# SAT Solving with Conflict Driven Clause Learning

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CS 7240 Final Project

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# Overview and Project Goals

- Satisfiability is the canonical NP-complete problem.
- Much work has been devoted to building efficient SAT solvers over last decades.
- **Project Goal:** Implement a basic SAT solver based on *conflict driven clause learning* (CDCL), the dominant core technique used in modern solvers.
  - Gain a deeper understanding of the DPLL and CDCL based algorithms for SAT solving
  - Use as a platform for potentially exploring new SAT solving techniques
  - E.g. learning heuristics using a data-driven approach, extending methods of *CrystalBall* [SKM19]

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CNF notation:

$$\{\{x_1, x_2\}, \{\neg x_3, \neg x_1\}\}$$

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- Basic idea of DPLL is to do a depth first, brute force search with backtracking along with some basic formula simplification as you go.
  - Also employs the *unit propagation rule*

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$$\{\} \quad \text{(SAT)}$$

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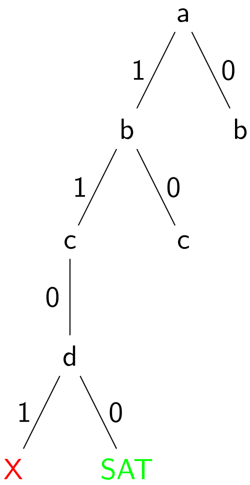
$\{\neg b, \neg c\}$

$\{c, \neg d\}$



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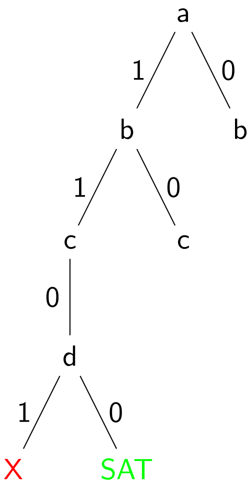

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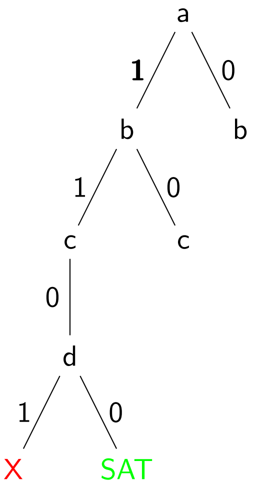

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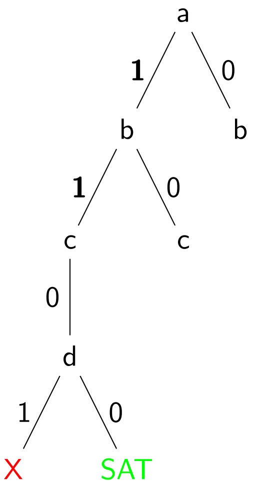

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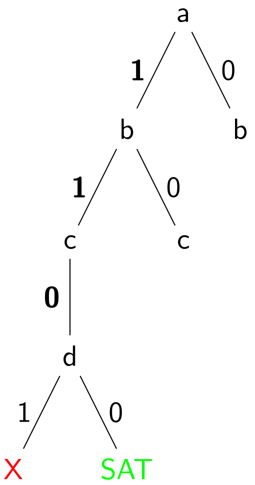

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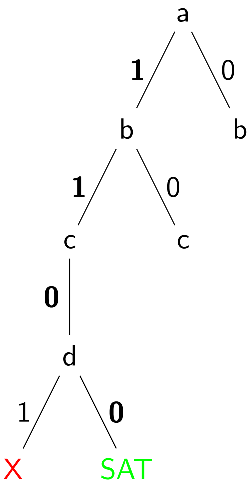

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- In addition, employ *non-chronological backtracking*

# CDCL Example

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| $c_1$ | $\{a, b\}$ |
| $c_2$ | $\{b, c\}$ |
| $c_3$ | $\{\neg a, \neg x, y\}$ |
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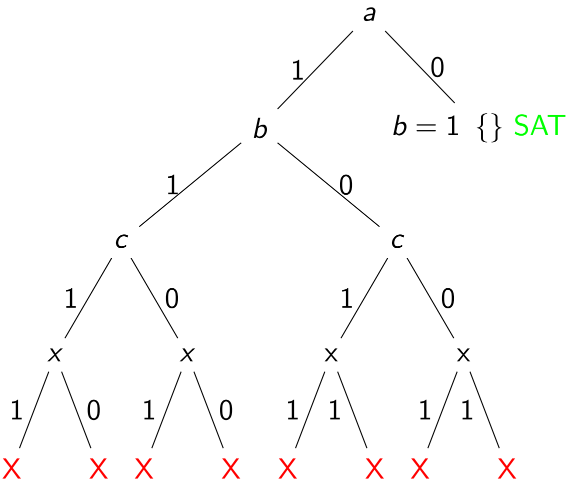


Figure: Basic DPLL termination tree. Explores large portion of left search tree.

