Abstract Interpretation

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If we want to analyze the behavior of a program, we typically perform some kind of *abstraction*. That is, we approximate the concrete semantics of the program in some way that is sufficient for analysis. *Abstract interpretation* provides a formal framework for defining and performing these types of program abstractions.

An *abstraction domain* is defined as follows:

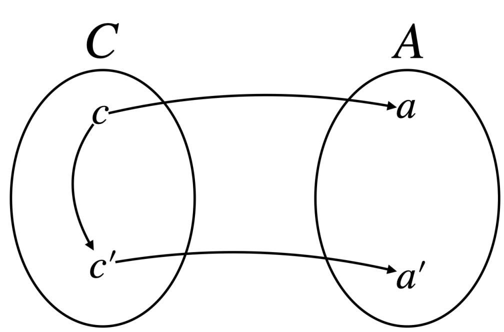
- C is a *concrete domain* i.e., a set of elements with an associated partial order $\leq$.
- A is an *abstract domain* with associated partial order $\sqsubseteq$ (elements of A form a lattice).

We then define mappings between these two sets:

- $\gamma : A \to C$: A **concretization function** that maps abstract values to sets of concrete elements.
- $\alpha : C \to A$: An **abstraction function** that maps sets of concrete elements to the most precise value in the abstract domain.

where α and γ must form a *Galois connection*. That is, they satisfy the following condition

$$\forall c \in C, \forall a \in A : \alpha(x) \sqsubseteq a \Leftrightarrow c \sqsubseteq \gamma(a)$$



TODO

Note that a *lattice* $L = (S, \sqsubseteq)$ is a partially ordered set where each pair of elements has a least upper bound (i.e. *join* $\sqcup$) and a greatest lower bound (i.e. *meet*, $\sqcap$).

Intuitively, the above requirements means that the abstraction and concretization functions respect the orderings of D and A. That is, if $\alpha(x)$, the abstraction of x, is ordered before some other $\hat{x} \in A$, then x should be ordered before $\gamma(\hat{x})$, the concretization of $\hat{x}$.