Non Parametric Filters: Grid Localization and Monte Carlo Localization

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Readings for this class

- Chapter 8: Grid and Monte Carlo Localization (pages 187-219) from *Probabilistic Robotics*, Sebastian Thrun

Introduction

Characteristics of Grid Localization and Monte Carlo Localization:

- They can process raw sensor measurements.
- They are non-parametric (they are not bound to a uni-modal distribution)
- They can solve global localization and - in some instances - kidnapped robot problems.
- Excellent performance in a number of fielded robotic systems.

Grid Localization

Grid Localization

- **Discrete** implementation of the Bayes Filter
- **Probability Distribution Functions (pdfs)** are represented through their histograms.
- Markov localization implemented with a histogram filter.
- Initial position is unknown.
- Global Localization.

Grid Localization

Issues that might arise:

- **Using a fine-grained grid**, the computation required for a naive implementation may make the algorithm unreasonably slow.

- **With a coarse grid**, the additional information loss through the discretization negatively affects the filter and - if not properly treated - may even prevent the filter from working.

Grid Localization: Robot faces a door example

Algorithm Bayes_filter($bel(x_{t-1}), u_t, z_t$**):**
for all x_t do
$\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1})\ bel(x_{t-1})\ dx$
$bel(x_t) = \eta\ p(z_t \mid x_t)\ \overline{bel}(x_t)$
endfor
return $bel(x_t)$

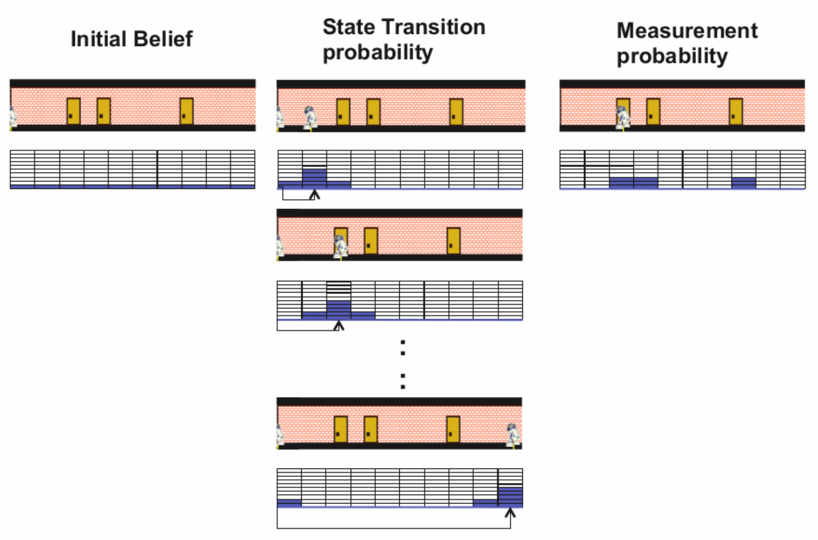
Algorithm Discrete_Bayes_filter($\{p_{k,t-1}\}, u_t, z_t$**):**
for all k do
$\bar{p}_{k,t} = \sum_i p(X_t = x_k \mid u_t, X_{t-1} = x_i)\ p_{i,t-1}$
$p_{k,t} = \eta\ p(z_t \mid X_t = x_k)\ \bar{p}_{k,t}$
endfor
return $\{p_{k,t}\}$

Grid Localization: Robot faces a door example

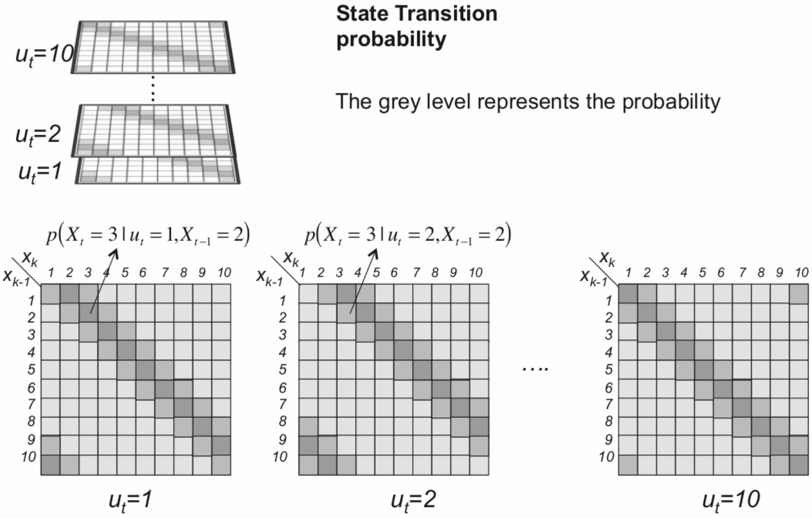
- $p(x_t \mid x_{t-1}, u_t)$ is the State Transition Probability
- $p(z_t \mid x_t)$ is called the Measurement Probability.
- The state transition probability and the measurement probability together describe the dynamical stochastic system of the robot and its environment.

Grid Localization: Robot faces a door example

Assumptions:

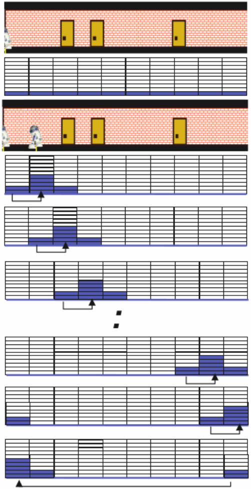


Assume you divide the environment into 10 equal areas.



Grid Localization: Robot faces a door example

Computing individual state transition probabilities for each of the grid cells (10 of them)



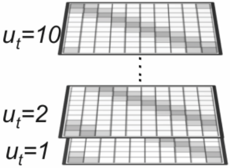
$$p(X_t \mid u_t = 1, X_{t-1} = x_1)$$

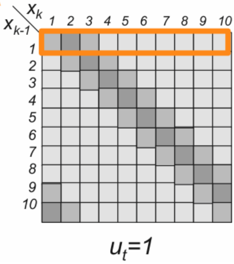
$$p(X_t \mid u_t = 1, X_{t-1} = x_2)$$

$$p(X_t \mid u_t = 1, X_{t-1} = x_3)$$

$\vdots$

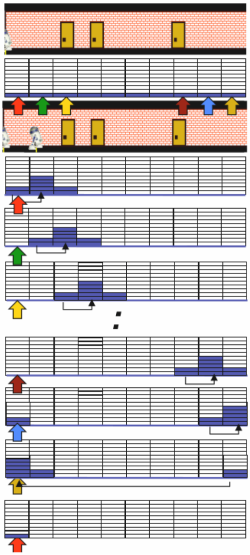
Algorithm Discrete_Bayes_filter($\{p_{k,t-1}\}, u_t, z_t$):
  for all k do
    $\bar{p}_{k,t} = \sum_i p(X_t = x_k \mid u_t, X_{t-1} = x_i)\, p_{i,t-1}$
    $p_{k,t} = \eta\, p(z_t \mid X_t = x_k)\, \bar{p}_{k,t}$
  endfor
  return $\{p_{k,t}\}$





Grid Localization: Robot faces a door example

```
Algorithm Discrete_Bayes_filter({p_{k,t-1}}, u_t, z_t):
    for all k do
        p̄_{k,t} = Σ_i p(X_t = x_k | u_t, X_{t-1} = x_i) p_{i,t-1}
        p_{k,t} = η p(z_t | X_t = x_k) p̄_{k,t}
    endfor
    return {p_{k,t}}
```



$$p_{1,t} = \sum_i p(X_t = x_1 \mid 1, X_{t-1} = x_i) p_{i,t-1}$$ K=1

$$p(X_t = x_1 \mid u_t = 1, X_{t-1} = x_1) p_{1,t-1} = 0,2 \cdot 0,1 = 0,02$$ i=1

$$p(X_t = x_1 \mid u_t = 1, X_{t-1} = x_2) p_{2,t-1} = 0,014 \cdot 0,1 = 0,0014$$ i=2

$$p(X_t = x_1 \mid u_t = 1, X_{t-1} = x_3) p_{3,t-1} = 0,014 \cdot 0,1 = 0,0014$$ i=3

