Formula Sheet for USAAAO/IOAA
version 1.6

Formula sheet baby! Unfortunately this does not cover everything.

1 Optics

Telescopes. Assume diameter D and focal length f.

1. Aperture ratio $A = D/f$, but often given as "f/Number" (e.g. f/8), where the focal ratio $n = f/D$. Note that intensity I has the following relation:
$$I = \frac{P}{A} = \frac{F}{\tan^2(\theta)}\frac{D^2}{f^2} \propto \frac{1}{(f/D)^2}$$
2. Plate scale
$$\frac{d\theta}{ds} \approx \frac{1}{f}$$
3. Rayleigh criterion
$$\theta_{min} = \frac{1.22\lambda}{D}$$
4. Field of View:
$$\text{FOV} = 2\arctan\left(\frac{w}{2f}\right)$$
where w is the sensor width. If the FOV of the eyepiece is given, but it is magnified by the primary lens:
$$\text{FOV} = \frac{\text{FOV}_{\text{eyepiece}}}{m}$$
where $m = f_p/f_{eye}$ is the magnification.
5. Light Gathering Power (amount of energy gathered per unit time):
$$P = F \times \pi(D/2)^2$$
6. Main types: Newtonian, satisfying $f_e = f_p$, Cassegrain, satisfying $f_e = (b/a)f_p$, and Coudé, equivalent to Cassegrain in effective focal length.

Interferometer. Assume detectors a distance D apart, aimed at an object $\theta \ll 1$ from the vertical.

1. $\theta = \lambda/D$ for constructive interference.
2. $\theta = \lambda/2D$ for destructive interference.

Lenses and mirrors. Lens equations:
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$
$$m = \frac{-s'}{s}$$
Sign conventions, assuming light comes from the left side:

1. s, object distance is + when O is left of lens/mirror (real object), − when O is right of lens/mirror (virtual object)
2. s', image distance is + when I is real (right of lens or left of mirror), − when I is virtual (left of lens or right of mirror).
3. f, focal length is + when converging (convex lens, concave mirror), − when diverging (concave lens, convex mirror)
4. m, magnification is > 0 when image is upright, < 0 when image is inverted

Things to note:

1. s' sign depends on lens or mirror, not always + on the right
2. By looking at the sign convention and magnification equation, one can tell that **virtual** images are always **upright** and **real** images are always **inverted**.

Lensmaker's equation. n_{surr} is index of refraction of surrounding material (like air) and R_i is + when converging and − for diverging:

$$\frac{1}{f} = \frac{n - n_{surr}}{n_{surr}} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

Flux and Luminosity.

1. luminosity L (also known as total flux) is energy/time.
2. flux F (also known as flux density) is energy/(time area).

$$L = 4\pi r^2 F$$

3. brightness

$$B = \frac{F}{\omega}$$

where ω is the solid angle, defined by A/r^2. B is oftentimes expressed in mag/arcsec^2.

Magnitudes.

1. apparent magnitude:

$$m_1 - m_2 = -2.5 \log \left(\frac{F_1}{F_2} \right)$$

$$\frac{F_1}{F_2} = 10^{-0.4(m_1 - m_2)}$$

2. absolute magnitude (app mag, but at 10 pc):

$$M_1 - M_2 = -2.5 \log \left(\frac{L_1}{L_2} \right)$$

3. magnitude of composite system:

$$m_{\text{sys}} = -2.5 \log \left(\sum_i 10^{-0.4 m_i} \right)$$

4. integrated magnitude:

$$m_{\text{int}} = B - 2.5 \log A$$

where B is the surface brightness (ex. mag/arcsec^2) and A is the area (ex. arcsec^2)

5. distance modulus:
$$m - M = 5\log\left(\frac{r}{10\text{ pc}}\right) + A$$
where A is the extinction, defined in the next section.

6. bolometric correction:
$$BC = M_{bol} - M_V$$
In general, BC will be large and negative for either very hot or very cold stars, since in that case a majority of the radiation will not be in the visual band.