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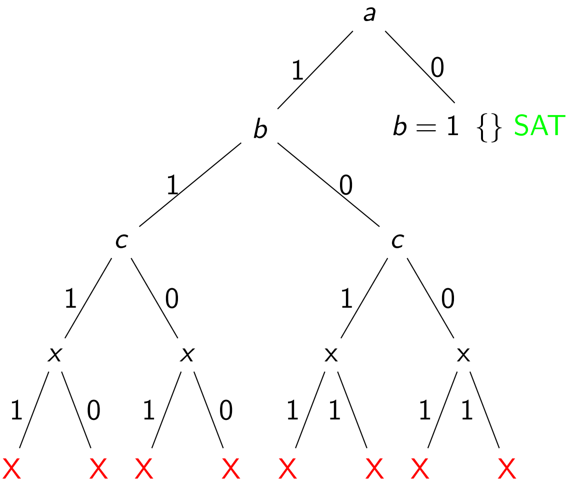


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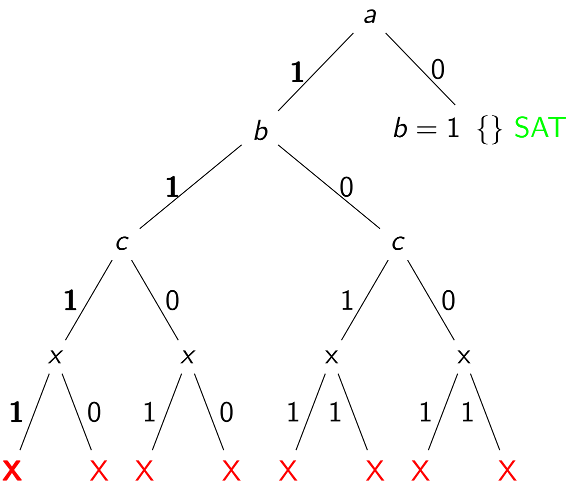


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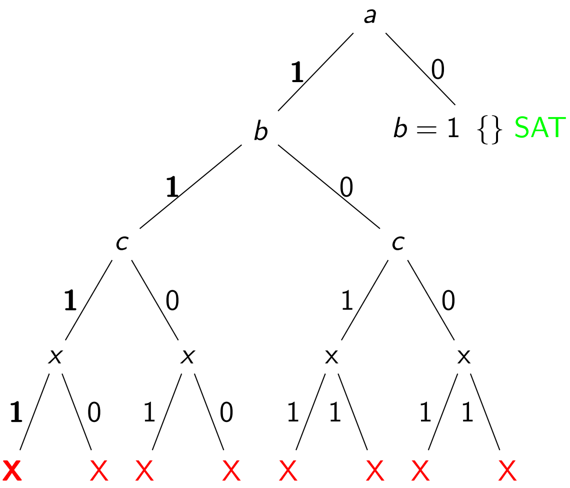
unit propagate $y$ ($c_3$)

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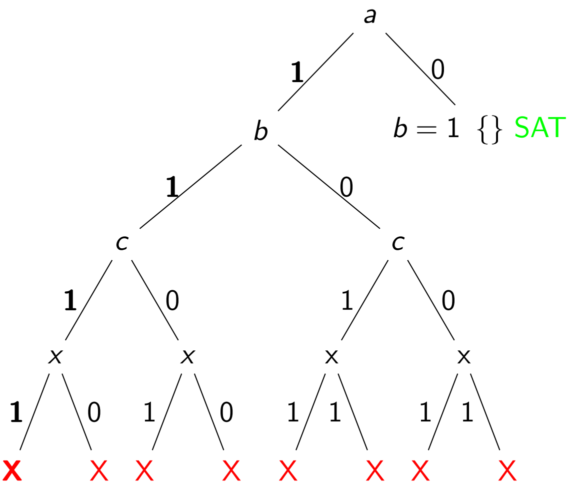
unit propagate $z$ ($c_5$)
Conflict!