:

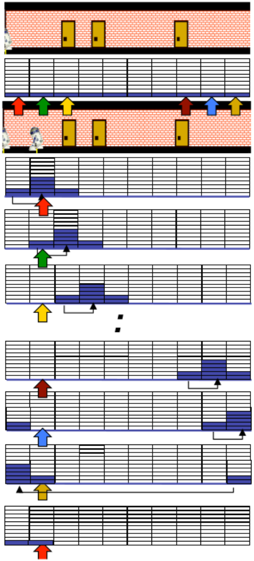
$$p(X_t = x_1 \mid u_t = 1, X_{t-1} = x_8) p_{8,t-1} = 0,014 \cdot 0,1 = 0,0014$$ i=8

$$p(X_t = x_1 \mid u_t = 1, X_{t-1} = x_9) p_{9,t-1} = 0,2 \cdot 0,1 = 0,02$$ i=9

$$p(X_t = x_1 \mid u_t = 1, X_{t-1} = x_{10}) p_{10,t-1} = 0,5 \cdot 0,1 = 0,05$$ i=10

$$p_{1,t} = 0,02 + 0,0014 + 0,0014 + \ldots + 0,0014 + 0,02 + 0,05 = 0,1$$

Grid Localization: Robot faces a door example



1: **Algorithm Discrete_Bayes_filter**($\{p_{k,t-1}\}, u_t, z_t$):
2: for all k do
3: $\bar{p}_{k,t} = \sum_i p(X_t = x_k \mid u_t, X_{t-1} = x_i)\; p_{i,t-1}$
4: $p_{k,t} = \eta\; p(z_t \mid X_t = x_k)\; \bar{p}_{k,t}$
5: endfor
6: return $\{p_{k,t}\}$

k=2
$$p_{2,t} = \sum_i p(X_t = x_1 \mid 1, X_{t-1} = x_i)\, p_{i,t-1}$$

i=1
$$p(X_t = x_2 \mid u_t = 1, X_{t-1} = x_1)\, p_{1,t-1} = 0{,}5 \cdot 0{,}1 = 0{,}05$$

i=2
$$p(X_t = x_2 \mid u_t = 1, X_{t-1} = x_2)\, p_{2,t-1} = 0{,}2 \cdot 0{,}1 = 0{,}02$$

i=3
$$p(X_t = x_2 \mid u_t = 1, X_{t-1} = x_3)\, p_{3,t-1} = 0{,}014 \cdot 0{,}1 = 0{,}0014$$

:

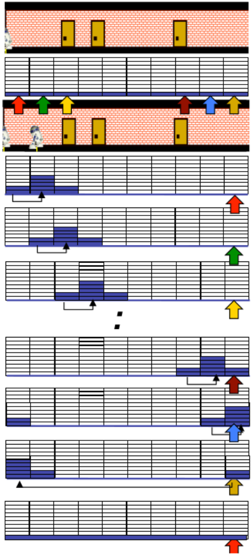
i=8
$$p(X_t = x_2 \mid u_t = 1, X_{t-1} = x_8)\, p_{8,t-1} = 0{,}014 \cdot 0{,}1 = 0{,}0014$$

i=9
$$p(X_t = x_2 \mid u_t = 1, X_{t-1} = x_9)\, p_{9,t-1} = 0{,}2 \cdot 0{,}1 = 0{,}02$$

i=10
$$p(X_t = x_2 \mid u_t = 1, X_{t-1} = x_{10})\, p_{10,t-1} = 0{,}2 \cdot 0{,}1 = 0{,}02$$

$$p_{2,t} = 0{,}05 + 0{,}02 + 0{,}0014 + \ldots + 0{,}0014 + 0{,}02 + 0{,}02 = 0{,}1$$

Grid Localization: Robot faces a door example



1: **Algorithm Discrete_Bayes_filter**($\{p_{k,t-1}\}, u_t, z_t$):
2: for all k do
3: $\bar{p}_{k,t} = \sum_i p(X_t = x_k \mid u_t, X_{t-1} = x_i)\, p_{i,t-1}$
4: $p_{k,t} = \eta\, p(z_t \mid X_t = x_k)\, \bar{p}_{k,t}$
5: endfor
6: return $\{p_{k,t}\}$

K=10

$$p_{10,t} = \sum_i p\left(X_t = x_1 \mid 1, X_{t-1} = x_i\right) p_{i,t-1}$$

i=1

$$p\left(X_t = x_{10} \mid u_t = 1, X_{t-1} = x_1\right) p_{1,t-1} = 0{,}014 \cdot 0{,}1 = 0{,}0014$$

i=2

$$p\left(X_t = x_{10} \mid u_t = 1, X_{t-1} = x_2\right) p_{2,t-1} = 0{,}014 \cdot 0{,}1 = 0{,}0014$$

i=3

$$p\left(X_t = x_{10} \mid u_t = 1, X_{t-1} = x_3\right) p_{3,t-1} = 0{,}014 \cdot 0{,}1 = 0{,}0014$$

i=8

$$p\left(X_t = x_{10} \mid u_t = 1, X_{t-1} = x_8\right) p_{8,t-1} = 0{,}2 \cdot 0{,}1 = 0{,}02$$

i=9

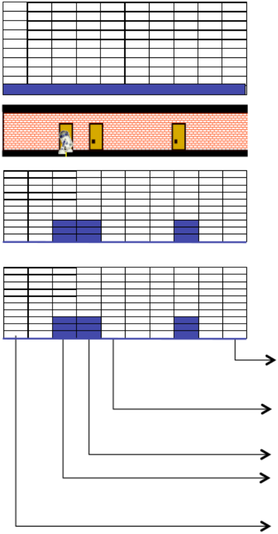
$$p\left(X_t = x_{10} \mid u_t = 1, X_{t-1} = x_9\right) p_{9,t-1} = 0{,}5 \cdot 0{,}1 = 0{,}05$$

i=10

$$p\left(X_t = x_{10} \mid u_t = 1, X_{t-1} = x_{10}\right) p_{10,t-1} = 0{,}2 \cdot 0{,}1 = 0{,}02$$

$$p_{10,t} = 0{,}0014 + 0{,}0014 + 0{,}0014 + \ldots + 0{,}02 + 0{,}05 + 0{,}02 = 0{,}1$$