Extinction.

1. Intensity I as a function of optical depth τ:
$$dI = -\kappa\rho I dr$$
$$I = I_0 e^{-\tau}$$
$$\left(\tau = \int \kappa\rho dr\right)$$

2. Extinction in distance modulus:
$$m - M = 5\log\left(\frac{r}{10\text{ pc}}\right) - 2.5\log\left(e^{-\tau}\right)$$
$$\equiv 5\log\left(\frac{r}{10\text{ pc}}\right) + A$$

3. Color Excess, ex. $E_{B-V} = A_B - A_V$:
$$V = V_0 + A_V,\, B = B_0 + A_V$$
$$B - V = (B - V)_0 + (A_B - A_V)$$
Note that $(B-V)_0$ is called the intrinsic color, while E_{B-V} is the color excess, and $E_{B-V} > 0$ indicates reddening.

4. Studies show that
$$R_V = \frac{A_V}{E_{B-V}} \approx 3.1$$

2 Radiation

Speed of light. $c = \lambda f = 1/\sqrt{\mu_0\epsilon_0}$

Kirchoff's Laws.

1. For a hot, dense gas or a solid, a continuous spectrum (governed by Planck's law) is emitted.
2. For a hot, diffuse gas, discrete emission lines are emitted.
3. For a cold, diffuse gas, discrete absorption lines (corresponding to the same λs as emission lines) are absorbed.

Planck's Law. For blackbody radiation.

1. Wavelength form:

$$B(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

$B(\lambda)$ is spectral radiance $((\mathrm{W/m^2}) \cdot \mathrm{sr^{-1}})$ per unit wavelength, so:

$$[B(\lambda)] = \frac{\mathrm{W}}{\mathrm{m^2}} \cdot \mathrm{m^{-1}} \cdot \mathrm{sr^{-1}}$$

$$F(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} = \frac{c_1}{\lambda^5} \frac{1}{e^{c_2/\lambda T} - 1}$$

$F(\lambda)$, on the other hand, is the flux density (i.e. flux $(\mathrm{W/m^2})$ per unit wavelength), which has the following units:

$$[F(\lambda)] = \frac{\mathrm{W}}{\mathrm{m^2}} \cdot \mathrm{m^{-1}}$$

If you wanted to find the net flux density $((\mathrm{W/m^2}) \cdot \mathrm{m^{-1}})$ of an object, the following expressions are equivalent:

$$S_\lambda = B_\lambda \Omega \text{ (solid angle)} = F_\lambda \cdot \frac{\text{emitting area (e.g. } 4\pi R^2)}{\text{receiving area (e.g. } 4\pi d^2)}$$

2. To convert from wavelength to frequency:

$$B_\nu d\nu = -B_\lambda d\lambda$$

3. Stefan-Boltzmann's Law:

$$F = \int_0^\infty F(\lambda, T) d\lambda = \sigma T^4$$

4. Wien approximation ($hc/\lambda kT \gg 1$):

$$B(\lambda, T) \approx \frac{2hc^2}{\lambda^5} e^{-hc/\lambda kT}$$

5. Rayleigh-Jeans approximation ($hc/\lambda kT \ll 1$):

$$B(\lambda, T) \approx \frac{2ckT}{\lambda^4}$$

This is the expression predicted by classical physics, which resulted in the UV catastrophe.

Properties of Isotropic Blackbody Radiation.

1. energy density (using $a = 4\sigma/c$):

$$u = aT^4$$

2. radiation pressure:

$$P_{rad} = \frac{1}{3} aT^4 = \frac{1}{3} u$$

Wien's Displacement Law. Note that $\nu_{max} \neq c/\lambda_{max}$.

$$\lambda_{max} = \frac{b}{T},\ b \approx 0.0029 \text{ K} \cdot \text{m}$$

Spectral Lines of H.

$$E_n = \frac{-13.6 \text{ eV}}{n^2},\ n \geq 1$$

$$\Delta E = hf = \frac{hc}{\lambda}$$

$$\frac{1}{\lambda} = \frac{\Delta E}{hc} = R_H \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

General Radiation Pressure

1. For absorption:
$$P_{rad} = \frac{I\cos\theta}{c}$$

2. For reflection:
$$P_{rad} = \frac{2I\cos^2\theta}{c}$$

   Note that the force due to radiation is given by $2IA\cos\theta/c$ instead of the $\cos^2\theta$ factor, since while the pressure is only in the normal direction, force includes the parallel direction.

