# PDF of Photoevaporation flow Density. 2025-11-12

H1

Use analytic $R(U)$ from Dyson 1968

All quantities normalized to $R=1$

$$n = \frac{1}{R^{\alpha} U} \quad \text{where} \quad \alpha = \begin{cases} 2 & \text{spherical} \\ 1 & \text{cylindrical} \end{cases}$$

Assume emissivity $e = n^2$

We want to calculate emission-weighted histogram (PDF) of $n$

Volume element $dV = R^{\alpha} dR$

We want to find the histogram $H(n)$ such that

$$\int H(n)\, dn = \int e\, dV \quad \text{for}$$

each bin. $= \int de$

where $de = e\, dV = U^{-2} R^{-\alpha}\, dR$

112

We can consider two cases for the velocity law:

A. Constant velocity $U = 1$

B. Dyson law: $R(U) = U^{-1/\alpha} e^{(U^2-1)/2\alpha}$

Do A first as a sanity check

With constant velocity

$$e\,dV = R^{-\alpha}\,dR$$

$$\text{Total emission} = \int^{\infty} R^{-\alpha}\,dR$$

$$= \frac{-1}{1-\alpha} = \frac{1}{\alpha-1} = \begin{cases} 1 & (\alpha=2) \\ \ln R_{max} & (\alpha=1) \end{cases}$$

Also $n = R^{-\alpha}$ so $\frac{dn}{dR} = -\alpha R^{-(\alpha+1)}$

$$\Rightarrow R = n^{-1/\alpha} \qquad = -\alpha n/R$$

$$= -\alpha n^{(1+1/\alpha)}$$

$$H(n)\,dn = e\,dV = R^{-\alpha}\,dR$$

$$\Rightarrow H(n) = R^{-\alpha}\left|\frac{dR}{dn}\right| \qquad -\alpha n^{\frac{1+\alpha}{\alpha}}$$

$$\Rightarrow H(n) = n^{\frac{1+\alpha}{\alpha}}$$

$$H(n) = \alpha n^{(1+\alpha)/\alpha} \qquad \begin{cases} \alpha = 1 : n^3 \\ \alpha = 2 : 2n^2 \end{cases}$$

$$dR/dn = \frac{-1}{\alpha} n^{-(1+\alpha)/\alpha} \qquad \int x^n\,dx = \frac{1}{n+1} x^{n+1}$$

H3

$$H(n) = \frac{n}{dn/dR} = \frac{n}{\alpha\, n^{(1+\alpha)/\alpha}}$$

$$1 - \frac{1+\alpha}{\alpha} = \frac{-1}{\alpha} \Rightarrow H(n) = \alpha^{-1} n^{-\alpha^{-1}}$$

$\alpha = 1$ : $H(n) = n^{-1}$

$\alpha = 2$ : $H(n) = \frac{1}{2} n^{-1/2}$

Double check the total integral.

We should have

$$\int_0^1 H(n)\, dn = \int_{R=1}^{R=\infty} e\, dV = \frac{1}{\alpha+1}$$

$$\alpha^{-1} \int_0^1 n^{-\alpha^{-1}}\, dn = \left[ \frac{\alpha^{-1}}{1-\alpha^{-1}} n^{1-\alpha^{-1}} \right]_0^1$$

