

# Wind accretion in a circular binary system

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## ABSTRACT

Commentary on Tejeda & Toalá (2025).

**Key words:** Binary stars – stellar winds – stellar accretion

## 1 INTRODUCTION

## 2 BINARY SYSTEM AND WIND PARAMETERS

Consider a binary system in which the secondary star with mass  $M_2$  accretes from the wind of the primary star with mass  $M_1$ . The orbit is assumed to be circular with separation  $r$ . The orbital speed of the secondary in the rest frame of the primary is  $v_0$ . Kepler's laws gives the orbital period  $T$  as

$$T = 2\pi G(M_1 + M_2)/v_0^3 = 2\pi r/v_0. \quad (1)$$

The isotropic stellar wind from the primary has mass-loss rate  $\dot{M}_w$  and hypersonic terminal velocity  $v_w$ . The undisturbed wind density  $\rho_w$  at the position of the secondary is therefore

$$\rho_w = \dot{M}_w/4\pi r^2 v_w. \quad (2)$$

The orbital velocity is purely tangential and thus perpendicular to the purely radial wind velocity. The relative speed between the wind and the secondary star is therefore

$$v_r = \left(v_w^2 + v_0^2\right)^{1/2}. \quad (3)$$

The system is characterized by two dimensionless parameters:

$$\text{Mass ratio: } q \equiv M_2/(M_1 + M_2), \quad (4)$$

$$\text{Velocity ratio: } w \equiv v_w/v_0. \quad (5)$$

## 3 BONDII-HOYLE-LYTTLETON (BHL) ACCRETION

Following Hoyle & Lyttleton (1939); Bondi & Hoyle (1944), the mass accretion rate is

$$\dot{M}_{\text{acc}} = \pi r_B^2 v_r \rho_w, \quad (6)$$

where

$$r_B = 2GM_2/v_r^2 \quad (7)$$

is the accretion radius. From equations (1, 3, 4, 5) we find that the ratio of the accretion radius to the orbital radius is

$$r_B/r = 2q/(1 + w^2). \quad (8)$$

We define a dimensionless accretion efficiency as the fraction of the stellar wind that is captured by the secondary:

$$\eta_B \equiv \dot{M}_{\text{acc}}/\dot{M}_w, \quad (9)$$

which from equations (2, 6) yields

$$\eta_B = \frac{1}{4} \left( \frac{v_r}{v_w} \right) \left( \frac{r_B}{r} \right)^2. \quad (10)$$

The boost of the relative wind speed due to the orbital motion is

$$v_r/v_w = w^{-1} (1 + w^2)^{1/2}, \quad (11)$$

which combine with equations (10, 8) implies

$$\eta_B = \frac{q^2}{w(1 + w^2)^{3/2}}. \quad (12)$$

The BHL analysis assumes that the wind density and velocity vector are constant over the entire accretion capture zone of radius  $r_B$ . For accretion from a wind, this is only true in the limit that  $r_B \ll r$ .

## 4 A PROBLEMATIC GEOMETRIC CORRECTION

Tejeda & Toalá (2025) point out an issue with equation (12) when the wind velocity is much smaller than the orbital velocity ( $w \ll 1$ ): in the limit  $w \rightarrow 0$  then  $\eta_B \rightarrow q^2/w$ , which can become larger than unity. This is clearly non-physical since the mass accretion rate cannot exceed the wind mass-loss rate. They propose to remedy this deficiency by making a geometric correction to the mass accretion rate.  $\dot{M}_{\text{acc}}$  is multiplied by a factor of  $\cos \theta$ , where  $\theta$  is the angle between the relative velocity vector and the radial direction from the primary, which accounts for

... the projected area of the accretion cylinder's cross section onto a sphere centered around the primary. This projection accounts for the effective area capturing the wind.

