

# Weakest Preconditions

William Schultz

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The notion of *weakest precondition* in program analysis derives from the work of Dijkstra [1]. He introduced the *Guarded Command Language* as a simple modeling language for program specification e.g.

Dijkstra introduced the Guarded Command Language (GCL) with the grammar

$$\begin{aligned} S ::= & \text{skip} \mid x := E \mid S_1; S_2 \\ & \mid \text{if } B_1 \rightarrow S_1 \parallel B_2 \rightarrow S_2 \parallel \dots \parallel B_n \rightarrow S_n \text{ fi} \\ & \mid \text{do } B_1 \rightarrow S_1 \parallel B_2 \rightarrow S_2 \parallel \dots \parallel B_n \rightarrow S_n \text{ od} \end{aligned}$$

where the  $B_i$  are Boolean expressions. The  $B_i$  are called *guards* because they guard the corresponding statements  $S_i$ . The symbol  $\parallel$  is the *nondeterministic choice operator* and is not to be confused with  $|$ . In if and do statements, a clause  $B_i \rightarrow S_i$  is said to be *enabled* if its guard  $B_i$  is true.

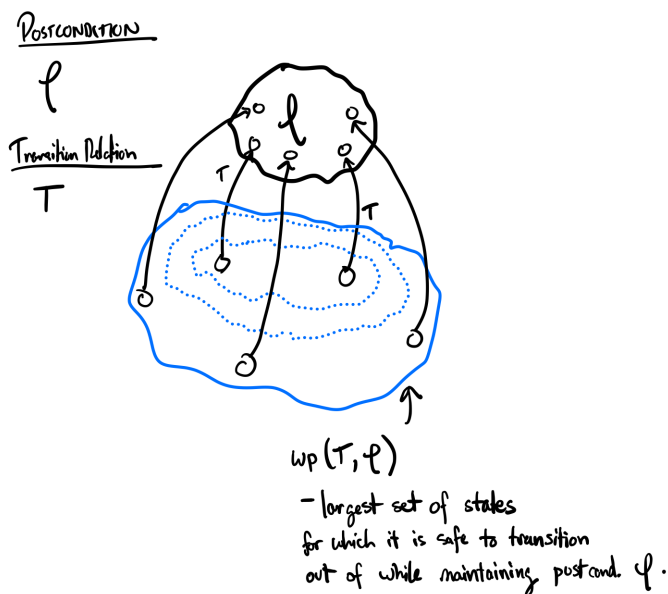
So, given a program  $S$  and a postcondition  $\varphi$ , we define the *weakest precondition* as the weakest property of the input state that guarantees that  $S$  will terminate with the postcondition  $\varphi$ , denoted  $wp(S, \varphi)$ . In the definition of GCL, we can provide definitions of how to compute the weakest precondition for various program statements. For example, for an assignment statement  $x := E$ , we have that

$$wp(x := E, \varphi) \equiv \varphi\{E/x\}$$

where  $\varphi\{E/x\}$  represents the property  $\varphi$  with appearances of  $x$  in  $\varphi$  replaced with  $E$ . For example,

$$\begin{aligned} wp(x := x + 1, x = 3) &\equiv (x = 3)\{x + 1/x\} \\ &\equiv (x + 1) = 3 \\ &\equiv x = 2 \end{aligned}$$

From a more semantic perspective, we can also think about weakest preconditions in terms of symbolic transition relations. If we have a transition relation  $T$  and a postcondition  $\varphi$  (i.e. a state predicate), the weakest precondition of  $T$  with respect to  $\varphi$  is the weakest predicate  $P$  (in other words, the largest set of states) such that a transition out of any state in  $P$  will uphold the property  $\varphi$ .



We can also consider weakest precondition computation as a kind of backwards symbolic execution. That is, we start from a given postcondition predicate, and a given transition relation, and execute the transition relation backwards to compute the states contained in the weakest precondition.

## References

- [1] Edsger W. Dijkstra. Guarded commands, nondeterminacy and formal derivation of programs. *Commun. ACM*, 18(8):453–457, aug 1975.