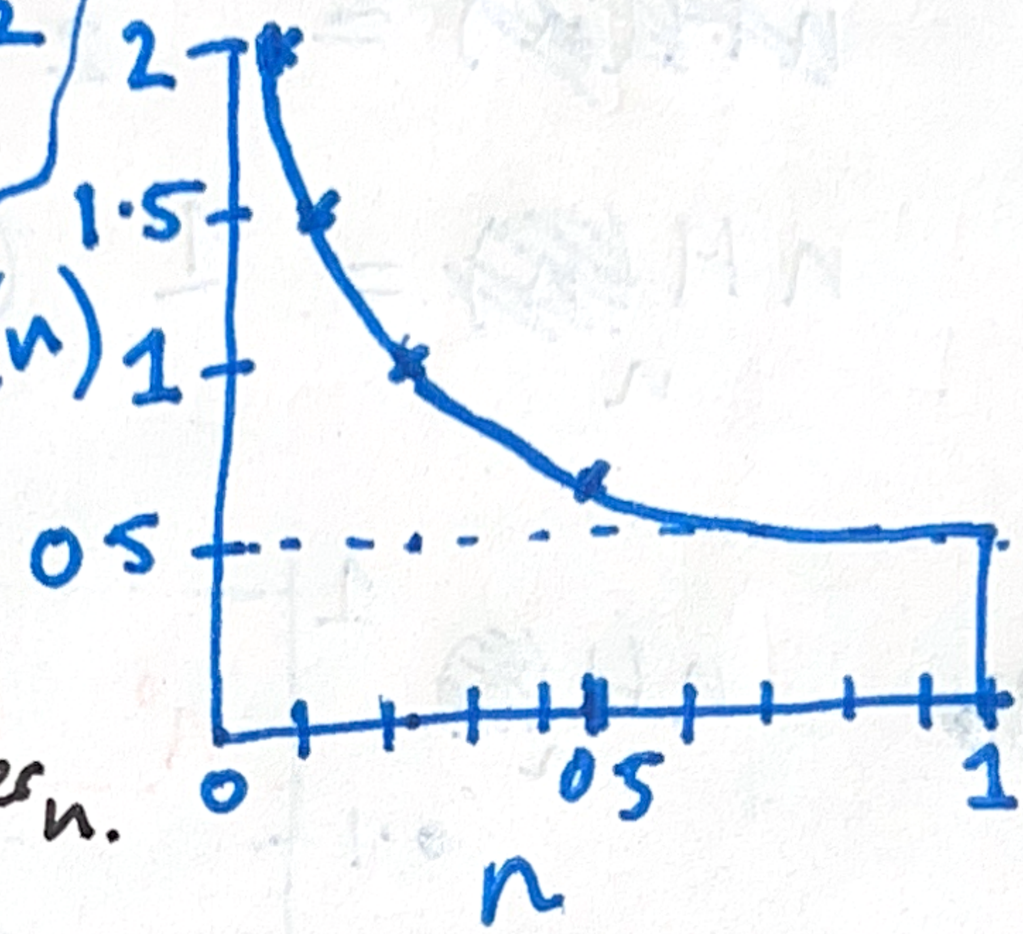
$$= \frac{1}{\alpha - 1}$$

So this works OK for spherical case ($\alpha = 2$).

But for ~~logarithmic~~ cylindrical case ($\alpha = 1$) we have a logarithmic divergence of the total emission: $\int e\, dV \propto \log(R_{out}/R_{in})$

So for spherical case

$$\boxed{H(n) = \frac{1}{2} n^{-1/2}}$$




we could instead write $H_n$ instead of $H(n)$ to emphasize it is per $n$.

This result is somewhat counter-intuitive (for me) since it gives a histogram that is weighted towards lower densities.

<u>However</u>, in the spherical case, the singularity at $n \to 0$ is integrable, so benign.

