Extra Topics for USAAAO/IOAA
version 1.3.1

Extra topics. These come up occasionally (for some, not at all yet) on the USAAAO and IOAA.

1 Optics

Minimum Magnification. Any less than the minimum magnification, and outgoing light has too large a diameter for the eye:

$$\omega_{min} = \frac{D}{d_{eye}}$$

Maximum Magnification. Any more than the max, and the resolution will be lower than that of the human eye. In other words, magnifying it further no longer serves any purpose. If the angular resolution of the human eye $e \approx 2'$ is compared to that of the telescope, R:

$$\omega_{max} = \frac{e}{R} \approx \frac{eD}{\lambda} \approx \frac{D}{1 \text{ mm}}$$

Atmospheric Refraction. Suppose α is the angle of incidence of a ray of light at the top of the atmosphere, and β is the angle of incidence at the bottom of the atmosphere. Thus:

$$n_0 \sin\beta = \sin\alpha$$

If the ray approaches from close to the zenith, $R \equiv \alpha - \beta \ll 1$. Thus:

$$R = (n_0 - 1)\tan\beta$$

Poynting Vector. The rate and direction in which energy is transferred by electromagnetic radiation (in units W/m^2) is given by the *Poynting Vector*:

$$\boldsymbol{S} = \frac{1}{\mu_0}\boldsymbol{E} \times \boldsymbol{B}$$

Definition of Flux. Radiative flux is found from intensity by the following:

$$F = \int I\cos\theta \, d\Omega = \int_0^{2\pi}\int_0^{\pi} I\cos\theta\sin\theta \, d\theta \, d\phi$$

and for isotropic radiation, $F = 0$.

2 Celestial Mechanics

Gauss's Law for Gravitation.

$$\oint_V \boldsymbol{g} \cdot d\boldsymbol{A} = -4\pi GM$$

Effective Potential.

$$\begin{aligned} E = K + U &= \frac{1}{2}mv^2 - \frac{GMm}{r} \\ &= \left(\frac{1}{2}mv_r^2 + \frac{1}{2}mv_\theta^2\right) - \frac{GMm}{r} \\ &= \left(\frac{L^2}{2mr^2} - \frac{GMm}{r}\right) + \frac{1}{2}mv_r^2 \end{aligned}$$

$$\equiv U_{\text{eff}} + \frac{1}{2}mv_r^2$$

Rocket Equation. Momentum Conservation yields:

$$mv = (m + dm)(v + dv) - (-dm)(v_e - v)$$

Solving the differential equation yields:

$$\Delta v = v_e \ln \left(\frac{m_i}{m_f} \right)$$

Hohmann Transfer Orbit. The most efficient way of transferring a satellite between two circular orbits. Given $\alpha \equiv R_2/R_1 > 1$, we have:

1. Initial velocity boost:

$$\Delta v_p = v_0 \left(\sqrt{\frac{2\alpha}{1+\alpha}} - 1 \right)$$

2. Final velocity boost:

$$\Delta v_a = v_f \left(1 - \sqrt{\frac{2}{1+\alpha}} \right)$$
$$= \frac{v_0}{\sqrt{\alpha}} \left(1 - \sqrt{\frac{2}{1+\alpha}} \right)$$

3. Time spent in Hohmann Transfer:

$$\Delta t = \frac{T}{2} = \frac{\pi (R_1 + R_2)^{3/2}}{\sqrt{8GM}}$$

where $v_0 = \sqrt{GM/R_1}$ and $v_f = \sqrt{GM/R_2}$.

3 Planetary Motion

Lengths of Time.

1. Sidereal Day: time it takes for a fixed star to return to the same meridian. Or, rotation period of Earth's axis. $T_{\text{sid}} = 86164.1$ seconds or $23^h 56^m 4^s$.

2. Solar Day: time it takes for the Sun to return to the same meridian. The average solar day is $T_{\text{solar}} = 86400$ seconds or 24^h. Note that the rotational velocity of the Sun with respect to Earth is

$$\omega_r = \omega - \Omega$$

where $\omega = 2\pi/T_{\text{sid}}$, $\Omega = 2\pi/T_{\text{sid,year}}$, and $\omega_r = 2\pi/T_{\text{solar}}$. Thus:

$$\frac{1}{T_{\text{solar}}} = \frac{1}{T_{\text{sid}}} - \frac{1}{T_{\text{sid,year}}}$$

3. Sidereal Year: time it takes for the Sun to return to the same position with respect to the stars. $T_{\text{sid,year}} = 365.25$ solar days.

