Program Synthesis

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In general, we can present the *synthesis* problem in contrast to the verification problem as follows:

- The **verification problem**: given system M and spec φ, check that $M \models \varphi$.
- The **synthesis problem**: given spec φ, find M such that $M \models \varphi$.

0.1 Functional Synthesis

The classic, *functional synthesis* problem, is defined with respect to programs that take some input x and transform it to output y. In this setting, if we are given a specification φ that prescribes the desired input/output relation, we can construct a program by means of establishing validity of the theorem

$$\forall x \, \exists y : \varphi(x, y)$$

Note that this is equivalent to the second order formula

$$\exists f, \forall x : \varphi(x, f(x))$$

where f is a concrete function that takes input x and returns the correct output y satisfying specification φ [PR89, WL69]. If we have a constructive way to prove this theorem, then we can construct f, from which we can construct a program that satisfies φ. This approach is also referred to as *deductive synthesis* [MW80].

0.2 Reactive Synthesis

The above approach is suitable for sequential programs, but if we want to move to concurrent programs, then we need a more expressive specification language in which to express φ. Temporal logic became the natural choice for this and the works of [CE82, MW84] essentially do concurrent program synthesis by showing satisfiability of of a particular temporal formula specification φ, and then using the model satisfying φ to construct a program implementing φ.

In [PR89], however, they claim that the approach taken in [CE82, MW84] is not quite sufficient, because it assumes that we are trying to synthesize *closed* systems. That is, systems for which we have full control over every component. They claim that if, for example, we aree synthesizing a system with two components, C_1 and C_2, the "hidden assumption" in [CE82] is that we have the power to construct both C_1 and C_2 in a way that will ensure the needed cooperation. But, if we are in a so-called *open system* setting, then C_1, for example, may represent the *environment* over which the implementor has no control, while C_2 is the body of the system itself (which we may refer to as a *reactive module*). In this case, we instead have to synthesize C_2 in such a way that it will work correctly in response to any possible behaviors of the environment C_1.

For example, if C_1 is a module that can only modify x (a shared variable for communication), and C_2 can only modify y, then they claim the synthesis problem should instead be stated as

$$\forall x \, \exists y : \varphi(x, y)$$

which they refer to as the *reactive synthesis* problem. Note that in the formal statement above, we should now interpret x and y as being quantified over behaviors of the computation, since we are now interpreting it over temporal logic. So, the statement is saying that for any possible sequence of values x (that can be produced by the environment C_1), there exists a sequence of values y (produced by the controllable system C_2) such that $\varphi(x, y)$ holds. They note that the approach of [CE82] can be viewed as a solution to the alternate problem statement $\exists x \, \exists y : \varphi(x, y)$.

0.3 Temporal Synthesis

Temporal synthesis considers specifications given in the form of LTL (or CTL), for example. An initial approach was to use satisfiability of a temporal formula as a way to derive M [CE82]. See also [MW84].

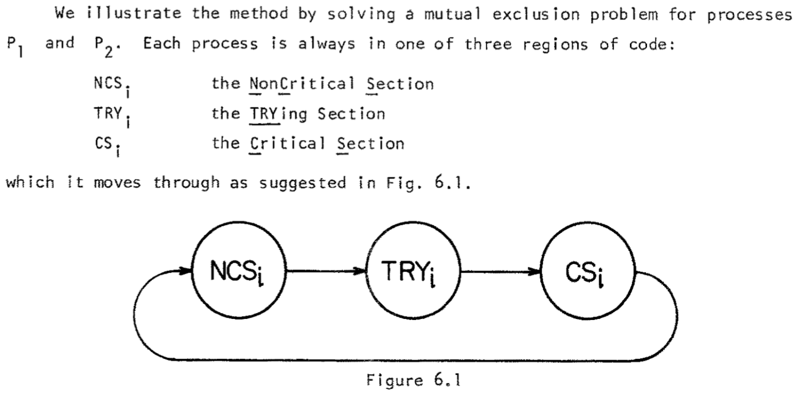
In [CE82] they consider concurrent systems consisting of a finite number of fixed processes $P_1, \ldots, P_m$ running in parallel. They treat parallelism in the usual sense i.e. nondeterministic interleaving of the sequential atomic actions of each process. They use CTL as a specification language, and consider the semantics of CTL with respect to a (Kripke) structure $M = (S, A_1, \ldots, A_k, L)$, where