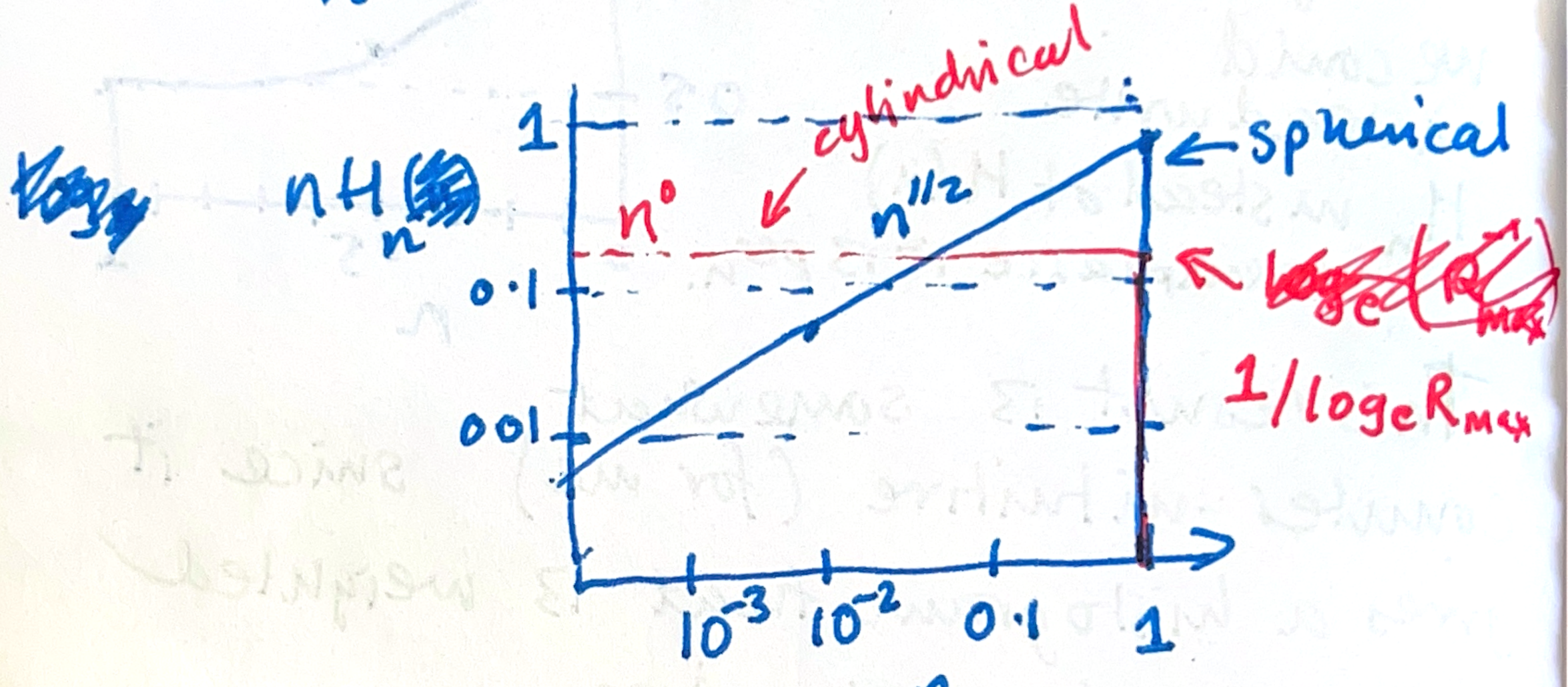
In fact, if we work in log space, then it will look better since $d\log_e n = \frac{dn}{n}$

$$H_n\, dn = n H_n\, d\log_e n$$

This is similar in spirit to using the SED: $\nu F_\nu = \lambda F_\lambda$

So $\frac{nH_n}{n} = \frac{1}{2} n^{1/2}$ (spherical).

$\frac{nH_n}{n} = 1$ (cylindrical).




We could call this the DED
Pycnic Energy Distribution
by analogy with the SED —
but that would not be a good idea.
Maybe "Density Energy Distribution"
or Densimetric? but we are not "measuring" DED here.

This shows that spherical flow is dominated by the peak density and thereabouts, while cylindrical flow has equal contributions from all densities down to minimum.