4. Tropical/Solar Year: time it takes for the Sun to return to the same position with respect to Earth, which is slightly less than sidereal year due to Earth's precession of equinoxes.

$$\frac{1}{T_{\text{trop,year}}} = \frac{1}{T_{\text{sid,year}}} + \frac{1}{T_{\text{pr}}}$$

where $T_{\text{pr}} = 25765$ years.

5. Sidereal Month: time it takes for the moon to return to the same position relative to stars. $T_{\text{sid,month}} = 27.32^d$.

6. Synodic Month: relative to Sun. $T_{\text{syn,month}} = 29.53^d$.

$$\frac{1}{T_{\text{syn,month}}} = \frac{1}{T_{\text{sid,month}}} - \frac{1}{T_{\text{year}}}$$

7. Draconic Month/Draconic Year: to be added.

Eclipses. To be added.

4 Stellar Physics

Sources of Opacity. Defined as $\kappa = \sigma/m$, where σ is the effective cross-sectional area and m is the mass of the particle. The four main sources of opacity are:

1. Bound-bound transitions. Electrons make transitions from one orbital to another.

2. Bound-free absorption/Photoionization. Occurs when the wavelength $\lambda \leq hc/\chi_n$, where χ_n is the ionization energy of the nth orbital. The inverse is free-bound emission.

3. Free-free absorption. Occurs when electron in the vicinity of an ion absorbs a photon. Is a source of continuum opacity. The inverse is free-free emission due to a decelerating electron, called *Bremsstrahlung* or braking radiation.

4. Scattering. If done by free electron, called *Thomson scattering*, and has $\sigma = \sigma_T$ (Thomson cross-section). If done by electron and $\lambda \ll$ the size of an atom, then it is termed *Compton scattering*.

Kramer's Opacity Law. Many compositions follow the following opacity law:

$$\bar{\kappa} \propto \kappa_0 \frac{\rho}{T^{3.5}}$$

where ρ is the density, T is the temperature, and κ_0 is some constant opacity. I used $\propto$ to indicate that this equation would otherwise be dimensionally incorrect.

Mean Free Path. From the standard formula sheet:

$$l = \frac{1}{n\sigma}$$

Note that since opacity $\kappa = \sigma/m$, one can also write the mean free path as:

$$l = \frac{1}{\kappa\rho}$$

Thus, the optical depth τ can be expressed as $\tau = r/l$, or the number of mean free paths from the surface. Further, since a random walk with N steps travels $d = l\sqrt{N}$ on average, the average number of steps needed to escape (for $\tau \gg 1$) is:

$$N = \tau^2$$

Transfer Equation. The radiative transfer equation can be derived as such:

$$dI = -\kappa\rho I ds + j\rho ds$$

where j is the emission coefficient. Simplifying:

$$-\frac{1}{\kappa\rho}\frac{dI}{ds} = I - S$$

where $S \equiv j/\kappa$ is the *source function*. For isotropic blackbody radiation in thermal equilibrium, $S_\lambda = B_\lambda$. Now, using a vertical optical depth $\tau_v = \tau/\cos\theta$, we have:

$$\frac{dI}{d\tau_v}\cos\theta = I - S$$

Applications of Transfer Equation.

1. Integrating the transfer equation over all $d\Omega$, we have:

$$\frac{d}{d\tau_v}\int I\cos\theta\,d\Omega = \int I\,d\Omega - S\int d\Omega$$

   Note that S is independent of direction. Simplifying, we have:

$$\frac{dF_{\text{rad}}}{d\tau_v} = 4\pi(\langle I\rangle - S)$$

   Note that in an equilibrium atmosphere, $dF_{\text{rad}}/d\tau_v = 0$, so $\langle I\rangle = S$.