3 Coordinates, Geometry, Time

Spherical Trig. A capital letter indicates a letter, a lowercase letter indicates a side.

1. Spherical law of Cosines:
$$\cos a = \cos b\cos c + \sin b\sin c\cos A$$

2. Spherical law of Sines:
$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

3. Four part formula (inner side i, outer side o, inner angle I, outer angle O):
$$\cos i\cos I = \cot o\sin i - \cot O\sin I$$

Tricky Definitions.

1. Azimuth A: clockwise from South, along the local horizon. Definition holds regardless of hemisphere. (Definitions may vary; this is my preferred definition.)

2. Right Ascension α: eastward from the vernal equinox on celestial equator, CCW as viewed from North Pole. OR number of hours behind the Sun on the vernal equinox.

3. Vernal Equinox: intersection of ecliptic and celestial equator when travelling eastward on the ecliptic (the ascending node). $\alpha = 0$.

4. Hour angle H: angle from meridian in equatorial frame. Increases in the direction of rotation of the sky.

5. The fictitious "mean sun": fictitious body that moves in a circular orbit with constant angular velocity around the celestial equator, rather than on ecliptic.

Location of the Sun. In (α, δ):

1. Vernal Equinox $(0^h, 0°)$

2. Summer Solstice $(6^h, +23.45°)$

3. Autumnal Equinox $(12^h, 0°)$

4. Winter Solstice $(18^h, -23.45°)$

5. Approximate RA of the Sun at all times, where Δt is time after vernal equinox:
$$\alpha_\odot \approx \frac{\Delta t}{1\text{ year}}24^h$$

   Note that this approximate expression is the exact expression for the RA of the mean Sun.

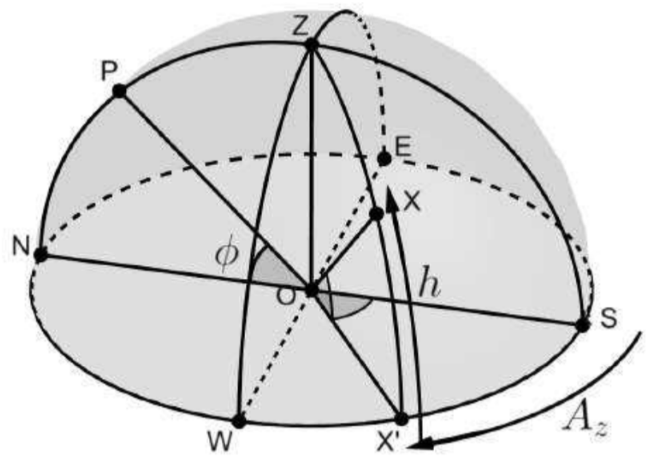


Figure 1: Horizontal Coordinates.

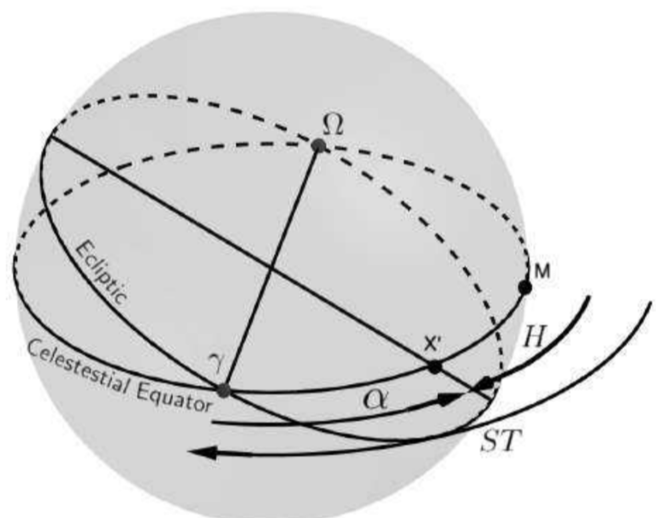


Figure 2: Hour Angle, Right ascension, and LST.