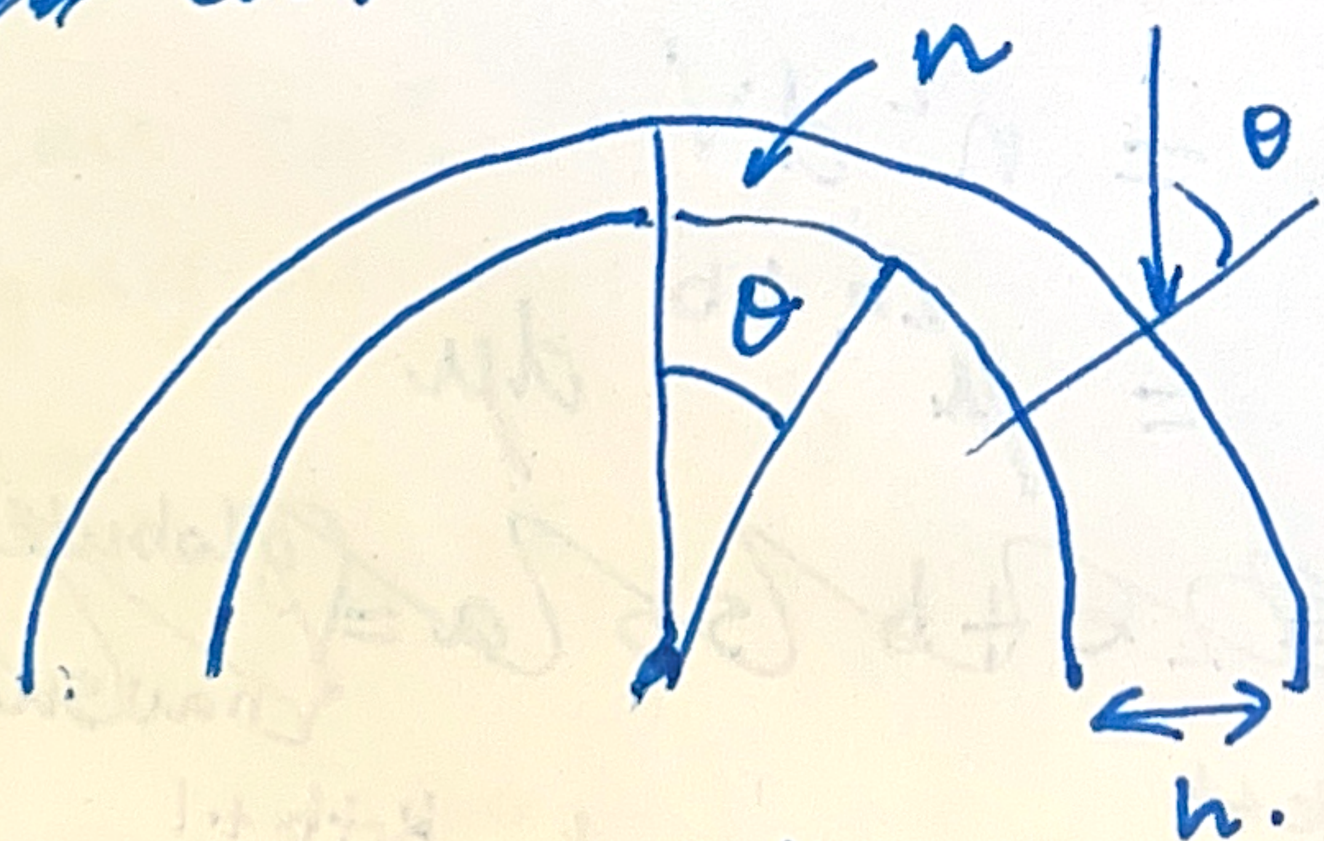
We still need to include

i) the angular variation

ii) the Dyson velocity law case

We will do (i) first.

This can be used not just for globule wind case, but also for bow shocks for instance, or for ~~gas~~ confined globules (strong external wind)




Take ~~a spherical~~ angle about the center of curvature and put $n(\theta) \sim (\cos\theta)^k$ for instance $k = \frac{1}{2}$ for globules

Also allow that the effective thickness varies with $\theta$.

But it can't be a negative power of $\cos\theta$ since that gives $h \to \infty$ as $\theta \to \frac{\pi}{2}$

but maybe that is OK

This is all for spherical case.