Grid Localization: Robot faces a door example



Algorithm Discrete_Bayes_filter$(\{p_{k,t-1}\}, u_t, z_t)$:
  for all k do
    $\bar{p}_{k,t} = \sum_i p(X_t = x_k \mid u_t, X_{t-1} = x_i)\, p_{i,t-1}$
    $p_{k,t} = \eta\, p(z_t \mid X_t = x_k)\, \bar{p}_{k,t}$
  endfor
  return $\{p_{k,t}\}$

$$p_{k,t} = \eta p\left(z_t \middle| X_t = x_k\right)\bar{p}_{k,t}$$

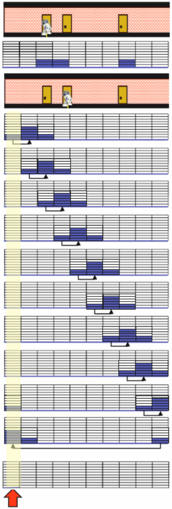
$$\eta = \frac{1}{0{,}0014 \cdot 7 + 0{,}3 \cdot 3} = 1{,}0020$$

$$\left.\begin{array}{l}
p_{10,t} = \eta p\left(z_t \middle| X_t = x_{10}\right)\bar{p}_{10,t} = 0{,}014 \cdot 0.1\eta = 0{,}0014\eta \\
p_{9,t} = \eta p\left(z_t \middle| X_t = x_9\right)\bar{p}_{9,t} = 0{,}014 \cdot 0.1\eta = 0{,}0014\eta \\
p_{8,t} = \eta p\left(z_t \middle| X_t = x_8\right)\bar{p}_{8,t} = 0{,}3 \cdot 0{,}1\eta = 0{,}03\eta \\
\vdots \\
p_{4,t} = \eta p\left(z_t \middle| X_t = x_4\right)\bar{p}_{4,t} = 0{,}3 \cdot 0{,}1\eta = 0{,}03\eta \\
p_{3,t} = \eta p\left(z_t \middle| X_t = x_3\right)\bar{p}_{3,t} = 0{,}3 \cdot 0{,}1\eta = 0{,}03\eta \\
p_{2,t} = \eta p\left(z_t \middle| X_t = x_2\right)\bar{p}_{2,t} = 0{,}014 \cdot 0.1\eta = 0{,}0014\eta \\
p_{1,t} = \eta p\left(z_t \middle| X_t = x_1\right)\bar{p}_{1,t} = 0{,}014 \cdot 0.1\eta = 0{,}0014\eta
\end{array}\right\} \Rightarrow \left\{\begin{array}{l}
p_{10,t} = 0{,}0014\eta = 0{,}0014 \\
p_{9,t} = 0{,}0014\eta = 0{,}0014 \\
p_{8,t} = 0{,}03\eta = 0{,}3 \\
\vdots \\
p_{4,t} = 0{,}03\eta = 0{,}3 \\
p_{3,t} = 0{,}03\eta = 0{,}3 \\
p_{2,t} = 0{,}0014\eta = 0{,}0014 \\
p_{1,t} = 0{,}0014\eta = 0{,}0014
\end{array}\right.$$

What you saw so far, was when the robot moved only once with $u = 1$. But the robot moves constantly, so the process is repeated every time the robot moves, using the previous computed values.

Grid Localization: Robot faces a door example

```
Algorithm Discrete_Bayes_filter({p_{k,t-1}}, u_t, z_t):
    for all k do
        p̄_{k,t} = Σ_i p(X_t = x_k | u_t, X_{t-1} = x_i) p_{i,t-1}
        p_{k,t} = η p(z_t | X_t = x_k) p̄_{k,t}
    endfor
    return {p_{k,t}}
```



$$p_{1,t} = \sum_i p(X_t = x_1 \mid 1, X_{t-1} = x_i) p_{i,t-1}$$ k=1

$$p(X_t = x_1 \mid u_t = 1, X_{t-1} = x_1) p_{1,t-1} = 0,2 \cdot 0,014 = 0,0028$$ i=1

$$p(X_t = x_1 \mid u_t = 1, X_{t-1} = x_2) p_{2,t-1} = 0,014 \cdot 0,014 = 0,000196$$ i=2

$$p(X_t = x_1 \mid u_t = 1, X_{t-1} = x_3) p_{3,t-1} = 0,014 \cdot 0,3 = 0,0042$$ i=3

$$p(X_t = x_1 \mid u_t = 1, X_{t-1} = x_4) p_{4,t-1} = 0,014 \cdot 0,3 = 0,0042$$ i=4

$$p(X_t = x_1 \mid u_t = 1, X_{t-1} = x_5) p_{5,t-1} = 0,014 \cdot 0,014 = 0,000196$$ i=5

$$p(X_t = x_1 \mid u_t = 1, X_{t-1} = x_6) p_{6,t-1} = 0,014 \cdot 0,014 = 0,000196$$ i=6

$$p(X_t = x_1 \mid u_t = 1, X_{t-1} = x_7) p_{7,t-1} = 0,014 \cdot 0,014 = 0,000196$$ i=7

$$p(X_t = x_1 \mid u_t = 1, X_{t-1} = x_8) p_{8,t-1} = 0,014 \cdot 0,3 = 0,0042$$ i=8

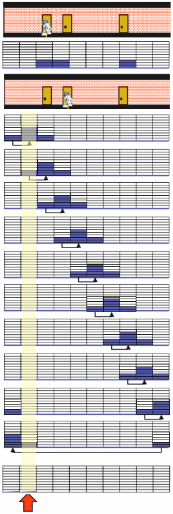
$$p(X_t = x_1 \mid u_t = 1, X_{t-1} = x_9) p_{9,t-1} = 0,2 \cdot 0,014 = 0,0028$$ i=9

$$p(X_t = x_1 \mid u_t = 1, X_{t-1} = x_{10}) p_{10,t-1} = 0,5 \cdot 0,014 = 0,007$$ i=10

$$p_{1,t} = 0,0028 \cdot 2 + 0,000196 \cdot 4 + 0,0042 \cdot 3 + 0,007 = 0,0259$$

Grid Localization: Robot faces a door example

Algorithm Discrete_Bayes_filter($\{p_{k,t-1}\}, u_t, z_t$):
   for all k do
      $\bar{p}_{k,t} = \sum_i p(X_t = x_k \mid u_t, X_{t-1} = x_i)\; p_{i,t-1}$
      $p_{k,t} = \eta\; p(z_t \mid X_t = x_k)\; \bar{p}_{k,t}$
   endfor
   return $\{p_{k,t}\}$



$$p_{2,t} = \sum_i p(X_t = x_2 \mid 1, X_{t-1} = x_i) p_{i,t-1}$$ K=2

$$p(X_t = x_2 \mid u_t = 1, X_{t-1} = x_1) p_{1,t-1} = 0{,}5 \cdot 0{,}014 = 0{,}007$$ i=1

$$p(X_t = x_2 \mid u_t = 1, X_{t-1} = x_2) p_{2,t-1} = 0{,}2 \cdot 0{,}014 = 0{,}0028$$ i=2

$$p(X_t = x_2 \mid u_t = 1, X_{t-1} = x_3) p_{3,t-1} = 0{,}014 \cdot 0{,}3 = 0{,}0042$$ i=3

$$p(X_t = x_2 \mid u_t = 1, X_{t-1} = x_4) p_{4,t-1} = 0{,}014 \cdot 0{,}3 = 0{,}0042$$ i=4

$$p(X_t = x_2 \mid u_t = 1, X_{t-1} = x_5) p_{5,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=5

$$p(X_t = x_2 \mid u_t = 1, X_{t-1} = x_6) p_{6,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=6

$$p(X_t = x_2 \mid u_t = 1, X_{t-1} = x_7) p_{7,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=7

$$p(X_t = x_2 \mid u_t = 1, X_{t-1} = x_8) p_{8,t-1} = 0{,}014 \cdot 0{,}3 = 0{,}0042$$ i=8

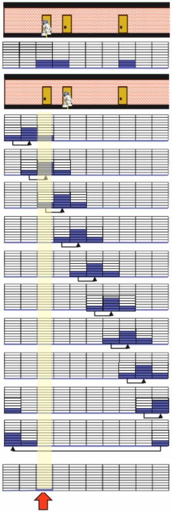
$$p(X_t = x_2 \mid u_t = 1, X_{t-1} = x_9) p_{9,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=9

$$p(X_t = x_2 \mid u_t = 1, X_{t-1} = x_{10}) p_{10,t-1} = 0{,}2 \cdot 0{,}014 = 0{,}0028$$ i=10

$$p_{1,t} = 0{,}0028 \cdot 2 + 0{,}000196 \cdot 4 + 0{,}0042 \cdot 3 + 0{,}007 = 0{,}0259$$