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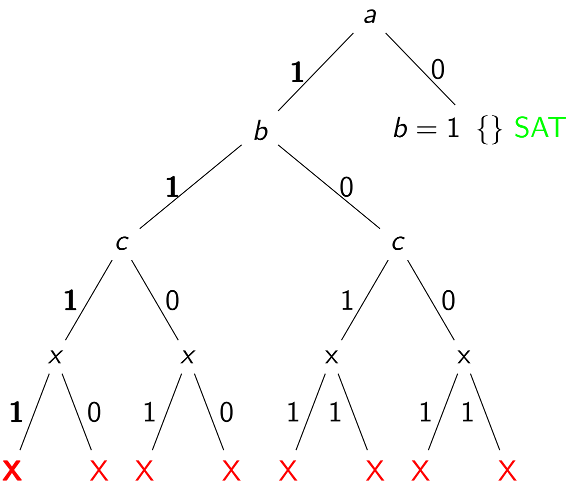
Note that $b$ and $c$ are irrelevant to the $c_7$ conflict. $(a \wedge y)$ (or $(a \wedge x)$) is sufficient.

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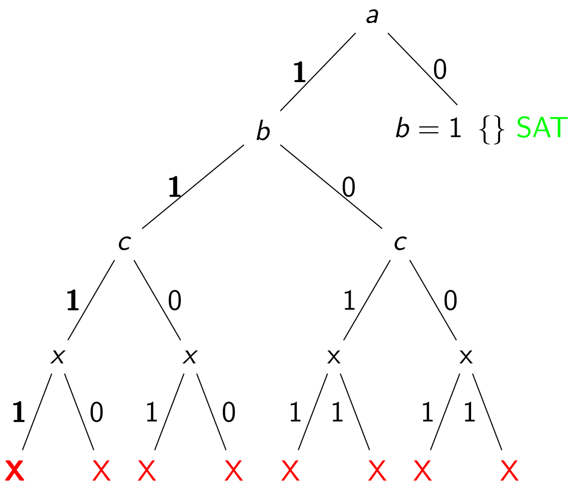
So, we can learn $\neg(a \wedge x) = (\neg a \vee \neg x)$ as a new constraint i.e. a *learned clause*.

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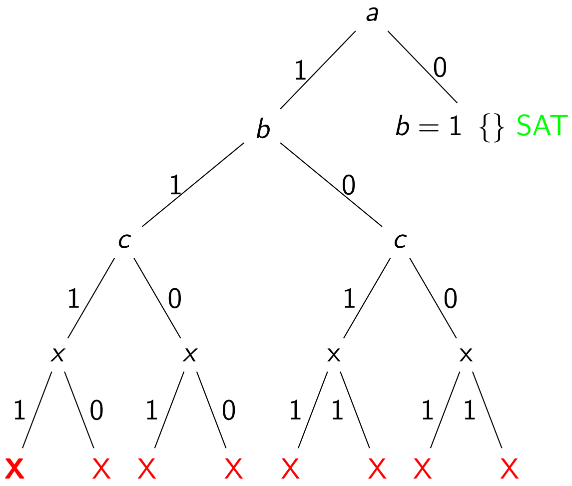


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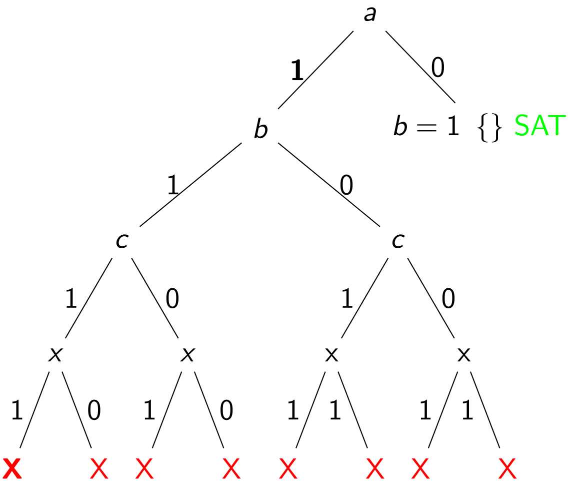


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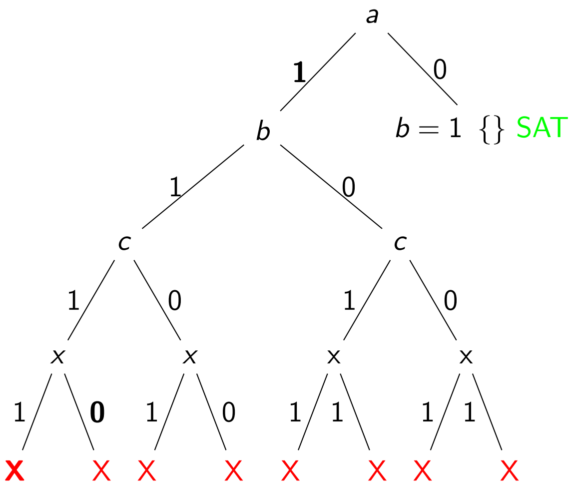
With the learned clause, we come to the conflict quickly.

