Your Name: ____________William Ard____________
(This is an INDIVIDUAL assignment)

CSC 4512, Optimization Approaches in CS: Algorithms and Applications

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Spring 2022 Semester

SUBMIT ALL PROJECTS Electronically via Moodle

A typical file name should have the following format:
"CSC4512_Spring2022_PROJ_2_YourFirstName_YourLastName.zip"
(Each submission involves more than one file; thus you will have to zip them into a single file.)

MAIN GOAL for PROJECT #2: Get familiarized with the process of developing algorithms for optimization problems.

Today's date: Thursday, March 10, 2022
Due date: Thursday, March 31, 2022. By 10:00 PM of that day via MOODLE
Maximum grade points = 100
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Recall that our TA is Augustine Orgah. His E-mail is: aorgah1@lsu.edu
CLEARLY EXPLAIN AND ORGANIZE YOUR ASNWERS! The TA may take points off otherwise.
Your answers must be presented sequentially.

NOTE: Always observe the Policy Statement for this course regarding Cheating / Academic Misconduct as it is stated on page 4 of the syllabus and described in the first day of classes.

1. Problem Description

Suppose that given are pairs of positive numbers, say V_1 and V_2, where $V_1, V_2 \geq 0$. We want such pairs to be reciprocals of each other. That is, their product to be equal to 1. An example is the pair $V_1 = 2$ and $V_2 = 0.5$ (i.e., the pair $\{2, 0.5\}$). If this condition does not hold for a given pair, then we would like to **minimally adjust** it so the reciprocal condition will hold on the adjusted pair. Formulate this problem and then solve it by considering the following pairs of numbers as test problems (you may want to solve even more problems to gain additional insight):

Pair #1:	{2,	1}
Pair #2:	{0.2,	7}
Pair #3:	{3,	1/3}
Pair #4:	{8,	2}
Pair #5:	{1,	2}

Pair #6:	{2,	4}
Pair #7:	{0.2	6}
Pair #8:	{4,	0.2}

You have to formulate this problem as an optimization one. Note that the objective function and/or some or all of the constraint(s) may or may not be linear. Ideally, you need to develop an exact optimal approach or as a compromise a heuristic approach. In such case, you may want to provide some insight on how well your heuristic performs on random problems. You may also want to explore some theoretical aspects such as the following questions: Does this problem always have a solution? Can it have alternative optimal solutions? Can you guarantee if a solution is optimal? Do you need to write a computer program, or a theoretical approach may suffice to determine the optimal solution? and so on.

2. Deliverables

Make sure your deliverables include the following items (use separate files if needed):

1. Clearly explain what your solution approach is and how well it works. Consider answering the previous questions or even additional ones regarding any theoretical aspects of it.

2. If you write a computer program, then you need to provide the source code (in a different file) with adequate info on how one can run it and reproduce your results. Write plenty of comments to explain your code.

As a final note: You will need to spend time to understand the problem and then develop the solution approach(es). Thus, start ASAP as this project may take more time than what you may anticipate now.

