Probability and Stats

William Schultz

December 2, 2023

1 Random Variables

We can model probabilistic events based on *outcomes*, which are the outcome of an experiment or a trial (e.g., getting a "3" on the roll of a die), and a *sample space*, which is a set of possible outcomes, and *events*, which are a subset of the sample space.

A **random variable** is then, formally, simply a mapping from outcomes in the sample space to the set of real numbers. For example consider a 6-sided die with sides $\{1, 2, 3, 4, 5, 6\}$. Some possible outcomes are 1, 3, or 5, and the sample space is $\{1, 2, 3, 4, 5, 6\}$. The probability of each outcome for a fair die, deteremined by some random variable, is $1/6$. We can define an event A representing the case that the roll is odd i.e., $A = \{1, 3, 5\}$.

Expected Value

The **expected value** of a random variable X can be thought of as the "average" or "mean" value attained by that random variable. Formally, expected value is defined as

$$E[X] = \sum_x x * P(X = x)$$

for every outcome x in the sample space of X. For example, for a fair, 6-sided die, the expected value is

$$\frac{1}{6} * 1 + \frac{1}{6} * 2 + \frac{1}{6} * 3 + \frac{1}{6} * 4 + \frac{1}{6} * 5 + \frac{1}{6} * 6 = 3.5$$

Or if we have, say, a 4-sided die where each roll has some "payoff", e.g.

- $1 \mapsto 1$
- $2 \mapsto 2$
- $3 \mapsto 3$
- $4 \mapsto 10$

our expected payoff from rolling this die is

$$\frac{1}{4} * 1 + \frac{1}{4} * 2 + \frac{1}{4} * 3 + \frac{1}{4} * 10 = 4$$