Grid Localization: Robot faces a door example

Algorithm Discrete_Bayes_filter$(\{p_{k,t-1}\}, u_t, z_t)$:
  for all k do
    $\bar{p}_{k,t} = \sum_i p(X_t = x_k \mid u_t, X_{t-1} = x_i)\, p_{i,t-1}$
    $p_{k,t} = \eta\, p(z_t \mid X_t = x_k)\, \bar{p}_{k,t}$
  endfor
  return $\{p_{k,t}\}$



$$p_{3,t} = \sum_i p(X_t = x_3 \mid 1, X_{t-1} = x_i)\, p_{i,t-1}$$ K=3

$$p(X_t = x_3 \mid u_t = 1, X_{t-1} = x_1)\, p_{1,t-1} = 0{,}2 \cdot 0{,}014 = 0{,}0028$$ i=1

$$p(X_t = x_3 \mid u_t = 1, X_{t-1} = x_2)\, p_{2,t-1} = 0{,}5 \cdot 0{,}014 = 0{,}007$$ i=2

$$p(X_t = x_3 \mid u_t = 1, X_{t-1} = x_3)\, p_{3,t-1} = 0{,}2 \cdot 0{,}3 = 0{,}06$$ i=3

$$p(X_t = x_3 \mid u_t = 1, X_{t-1} = x_4)\, p_{4,t-1} = 0{,}014 \cdot 0{,}3 = 0{,}0042$$ i=4

$$p(X_t = x_3 \mid u_t = 1, X_{t-1} = x_5)\, p_{5,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=5

$$p(X_t = x_3 \mid u_t = 1, X_{t-1} = x_6)\, p_{6,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=6

$$p(X_t = x_3 \mid u_t = 1, X_{t-1} = x_7)\, p_{7,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=7

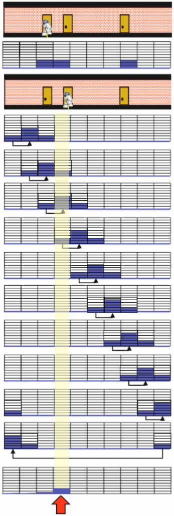
$$p(X_t = x_3 \mid u_t = 1, X_{t-1} = x_8)\, p_{8,t-1} = 0{,}014 \cdot 0{,}3 = 0{,}0042$$ i=8

$$p(X_t = x_3 \mid u_t = 1, X_{t-1} = x_9)\, p_{9,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=9

$$p(X_t = x_3 \mid u_t = 1, X_{t-1} = x_{10})\, p_{10,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=10

$$p_{3,t} = 0{,}0028 + 0{,}007 + 0{,}06 + 0{,}0042 \cdot 2 + 0{,}000196 \cdot 5 = 0{,}0792$$

Grid Localization: Robot faces a door example



Algorithm Discrete_Bayes_filter($\{p_{k,t-1}\}, u_t, z_t$):
  for all k do
    $\bar{p}_{k,t} = \sum_i p(X_t = x_k \mid u_t, X_{t-1} = x_i)\, p_{i,t-1}$
    $p_{k,t} = \eta\, p(z_t \mid X_t = x_k)\, \bar{p}_{k,t}$
  endfor
  return $\{p_{k,t}\}$

K=4

$$p_{4,t} = \sum_i p(X_t = x_4 \mid 1, X_{t-1} = x_i) p_{i,t-1}$$

i=1

$$p(X_t = x_4 \mid u_t = 1, X_{t-1} = x_1) p_{1,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$

i=2

$$p(X_t = x_4 \mid u_t = 1, X_{t-1} = x_2) p_{2,t-1} = 0{,}2 \cdot 0{,}014 = 0{,}0028$$

i=3

$$p(X_t = x_4 \mid u_t = 1, X_{t-1} = x_3) p_{3,t-1} = 0{,}5 \cdot 0{,}3 = 0{,}15$$

i=4

$$p(X_t = x_4 \mid u_t = 1, X_{t-1} = x_4) p_{4,t-1} = 0{,}2 \cdot 0{,}3 = 0{,}06$$

i=5

$$p(X_t = x_4 \mid u_t = 1, X_{t-1} = x_5) p_{5,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$

i=6

$$p(X_t = x_4 \mid u_t = 1, X_{t-1} = x_6) p_{6,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$

i=7

$$p(X_t = x_4 \mid u_t = 1, X_{t-1} = x_7) p_{7,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$

i=8

$$p(X_t = x_4 \mid u_t = 1, X_{t-1} = x_8) p_{8,t-1} = 0{,}014 \cdot 0{,}3 = 0{,}0042$$

i=9

$$p(X_t = x_4 \mid u_t = 1, X_{t-1} = x_9) p_{9,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$

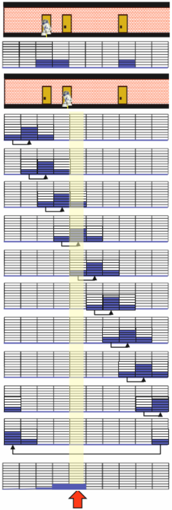
i=10

$$p(X_t = x_4 \mid u_t = 1, X_{t-1} = x_{10}) p_{10,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$

$$p_{4,t} = 0{,}000196 \cdot 6 + 0{,}0028 + 0{,}15 + 0{,}06 + 0{,}0042) = 0{,}2182$$

Grid Localization: Robot faces a door example

Algorithm Discrete_Bayes_filter($\{p_{k,t-1}\}, u_t, z_t$):
  for all k do
    $\bar{p}_{k,t} = \sum_i p(X_t = x_k \mid u_t, X_{t-1} = x_i)\; p_{i,t-1}$
    $p_{k,t} = \eta\; p(z_t \mid X_t = x_k)\; \bar{p}_{k,t}$
  endfor
  return $\{p_{k,t}\}$



$$p_{5,t} = \sum_i p(X_t = x_5 \mid 1, X_{t-1} = x_i)\, p_{i,t-1}$$

K=5

$p(X_t = x_5 \mid u_t = 1, X_{t-1} = x_1)\, p_{1,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$ i=1

$p(X_t = x_5 \mid u_t = 1, X_{t-1} = x_2)\, p_{2,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$ i=2

$p(X_t = x_5 \mid u_t = 1, X_{t-1} = x_3)\, p_{3,t-1} = 0{,}2 \cdot 0{,}3 = 0{,}06$ i=3

$p(X_t = x_5 \mid u_t = 1, X_{t-1} = x_4)\, p_{4,t-1} = 0{,}5 \cdot 0{,}3 = 0{,}15$ i=4

$p(X_t = x_5 \mid u_t = 1, X_{t-1} = x_5)\, p_{5,t-1} = 0{,}02 \cdot 0{,}014 = 0{,}0028$ i=5

$p(X_t = x_5 \mid u_t = 1, X_{t-1} = x_6)\, p_{6,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$ i=6

$p(X_t = x_5 \mid u_t = 1, X_{t-1} = x_7)\, p_{7,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$ i=7

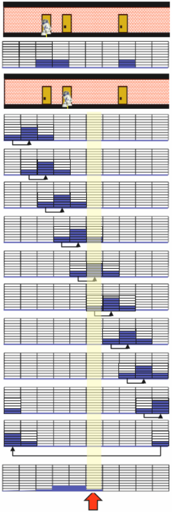
$p(X_t = x_5 \mid u_t = 1, X_{t-1} = x_8)\, p_{8,t-1} = 0{,}014 \cdot 0{,}3 = 0{,}0042$ i=8