Attach this form with your name filled in on front of your report

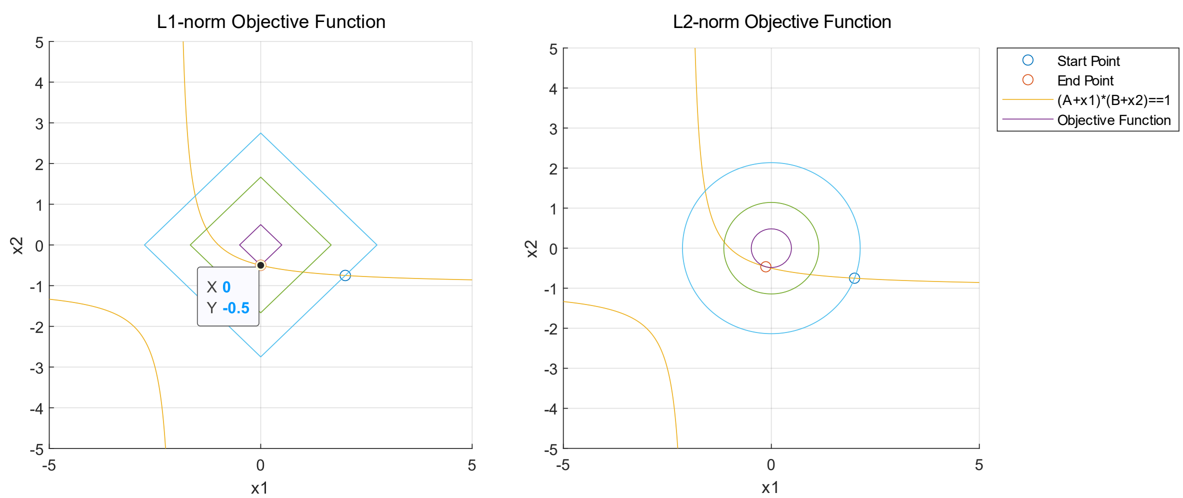
The optimization problem was formulated as:

$$\boxed{\begin{array}{l}\text{Minimize } z = |x|_1 = |x_1| + |x_2| \\ \qquad\qquad \text{Subject to: } (A + x_1)(B + x_2) = 1\end{array}}$$

An alternative formulation was also used, where instead of the L1 norm being used for the objective function, the L2 norm was used:

$$\boxed{\begin{array}{l}\text{Minimize } z = |x|_2 = \sqrt{x_1^2 + x_2^2} \\ \qquad\qquad \text{Subject to: } (A + x_1)(B + x_2) = 1\end{array}}$$

Graphically, the problem can be visualized as:



Both results are presented on a table on the following pages.

The objective function is a function of x_1 and x_2:

$$z(x_1, x_2) = |x_1| + |x_2| \tag{1}$$

The algorithm works by first selecting an arbitrary value for x_1 (let $x_1 = A$) that lies on the constraint equation. x_2 is expressed in terms of x_1 as:

$$x_2 = \frac{1}{A + x_1} - B \tag{2}$$

The objective function can then be taken to be a function of x_1 only:

$$z(x_1) = |x_1| + \left|\frac{1}{A + x_1} - B\right| \tag{3}$$

Incrementing x_1 by ± 0.00001, the direction to minimize the objective function can then be found by comparing:

$$z(x_1 + 0.00001) \text{ and } z(x_1 - 0.00001)$$

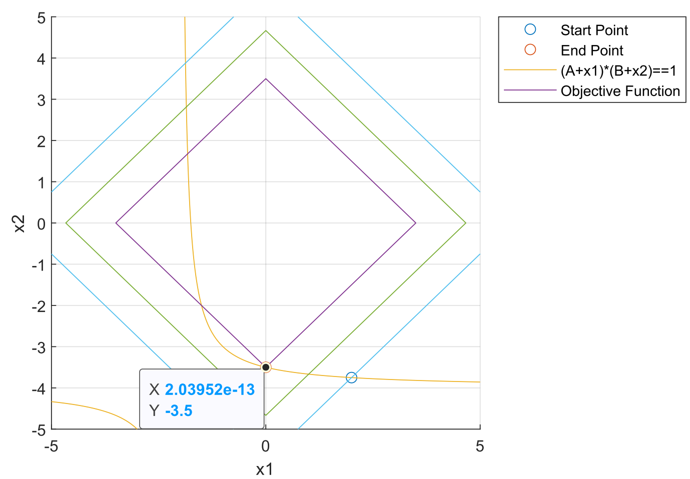
Once the direction to move along the constraint equation is found, the algorithm is then:

```
while delta_z > 0:
      z_current = z(x1)
      z_new = z(x1 + 0.0001)
      delta_z = z_current - z_new
      x1 = x1 + delta
```