H6

For cylinder case: $dV = h\,d\theta$

Volume element in this case (if $\alpha = 2$) is $h\,d\mu = dV$

So if we put $n = \mu^k$ $\quad \frac{dn}{d\mu} = k\mu^{k-1}$

$$h = \mu^b$$

For globules: $k = 1/2$, $b = 0$

For bow shock: $k = 2$, $b = -1$ for instance

↑ flaring

Again, we set

$$H_n\,dn = n^2\,dV$$

$$= \mu^{2k+b}\,d\mu$$

~~Define $a \equiv 2k+b$ so $a = \begin{cases} \text{globule}: 1 \\ \text{bowshock}: 3 \end{cases}$~~

$$H_n = \frac{\mu^{2k+b}}{dn/d\mu} = k^{-1}\mu^{k+b+1}$$

$$2k+b-(k-1) = k+b+1$$

$$\mu = n^{1/k} \Rightarrow H_n = \frac{1}{k}\,n^{1+\frac{b+1}{k}}$$

So $nH_n = \frac{1}{k} n^{\left(2+\frac{b+1}{k}\right)}$

For spherical globule: $2+\frac{b+1}{k} = 4$

For bow shock: $2+\frac{b+1}{k} = 2$ (or $2.825$ with $b=-\frac{1}{2}$)

$b=-1$

so it does not make much difference

Normalization

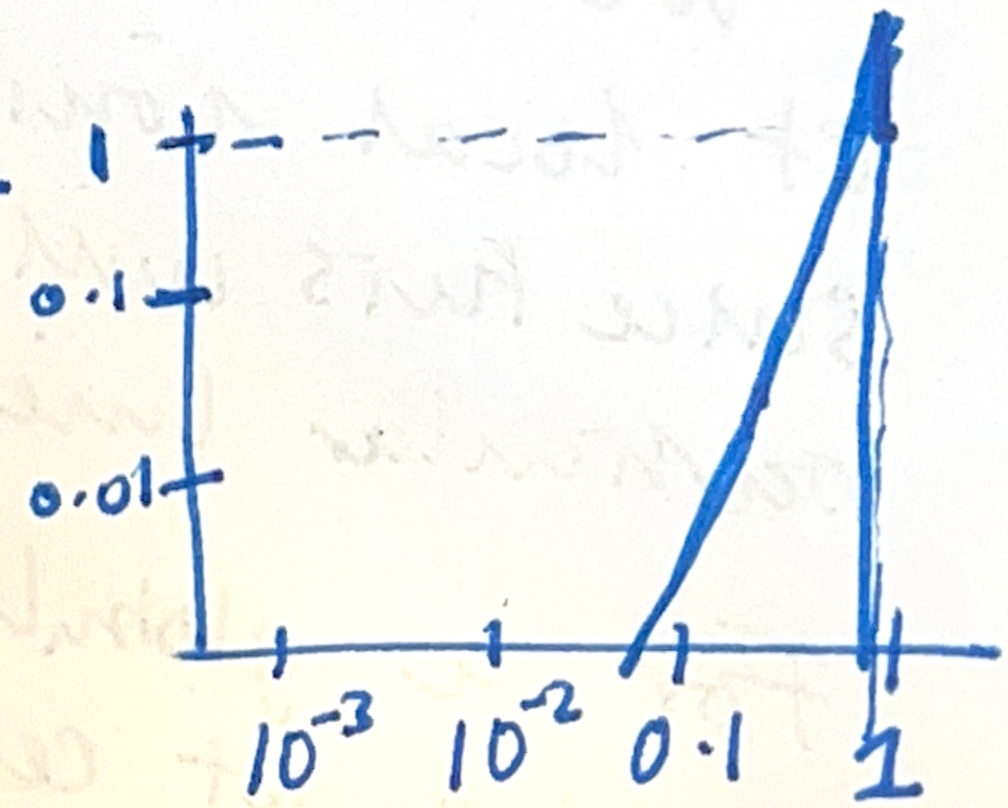
$$\int n^2 dV = \int_0^1 \mu^{2k+b} d\mu$$

$$= \frac{1}{1+2k+b}$$

We can call this $m_0$

Applying normalization we have

$$nH_n = m_0 n^{m_0}$$




$m_0 = 4$ (globule) or $\simeq 2$ (bow shock)