Local Sidereal Time (LST). Time that $= 0^h$ when vernal equinox crosses meridian. Is the same throughout the year. Often represented with Θ.

$$H + \alpha = \Theta$$

Hours behind the Sun. For a star with right ascension α, the number of hours behind the sun are:

$$\Delta H = \alpha - \alpha_\odot$$

Greenwich Mean Sidereal Time (GMST). λ is longitude in this case, with $\lambda > 0$ towards the East.

$$\Theta = \text{GMST} + \lambda$$

Mean Solar Time. When hour angle of the Sun is 0^h, it is noon solar time. In general:

$$T_M = H_M + 12^h$$

$$T_M = \Theta - \alpha_M + 12^h = \Theta + 12^h - \frac{\Delta t}{1 \text{ year}} 24^h$$

Equation of Time. Difference between true solar time (T) and mean solar time (T_M):

$$ET = T - T_M = H - H_M = \alpha_M - \alpha$$

Analemma. An analemma is a diagram showing the position of the Sun in the sky as seen from a fixed location on Earth at the same mean solar time, as that position varies over the course of a year. The azimuth varies with the equation of time ET, while the altitude varies with the declination of the Sun.

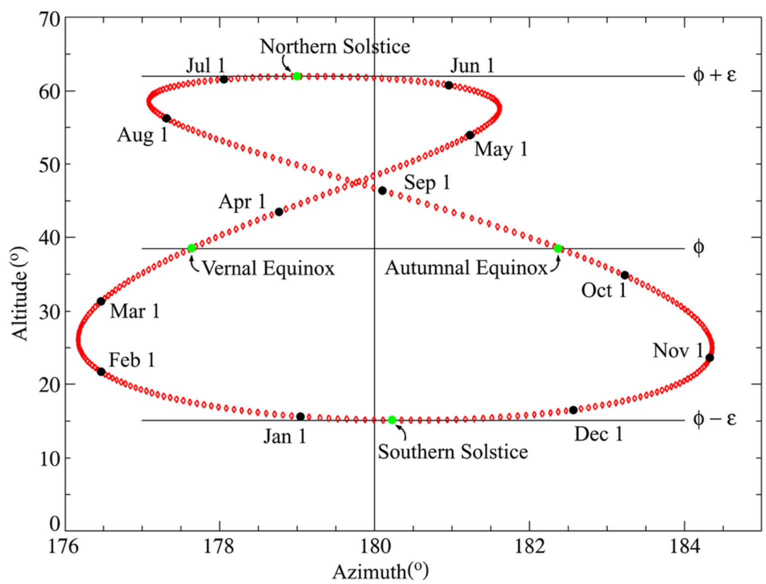


Figure 3: An example analemma.

Ecliptic Coordinate System. Coordinates based on the Earth's orbital plane around the Sun (or the apparent motion of the Sun). The ecliptic latitude β and the ecliptic longitude λ are defined as in the diagram.

Galactic Coordinate System. Coordinates based on the center of the Milky Way. The galactic latitude b and longitude l are defined as in the diagram.

Miscellaneous Geometry.

1. Superior planets can exist in opposition, conjunction, or quadrature, while inferior planets can have conjunctions and greatest elongations.