2. Integrating the transfer equation multiplied by $\cos\theta$ over $d\Omega$, we have:

$$\frac{d}{d\tau_v}\int I\cos^2\theta\,d\Omega = \int I\cos\theta\,d\Omega - S\int\cos\theta\,d\Omega$$

   Note that $\int\cos\theta\,d\Omega = 0$, so after simplifying we have:

$$\frac{dP_{\text{rad}}}{d\tau_v} = \frac{1}{c}F_{\text{rad}}$$

   In an equilibrium atmosphere, F_{rad} is constant, so

$$P_{\text{rad}} = \frac{1}{c}F_{\text{rad}}\tau_v + C$$

Eddington Approximation. To relate $\langle I\rangle$ to P_{rad}, we can make the following approximation:

$$I = \begin{cases} I_{\text{out}} & \text{for upper hemisphere} \\ I_{\text{in}} & \text{for lower hemisphere} \end{cases}$$

This means the following:

$$\langle I\rangle = \int I d\Omega = \frac{1}{2}(I_{\text{out}} + I_{\text{in}})$$

$$F_{\text{rad}} = \int I\cos\theta\,d\Omega = \pi(I_{\text{out}} - I_{\text{in}})$$

$$P_{\text{rad}} = \frac{1}{c}\int I\cos^2\theta\,d\Omega = \frac{2\pi}{3c}(I_{\text{out}} + I_{\text{in}}) = \frac{4\pi}{3c}\langle I\rangle$$

Thus, using application equation 2, we have:

$$\frac{4\pi}{3}\langle I\rangle = \frac{1}{c}F_{\text{rad}}\tau_v + C$$

At $\tau_v = 0$, $I_{\text{in}} = 0$, so $\langle I(\tau_v = 0)\rangle = F_{\text{rad}}/2\pi$. Thus, $C = 2F_{\text{rad}}/3$. Since $F_{\text{rad}} = \sigma T_e^4$, where T_e is the effective temperature at the surface, and $\langle I\rangle = S = B = \sigma T^4/\pi$, the final equation becomes:

$$\boxed{T^4 = \frac{3}{4}T_e^4\left(\tau_v + \frac{2}{3}\right)}$$

Note that the 'surface' with temperature T_e is at optical depth $\tau_v = 2/3$.

Limb Darkening. To be added.

Sources of Line Broadening. To be added.

5 Interstellar Medium

Opacity Continued. For the following scattering processes, σ varies by:

1. Geometric scattering, $\lambda \ll R$:
$$\sigma \sim \pi R^2$$
2. Mie scattering, $\lambda \sim R$:
$$\sigma \propto \lambda^{-1}$$
3. Rayleigh scattering, $\lambda \gg R$:
$$\sigma \propto \lambda^{-4}$$

Cloud Collapse Timescales.

1. By dimensional analysis, the time for gravitational collapse, called the free-fall time τ_{ff}, is:
$$\tau_{ff} \sim \frac{1}{\sqrt{\rho_0 G}}$$
2. The time in which pressure can react, τ_p, is given using the speed of sound $v_s = \sqrt{\gamma kT/m}$:
$$\tau_p = \frac{R}{v_s}$$

Jeans Mass and Radius. Cloud collapses if $U_{grav} + K_{thermal} < 0$. For a uniform density and temperature monatomic gas:

$$-\frac{3GM^2}{5R} < \frac{3}{2}NkT = \frac{3}{2}\frac{M}{m}kT$$

Using $M_J = (4/3)\pi R_J^3 \rho$, we find:

$$R_J = \sqrt{\frac{15kT}{8\pi G\rho m}}$$

The Jeans Radius R_J can also be thought of as the radius at which the free-fall time is much less than the pressure timescale, such that a cloud can no longer react.

$$R_J \gg \frac{v_s}{\sqrt{\rho_0 G}}$$

6 Statistical Mechanics

Boltzmann Equation. From statistical mechanics, the ratio of probabilities that a system be in state A or B is

$$\frac{P(A)}{P(B)} = \frac{g_a}{g_b} e^{-(E_a - E_b)/kT}$$

where g_i is the *statistical weight*, or the number of states with the same energy (the number of degenerate states).

Saha Equation. The Saha equation describes the ratio of atoms in the $i+1$ versus i ionization state. First defining the *partition function* Z as

$$Z = \sum_{j=1}^{\infty} g_j e^{-(E_j - E_1)/kT}$$

Z can be thought of as proportional to the total probability of being in a particular ionization. The true ratio of probabilities, which is in turn equal to the ratio of ions, is the Saha equation:

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i} \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_i/kT}$$

where χ_i is the ith ionization energy. Note that N_I is the non-ionized ion. A derivation of the Saha equation from the Boltzmann equation can be found here.