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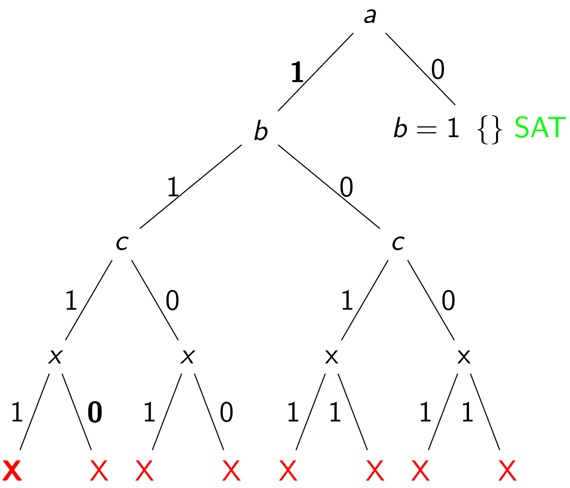


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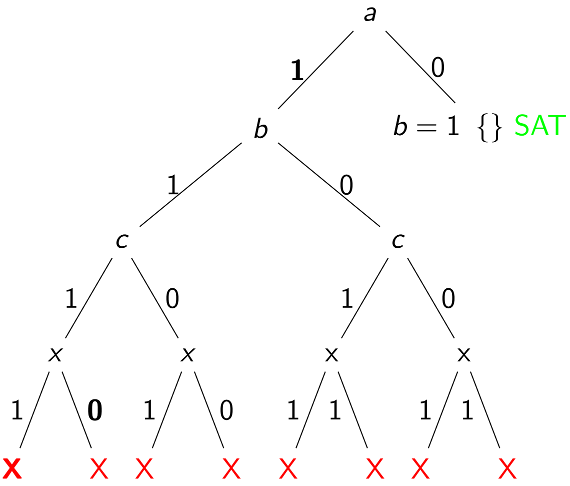
This time, $a$ is sufficient to cause the conflict, so we learn $\neg a$.

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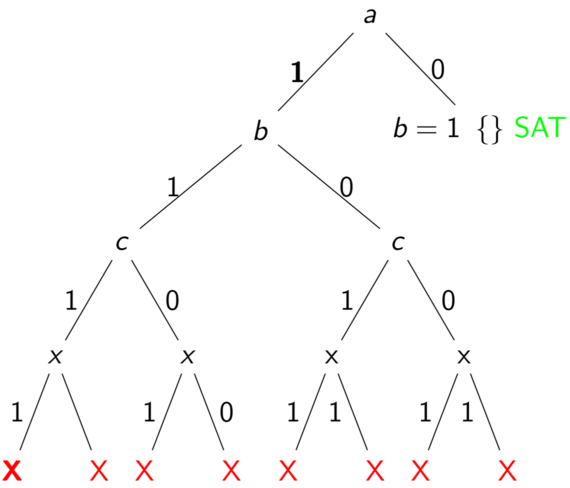


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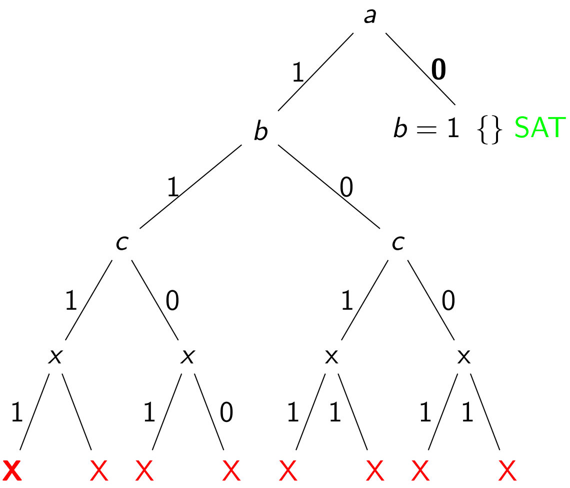