2. Parallax: for r in pc and θ in arcseconds:
$$r = \frac{1}{\theta}$$

3. Proper Motion:
$$\mu = \frac{v_t}{r} = \sqrt{\mu_\alpha^2 \cos^2(\delta) + \mu_\delta^2}$$
$$\mu_\alpha = \frac{d\alpha}{dt},\ \mu_\delta = \frac{d\delta}{dt}$$

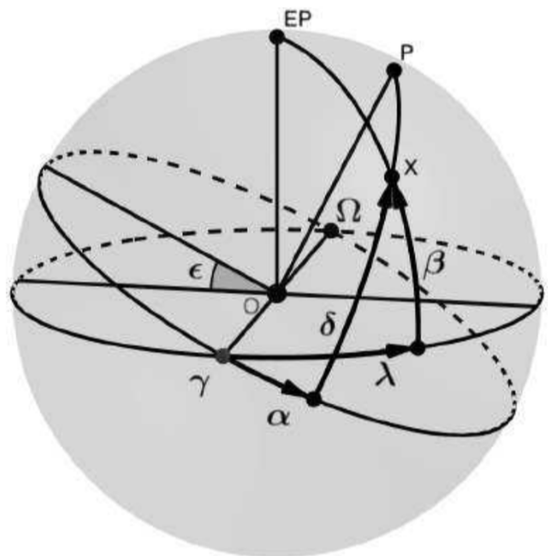


Figure 4: Ecliptic Coordinates.

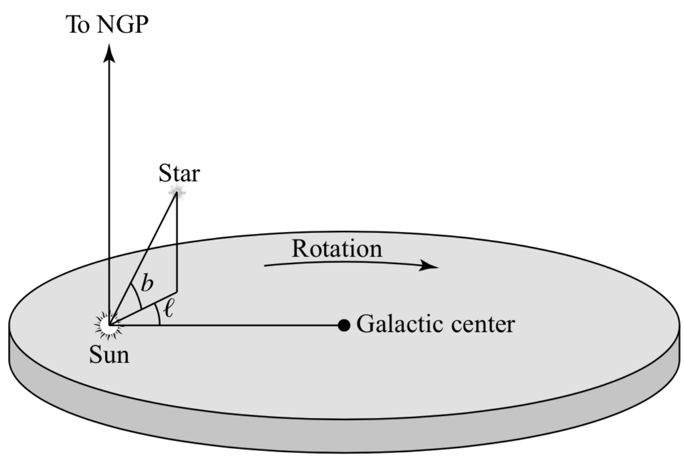


Figure 5: Galactic Coordinates.

4. Sidereal vs. Solar/Synodic days: Assume τ_* is sidereal and τ is synodic, and orbital period around the Sun is P:
$$\frac{1}{\tau} = \frac{1}{\tau_*} - \frac{1}{P} \text{ for prograde rotation}$$
$$\frac{1}{\tau} = \frac{1}{\tau_*} + \frac{1}{P} \text{ for retrograde rotation}$$
For the Earth-Sun system, $P = 365.2564$ d, $\tau = 1$ d, $\tau_* = 23^h 56^m 4^s$ in solar time.

5. Horizon distance and angle: At a height h on a planet of radius R, the horizon distance d and the angle below the horizontal a can be given by:
$$d = \sqrt{2hR + h^2} \approx \sqrt{2hR}$$
$$\cos a = \frac{R}{R+h},\ a \approx \sqrt{\frac{2h}{R}}$$