This yields a different equation for the accretion efficiency:

$$\eta_T = \eta_B = \left( \frac{q}{1 + w^2} \right)^2. \quad (13)$$

This clearly resolves the issue mentioned above since as  $w \rightarrow 0$  then  $\eta_T \rightarrow q^2$ , guaranteeing that  $\eta_T < 1$ . On the other hand, in the

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opposite limit of large wind velocity the two efficiencies agree: as  $w \rightarrow \infty$  then  $\eta_T \rightarrow \eta_B \rightarrow q^2/w^4$ .

However, the physical basis for making this “correction” is unclear. Unlike the BHL theory, which is entirely local to the rest frame of the accreting secondary, the correction factor introduces quantities from the rest frame of the primary, which casts doubt on its validity.

For circular orbits, one finds

$$\cos \theta = v_w/v_T, \quad (14)$$

so an alternative way of writing the Tejada efficiency is

$$\eta_T = \frac{1}{4} \left( \frac{r_B}{r} \right)^2 = \frac{\pi r_B^2}{4\pi r^2}, \quad (15)$$

which is simply the area covering factor of the BHL capture zone of a *stationary* accretor. However, this is inconsistent with the well-defined physical limit for a fast-orbiting accretor, as we will show in the following section.

## 5 ASYMPTOTIC EFFICIENCY OF A FAST-ORBITING ACCRETOR IN A SLOW WIND

During one orbital period,  $T$ , the accretion capture zone will sweep out a torus that fully encircles the primary star. If  $T$  is sufficiently short compared with the time  $2r_B \sin \theta/v_w$  for the wind to cross the accretion capture zone, then all of the wind that passes within a distance  $r_B$  of the orbital path will be captured. This corresponds to the limit  $w \ll 1$ ,  $\theta \approx \pi/2$ , for which the solid angle subtended at the primary by the torus is

$$\Omega = 2\pi \int_{-r_B/r}^{r_B/r} d\mu = 4\pi r_B/r \quad (16)$$

Therefore, the limiting accretion efficiency is

$$\eta_{\text{lim}} = \Omega/4\pi = \lim_{w \rightarrow 0} \frac{r_B}{r} = 2q. \quad (17)$$

Note that this is inconsistent with the Tejada result,  $\eta_T \approx q^2$ , in the same limit, casting further doubt on the correctness of that result.

It is also inconsistent with the naive BHL result,  $\eta_B \rightarrow \infty$ , so we clearly require *some* correction to BHL. In the following section we outline a physically motivated correction that is consistent with  $\eta_{\text{lim}}$ .

## 6 STARVATION BY FINITE REFILL TIME

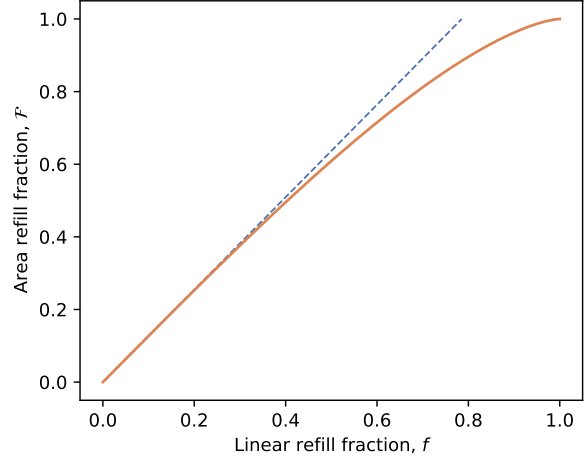
Tejada & Toalá (2025) discuss the refill time, which is the time needed for the wind to replenish the material inside the accretion torus (see previous section) but they do not explicitly calculate its influence on the accretion efficiency. However, we will show that accounting for this effect is entirely sufficient to prevent the divergence of  $\eta_B$  for small  $w$ .