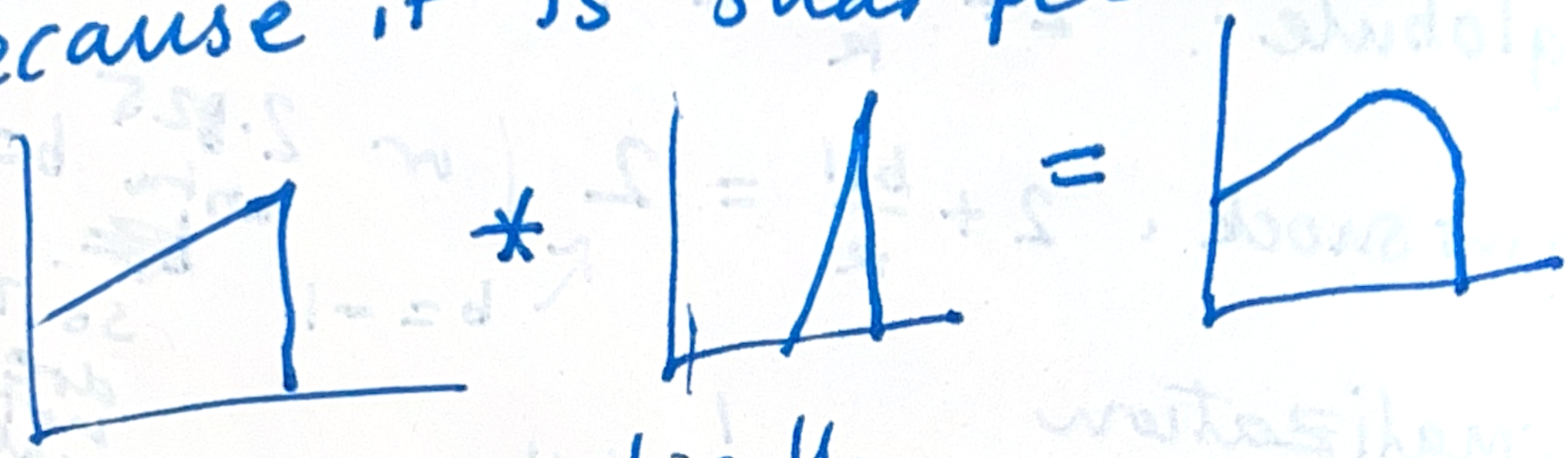
We could do better bow shock models with a parabola

Note that radial DED can be written in same form: $nH_n = m_r n^{m_r}$

with $m_r = \frac{1}{2}$

We now need to convolve these together.

Convolution will mainly effect the high-density end because it is sharpest.



Schematically.

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We can also do distribution of local ionization parameter, since this will be important for particular lines (e.g. [SII]).