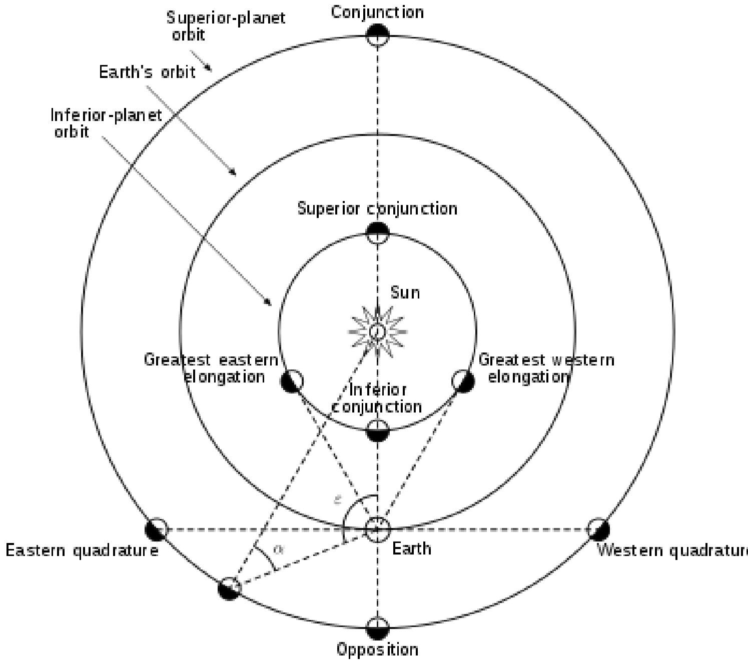


Figure 6: Possible Positions of Planets.

4 Celestial Mechanics

Universal Law of Gravitation.

$$F = -\frac{Gm_1m_2}{r^2}$$

Gravitational Potential

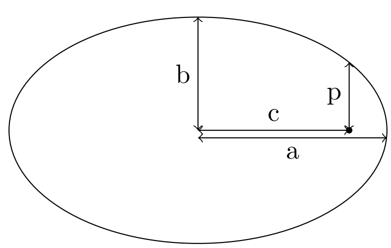
1. For two point masses m_1 and m_2, the potential of the pair is:

$$U = -\frac{Gm_1m_2}{r}$$

2. For a uniform density sphere of mass M, its potential is:

$$U = -\frac{3GM^2}{5R}$$

Geometry of an Ellipse.



1. sum of the distances between the two foci = $2a$.
2. $a^2 = b^2 + c^2$
3. $e = c/a$ can be given by:

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

4. semi-latus rectum $p = b^2/a = a(1-e^2)$
5. $r_p = a(1-e)$, $r_a = a(1+e)$
6. $A = \pi ab$
7. polar form of an ellipse (using angle from the periapsis θ):

$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$

Geometry of a Parabola. [insert diagram here]

1. $a = \infty$, b and c are now irrelevant
2. $e = 1$
3. semi-latus rectum $p = 2r_p$
4. polar form of a parabola:

$$r = \frac{p}{1+\cos\theta}$$

Geometry of a Hyperbola. [insert diagram here]

1. difference between the distances between the two foci $= 2a$.
2. $c^2 = a^2 + b^2$
3. $e = c/a$ can be given by:

$$e = \sqrt{1+\frac{b^2}{a^2}}$$

4. semi-latus rectum $p = b^2/a = a(e^2-1)$
5. $r_p = a(e-1)$
6. perpendicular distance from focus to asymptote $= b$
7. polar form of a hyperbola:

$$r = \frac{a(e^2-1)}{1+e\cos\theta}$$

Useful Expressions. Use reduced mass μ.

1. Energy (ellipse/parabola/hyperbola):

$$E = -\frac{Gm\mu}{2a}$$
$$E = 0$$
$$E = +\frac{Gm\mu}{2a}$$

2. Angular Momentum (ellipse/parabola/hyperbola):

$$L = \mu b\sqrt{\frac{GM}{a}}$$

$$L = \mu r_p\sqrt{\frac{2GM}{r_p}}$$

$$L = \mu b\sqrt{\frac{GM}{a}}$$

3. Vis-Viva (ellipse/parabola/hyperbola):

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

$$v^2 = \frac{2GM}{r}$$

$$v^2 = GM\left(\frac{2}{r} + \frac{1}{a}\right)$$

Kepler's Laws.

1. ellipse, with sun at one focus
2. $dA/dt = L/2\mu$.
3. $T^2 \propto a^3$. In particular:

$$T^2 = \frac{4\pi^2}{GM}a^3$$

Lagrange Points. [insert diagram here]

1. To find the locations, enter the rotating reference frame and balance gravity with centrifugal.
2. For points 1 and 2:

$$\Delta r \approx \pm r\left(\frac{M_E}{3M_\odot}\right)^{1/3}$$

3. For point 3:

$$\Delta r \approx -r\left(\frac{7M_E}{12M_\odot}\right)$$

4. Lagrange points 4 and 5 are on Earth's orbit, exactly 60° ahead and behind.

Virial Theorem. For gravitationally bound systems only, since there you can apply a time average such that the virial theorem applies.

