

Abstract Interpretation

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If we want to analyze the behavior of a program, we typically perform some kind of *abstraction*. That is, we approximate the concrete semantics of the program in some way that is sufficient for analysis. *Abstract interpretation* provides a formal framework for defining and performing these types of program abstractions.

Background

A *lattice* $L = (S, \sqsubseteq)$ is a partially ordered set where each pair of elements has a least upper bound (i.e. *join* \sqcup) and a greatest lower bound (i.e. *meet*, \sqcap).

Abstraction Domains

An *abstraction domain* is defined as follows, where D is a *concrete domain* and \hat{D} is an *abstract domain*, and elements of \hat{D} form a lattice:

- $\gamma : \hat{D} \rightarrow D$: **Concretization function** that maps abstract values to sets of concrete elements.
- $\alpha : D \rightarrow \hat{D}$: **Abstraction function** that maps sets of concrete elements to the most precise value in the abstract domain.

where α and γ must form a *Galois connection*. That is, they satisfy the following condition

$$\forall x \in D, \forall \hat{x} \in \hat{D} : \alpha(x) \sqsubseteq \hat{x} \Leftrightarrow x \sqsubseteq \gamma(\hat{x})$$

Intuitively, this means that the abstraction and concretization functions respect the orderings of D and \hat{D} . That is, if $\alpha(x)$, the abstraction of x , is ordered before some other $\hat{x} \in \hat{D}$, then x should be ordered before $\gamma(\hat{x})$, the concretization of \hat{x} .

Note that we also have *abstract transformers*, which are functions $T : \hat{D} \rightarrow \hat{D}$, and which are *sound* if they respect the over-approximation property i.e., if an original abstract element d is an overapproximation of its concretization, then after applying the transformer to d , it will remain an overapproximation of the concretization of the transformed value. TODO: Make this clearer.