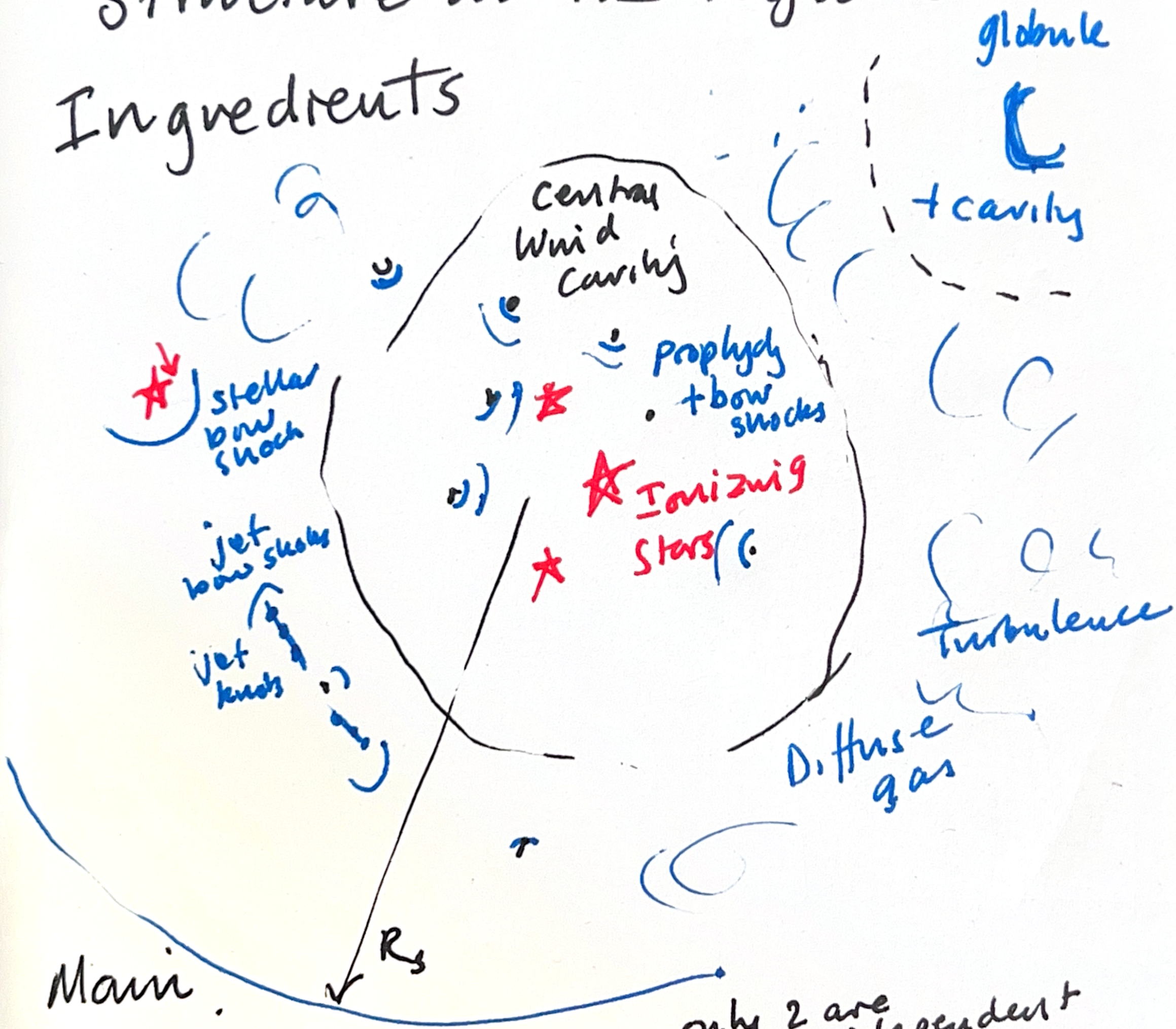
For a globule that is situated at certain fraction of $R_s$, we can find $\Upsilon/\Upsilon_0$ as function of $n$

where $\Upsilon_0$ is the global value $\frac{Q}{4\pi R_s^2 n_s}$

We can then find ion fractions as function of $\Upsilon$ for a given ionizing SED.

# Global models of density structure in HII regions

## Ingredients




Main parameters:

only 2 are independent

HII region: $Q_H$, $R_S$, $n_S$

$M_{tot}$ determines $\sigma_p$ of BG gas

Proplyds: spatial distribution of stars $\left(1+(r/r_c)^2\right)^{-1}$ or similar.

we could have proplyd fraction with central hole, or could have size distrib with upper bound that drops towards center.