1. $\langle K\rangle = -\frac{1}{2}\langle U\rangle$
2. $\langle E\rangle = \frac{1}{2}\langle U\rangle$
3. For a general power law potential, $V \propto r^n$, we have:

$$\langle K\rangle = -\frac{n}{2}\langle U\rangle$$

5 Cosmology

Redshift. Redshift is defined as

$$z \equiv \frac{\lambda - \lambda_0}{\lambda_0}$$

1. For small speeds: $z = v/c$
2. For relativistic speeds ($\beta \equiv v/c$):

$$1 + z = \sqrt{\frac{1+\beta}{1-\beta}}$$

$$\beta = \frac{(1+z)^2 - 1}{(1+z)^2 + 1}$$

Scale Factor. Defined as $r/r_0 = a(t)$, with $a(0) = 1$ and $t = 0$ indicating today.

1. Relation between z and a:
$$\frac{\lambda}{\lambda_0} = \boxed{1 + z = \frac{1}{a}}$$

2. Relation between z and time:
$$\frac{\lambda}{\lambda_0} = \frac{f_0}{f} = \boxed{\frac{\Delta t}{\Delta t_0} = 1 + z}$$

CMB Temperature. $T \propto 1/a$.

Hubble's Law. $v = Hr$, where $H = \dot{a}/a$.

Distance Measures.

1. Comoving distance: d_C or χ

2. Physical/Proper distance:
$$d_P = a_0 \chi$$
where a_0 is the scale factor at the present time.

3. Luminosty distance:
$$d_L \equiv \sqrt{\frac{L}{4\pi F}}$$
$$d_L = a_0 \chi (1 + z) = (a_0/a)\chi$$
where a is the scale factor at a time in the past.

4. Angular distance:
$$d_A \equiv \frac{D}{\theta}$$
$$d_A = \frac{\chi}{1 + z} = a\chi$$
where a is the scale factor at a time in the past.

Photon Traversing Space. For a photon, $a d\chi = c dt$ or $d\chi = c dz/H$.

Friedmann Equation and Similar.

1. Official version:
$$H^2 = \frac{8\pi G}{3c^2}\epsilon(t) - \frac{\kappa c^2}{R_0^2 a^2} + \frac{\Lambda}{3}$$

2. Energy density version, using $\Omega_0 = \epsilon_0/\epsilon_c$.
$$H^2 = H_0^2 \left(\frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1 - \Omega_0}{a^2} \right)$$
Note that
$$\epsilon_c = \frac{3H_0^2 c^2}{8\pi G}$$

3. Energy Scale-Factor Relation: For the equation of state $P = w\epsilon$, we have:
$$\epsilon(t) = \epsilon_0 a^{-3(1+w)}$$
where $w = 0$ for non-relativistic matter, $w = 1/3$ for relativistic particles (like photons), and $w = -1$ for dark energy.

6 Other Physics

Visual Binaries.

1. Considering the frame in which the center of mass is stationary, $m_1 r_1 = m_2 r_2$ and $m_1 v_1 = m_2 v_2$
2. Mass function:
$$\frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2} = \frac{T v_{1r}^3}{2\pi G}$$

Eclipsing Binaries. [insert diagram here.]

1. Primary minimum (when hotter star is eclipsed), secondary minimum (when colder star is eclipsed)
2. Radii of stars (v is relative velocity):
$$r_s = \frac{v}{2}(t_b - t_a)$$
$$r_l = \frac{v}{2}(t_c - t_a)$$
3. Temperatures given brightness dips:
$$\frac{B_0 - B_p}{B_0 - B_s} = \frac{F_1}{F_2} = \left(\frac{T_1}{T_2}\right)^4$$

Stellar Atmospheres.

