

CS 3823 - Theory of Computation: Homework Assignment 1

FALL 2019

Due: Friday, September 6, 2019

Related Reading. Chapter 1.1 and Chapter 2

Instructions. Near the top of the first page of your solutions please list clearly **all** the members of the group (please see the syllabus for the collaboration policy) who have created the solutions that you are submitting. Listing the names of the people in the group implies their full name and their 4x4 IDs. Alternatively, you can use the space below and provide the relevant information in case you submit the solutions using this document.

Student Information for the Solutions Submitted

	Lastname, Firstname	4x4 ID (e.g., dioc0000)
1		
2		
3		
4		
5		
6		

Grade

Exercise	Pages	Your Score	Max
1	2		4
2	3		4
3	4		4
4	5-6		12
5	7-8		12
6	9		4
Total	2-9		40

Additional Help and Resources. Did you use help and/or resources other than the textbook? Please indicate below.

1 Set Theory [4 points]

Let A be the set $\{x, y, z\}$ and B be the set $\{x, y\}$.

- (i) Is A a subset of B and why?
- (ii) What is $A \cap B$?
- (iii) What is $A \times B$?
- (iv) What is the powerset of B ?

2 Induction [4 points]

Prove by induction on k that for all integers $k \geq 4$ it holds that $k! > 2^k$.

3 Strings and Sequences [4 points]

Imagine the following sequence over the alphabet $\Sigma = \{a, b\}$ where we say that a and b are complementing each other: begin with the symbol a and continually append the string that complements the sequence of symbols seen so far. Hence, the sequence begins as

abbabaabbaababba...

One can think of the elements, t_i , of the sequence individually, starting with t_0 . So, we have

$$t_0 = a, t_1 = b, t_2 = b, t_3 = a, t_4 = b, t_5 = a, \dots$$

What is the value of t_{129} ? How did you arrive at your answer?

4 Drawing State Diagrams [12 points; 4 points each]

Draw state diagrams for DFAs recognizing the following languages:

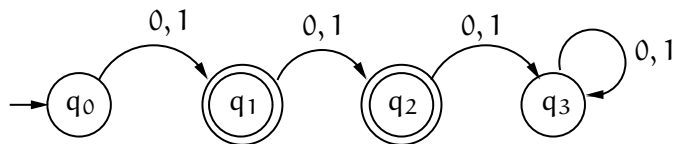
- (i) $L_1 = \{w \mid \text{length of } w \text{ is odd}\}, \Sigma = \{1\}$.
- (ii) $L_2 = \{w \mid w \text{ has at most two occurrences of the symbol } b\}, \Sigma = \{a, b\}$.
- (iii) $L_3 = \{w \mid w \text{ starts with an } 1 \text{ or ends with a } 0\}, \Sigma = \{0, 1\}$.

(blank space in case you need it for exercise 4)

5 Reading State Machines [12 points]

Let $\Sigma = \{0, 1\}$. For each of the following DFAs explain what language they recognize.

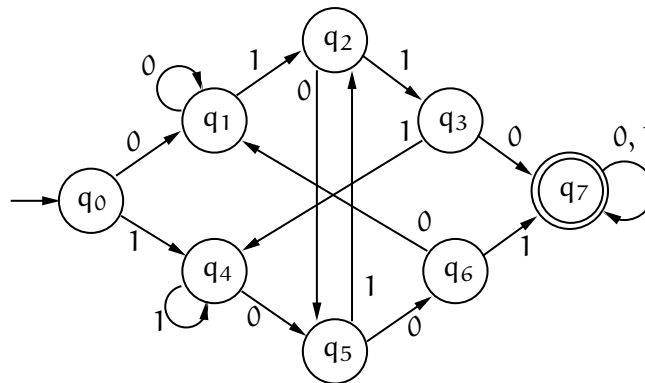
- (i) [6 points] Please see the DFA of machine M_1 below. For this machine M_1 , also give its formal



description as a 5-tuple. You do not need to do this for the machine M_2 that follows in part (ii).

(continuation of exercise 5)

(ii) [6 points] Please see the DFA of machine M_2 below.



6 Closure [4 points]

Let A and B be regular languages. Show that $A \setminus B$ is also regular.

(Recall that $A \setminus B = \{x \in A \mid x \notin B\}$. In other words, this operation removes from A all the strings that are also in B .)