w$, which can become larger than unity. This is clearly non-physical since the mass accretion rate cannot exceed the wind mass-loss rate. They propose to remedy this deficiency by making a geometric correction to the mass accretion rate. $\dot{M}_{acc}$ is multiplied by a factor of $\cos\theta$, where θ is the angle between the relative velocity vector and the radial direction from the primary, which accounts for

> *. . . the projected area of the accretion cylinder's cross section onto a sphere centered around the primary. This projection accounts for the effective area capturing the wind.*

This yields a different equation for the accretion efficiency:

$$\eta_T = \eta_B = \left(\frac{q}{1 + w^2}\right)^2. \tag{13}$$

This clearly resolves the issue mentioned above since as $w \to 0$ then $\eta_T \to q^2$, guaranteeing that $\eta_T < 1$. On the other hand, in the

★ w.henney@irya.unam.mx

opposite limit of large wind velocity the two efficiencies agree: as $w \to \infty$ then $\eta_T \to \eta_B \to q^2/w^4$.

However, the physical basis for making this "correction" is unclear. Unlike the BHL theory, which is entirely local to the rest frame of the accreting secondary, the correction factor introduces quantities from the rest frame of the primary, which casts doubt on its validity.

For circular orbits, one finds

$$\cos\theta = v_W/v_r, \tag{14}$$

so an alternative way of writing the Tejeda efficiency is

$$\eta_T = \frac{1}{4}\left(\frac{r_B}{r}\right)^2 = \frac{\pi r_B^2}{4\pi r^2}, \tag{15}$$

which is simply the area covering factor of the BHL capture zone of a *stationary* accretor. However, this is inconsistent with the well-defined physical limit for a fast-orbiting accretor, as we will show in the following section.

5 ASYMPTOTIC EFFICIENCY OF A FAST-ORBITING ACCRETOR IN A SLOW WIND

During one orbital period, T, the accretion capture zone will sweep out a torus that fully encircles the primary star. If T is sufficiently short compared with the time $2r_B \sin\theta/v_W$ for the wind to cross the accretion capture zone, then all of the wind that passes within a distance r_B of the orbital path will be captured. This corresponds to the limit $w \ll 1$, $\theta \approx \pi/2$, for which the solid angle subtended at the primary by the torus is

$$\Omega = 2\pi \int_{-r_B/r}^{r_B/r} d\mu = 4\pi\, r_B/r \tag{16}$$

Therefore, the limiting accretion efficiency is

$$\eta_{\rm lim} = \Omega/4\pi = \lim_{w\to 0} \frac{r_B}{r} = 2q. \tag{17}$$

Note that this is inconsistent with the Tejeda result, $\eta_T \approx q^2$, in the same limit, casting further doubt on the correctness of that result.

It is also inconsistent with the naive BHL result, $\eta_B \to \infty$, so we clearly require *some* correction to BHL. In the following section we outline a physically motivated correction that is consistent with $\eta_{\rm lim}$.

6 STARVATION BY FINITE REFILL TIME

Tejeda & Toalá (2025) discuss the refill time, which is the time needed for the wind to replenish the material inside the accretion torus (see previous section) but they do not explicitly calculate its influence on the accretion efficiency. However, we will show that accounting for this effect is entirely sufficient to prevent the divergence of η_B for small w.

During a single orbital period, the wind will propagate a distance

$$x = v_W T = 2\pi w r, \tag{18}$$

where the second equality makes use of equations (1, 5). The radial thickness in the orbital plane of the accretion torus can be found by applying the cosine law and Taylor expansion in powers of r_B/r to yield

$$h = 2r_B \sin\theta \left[1 - \frac{1}{2}(r_B/r)^2 \cos^2\theta + O(r_B/r)^4\right]. \tag{19}$$

For small $(r_B/r)\cos\theta$ the term in square brackets is approximately unity, so from equations (11, 14) we have

$$h \approx 2r_B \sin\theta = 2r_B\left(1+w^2\right)^{-1/2}. \tag{20}$$

Therefore the fraction of the thickness refilled by the wind is

$$f = x/h = \frac{\pi w\left(1+w^2\right)^{3/2}}{2q}. \tag{21}$$

If $f < 1$, then a portion of the accretion surface πr_B^2 is empty of wind, resulting in a reduction in the effective area by a factor $\mathcal{F}$, which can be found for small (r_B/r) from the standard formula for the area of a circular segment, yielding

$$\mathcal{F} = 1 - \frac{2}{\pi}\left[\cos^{-1}(f) - f\left(1-f^2\right)^{1/2}\right] \approx \frac{4f}{\pi}, \tag{22}$$

where the final approximate equality is accurate for $f < 0.5$.

In this refill-limited case, the accretion efficiency is therefore given from equations (12, 21, 22) as

$$\eta_\star = \mathcal{F}\eta_B \approx 2q, \tag{23}$$

which applies when $w \lesssim 0.64q$. Note that this is exactly the same efficiency as the limiting value $\eta_{\rm lim}$ from section 5. We have therefore shown how the asymptotic behavior of the accretion efficiency when the wind speed is small compared with the orbital speed can arise naturally from a reduction in the accretion area due to the "hole" in the wind that was cleared out during the previous orbit. By using the exact expression for $\mathcal{F}$ in equation (22), we achieve a smooth transition from $\eta_\star$ to η_B for $w > q$.

7 GRAVITATIONAL INFLUENCE OF THE PRIMARY

References

Bondi H., Hoyle F., 1944, MNRAS, 104, 273
Hoyle F., Lyttleton R. A., 1939, Proceedings of the Cambridge Philosophical Society, 35, 405
Tejeda E., Toalá J. A., 2025, ApJ, 980, 226

This paper has been typeset from a TEX/LATEX file prepared by the author.