Applications. The Boltzmann and Saha equation give the ratios of energy states and ionization states, respectively. A combination of these equations is often required for applications.

For example, to find the ratio of the second energy state to the total N_2/N_{total}, we can take the approximation:

$$\frac{N_2}{N_{\text{total}}} \approx \left(\frac{N_2}{N_1 + N_2}\right)\left(\frac{N_{\text{I}}}{N_{\text{total}}}\right) \approx \left(\frac{N_2/N_1}{1 + N_2/N_1}\right)\left(\frac{1}{1 + N_{\text{II}}/N_{\text{I}}}\right)$$

where we have that N_1 and N_2 indicate energy states of the un-ionized atom, and N_{I} and N_{II} indicate ionization states. The approximations used are $N_{\text{I}} \approx N_1 + N_2$, since most of the un-ionized atoms are likely to be in the two lowest energy states, and $N_{\text{total}} = N_{\text{I}} + N_{\text{II}}$, for a similar reason. From the final expression, one can find N_2/N_{total} since N_2/N_1 and $N_{\text{II}}/N_{\text{I}}$ can be found by the Boltzmann and Saha equation, respectively.

7 Clusters

RMS velocity. v_{rms} can be found using the virial theorem to be:

$$\langle v^2 \rangle = \frac{3GM}{5R}$$

Virial Theroem Validity. The virial theorem is only valid if 1) the system is gravitationally bound, and 2) the system is in equilibrium. Equilibrium or "dynamically relaxed" means that the exchange in energy between stars is much faster than the evolution of the cluster. See timescales below.

Timescales.

1. Crossing time. Typical time for a star to travel a distance R, usually the half-mass radius:

$$t_{cr} \sim \frac{R}{v}$$

2. Relaxation time. Typical time where star's velocity is changed (due to gravitational interaction) by a comparable amount to its original velocity.

$$t_{rel} = \frac{\text{mean free path}}{v} = \frac{1}{n\sigma v}$$

$\sigma = \pi R^2$, where R is an appropriate distance in which gravitational interactions are important. This can be taken as the value where two stars are bound to each other gravitationally, or

$$R \sim \frac{GM}{v^2}$$

Using $n = N/(4/3\pi R^3) = (M/m)/(4/3\pi R^3)$, $R = 2GM/v^2$, and $\sigma = \pi R^2$:

$$t_{rel} = \frac{v^3 R^3}{3G^2 mM}$$

Using $v^2 = 3GM/5R$:

$$t_{rel} \sim \frac{ND}{v} = Nt_{cr}$$

3. More Accurate Relaxation Time. Divide by $\ln N$??

4. Evaporation time. Time required for cluster to dissolve through gradual loss of stars. Every relaxation time, the velocity of every star is re-calibrated independent of its original velocity, so after every t_{rel} there will be some stars that are above the escape velocity. Assume that a constant fraction of the stars in the cluster α is lost every relaxation time, or

$$\frac{dN}{dt} = -\frac{\alpha N}{t_{rel}} = -\frac{N}{t_{ev}}$$

α is given by the fraction of stars with $v > 2v_{rms}$, and assuming a Maxwell-Boltzmann distribution, $\alpha = 7.4 \times 10^{-3}$. Thus:

$$t_{ev} = \frac{t_{rel}}{\alpha} \approx 136 t_{rel}$$

Virial Mass. For a dynamically relaxed cluster,

$$M = \frac{5R\langle v^2 \rangle}{3G} = \frac{5R\langle v_r^2 \rangle}{G}$$

8 Galaxies

Rotation Curve. For circular orbits:

$$M(r) = \frac{rv^2(r)}{G}$$

After a sufficient radius, $v(r)$ is observed to be roughly constant, due to the presence of a dark matter halo.

Types of Galaxies.

1. Spiral Galaxies. Gas and dust, young and old stars, contain a disk. Surface luminosity at large radii decreases as

$$L = L_0 e^{-r/r_0}$$

with $r_0 \sim$ 5 kpc.

2. Elliptical Galaxies. Elliptical, low gas and old stars, stars move randomly within. Surface luminosity decreases as

$$L = L_0 (e^{-r/r_0})^{1/4}$$

with very variable r_0.