$p(X_t = x_5 \mid u_t = 1, X_{t-1} = x_9)\, p_{9,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$ i=9

$p(X_t = x_5 \mid u_t = 1, X_{t-1} = x_{10})\, p_{10,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$ i=10

$$p_{5,t} = 0{,}000196 \cdot 6 + 0{,}0028 + 0{,}15 + 0{,}06 + 0{,}0042) = 0{,}2182$$

Grid Localization: Robot faces a door example



Algorithm Discrete_Bayes_filter($\{p_{k,t-1}\}, u_t, z_t$):
for all k do
$\bar{p}_{k,t} = \sum_i p(X_t = x_k \mid u_t, X_{t-1} = x_i)\, p_{i,t-1}$
$p_{k,t} = \eta\, p(z_t \mid X_t = x_k)\, \bar{p}_{k,t}$
endfor
return $\{p_{k,t}\}$

$$p_{6,t} = \sum_i p(X_t = x_6 \mid 1, X_{t-1} = x_i)\, p_{i,t-1}$$ K=6

$$p(X_t = x_6 \mid u_t = 1, X_{t-1} = x_1)\, p_{1,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=1

$$p(X_t = x_6 \mid u_t = 1, X_{t-1} = x_2)\, p_{2,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=2

$$p(X_t = x_6 \mid u_t = 1, X_{t-1} = x_3)\, p_{3,t-1} = 0{,}014 \cdot 0{,}3 = 0{,}0042$$ i=3

$$p(X_t = x_6 \mid u_t = 1, X_{t-1} = x_4)\, p_{4,t-1} = 0{,}2 \cdot 0{,}3 = 0{,}06$$ i=4

$$p(X_t = x_6 \mid u_t = 1, X_{t-1} = x_5)\, p_{5,t-1} = 0{,}5 \cdot 0{,}014 = 0{,}07$$ i=5

$$p(X_t = x_6 \mid u_t = 1, X_{t-1} = x_6)\, p_{6,t-1} = 0{,}2 \cdot 0{,}014 = 0{,}0028$$ i=6

$$p(X_t = x_6 \mid u_t = 1, X_{t-1} = x_7)\, p_{7,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=7

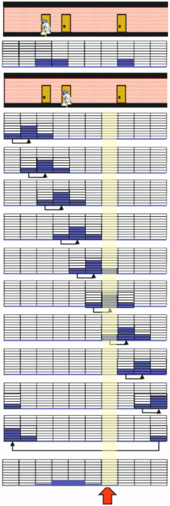
$$p(X_t = x_6 \mid u_t = 1, X_{t-1} = x_8)\, p_{8,t-1} = 0{,}014 \cdot 0{,}3 = 0{,}0042$$ i=8

$$p(X_t = x_6 \mid u_t = 1, X_{t-1} = x_9)\, p_{9,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=9

$$p(X_t = x_6 \mid u_t = 1, X_{t-1} = x_{10})\, p_{10,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=10

$$p_{6,t} = 0{,}000196 \cdot 5 + 0{,}0028 + 0{,}07 + 0{,}06 + 0{,}0042 \cdot 2 = 0.0792$$

```
Algorithm Discrete_Bayes_filter({p_{k,t-1}}, u_t, z_t):
    for all k do
        p̄_{k,t} = Σ_i p(X_t = x_k | u_t, X_{t-1} = x_i) p_{i,t-1}
        p_{k,t} = η p(z_t | X_t = x_k) p̄_{k,t}
    endfor
    return {p_{k,t}}
```



$$p_{7,t} = \sum_i p(X_t = x_7 \mid 1, X_{t-1} = x_i)\, p_{i,t-1}$$ K=7

$$p(X_t = x_7 \mid u_t = 1, X_{t-1} = x_1)\, p_{1,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=1

$$p(X_t = x_7 \mid u_t = 1, X_{t-1} = x_2)\, p_{2,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=2

$$p(X_t = x_7 \mid u_t = 1, X_{t-1} = x_3)\, p_{3,t-1} = 0{,}014 \cdot 0{,}3 = 0{,}0042$$ i=3

$$p(X_t = x_7 \mid u_t = 1, X_{t-1} = x_4)\, p_{4,t-1} = 0{,}014 \cdot 0{,}3 = 0{,}06$$ i=4

$$p(X_t = x_7 \mid u_t = 1, X_{t-1} = x_5)\, p_{5,t-1} = 0{,}2 \cdot 0{,}014 = 0{,}0028$$ i=5

$$p(X_t = x_7 \mid u_t = 1, X_{t-1} = x_6)\, p_{6,t-1} = 0{,}5 \cdot 0{,}014 = 0{,}007$$ i=6

$$p(X_t = x_7 \mid u_t = 1, X_{t-1} = x_7)\, p_{7,t-1} = 0{,}2 \cdot 0{,}014 = 0{,}0028$$ i=7

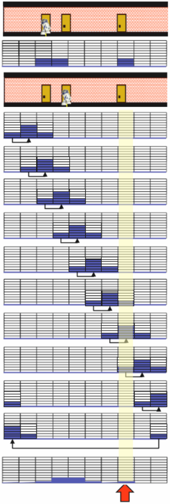
$$p(X_t = x_7 \mid u_t = 1, X_{t-1} = x_8)\, p_{8,t-1} = 0{,}014 \cdot 0{,}3 = 0{,}0042$$ i=8

$$p(X_t = x_7 \mid u_t = 1, X_{t-1} = x_9)\, p_{9,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=9

$$p(X_t = x_7 \mid u_t = 1, X_{t-1} = x_{10})\, p_{10,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=10

$$p_{7,t} = 0{,}000196 \cdot 4 + 0{,}0028 \cdot 2 + 0{,}007 + 0{,}06 + 0{,}0042 \cdot 2 = 0{,}0259$$

Grid Localization: Robot faces a door example



Algorithm Discrete_Bayes_filter$(\{p_{k,t-1}\}, u_t, z_t)$:
  for all k do
    $\bar{p}_{k,t} = \sum_i p(X_t = x_k \mid u_t, X_{t-1} = x_i)\, p_{i,t-1}$
    $p_{k,t} = \eta\, p(z_t \mid X_t = x_k)\, \bar{p}_{k,t}$
  endfor
  return $\{p_{k,t}\}$

$$p_{8,t} = \sum_i p(X_t = x_8 \mid 1, X_{t-1} = x_i)\, p_{i,t-1}$$ K=8

$$p(X_t = x_8 \mid u_t = 1, X_{t-1} = x_1)\, p_{1,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=1

$$p(X_t = x_8 \mid u_t = 1, X_{t-1} = x_2)\, p_{2,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=2

$$p(X_t = x_8 \mid u_t = 1, X_{t-1} = x_3)\, p_{3,t-1} = 0{,}014 \cdot 0{,}3 = 0{,}0042$$ i=3

$$p(X_t = x_8 \mid u_t = 1, X_{t-1} = x_4)\, p_{4,t-1} = 0{,}014 \cdot 0{,}3 = 0{,}0042$$ i=4

$$p(X_t = x_8 \mid u_t = 1, X_{t-1} = x_5)\, p_{5,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=5

$$p(X_t = x_8 \mid u_t = 1, X_{t-1} = x_6)\, p_{6,t-1} = 0{,}2 \cdot 0{,}014 = 0{,}0028$$ i=6

$$p(X_t = x_8 \mid u_t = 1, X_{t-1} = x_7)\, p_{7,t-1} = 0{,}5 \cdot 0{,}014 = 0{,}007$$ i=7