During a single orbital period, the wind will propagate a distance

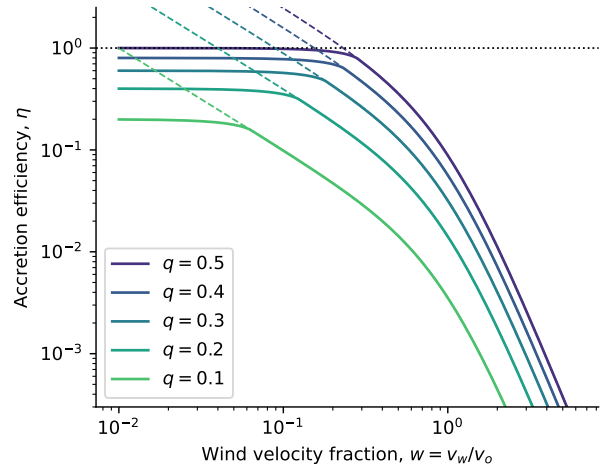
$$x = v_w T = 2\pi w r, \quad (18)$$

where the second equality makes use of equations (1, 5). The radial thickness in the orbital plane of the accretion torus can be found by applying the cosine law and Taylor expansion in powers of  $r_B/r$  to yield

$$h = 2r_B \sin \theta \left[ 1 - \frac{1}{2} \left( \frac{r_B}{r} \right)^2 \cos^2 \theta + \mathcal{O}(r_B/r)^4 \right]. \quad (19)$$



**Figure 1.** Refill fraction of the accretion surface area  $\mathcal{F}$  from equation (22). The dashed line shows the linear approximation  $\mathcal{F} = 4f/\pi$ .



**Figure 2.** Accretion efficiency  $\eta_*$  accounting for finite refill time (solid lines) from equations (21, 22, 23). Dashed lines show the naive BHL efficiency  $\eta_B$  from equation (12).

For small  $(r_B/r) \cos \theta$  the term in square brackets is approximately unity, so from equations (11, 14) we have

$$h \approx 2r_B \sin \theta = 2r_B (1 + w^2)^{-1/2}. \quad (20)$$

Therefore the fraction of the thickness refilled by the wind is

$$f = x/h = \frac{\pi w (1 + w^2)^{3/2}}{2q}. \quad (21)$$

If  $f < 1$ , then a portion of the accretion surface  $\pi r_B^2$  is empty of wind, resulting in a reduction in the effective area by a factor  $\mathcal{F}$ , which can be found for small  $(r_B/r)$  from the standard formula for the area of a circular segment, yielding

$$\mathcal{F} = 1 - \frac{2}{\pi} \left[ \cos^{-1}(f) - f (1 - f^2)^{1/2} \right] \approx \frac{4f}{\pi}, \quad (22)$$

where the final approximate equality is accurate for  $f \lesssim 0.4$ . This function is shown in Figure 1.

In this refill-limited case, the accretion efficiency is therefore given from equations (12, 21, 22) as

$$\eta_{\star} = \mathcal{F} \eta_{\text{B}} \approx 2q, \quad (23)$$

which applies when  $w \lesssim 0.64q$ . Note that this is exactly the same efficiency as the limiting value  $\eta_{\text{lim}}$  from section 5. We have therefore shown how the asymptotic behavior of the accretion efficiency when the wind speed is small compared with the orbital speed can arise naturally from a reduction in the accretion area due to the “hole” in the wind that was cleared out during the previous orbit. By using the exact expression for  $\mathcal{F}$  in equation (22), we achieve a smooth transition from  $\eta_{\star}$  to  $\eta_{\text{B}}$  for  $w > q$ . The resultant efficiency is plotted against  $w$  for different values of  $q$  in Figure 2.

## 7 GRAVITATIONAL INFLUENCE OF THE PRIMARY

The derivation of the Bondi–Hoyle–Lyttleton accretion rate (section 3) treats the gravity of the accreting star in isolation. However, in a binary system this is only valid within the “gravitational sphere of influence” of the secondary (Souami et al. 2020 and references therein). We will quantify this by considering the radius of the “Hill sphere”, which corresponds to the Roche lobe of the secondary, extending between the Lagrange points  $L_1$  and  $L_2$ .

at sufficiently small radii  $r \ll r_{\text{H}}$  from the secondary, where  $r_{\text{H}}$  is the Hill radius

z

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