3. Lenticular Galaxies, Irregular Galaxies.

Galactic Evolution.

1. Dynamical Friction. By dimensional analysis:

$$F = C\frac{\rho(GM)^2}{v^2}$$

2. More.

Radio Galaxies.

1. Superluminal motion. More.

2. Relativistic jet beaming: initially isotropic, angle gets smaller with increasing speed. More.

Other Active Galactic Nuclei.

1. Seyferts. "Bright pinpoint nuclei ... They do not have radio lobes. Most are powerful sources of infrared radiation ... Seyfert 1 galaxies have hydrogen emission features with very large widths, indicating that the gas in the galaxy's central regions is moving with velocities of several thousand km/sec (Seyfert 1 galaxies show velocities up to almost 0.1c). Seyfert 2 galaxies have much narrower emission features implying much lower velocities"

2. Quasars. Bright, star-like when observing, strong radio source. Also known as quasi-stellar objects. Luminosity comes from accretion disk.

3. Accretion onto supermassive black holes. Maximum energy for mass m obtained is $E_{max} = GMm/R_S$, where $R_S = 2GM/c^2$. By combining these two equations:

$$E_{max} = \frac{1}{2}mc^2$$

   Usually the accretion process has an efficiency of $\epsilon \sim 0.1$. Thus, luminosity is

$$L = \frac{1}{2}\epsilon c^2\frac{dm}{dt}$$

4. Unified picture. Note that most AGNs are the same kind of galaxy, just looked at from a different angle.

9 Cosmology

Newtonian Friedmann Equation Derivation. By Energy Conservation, for a galaxy of mass m a distance R away:

$$E = \frac{1}{2}m\dot{R}^2 - \frac{GmM(R)}{R}$$

Applying Hubble's law and assuming homogeneity and isotropy:

$$E = \frac{1}{2}m(HR)^2 - \frac{4\pi}{3}Gm\rho_m R^2$$

or

$$\frac{2E}{mR^2} = H^2 - \frac{8\pi}{3}G\rho_m$$

Note that based on our assumptions, H and ρ_m are not functions of R, so $2E/mR^2$ doesn't depend on R either. However, the left hand side is time dependent as R changes with time. Since m is arbitrary, we can choose it such that $|2E/mc^2| = 1$ holds an arbitrary moment, as long as $E \neq 0$. Thus:

$$\frac{kc^2}{R^2} = H^2 - \frac{8\pi}{3}G\rho_m$$

where $k = 0, \pm 1$ is constant since E is constant. To generalize, replace density with energy density, and add a cosmological constant:

$$H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\epsilon/c^2 + \frac{kc^2}{R^2} + \frac{\Lambda}{3}$$

More Convenient Friedmann. Converting the previous section into a more convenient Friedmann equation, we can slap

$$\left(\frac{H}{H_0}\right)^2 = \Omega$$

where Ω encapsulates all energy densities, which may or may not change with a, and is the ratio between energy density and the critical energy density:

$$\epsilon_c = \frac{3H_0^2c^2}{8\pi G}$$

Hubble's Law (and other things) exposed. The scale factor a is almost always a complicated function with time. In general, it would be given by:

$$\frac{\dot{a}}{a} = H_0\sqrt{\Omega}$$

However, using a Taylor approximation to the first order, it can be approximated as:

$$\begin{aligned} a(t) &\approx a(t_0) + (t - t_0)\dot{a}(t_0) \\ &= a(t_0)\left(1 + (t - t_0)\frac{\dot{a}(t_0)}{a(t_0)}\right) \end{aligned}$$

Using $H = \dot{a}/a$ and $a(t_0) = 1$, while appreviating $t - t_0$ as just t:

$$a(t) \approx 1 + H_0 t$$

Now using the relation $1 + z = 1/a$:

$$1 + z \approx \frac{1}{1 + H_0 t} = 1 - H_0 t$$

or

$$z \approx -H_0 t$$

Note that since t is really $t - t_0$, a negative t means a time in the past, as expected. Replacing $-t$ with r/c, which is another approximation in itself, we get:

$$\begin{aligned} z &\approx \frac{H_0 r}{c} \\ cz &\approx H_0 r \end{aligned}$$

where $v = cz$ gets you Hubble's law. Exposé number 2 is that the procedure of obtaining an object's distance from redshift using:

1. $1 + z = \sqrt{(1 + \beta)/(1 - \beta)}$
2. $v = H_0 r$

from which you obtain:

$$r = \frac{c}{H_0}\frac{(1+z)^2-1}{(1+z)^2+1}$$

Note that $r = cz/H_0$ is an alright approximation for $z \ll 1$, and the relativistic expression differs from the true expression by less than 5% for $z < 2$, but both attempts are just plain wrong for the true value, despite the attempt at accuracy using the relativistic Doppler. This is clearly wrong, because:

1. This method attempts to use the wrong theory (relativity) to calculate cosmological redshift.
2. Thus, in this approximation, v is limited to a value of c, even though cosmological v can exceed c since the *expansion of space* is not limited by c.
3. In this approximation, no matter how high the redshift is, the upper bound distance is c/H_0. This is in contrast with the fact that the radius of the observable universe is $\sim 3c/H_0$.

However, this is what the USAAAO (first round only) wants, so just do it.

10 Modern Physics

Nuclear Binding Energy. For a nucleus with Z protons and N neutrons, the binding energy is:

$$E = \Delta mc^2 = (Zm_p + Nm_n - m_{nuc})c^2$$

Photoelectric Effect. When certain electromagnetic radiation is shined on metals, electrons are emitted. Only frequencies above a threshold frequency cause electrons to be emitted, and higher frequencies cause greater kinetic energy. More intensity, on the other hand, causes only more electrons to be emitted, instead of greater kinetic energies.

The max kinetic energy is given by

$$K_{\max} = \frac{hc}{\lambda} - \phi$$

where ϕ is the work function.

Wave-Particle Duality. All particles have a de-Broglie wavelength of

$$\lambda = \frac{h}{p}$$

Compton Effect. As proof that photons act as particles, photons are observed to have momentum and have collisions with electrons. In particular, given the angle between the new and original photon's path θ, the photon increases wavelength by:

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta)$$

Heisenberg Uncertainty Principle. Due to the wave-like nature of all matter, all matter must obey Schrödinger's equation and thus have fundamental uncertainty. Mathematically:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta E \Delta t \approx \hbar$$

Quantum Numbers. The four quantum numbers that describe a system that obeys Schrödinger's equation are n, l, m_l, and m_s.

1. Principal Quantum Number (n): specifies the energy. n is an integer from 1 to ∞. In hydrogen:

$$E_n = -\frac{m_e e^4}{32\pi^2 \epsilon_0^2 \hbar^2} \frac{1}{n^2} = -\frac{13.6 \text{ eV}}{n^2}$$

2. Azimuthal Quantum Number (l): specifies the angular momentum of a subshell. l is an integer ranging from 0 to $n-1$, and gives the magnitude of the angular momentum through the following relation:

$$L = \hbar\sqrt{l(l+1)}$$

3. Magnetic Quantum Number (m_l): specifies the projection of angular momentum along a specified axis, denoted z. An integer ranging from $-l$ to l with the following relation with L_z:

$$L_z = m_l \hbar$$

   Note that energy values of isolated hydrogen are independent of l and m_l, meaning subshells of same n are *degenerate*.

4. Spin Quantum Number (m_s): specifies the angular momentum of a purely quantum effect, termed 'spin'. m_s has a value of $\pm 1/2$ for electrons. $|\boldsymbol{S}| = \sqrt{3}\hbar/2$ and has a projection on the z-axis:

$$S_z = m_s \hbar$$

Pauli Exclusion Principle. No two electrons (or in general, fermions) may share the same four quantum numbers. Note that fermions have an odd half-integer spin, like $(1/2)\hbar$ or $(3/2)\hbar$, whereas bosons have integral spin and do not obey the Pauli Exclusion Principle.

Selection Rules. To be added.

11 Topics Not Listed

Fairly Common:

1. Boltzmann distribution
2. Binding energy **(DONE)**
3. Relativity
4. Minimum, maximum magnification for the eye **(DONE)**

Advanced:

1. radiative transfer
2. Jeans mass/radius **(DONE)**
3. Kepler's equation ($E - e\sin E = M$)
4. Temperature gradients for radiation or convection
5. Pauli Exclusion Principle (white dwarf/degenerate matter physics)
6. Oort Constant
7. Refraction in atmosphere **(DONE)**
8. A ton more