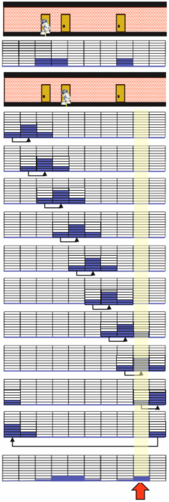
$$p(X_t = x_8 \mid u_t = 1, X_{t-1} = x_8)\, p_{8,t-1} = 0{,}2 \cdot 0{,}3 = 0{,}06$$ i=8

$$p(X_t = x_8 \mid u_t = 1, X_{t-1} = x_9)\, p_{9,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=9

$$p(X_t = x_8 \mid u_t = 1, X_{t-1} = x_{10})\, p_{10,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=10

$$p_{8,t} = 0{,}000196 \cdot 5 + 0{,}0028 + 0{,}007 + 0{,}06 \cdot 2 + 0{,}0042 \cdot 2 = 0{,}0792$$

Grid Localization: Robot faces a door example



Algorithm Discrete_Bayes_filter$(\{p_{k,t-1}\}, u_t, z_t)$:
- for all k do
  - $\bar{p}_{k,t} = \sum_i p(X_t = x_k \mid u_t, X_{t-1} = x_i)\, p_{i,t-1}$
  - $p_{k,t} = \eta\, p(z_t \mid X_t = x_k)\, \bar{p}_{k,t}$
- endfor
- return $\{p_{k,t}\}$

$$p_{9,t} = \sum_i p(X_t = x_9 \mid 1, X_{t-1} = x_i)\, p_{i,t-1}$$ K=9

$$p(X_t = x_9 \mid u_t = 1, X_{t-1} = x_1)\, p_{1,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=1

$$p(X_t = x_9 \mid u_t = 1, X_{t-1} = x_2)\, p_{2,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=2

$$p(X_t = x_9 \mid u_t = 1, X_{t-1} = x_3)\, p_{3,t-1} = 0{,}014 \cdot 0{,}3 = 0{,}0042$$ i=3

$$p(X_t = x_9 \mid u_t = 1, X_{t-1} = x_4)\, p_{4,t-1} = 0{,}014 \cdot 0{,}3 = 0{,}0042$$ i=4

$$p(X_t = x_9 \mid u_t = 1, X_{t-1} = x_5)\, p_{5,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=5

$$p(X_t = x_9 \mid u_t = 1, X_{t-1} = x_6)\, p_{6,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=6

$$p(X_t = x_9 \mid u_t = 1, X_{t-1} = x_7)\, p_{7,t-1} = 0{,}2 \cdot 0{,}014 = 0{,}0028$$ i=7

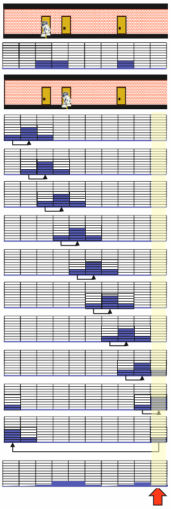
$$p(X_t = x_9 \mid u_t = 1, X_{t-1} = x_8)\, p_{8,t-1} = 0{,}5 \cdot 0{,}3 = 0{,}15$$ i=8

$$p(X_t = x_9 \mid u_t = 1, X_{t-1} = x_9)\, p_{9,t-1} = 0{,}2 \cdot 0{,}014 = 0{,}0028$$ i=9

$$p(X_t = x_9 \mid u_t = 1, X_{t-1} = x_{10})\, p_{10,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=10

$$p_{9,t} = 0{,}000196 \cdot 5 + 0{,}0028 \cdot 2 + 0.15 + 0{,}0042 \cdot 2 = 0{,}1649$$

Grid Localization: Robot faces a door example



Algorithm Discrete_Bayes_filter$(\{p_{k,t-1}\}, u_t, z_t)$:
for all k do
$\bar{p}_{k,t} = \sum_i p(X_t = x_k \mid u_t, X_{t-1} = x_i)\, p_{i,t-1}$
$p_{k,t} = \eta\, p(z_t \mid X_t = x_k)\, \bar{p}_{k,t}$
endfor
return $\{p_{k,t}\}$

$$p_{10,t} = \sum_i p(X_t = x_{10} \mid 1, X_{t-1} = x_i)\, p_{i,t-1}$$ k=10

$$p(X_t = x_{10} \mid u_t = 1, X_{t-1} = x_1)\, p_{1,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=1

$$p(X_t = x_{10} \mid u_t = 1, X_{t-1} = x_2)\, p_{2,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=2

$$p(X_t = x_{10} \mid u_t = 1, X_{t-1} = x_3)\, p_{3,t-1} = 0{,}014 \cdot 0{,}3 = 0{,}0042$$ i=3

$$p(X_t = x_{10} \mid u_t = 1, X_{t-1} = x_4)\, p_{4,t-1} = 0{,}014 \cdot 0{,}3 = 0{,}0042$$ i=4

$$p(X_t = x_{10} \mid u_t = 1, X_{t-1} = x_5)\, p_{5,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=5

$$p(X_t = x_{10} \mid u_t = 1, X_{t-1} = x_6)\, p_{6,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=6

$$p(X_t = x_{10} \mid u_t = 1, X_{t-1} = x_7)\, p_{7,t-1} = 0{,}014 \cdot 0{,}014 = 0{,}000196$$ i=7

$$p(X_t = x_{10} \mid u_t = 1, X_{t-1} = x_8)\, p_{8,t-1} = 0{,}2 \cdot 0{,}3 = 0{,}06$$ i=8

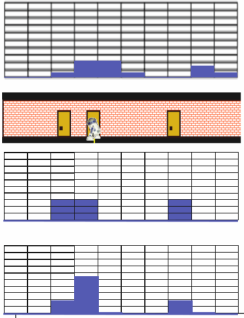
$$p(X_t = x_{10} \mid u_t = 1, X_{t-1} = x_9)\, p_{9,t-1} = 0{,}5 \cdot 0{,}014 = 0{,}007$$ i=9

$$p(X_t = x_{10} \mid u_t = 1, X_{t-1} = x_{10})\, p_{10,t-1} = 0{,}2 \cdot 0{,}014 = 0{,}0028$$ i=10

$$p_{10,t} = 0{,}000196 \cdot 5 + 0{,}0028 + 0.06 + 0{,}007 + 0{,}0042 \cdot 2 = 0{,}0792$$

Grid Localization: Robot faces a door example

Algorithm Discrete_Bayes_filter($\{p_{k,t-1}\}, u_t, z_t$):
  for all k do
    $\bar{p}_{k,t} = \sum_i p(X_t = x_k \mid u_t, X_{t-1} = x_i)\, p_{i,t-1}$
    $p_{k,t} = \eta\, p(z_t \mid X_t = x_k)\, \bar{p}_{k,t}$
  endfor
  return $\{p_{k,t}\}$



$$p_{k,t} = \eta p\left(z_t \middle| X_t = x_k\right)\bar{p}_{k,t}$$

$$\eta = \frac{1}{0.0011 \cdot 2 + 0.0023 + 0.0238 \cdot 2 + (3.6e-4) \cdot 3 + 0.0031 + 0.0655} = 8.2183$$