- S: countable set of system states
- $A_i \subseteq S \times S$: transition relation of process i
- L: assignment of atomic propositions to each state

They use a decision procedure for satisfiability of CTL formulae (similar to one described in [BAMP81]) as part of their synthesis procedure. Given a CTL formula f_0, the decision procedure returns either "Yes, f_0 is satisfiable or "No, f_0 is unsatisfiable". If f_0 is satisfiable, then a finite model (structure) is also constructed. Their overall synthesis algorithm consists of the following high level steps:

1. Specify the desired behavior of the concurrent system using a CTL formula φ.
2. Apply the decision procedure to the formula φ to obtain a finite model of the formula.
3. Factor out the synchronization skeletons of the individual processes from the global system flowgraph defined by the model.

They demonstrate this procedure on a simple, 2 process mutual exclusion example. Below is shown the description of the abstract states of each process, $\{NCS_i, TRY_i, CS_i\}$:



We illustrate the method by solving a mutual exclusion problem for processes P_1 and P_2. Each process is always in one of three regions of code:

NCS_i the NonCritical Section
TRY_i the TRYing Section
CS_i the Critical Section

which it moves through as suggested in Fig. 6.1.

NCS$_i$
TRY$_i$
CS$_i$

Figure 6.1

and they give the specification of the mutual exclusion problem in CTL as follows:

1. start state

   $NCS_1 \wedge NCS_2$

2. mutual exclusion

   $AG(\sim(CS_1 \wedge CS_2))$

3. absence of starvation for P_i

   $AG(TRY_i \rightarrow AF\ CS_i)$

4. each process P_i is always in exactly one of the three code regions

   $AG(NCS_i \vee TRY_i \vee CS_i)$

   $AG(NCS_i \rightarrow \sim(TRY_i \vee CS_i))$

   $AG(TRY_i \rightarrow \sim(NCS_i \vee CS_i))$

   $AG(CS_i \rightarrow \sim(NCS_i \vee TRY_i))$

5. it is always possible for P_i to enter its trying region from its noncritical region

   $AG(NCS_i \rightarrow EX_i TRY_i)$

6. it is always the case that any move P_i makes from its trying region is into the critical region

   $AG(TRY_i \wedge EX_i True \rightarrow AX_i CS_i)$

7. it is always possible for P_i to re-enter its noncritical region from its critical region

   $AG(CS_i \rightarrow EX_i NCS_i)$

8. a transition by one process cannot cause a move by the other

   $AG(NCS_i \rightarrow AX_j NCS_i)$

   $AG(TRY_i \rightarrow AX_j TRY_i)$

   $AG(CS_i \rightarrow AX_j CS_i)$

9. some process can always move

   $AG(EX\ true)$

From this they then construct a tableau T using their decision procedure:

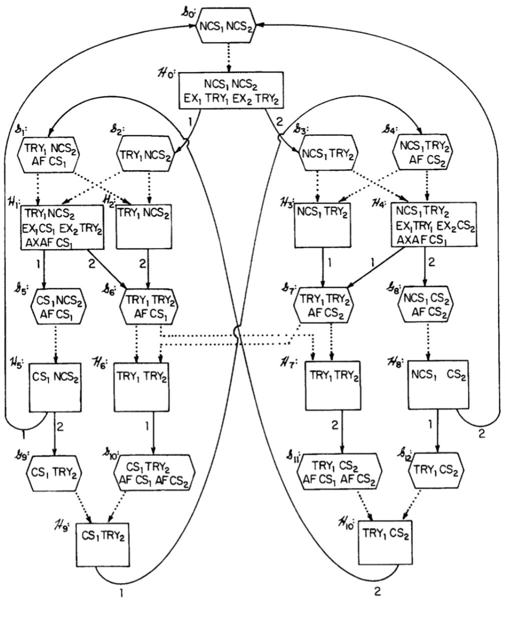

$\mathcal{D}_0$: NCS₁ NCS₂
$\mathcal{H}_0$: NCS₁ NCS₂ EX₁ TRY₁ EX₂ TRY₂
1
2
$\mathcal{D}_1$: TRY₁ NCS₂ AF CS₁
$\mathcal{D}_2$: TRY₁ NCS₂
$\mathcal{D}_3$: NCS₁ TRY₂
$\mathcal{D}_4$: NCS₁ TRY₂ AF CS₂
$\mathcal{H}_1$: TRY₁ NCS₂ EX₁ CS₁ EX₂ TRY₂ AXAF CS₁
$\mathcal{H}_2$: TRY₁ NCS₂
$\mathcal{H}_3$: NCS₁ TRY₂
$\mathcal{H}_4$: NCS₁ TRY₂ EX₁ TRY₁ EX₂ CS₂ AXAF CS₁
$\mathcal{D}_5$: CS₁ NCS₂ AF CS₁
$\mathcal{D}_6$: TRY₁ TRY₂ AF CS₁
$\mathcal{D}_7$: TRY₁ TRY₂ AF CS₂
$\mathcal{D}_8$: NCS₁ CS₂ AF CS₂
$\mathcal{H}_5$: CS₁ NCS₂
$\mathcal{H}_6$: TRY₁ TRY₂
$\mathcal{H}_7$: TRY₁ TRY₂
$\mathcal{H}_8$: NCS₁ CS₂
$\mathcal{D}_9$: CS₁ TRY₂
$\mathcal{D}_{10}$: CS₁ TRY₂ AF CS₁ AF CS₂
$\mathcal{D}_{11}$: TRY₁ CS₂ AF CS₁ AF CS₂
$\mathcal{D}_{12}$: TRY₁ CS₂
$\mathcal{H}_9$: CS₁ TRY₂
$\mathcal{H}_{10}$: TRY₁ CS₂

Figure 6.2

and then from T they extract a finite model of the global program behavior:

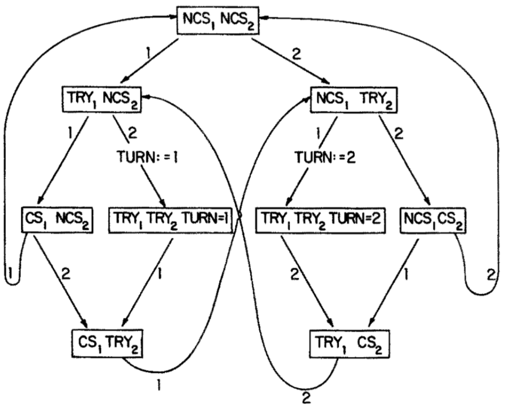


Figure 6.6

Note they manually introduced an auxiliary variable $TURN$ in order to distinguish states H_6 and H_7 in the tableau, which carries over into the extracted model.

After constructing the model representing the global program behavior, they extract "skeletons" for each individual process, which they seem to describe in a somewhat ad hoc manner i.e. they don't give a formal algorithmic procedure for this. Note that this is pointed out in [AE01], which appears to give a more formal treatment of this extraction procedure. The final, extracted skeletons for process P_1 and P_2 are shown as follows:

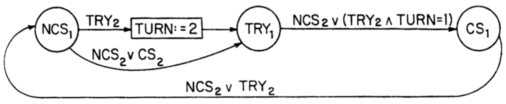


Figure 6.7 (a)

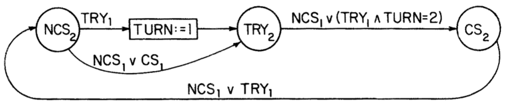


Figure 6.7 (b)

References

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