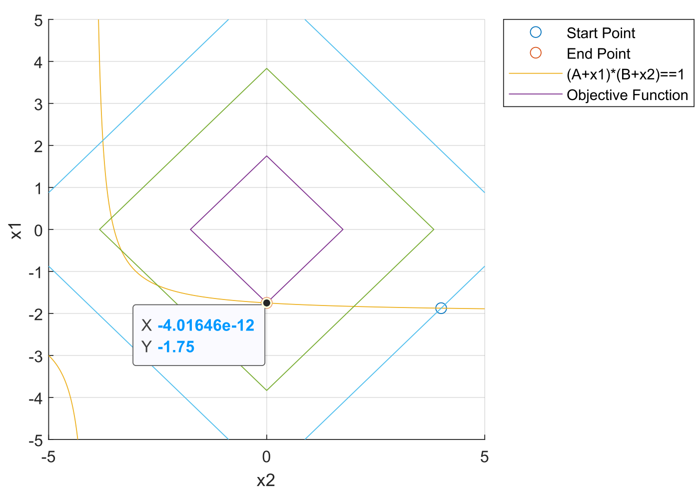
When Δz becomes negative, the objective function has begun to increase, so this marks the terminating condition for the algorithm. The minimum of the objective function has been found, and x_2 is related to x_1 by equation 3, so the value for minimal adjustment has been found.

It should be noted that there are cases where the algorithm will reach a local minimum and terminate early. This is due to the search being performed in terms of x_1. An example of this is shown below:

For the pair [A, B] = [2, 4], when starting at $x_1 = 2$, the algorithm converges to a local minima:



To correct this issue, it's necessary to perform the search from the opposite direction, expressing x_1 in terms of x_2:



By performing both searches, and comparing the objective function for both cases, the global minimum can always be found, as long as the inputs are cases for which a real-valued reciprocal exists (if one of the inputs was 0, there would be no reciprocal). In this case, for the first search, $z = 3.5$, and for the 2nd search, $z = 1.75$, it's clear that the minimal adjustment for pair [A, B] = [2, 4] is [2-1.75, 4-0] = [0.25, 4.0].

Multiple solutions could exist in the case where the pair is something like: [-A, A], as it would be possible to decrease one or increase the other by the same amount to get the same optimal result. Take [-2, 2] as an example pair. The first term could be increased by 2.5, giving: [-2+2.5, 2] = [0.5, 2], or the 2nd term could be decreased by the same amount. Both results produce the same minimal adjustment of 2.5.

It should be noted that accuracy of the solver can be increased by using a smaller delta for the solver increment, but this comes at the expense of computation time.

MATLAB was used for the generation of plots for this project, and Python was used for the final version of the solver.

Both the MATLAB code and the Python code are included with this project. To get the results, run the `get_results.py` file included in the archive.

Results using the L-1 Norm as the objective function

Pair		$z = \|x\|_1$		Adjusted Pair	
A	B	x_1	x_2	$A + x_1$	$B + x_2$
2	1	0.0000	-0.5000	2	0.5
0.2	7	-0.0571	0.0000	0.14286	7
3	1/3	0.0000	0.0000	3	1/3
8	2	0.0000	-0.1750	8	0.125
1	2	-0.5000	0.0000	0.5	2
2	4	-1.7500	0	0.25	4
0.2	6	-0.0333	-0.0012	0.16667	6
4	0.2	0.0000	0.0500	4	0.25

Results using the L-2 Norm as the objective function:

Pair		$z = \|x\|_2$		Adjusted Pair	
A	B	x_1	x_2	$A + x_1$	$B + x_2$
2	1	-0.1332	-0.4643	1.8668	0.5357
0.2	7	-0.0571	-0.0012	0.1429	6.9988
3	1/3	0.0000	0.0000	3.0000	0.3333
8	2	-0.0295	-1.8745	7.9705	0.1255
1	2	-0.4643	-0.1332	0.5357	1.8668
2	4	-1.7426	-0.1155	0.2574	3.8845
0.2	6	-0.0333	-0.0009	0.1667	5.9991
4	0.2	0.0031	0.0498	4.0031	0.2498