$$\left.\begin{aligned}
p_{10,t} &= \eta p\left(z_t \middle| X_t = x_{10}\right)\bar{p}_{10,t} = 0{,}014 \cdot 0{,}0792\eta = 0.0011\eta \\
p_{9,t} &= \eta p\left(z_t \middle| X_t = x_9\right)\bar{p}_{9,t} = 0{,}014 \cdot 0{,}1649\eta = 0.0023\eta \\
p_{8,t} &= \eta p\left(z_t \middle| X_t = x_8\right)\bar{p}_{8,t} = 0{,}3 \cdot 0{,}0792\eta = 0{,}0238\eta \\
p_{7,t} &= \eta p\left(z_t \middle| X_t = x_7\right)\bar{p}_{7,t} = 0{,}014 \cdot 0{,}0259\eta = 3{,}6e-4\eta \\
p_{6,t} &= \eta p\left(z_t \middle| X_t = x_6\right)\bar{p}_{6,t} = 0{,}014 \cdot 0{,}0792\eta = 0{,}0011\eta \\
p_{5,t} &= \eta p\left(z_t \middle| X_t = x_5\right)\bar{p}_{5,t} = 0{,}014 \cdot 0{,}2182\eta = 0{,}0031\eta \\
p_{4,t} &= \eta p\left(z_t \middle| X_t = x_4\right)\bar{p}_{4,t} = 0{,}3 \cdot 0{,}2182\eta = 0{,}0655\eta \\
p_{3,t} &= \eta p\left(z_t \middle| X_t = x_3\right)\bar{p}_{3,t} = 0{,}3 \cdot 0{,}0792\eta = 0{,}0238\eta \\
p_{2,t} &= \eta p\left(z_t \middle| X_t = x_2\right)\bar{p}_{2,t} = 0{,}014 \cdot 0{,}259\eta = 3{,}6e-4\eta \\
p_{1,t} &= \eta p\left(z_t \middle| X_t = x_1\right)\bar{p}_{1,t} = 0{,}014 \cdot 0{,}0259\eta = 3{,}6e-4\eta
\end{aligned}\right\} \Rightarrow \left\{\begin{aligned}
p_{10,t} &= 0.0011\eta = 0.009 \\
p_{9,t} &= 0.0023\eta = 0{,}0189 \\
p_{8,t} &= 0{,}0238\eta = 0{,}1956 \\
p_{7,t} &= 3{,}6e-4\eta = 0{,}003 \\
p_{6,t} &= 0{,}0011\eta = 0{,}009 \\
p_{5,t} &= 0{,}0031\eta = 0{,}0255 \\
p_{4,t} &= 0{,}0655\eta = 0{,}5383 \\
p_{3,t} &= 0{,}0238\eta = 0{,}1956 \\
p_{2,t} &= 3{,}6e-4\eta = 0{,}003 \\
p_{1,t} &= 3{,}6e-4\eta = 0{,}003
\end{aligned}\right.$$

Grid Localization: Real-Environment example

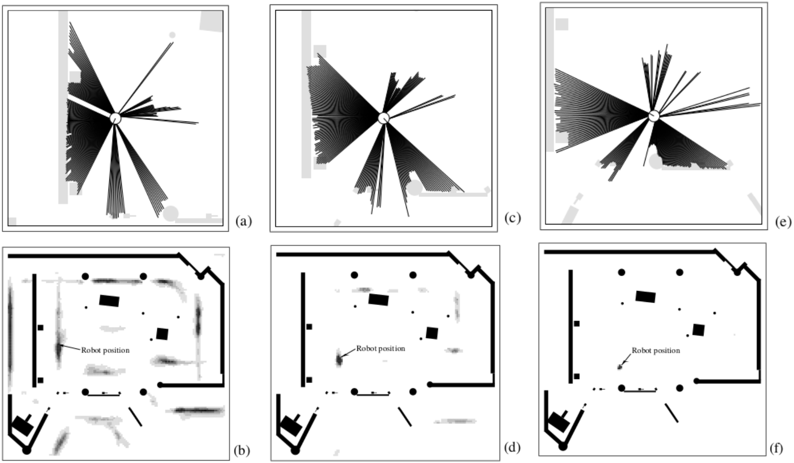


Figure: Global localization using laserdata. (a) Scan of the laser range-finders taken at the start position of the robot. (b) shows the situation after incorporating this laser scan, starting with the uniform distribution. (c) Second scan and (d) resulting belief. After integrating the final scan shown in (e), the robot's belief is centered at its actual location (see (f)).

Monte Carlo Localization

Monte Carlo Localization (MCL)

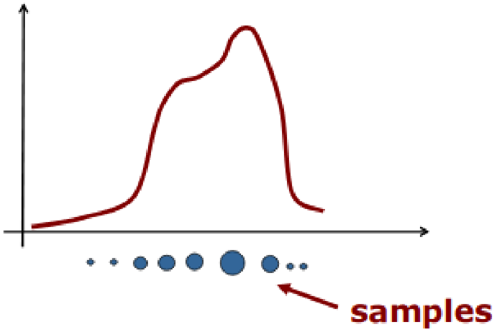
- MCL is applicable to both local and global localization problems.
- MCL one of the most popular localization algorithms in robotics.
- It is easy to implement, and tends to work well across a broad range of localization problems.

Monte Carlo Localization (MCL)

- The algorithm uses a **particle filter** to represent the distribution of likely states, with each particle representing a possible state, i.e., a hypothesis of where the robot is.
- PDF is represented by a set of samples.
- The higher the density of the particles, the higher the probability.
- Initial position is unknown.

Particle filter: Key Idea

- Use multiple weighted samples to represent arbitrary distributions.



- This is only an approximation.
- A sufficient number of samples is needed.

Particle filter: Key Idea

- Particle representation is a set of weighted samples.

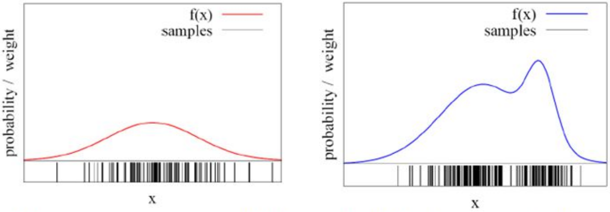
$$\mathcal{X} = \{\langle x^{[i]}, w^{[i]} \rangle\}_{i=1,\ldots N}$$

  where x_i is the *state hypothesis* (it can be a vector) and w_i is the *importance weight* (a real number).
- The samples represent the posterior

$$p(x) = \sum_{i=1}^{N} w^{[j]} \delta_{x[j]}(x)$$

  parameteric representation where δ is the Dirac function.

Particle filter: Key Idea

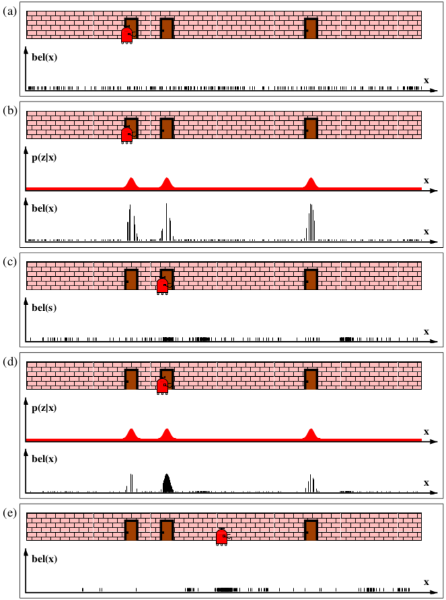


- The more particles fall into a region, the higher the probability of the region.

Particle filter: Steps

1. A uniform random distribution of particles are spread over the configuration space (the robot has no information about where it is and assumes it is equally likely to be at any point in space).
2. The robot moves, it shifts the particles to predict its new state after the movement.
3. The robot senses something, the particles are resampled based on recursive Bayesian estimation, i.e., how well the actual sensed data correlate with the predicted state
4. Ultimately, the particles should converge towards the actual position of the robot.