1. Hydrostatic Equilibrium:
$$\frac{dP}{dr} = -\rho g = \frac{-GM_r \rho}{r^2}$$
2. Mass Conservation:
$$\frac{dM}{dr} = 4\pi r^2 \rho$$
3. Scale height H_P:
$$\frac{1}{H_P} \equiv -\frac{1}{P}\frac{dP}{dr}$$
$$P = P_0 e^{-r/H_P}$$

Stellar Mass Relations.

1. Mass-Luminosity Relation:
$$L \propto M^3$$
2. Stellar Lifetime:
$$\tau \propto \frac{M}{L} \propto M^{-2}$$

Eddington Luminosity. The maximum luminosity such that $F_{rad} \leq F_G$. Using the opacity $\kappa \equiv \sigma_T / m_e$, where where σ_T is the Thomson scattering cross-section of electrons and m_e is the electron mass:

$$L_{Edd} = \frac{4\pi GM}{\kappa}$$

Main Sequence Turnoff. All stars above this point will have evolved off of the main sequence. Location of point determines age of cluster by stellar mass relations.

Thermodynamics.

1. $PV = nRT = Nk_B T$

2. $K = \frac{3}{2}NkT$ for monatomic, $\frac{5}{2}NkT$ for diatomic, $\frac{x}{2}NkT$ for x degrees of freedom (equipartition theorem).

3. Velocities:

$$v_{mp} = \sqrt{\frac{2kT}{m}}$$

$$v_{av} = \sqrt{\frac{8kT}{\pi m}}$$

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

4. Mean Free Path (for cross-section σ and number density n):

$$l = \frac{1}{n\sigma}$$

5. Random Walk: For a random walk with step-size l and number of steps N, the distance is given by:

$$d = l\sqrt{N}$$

Schwarzschild Radius.

$$r_s = \frac{2GM}{c^2}$$

Mass-Energy Equivalence. $E = mc^2$

7 Math

Error Propogation. Assume everything is a Gaussian distribution.

1. For $z = x \pm y$:

$$\Delta z = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

2. For $z = xy$ or $z = x/y$:

$$\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

3. For $z = x^n$:

$$\frac{\Delta z}{|z|} = |n|\frac{\Delta x}{|x|}$$

4. In general, for a function $f(x, y, ...)$, assuming each variable is independent:

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2 + ...}$$

Taylor Approximations. For an arbitrary function $f(x)$, it can be written as the following polynomial:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + ... = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Typically, only one or two terms is used to approximate the function as a polynomial. Common approximations:

1. $(1+x)^\alpha \approx 1+\alpha x$
$$= 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + ... = \sum_{n=0}^{\infty}\binom{\alpha}{n}x^n$$
A special case of this is the sum of a geometric series:
$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + ...$$
2. $e^x \approx 1+x$
$$= 1 + x + \frac{x^2}{2!} + ... = \sum_{n=0}^{\infty}\frac{x^n}{n!}$$
3. $\ln(1+x) \approx x$
$$= x - \frac{x^2}{2} + \frac{x^3}{3} - ... = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$$
4. $\sin x \approx x$
$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - ... = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$$
5. $\cos x \approx 1 - \frac{x^2}{2}$
$$= 1 - \frac{x^2}{2!} + \frac{x^2}{4!} - ... = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$$
6. $\arctan x \approx x$
$$= x - \frac{x^3}{3} + \frac{x^5}{5} - ... = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$$

8 Topics Not Listed

Fairly Common:

1. Boltzmann distribution
2. Binding energy
3. Relativity
4. Minimum, maximum magnification for the eye

Advanced:

1. radiative transfer
2. Jeans mass/radius
3. Kepler's equation ($E - e\sin E = M$)
4. Temperature gradients for radiation or convection
5. Pauli Exclusion Principle (white dwarf/degenerate matter physics)
6. Oort Constant
7. Refraction in atmosphere
8. A ton more