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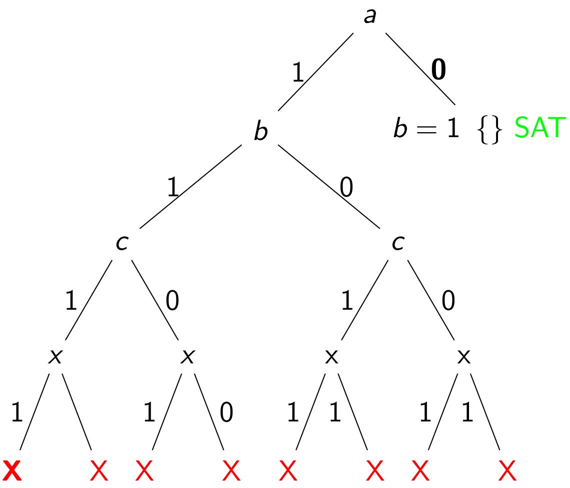


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  - Nodes of this graph represent variable assignments in the current search path. Edges to dependencies between these assignments.
  - Cuts in the graph correspond to a conflict set and, by negating it, a potential clause to learn

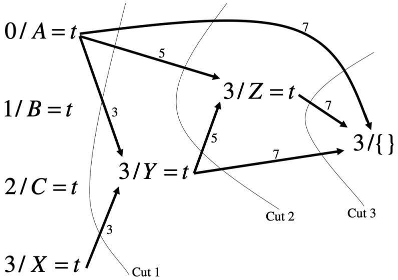


**Figure 3.8.** Three cuts in an implication graph, leading to three conflict sets.

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- Implementing my own CDCL SAT solver as a framework for exploring future potential SAT enhancements
- Around 1500 lines of C++, tested on a variety of easy to medium SAT benchmark problems
  - `https://github.com/will62794/mysat`
- Still order of magnitude slower than modern solvers (e.g. MiniSAT)

# SAT Solver Implementation

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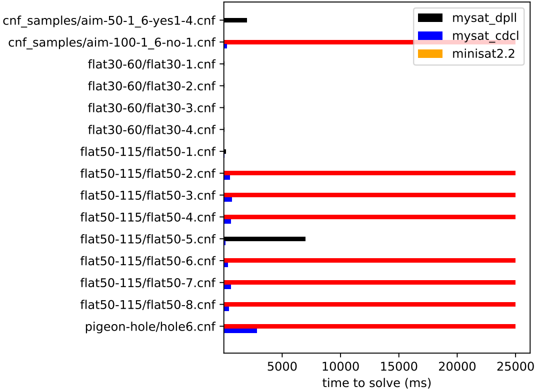
- e.g. runtime on some benchmarks with ≈ 50-200 variables, time budget of 25 seconds (red bar indicates timeout)



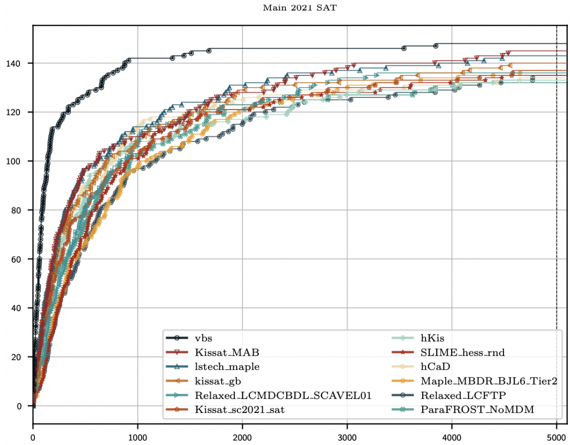



Figure: Cactus plot for top 10 solvers of SAT 2021 competition (SAT instances).

# Future Extensions: Learning Heuristics

- Modern CDCL based SAT solvers employ many heuristics e.g. for determining:
  1. Variable ordering
  2. Learned clause deletion policies
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- Modern CDCL based SAT solvers employ many heuristics e.g. for determining:
    1. Variable ordering
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    3. Random restarts
- Often these are "expertly tuned", based on experience/intuition

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- DRAT resolution proofs serve as a good source of data
  - Proofs based on resolution inference rule i.e. if

$$C_1 = (x \vee a_1 \vee \cdots \vee a_n)$$
$$C_2 = (\neg x \vee b_1 \vee \cdots \vee b_m)$$

    the clause

$$C = C_1 \bowtie C_2 = (a_1 \vee \cdots \vee a_n \vee b_1 \vee \cdots \vee b_m)$$

    can be inferred.

# Future Goals

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- Capture more statistics from solving runs as a basis for learning new heuristics e.g. clause activity

Questions?

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