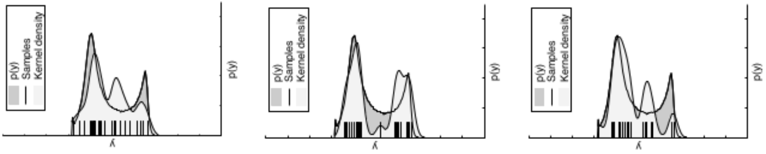
Particle filter: Steps



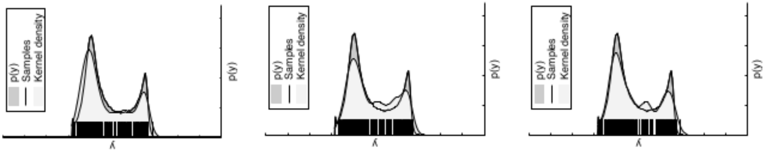
Particle filter

The more samples you use, the better the approximation. Here is an example of 3 pdf approximations with 25 and 250 random samples.

- Observe the variance in the PDF approximation due to random samples.



25 samples



250 samples

Particle filter - the algorithm

Algorithm Particle_filter$(\mathcal{X}_{t-1}, u_t, z_t)$**:**
  $\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$
  for $m = 1$ to M do
    sample $x_t^{[m]} \sim p(x_t \mid u_t, x_{t-1}^{[m]})$
    $w_t^{[m]} = p(z_t \mid x_t^{[m]})$
    $\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$
  endfor
  for $m = 1$ to M do
    draw i with probability $\propto w_t^{[i]}$
    add $x_t^{[i]}$ to $\mathcal{X}_t$
  endfor
  return $\mathcal{X}_t$

Algorithm MCL$(\mathcal{X}_{t-1}, u_t, z_t, m)$**:**
  $\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$
  for $m = 1$ to M do
    $x_t^{[m]} =$ **sample_motion_model**$(u_t, x_{t-1}^{[m]})$
    $w_t^{[m]} =$ **measurement_model**$(z_t, x_t^{[m]}, m)$
    $\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$
  endfor
  for $m = 1$ to M do
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- **PF algorithm**

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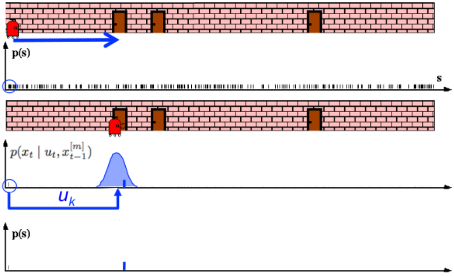

Particle filter - the algorithm

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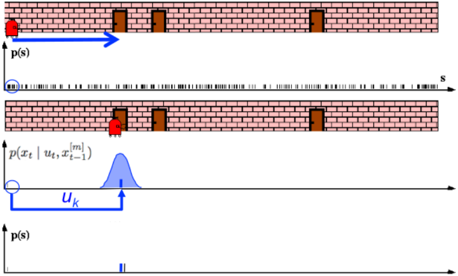

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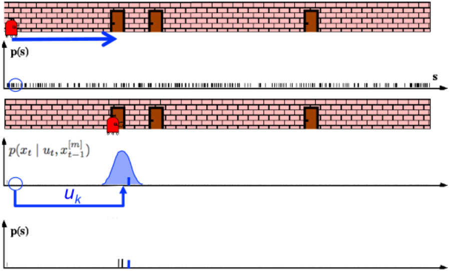

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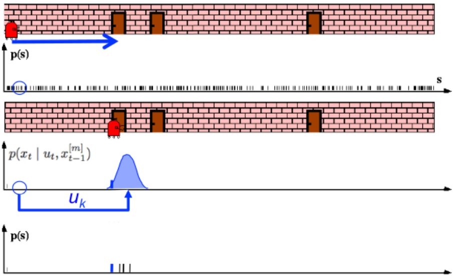

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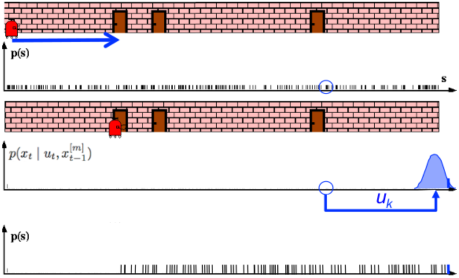

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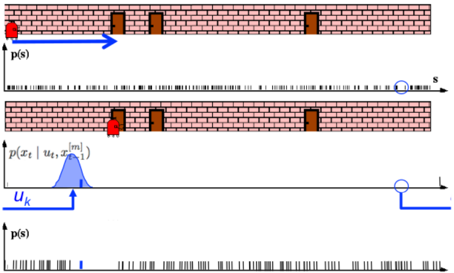

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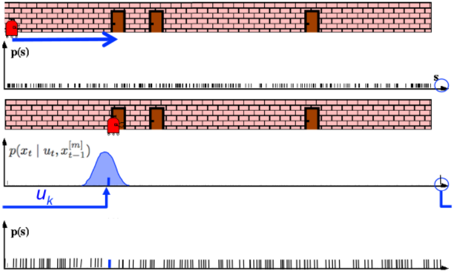

$x_t = x_{t-1} + u_t + w_t$
$w_t = N(0, \sigma_w)$

Particle filter - the algorithm

- **PF algorithm**

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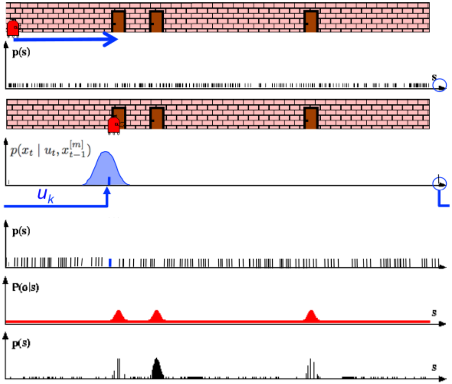

Importance Factor

- **PF algorithm**

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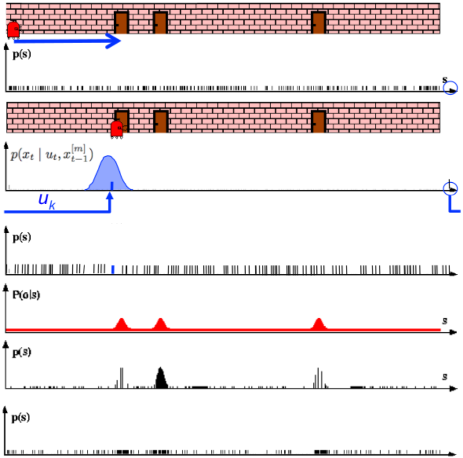
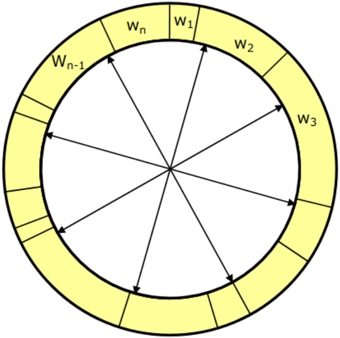
Resampling



Particle filter: Resampling

- Survival of the fittest: Replace unlikely samples by more likely ones
- ”Trick” to avoid that many samples cover unlikely states!
- Needed as we have a limited number of samples.

Particle filter: Resampling



- Stochastic universal sampling
- Systematic resampling
- Linear time complexity
- Easy to implement, low variance

Low_variance_resampling$(\mathcal{X}_t, \mathcal{W}_t)$**:**

$\bar{\mathcal{X}}_t = \emptyset$

$r = \text{rand}(0; J^{-1})$

$c = w_t^{[1]}$

$i = 1$

for $j = 1$ *to* J *do*

  $U = r + (j-1)J^{-1}$

  while $U > c$

    $i = i + 1$

    $c = c + w_t^{[i]}$

  endwhile

  add $x_t^{[i]}$ *to* $\bar{\mathcal{X}}_t$

endfor

return $\bar{\mathcal{X}}_t$

Questions?